題號 14-5.6

29.5-6.

For an (s, S) policy with $s = 2$ and $S = 3$:

$$c(x_{t-1}, D_t) = \begin{cases} 10 + 25(3 - x_{t-1}) + 50\max(D_t - 3, 0) & \text{for } x_{t-1} < 2 \\ 50\max(D_t - x_{t-1}, 0) & \text{for } x_{t-1} \geq 2. \end{cases}$$

$K(0) = E[c(0, D_t)] = 85 + 50[\sum_{j=4}^{\infty}(j-3) \cdot P(D_t = j)] \simeq 86.2,$

$K(1) = E[c(1, D_t)] = 60 + 50[\sum_{j=4}^{\infty}(j-3) \cdot P(D_t = j)] \simeq 61.2,$

$K(2) = E[c(2, D_t)] = 0 + 50[\sum_{j=4}^{\infty}(j-2) \cdot P(D_t = j)] \simeq 5.2,$

$K(3) = E[c(3, D_t)] = 0 + 50[\sum_{j=4}^{\infty}(j-2) \cdot P(D_t = j)] \simeq 1.2.$

$$x_{t+1} = \begin{cases} \max(3 - D_{t+1}, 0) & \text{for } x_t < 2 \\ \max(x_t - D_{t+1}, 0) & \text{for } x_t \geq 2 \end{cases}$$

$$\mathrm{P} = \begin{pmatrix} 0.080 & 0.184 & 0.368 & 0.368 \\ 0.080 & 0.184 & 0.368 & 0.368 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{pmatrix}$$

Solving the steady-state equations gives $(\pi_0, \pi_1, \pi_2, \pi_3) = (0.148, 0.252, 0.368, 0.232)$. Then the long-run average cost per week is $\sum_{j=0}^{3} K(j) \cdot \pi_j = 30.37$.

題號 14-5.7

29.5-7.

(a)

$$x_{t+1} = \begin{cases} \max(x_t + 2 - D_{t+1}, 0) & \text{for } x_t \leq 1 \\ \max(x_t - D_{t+1}, 0) & \text{for } x_t \geq 2 \end{cases}$$

$$\mathrm{P} = \begin{pmatrix} 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{pmatrix}$$

Solving the steady-state equations gives $(\pi_0, \pi_1, \pi_2, \pi_3) = (0.182, 0.285, 0.368, 0.165)$.

(b) $\lim_{n\to\infty} E\left(\frac{1}{n}\sum_{t=1}^{n} c(x_t)\right) = 0 \cdot \pi_0 + 2 \cdot \pi_1 + 8 \cdot \pi_2 + 18 \cdot \pi_3 = 6.48.$

題號 14-6.4

29.6-4.

(a)

$$P = \begin{pmatrix} 0.25 & 0.5 & 0.25 \\ 0.75 & 0.25 & 0 \\ 0.25 & 0.5 & 0.25 \end{pmatrix}$$

(b)

$$P^{(2)} = \begin{pmatrix} 0.5 & 0.375 & 0.125 \\ 0.375 & 0.438 & 0.188 \\ 0.5 & 0.375 & 0.125 \end{pmatrix}$$

$$P^{(5)} = \begin{pmatrix} 0.449 & 0.4 & 0.15 \\ 0.451 & 0.399 & 0.149 \\ 0.449 & 0.4 & 0.15 \end{pmatrix}$$

$$P^{(10)} = \begin{pmatrix} 0.45 & 0.4 & 0.15 \\ 0.45 & 0.4 & 0.15 \\ 0.45 & 0.4 & 0.15 \end{pmatrix}$$

(c)
$$\begin{aligned} \mu_{00} &= 1 + 0.5\mu_{10} + 0.25\mu_{20} \\ \mu_{10} &= 1 + 0.25\mu_{10} \\ \mu_{20} &= 1 + 0.5\mu_{10} + 0.25\mu_{20} \end{aligned}$$

$\Rightarrow \mu_{00} = 20/9, \mu_{10} = 4/3, \mu_{20} = 20/9$

$$\begin{aligned} \mu_{01} &= 1 + 0.25\mu_{01} + 0.25\mu_{21} \\ \mu_{11} &= 1 + 0.75\mu_{01} \\ \mu_{21} &= 1 + 0.25\mu_{01} + 0.25\mu_{21} \end{aligned}$$

$\Rightarrow \mu_{01} = 2, \mu_{11} = 2\frac{1}{2}, \mu_{21} = 2$

$$\begin{aligned} \mu_{02} &= 1 + 0.25\mu_{02} + 0.5\mu_{12} \\ \mu_{12} &= 1 + 0.75\mu_{02} + 0.25\mu_{12} \\ \mu_{22} &= 1 + 0.25\mu_{02} + 0.5\mu_{12} \end{aligned}$$

$\Rightarrow \mu_{02} = 20/3, \mu_{12} = 8, \mu_{22} = 20/3$

(d) The steady-state probability vector is $(0.45 \;\; 0.4 \;\; 0.15)$.

