第四章 線性規劃問題求解法：單行法

題目 4.4-5 (a) (b)

4.4-5.

(a) Set $x_1 = x_2 = x_3 = 0$.

(0) $\quad Z - 2x_1 - 4x_2 - 3x_3 = 0$

(1) $\quad x_1 + 3x_2 + 2x_3 + x_4 = 80 \Rightarrow x_4 = 80$

(2) $\quad 3x_1 + 4x_2 + 2x_3 + x_5 = 60 \Rightarrow x_5 = 60$

(3) $\quad 2x_1 + x_2 + 2x_3 + x_6 = 40 \Rightarrow x_6 = 40$

Optimality Test: The coefficients of all nonbasic variables are negative, so the solution $(0, 0, 0, 80, 60, 40)$ is not optimal.

Choose x_2 as the entering basic variable, since it has the largest coefficient.

(1) $\quad x_1 + 3x_2 + 2x_3 + x_4 = 80 \Rightarrow x_4 = 80 - 3x_2 \Rightarrow x_2 \leq 26.67$

(2) $\quad 3x_1 + 4x_2 + 2x_3 + x_5 = 60 \Rightarrow x_5 = 60 - 4x_2 \Rightarrow x_2 \leq 15 \leftarrow$ minimum

(3) $\quad 2x_1 + x_2 + 2x_3 + x_6 = 40 \Rightarrow x_6 = 40 - x_2 \Rightarrow x_2 \leq 40$

We choose x_5 as the leaving basic variable. Set $x_1 = x_5 = x_3 = 0$.

(0) $\quad Z + x_1 - x_3 + x_5 = 60$

(1) $\quad -1.25x_1 + 0.5x_3 + x_4 - 0.75x_5 = 35 \Rightarrow x_4 = 35$

(2) $\quad 0.75x_1 + x_2 + 0.5x_3 - 0.25x_5 = 15 \Rightarrow x_2 = 15$

(3) $\quad 1.25x_1 + 1.5x_3 - 0.25x_5 + x_6 = 25 \Rightarrow x_6 = 25$

Optimality Test: The coefficient of x_3 is negative, so the solution $(0, 15, 0, 35, 0, 25)$ is not optimal.

Let x_3 be the entering basic variable.

(1) $\quad -1.25x_1 + 0.5x_3 + x_4 - 0.75x_5 = 35 \Rightarrow x_4 = 35 - 0.5x_3 \Rightarrow x_3 \leq 70$

(2) $\quad 0.75x_1 + x_2 + 0.5x_3 + 0.25x_5 = 15 \Rightarrow x_2 = 15 - 0.5x_3 \Rightarrow x_3 \leq 30$

(3) $\quad 1.25x_1 + 1.5x_3 - 0.25x_5 + x_6 = 25 \Rightarrow x_6 = 25 - 1.5x_3 \Rightarrow x_3 \leq 16.67 \leftarrow$ min

We choose x_6 as the leaving basic variable. Set $x_1 = x_5 = x_6 = 0$.

(0) $\quad Z + 1.83x_1 + 0.83x_5 + 0.67x_6 = 76.67$

(1) $\quad -1.67x_1 + x_4 - 0.67x_5 - 0.33x_6 = 26.67 \Rightarrow x_4 = 26.67$

(2) $\quad 0.33x_1 + x_2 + 0.33x_5 - 0.33x_6 = 6.67 \Rightarrow x_2 = 6.67$

(3) $\quad 0.83x_1 + x_3 - 0.17x_5 + 0.67x_6 = 16.67 \Rightarrow x_3 = 16.67$

Optimality Test: All of the coefficients are positive, so the solution (0, 6.67, 16.67, 26.67, 0, 0) is optimal. $Z^* = 76.67$.

