Algorithms and Complexity Group
Institute for Computer Science
Goethe University Frankfurt

Summer Semester 2023

**Seminar Approximation Algorithms**

# ANSWuSVþ(U)M

Zeno Adrian Weil

16th May 2023

Supervisor: Dr Giovanna Varricchio

## Abstract

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

# 1 Introduction

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

**Function** SMatch for the Asymmetric Additive NSW problem

**Input:** set $\mathcal{A} = \{1, \dots, n\}$ of agents with weights $\eta_i \forall i \in \mathcal{A}$, set $\mathcal{G} = \{1, \dots, m\}$ indivisible items, additive valuations $v_i : \mathcal{P}(\mathcal{G}) \to \mathbb{R}^+_{>0}$ where $v_i(\mathcal{S})$ is the valuation of agent $i \in \mathcal{A}$ for each item set $\mathcal{S} \subset \mathcal{G}$

**Output:** $\frac{1}{2n}$-approximation $\boldsymbol{x} = (\boldsymbol{x}_1, \dots, \boldsymbol{x}_n)$ of an optimal allocation

$\boldsymbol{x}_i \leftarrow \emptyset \quad \forall i \in \mathcal{A}$

$u_i \leftarrow v_i\big(\mathcal{G}_{i,[2n+1:m]}\big) \quad \forall i \in \mathcal{A}$

$\mathcal{W} \leftarrow \{\, \eta_i \cdot \log\big(v_i(j) + \frac{u_i}{n}\big) \,\big|\, i \in \mathcal{A}, j \in \mathcal{G} \,\}$ ▷ *graph weights*

$G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$ ▷ *bipartite graph*

$\mathcal{M} \leftarrow$ `max_weight_matching`$(G)$

$\boldsymbol{x}_i \leftarrow \{\, j \mid (i, j) \in \mathcal{M} \,\} \quad \forall i \in \mathcal{A}$

$\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G} \setminus \{\, j \mid (i, j) \in \mathcal{M} \,\}$

**while** $\mathcal{G}^{\text{rem}} \neq \emptyset$ **do**

    $\mathcal{W} \leftarrow \{\, \eta_i \cdot \log\big(v_i(j) + v_i(\boldsymbol{x}_i)\big) \,\big|\, i \in \mathcal{A}, j \in \mathcal{G}^{\text{rem}} \,\}$

    $G \leftarrow (\mathcal{A}, \mathcal{G}^{\text{rem}}, \mathcal{W})$

    $\mathcal{M} \leftarrow$ `max_weight_matching`$(G)$

    $\boldsymbol{x}_i \leftarrow \boldsymbol{x}_i \cup \{\, j \mid (i, j) \in \mathcal{M} \,\} \quad \forall i \in \mathcal{A}$

    $\mathcal{G}^{\text{rem}} \leftarrow \mathcal{G}^{\text{rem}} \setminus \{\, j \mid (i, j) \in \mathcal{M} \,\}$

**end while**

**return** $\boldsymbol{x}$

**Function** RepReMatch for the Asymmetric Submodular NSW problem

**Input:** set $\mathcal{A} = \{1, \dots, n\}$ of agents with weights $\eta_i \forall i \in \mathcal{A}$, set $\mathcal{G} = \{1, \dots, m\}$ indivisible items, additive valuations $v_i : \mathcal{P}(\mathcal{G}) \to \mathbb{R}^+_{>0}$ where $v_i(\mathcal{S})$ is the valuation of agent $i \in \mathcal{A}$ for each item set $\mathcal{S} \subset \mathcal{G}$

**Output:** $\frac{1}{2n \log n}$-approximation $\boldsymbol{x}^{\mathrm{III}} = (\boldsymbol{x}_1^{\mathrm{III}}, \dots, \boldsymbol{x}_n^{\mathrm{III}})$ of an optimal allocation

*Phase I:*

$\boldsymbol{x}_i^{\mathrm{I}} \leftarrow \emptyset \quad \forall i \in \mathcal{A}$
$\mathcal{G}^{\mathrm{rem}} \leftarrow \mathcal{G}$
**for** $t = 0, \dots, \lceil \log n \rceil - 1$ **do**
  **if** $\mathcal{G}^{\mathrm{rem}} \neq \emptyset$ **then**
    $\mathcal{W} \leftarrow \{\, \eta_i \cdot \log(v_i(j)) \mid i \in \mathcal{A}, j \in \mathcal{G} \,\}$
    $G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$
    $\mathcal{M} \leftarrow$ `max_weight_matching`$(G)$
    $\boldsymbol{x}_i^{\mathrm{I}} \leftarrow \boldsymbol{x}_i^{\mathrm{I}} \cup \{j\} \quad \forall (i,j) \in \mathcal{M}$
    $\mathcal{G}^{\mathrm{rem}} \leftarrow \mathcal{G}^{\mathrm{rem}} \setminus \{\, j \mid (i,j) \in \mathcal{M} \,\}$
  **end if**
**end for**

*Phase II:*

$\boldsymbol{x}_i^{\mathrm{II}} \leftarrow \emptyset \quad \forall i \in \mathcal{A}$
**while** $\mathcal{G}^{\mathrm{rem}} \neq \emptyset$ **do**
  $\mathcal{W} \leftarrow \{\, \eta_i \cdot \log(v_i(\boldsymbol{x}_i^{\mathrm{II}} \cup \{j\})) \mid i \in \mathcal{A}, j \in \mathcal{G} \,\}$
  $G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$
  $\mathcal{M} \leftarrow$ `max_weight_matching`$(G)$
  $\boldsymbol{x}_i^{\mathrm{II}} \leftarrow \boldsymbol{x}_i^{\mathrm{II}} \cup \{j\} \quad \forall (i,j) \in \mathcal{M}$
  $\mathcal{G}^{\mathrm{rem}} \leftarrow \mathcal{G}^{\mathrm{rem}} \setminus \{\, j \mid (i,j) \in \mathcal{M} \,\}$
**end while**

*Phase III:*

$\mathcal{G}^{\mathrm{rem}} \leftarrow \bigcup_{i \in \mathcal{A}} \boldsymbol{x}_i^{\mathrm{I}}$ ▷ *release items allocated in first phase*
$\mathcal{W} \leftarrow \{\, \eta_i \cdot \log(v_i(\boldsymbol{x}_i^{\mathrm{II}} \cup \{j\})) \mid i \in \mathcal{A}, j \in \mathcal{G} \,\}$
$G \leftarrow (\mathcal{A}, \mathcal{G}, \mathcal{W})$
$\mathcal{M} \leftarrow$ `max_weight_matching`$(G)$
$\boldsymbol{x}_i^{\mathrm{III}} \leftarrow \boldsymbol{x}_i^{\mathrm{II}} \cup \{j\} \quad \forall (i,j) \in \mathcal{M}$
$\mathcal{G}^{\mathrm{rem}} \leftarrow \mathcal{G}^{\mathrm{rem}} \setminus \{\, j \mid (i,j) \in \mathcal{M} \,\}$
$\boldsymbol{x}^{\mathrm{III}} \leftarrow$ `arbitrary_allocation`$(\mathcal{A}, \mathcal{G}^{\mathrm{rem}}, \boldsymbol{x}^{\mathrm{III}}, (v_i)_{i \in \mathcal{A}})$
**return** $\boldsymbol{x}^{\mathrm{III}}$