# Volatility-Managed Portfolio: *Does It Really Work?*

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**KEY FINDINGS**

- A typical application of volatility-timing strategies to the aggregate stock market, instead of at the stock level, can be more susceptible to look-ahead bias.
- After correcting the bias, the strategy is difficult to implement due to large drawdowns, and it performs poorly over time except during the recent financial crisis period when volatility was extremely high.
- One cannot easily beat the market via volatility-timing the market alone.

**ABSTRACT:** *In this article, the authors find that a typical application of volatility-timing strategies to the stock market suffers from look-ahead bias, despite existing evidence on the success of the strategies at the stock level. After correcting this bias, the strategy becomes very difficult to implement in practice because its maximum drawdown is 68%–93% in almost all cases. Moreover, the strategy outperforms the market only during the financial crisis period. The authors also consider three alternative volatility-timing strategies and find that they do not outperform the market either. Their results show that one cannot easily beat the market via timing the market alone.*

**TOPICS:** ***Portfolio construction, statistical methods, risk management****

Because it is difficult to estimate expected stock returns, the global minimum variance portfolio has been widely used in practice (e.g., Basak, Jagannathan, and Ma 2009). Under some simplifying assumptions, the portfolio essentially puts smaller weights on stocks with greater volatility and larger weights on those with less volatility. Early studies by Fleming, Kirby, and Ostdiek (2001, 2003) considered daily asset allocation across stocks and found supportive evidence for the economic value of volatility timing—that is, using volatility information to improve portfolio performance. Recent studies, such as by Barroso and Santa-Clara (2015) and Han, Huang, and Zhou (2019), confirmed the usefulness of volatility timing at the stock level. Related to these studies, Moreira and Muir (2017) provided a *market* volatility-timing strategy that exploits the well-known property of volatility persistence. The volatility-managed portfolio of Moreira and Muir (2017) is a leverage of the market, with a greater weight assigned to the market when recent volatility is low and a lower weight assigned when recent volatility is high. They showed that their strategy beats the market,

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yielding a positive alpha, a greater Sharpe ratio (SR), and sizable utility gains for mean–variance investors who follow their volatility-timing strategy versus buy-and-hold. They also showed that their strategy works for factor portfolios that are sorted on value, momentum, profitability, return on equity investment, and so on, as well as for currency carry trade.

In this article, we identify a *look-ahead bias* in the strategy of Moreira and Muir (2017). In their empirical results, they calibrated the weight parameter based on the unconditional volatility over the entire sample period. This appears innocuous at first, but a careful examination reveals that the results are highly sensitive to the calibrated parameter. To avoid the look-ahead bias and estimate the weight parameter at each time with only the data available at that time, we adopt two popular estimation windows: a fixed-window approach and a rolling-window approach. With both approaches and with the window size ranging from 5 to 20 years, we find that the maximum drawdown (MDD) of the volatility-managed portfolio is 68%–93% in most cases and thus likely infeasible in practice. In contrast, without correcting the look-ahead bias, the MDD is only 56%, comparable to 50% of the market.

Assuming the strategy is feasible in spite of the large drawdowns, its performance is nonetheless disappointing. In terms of the SR, the volatility-managed portfolio does not outperform the market over the sample period from August 1936 to December 2017. For all estimation cases, the SR is only marginally higher (lower in one case), and the difference is never statistically significant.[1] In the real world, consideration of trading costs clearly makes the strategy even less appealing.

Breaking down the entire sample period into roughly 20-year subperiods, we find that the volatility-managed portfolio underperforms the market almost half of the time. However, it does outperform the market in the most recent period, driven largely by the extremely high volatility during the recent financial crisis. Because financial crises are rare and difficult to predict, the superior performance of the volatility-managed portfolio in this period alone does not support it as a superior investment strategy overall for two reasons. First, an investment strategy that delivers no superior performance over long periods is unlikely to be adopted in practice. Second, the large drawdowns are of concern. The strategy can suffer a forced liquidation before a financial crisis (whose arrival time is difficult to anticipate) happens.

We argue that the SR is a more informative measure than the alpha in the current context (though the conclusions are similar based on the alpha). A strategy that has a positive alpha will not necessarily add to the investment value for an investor; the investment value increases only if the strategy yields a greater SR or higher investor utility when it is combined with the market or the investor's existing portfolio. Because the volatility-managed portfolio is already a portfolio of the market, just with time-varying weights, alpha is uninformative here. The fact that the volatility-managed portfolio does not generally yield a higher SR than the market suggests that its value is limited.

An important question is whether there are similar volatility-timing strategies that can perform better. To this end, we consider three alternative volatility-timing strategies in the literature, none of which seem to suffer a look-ahead bias. The first is a volatility-targeting strategy proposed by Barroso and Santa-Clara (2015), the second is an allocation strategy under estimation risk proposed by Kan and Zhou (2007), and the third is an optimal portfolio strategy using conditional information proposed by Ferson and Siegel (2001). Applying all three strategies to real data, we find that they fail to outperform the market over the sample period from August 1936 to December 2017.

Our article is related to a few emerging studies on volatility-managed portfolios. Independently, Cederburg et al. (2019) studied volatility-managed portfolios for 103 spread portfolios. In contrast to their study, we focus on market timing and alternative volatility strategies. For volatility-managed portfolios to be profitable in the real world, transaction costs must be taken into consideration. In this case, Barroso and Detzel (2018) showed that there are no significant alphas even if we ignore the look-ahead bias.

Overall, our results suggest that it is extremely difficult to beat the market by simply trading the market alone via volatility timing. The volatility-managed strategy of Moreira and Muir (2017) fails to do so, as do the three alternative strategies. In particular, even the unconditional optimal portfolio with conditional information by Ferson and Siegel (2001) fails to outperform

[1] An online supplement to this article provides similar results for factor portfolios such as size and value.

the market, although it was theoretically designed to be the optimal strategy. This result is due to estimation errors that make the strategy less effective in practice. However, it should be pointed out that we do not exclude the possibility of beating the market by trading individual stocks using volatility information.

## VOLATILITY-TIMING STRATEGIES

This section reviews four volatility-timing strategies in the literature. The first is the volatility-managed portfolio of Moreira and Muir (2017), the second is the volatility-targeting strategy of Barroso and Santa-Clara (2015), the third is the Kan and Zhou (2007) mean–variance portfolio allocation strategy under estimation risk, and the fourth is the unconditional optimal portfolio with conditional information by Ferson and Siegel (2001).

### Volatility-Managed Portfolio

Moreira and Muir (2017) proposed a volatility-managed portfolio constructed by scaling the excess return of the market or a factor portfolio by the inverse of the previous month's realized return variance. This strategy is motivated by the observation that changes in volatility over time are not offset by proportional changes in expected returns. Moreira and Muir (2017) found that this strategy improves investment performance relative to the original portfolio by reducing risk exposure when volatility is high.

Theoretically, this volatility-managed strategy can be understood from the perspective of an optimal capital allocation problem between a risky portfolio and a risk-free asset. Suppose there exists a risky portfolio whose return in excess of the risk-free rate, $r_f$, is a random variable $\tilde{r}_t$ over month $t$. The expected excess return is $\mu_t$, and the return volatility is $\sigma_t$. Consider a mean–variance investor with risk aversion $A$ who chooses a weight $w_t$ invested in the risky portfolio and $1 - w_t$ invested in the risk-free asset. Note that $w_t$ can be viewed as the leverage level of this strategy. The investor faces the following mean–variance utility maximization problem:

$$\max_{w_t} U(w_t) = r_f + w_t\mu_t - \frac{A}{2}w_t^2\sigma_t^2 \tag{1}$$

Solving Equation 1 yields the optimal leverage

$$w_t = \frac{1}{A}\frac{\mu_t}{\sigma_t^2}$$

Therefore, if $\mu_t$ and $\sigma_t$ are known, the optimal leverage should be proportional to $\mu_t/\sigma_t^2$.