(b) Optimal solution: $(x_1^*, x_2^*, x_3^*) = (0, 6.67, 16.67)$ and $Z^* = 76.67$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	-2	-4	-3	0	0	0	0
X4	1	0	1	3	2	1	0	0	80
X5	2	0	3	4*	2	0	1	0	60
X6	3	0	2	1	2	0	0	1	40

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	1	0	-1	0	1	0	60
X4	1	0	-1.25	0	0.5	1	-0.75	0	35
X2	2	0	0.75	1	0.5	0	0.25	0	15
X6	3	0	1.25	0	1.5*	0	-0.25	1	25

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	1.833	0	0	0	0.833	0.667	76.67
X4	1	0	-1.67	0	0	1	-0.67	-0.33	26.67
X2	2	0	0.333	1	0	0	0.333	-0.33	6.667
X3	3	0	0.833	0	1	0	-0.17	0.667	16.67

題目 4.4-8

4.4-8.

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (6\frac{2}{3}, 0, 36\frac{2}{3})$ and $Z^* = 66\frac{2}{3}$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	1	-1	-2	0	0	0	0
X_4	1	0	1	2	-1	1	0	0	20
X_5	2	0	-2	4	2	0	1	0	60
X_6	3	0	2	3	1	0	0	1	50

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-1	3	0	0	1	0	60
X_4	1	0	0	4	0	1	0.5	0	50
X_3	2	0	-1	2	1	0	0.5	0	30
X_6	3	0	3	1	0	0	-0.5	1	20

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	3.3333	0	0	0.8333	0.3333	66.6667
X_4	1	0	0	4	0	1	0.5	0	50
X_3	2	0	0	2.3333	1	0	0.3333	0.3333	36.6667
X_1	3	0	1	0.3333	0	0	-0.167	0.3333	6.66667

題目 4.6-7 (a) (b) (c) (d) (e) (f)

4.6-7.

(a) Initial artificial BF solution: $(0, 0, 0, 0, 20, 50)$

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
			-3M	-2M	-2M				
Z	0	1	-2	-5	-3	1M	0	0	-70M
$\bar{x}_5$	1	0	1	-2	1	-1	1	0	20
$\bar{x}_6$	2	0	2	4	1	0	0	1	50

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 0, 50)$ and $Z^* = 150$

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
				-8M	1M	-2M	3M		-10M
Z	0	1	0	-9	-1	-2	+2	0	+40
x_1	1	0	1	-2	1	-1	1	0	20
$\bar{x}_6$	2	0	0	8	-1	2	-2	1	10

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
							1M	1M	
Z	0	1	0	0	-2.125	0.25	-0.25	+1.125	51.25
x_1	1	0	1	0	0.75	-0.5	0.5	0.25	22.5
x_2	2	0	0	1	-0.125	0.25	-0.25	0.125	1.25

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
							1M	1M	
Z	0	1	2.8333	0	0	-1.167	+1.167	+1.833	115
x_3	1	0	1.3333	0	1	-0.667	0.6667	0.3333	30
x_2	2	0	0.1667	1	0	0.1667	-0.167	0.1667	5

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
								1M	
Z	0	1	4	7	0	0	1M	+3	150
x_3	1	0	2	4	1	0	0	1	50
x_4	2	0	1	6	0	1	-1	1	30

(c) Initial artificial BF solution: $(0, 0, 0, 0, 20, 50)$

Phase 1:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-3	-2	-2	1	0	0	-70
$\bar{X}_5$	1	0	1	-2	1	-1	1	0	20
$\bar{X}_6$	2	0	2	4	1	0	0	1	50

(d)

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	-8	1	-2	3	0	-10
X_1	1	0	1	-2	1	-1	1	0	20
$\bar{X}_6$	2	0	0	8	-1	2	-2	1	10

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	0	0	0	1	1	0
X_1	1	0	1	0	0.75	-0.5	0.5	0.25	22.5
X_2	2	0	0	1	-0.125	0.25	-0.25	0.125	1.25

(e) - (f) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 0, 50)$ and $Z^* = 150$

Phase 2:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	0	0	-2.125	0.25	51.25
X_1	1	0	1	0	0.75	-0.5	22.5
X_2	2	0	0	1	-0.125	0.25	1.25

