# 4.1 節

4.1 有一個 12 人的樣本，被詢問他們的口袋和錢包中有多少零錢。他們的回應 (以美分計) 是

    52   25   15   0    104  44

    60   30   33   81   40   5

計算這些資料的平均數、中位數和眾數。

4.1 a $\bar{x} = \dfrac{\sum x_i}{n} = \dfrac{52+25+15+0+104+44+60+30+33+81+40+5}{12} = \dfrac{489}{12} = 40.75$

Ordered data: 0, 5, 15, 25, 30, 33, 40, 44, 52, 60, 81, 104; Median = (33 + 40)/2 = 36.5

Mode = all

4.2 有一個 12 位慢跑者的隨機樣本，被要求追蹤和報告他們上週跑步的哩數。他們的回應是

    5.5  7.2  1.6   22.0  8.7  2.8

    5.3  3.4  12.5  18.6  8.3  6.6

a. 計算測量中央位置的三種統計量。

b. 簡要說明每一種統計量告訴你什麼訊息。

4.2 a $\bar{x} = \dfrac{\sum x_i}{n} = \dfrac{5.5+7.2+1.6+22.0+8.7+2.8+5.3+3.4+12.5+18.6+8.3+6.6}{12} = \dfrac{102.5}{12} = 8.54$

Ordered data: 1.6, 2.8, 3.4, 5.3, 5.5, 6.6, 7.2, 8.3, 8.7, 12.5, 18.6, 22.0; Median = 6.9

Mode = all

b The mean number of miles jogged is 8.54. Half the sample jogged more than 6.9 miles and half

jogged less.

4.3 Wilfrid Laurier 大學的教授，被要求在學期結束前 10 天到註冊組繳交他們的期末考試卷。

試務統籌人員對 20 位教授取樣，並且記錄每一位教授在期末考之前提交試卷的天數。紀錄的

結果是：

　　14　8　3　2　6　4　9　13　10　12

　　7　4　9　13　15　8　11　12　4　0

a. 計算平均數、中位數和眾數。

b. 簡要說明每一種統計量告訴你什麼訊息。

4.3 a $\overline{x} = \dfrac{\sum x_i}{n} = \dfrac{14+8+3+2+6+4+9+13+10+12+7+4+9+13+15+8+11+12+4+0}{20}$

$\qquad = \dfrac{164}{20} = 8.2$

Ordered data: 0, 2, 3, 4, 4, 4, 6, 7, 8, 8, 9, 9, 10, 11, 12, 12, 13, 13, 14, 15; Median = 8.5

Mode = 4

b The mean number of days to submit grades is 8.2, the median is 8.5, and the mode is 4.

# 4.2 節

4.12 計算下列樣本的變異數。

4　5　3　6　5　6　5　6

4.12 $\bar{x} = \dfrac{\sum x_i}{n} = \dfrac{4+5+3+6+5+6+5+6}{8} = \dfrac{40}{8} = 5$

$s^2 = \dfrac{\sum (x_i - \bar{x})^2}{n-1} = \dfrac{[(4-5)^2 + (5-5)^2 + ... + (6-5)^2]}{8-1} = \dfrac{8}{7} = 1.14$

4.13 求出下列樣本的變異數與標準差。

0　−5　−3　6　4　−4　1　−5　0　3

4.13 $\bar{x} = \dfrac{\sum x_i}{n} = \dfrac{0+(-5)+(-3)+6+4+(-4)+1+(-5)+0+3}{10} = \dfrac{-3}{10} = -.30$

$s^2 = \dfrac{\sum (x_i - \bar{x})^2}{n-1} = \dfrac{[(0-(-.3))^2 + ((-5)-(-.3))^2 + ... + (3-(-.3))^2]}{10-1} = \dfrac{136.1}{9} = 15.12$

$s = \sqrt{s^2} = \sqrt{15.12} = 3.89$

4.16 有一組資料，它的直方圖是鐘形的，平均數與標準差分別是 50 與 4。大約多少比率的觀測值
a. 是介於 46 和 54 之間？
b. 是介於 42 和 58 之間？
c. 是介於 38 和 62 之間？

4.16

a. About 68%

b. About 95%

c. About 99.7%

4.18 有一組資料，它的直方圖是極端的偏斜，平均數與標準差分別是 70 與 12。至少有多少比率的觀測值

a. 是介於 46 和 94 之間？

b. 是介於 34 和 106 之間？

4.18

a at least 75%

b at least 88.9%

# 4.3 節

4.23 求出下列樣本的第一、第二和第三個四分位數。

5    8    2    9    5    3    7    4    2    7    4    10    4    3    5

4.23 First quartile: $L_{25} = (15+1)\dfrac{25}{100}$ = (16)(.25) = 4; the fourth number is 3.

Second quartile: $L_{50} = (15+1)\dfrac{50}{100}$ = (16)(.5) = 8; the eighth number is 5.

