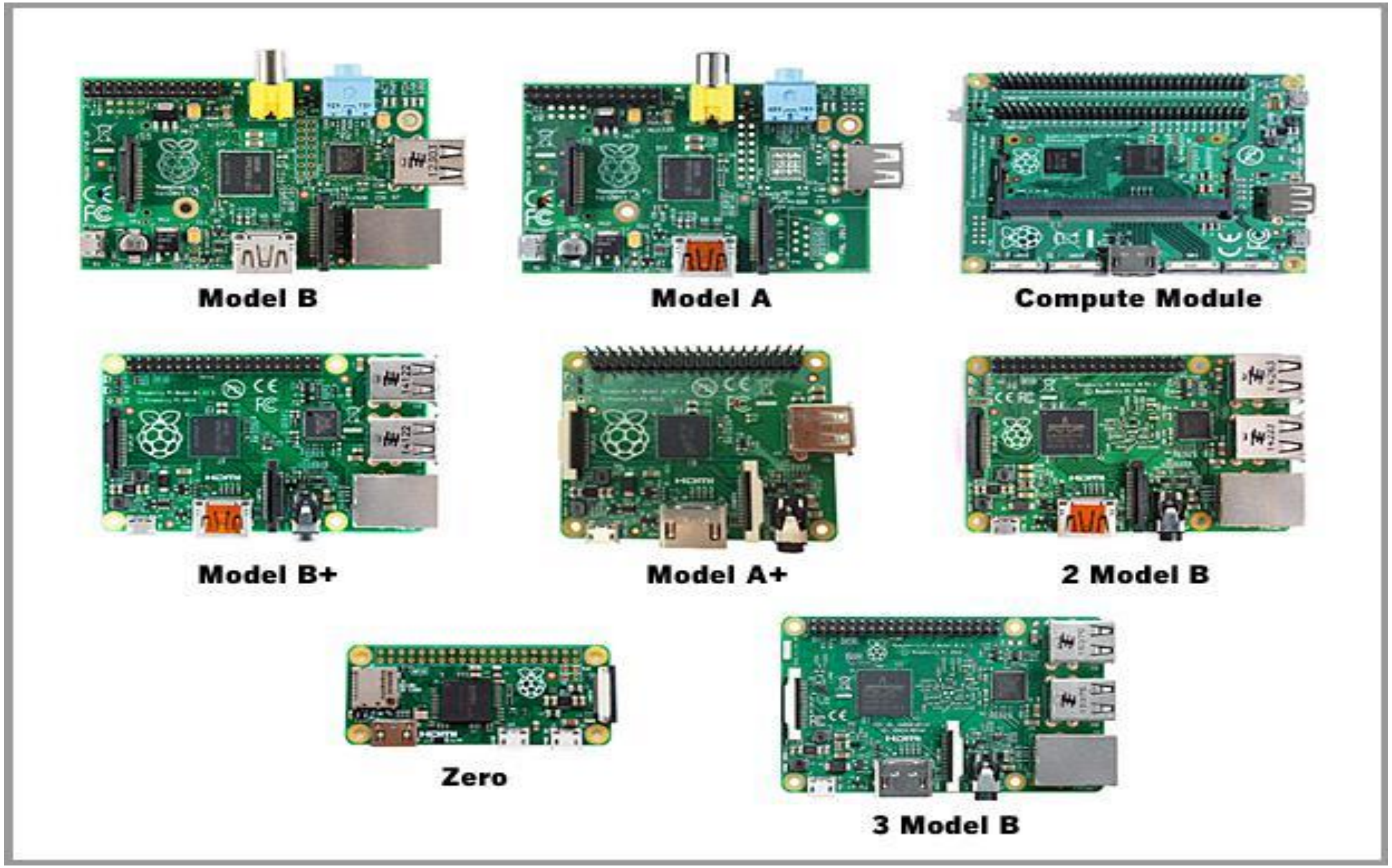
2019 OAI Bootcamp

100 OAI Raspberry Pi and IoT Curriculum



Rpi Board Features Summary

Raspberry Pi Boards Feature Summary

<table>
<tr><th></th><th>Model B</th><th>Model A</th><th>Compute Module</th><th>Model B+</th><th>Model A+</th><th>2 Model B</th><th>Zero</th><th>3 Model B</th></tr>
<tr><td>Release</td><td>Apr. 2012</td><td>Feb. 2013</td><td>Apr. 2014</td><td>Jul. 2014</td><td>Nov. 2014</td><td>Feb. 2015</td><td>Nov. 2015</td><td>Feb. 2016</td></tr>
<tr><td>RAM</td><td>512MB</td><td>256MB</td><td colspan="3">512MB</td><td>1GB</td><td>512MB</td><td>1GB</td></tr>
<tr><td>System on a Chip</td><td colspan="5">Broadcom BCM2835</td><td>Broadcom BCM2836</td><td>Broadcom BCM2835</td><td>Broadcom BCM2837</td></tr>
<tr><td>Processor</td><td colspan="5">ARMv6 (32-bit)</td><td>ARMv7 (32-bit)</td><td>ARMv6 (32-bit)</td><td>ARMv8 (64-bit)</td></tr>
<tr><td>Cores</td><td colspan="5">1</td><td>4</td><td>1</td><td>4</td></tr>
<tr><td>Clock</td><td>700Mhz</td><td>700Mhz</td><td>700Mhz</td><td>700Mhz</td><td>700Mhz</td><td>900MHz</td><td>1GHz</td><td>1.2GHz</td></tr>
<tr><td>USB 2.0</td><td>2</td><td colspan="2">1</td><td>4</td><td>1</td><td>4</td><td>1</td><td>4</td></tr>
<tr><td>Ethernet</td><td>10/100Mbps</td><td>None</td><td colspan="2">10/100Mbps</td><td>None</td><td>10/100Mbps</td><td>None</td><td>10/100Mbps</td></tr>
<tr><td>Wi-Fi 802.11n</td><td colspan="7">No</td><td>Yes</td></tr>
<tr><td>Bluetooth 4.1</td><td colspan="7">No</td><td>Yes</td></tr>
<tr><td>On board storage</td><td colspan="2">SD, MMC, SDIO card slot</td><td>4 GB eMMC flash</td><td colspan="5">MicroSDHC slot</td></tr>
<tr><td>GPIO</td><td colspan="2">8</td><td>46</td><td colspan="3">17</td><td>40</td><td>17</td></tr>
<tr><td>HAT ID bus support</td><td colspan="3">No</td><td colspan="3">Yes</td><td>No</td><td>yes</td></tr>
<tr><td>GPU</td><td colspan="8">Broadcom VideoCore IV, OpenGL ES 2.0, MPEG-2 and VC-1 , H.264/MPEG-4 AVC</td></tr>
<tr><td>1080p video</td><td colspan="7">30fps</td><td>60fps</td></tr>
</table>

Raspberry Pi Pricing

Generation	Release	Model	Price
1	Apr. 2012	Model B	$35
1	Feb. 2013	Model A	$25
n/a	Apr. 2014	Compute Module	$30*
1	Jul. 2014	Model B+	$25
1	Nov. 2014	Model A+	$20
2	Feb. 2015	2 Model B	$35
n/a	Nov. 2015	Zero	$5
3	Feb. 2016	3 Model B	$35

* 100 off price

Global Challenges

Level 1

Climate Change

Water Scarcity

Energy Security

Cyber Security

Global financial structure

Biodiversity and Ecosystem losses

Fisheries Depletion

Deforestation

Infectious Disease

Eliminate our way of life

Level 2

Poverty

Education

The Digital Divide

Urbanization

Intellectual property

International labor and migration

E-Commerce rules

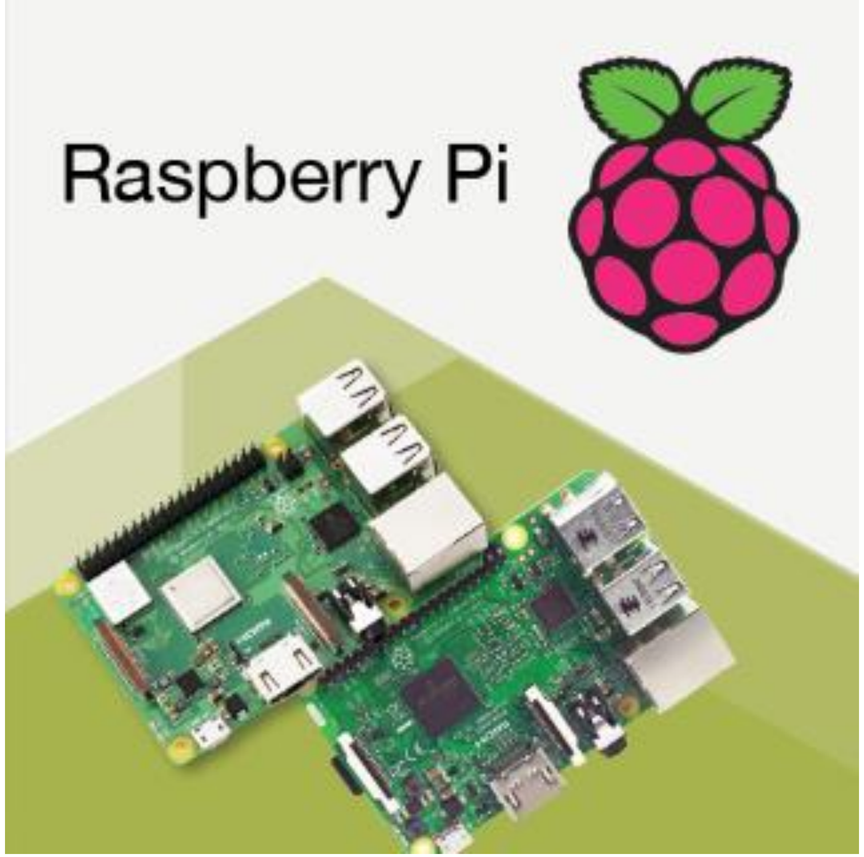
Biotechnology rules

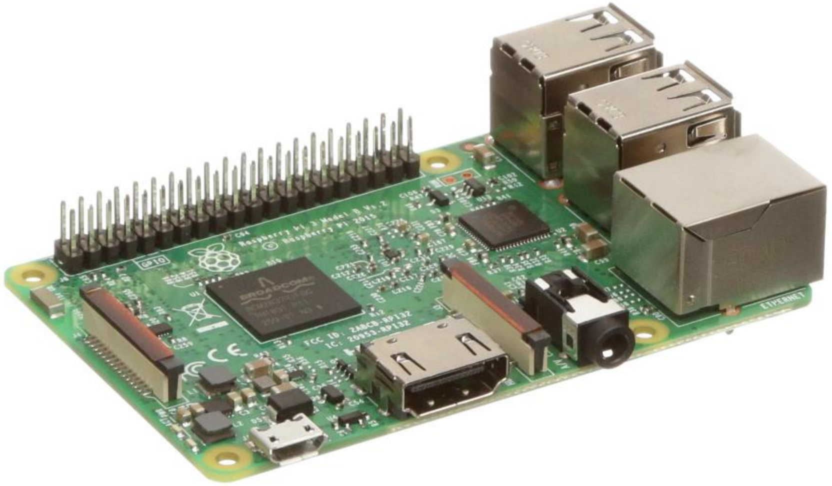
Maritime Safety and Pollution

Disruptive to our way of life

Questions

- Introductions
- What do you know about programming and Raspberry Pi?





Global Competition: SINGAPORE

http://www.country-data.com/cgi-bin/query/r-11828.html

Global Competition: SINGAPORE Today

http://inhabitat.com/marina-bay-sail-by-nbbj-architects/

Accelerated Change!

Years to 50 Million users

Radio – 38 years

Television – 13 years

Cell phone – 7 years

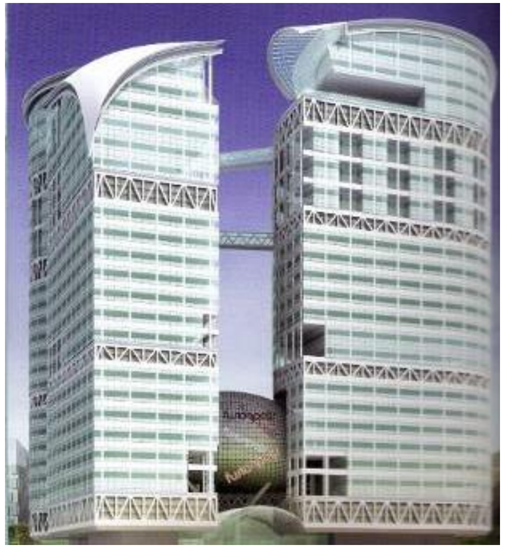
Internet – 4 years

IPOD – 3 years

Facebook – 2 years

More than half of the top 10 in demand jobs in 2014 did not exist in 2004

Shanghai - Pudong



Singapore Science Park

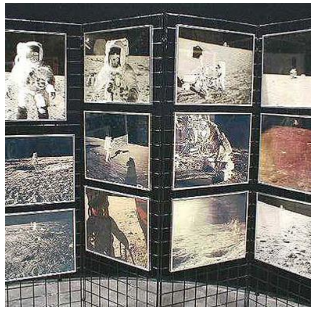
A world of constant disruption…

It took two centuries to fill the US Library of Congress with more than:

29 million
Books and periodicals

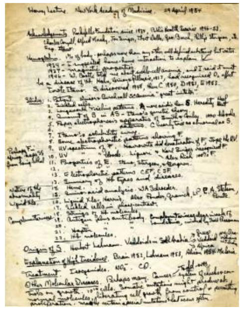
2.4 million
Recordings

12 million
Photographs



4.8 million
Maps

57 million
Manuscripts



Today, it takes about 5 minutes for the world to churn the equivalent amount of new digital information

Grand Opportunities in STEM

In the next 5 years you will no longer need

IDs

Money

Credit Cards

Store cards

Business Cards

Photos

Mail/Mailman

Paper and Hardback Books

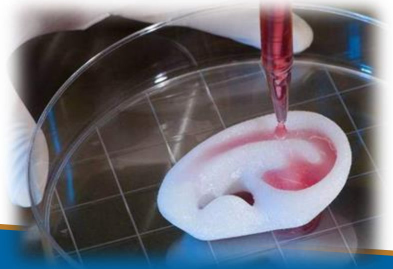
Bills and notices

Paper

Steering Wheels

Organ Donors?

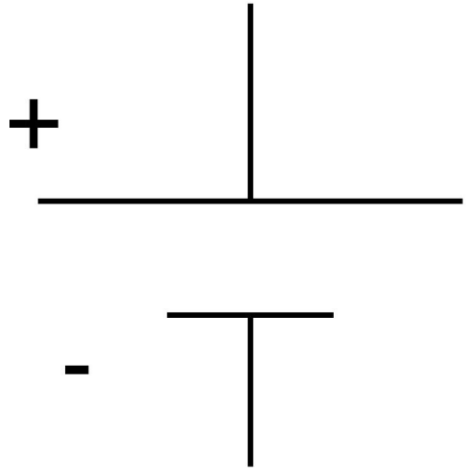
Classrooms?



Introduction to Simple Circuits

Reading Schematics:

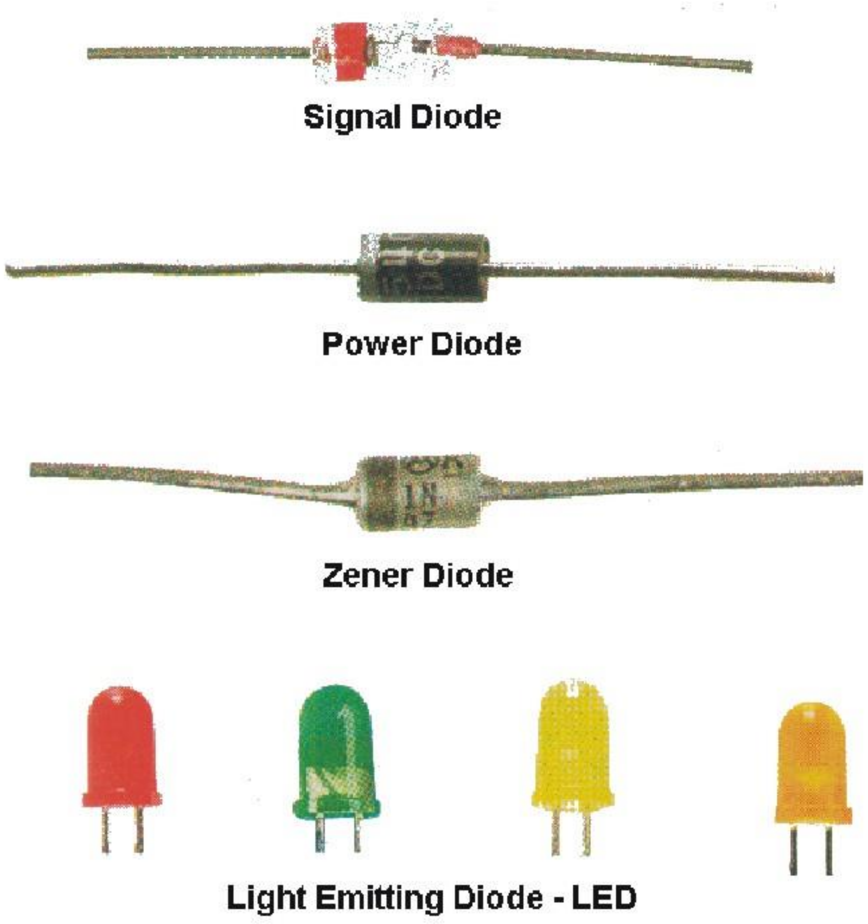
What's this?



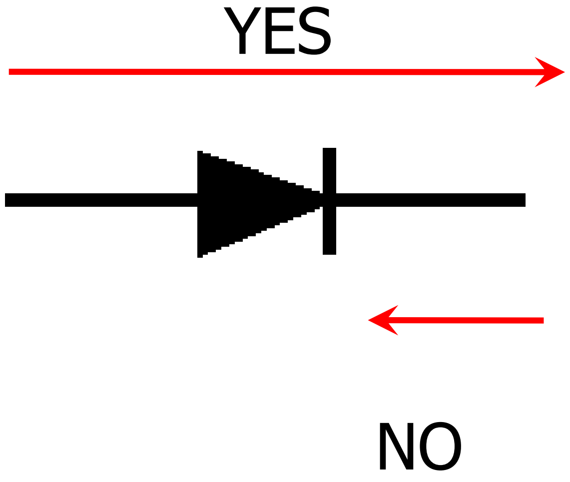
- Battery

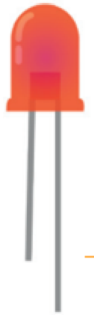
How do electrons start moving in the first place? Well, the force that drives electrons to move, creating current, is called voltage (measured in volts, V). Usually the black terminal is associated with the negative

What is this?



