

Seminar Approximation Algorithms

ANSWuSVp(U)M

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Abstract

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1 Introduction

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Function SMatch for the Asymmetric Additive NSW problem

Input: set $\mathcal{A} = \{1, \dots, n\}$ of agents with weights $\eta_i \forall i \in \mathcal{A}$, set $\mathcal{G} = \{1, \dots, m\}$ indivisible items, additive valuations $v_i: \mathcal{P}(\mathcal{G}) \rightarrow \mathbb{R}_{>0}^+$ where $v_i(\mathcal{S})$ is the valuation of agent $i \in \mathcal{A}$ for each item set $\mathcal{S} \subset \mathcal{G}$

Output: $\frac{1}{2n}$ -approximation $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$ of an optimal allocation

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1  $\mathbf{x}_i \leftarrow \emptyset \quad \forall i \in \mathcal{A}$ 
2  $u_i \leftarrow v_i(\mathcal{G}_{i,[2n+1:m]}) \quad \forall i \in \mathcal{A}$ 
3  $\mathcal{W} \leftarrow \{ \eta_i \cdot \log(v_i(j) + \frac{u_i}{n}) \mid i \in \mathcal{A}, j \in \mathcal{G} \}$  ▷graph weights
4  $G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$  ▷bipartite graph
5  $\mathcal{M} \leftarrow \text{max\_weight\_matching}(G)$ 
6  $\mathbf{x}_i \leftarrow \{ j \mid (i, j) \in \mathcal{M} \} \quad \forall i \in \mathcal{A}$ 
7  $\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G} \setminus \{ j \mid (i, j) \in \mathcal{M} \}$ 
8 while  $\mathcal{G}^{\text{rem}} \neq \emptyset$  do
9    $\mathcal{W} \leftarrow \{ \eta_i \cdot \log(v_i(j) + v_i(\mathbf{x}_i)) \mid i \in \mathcal{A}, j \in \mathcal{G}^{\text{rem}} \}$ 
10   $G \leftarrow (\mathcal{A}, \mathcal{G}^{\text{rem}}, \mathcal{W})$ 
11   $\mathcal{M} \leftarrow \text{max\_weight\_matching}(G)$ 
12   $\mathbf{x}_i \leftarrow \mathbf{x}_i \cup \{ j \mid (i, j) \in \mathcal{M} \} \quad \forall i \in \mathcal{A}$ 
13   $\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G}^{\text{rem}} \setminus \{ j \mid (i, j) \in \mathcal{M} \}$ 
14 end while
15 return  $\mathbf{x}$ 

```

Function RepReMatch for the Asymmetric Submodular NSW problem

Input: set $\mathcal{A} = \{1, \dots, n\}$ of agents with weights $\eta_i \forall i \in \mathcal{A}$, set $\mathcal{G} = \{1, \dots, m\}$ indivisible items, additive valuations $v_i: \mathcal{P}(\mathcal{G}) \rightarrow \mathbb{R}_{>0}^+$ where $v_i(\mathcal{S})$ is the valuation of agent $i \in \mathcal{A}$ for each item set $\mathcal{S} \subset \mathcal{G}$

Output: $\frac{1}{2n \log n}$ -approximation $\mathbf{x}^{\text{III}} = (\mathbf{x}_1^{\text{III}}, \dots, \mathbf{x}_n^{\text{III}})$ of an optimal allocation

Phase I:

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1  $\mathbf{x}_i^{\text{I}} \leftarrow \emptyset \quad \forall i \in \mathcal{A}$ 
2  $\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G}$ 
3 for  $t = 0, \dots, \lceil \log n \rceil - 1$  do
4   if  $\mathcal{G}^{\text{rem}} \neq \emptyset$  then
5      $\mathcal{W} \leftarrow \{ \eta_i \cdot \log(v_i(j)) \mid i \in \mathcal{A}, j \in \mathcal{G} \}$ 
6      $G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$ 
7      $\mathcal{M} \leftarrow \text{max\_weight\_matching}(G)$ 
8      $\mathbf{x}_i^{\text{I}} \leftarrow \mathbf{x}_i^{\text{I}} \cup \{j\} \quad \forall (i, j) \in \mathcal{M}$ 
9      $\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G}^{\text{rem}} \setminus \{j \mid (i, j) \in \mathcal{M}\}$ 
10  end if
11 end for
```

Phase II:

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12  $\mathbf{x}_i^{\text{II}} \leftarrow \emptyset \quad \forall i \in \mathcal{A}$ 
13 while  $\mathcal{G}^{\text{rem}} \neq \emptyset$  do
14    $\mathcal{W} \leftarrow \{ \eta_i \cdot \log(v_i(\mathbf{x}_i^{\text{II}} \cup \{j\})) \mid i \in \mathcal{A}, j \in \mathcal{G} \}$ 
15    $G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$ 
16    $\mathcal{M} \leftarrow \text{max\_weight\_matching}(G)$ 
17    $\mathbf{x}_i^{\text{II}} \leftarrow \mathbf{x}_i^{\text{II}} \cup \{j\} \quad \forall (i, j) \in \mathcal{M}$ 
18    $\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G}^{\text{rem}} \setminus \{j \mid (i, j) \in \mathcal{M}\}$ 
19 end while
```

Phase III:

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20  $\mathcal{G}^{\text{rem}} \leftarrow \bigcup_{i \in \mathcal{A}} \mathbf{x}_i^{\text{I}}$  ▷ release items allocated in first phase
21  $\mathcal{W} \leftarrow \{ \eta_i \cdot \log(v_i(\mathbf{x}_i^{\text{II}} \cup \{j\})) \mid i \in \mathcal{A}, j \in \mathcal{G} \}$ 
22  $G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$ 
23  $\mathcal{M} \leftarrow \text{max\_weight\_matching}(G)$ 
24  $\mathbf{x}_i^{\text{III}} \leftarrow \mathbf{x}_i^{\text{II}} \cup \{j\} \quad \forall (i, j) \in \mathcal{M}$ 
25  $\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G}^{\text{rem}} \setminus \{j \mid (i, j) \in \mathcal{M}\}$ 
26  $\mathbf{x}^{\text{III}} \leftarrow \text{arbitrary\_allocation}(\mathcal{A}, \mathcal{G}^{\text{rem}}, \mathbf{x}^{\text{III}}, (v_i)_{i \in \mathcal{A}})$ 
27 return  $\mathbf{x}^{\text{III}}$ 
```