Third quartile: $L_{75} = (15+1)\dfrac{75}{100}$ = (16)(.75) = 12; the twelfth number is 7.

4.25 求出下列資料組的第一和第二個五分位數（第 20 和第 40 個百分位數）。

52    61    88    43    64    71    39    73    51    60

4.25    20th percentile: $L_{20} = (10+1)\dfrac{20}{100}$ = (11)(.20) = 2.2; the 20th percentile is 43 + .2(51–43) = 44.6.

40th percentile: $L_{40} = (10+1)\dfrac{40}{100}$ = (11)(.40) = 4.4; the 40th percentile is 52 +.4(60–52) = 55.2.

4.26 計算下列資料的第一、第二和第三個四分位數。

10.5　14.7　15.3　17.7　15.9　12.2　10.0

14.1　13.9　18.5　13.9　15.1　14.7

4.26　First quartile: $L_{25} = (13+1)\dfrac{25}{100}$ = (14)(.25) = 3.5; the first quartile is 13.05.

Second quartile: $L_{50} = (13+1)\dfrac{50}{100}$ = (14)(.5) = 7; the second quartile is 14.7.

Third quartile: $L_{75} = (13+1)\dfrac{75}{100}$ = (14)(.75) = 10.5; the third quartile is 15.6.

4.27 參考練習題 4.26，計算四分位距。

4.27　Interquartile range = 15.6 –13.05 = 2.55

4.28 計算下列資料的四分位距。

5　8　14　6　21　11　9　10　18　2

4.28　First quartile = 5.75, third quartile = 15; interquartile range = 15 – 5.75 = 9.25

**4.4 節**

4.33 Xr04-85 一位零售商想估計每月固定和變動的銷售費用。首先，她蒐集過去 8 個月的資料。總銷售費用(以$1,000 為單位) 以及總銷售額( 以$1,000 為單位) 被記錄下來並且在下面列出。

a. 計算共變異數、相關係數和判定係數並且描述這些統計量告訴你什麼訊息。

b. 確定最小平方線並且用它產生零售商想要的估計值。

| 總銷售額 | 銷售費用 |
|---|---|
| 20 | 14 |
| 40 | 16 |
| 60 | 18 |
| 50 | 17 |
| 50 | 18 |
| 55 | 18 |
| 60 | 18 |
| 70 | 20 |

| 4.33a. | $x_i$ | $y_i$ | $x_i^2$ | $y_i^2$ | $x_i y_i$ |
|---|---|---|---|---|---|
| | 20 | 14 | 400 | 196 | 280 |
| | 40 | 16 | 1600 | 256 | 640 |
| | 60 | 18 | 3600 | 324 | 1080 |
| | 50 | 17 | 2500 | 289 | 850 |
| | 50 | 18 | 2500 | 324 | 900 |
| | 55 | 18 | 3025 | 324 | 990 |
| | 60 | 18 | 3600 | 324 | 1080 |
| | 70 | 20 | 4900 | 400 | 1400 |
| Total | 405 | 139 | 22,125 | 2,437 | 7,220 |

$$\sum_{i=1}^{n} x_i = 405 \quad \sum_{i=1}^{n} y_i = 139 \quad \sum_{i=1}^{n} x_i^2 = 22,125 \quad \sum_{i=1}^{n} y_i^2 = 2,437 \quad \sum_{i=1}^{n} x_i y_i = 7,220$$

$$s_{xy} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}\right] = \frac{1}{8-1}\left[7,220 - \frac{(405)(139)}{8}\right] = 26.16$$

$$s_x^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right] = \frac{1}{8-1}\left[22,125 - \frac{(405)^2}{8}\right] = 231.7$$