- Diode
  - Lets current flow one direction but not the other

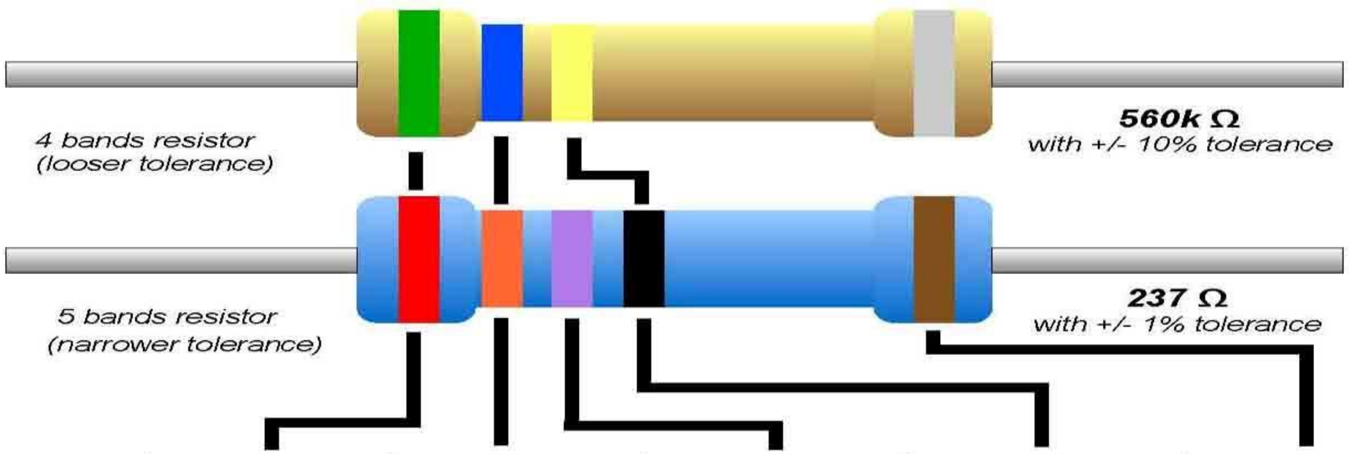




Anode - current flows in (long leg)
Cathode – current flows out

LED

Resistor Color Code



Color	1st Band	2nd Band	3rd Band	Multiplier	Tolerance
Black	0	0	0	x 1 Ω	
Brown	1	1	1	x 10 Ω	+/- 1%
Red	2	2	2	x 100 Ω	+/- 2%
Orange	3	3	3	x 1K Ω	
Yellow	4	4	4	x 10K Ω	
Green	5	5	5	x 100K Ω	+/- .5%
Blue	6	6	6	x 1M Ω	+/- .25%
Violet	7	7	7	x 10M Ω	+/- .1%
Grey	8	8	8		+/- .05%
White	9	9	9		
Gold				x .1 Ω	+/- 5%
Silver				x .01 Ω	+/- 10%

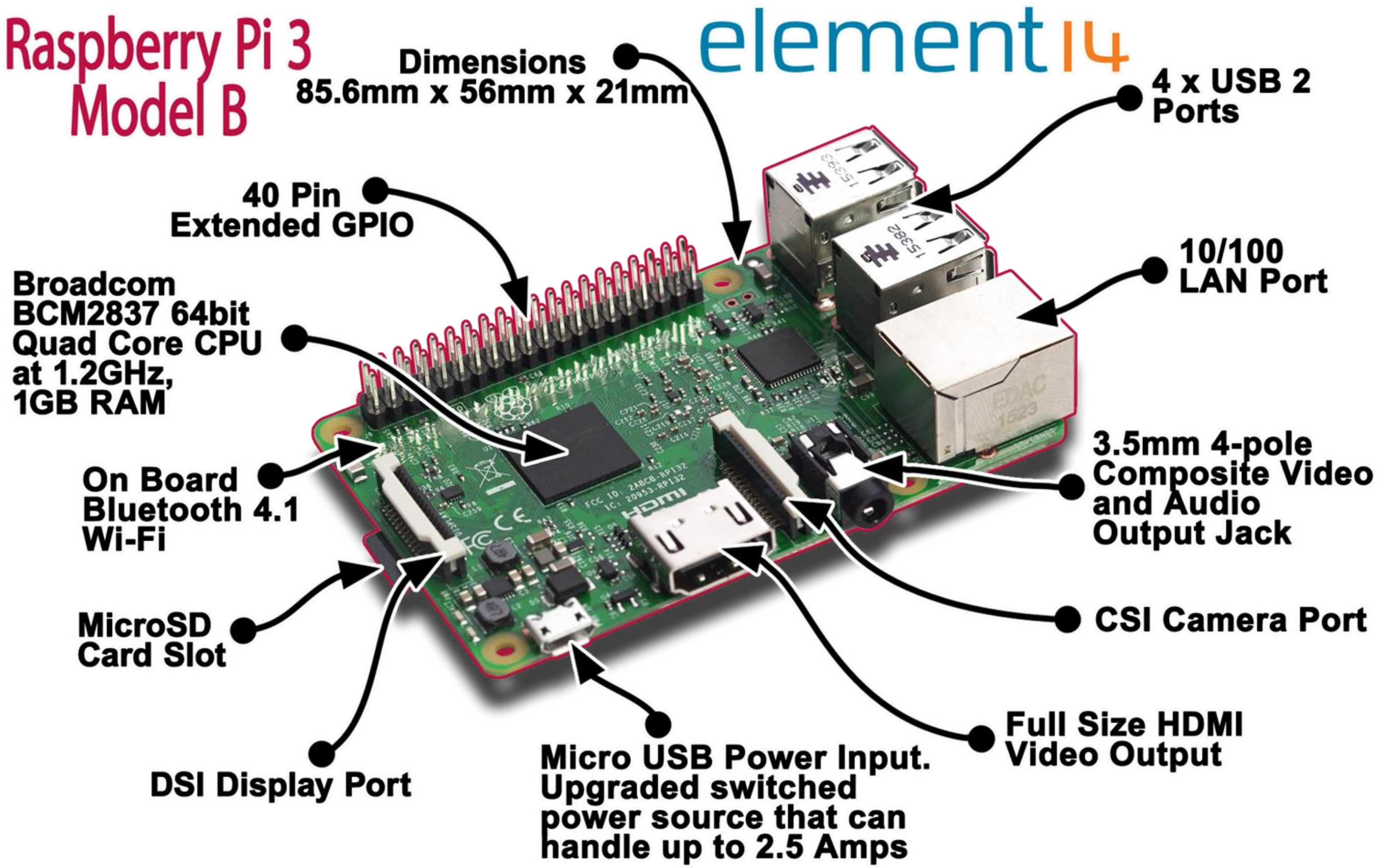
1st band = Orange 3
2nd band = Blue 6 = 360 Ohm Resistor
Multiplier band = Brown x10 Ohms

Raspberry Pi 3 GPIO Header

Pin#	NAME	NAME	Pin#
01	3.3v DC Power	DC Power 5v	02
03	GPIO02 (SDA1 , I²C)	DC Power 5v	04
05	GPIO03 (SCL1 , I²C)	Ground	06
07	GPIO04 (GPIO_GCLK)	(TXD0) GPIO14	08
09	Ground	(RXD0) GPIO15	10
11	GPIO17 (GPIO_GEN0)	(GPIO_GEN1) GPIO18	12
13	GPIO27 (GPIO_GEN2)	Ground	14
15	GPIO22 (GPIO_GEN3)	(GPIO_GEN4) GPIO23	16
17	3.3v DC Power	(GPIO_GEN5) GPIO24	18
19	GPIO10 (SPI_MOSI)	Ground	20
21	GPIO09 (SPI_MISO)	(GPIO_GEN6) GPIO25	22
23	GPIO11 (SPI_CLK)	(SPI_CE0_N) GPIO08	24
25	Ground	(SPI_CE1_N) GPIO07	26
27	ID_SD (I²C ID EEPROM)	(I²C ID EEPROM) ID_SC	28
29	GPIO05	Ground	30
31	GPIO06	GPIO12	32
33	GPIO13	Ground	34
35	GPIO19	GPIO16	36
37	GPIO26	GPIO20	38
39	Ground	GPIO21	40

Rev. 2
29/02/2016

www.element14.com/RaspberryPi



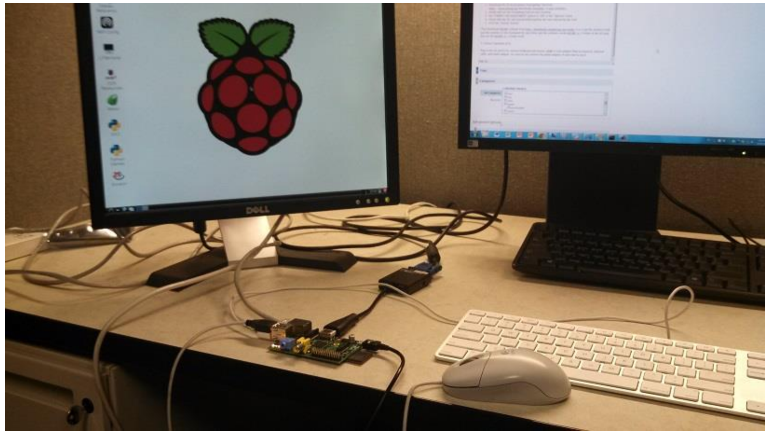
- Connect your Raspberry Pi to your monitor using the HDMI cable. Turn Monitor on.
- Connect your keyboard and mouse
- Connect your power supply

What is Raspberry Pi?

Raspberry Pi is a computer built on a single circuit board with microprocessors, memory, and input/output.

Credit-card sized computer that is used to connect sensors and actuators

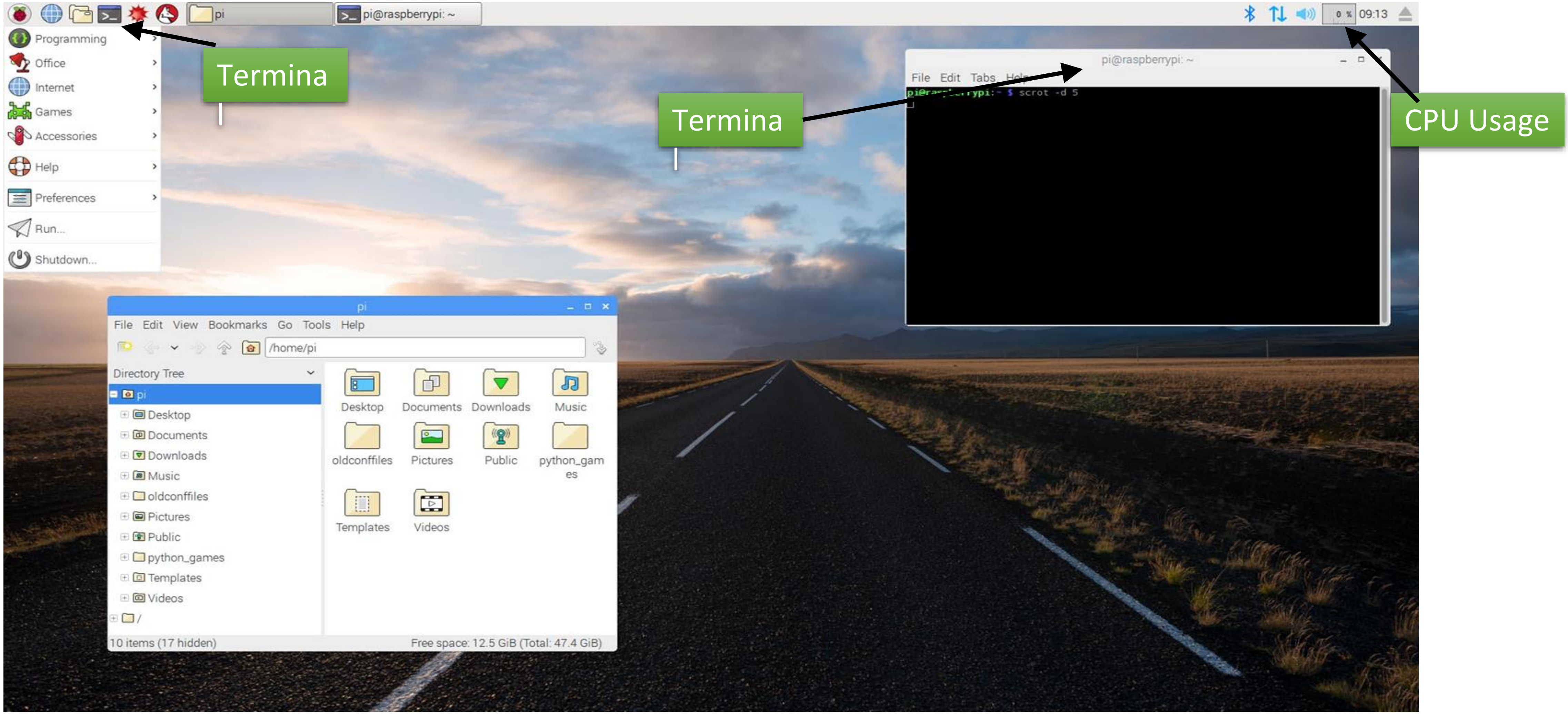
It is used as embedded computer controllers.



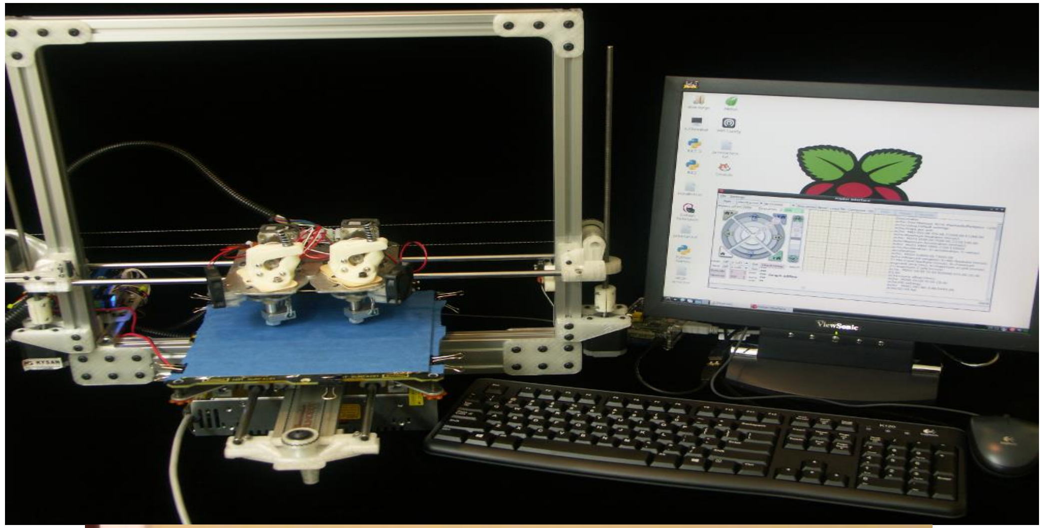
Linux

- Open-source Operating system
- For instance, Windows and MacOS
- Raspberry Pi uses Raspbian Jessie, a Linux-based Operating System

Raspbian Operating System



Some Raspberry-PI Projects



Click on the images to see the videos on YouTube.

Let’s Get Physical

Scratch & Python

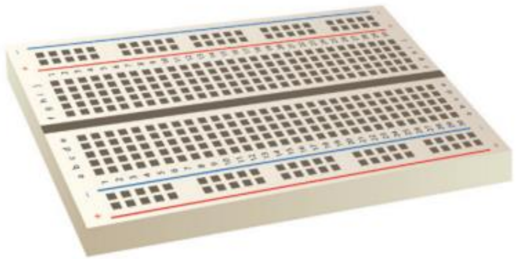
Goals

- Why physical computing?
- Build a simple circuit using Raspberry Pi
- Control an output using Scratch
- Create more complex circuits & programs with Python

Workshop outline

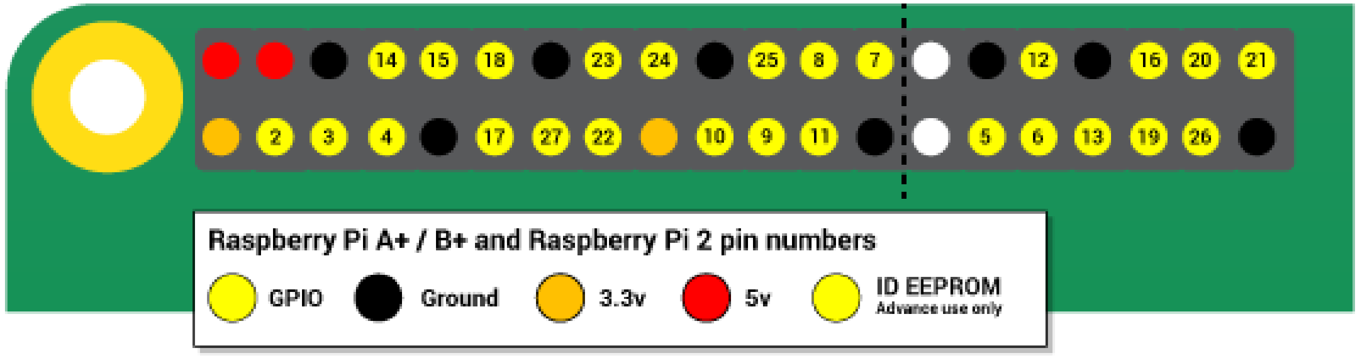
You will need:

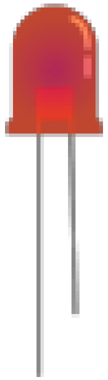
x 2

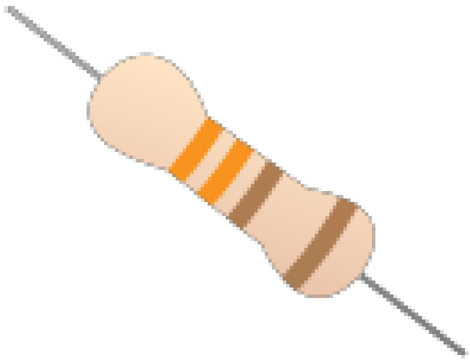


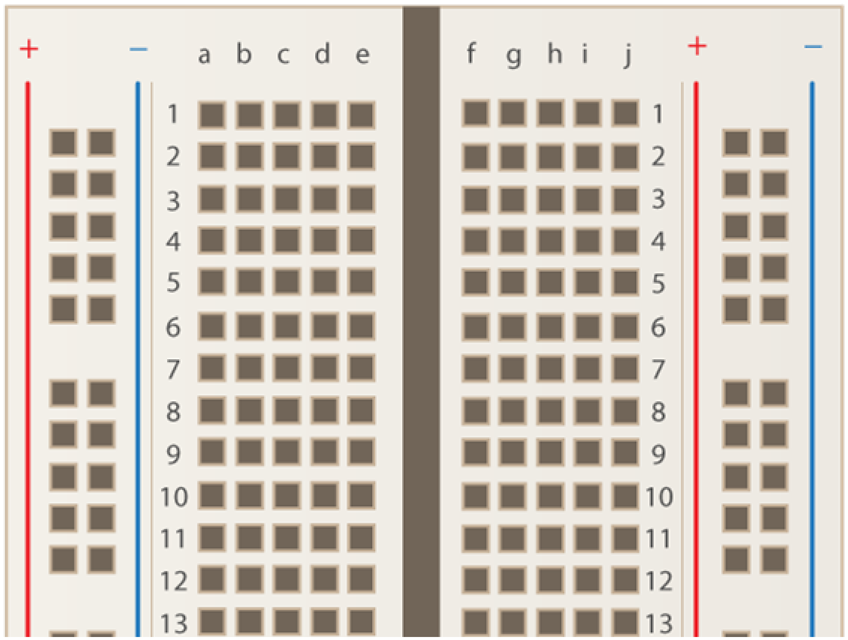
Physical Computing

Enables creators to connect the virtual world with the physical, great for engagement.