Observing that volatility is highly persistent over time and that returns are difficult to predict, Moreira and Muir (2017) constructed a volatility-managed portfolio by choosing

$$w_t^{MM} = \frac{L}{\hat{\sigma}_{t-1}^2} \tag{2}$$

where $L$ is a constant, and $\hat{\sigma}_{t-1}^2$ is the realized return variance in month $t-1$ computed from daily returns over the month (used as a proxy of $\sigma_t^2$ based on information available as of month $t-1$). Specifically, $\hat{\sigma}_{t-1}^2$ can be computed as

$$\hat{\sigma}_{t-1}^2 = \frac{22}{D_{t-1}}\sum_{d=1/D_{t-1}}^{1}\left(r_{t-1,d} - \frac{1}{D_{t-1}}\sum_{d=1/D_{t-1}}^{1} r_{t-1,d}\right)^2$$

where $D_{t-1}$ is the number of trading days in month $t-1$, $r_{t-1,d}$ is the excess return of the risky portfolio on date $d$ of month $t-1$, and a multiplier of 22 is included to convert daily variances into monthly values. The excess return of this strategy is thus given by

$$\tilde{r}_t^{MM} = \frac{L}{\hat{\sigma}_{t-1}^2}\tilde{r}_t \tag{3}$$

The volatility-managed strategy is not implementable unless $L$ is given a priori. Moreira and Muir (2017) chose $L$ so that their strategy had the same unconditional volatility as the original portfolio over the full sample. The choice of $L$ appears innocuous because it does not affect the SR. However, it introduces more complex issues in practice. First, choosing $L$ based on the unconditional volatility over the entire period is an in-sample approach and is thus subject to look-ahead bias. In practice, one must choose an out-of-sample method (i.e., using only information available at each time point) to estimate $L$ to implement the strategy in real time. Different estimation methods lead to different outcomes, and it is unknown ex ante which estimation method will perform the best in the future. Second, the $L$ estimate can potentially expose the strategy to too much tail risk in terms of drawdowns. In such cases, the strategy may be impossible to carry out to the end of the period because investors will almost surely choose to liquidate when the drawdown reaches a level that is too high.

In short, choosing $L$ based on the unconditional volatility over the full sample is subject to look-ahead bias. To avoid the bias, one must estimate $L$ using an out-of-sample approach, which may render the strategy very difficult to implement in practice because of the large drawdowns. In addition, even ignoring the drawdowns, we find that the volatility-managed portfolio cannot outperform the market in general, as will be demonstrated empirically in the third section.

### Volatility-Targeting Strategy

Barroso and Santa-Clara (2015) proposed a volatility-timing strategy by setting a volatility target for the portfolio under consideration. They showed that the strategy is effective in avoiding momentum crashes. Their strategy is based on the same idea of scaling an excess return by its conditional volatility. The key difference is that, instead of choosing the leverage size by requiring the unconditional volatility of the strategy to be equal to that of the original portfolio, they set some specific ex ante target volatility $\sigma_{target}$:

$$\tilde{r}_t^{BS} = \frac{\sigma_{target}}{\hat{\sigma}_{t-1}} \tilde{r}_t \tag{4}$$

The leverage level of this strategy is thus

$$w_t^{BS} = \frac{\sigma_{target}}{\hat{\sigma}_{t-1}} \tag{5}$$

Clearly, the choice of the target volatility is not subject to look-ahead bias.

Similar to Moreira and Muir (2017), Barroso and Santa-Clara (2015) observed that the choice of $\sigma_{target}$ does not affect the SR of the strategy and is thus unimportant. For implementation, they chose a target level corresponding to an annualized volatility of 12%. However, as in the case of Moreira and Muir (2017), the choice of $\sigma_{target}$ has strong implications for the drawdowns.

### Mean–Variance Portfolio Allocation under Estimation Risk

The preceding strategies do not exploit any information on expected returns. We now examine whether such information may be incorporated to improve the performance of volatility timing. Because the expected return is unknown and must be estimated from data, using the estimated expected return (i.e., sample mean) introduces an estimation risk. The greater the volatility, the more difficult it is to estimate the expected return and thus the greater the estimation risk. Intuitively, when faced with estimation risk, mean–variance investors should lower their investment in the risky portfolio. This is qualitatively similar to the volatility-timing strategies of both Moreira and Muir (2017) and Barroso and Santa-Clara (2015).

Consider an estimated allocation strategy:

$$w_t^{KZ} = \frac{c}{A} \frac{\hat{\mu}_{t-1}}{\hat{\sigma}_{t-1}^2} \tag{6}$$

where $c$ is a prespecified constant, and $\hat{\mu}_{t-1}$ and $\hat{\sigma}_{t-1}$ are the estimated expected excess return and volatility of the risky portfolio at $t - 1$. When $c = 1$, this is the standard optimal portfolio rule. Intuition suggests that $c$ should be less than 1 when $\hat{\sigma}_{t-1} > 0$ owing to risk involved in estimating $\hat{\mu}_{t-1}$.

Assume that the excess return of the risky portfolio follows a normal distribution:

$$\tilde{r}_t \sim N(\mu_t, \sigma_t^2)$$

Kan and Zhou (2007) studied the optimal allocation problem under estimation risk, and Zhou (2008) applied the results to active portfolio management. Here, using the fact that volatility is highly persistent and much more predictable than returns, for simplicity, we assume further that the volatility is known and is equal to the past realized volatility. Then, based on Kan and Zhou (2007), the optimal $c$ that minimizes utility loss associated with estimation risk is

$$c = \frac{\hat{\theta}_{t-1}^2}{\hat{\theta}_{t-1}^2 + 1/T}$$

where

$$\hat{\theta}_{t-1}^2 = \frac{\hat{\mu}_{t-1}^2}{\hat{\sigma}_{t-1}^2}$$

and $T$ is the sample size used to estimate $\hat{\mu}_{t-1}$. The analytical formula makes intuitive sense. Everything else equal, greater volatility yields a lower $c$, which in turn reduces exposure to the risky portfolio. As the sample size approaches infinity, the estimation risk disappears, and the strategy converges to the standard optimal allocation rule.

### Unconditional Optimal Portfolio with Conditional Information

Ferson and Siegel (2001) showed that conditional information can be used to improve the unconditional performance of portfolios. In particular, assume that the conditional expected excess return $\tilde{\mu}_{t-1}$ and the conditional volatility $\tilde{\sigma}_{t-1}^2$ follow stationary processes so that the expectation

$$\zeta = \mathrm{E}\left(\frac{\tilde{\mu}_{t-1}^2}{\tilde{\mu}_{t-1}^2 + \tilde{\sigma}_{t-1}^2}\right)$$

is well defined. Then, for any given unconditional target expected excess return $\mu_{target}$, the optimal leverage yielding minimum unconditional variance is

$$w_t^{FS} = \frac{\mu_{target}}{\zeta}\left(\frac{\hat{\mu}_{t-1}}{\hat{\mu}_{t-1}^2 + \hat{\sigma}_{t-1}^2}\right)$$

where $\hat{\mu}_{t-1}$ and $\hat{\sigma}_{t-1}$ are the sample conditional expected excess return and conditional volatility estimated as before.