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	2.8333	0	0	-1.167	115
X_3	1	0	1.3333	0	1	-0.667	30
X_2	2	0	0.1667	1	0	0.1667	5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	4	7	0	0	150
X_3	1	0	2	4	1	0	50
X_4	2	0	1	6	0	1	30

題目 4.6-9 (a) (b)

4.6-9.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 15, 15)$ and $Z^* = 90$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-5M +3	-4M +2	-8M +4	1M	0	0	-180M
$\bar{X}_5$	1	0	2	1	3	0	1	0	60
$\bar{X}_6$	2	0	3	3	5	-1	0	1	120

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0.333M +0.333	-1.33M +0.667	0	1M	2.667M -1.333	0	-20M -80
X_3	1	0	0.6667	0.3333	1	0	0.3333	0	20
$\bar{X}_6$	2	0	-0.333	1.3333	0	-1	-1.667	1	20

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0.5	0	0	0.5	1M -0.5	1M -0.5	-90
X_3	1	0	0.75	0	1	0.25	0.75	-0.25	15
X_2	2	0	-0.25	1	0	-0.75	-1.25	0.75	15

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 15, 15)$ and $Z^* = 90$

Phase 1:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-5	-4	-8	1	0	0	-180
$\bar{X}_5$	1	0	2	1	3	0	1	0	60
$\bar{X}_6$	2	0	3	3	5	-1	0	1	120

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0.3333	-1.333	0	1	2.6667	0	-20
X_3	1	0	0.6667	0.3333	1	0	0.3333	0	20
$\bar{X}_6$	2	0	-0.333	1.3333	0	-1	-1.667	1	20

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-3e-20	0	0	0	1	1	0
X_3	1	0	0.75	0	1	0.25	0.75	-0.25	15
X_2	2	0	-0.25	1	0	-0.75	-1.25	0.75	15

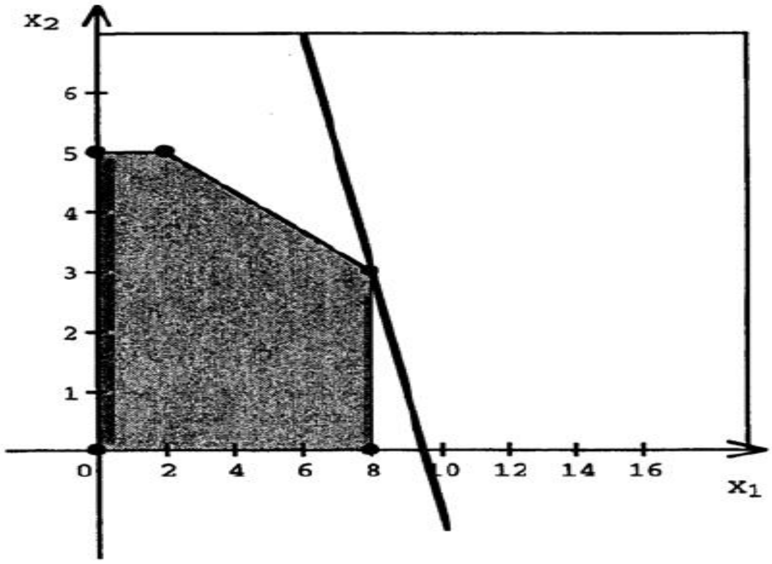
Phase 2:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	0.5	0	0	0.5	-90
X_3	1	0	0.75	0	1	0.25	15
X_2	2	0	-0.25	1	0	-0.75	15