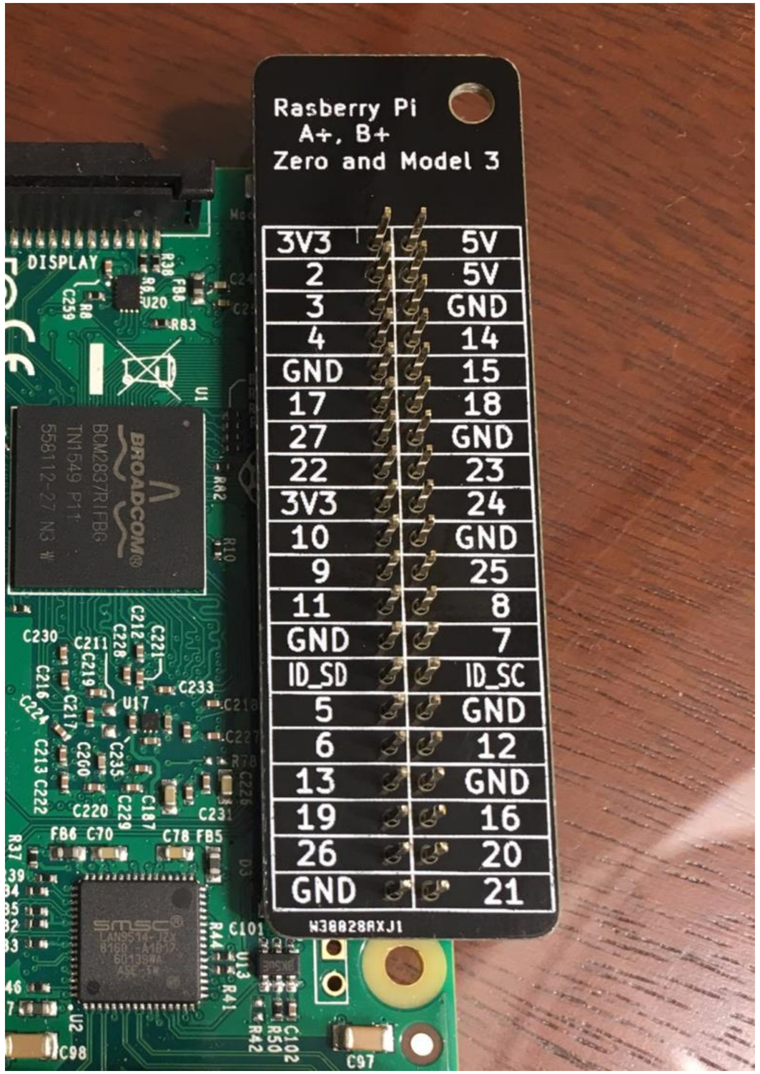


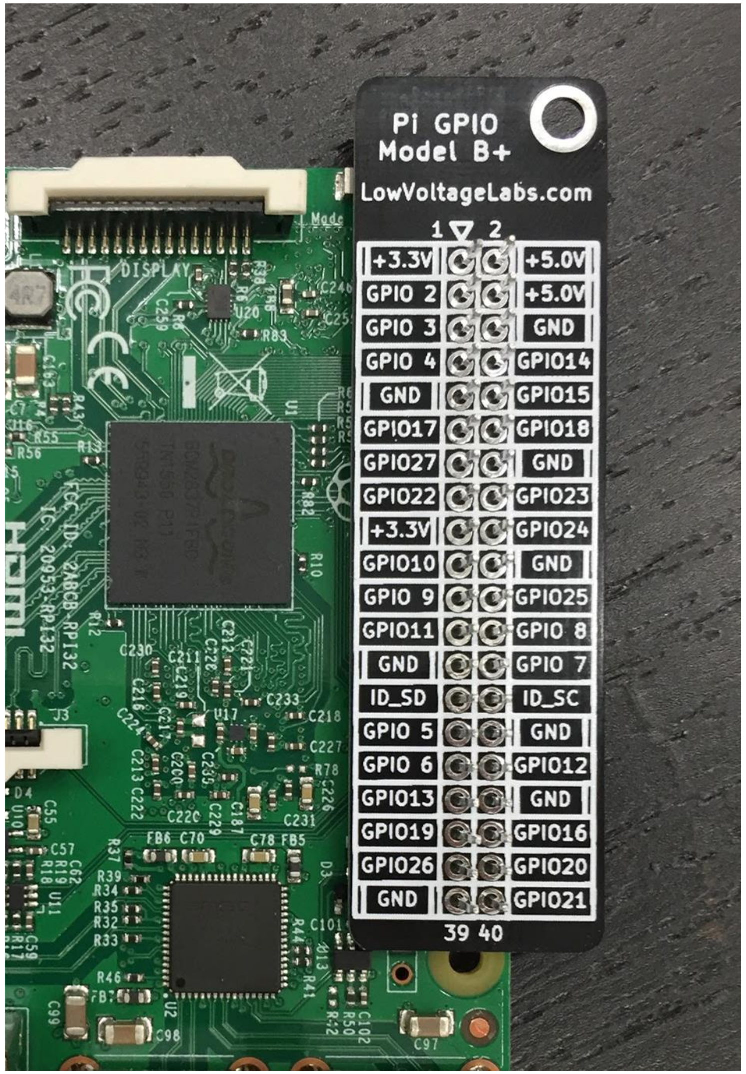






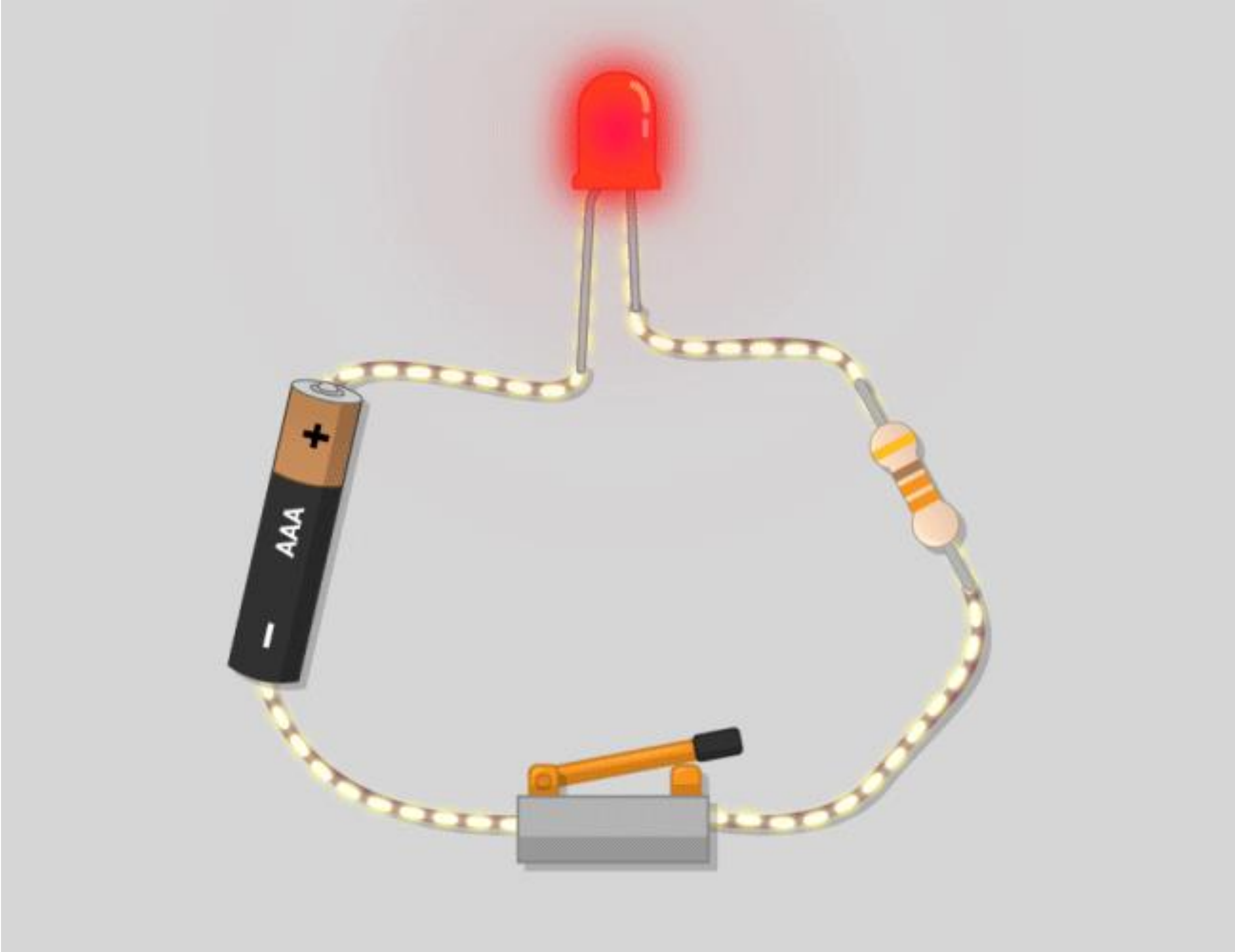
GPIO reference board



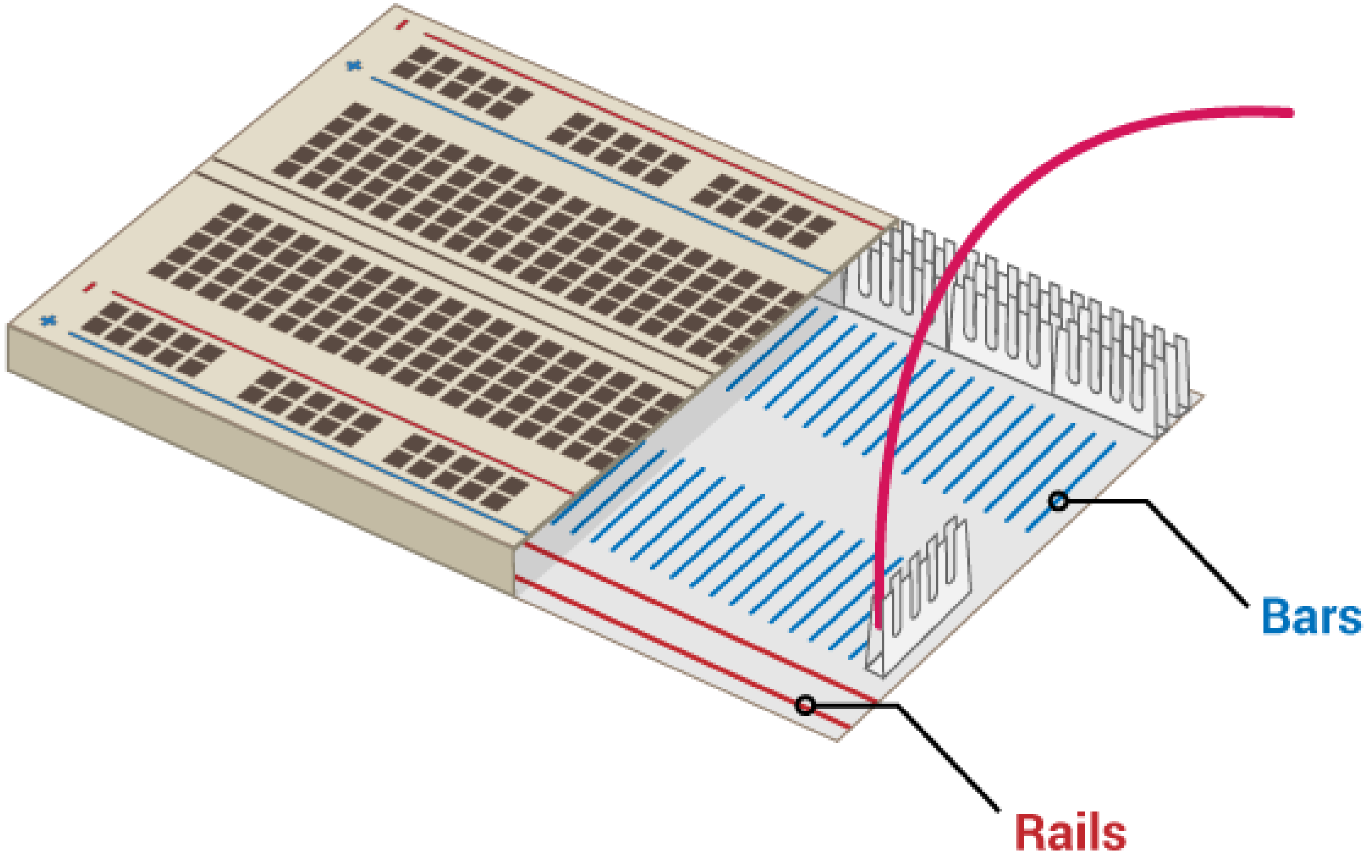


Simple Circuit

Your Raspberry Pi can act as the power supply for simple circuits, we can use this test our hardware is working.



Inside a breadboard



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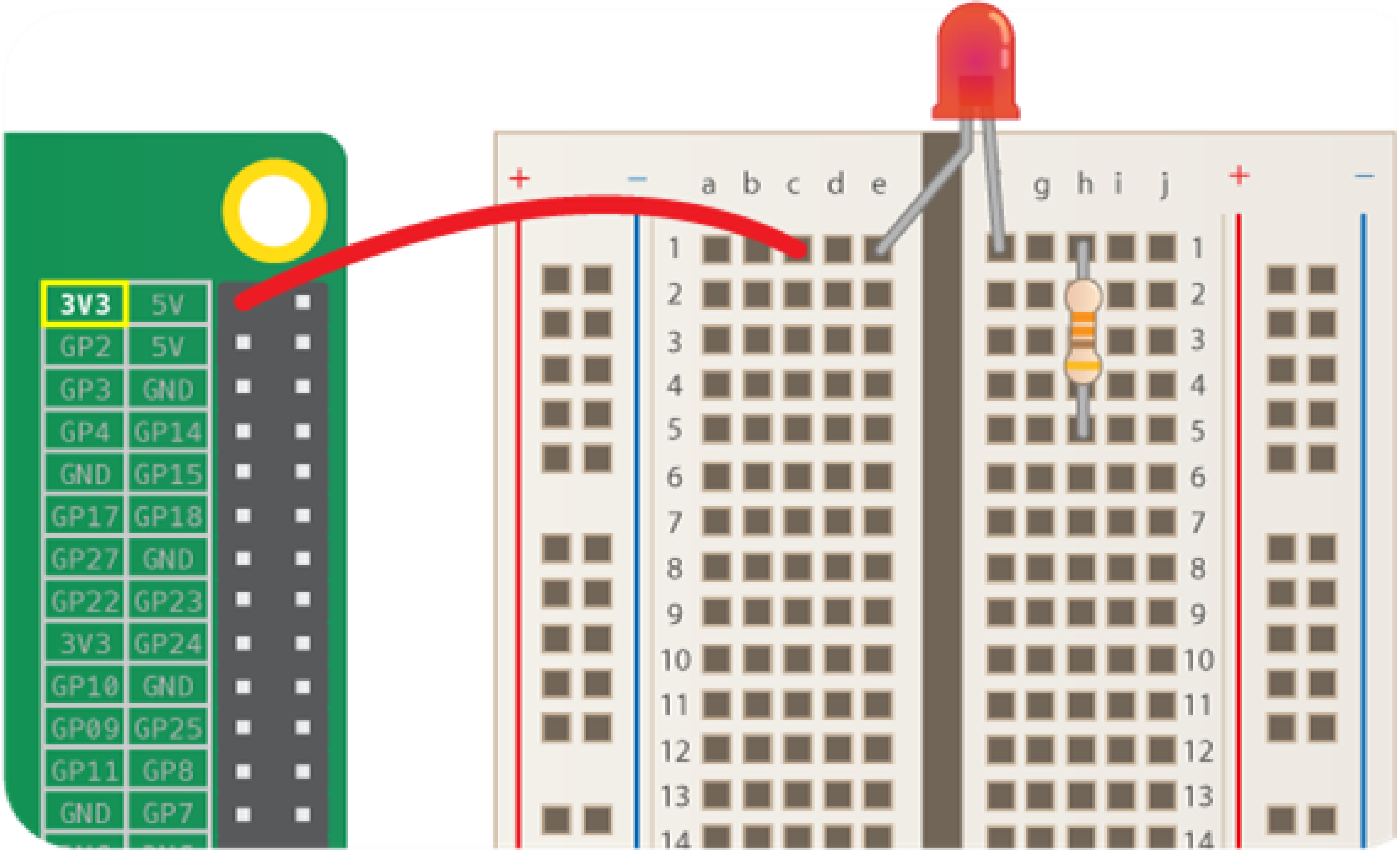

Simple Circuit

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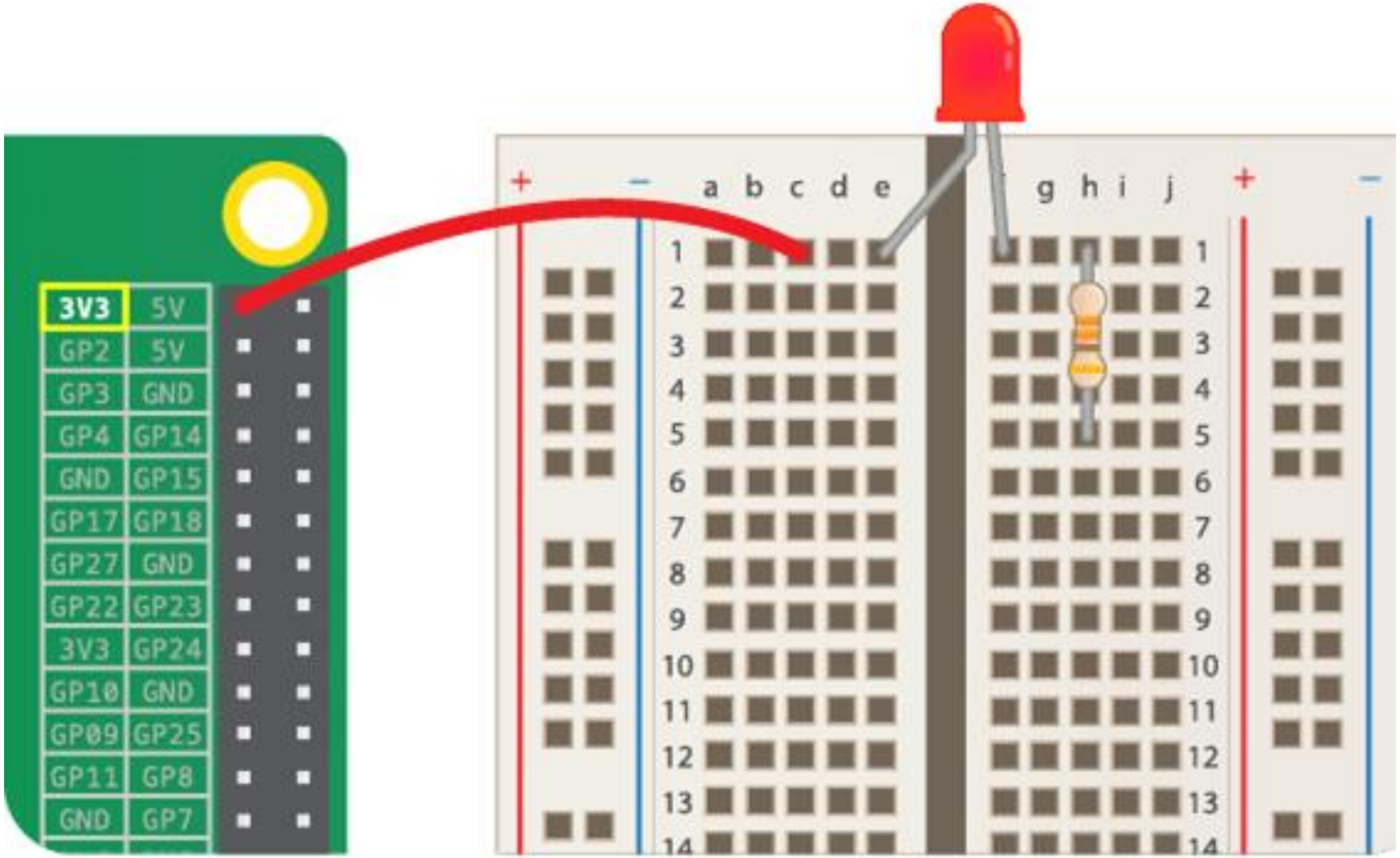
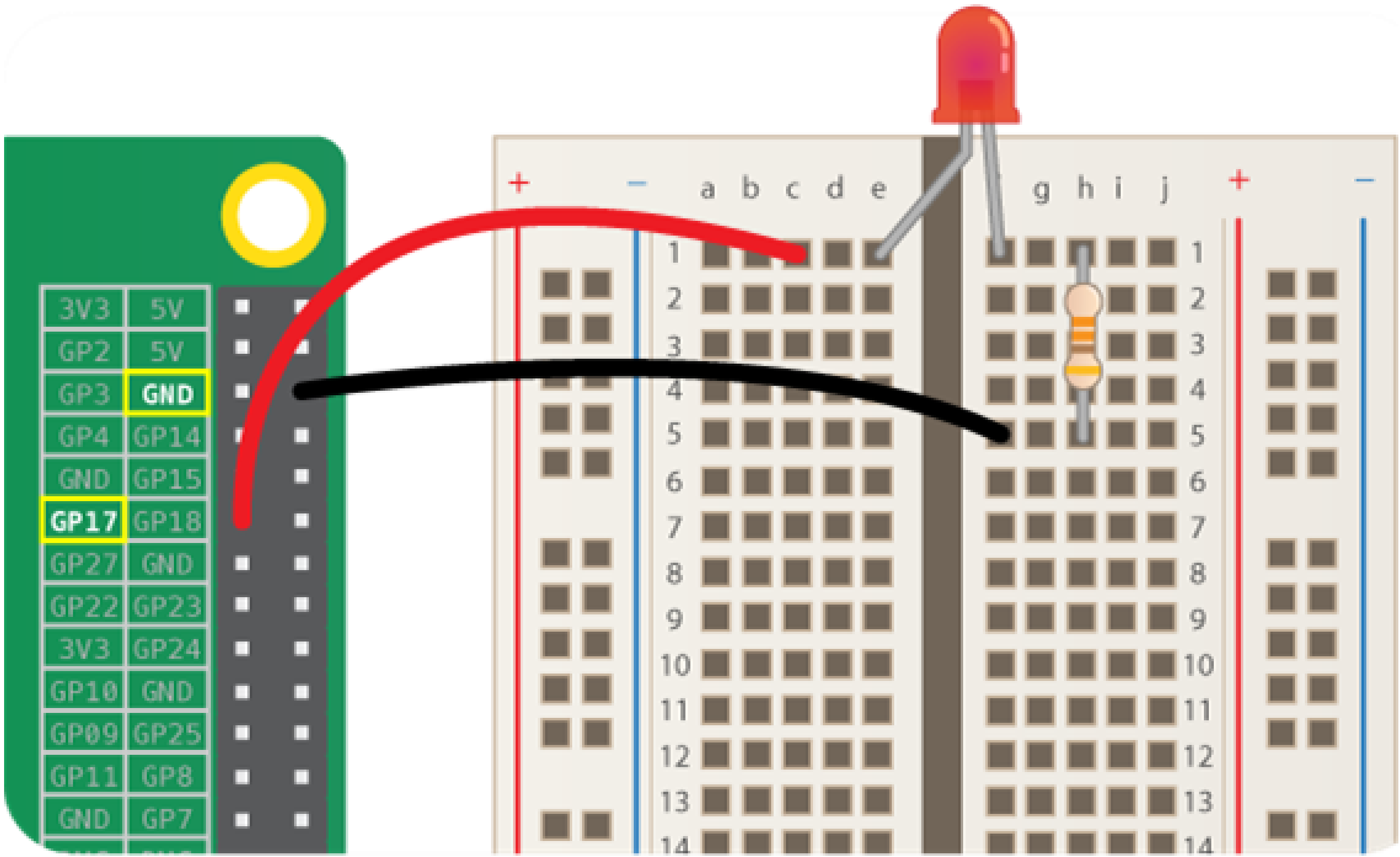


