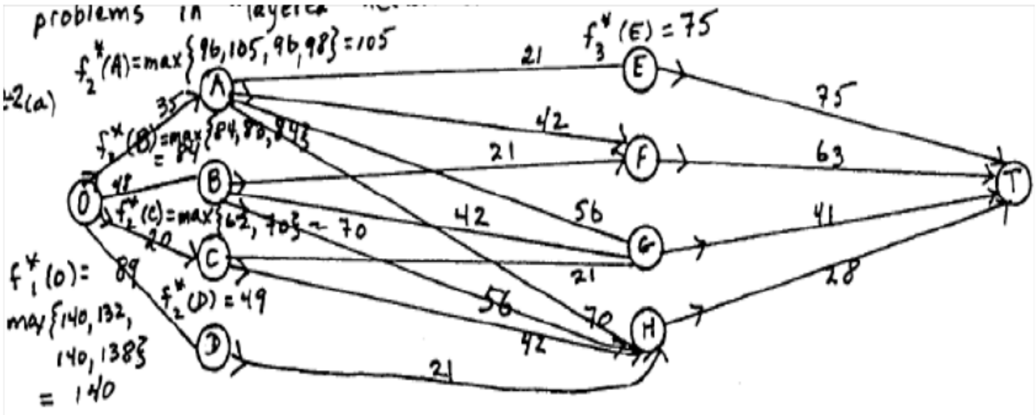
第 10 章 動態規劃

題號 10.2-2

11.2-2.

(a)



The optimal routes are $O-A-F-T$ and $O-C-H-T$, the associated sales income is 140. The route $O-A-F-T$ corresponds to assigning 1, 2, and 3 salespeople to regions 1, 2, and 3 respectively. The route $O-C-H-T$ corresponds to assigning 3, 2, and 1 salespeople to regions 1, 2, and 3 respectively.

(b) The regions are the stages and the number of salespeople remaining to be allocated at stage n are possible states at stage n, say s_n. Let x_n be the number of salespeople assigned to region n and $c_n(x_n)$ be the increase in sales in region n if x_n salespeople are assigned to it. Number of stages: 3.

s_3	$f_3^*(s_3)$	x_3^*
1	28	1
2	41	2
3	63	3
4	75	4

	$f_2(s_2, x_2)$					
s_2	1	2	3	4	$f_2^*(s_2)$	x_2^*
2	49	–	–	–	49	1
3	62	70	–	–	70	2
4	84	83	84	–	84	1,3
5	96	105	97	98	105	2

	$f_1(s_1, x_1)$					
s_1	1	2	3	4	$f_1^*(s_1)$	x_1^*
6	140	132	140	138	140	1, 3

The optimal solutions are $(x_1^* = 1, x_2^* = 2, x_3^* = 3)$ and $(x_1^* = 3, x_2^* = 2, x_3^* = 1)$.

題號 10.3-3

11.3-3.

Let x_n be the number of study days allocated to course n, $p_n(x_n)$ be the number of grade points expected when x_n days are allocated to course n and s_n be the number of study days remaining to be allocated to courses $k \geq n$. Then

$$f_n^*(s_n) = \max_{1 \leq x_n \leq \min(s_n, 4)} [p_n(x_n) + f_{n+1}^*(s_n - x_n)].$$

Number of stages: 4

s_4	$f_4^*(s_4)$	x_4^*
1	6	1
2	7	2
3	9	3
4	9	4

	$f_3(s_3, x_3)$					
s_3	1	2	3	4	$f_3^*(s_3)$	x_3^*
2	8	—	—	—	8	1
3	9	10	—	—	13	2
4	11	11	13	—	13	3
5	11	13	14	14	14	3, 4

	$f_2(s_2, x_2)$					
s_2	1	2	3	4	$f_2^*(s_2)$	x_2^*
3	13	—	—	—	13	1
4	15	13	—	—	15	1
5	18	15	14	—	18	1
6	19	18	16	17	19	1

	$f_1(s_1, x_1)$					
s_1	1	2	3	4	$f_1^*(s_1)$	x_1^*
7	22	23	21	20	23	2

Optimal Solution	x_1^*	x_2^*	x_3^*	x_4^*
1	2	1	3	1

題號 10.3-8

11.3-8.

Let x_n be the number of parallel units of component n that are installed, $p_n(x_n)$ be the probability that the component will function if it contains x_n parallel units, $c_n(x_n)$ be the cost of installing x_n units of component n, s_n be the amount of money remaining in hundreds of dollars. Then

$$f_n^*(s_n) = \max_{x_n=0,\ldots,\min(3,\alpha_{s_n})} [p_n(x_n) f_{n+1}^*(s_n - c_n(x_n))]$$

where $\alpha_{s_n} \equiv \max\{\alpha : c_n(\alpha) \leq s_n, \alpha \text{ integer}\}$.

s_4	$f_4^*(s_4)$	x_4^*
0, 1	0	0
2	0.5	1
3	0.7	2
$4 \leq s_4 \leq 10$	0.9	3

$f_3(s_3, x_3) = P_3(x_3) f_4^*(s_3 - c_3(x_3))$

	$f_3(s_3, x_3)$					
s_3	0	1	2	3	$f_3^*(s_3)$	x_3^*
0	0	—	—	—	0	0
1, 2	0	0	—	—	0	0, 1
3	0	0.35	0	—	0.35	1
4	0	0.49	0	0	0.49	1
5	0	0.63	0.40	0	0.63	1
6	0	0.63	0.56	0.45	0.63	1
7	0	0.63	0.72	0.63	0.72	2
$8 \leq s_3 \leq 10$	0	0.63	0.72	0.81	0.81	3