题目 4.7-3

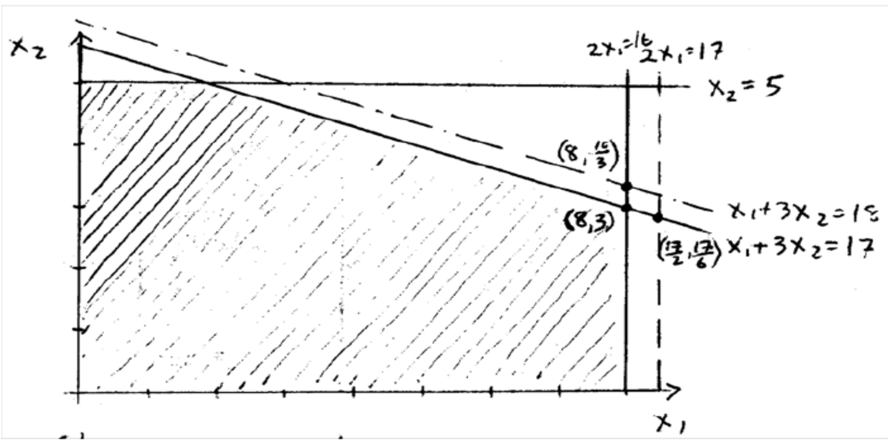
4.7-3.

(a) Optimal Solution: $(x_1^*, x_2^*) = (8, 3)$ and $Z^* = 38$



Corner Point	Z
$(8, 3)$	$38*$
$(8, 0)$	32
$(2, 5)$	18
$(0, 5)$	10
$(0, 0)$	0

(b)



Increasing resource 1 to 17 units increases Z to $4(8.5) + 2(2.83) = 39.67$, so $\Delta Z = y_1^* = 1.67$.
Increasing resource 2 to 18 units increases Z to $4(8) + 2(3.33) = 38.33$, so $\Delta Z = y_2^* = 0.67$.
The third constraint is not binding, so $y_3^* = 0$.

(c) To increase Z by 15, resource 1 should be increased by $\frac{15}{y_1^*} = \frac{15}{1.67} \approx 9$. Solving the LP problem with resource 1 set to $16 + 9 = 25$ returns the result $Z = 53$.

題目 4.7-4 (a) (b)

4.7-4.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0.5, 0, 4.5)$ and $Z^* = 14$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-1	7	-3	0	0	0	0
X_4	1	0	2	1	-1	1	0	0	4
X_5	2	0	4	-3	0	0	1	0	2
X_6	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-10	13	0	0	0	3	9
X_4	1	0	-1	3	0	1	0	1	7
X_5	2	0	4	-3	0	0	1	0	2
X_3	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	5.5	0	0	2.5	3	14
X_4	1	0	0	2.25	0	1	0.25	1	7.5
X_1	2	0	1	-0.75	0	0	0.25	0	0.5
X_3	3	0	0	-0.25	1	0	0.75	1	4.5

(b) The shadow prices for the three resources are given by the reduced costs (in the objective function) for the corresponding slack variables. These values are circled in the table above. The shadow prices for resources 1, 2 and 3 are 0, 2.5 and 3 respectively. They represent the rate at which the objective function value z increases as the corresponding resource is increased. For instance, increasing resource 3 by one unit increases Z by 3, provided that no other constraints cause any trouble.

題目 4.7-5(a) (b)

4.7-5.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 1, 3)$ and $Z^* = 7$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-2	2	-3	0	0	0	0
X_4	1	0	-1	1	1	1	0	0	4
X_5	2	0	2	-1	1	0	1	0	2
X_6	3	0	1	1	3	0	0	1	12

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	4	-1	0	0	3	0	6
X_4	1	0	-3	2	0	1	-1	0	2
X_3	2	0	2	-1	1	0	1	0	2
X_6	3	0	-5	4	0	0	-3	1	6

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	2.5	0	0	0.5	2.5	0	7
X_2	1	0	-1.5	1	0	0.5	-0.5	0	1
X_3	2	0	0.5	0	1	0.5	0.5	0	3
X_6	3	0	1	0	0	-2	-1	1	2

(b) The shadow prices are $y_1^* = 0.5$, $y_2^* = 2.5$ and $y_3^* = 0$. They are the marginal values of resources 1, 2 and 3 respectively.