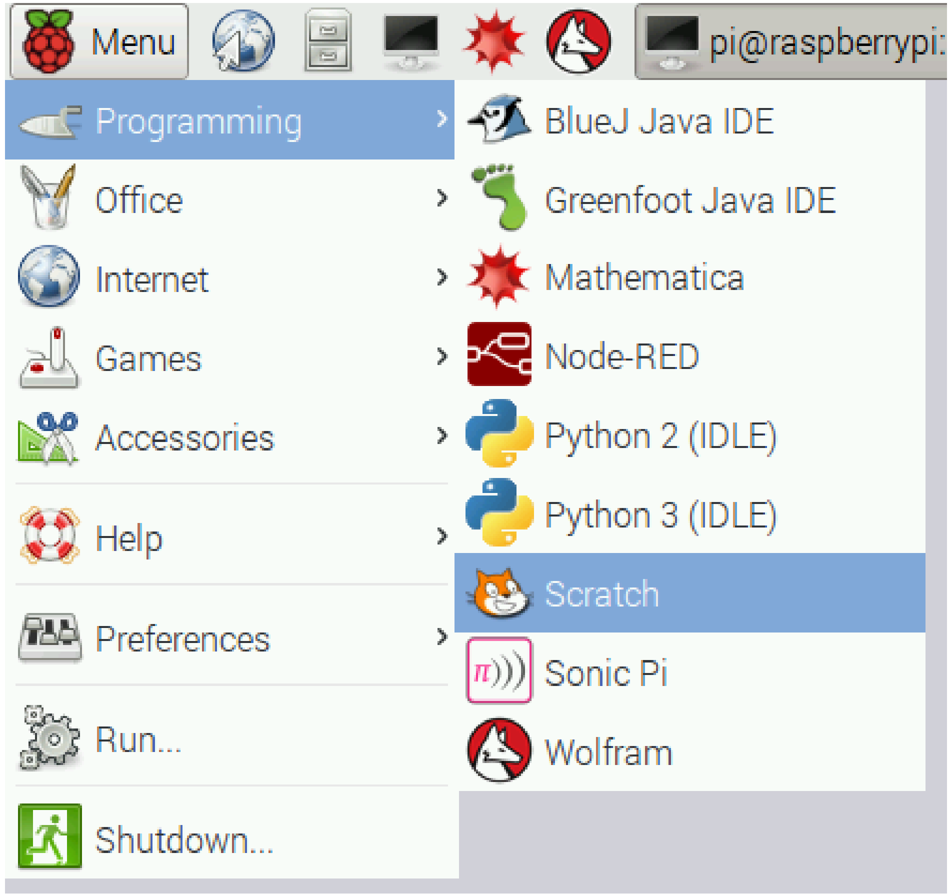
Switching Circuit

Your Raspberry Pi can act as a switch in the circuit, which can be controlled in software. Connect the positive leg of the LED to any other GPIO pin on your Raspberry Pi.



Scratch on the Raspberry Pi

Open Scratch and activate the GPIO server.



gpioserveron

Blinking LED

Setup your pin as an output.	broadcast config17out	config17out
Switch pin 17 on	broadcast gpio17on	gpio17on
Pause	wait 1 secs	
Switch pin 17 off	broadcast gpio17off	gpio17off
Pause	wait 1 secs	
Loop forever	forever	

Extra Challenge!

Can you:

- Flash your LED at different speeds, how fast can you make it flash?
- Can you make a dot (short flash) and dash (long flash) and use to make a distress beacon. S(...) O(---) S(...)

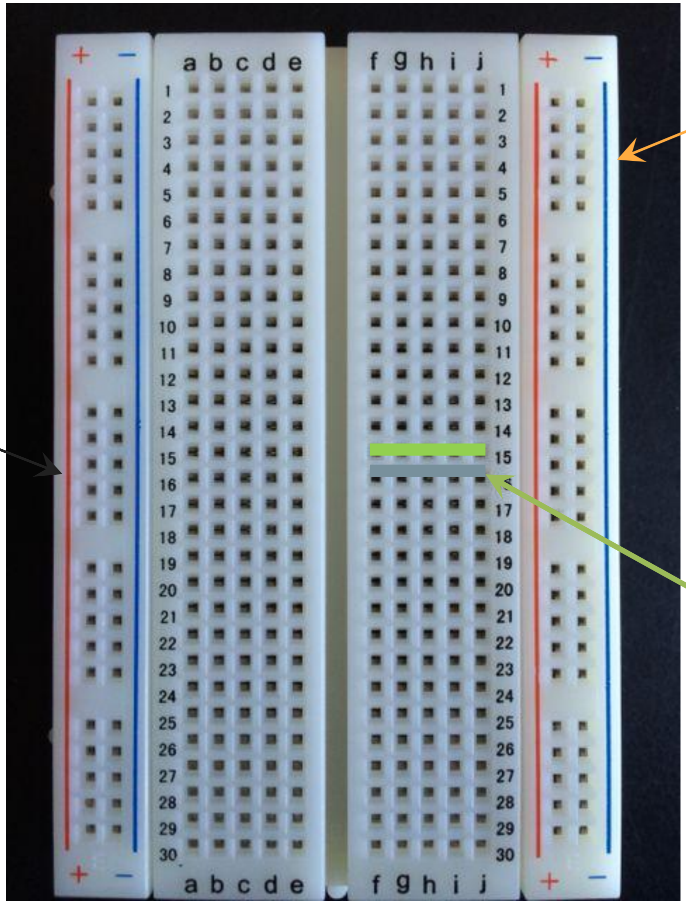
Programming in Python

- Python is a high-level programming language. By high level we mean that it is more removed from the actual language and details of a computer and utilizes more natural language in its operation.
- This high-level language makes it easier to use
- We will learn some basic Python in this class. For those wanting to know more I recommend going through the online tutorial:

https://www.codecademy.com/learn/python

The Breadboard

Every hole along the red line (this column) is electrically connected vertically, but not horizontally

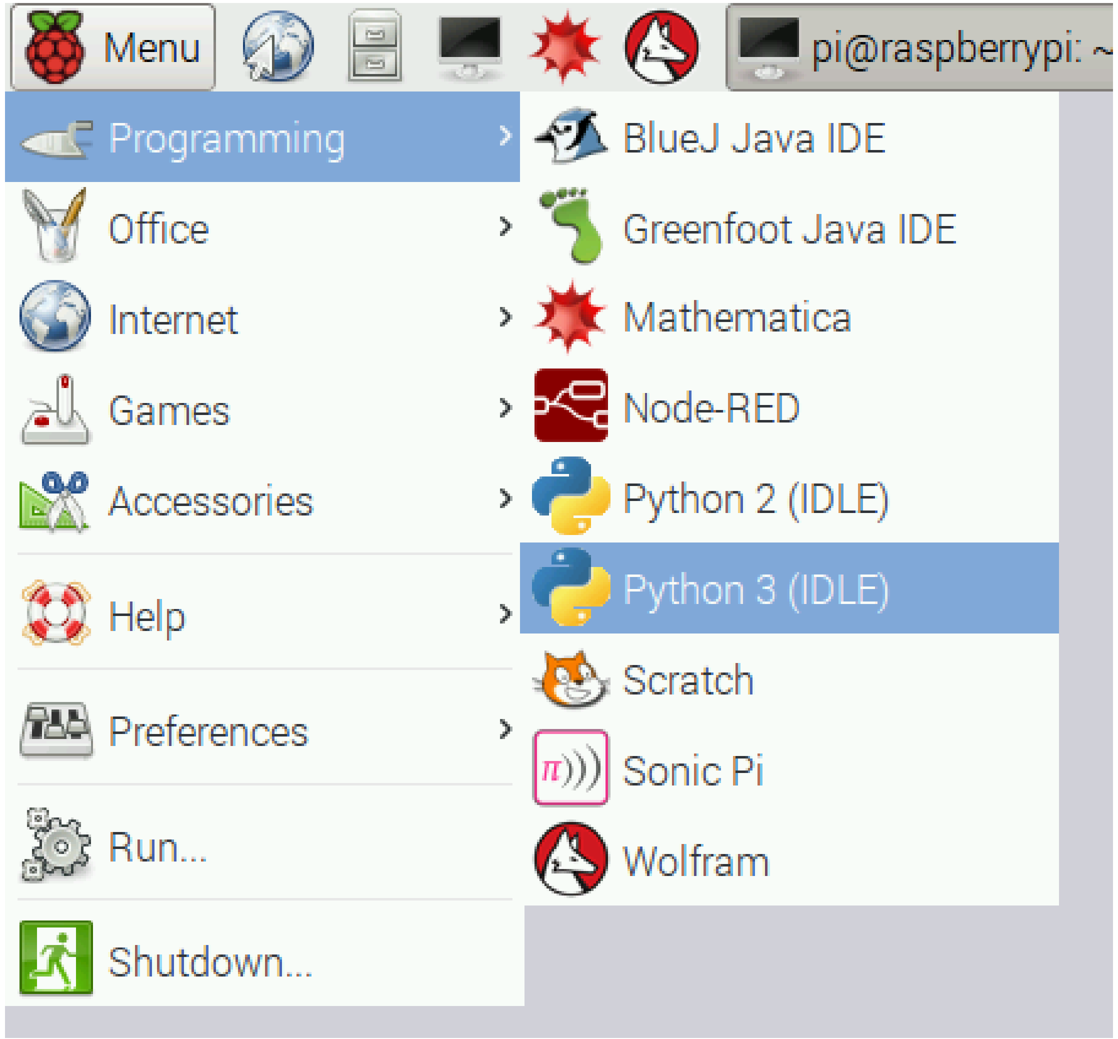


Every hole along the blue line (this column) is electrically connected vertically, but not horizontally

Every hole along a given row (green line) is electrically connected horizontally, but NOT vertically

The two red and two blue columns are called buses

Create New Python 3 File



Coding Your LED Light

```
# Flashing LED


from gpiozero import LED
from time import sleep

myled = LED(17)

while True:
    myled.on()
    sleep(1)
    myled.off()
    sleep(1)
```

Example Programs

```
# Flashing LED

from gpiozero import LED

led = LED(17)

led.blink()
```

Try some other values in `led.blink()`, what would these do:

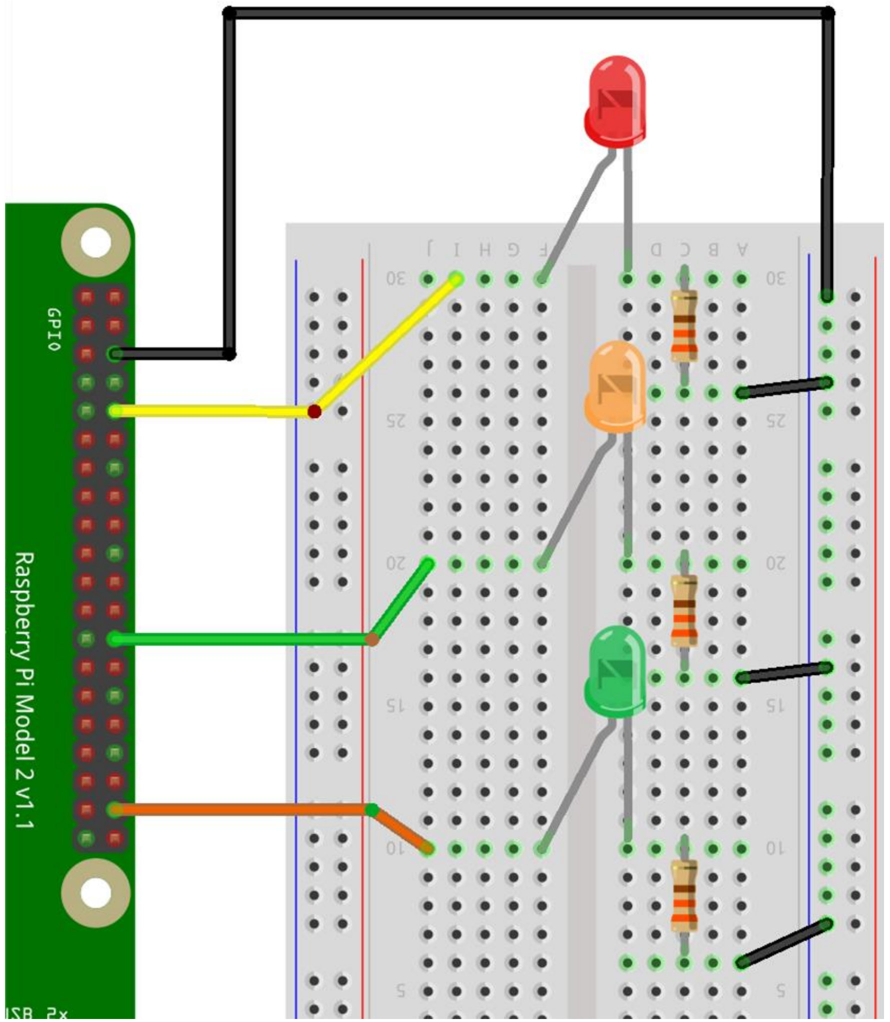
- `led.blink(5)`
- `led.blink(2, 0.5)`
- `led.blink(0.1, 10)`
- `led.blink(0.5, 0.5, 5, False)`

Example Programs

```
# LED methods from the docs:
https://gpiozero.readthedocs.io

led.on() - Switches the Pin high

led.off() - Switches the Pin low

led.blink() - Makes the LED blink

led.toggle() - Change the state of the LED

led.pin.number - Returns the pin number

led.is_lit - Returns the current state
```

Abstraction

```
# Traffic Lights 1

from gpiozero import LED
from time import sleep

red = LED(21)
amber = LED(20)
green = LED(16)

red.on()
sleep(3)
red.off()
amber.on()
…
```



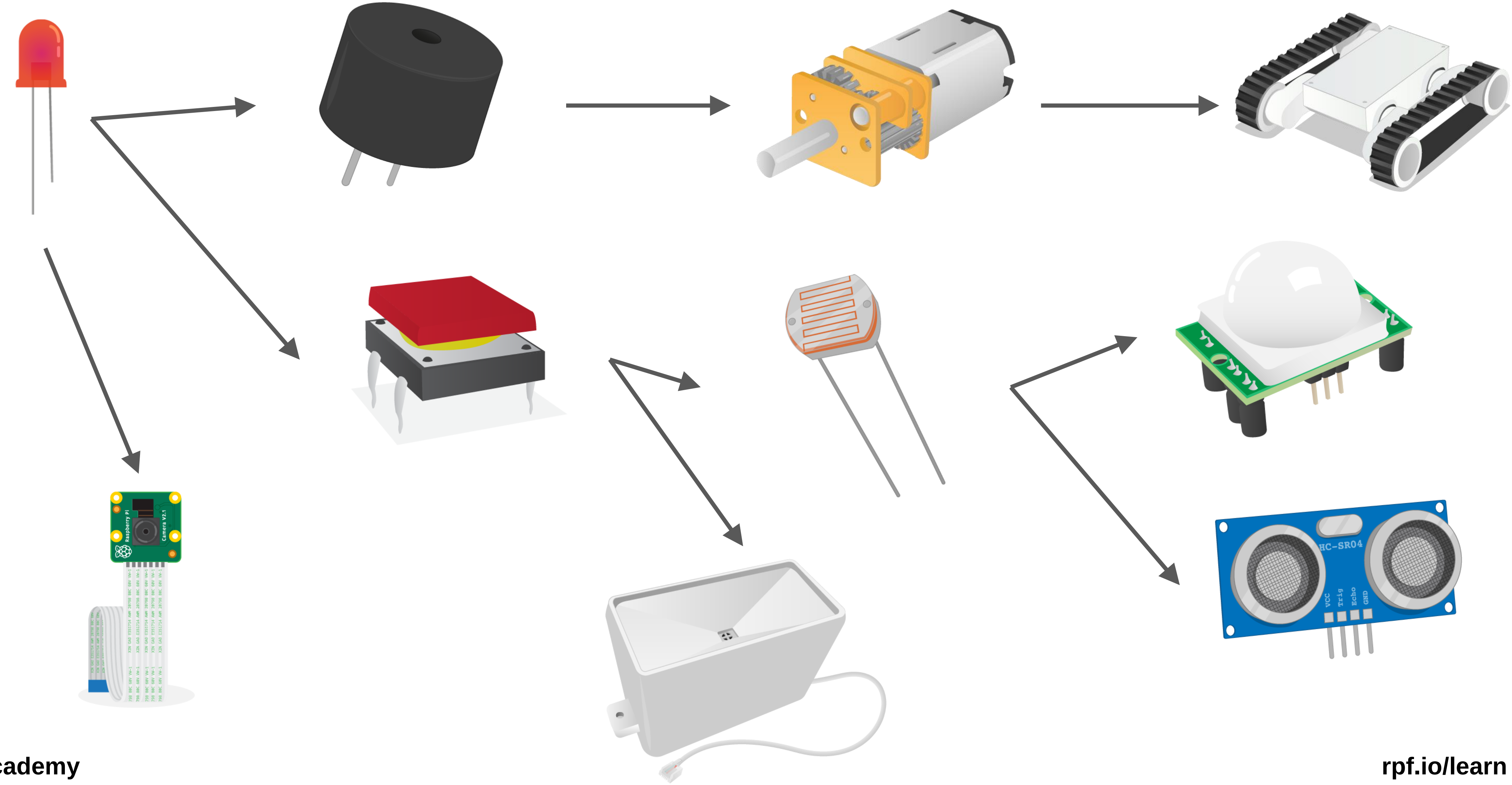
Abstraction

Removing complexity to make a task more accessible

```
# Traffic Lights 1

from gpiozero import LED
from time import sleep

red = LED(21)
amber = LED(20)
green = LED(16)

red.on()
sleep(3)
red.off()
amber.on()
…
```

```
# Traffic Lights 2

from gpiozero import TrafficLights
from time import sleep

lights = TrafficLights(21, 20, 16)



lights.red.on()
sleep(3)
lights.red.off()
lights.amber.on()
…
```

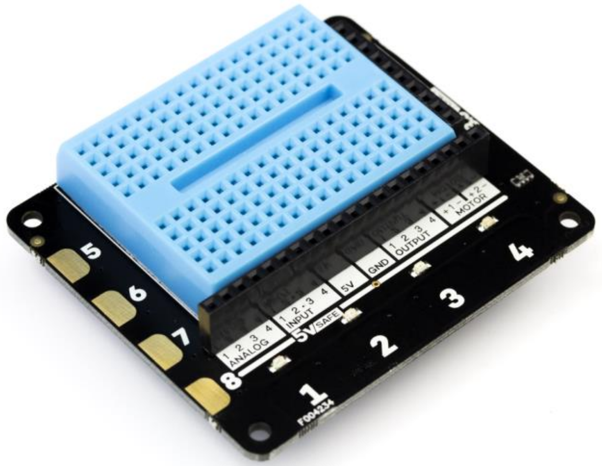
Where next?



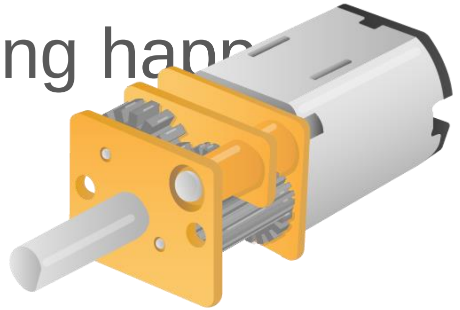
Make things SPIN!