For implementation, we estimate $\zeta$ as the historical mean of $\hat{\mu}_{t-1}^2/(\hat{\mu}_{t-1}^2 + \hat{\sigma}_{t-1}^2)$. Clearly, it is of interest to develop better econometric models for estimating $\hat{\sigma}_{t-1}$, $\hat{\mu}_{t-1}$, and $\zeta$. However, because the focus of this article is to point out the look-ahead bias in the study by Moreira and Muir (2017) and to demonstrate the empirical fact that all four simple volatility-timing strategies in the literature fail to beat the market, we leave the search for more sophisticated models of conditional information as future work.

## EMPIRICAL RESULTS

In this section, we examine the empirical performance of the volatility-timing strategies and show how their performance depends critically on the level of leverage. We focus on applying the strategies to the market excess return, while leaving discussions for other factor portfolios to the online supplement.

We start by comparing the performance of the volatility-managed portfolio of Moreira and Muir (2017) with $L$ estimated in sample (i.e., based on the unconditional volatility over the entire period) against that of the market excess return. We obtain the market excess returns from CRSP for the sample period from August 1926 to December 2017. The estimated value of $L$ is around 0.0010. Because the volatility-managed strategy involves leverage, which may be subject to constraints in practice, we follow Moreira and Muir (2017) and consider two cases, one with unlimited leverage and one with limited leverage. In the limited leverage case, we impose a constraint that the leverage does not exceed 2; that is, $w_t^{MM} \leq 2$. If $w_t^{MM}$ estimated from Equation 2 is higher than 2, we then truncate it at 2. Similar constraints are also imposed for other volatility-timing strategies.

Exhibit 1 provides summary statistics of the market excess return and the corresponding volatility-managed portfolio for the full sample and for subsamples.[2] Over the full sample, the annualized SRs of the volatility-managed portfolio with unlimited and limited leverage are 0.5143 and 0.5282, respectively. Neither is significantly different from the SR of the market, 0.4996, based on heteroskedasticity- and autocorrelation-consistent (HAC) standard errors (Ledoit and Wolf 2008). We also examine the MDD of the portfolios. Under unlimited and limited leverage, the MDD values of the volatility-managed strategy are 0.5642 and 0.5459, respectively. Although they are larger than that of the market index, 0.5039, they do not seem too extreme. We also estimate the risk-adjusted alphas of the strategy with respect to the market index; the alphas are weakly significant based on Newey–West standard errors.

[2] The full sample results would be the same as those from Moreira and Muir (2017) if the data start in August 1926. We use the period from August 1936 to December 2017 to be consistent with our out-of-sample results later, in which the first 10 years' data are used to estimate $L$.

## EXHIBIT 1
### In-Sample Performance of Volatility-Managed Portfolio for Market

**Panel A: Market Index**

| | Full Sample | August 1936–December 1960 | January 1961–December 1980 | January 1981–December 2000 | January 2001–December 2017 |
|---|---|---|---|---|---|
| No. of Obs | 977 | 293 | 240 | 240 | 204 |
| Mean | 0.0065 | 0.0091 | 0.0034 | 0.0073 | 0.0054 |
| Vol | 0.0451 | 0.0478 | 0.0444 | 0.0445 | 0.0427 |
| Min | –0.2382 | –0.2382 | –0.1290 | –0.2324 | –0.1723 |
| Max | 0.2387 | 0.2387 | 0.1610 | 0.1247 | 0.1135 |
| MDD | 0.5039 | 0.4935 | 0.4642 | 0.2991 | 0.5039 |
| SR | 0.4996 | 0.6610 | 0.2671 | 0.5710 | 0.4380 |

**Panel B: Volatility-Managed Portfolio**

| | Full Sample | | August 1936–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0081 | 0.0066 | 0.0111 | 0.0096 | 0.0062 | 0.0036 | 0.0082 | 0.0069 | 0.0061 | 0.0057 |
| Vol | 0.0548 | 0.0435 | 0.0640 | 0.0520 | 0.0687 | 0.0483 | 0.0443 | 0.0398 | 0.0256 | 0.0239 |
| Min | –0.3125 | –0.3125 | –0.3125 | –0.3125 | –0.2300 | –0.1436 | –0.1154 | –0.1154 | –0.0603 | –0.0542 |
| Max | 0.3577 | 0.1876 | 0.2909 | 0.1876 | 0.3577 | 0.1576 | 0.1662 | 0.1370 | 0.1198 | 0.0972 |
| MDD | 0.5642 | 0.5459 | 0.5642 | 0.5459 | 0.4934 | 0.3933 | 0.1481 | 0.1410 | 0.1680 | 0.1680 |
| Alpha | 0.0027* | 0.0017* | 0.0023 | 0.0017 | 0.0026 | 0.0006 | 0.0025 | 0.0014 | 0.0040** | 0.0037*** |
| SR | 0.5143 | 0.5282 | 0.6001 | 0.6370 | 0.3117 | 0.2583 | 0.6417 | 0.5971 | 0.8264 | 0.8296 |
| ΔSR | 0.0148 | 0.0286 | –0.0608 | –0.0239 | 0.0446 | –0.0088 | 0.0708 | 0.0261 | 0.3884* | 0.3916* |

*Notes: This exhibit reports summary statistics of the market excess return (Panel A) and the volatility-managed portfolio of Moreira and Muir (2017) applied to the market excess return (Panel B) with* L *in Equation 3 estimated in sample. We report results for the full sample from August 1936 to December 2017 and for four subsample periods. The summary statistics include the number of observations, mean, volatility, minimum, maximum, MDD, and the SR. For the volatility-managed portfolio, we consider cases with unlimited leverage (UL) and limited leverage (LL) separately. We also report the risk-adjusted alpha of the volatility-managed portfolio with respect to the market index and the difference in the SR between the volatility-managed portfolio and the market index.*

****, **, and * denote statistical significance based on Newey–West standard errors (for alpha) or HAC standard errors (for the SR test) at the 1%, 5%, and 10% levels, respectively.*

Breaking down the full sample into roughly 20-year subperiods shows that 2001 to 2017 is the only period during which the volatility-managed portfolio clearly dominates the market index, driven largely by the financial crisis. For the other subperiods, there is no clear evidence that volatility timing improves performance even when $L$ is estimated in sample.

We turn to estimating $L$ out of sample and see how this would affect the performance of the volatility-managed strategy. We use two out-of-sample estimation approaches. The first is a fixed-window approach, which uses the first 10 years between August 1926 and July 1936 as a training window. We choose $L$ such that the volatility-managed portfolio has the same unconditional volatility as the market index over the training window, and we use this same value of $L$ to determine leverage for all remaining months. The second approach is to estimate $L$ using 10-year rolling windows, allowing time variations in the value of $L$. For example, we estimate $L$ based on returns from August 1926 to July 1936 and use this $L$ to determine the leverage size for August 1936, and so on.