$$s_x = \sqrt{s_x^2} = \sqrt{231.7} = 15.22$$

$$s_y^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right] = \frac{1}{8-1}\left[2{,}437 - \frac{(139)^2}{8}\right] = 3.13$$

$$s_y = \sqrt{s_y^2} = \sqrt{3.13} = 1.77$$

$$r = \frac{s_{xy}}{s_x s_y} = \frac{26.16}{\sqrt{(15.22)(1.77)}} = .9711 \quad = \frac{26.16}{15.22 \times 1.77}$$

$$R^2 = r^2 = .9711^2 = .9430$$

The covariance is 26.16, the coefficient of correlation is .9711 and the coefficient of determination is .9430.

94.30% of the variation in expenses is explained by the variation in total sales.

b.  $$b_1 = \frac{s_{xy}}{s_x^2} = \frac{26.16}{231.7} = .113$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{405}{8} = 50.63$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{139}{8} = 17.38$$

$$b_0 = \bar{y} - b_1\bar{x} = 17.38 - (.113)(50.63) = 11.66$$

The least squares line is

$$\hat{y} = 11.66 + .113x$$

The estimated variable cost is .113 and the estimated fixed cost is 11.66.


4.34 Xr04-86 學生獲得的分數是否與其研讀該科目的時間相關？為了調查這項神秘的可能性，一名學生隨機選取上學期註冊會計課 10 位學生的樣本。他要求每一位學生告知他或她的分數以及用於研讀會計科目的時間。資料如下所列：

研讀時間　40　42　37　47　25　44　41　48　35　28

分數　　　77　63　79　86　51　78　83　90　65　47

a. 計算共變異數。

b. 計算相關係數。

c. 計算判定係數。

d. 決定最小平方線。

e. 這些被計算出來的統計量告訴你什麼有關分數和研讀時間之間的關係？

| 4.34 | $x_i$ | $y_i$ | $x_i^2$ | $y_i^2$ | $x_iy_i$ |
|------|-------|-------|---------|---------|----------|
| | 40 | 77 | 1,600 | 5,929 | 3,080 |
| | 42 | 63 | 1,764 | 3,969 | 2,646 |
| | 37 | 79 | 1,369 | 6,241 | 2,923 |
| | 47 | 86 | 2,209 | 7,396 | 4,041 |
| | 25 | 51 | 625 | 2,601 | 1,276 |
| | 44 | 78 | 1,936 | 6,084 | 3,432 |
| | 41 | 83 | 1,681 | 6,889 | 3,403 |
| | 48 | 90 | 2,304 | 8,100 | 4,320 |
| | 35 | 65 | 1,225 | 4,225 | 2,275 |
| | 28 | 47 | 784 | 2,209 | 1,316 |
| Total | 387 | 719 | 15,497 | 53,643 | 28,712 |

$$\sum_{i=1}^{n} x_i = 387 \qquad \sum_{i=1}^{n} y_i = 719 \qquad \sum_{i=1}^{n} x_i^2 = 15,497 \qquad \sum_{i=1}^{n} y_i^2 = 53,643 \qquad \sum_{i=1}^{n} x_iy_i = 28,712$$

a
$$s_{xy} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_iy_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}\right] = \frac{1}{10-1}\left[28,712 - \frac{(387)(719)}{10}\right] = 98.52$$

$$s_x^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right] = \frac{1}{10-1}\left[15,497 - \frac{(387)^2}{10}\right] = 57.79$$

$$s_y^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right] = \frac{1}{10-1}\left[53,643 - \frac{(719)^2}{10}\right] = 216.32$$

b
$$r = \frac{s_{xy}}{s_x s_y} = \frac{98.52}{\sqrt{(57.79)(216.32)}} = .8811$$

c
$$R^2 = r^2 = .8811^2 = .7763$$

d
$$b_1 = \frac{s_{xy}}{s_x^2} = \frac{98.52}{57.79} = 1.705$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{387}{10} = 38.7$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{719}{10} = 71.9$$

$$b_0 = \bar{y} - b_1\bar{x} = 71.9 - (1.705)(38.7) = 5.917$$

The least squares line is

$$\hat{y} = 5.917 + 1.705x$$

e. There is a strong positive linear relationship between marks and study time. For each additional hour of study time marks increased on average by 1.705.