Explorer Hat & Python

What you will learn

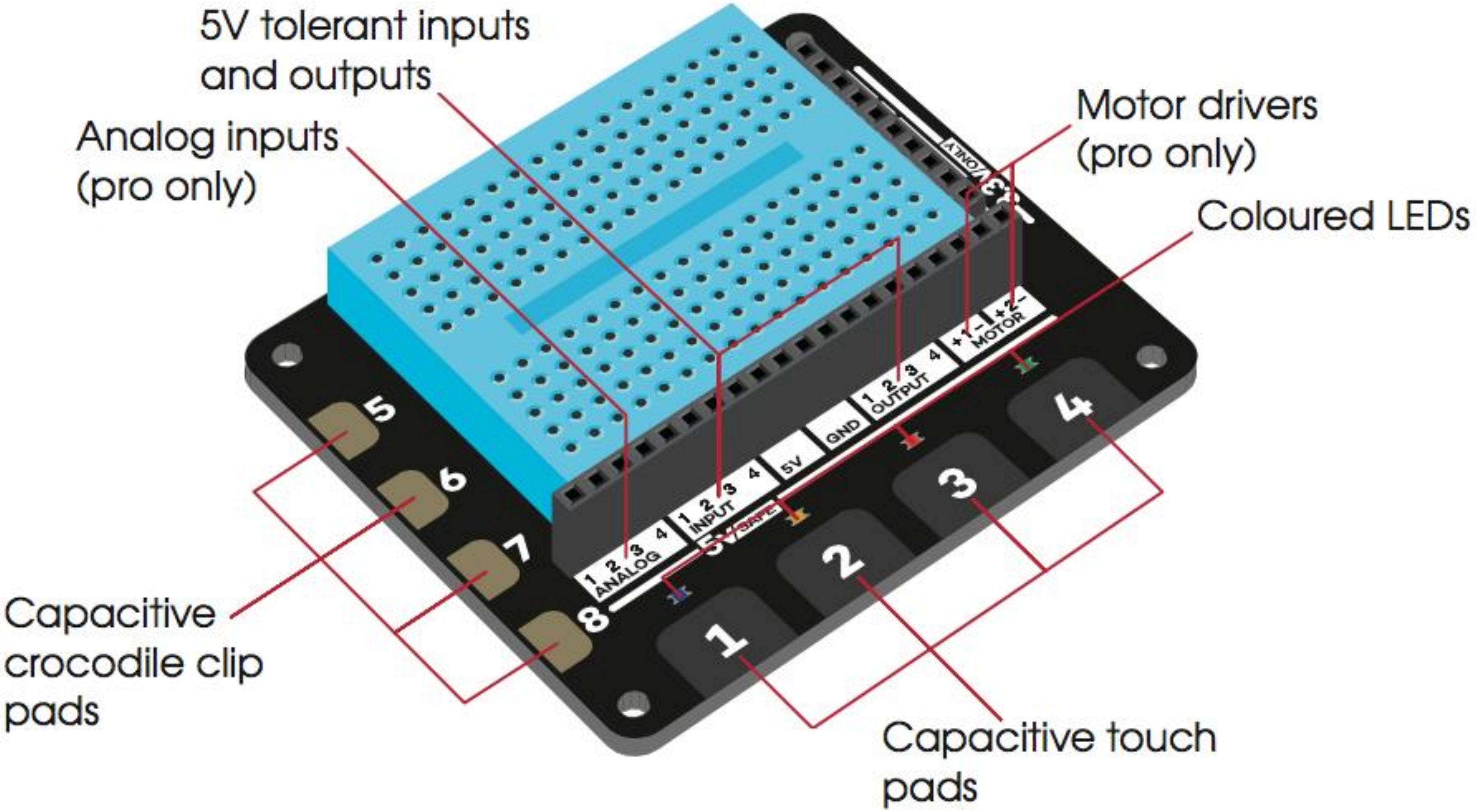


- What is a HAT?
- To turn on and off LEDs with Python
- How to control motors
- To use capacitive touch buttons to make something hap
- Bring it all together and create an invention



Explorer HAT Pro by Pimoroni

Add-on board for physical computing with Raspberry Pi.



Putting your HAT on

First power down and unplug the power cable. Then:

Testing the Explorer HAT in Python

```
>>> import explorerhat
```

It should display a message
"Explorer HAT Pro detected…"

Testing the Explorer HAT in Python

```
>>> explorerhat.light.red.on()
>>> explorerhat.light.red.off()

>>> explorerhat.light.red.toggle()
>>> explorerhat.light.red.blink(0.5, 0.2)

>>> explorerhat.light.on()
>>> explorerhat.light.off()

>>> explorerhat.light.blue.pulse(0.2, 0.2, 0.5, 0.5)
```

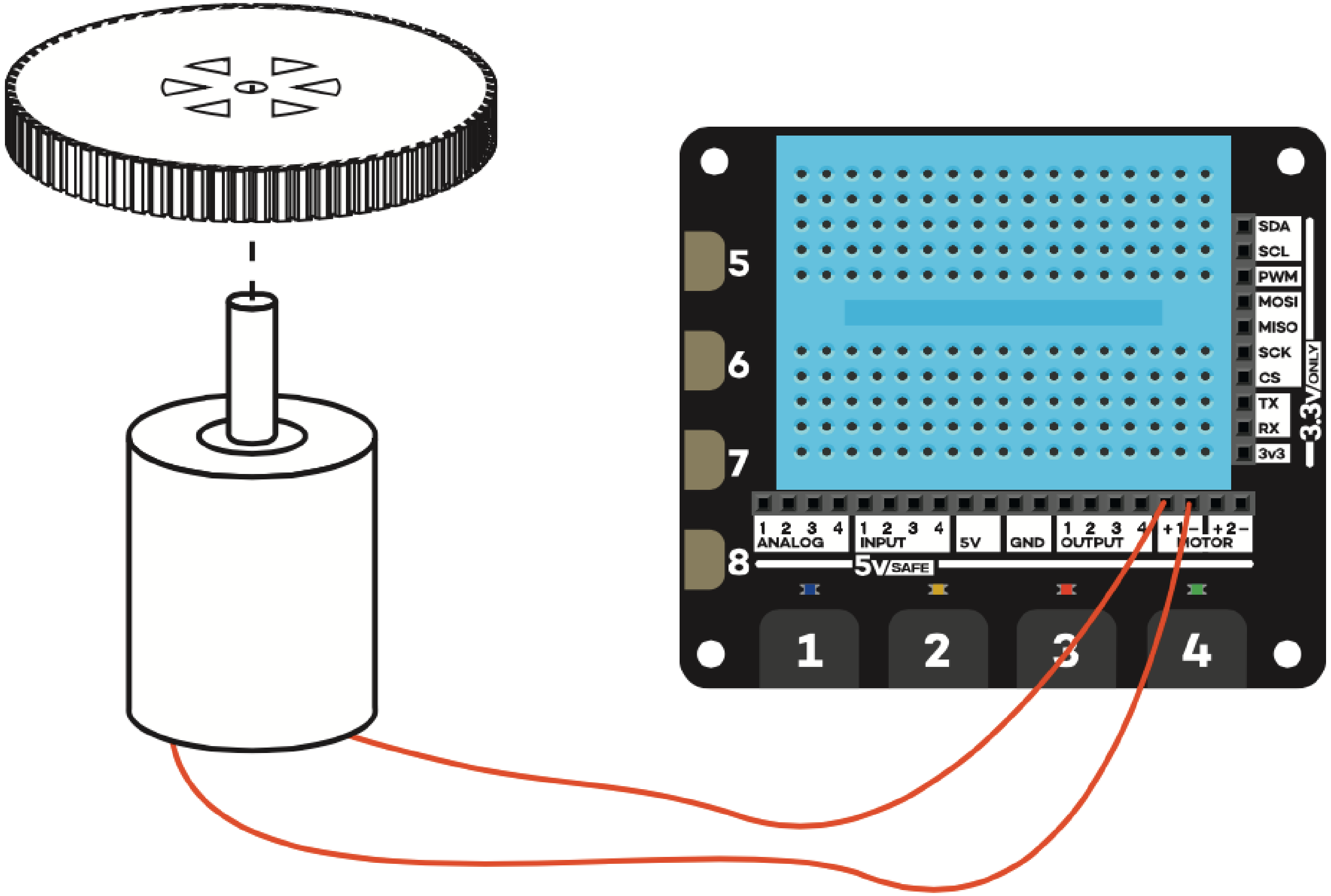
Challenges

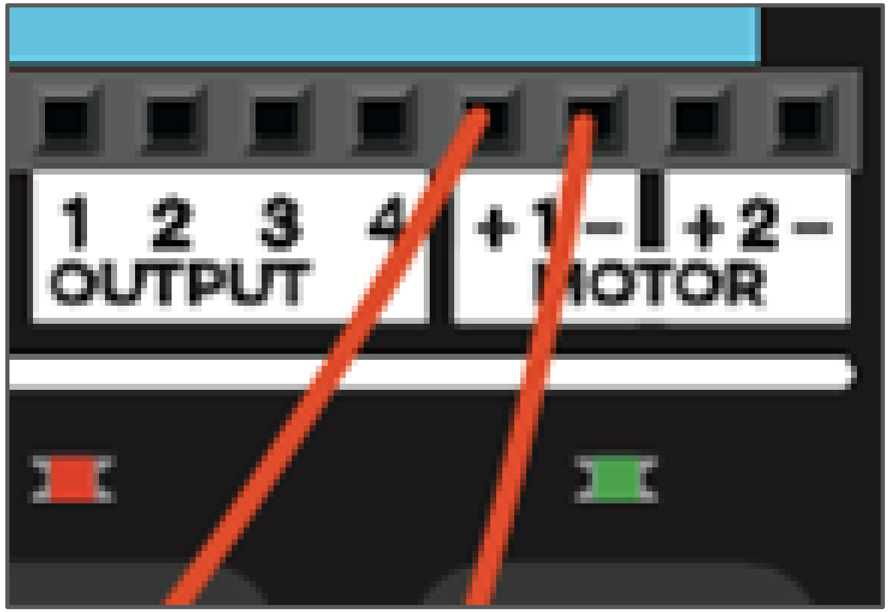
Play around with the other colours “yellow”, “green”, “blue”:

- Can you light these LEDs up?
- Open a **New Window**, via the **File** menu and create a light sequence.
- What activities could your students do with this?

Connecting a motor

The Explorer HAT Pro has some circuitry to make controlling motors easier.





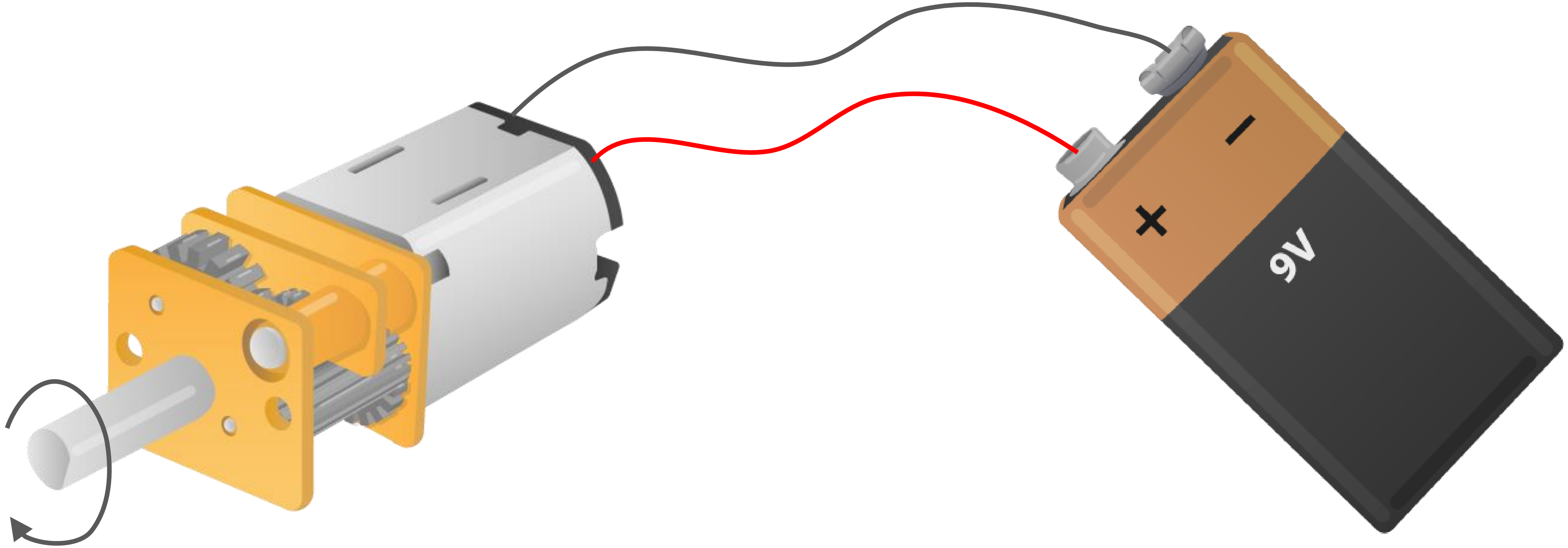
Code it

In a **new Python 3 file** window type:

```
import explorerhat
from time import sleep

explorerhat.motor.one.forward(100)

sleep(5)

explorerhat.motor.one.stop()
```

Why do we need the Explorer HAT for this?

The HAT has an H-Bridge. Here's what it's for:



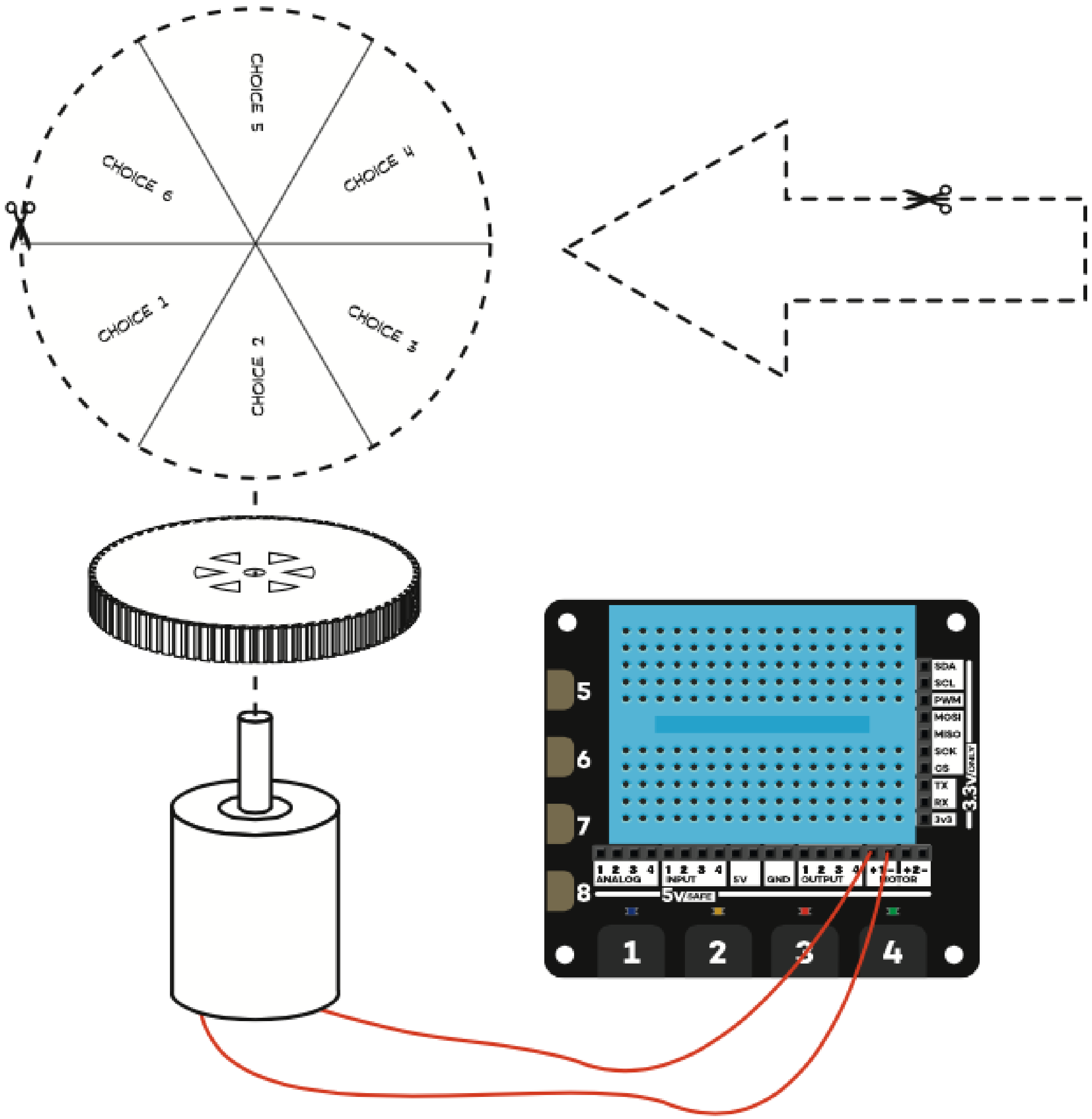
Touch input and Motor Control

```
import explorerhat
from time import sleep

def wheel (channel, event):
    explorerhat.motor.one.forward(100)
    sleep(5)
    explorerhat.motor.one.stop()

explorerhat.touch.one.pressed(wheel)
```

Touch input and Random Motor Control

```
import explorerhat
from time import sleep

from random import randint

def wheel (channel, event):
    duration = randint(1, 10)
    print(duration)
    explorerhat.motor.one.forward(100)
    sleep(duration)
    explorerhat.motor.one.stop()

explorerhat.touch.one.pressed(wheel)
```

Make something!

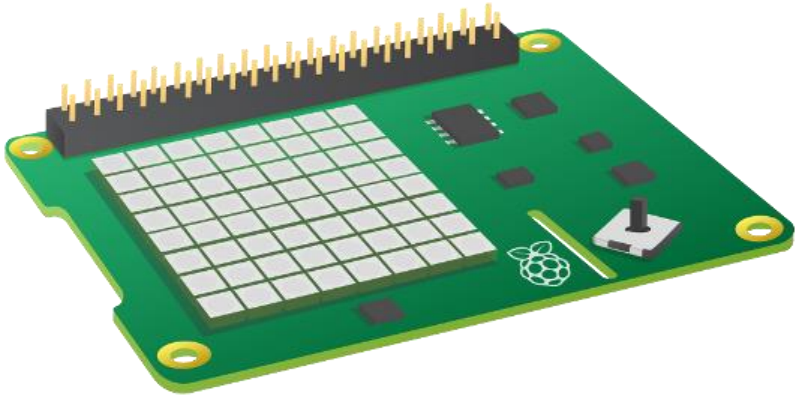


Sense HAT

...and sense-ability!

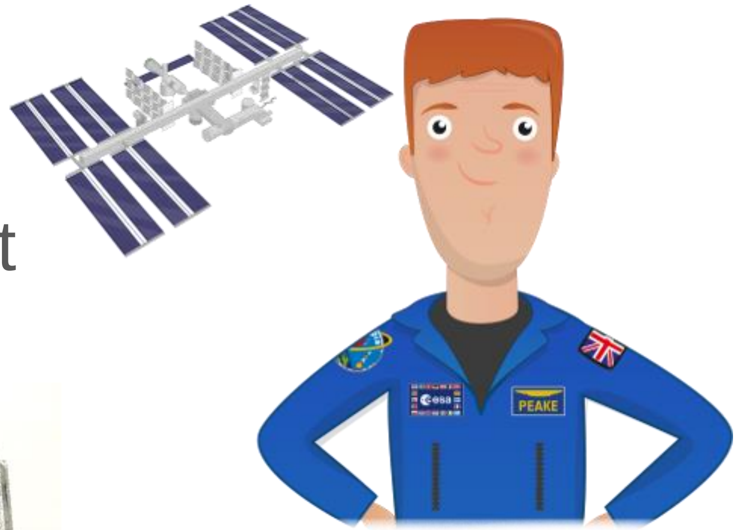
What you will learn

- Hardware, Emulator, and Web
- Displaying emojis using RGB LEDs
- Sensing the orientation of the Sense HAT
- Using Python to sense the environment