Panel A of Exhibit 2 reports results based on the 10-year fixed-window approach. The estimated $L$ in this case is 0.0023, higher than the in-sample value of $L$. This leads to proportionally higher leverage in

## EXHIBIT 2

### Out-of-Sample Performance of Volatility-Managed Portfolio for Market: Fixed Window

**Panel A: 10-Year Fixed Window**

| | Full Sample | | August 1936–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0194 | 0.0096 | 0.0265 | 0.0143 | 0.0148 | 0.0044 | 0.0196 | 0.0099 | 0.0146 | 0.0088 |
| Vol | 0.1308 | 0.0654 | 0.1528 | 0.0721 | 0.1640 | 0.0704 | 0.1057 | 0.0681 | 0.0612 | 0.0418 |
| Min | –0.7461 | –0.4390 | –0.7461 | –0.4390 | –0.5491 | –0.2382 | –0.2754 | –0.2754 | –0.1439 | –0.1194 |
| Max | 0.8539 | 0.2494 | 0.6944 | 0.1876 | 0.8539 | 0.1995 | 0.3968 | 0.2494 | 0.2861 | 0.1114 |
| MDD | 0.9293 | 0.7596 | 0.9293 | 0.7596 | 0.9222 | 0.5800 | 0.8545 | 0.3765 | 0.4414 | 0.4088 |
| Alpha | 0.0065* | 0.0013 | 0.0055 | 0.0022 | 0.0062 | –0.0006 | 0.0059 | –0.0005 | 0.0096** | 0.0047** |
| SR | 0.5143 | 0.5103 | 0.6001 | 0.6891 | 0.3117 | 0.2149 | 0.6417 | 0.5028 | 0.8264 | 0.7297 |
| ΔSR | 0.0148 | 0.0107 | –0.0608 | 0.0281 | 0.0446 | –0.0522 | 0.0708 | –0.0682 | 0.3884* | 0.2916 |

**Panel B: 5-Year Fixed Window**

| | Full Sample | | August 1931–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| No. of Obs | 1,037 | 1,037 | 353 | 353 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0102 | 0.0076 | 0.0128 | 0.0100 | 0.0080 | 0.0040 | 0.0106 | 0.0081 | 0.0079 | 0.0069 |
| Vol | 0.0694 | 0.0498 | 0.0767 | 0.0561 | 0.0892 | 0.0549 | 0.0575 | 0.0486 | 0.0333 | 0.0291 |
| Min | –0.4057 | –0.4057 | –0.4057 | –0.4057 | –0.2986 | –0.1436 | –0.1498 | –0.1498 | –0.0782 | –0.0649 |
| Max | 0.4643 | 0.1876 | 0.3776 | 0.1876 | 0.4643 | 0.1630 | 0.2158 | 0.1778 | 0.1556 | 0.0972 |
| MDD | 0.6781 | 0.6527 | 0.6781 | 0.6527 | 0.6095 | 0.4558 | 0.2140 | 0.2140 | 0.2317 | 0.2317 |
| Alpha | 0.0047** | 0.0028** | 0.0065* | 0.0047* | 0.0033 | 0.0004 | 0.0032 | 0.0012 | 0.0052** | 0.0043*** |
| SR | 0.5113 | 0.5259 | 0.5785 | 0.6205 | 0.3117 | 0.2507 | 0.6417 | 0.5762 | 0.8264 | 0.8170 |
| ΔSR | 0.0543 | 0.0690 | 0.0587 | 0.1007 | 0.0446 | –0.0164 | 0.0708 | 0.0052 | 0.3884* | 0.3790* |

**Panel C: 20-Year Fixed Window**

| | Full Sample | | August 1946–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| No. of Obs | 857 | 857 | 173 | 173 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0119 | 0.0082 | 0.0180 | 0.0143 | 0.0093 | 0.0040 | 0.0124 | 0.0087 | 0.0092 | 0.0073 |
| Vol | 0.0805 | 0.0522 | 0.0962 | 0.0583 | 0.1038 | 0.0587 | 0.0669 | 0.0543 | 0.0387 | 0.0321 |
| Min | –0.3886 | –0.1860 | –0.3886 | –0.1860 | –0.3475 | –0.1658 | –0.1743 | –0.1743 | –0.0910 | –0.0755 |
| Max | 0.5403 | 0.2056 | 0.3300 | 0.1876 | 0.5403 | 0.1630 | 0.2511 | 0.2056 | 0.1810 | 0.0972 |
| MDD | 0.6785 | 0.4965 | 0.4543 | 0.3268 | 0.6785 | 0.4965 | 0.3745 | 0.2629 | 0.2746 | 0.2746 |
| Alpha | 0.0037* | 0.0017 | –0.0037 | –0.0006 | 0.0039 | 0.0000 | 0.0038 | 0.0009 | 0.0061** | 0.0044** |
| SR | 0.5131 | 0.5430 | 0.6494 | 0.8493 | 0.3117 | 0.2359 | 0.6417 | 0.5552 | 0.8264 | 0.7881 |
| ΔSR | 0.0018 | 0.0317 | –0.2932 | –0.0933 | 0.0446 | –0.0312 | 0.0708 | –0.0158 | 0.3884* | 0.3501* |

*Notes: This exhibit reports summary statistics of the volatility-managed portfolio of Moreira and Muir (2017) applied to the market excess return with* L *in Equation 3 estimated out of sample using a fixed training window at the beginning of our sample period from August 1926 to December 2017. We report results for the full sample and for four subsample periods. We use 10-year (Panel A), 5-year (Panel B), and 20-year (Panel C) training windows. We consider cases with UL and LL separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, MDD, and the SR. We also report the risk-adjusted alpha of the volatility-managed portfolio with respect to the market index and the difference in the SR between the volatility-managed portfolio and the market index.*

****, **, and * denote statistical significance based on Newey–West standard errors (for alpha) or HAC standard errors (for the SR test) at the 1%, 5%, and 10% levels, respectively.*

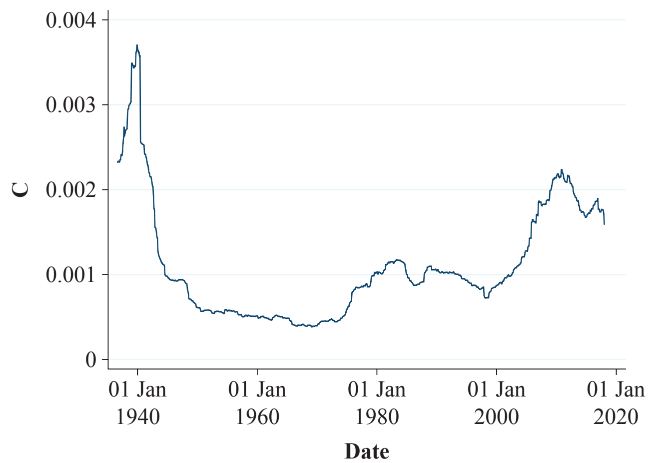
every single month, which does not affect the SR of the strategy without leverage constraint. However, it does have a substantial effect on the MDD of the strategy. Over the full sample, the MDD value is 0.9293 with unlimited leverage and 0.7596 with limited leverage, much higher than that of the market. In particular, an MDD of 0.9293 means that the strategy loses about 93% of its value during the worst decline, which would be unacceptable to investors in practice. To see how the performance of volatility timing is affected by the choice of the training window, Panels B and C report results using 5-year and 20-year windows for the estimation of *L*. The MDD values are 0.6781 and 0.6785, respectively. Both are substantially larger than the MDD of the market.