$f_2(s_2, x_2) = P_2(x_2) f_3^*(s_2 - c_2(x_2))$

	$f_2(s_2, x_2)$					
s_2	0	1	2	3	$f_2^*(s_2)$	x_2^*
0, 1	0	—	—	—	0	0
2, 3	0	0	—	—	0	0, 1
4	0	0	0	—	0	0, 1, 2
5	0	0.210	0	0	0.210	1
6	0	0.294	0	0	0.294	1
7	0	0.378	0.245	0	0.378	1
8	0	0.378	0.343	0.280	0.378	1
9	0	0.432	0.441	0.392	0.441	2
10	0	0.486	0.441	0.504	0.504	3

$f_1(s_1, x_1) = P_1(x_1) f_2^*(s_1 - c_1(x_1))$

	$f_1(s_1, x_1)$					
s_1	0	1	2	3	$f_1^*(s_1)$	x_1^*
10	0	0.22	0.227	0.302	0.302	3

The optimal solution is $x_1^* = 3$, $x_2^* = 1$, $x_3^* = 1$ and $x_4^* = 3$, yielding a system reliability of 0.3024.

題號 10.4-1

11.4-1.

Let s_n be the current fortune of the player, A be the event to have \$100 at the end and X_n be the amount bet at the nth match.

$$f_3^*(s_3) = \max_{0 \le x_3 \le s_3} \{\mathrm{P}\{A|s_3\}\}$$

$0 \le s_3 < 50$, $f_3^*(s_3) = 0$.

$$50 \le s_3 < 100,\ f_3^*(s_3) = \begin{cases} 0 & \text{if } x_3^* \ne 100 - s_3 \\ 1/2 & \text{if } x_3^* = 100 - s_3 \end{cases}$$

$$s_3 = 100,\ f_3^*(s_3) = \begin{cases} 0 & \text{if } x_3^* > 0 \\ 1 & \text{if } x_3^* = 0 \end{cases}$$

$$s_3 > 100,\ f_3^*(s_3) = \begin{cases} 0 & \text{if } x_3^* \ne s_3 - 100 \\ 1/2 & \text{if } x_3^* = s_3 - 100 \end{cases}$$

s_3	$f_3^*(s_3)$	x_3^*
$0 \leq s_3 < 50$	0	$0 \leq x_3^* \leq 50$
$50 \leq s_3 < 100$	$1/2$	$100 - s_3$
$s_3 = 100$	1	0
$100 < s_3$	$1/2$	$s_3 - 100$

$$f_2^*(s_2) = \max_{0 \leq x_2 \leq s_2} \left[\tfrac{1}{2} f_3^*(s_2 - x_2) + \tfrac{1}{2} f_3^*(s_2 + x_2)\right]$$

s_2	$f_2^*(s_2)$	x_2^*
$0 \leq s_2 < 25$	$\mathbf{0}$	$0 \leq x_2 \leq s_2$
$25 \leq s_2 < 50$	0	$0 \leq x_2 \leq 50 - s_2$
	$\mathbf{1/4}$	$50 - s_2 \leq x_2 \leq s_2$
$s_2 = 50$	$1/4$	$0 \leq x_2 < 50$
	$\mathbf{1/2}$	$x_2 = 50$
$50 < s_2 < 75$	$\mathbf{1/2}$	$0 \leq x_2 < s_2 - 50$
	$1/4$	$s_2 - 50 < x_2 < 100 - s_2$
	$\mathbf{1/2}$	$x_2 = 100 - s_2$
	$1/4$	$100 - s_2 < x_2 \leq s_2$
$s_2 = 75$	$1/2$	$0 \leq x_2 < 25$
	$\mathbf{3/4}$	$x_2 = 25$
	$1/4$	$25 \leq x_2 \leq 75$
$75 < s_2 < 100$	$1/2$	$0 \leq x_2 < 100 - s_2$
	$\mathbf{3/4}$	$x_2 = 100 - s_2$
	$1/2$	$100 - s_2 < x_2 \leq s_2 - 50$
	$1/4$	$s_2 - 50 < x_2 \leq s_2$
$s_2 = 100$	$\mathbf{1}$	$x_2 = 0$
	$1/2$	$0 < x_2 \leq 50$
	$1/4$	$50 \leq x_2 \leq 100$
$100 < s_2$	$1/2$	$0 \leq x_2 \leq s_2 - 100$
	$\mathbf{3/4}$	$x_2 = s_2 - 100$
	$1/2$	$s_2 - 100 < x_2 \leq s_2 - 50$
	$1/4$	$s_2 - 50 < x_2 \leq s_2$

The entries in bold represent the maximum value in each case.

$$f_1^*(75) = \max_{0 \leq x_1 \leq 75} \left[\tfrac{1}{2} f_2^*(75 - x_1) + \tfrac{1}{2} f_2^*(75 + x_1)\right]$$

$$f_1(75, x_1) = \begin{cases} 3/4 & \text{if } x_1 = 0 \\ 5/8 & \text{if } 0 < x_1 < 25 \\ 3/4 & \text{if } x_1 = 25 \\ 1/2 & \text{if } 25 < x_1 \leq 50 \\ 3/8 & \text{if } 50 < x_1 \leq 75 \end{cases}$$

s_1	$f_1^*(s_1)$	x_1^*
75	$3/4$	0 or 25

Policy	x_1	won 1st bet	lost 1st bet	won 2nd bet	lost 2nd bet
1	0	25	25	0	50
2	25	0	50	0	0