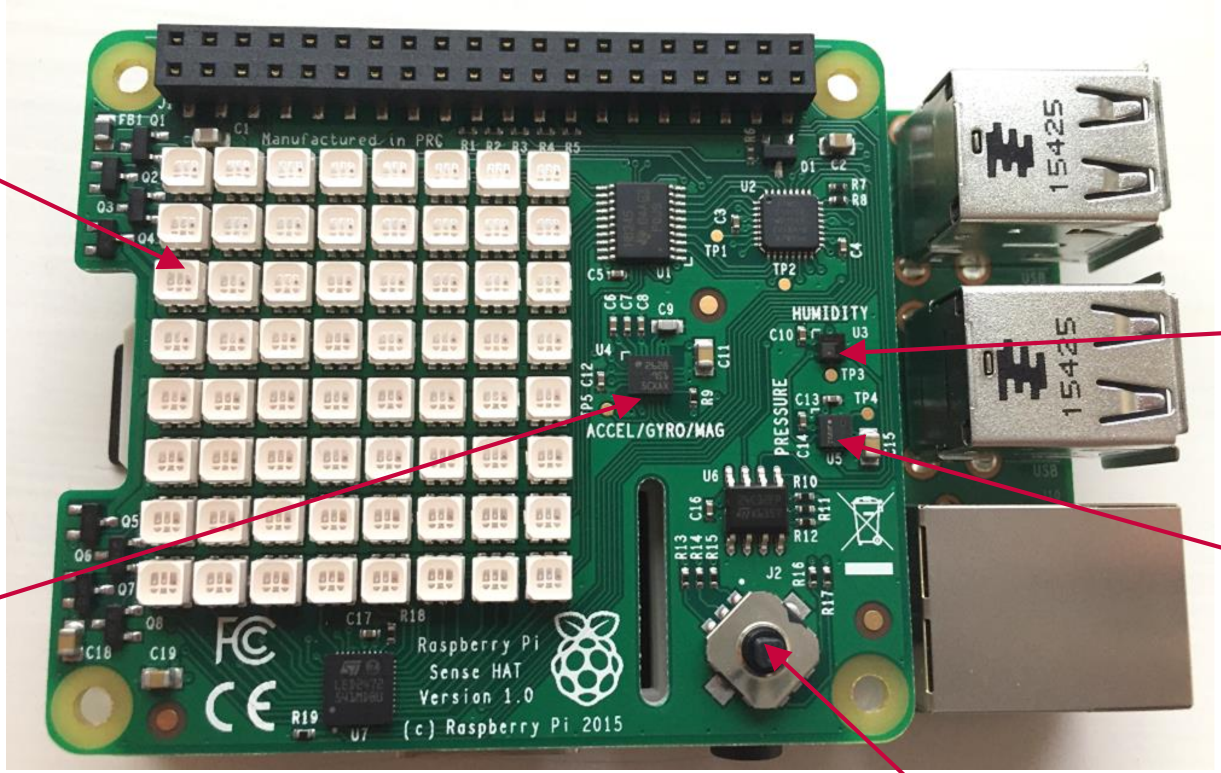


Features of the Sense HAT

The Sense HAT was designed for the Astro-Pi project



8x8 RGB LED matrix



Temperature and humidity sensor

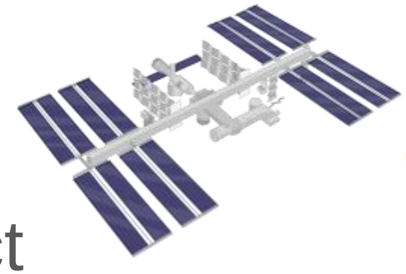
Barometric pressure sensor

Gyroscope, accelerometer and magnetometer

Mini joystick

Astro PI

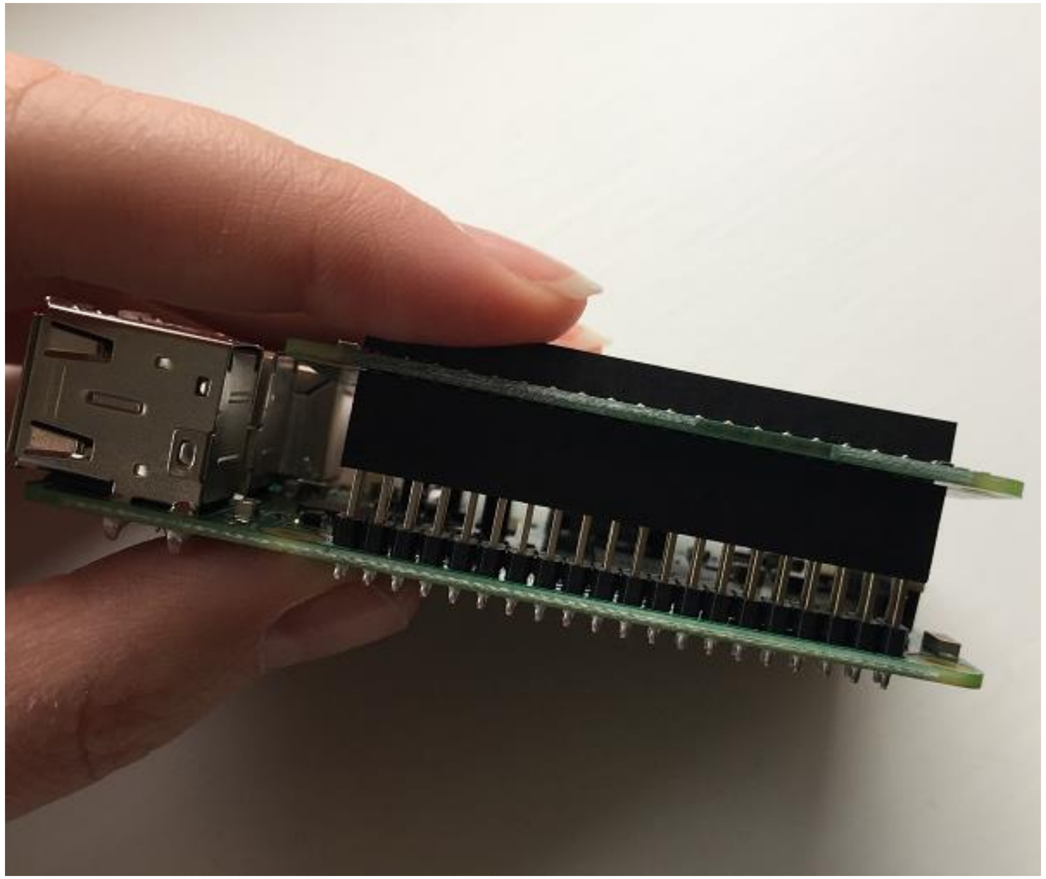
The Sense HAT was designed for the Astro-Pi project

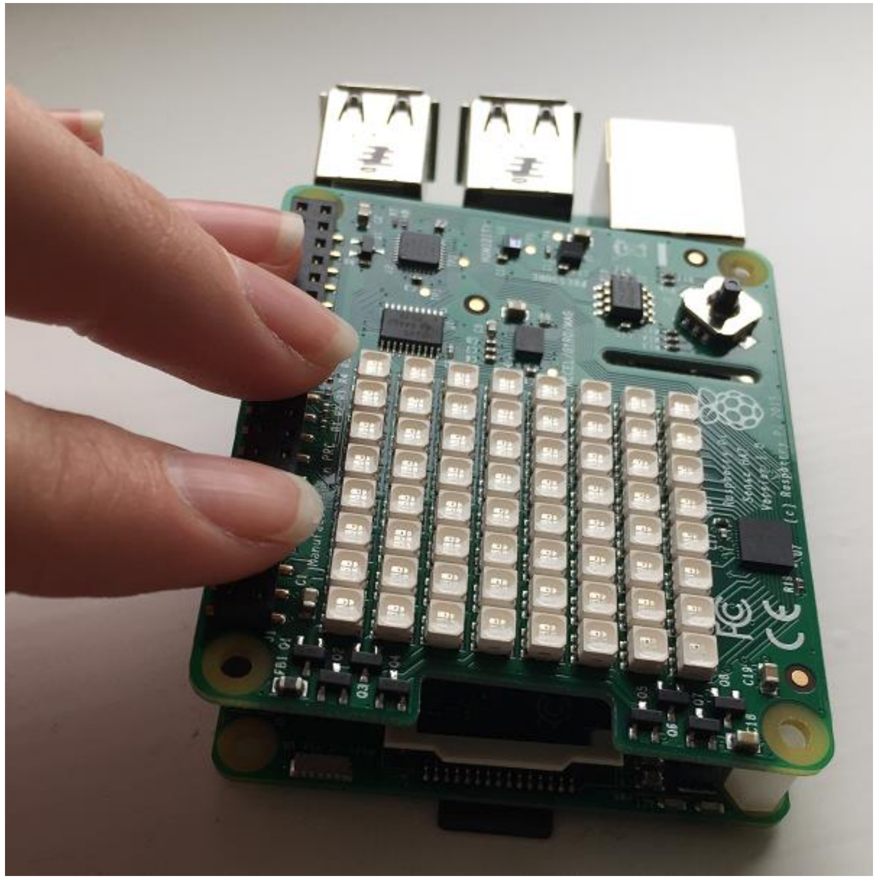


Attaching your Sense HAT

Power down and unplug your Raspberry Pi

Align the pins with the Sense HAT and push gently



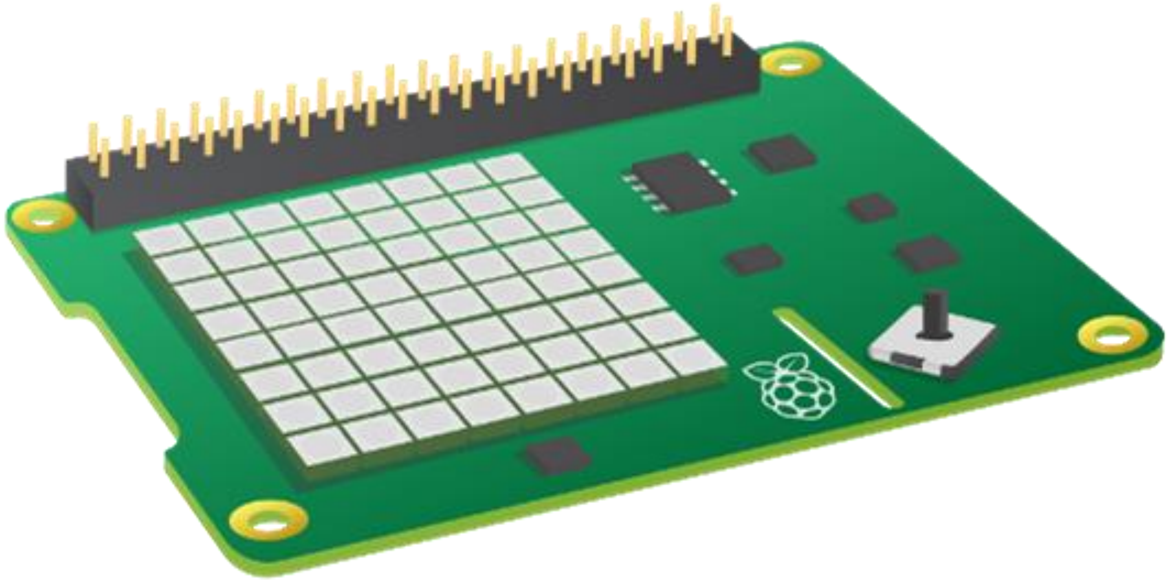


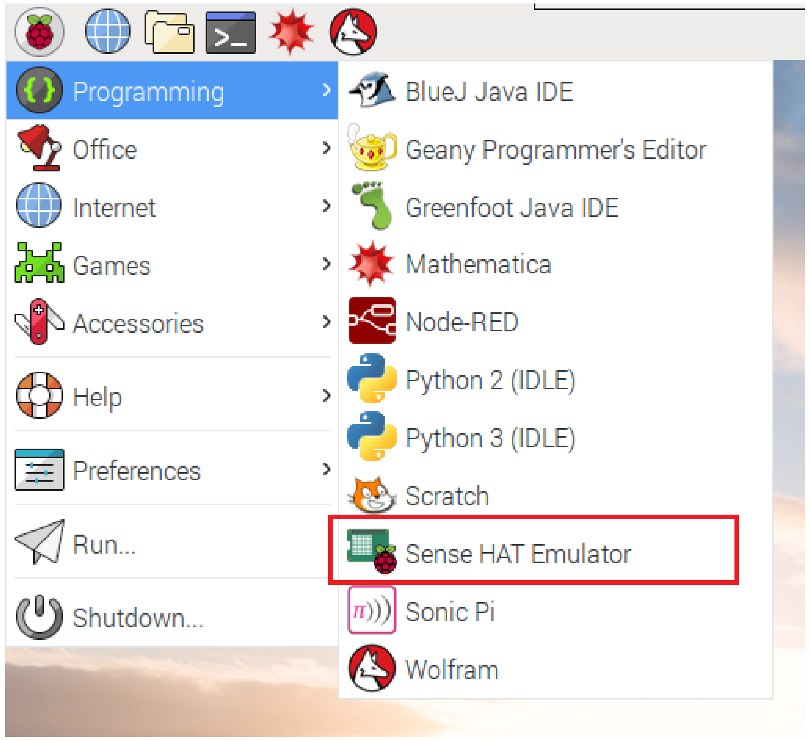
Sense HAT

Add-on board for physical computing with Raspberry Pi

Software emulator on the Raspberry Pi

Web-based emulator - https://trinket.io/sense-hat





Programming the Sense HAT with Python

Programming → Python 3 (IDLE)

```
from sense_hat import SenseHat

sense = SenseHat()

sense.show_message("Picademy")
```

While I love how the emulator allows for different experimentation and doesn't require a board purchase, I fall for those shiny LEDs every time.

Displaying colours (US: colors)

In a new Python 3 file window type:

```
from sense_hat import SenseHat
from time import sleep

sense = SenseHat()

red = (255, 0, 0)
sense.clear(red)                    # Fill whole screen red
sleep(1)
sense.clear()                       # Fill whole screen blank
sense.set_pixel(4, 5, red)  # Set specific pixel red
```

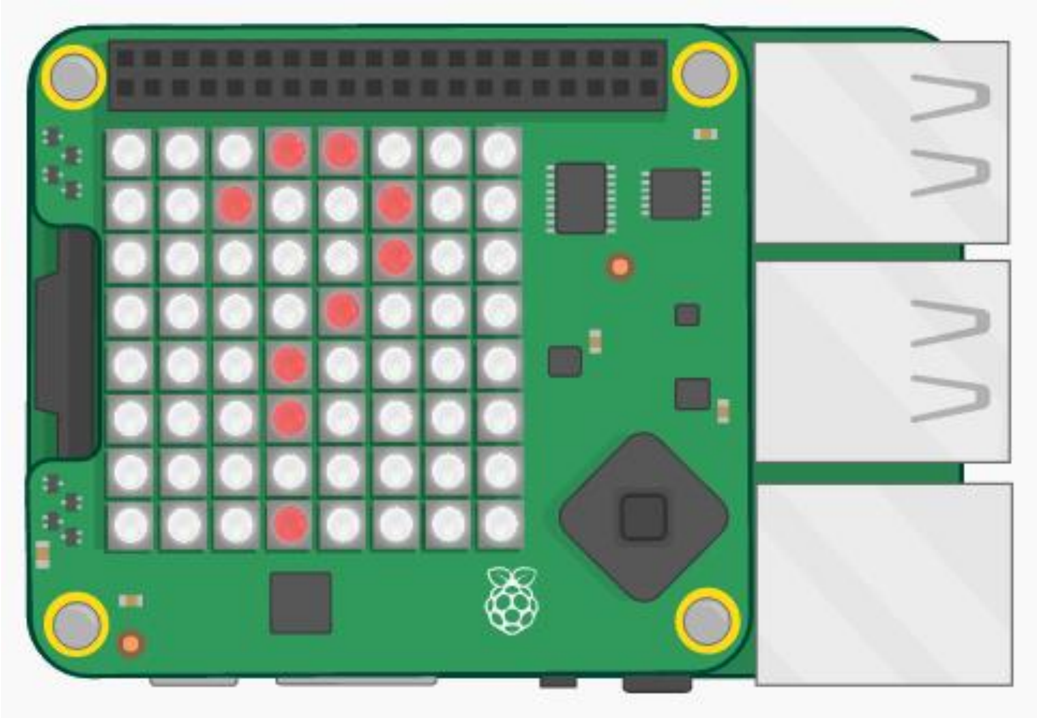
Ca

Yo

Displaying emojis

https://pythonhosted.org/sense-hat/api/ - set_pixels

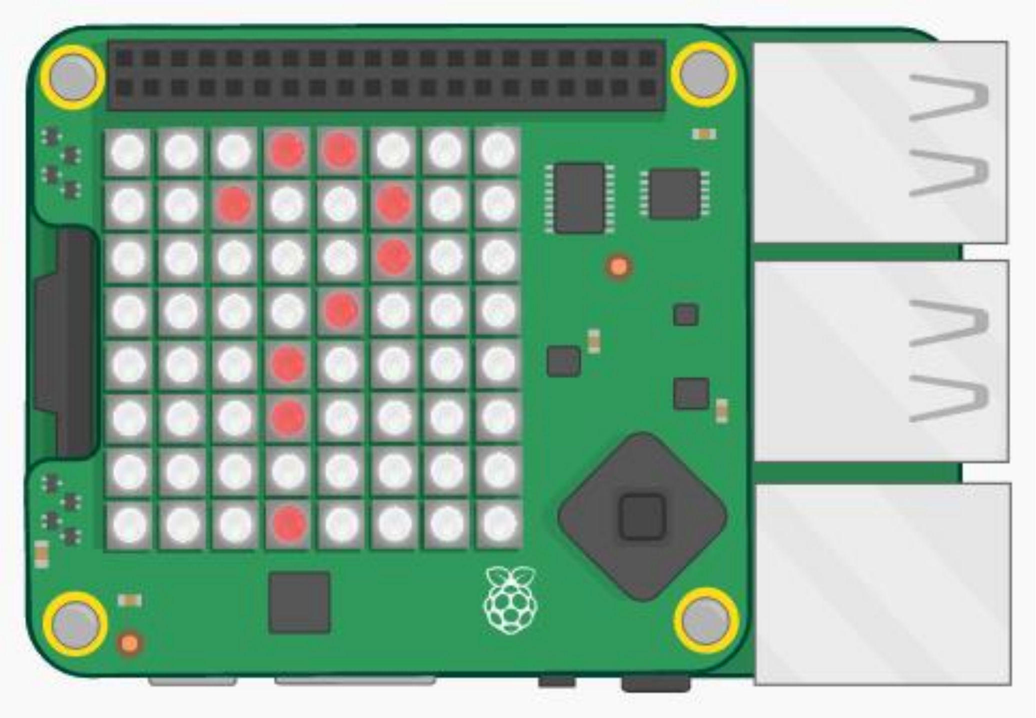
```
X = (255, 0, 0)                    #
Red
O = (255, 255, 255)          # White

question_mark = [
O, O, O, X, X, O, O, O,
O, O, X, O, O, X, O, O,
O, O, O, O, O, X, O, O,
O, O, O, O, X, O, O, O,
O, O, O, X, O, O, O, O,
O, O, O, X, O, O, O, O,
O, O, O, O, O, O, O, O,
O, O, O, X, O, O, O, O
]

sense.set_pixels(question_mark)
```



Getting emoji-onal

Can you write a program that will:

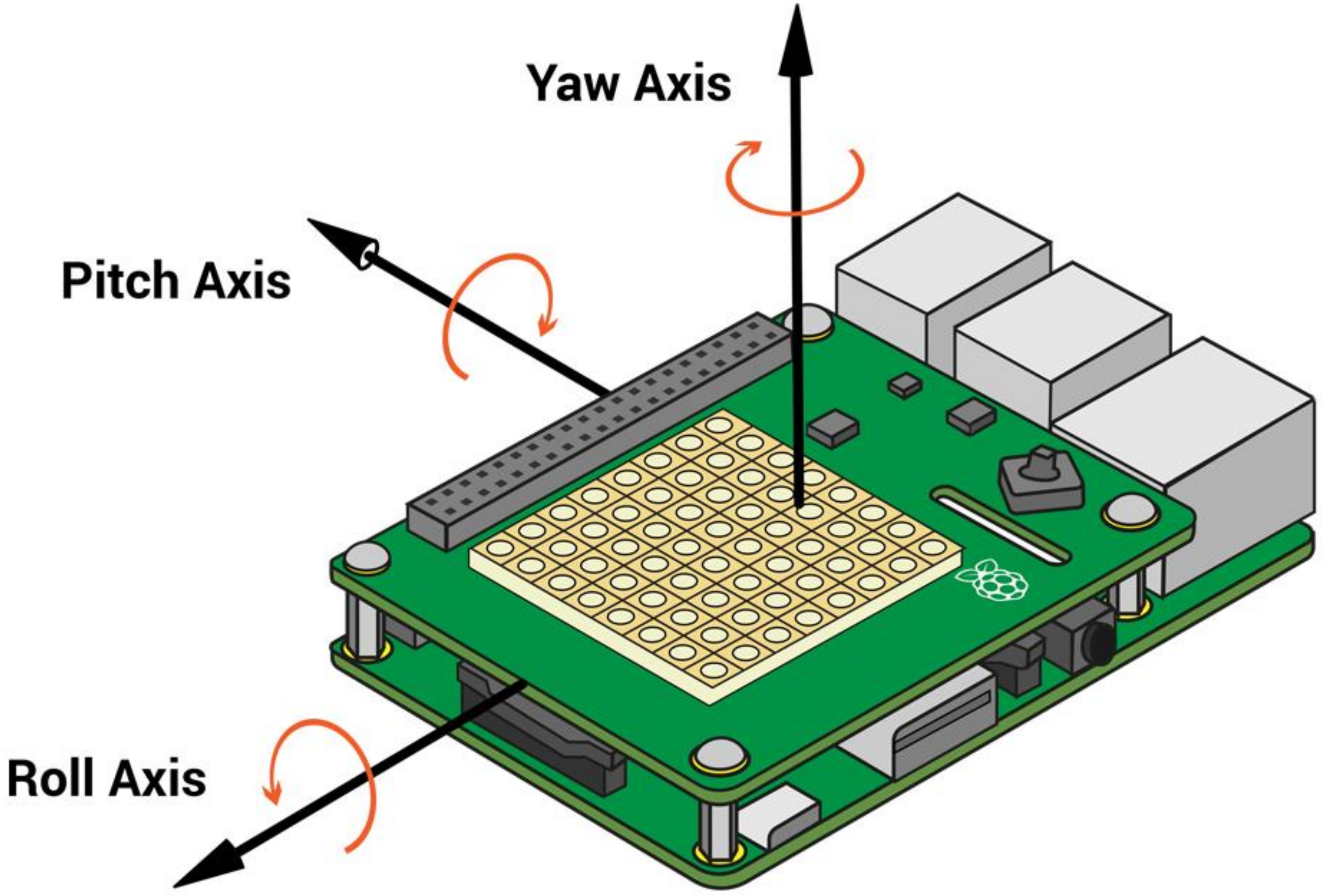
- Display a different emoji
- ...depending on either the humidity
- ...or the orientation of the Sense HAT?
- Make a marble maze
- Animate a pixel creature



For the curious:
https://pythonhosted.org/sense-hat/api/

Sensing the orientation of the Sense HAT

The Sense HAT can report **pitch**, **roll** and **yaw**



Sensing the orientation of the Sense HAT

The Sense HAT can report **pitch**, **roll** and **yaw**

```
from sense_hat import SenseHat

sense = SenseHat()

data = sense.get_orientation()
pitch = data['pitch']

print( pitch )
```

Sensing the orientation in ACTION!