We then turn to estimating *L* out-of-sample and see how this would affect the performance of the volatility-managed strategy. We use two out-of-sample estimation approaches. The first is a fixed-window approach, which uses the first 10 years between August 1926 and July 1936 as a training window. We choose *L* such that the volatility-managed portfolio has the same unconditional volatility as the market index over the training window, and we use this same value of *L* to determine leverage for all remaining months. The second approach is to estimate *L* using 10-year rolling windows, allowing time variations in the value of *L*. For example, we estimate *L* based on returns from August 1926 to July 1936 and use this *L* to determine the leverage size for August 1936, and so on.

Panel A of Exhibit 2 reports results based on the 10-year fixed-window approach. The estimated *L* in this case is 0.0023, higher than the in-sample value of *L*. This leads to proportionally higher leverage in every single month, which does not affect the SR of the strategy without leverage constraint. However, this does have a substantial effect on the MDD of the strategy. Over the full sample, the MDD value is 0.9293 with unlimited leverage and 0.7596 with limited leverage, much higher than that of the market. In particular, an MDD of 0.9293 means that the strategy loses about 93% of its value during the worst decline, which would be unacceptable to investors in practice. To see how the performance of volatility timing is affected by the choice of the training window, Panels B and C report results using 5-year and 20-year windows for the estimation of *L*. The MDD values are 0.6781 and 0.6785,

## Exhibit 3

**Rolling-Window Estimation of L for Volatility-Managed Portfolio**




*Notes: This exhibit plots the value of* L *for the volatility-managed portfolio, Equation 3, of Moreira and Muir (2017) applied to the market excess return with* L *estimated out of sample using 10-year rolling windows. The sample period is from August 1936 to December 2017.*

respectively. Both are substantially larger than the MDD of the market.

We now discuss results using rolling-window estimation of *L*. Exhibit 3 shows the time variation of *L* under the 10-year rolling-window approach. The value ranges between 0.0004 and 0.0037, with a mean of 0.0011 and a median of 0.0007. It peaks around 1940, quickly drops below the mean and stays low until 2000, and then picks up again. With time-varying values of *L*, the SR now differs from the in-sample level. As reported in Exhibit 4, over the full sample, the SR of the volatility-managed portfolio with 10-year rolling windows is only 0.3866, much lower than that of the market. The worst monthly return is −115%, which happened in May 1940, indicating that an investor using the volatility-managed strategy would go bankrupt in that month. Imposing the leverage constraint slightly improves performance: The SR becomes 0.4632, and the MDD is 0.7640, both of which are still worse than the market. Breaking down the full sample into subsamples shows that the poor performance mainly comes from the early years. From August 1936 to December 1960 especially, the SRs of the volatility-managed strategy under both unlimited and limited leverage are significantly lower than that of the market at the 5% level. This is

## EXHIBIT 4

### Out-of-Sample Performance of Volatility-Managed Portfolio for Market: Rolling Window

**Panel A: 10-Year Rolling Window**

| | Full Sample | | August 1936–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0074 | 0.0060 | 0.0066 | 0.0064 | 0.0040 | 0.0030 | 0.0078 | 0.0066 | 0.0118 | 0.0083 |
| Vol | 0.0660 | 0.0450 | 0.1012 | 0.0602 | 0.0380 | 0.0345 | 0.0445 | 0.0402 | 0.0461 | 0.0348 |
| Min | –1.1501 | –0.4390 | –1.1501 | –0.4390 | –0.1111 | –0.1111 | –0.1086 | –0.1086 | –0.1143 | –0.0966 |
| Max | 0.4421 | 0.1843 | 0.4421 | 0.1843 | 0.1664 | 0.1465 | 0.1669 | 0.1591 | 0.2112 | 0.0972 |
| MDD | Broke | 0.7640 | Broke | 0.7640 | Broke | 0.2056 | Broke | 0.1813 | Broke | 0.2525 |
| Alpha | 0.0012 | 0.0009 | –0.0070 | –0.0034 | 0.0019 | 0.0010 | 0.0021 | 0.0011 | 0.0082*** | 0.0052*** |
| SR | 0.3866 | 0.4632 | 0.2259 | 0.3703 | 0.3663 | 0.2999 | 0.6103 | 0.5670 | 0.8886 | 0.8307 |
| ΔSR | –0.1130 | –0.0363 | –0.4351** | –0.2906** | 0.0993 | 0.0328 | 0.0393 | –0.0040 | 0.4506** | 0.3927* |

**Panel B: 5-Year Rolling Window**

| | Full Sample | | August 1931–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| No. of Obs | 1,037 | 1,037 | 353 | 353 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0087 | 0.0066 | 0.0113 | 0.0074 | 0.0037 | 0.0033 | 0.0071 | 0.0061 | 0.0120 | 0.0096 |
| Vol | 0.0614 | 0.0461 | 0.0843 | 0.0587 | 0.0445 | 0.0379 | 0.0418 | 0.0384 | 0.0495 | 0.0376 |
| Min | –0.6935 | –0.4390 | –0.6935 | –0.4390 | –0.1577 | –0.1312 | –0.1035 | –0.1035 | –0.1649 | –0.0842 |
| Max | 0.5133 | 0.1812 | 0.5133 | 0.1812 | 0.2212 | 0.1576 | 0.1645 | 0.1645 | 0.1862 | 0.1114 |
| MDD | 0.9110 | 0.7658 | 0.9110 | 0.7658 | 0.3905 | 0.2292 | 0.1902 | 0.1750 | 0.3111 | 0.3111 |
| Alpha | 0.0036* | 0.0022* | 0.0040 | 0.0014 | 0.0012 | 0.0010 | 0.0018 | 0.0009 | 0.0079*** | 0.0061*** |
| SR | 0.4923 | 0.4952 | 0.4659 | 0.4369 | 0.2882 | 0.3041 | 0.5917 | 0.5513 | 0.8365 | 0.8812 |
| ΔSR | 0.0354 | 0.0383 | –0.0539 | –0.0830 | 0.0211 | 0.0371 | 0.0207 | –0.0197 | 0.3985* | 0.4432** |

**Panel C: 20-Year Rolling Window**

| | Full Sample | | August 1946–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| No. of Obs | 857 | 857 | 173 | 173 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0078 | 0.0063 | 0.0113 | 0.0101 | 0.0036 | 0.0023 | 0.0079 | 0.0066 | 0.0095 | 0.0073 |
| Vol | 0.0468 | 0.0368 | 0.0713 | 0.0504 | 0.0360 | 0.0314 | 0.0413 | 0.0365 | 0.0370 | 0.0284 |
| Min | –0.2973 | –0.1860 | –0.2973 | –0.1860 | –0.1231 | –0.1231 | –0.1055 | –0.1055 | –0.0861 | –0.0767 |
| Max | 0.3457 | 0.1815 | 0.3457 | 0.1815 | 0.1863 | 0.0957 | 0.1565 | 0.1220 | 0.2171 | 0.0972 |
| MDD | 0.4610 | 0.3278 | 0.4610 | 0.3278 | 0.2225 | 0.2225 | 0.1446 | 0.1379 | 0.1728 | 0.1728 |
| Alpha | 0.0032** | 0.0021** | –0.0042 | –0.0022 | 0.0017 | 0.0005 | 0.0027 | 0.0016 | 0.0069*** | 0.0048*** |
| SR | 0.5760 | 0.5903 | 0.5485 | 0.6958 | 0.3478 | 0.2576 | 0.6652 | 0.6272 | 0.8937 | 0.8842 |
| ΔSR | 0.0647 | 0.0791 | –0.3940** | –0.2468* | 0.0808 | –0.0094 | 0.0942 | 0.0562 | 0.4557** | 0.4462** |