(e) $\pi \cdot C = 0(0.45) + 2(0.4) + 8(0.15) = \$\ 2$ / week

題號 14-6.5

29.6-5.

(a)

$$P = \begin{pmatrix} 0 & 0.875 & 0.062 & 0.062 \\ 0 & 0.75 & 0.125 & 0.125 \\ 0 & 0 & 0.5 & 0.5 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$\pi_0 = 0.154$, $\pi_1 = 0.538$, $\pi_2 = 0.154$, and $\pi_3 = 0.154$

(b) $\pi \cdot C = 1(0.538) + 3(0.154) + 6(0.154) = \$\ 1923.08$

(c)
$$\begin{aligned} \mu_{00} &= 1 + 0.875\mu_{10} + 0.0625\mu_{20} + 0.0625\mu_{30} \\ \mu_{10} &= 1 + 0.75\mu_{10} + 0.125\mu_{20} + 0.125\mu_{30} \\ \mu_{20} &= 1 + 0.5\mu_{20} + 0.5\mu_{30} \\ \mu_{30} &= 1 + 0 \end{aligned}$$

So the expected recurrence time for state 0 is $\mu_{00} = 6.5$.

題號 14-7.1

29.7-1.

(a) $P_{00} = P_{TT} = 1$; $P_{i,i-1} = q$; $P_{i,i+1} = p$; $P_{i,k} = 0$ else.

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & & & \\ q & 0 & p & 0 & \cdots & & & \\ & \vdots & & \ddots & & & & \\ & & & & q & 0 & p & 0 \\ & & & & 0 & q & 0 & p \\ & & & & 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Class 1: $\{0\}$ absorbing
Class 2: $\{T\}$ absorbing
Class 3: $\{1, 2, \ldots, T-1\}$ transient

(c) Let $f_{iK} = P(\text{absorption at } K \text{ starting at } i)$. Then $f_{00} = f_{33} = 1$, $f_{30} = f_{03} = 0$.
Since $P_{ij} = 0$ for $|i-j| \neq 1$ and $P_{i,i+1} = p$, $P_{i,i-1} = q$, we get:

$$\begin{aligned} f_{10} &= q + pf_{20} \\ f_{13} &= 1 - f_{10} \\ f_{20} &= qf_{10} \\ f_{23} &= 1 - f_{20} \end{aligned}$$

Solving this system gives

$$f_{10} = \frac{q}{1-pq} = 0.886,\ f_{13} = 0.114,\ f_{20} = 0.62,\ f_{23} = 0.38.$$

(d) Plugging in $p = 0.7$ in the formulas in part (c), we obtain

$$f_{10} = 0.38,\ f_{13} = 0.62,\ f_{20} = 0.114,\ f_{23} = 0.886.$$

Observe that when $p > 1/2$, the drift is towards T and when $p < 1/2$, it is towards 0.

題號 14-7.2

29.7-2.

(a) $0 =$ Have to honor warranty
$1 =$ Reorder in 1st year
$2 =$ Reorder in 2nd year
$3 =$ Reorder in 3rd year

$$\mathrm{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.01 & 0 & 0.99 & 0 \\ 0.05 & 0 & 0 & 0.95 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) The probability that the manufacturer has to honor the warranty is $f_{10.}$

$$f_{10} = 0.01 f_{00} + 0 f_{10} + 0.99 f_{20} + 0 f_{30}$$
$$f_{20} = 0.05 f_{00} + 0 f_{10} + 0 f_{20} + 0.95 f_{30}$$
$$f_{00} = 1 \text{ and } f_{30} = 0$$
$$\Rightarrow f_{10} = 0.01 + 0.99 f_{20} \text{ and } f_{20} = 0.05$$
$$\Rightarrow f_{10} = 0.0595 = 5.95\%.$$