The Sense HAT can report **pitch**, **roll** and **yaw**

```
from sense_hat import SenseHat

sense = SenseHat()

while True:
    data = sense.get_orientation()
    pitch = data['pitch']

    print( pitch )
```

Wh ... ge?

```
blank = (0,0,0)                        #don't forget to import your
white = (255,255,255)                  #modules
sense.clear()
x = 1
y = 1
while True:
    data = sense.get_orientation()
      pitch = data['pitch']

    if 359 > pitch > 179 and x != 7 :
        sense.set_pixel(x, y, blank)        # Clear pixel
        x += 1                                                #
Move x coord
        sense.set_pixel(x, y, white)        # Set pixel

    elif 1 < pitch < 179 and x != 0:
        # What should you do here?
```

Python - Sensing humidity

```
from sense_hat import SenseHat
from time import sleep

sense = SenseHat()

while True:
    humidity = sense.get_humidity()

    if humidity > 29:           #change value based on room
        sense.clear(255,0,0)
    else:
        sense.clear(0,0,255)

    sleep(0.1)
```

Sensing Pressure and Humidity

The Sense Hat can also display **pressure** and **humidity**

```
from sense_hat import SenseHat
Sense = SenseHat()

While True:
          t = sense.get_temperature()
          p = sense.get_pressure()
          h = sense.get_humidity()

          t = round(t, 1)
          p = round(p, 1)
          h = round(h, 1)

          msg = ‘Temperature = {0}, Pressure = {1}, Humidity = {2}’.format(t, p, h)

          sense.show_message(msg, scroll_speed = 0.05)
```

Common Sense Hat Commands

https://pythonhosted.org/sense-hat/api/

*.show_message(”text”) #Can add conditions

*.clear() #sets all pixels to condition

*.pixel(x, y, *variable*) #sets x, y, pixel to *v*

```
Conditions:
     scroll_speed=x.xx
     text_colour[xxx,xxx,xxx]
     back_colour[xxx,xxx,xxx]
```

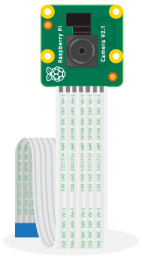
Raspberry Pi Camera

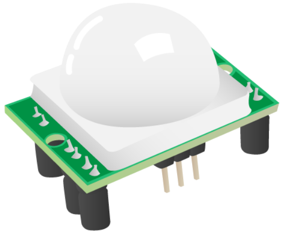
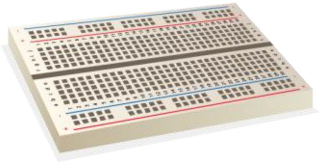
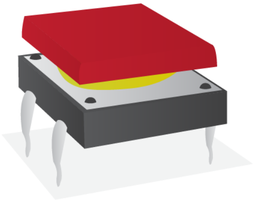
and other inputs

What you will learn

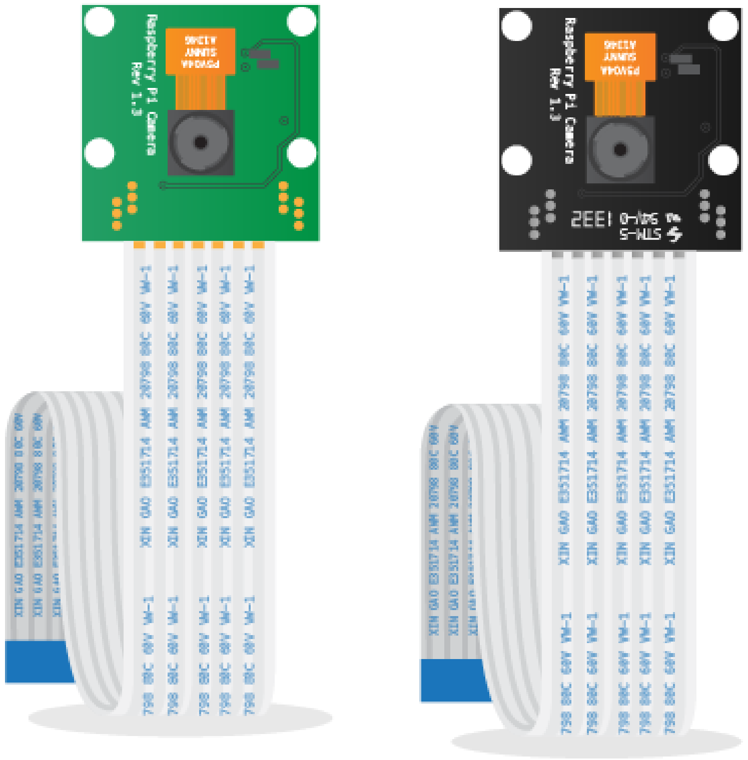
- How to connect the camera module
- How to use Python to take pictures
- How to add physical components to your project
- How to use loops to repeat commands

You will need:



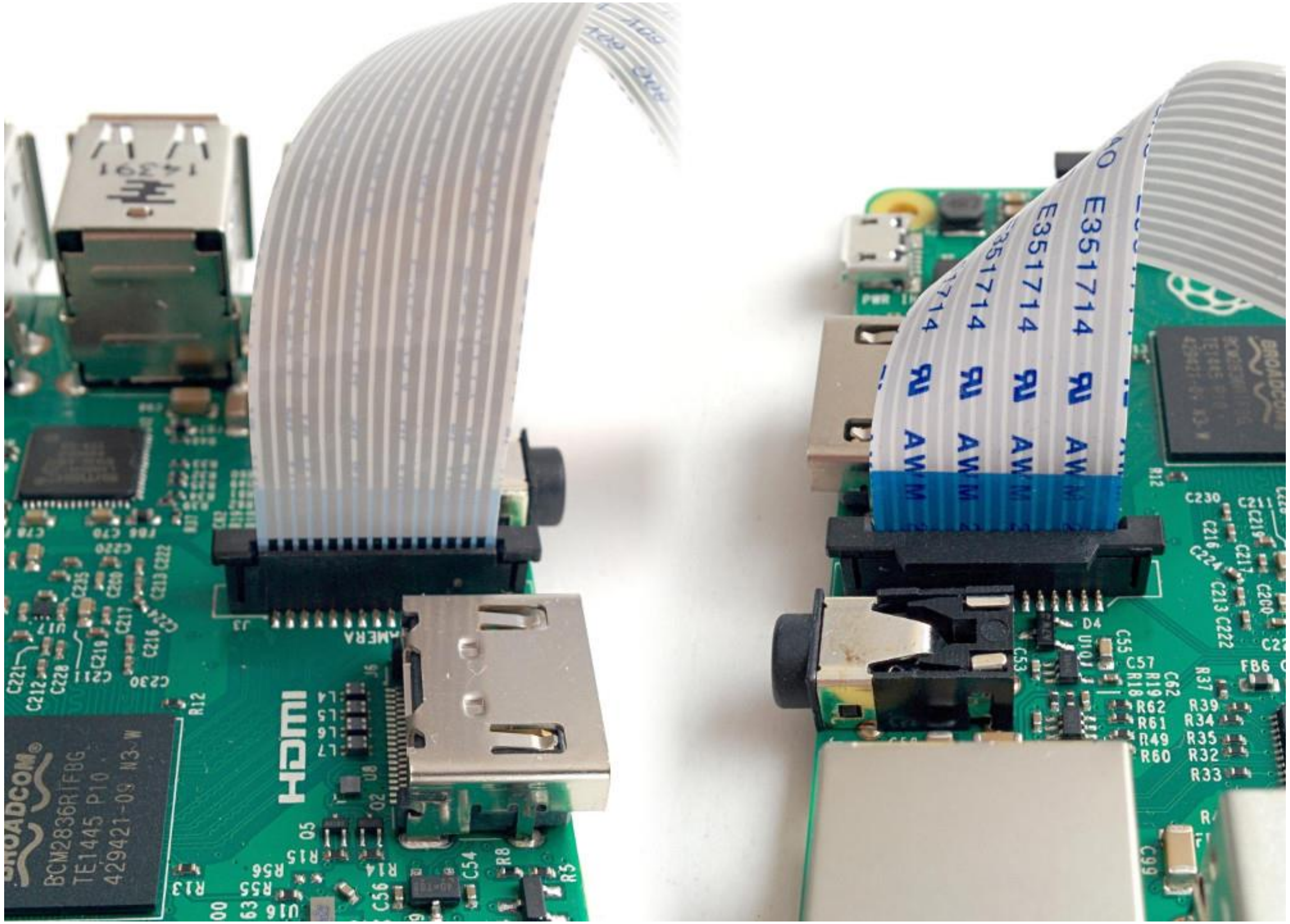





Raspberry Pi camera module



- 5Mpx / 8Mpx
- Full HD
- Photo & video
- Command line
- Python module
- Infra-red camera

Connect the camera



What does it add?

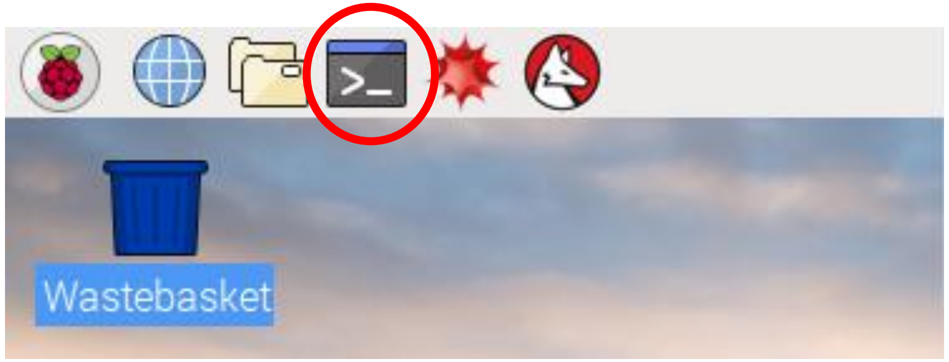
Timelapse

High Speed recording

Sensor & Images

Infrared photography

Test your camera

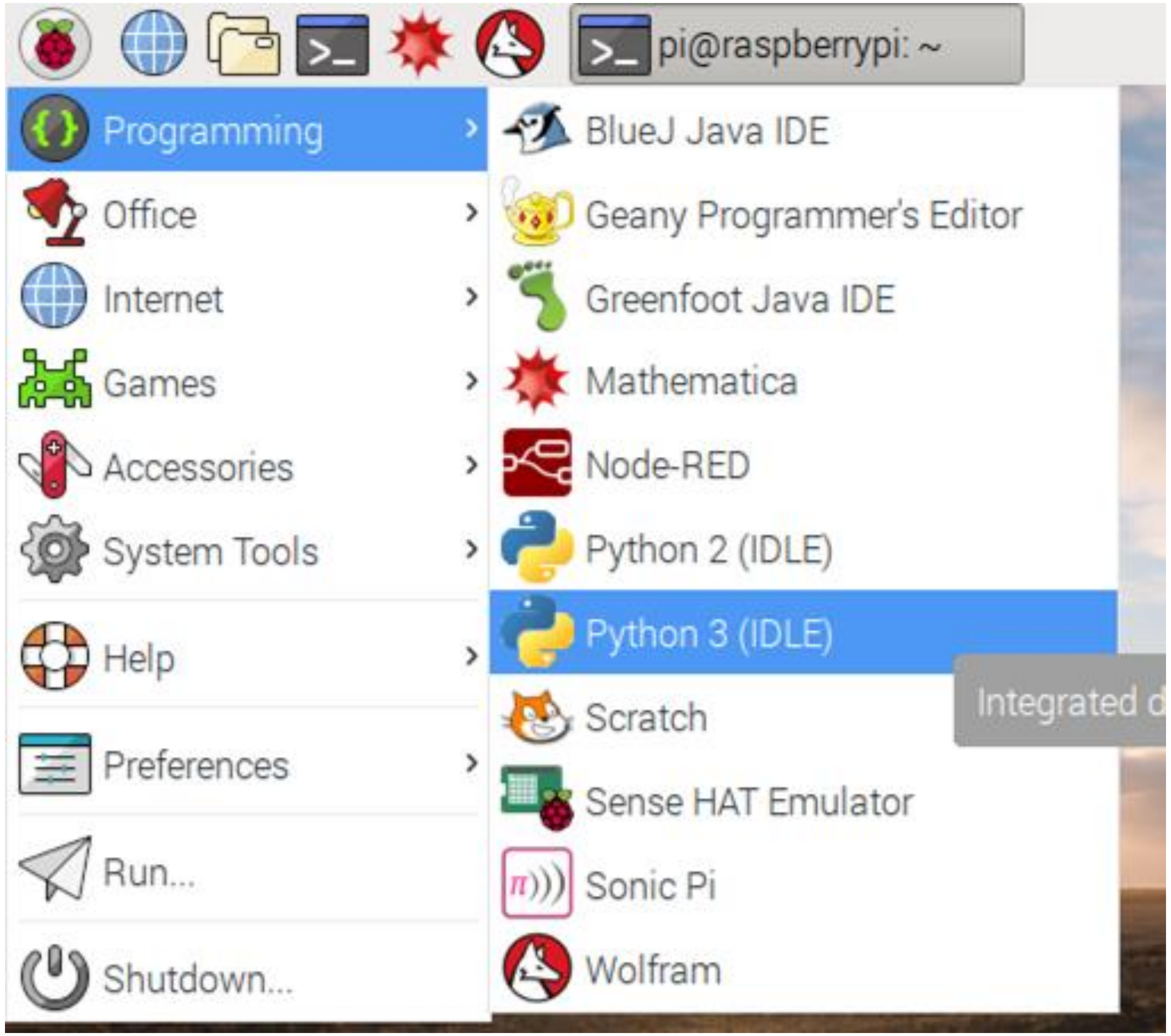



```
pi@raspberrypi:~ $ raspistill -k
```

Ctrl + C to close preview

```
pi@raspberrypi:~ $ raspistill -o image.jpg
```

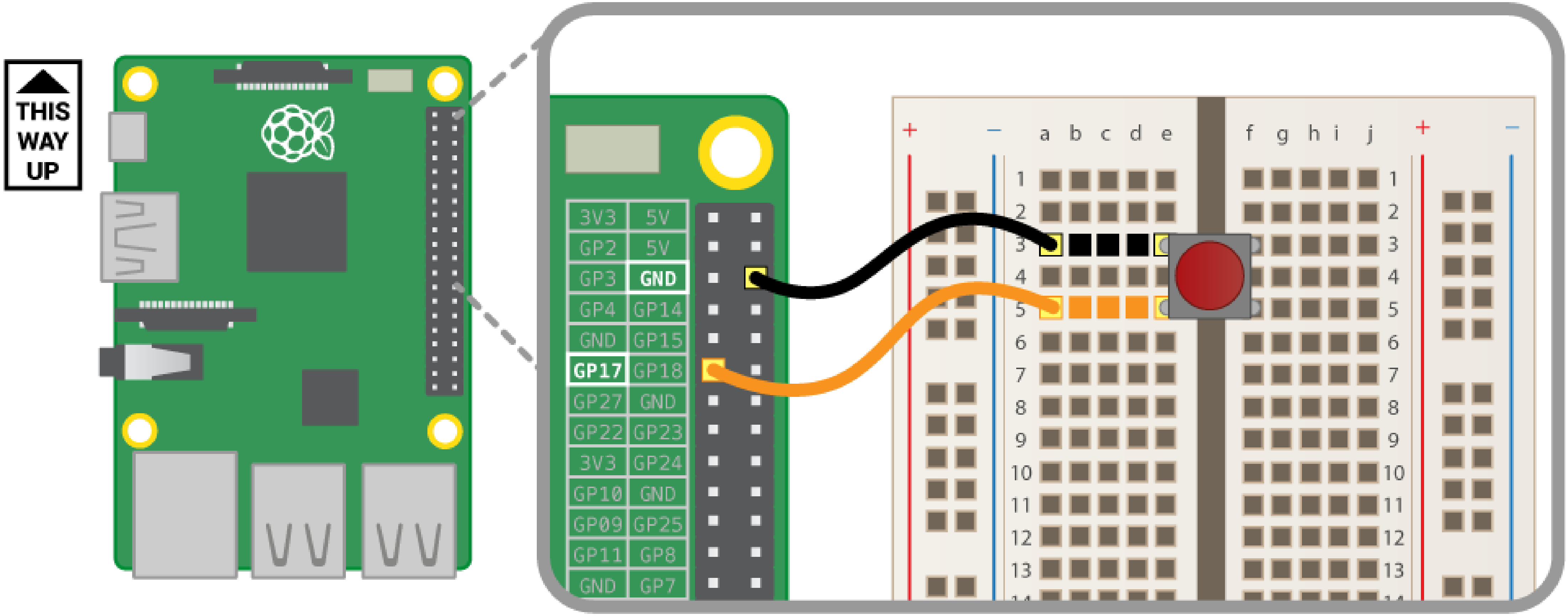
Create New Python 3 File



Take a selfie

```python
## selfie.py
from picamera import PiCamera
from time import sleep

camera = PiCamera()

camera.start_preview(alpha=192)
sleep(3)
camera.capture("/home/pi/image.jpg")
camera.stop_preview()
```

Add a GPIO Button



Add GPIO Button code

```
## button.py
from picamera import PiCamera
from gpiozero import Button
from time import sleep

camera = PiCamera()
button = Button(17)

camera.start_preview(alpha=192)
button.wait_for_press()
sleep(3)
camera.capture("/home/pi/button.jpg")
camera.stop_preview()
```

Add a loop

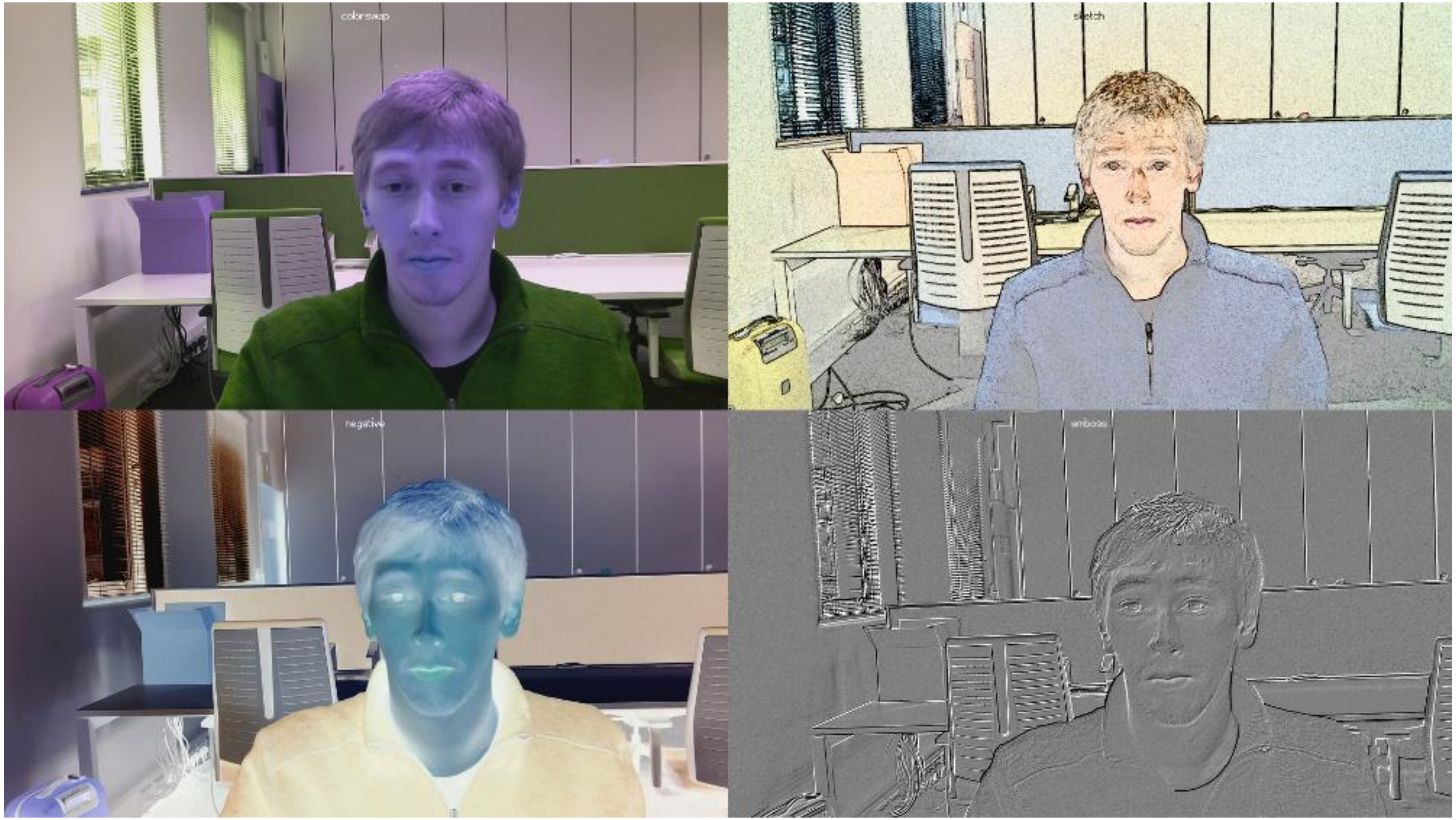
```
## loop.py
from picamera import PiCamera
from gpiozero import Button
from time import sleep

camera = PiCamera()
button = Button(17)

camera.start_preview(alpha=192)
for i in range(5):
    button.wait_for_press()
    sleep(1)
    camera.capture("/home/pi/button{0}.jpg".format(i))
camera.stop_preview()
```