*Notes: This exhibit reports summary statistics of the volatility-managed portfolio of Moreira and Muir (2017) applied to the market excess return with* L *in Equation 3 estimated out of sample using rolling windows. We report results for the full sample from August 1926 to December 2017 and for four subsample periods. We use 10-year (Panel A), 5-year (Panel B), and 20-year (Panel C) rolling windows. We consider cases with UL and LL separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, MDD, and the SR. We also report the risk-adjusted alpha of the volatility-managed portfolio with respect to the market index and the difference in the SR between the volatility-managed portfolio and the market index.*

****, **, and * denote statistical significance based on Newey–West standard errors (for alpha) or HAC standard errors (for the SR test) at the 1%, 5%, and 10% levels, respectively.*

# Exhibit 5

**Performance of Volatility Targeting for Market**

| | Full Sample | | August 1936–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| **Panel A: Annualized Target Volatility 12%** | | | | | | | | | | |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0073 | 0.0071 | 0.0103 | 0.0102 | 0.0042 | 0.0037 | 0.0078 | 0.0076 | 0.0062 | 0.0062 |
| Vol | 0.0459 | 0.0452 | 0.0523 | 0.0518 | 0.0507 | 0.0492 | 0.0438 | 0.0436 | 0.0295 | 0.0294 |
| Min | –0.2910 | –0.2910 | –0.2910 | –0.2910 | –0.1368 | –0.1318 | –0.1819 | –0.1819 | –0.0742 | –0.0742 |
| Max | 0.1590 | 0.1590 | 0.1590 | 0.1590 | 0.1248 | 0.1248 | 0.1398 | 0.1398 | 0.0804 | 0.0804 |
| MDD | 0.5230 | 0.5225 | 0.5230 | 0.5225 | 0.4177 | 0.4037 | 0.1981 | 0.1981 | 0.3018 | 0.3018 |
| Alpha | 0.0014* | 0.0012 | 0.0014 | 0.0013 | 0.0006 | 0.0002 | 0.0010 | 0.0009 | 0.0029** | 0.0029** |
| SR | 0.5527 | 0.5445 | 0.6822 | 0.6799 | 0.2855 | 0.2584 | 0.6146 | 0.6061 | 0.7275 | 0.7257 |
| ΔSR | 0.0531 | 0.0449 | 0.0213 | 0.0189 | 0.0184 | –0.0086 | 0.0437 | 0.0351 | 0.2895** | 0.2877** |
| **Panel B: Annualized Target Volatility 16%** | | | | | | | | | | |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0098 | 0.0091 | 0.0137 | 0.0129 | 0.0056 | 0.0047 | 0.0104 | 0.0097 | 0.0083 | 0.0080 |
| Vol | 0.0612 | 0.0573 | 0.0698 | 0.0649 | 0.0675 | 0.0612 | 0.0584 | 0.0569 | 0.0393 | 0.0387 |
| Min | –0.3880 | –0.3880 | –0.3880 | –0.3880 | –0.1824 | –0.1694 | –0.2426 | –0.2426 | –0.0989 | –0.0989 |
| Max | 0.2121 | 0.1876 | 0.2121 | 0.1876 | 0.1664 | 0.1576 | 0.1864 | 0.1864 | 0.1072 | 0.0972 |
| MDD | 0.6519 | 0.6442 | 0.6519 | 0.6442 | 0.5387 | 0.4980 | 0.2692 | 0.2692 | 0.3999 | 0.3999 |
| Alpha | 0.0019* | 0.0015* | 0.0018 | 0.0016 | 0.0009 | 0.0003 | 0.0014 | 0.0008 | 0.0039** | 0.0037** |
| SR | 0.5527 | 0.5482 | 0.6822 | 0.6889 | 0.2855 | 0.2675 | 0.6146 | 0.5885 | 0.7275 | 0.7172 |
| ΔSR | 0.0531 | 0.0487 | 0.0213 | 0.0279 | 0.0184 | 0.0005 | 0.0437 | 0.0175 | 0.2895** | 0.2792** |
| **Panel C: Annualized Target Volatility 20%** | | | | | | | | | | |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0122 | 0.0103 | 0.0172 | 0.0147 | 0.0070 | 0.0051 | 0.0130 | 0.0110 | 0.0103 | 0.0092 |
| Vol | 0.0764 | 0.0664 | 0.0872 | 0.0735 | 0.0844 | 0.0699 | 0.0730 | 0.0682 | 0.0491 | 0.0466 |
| Min | –0.4850 | –0.4390 | –0.4850 | –0.4390 | –0.2280 | –0.2117 | –0.3032 | –0.3032 | –0.1236 | –0.1236 |
| Max | 0.2651 | 0.2329 | 0.2651 | 0.2164 | 0.2079 | 0.1867 | 0.2329 | 0.2329 | 0.1340 | 0.1016 |
| MDD | 0.7567 | 0.7173 | 0.7567 | 0.7173 | 0.6610 | 0.5821 | 0.3649 | 0.3612 | 0.4867 | 0.4867 |
| Alpha | 0.0024* | 0.0013 | 0.0023 | 0.0016 | 0.0011 | –0.0000 | 0.0017 | 0.0003 | 0.0049** | 0.0039** |
| SR | 0.5527 | 0.5365 | 0.6822 | 0.6927 | 0.2855 | 0.2535 | 0.6146 | 0.5606 | 0.7275 | 0.6819 |
| ΔSR | 0.0531 | 0.0369 | 0.0213 | 0.0317 | 0.0184 | –0.0136 | 0.0437 | –0.0104 | 0.2895** | 0.2439* |

*Notes: This exhibit reports summary statistics of the volatility-targeting strategy of Barroso and Santa-Clara (2015) applied to the market excess return with annualized target volatility of 12% (Panel A), 16% (Panel B), and 20% (Panel C). We report results for the full sample from August 1936 to December 2017 and for four subsample periods. We consider cases with UL and LL separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, MDD, and the SR. We also report the risk-adjusted alpha of the volatility-targeting strategy with respect to the market index and the difference in the SR between the volatility-targeting strategy and the market index.*

*** and * denote statistical significance based on Newey–West standard errors (for alpha) or HAC standard errors (for the SR test) at the 5% and 10% levels, respectively.*

driven by the higher leverage during that period due to higher estimates of *L*. Using 5-year and 20-year rolling windows yields similar results.

Exhibit 5 reports the performance of the volatility-targeting strategy of Barroso and Santa-Clara (2015) with various target volatility levels, ranging between 12% and 20% annually. For all target levels, the SR of the strategy without leverage constraint is 0.5527, which is not significantly different from that of the market index. On the other hand, the MDD is very sensitive to the volatility target because the choice of the target directly affects the overall level of leverage (see Equation 5). When a target of

## EXHIBIT 6

**Performance of Portfolio Allocation under Estimation Risk for Market**

| | Full Sample | | August 1936–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| **Panel A: Risk Aversion $A = 3$** | | | | | | | | | | |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0222 | 0.0087 | 0.0400 | 0.0154 | 0.0175 | 0.0032 | 0.0160 | 0.0083 | 0.0092 | 0.0059 |
| Vol | 0.1547 | 0.0534 | 0.2161 | 0.0667 | 0.1735 | 0.0456 | 0.0907 | 0.0572 | 0.0447 | 0.0299 |
| Min | –1.1402 | –0.2714 | –1.1402 | –0.2714 | –0.8301 | –0.1730 | –0.2487 | –0.2487 | –0.1247 | –0.0937 |
| Max | 0.8775 | 0.1876 | 0.8669 | 0.1876 | 0.8775 | 0.1630 | 0.4359 | 0.1752 | 0.2777 | 0.0972 |
| MDD | Broke | 0.6905 | Broke | 0.6905 | Broke | 0.4313 | Broke | 0.2704 | Broke | 0.3630 |
| Alpha | 0.0112** | 0.0032** | 0.0158 | 0.0055** | 0.0120 | 0.0011 | 0.0060 | 0.0004 | 0.0067** | 0.0038* |
| SR | 0.4967 | 0.5622 | 0.6419 | 0.7974 | 0.3496 | 0.2446 | 0.6127 | 0.5034 | 0.7139 | 0.6802 |
| ΔSR | –0.0029 | 0.0627 | –0.0191 | 0.1364 | 0.0825 | –0.0225 | 0.0417 | –0.0676 | 0.2759 | 0.2422 |
| **Panel B: Risk Aversion $A = 5$** | | | | | | | | | | |
| No. of Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0133 | 0.0074 | 0.0240 | 0.0138 | 0.0105 | 0.0026 | 0.0096 | 0.0066 | 0.0055 | 0.0047 |
| Vol | 0.0928 | 0.0448 | 0.1296 | 0.0593 | 0.1041 | 0.0402 | 0.0544 | 0.0421 | 0.0268 | 0.0223 |
| Min | –0.6841 | –0.1860 | –0.6841 | –0.1860 | –0.4980 | –0.1730 | –0.1492 | –0.1492 | –0.0748 | –0.0562 |
| Max | 0.5265 | 0.1876 | 0.5202 | 0.1876 | 0.5265 | 0.1373 | 0.2615 | 0.1250 | 0.1666 | 0.0972 |
| MDD | 0.7676 | 0.5578 | 0.7324 | 0.5578 | 0.7676 | 0.3691 | 0.1897 | 0.1605 | 0.2825 | 0.2581 |
| Alpha | 0.0067** | 0.0031** | 0.0095 | 0.0056** | 0.0072 | 0.0009 | 0.0036 | 0.0012 | 0.0040** | 0.0033** |
| SR | 0.4967 | 0.5724 | 0.6419 | 0.8081 | 0.3496 | 0.2273 | 0.6127 | 0.5443 | 0.7139 | 0.7282 |
| ΔSR | –0.0029 | 0.0728 | –0.0191 | 0.1472 | 0.0825 | –0.0397 | 0.0417 | –0.0267 | 0.2759 | 0.2902 |

*Notes: This exhibit reports summary statistics of the mean–variance portfolio allocation strategy under the estimation risk of Kan and Zhou (2007) applied to the market excess return with risk aversion levels of* A = 3 *(Panel A) and* A = 5 *(Panel B). We report results for the full sample from August 1936 to December 2017 and for four subsample periods. We consider cases with UL and LL separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, MDD, and the SR. We also report the risk-adjusted alpha of the portfolio allocation strategy with respect to the market index and the difference in the SR between the portfolio allocation strategy and the market index.*

*** and * denote statistical significance based on Newey–West standard errors (for alpha) or HAC standard errors (for the SR test) at the 5% and 10% levels.*

12% is used, the MDD of the strategy is 0.5230, which is comparable to that of the market. When we increase the annualized volatility target to 16%, the MDD becomes 0.6519. When we further increase the target to 20%, the MDD becomes 0.7567, about 50% higher than that of the market. Imposing the leverage constraint barely changes the result.

Exhibit 6 reports the performance of the portfolio allocation strategy of Kan and Zhou (2007), accounting for estimation risk, with the expected returns $\hat{\mu}_{t-1}$ estimated as 10-year rolling sample means ($T = 120$). With risk version of $A = 3$ and $A = 5$, the strategy fails to outperform the market. Although the SR of the strategy is higher than that of the market over the last period, the difference is not statistically significant. More importantly, the MDD makes the strategy even less attractive. With risk aversion of $A = 3$, the strategy goes bankrupt, rendering it infeasible in practice. With $A = 5$, the MDD improves to some extent but remains substantially higher than that of the market.

Exhibit 7 reports the performance of the unconditional optimal portfolio of Ferson and Siegel (2001) in the presence of conditional information. With annualized target expected excess returns of 6% and 10%, this strategy again fails to improve the SR, and the MDD is always higher than that of the market index. Our evidence indicates that, although conditional information can theoretically be used to improve unconditional performance, empirically this fails because of estimation errors.

In summary, none of the four volatility-timing strategies delivers superior out-of-sample performance.

## EXHIBIT 7
**Performance of Unconditional Optimal Portfolio with Conditional Information for Market**

| | Full Sample | | August 1936–December 1960 | | January 1961–December 1980 | | January 1981–December 2000 | | January 2001–December 2017 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | UL | LL | UL | LL | UL | LL | UL | LL | UL | LL |
| **Panel A: Annualized Target Expected Return 6%** | | | | | | | | | | |
| #Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0030 | 0.0030 | 0.0055 | 0.0054 | 0.0012 | 0.0012 | 0.0026 | 0.0026 | 0.0019 | 0.0019 |
| Vol | 0.0272 | 0.0254 | 0.0453 | 0.0416 | 0.0143 | 0.0143 | 0.0148 | 0.0148 | 0.0092 | 0.0092 |
| Min | –0.3295 | –0.3295 | –0.3295 | –0.3295 | –0.0670 | –0.0670 | –0.0477 | –0.0477 | –0.0233 | –0.0233 |
| Max | 0.2120 | 0.1415 | 0.2120 | 0.1415 | 0.0556 | 0.0556 | 0.0558 | 0.0558 | 0.0482 | 0.0482 |
| MDD | 0.6679 | 0.6098 | 0.6679 | 0.6098 | 0.1255 | 0.1255 | 0.0443 | 0.0443 | 0.0516 | 0.0516 |
| Alpha | 0.0008 | 0.0008 | –0.0004 | –0.0003 | 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0013** | 0.0013** |
| SR | 0.3808 | 0.4064 | 0.4167 | 0.4504 | 0.2991 | 0.2975 | 0.6173 | 0.6173 | 0.7253 | 0.7253 |
| ΔSR | –0.1188 | –0.0931 | –0.2443 | –0.2106 | 0.0320 | 0.0304 | 0.0463 | 0.0463 | 0.2873 | 0.2873 |
| **Panel B: Annualized Target Expected Return 10%** | | | | | | | | | | |
| #Obs | 977 | 977 | 293 | 293 | 240 | 240 | 240 | 240 | 204 | 204 |
| Mean | 0.0050 | 0.0045 | 0.0091 | 0.0081 | 0.0021 | 0.0017 | 0.0044 | 0.0042 | 0.0032 | 0.0031 |
| Vol | 0.0453 | 0.0353 | 0.0756 | 0.0556 | 0.0238 | 0.0229 | 0.0246 | 0.0240 | 0.0154 | 0.0149 |
| Min | –0.5491 | –0.4390 | –0.5491 | –0.4390 | –0.1116 | –0.1116 | –0.0795 | –0.0795 | –0.0388 | –0.0388 |
| Max | 0.3534 | 0.1483 | 0.3534 | 0.1483 | 0.0926 | 0.0926 | 0.0929 | 0.0803 | 0.0803 | 0.0682 |
| MDD | 0.8745 | 0.7298 | 0.8745 | 0.7298 | 0.2153 | 0.2153 | 0.0805 | 0.0805 | 0.1280 | 0.1280 |
| Alpha | 0.0013 | 0.0013 | –0.0007 | –0.0000 | 0.0012 | 0.0008 | 0.0014 | 0.0012 | 0.0022** | 0.0021** |
| SR | 0.3808 | 0.4428 | 0.4167 | 0.5021 | 0.2991 | 0.2604 | 0.6173 | 0.6014 | 0.7253 | 0.7228 |
| ΔSR | –0.1188 | –0.0568 | –0.2443 | –0.1588 | 0.0320 | –0.0066 | 0.0463 | 0.0304 | 0.2873 | 0.2848 |

*Notes: This exhibit reports summary statistics of the unconditional optimal portfolio with conditional information of Ferson and Siegel (2001) applied to the market excess return with annualized target expected returns of 6% (Panel A) and 10% (Panel B). We report results for the full sample from August 1936 to December 2017 and for four subsample periods. We consider cases with UL and LL separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, MDD, and the SR. We also report the risk-adjusted alpha of the unconditional optimal portfolio with respect to the market index and the difference in the SR between the unconditional optimal portfolio and the market index.*

*** denotes statistical significance based on Newey–West standard errors (for alpha) or HAC standard errors (for the SR test) at the 5% level.*

The SR is never significantly improved except for during the financial crisis period. The full-sample SRs are at most slightly better (in some cases even lower), and the differences are statistically insignificant. The drawdowns are generally much worse than the market, making the strategies infeasible in most cases in the real world.

## CONCLUSION

In this article, we provide a fairly comprehensive examination of volatility-managed portfolio strategies. We find a look-ahead bias in a typical application of such a strategy to the market; after correcting this bias, the strategy's performance worsens substantially. In general, the strategy cannot be easily implemented because of large drawdowns. Even if we ignore the drawdowns and assume that the strategy were implementable, it cannot outperform the market in general. For deeper understanding, we further examine the performance of three alternative volatility-timing strategies and find that they are also unable to beat the market. Overall, although volatility timing is an appealing idea and seems to work well at the stock level, it remains a challenging task to show it can work for the market.

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## REFERENCES


Barroso, P., and A. Detzel. 2018. "Transaction Costs and Volatility-Managed Portfolios." Working paper.

Barroso, P., and P. Santa-Clara. 2015. "Momentum Has Its Moments." *Journal of Financial Economics* 116 (1): 111–120.

Basak, G. K., R. Jagannathan, and T. Ma. 2009. "Jackknife Estimator for Tracking Error Variance of Optimal Portfolios." *Management Science* 55 (6): 990–1002.

Cederburg, S., M. S. O'Doherty, F. Wang, and X. S. Yan. 2019. "On the Performance of Volatility-Managed Portfolios." *Journal of Financial Economics*, forthcoming.

Ferson, W. E., and A. F. Siegel. 2001. "The Efficient Use of Conditioning Information in Portfolios." *The Journal of Finance* 56 (3): 967–982.

Fleming, J., C. Kirby, and B. Ostdiek. 2001. "The Economic Value of Volatility Timing." *The Journal of Finance* 56 (1): 329–352.

——. 2003. "The Economic Value of Volatility Timing Using 'Realized' Volatility." *Journal of Financial Economics* 67 (3): 473–509.

Han, Y., D. Huang, and G. Zhou. 2019. "Anomalies Enhanced: A Portfolio Re-Balancing Approach." Working paper.

Kan, R., and G. Zhou. 2007. "Optimal Portfolio Choice with Parameter Uncertainty." *Journal of Financial and Quantitative Analysis* 42 (3): 621–656.

Ledoit, O., and M. Wolf. 2008. "Robust Performance Hypothesis Testing with the Sharpe Ratio." *Journal of Empirical Finance* 15 (5): 850–859.

Moreira, A., and T. Muir. 2017. "Volatility-Managed Portfolios." *The Journal of Finance* 72 (4): 1611–1644.

Zhou, G. 2008. "On the Fundamental Law of Active Portfolio Management: What Happens If Our Estimates Are Wrong?" *The Journal of Portfolio Management* 34 (4): 26–33.




## ADDITIONAL READING

### On the Fundamental Law of Active Portfolio Management

**What Happens If Our Estimates Are Wrong?**

Guofu Zhou

*The Journal of Portfolio Management*

**https://jpm.pm-research.com/content/34/4/26**

**ABSTRACT:** *The fundamental law of active portfolio management provides profound insights on the value creation process of managed funds and shows how forecasts of alphas or forecasting skills can be transformed into the value-added of an active portfolio. A key weakness of the law and its various extensions is that they ignore the estimation errors associated with the parameter inputs of the law. In this article, the author shows that the estimation errors have a substantial impact on the value-added and can easily destroy all the value promised by the law if not dealt with carefully. To minimize the impact of the estimation errors, the author proposes two methods—scaling and diversification. The scaling method scales the estimated optimal active portfolio by a suitable proportion to maximize the valued-added. The diversification approach suggests holding other portfolios along with the estimated optimal active portfolio in order to diversify away much of the estimation errors. The author shows that the two methods can be effectively used to minimize the impact of the estimation errors so as to substantially improve the valued-added.*

### Linear Trading Rules for Portfolio Management

Richard Grinold

*The Journal of Portfolio Management*

**https://jpm.pm-research.com/content/44/6/109**

**ABSTRACT:** *The author develops a workable procedure to make quantitative portfolio management more dynamic. Under several simplifying assumptions, an optimal portfolio management policy can be described by a linear trading rule in which the manager moves part of the way from the current position toward a target position. The author views the linear trading rule as the entry to the dynamic portfolio management space. A linear trading rule for one asset is governed by a parameter called the trade rate. The trade rate controls the rate of trading and the weight given to the sources of alpha for that asset. With the trade rate, one can calculate expectations of risk, transactions costs, turnover, and the asset's exposure to the sources of alpha. This leads to a choice of each asset's trade rate to maximize that asset's contribution to an overall objective. This, in turn, yields transactions cost–sensitive signal weighting at the asset level. Finally, the knowledge gained provides a dynamic component to more traditional single or multistage portfolio optimization.*

**Donuts:** ***A Picture of Optimization Applied to Fundamental Portfolios***

IAN DOMOWITZ AND AMEYA MOGHE

*The Journal of Portfolio Management*
**https://jpm.pm-research.com/content/44/3/103**

**ABSTRACT:** *The authors ask the question: How does one incorporate the benefits of portfolio optimization without disturbing the core beliefs of the fundamental manager? A solution is characterized graphically, and by example, capturing a variety of portfolio strategies. The simplicity of the framework permits the evaluation of multiple measures of portfolio performance along only two dimensions. Applied to risk-reduction programs, risk-adjusted return is flat to rising across augmented portfolios, and the cost of implementing rebalancing decisions falls sharply, increasing fund capacity. The framework is applied to an evaluation of portfolios relative to new Securities and Exchange Commission liquidity-risk regulations. Risk-adjusted return increases monotonically with the degree of liquidity enhancement. Benefits increase as assets-under-management rise, consistent with greater savings in the form of implementation costs. Even marginal increases in portfolio liquidity produce appreciable improvements in risk-adjusted returns.*