What's the difference?

```
…

for i in range(5):
    button.wait_for_press()
    sleep(3)
    camera.capture("/home/pi/button{0}.jpg".format(i))
```

```
…

button.wait_for_press()
for i in range(5):
    sleep(3)
    camera.capture("/home/pi/button{0}.jpg".format(i))
```

Picamera effects



Picamera effects

```
...

camera.start_preview(alpha=192)
button.wait_for_press()
camera.image_effect = 'negative'
sleep(5)
camera.capture("/home/pi/negative.jpg")
camera.stop_preview()
```

1. Start with **selfie.py**
2. Save As **effect.py**

Try more effects:

- negative
- colorswap
- sketch
- emboss

Capturing video

```
##video.py
from picamera import PiCamera

camera = PiCamera()
camera.start_preview(alpha=192)
camera.framerate = 24
camera.start_recording('my_video.h264')
camera.wait_recording(15)
camera.stop_recording()
camera.stop_preview()
```

Play your video

Open a Terminal window (Ctrl+Alt+T)

```
pi@raspberrypi:~ $ omxplayer myvideo.h264
```

Button → Motion Sensor

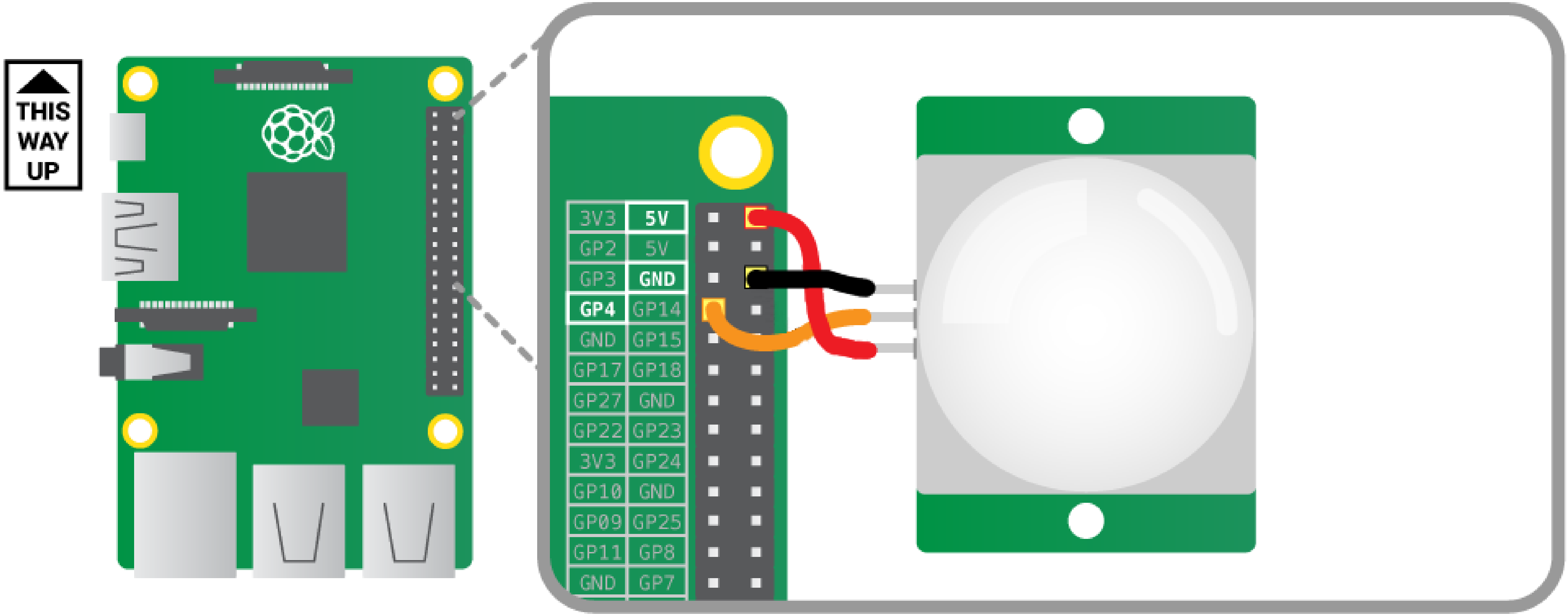
```python
# button.py
from picamera import PiCamera
from gpiozero import Button
from time import sleep

camera = PiCamera()
button = Button(17)

camera.start_preview(alpha=192)
button.wait_for_press()
sleep(3)
camera.capture("/home/pi/button.jpg"
)
camera.stop_preview()
```

```python
## motion.py
from picamera import PiCamera
from gpiozero import MotionSensor
from time import sleep

camera = PiCamera()
sensor = MotionSensor(4)

camera.start_preview(alpha=192)
sensor.wait_for_motion()
sleep(3)
camera.capture("/home/pi/pir.jpg"
)
camera.stop_preview()
```

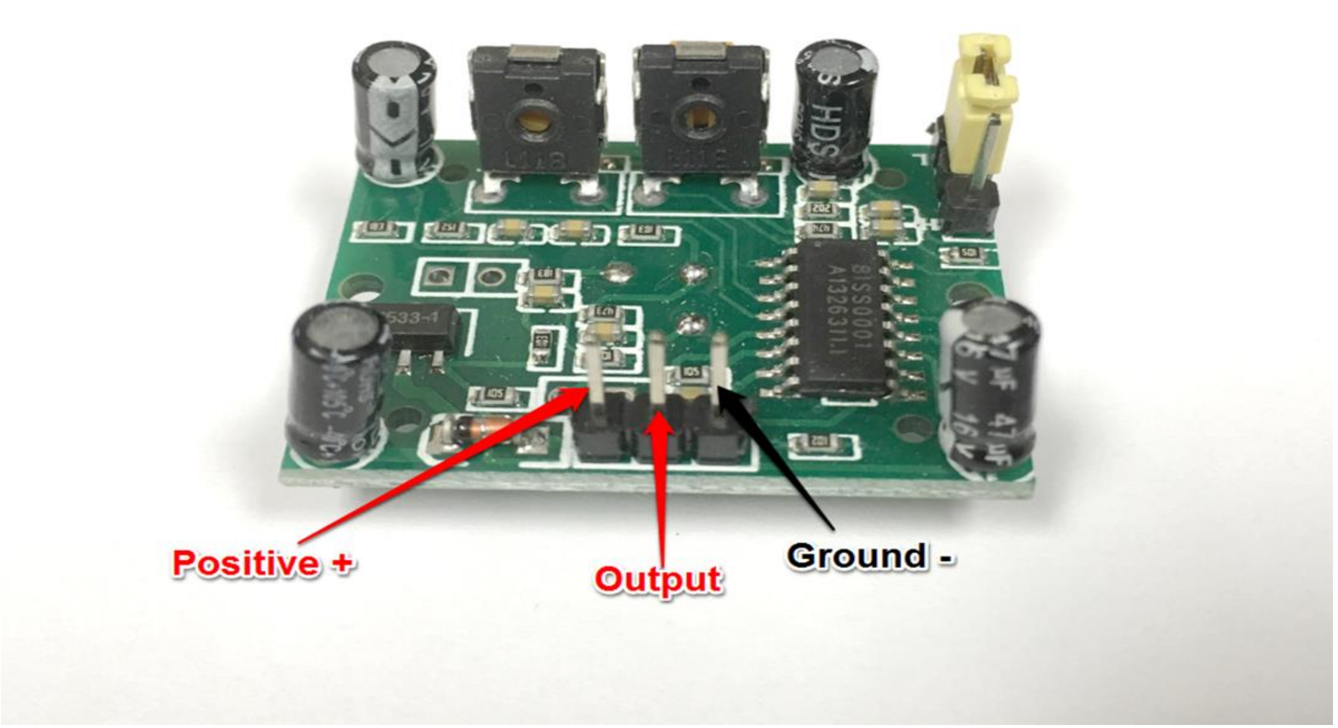
Capturing a time lapse

```
##timelapse.py
from picamera import PiCamera
from time import sleep

camera = PiCamera()
for num in range(1440):
     sleep(60)
     camera.start_preview(alpha=192)
     camera.capture("/home/pi/timelapse{0}.jpg".format(num))
     camera.stop_preview()
```

Connect a PIR Motion Sensor



Back of sensor



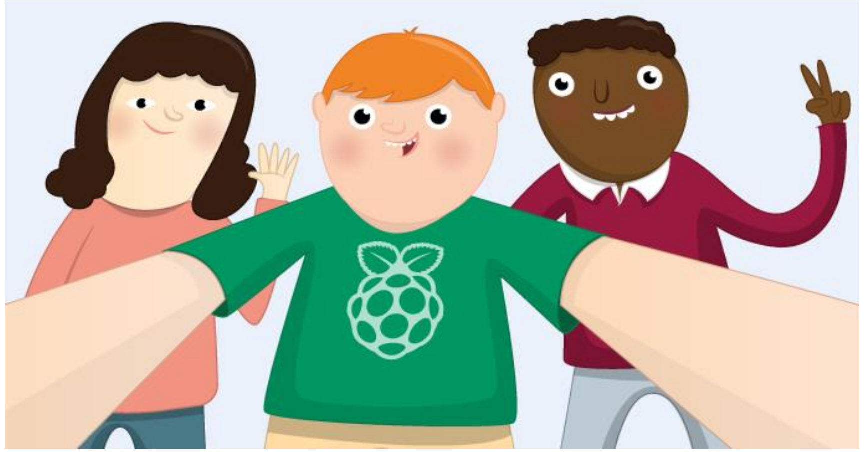
Documentation and help guides

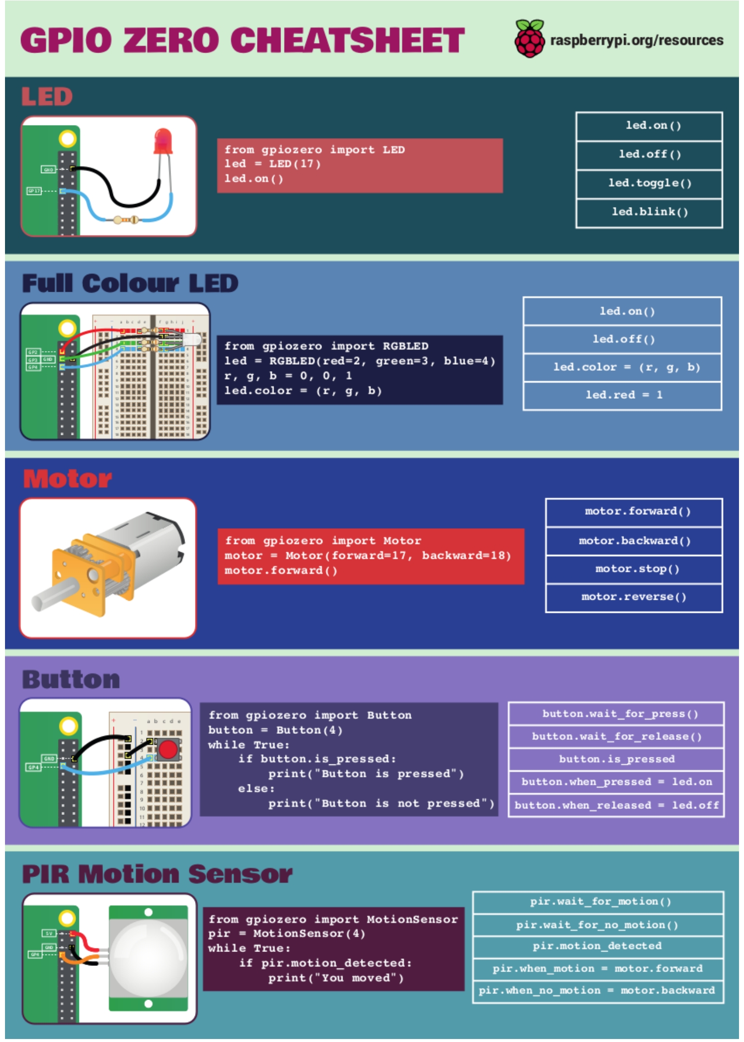
picamera.readthedocs.io

gpiozero.readthedocs.io

raspberrypi.org/resources

raspberrypi.org/education/downloads





Introduction to Python

What is my computer doing?

Programming comes down to reading and changing the ones and zeros in a computer's memory and that of the devices attached to your computer (hard disk, displays, keyboards).

You write code and save it to the disk.

Your computer loads the instructions into memory.

The processor reads an instruction performs it and looks at the next instruction.

What do these instructions look like?

Assembly / Machine Code

The lower level of instructions fed to a CPU.

Examples instructions:

ADD AL, *imm8*

BSWAP *r32*

CMPS *m64, m64*

Printing hello world in Assembly

```
section     .text
global      _start                              ;must be declared for linker (ld)

_start:                                       ;tell linker entry point

    mov     edx,len                            ;message length
    mov     ecx,msg                             ;message to write
    mov     ebx,1                              ;file descriptor (stdout)
    mov     eax,4                              ;system call number (sys_write)
    int     0x80                              ;call kernel

    mov     eax,1                              ;system call number (sys_exit)
    int     0x80                              ;call kernel
```

C and Compiled Languages

Luckily we don’t need to write code in assembly.

Some genius invented compiled languages like C.

Printing Hello World in C

```
#include <stdio.h>

int main(void) {

  printf("Hello, world.\n");

  return 0;

}
```

The compiler is a program that takes your code as a whole and converts it into machine code or assembly. “Building” is another word for compilation.

You can write C on any computer, but to say you have C or some other compiled language on your computer is to say you have the compiler.

Python and interpreted languages

Some other geniuses wrote the python interpreter in C.

Printing Hello World in Python

```
print(“Hello World”)
```

A interpreter is a program that reads, parses, and executes code line by line.

It doesn’t require a build step because your python code never gets turned into machine code.

Interpreted code runs MUCH SLOWER than compiled code.

- Python is a widely used general-purpose, high-level programming language
  - Easy to learn
  - Easy to read
  - IT’S FUN

Let’s start CODING!

Basic Variable Types

- Numbers
    - Integers
        - 1, -10, 0
    - Float
        - 5.3, 1.0
- String - a contiguous set of characters
    - “Hello”, “123”, “@!$”
- List - a sequence of items (of any data type)
    - [0, “stop”, 1.5]

Variable

- Assigning Variables
  - Equal sign (=) used to assign variables
  - Identifier to the left side, value to the right side

Numbers: Ints & Floats

```
>> myint = 7

>> print(myint)

>> myfloat = 7.0

>> print(myfloat)
```

Printing Strings & Variables

>>> print(“Hello World”)

a = 10

>> print(a)

Exercise:

Create an integer and float number. The int should be named myint. The float should be named myfloat. Then print both.

Number Arithmetic

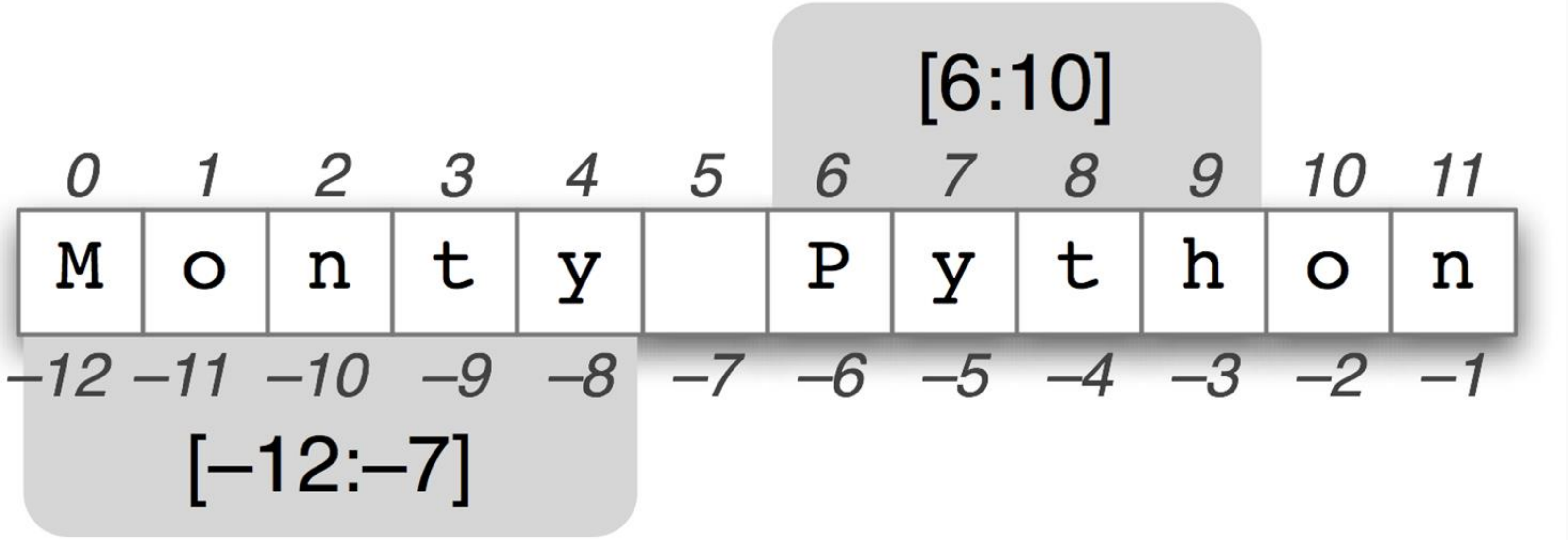
a = 10

b = 20

Operator	Description	Example
+ Addition	Adds values on either side of the operator.	a + b = 30
- Subtraction	Subtracts right hand operand from left hand operand.	a – b = -10
* Multiplication	Multiplies values on either side of the operator	a * b = 200
/ Division	Divides left hand operand by right hand operand	b / a = 2
% Modulus	Divides left hand operand by right hand operand and returns remainder	b % a = 0
** Exponent	Performs exponential (power) calculation on operators	a**b =10 to the power 20

Strings

- Anything in between quotation marks is a string
- Strings can be accessed by index
  - The first item is index 0!!!
  - You can select parts of string with [ ]



Exercise

```
str = 'Hello World!'

print str           # Prints complete string
print str[0]        # Prints first character of the string
print str[2:5]      # Prints characters starting from 3rd to 5th
print str[2:]       # Prints string starting from 3rd character
print str * 2       # Prints string two times
print str + "TEST" # Prints concatenated string
```

Exercise

- str = ‘Hello World!’
- print(str) # Prints complete string
- print(str[0]) # Prints first character of the string
- print(str[2:5]) # Prints characters starting from 3rd to 5th
- print(str[2:]) # Prints string starting from 3rd character
- print(str * 2) # Prints string two times
- print(str + “Test”) # Prints concatenated string

Adding / Concatenating Strings

```
>> helloworld = "hello" + " " + "world"

>> print(helloworld)

>> p = “pass”

>> s = “word”

>> print(p + s)
```

Exercise:
Create a string named mystring, like your name, favorite rap lyric, or sports team.

Now print the fourth letter of your string.

Lists

- A list contains items separated by commas and enclosed within square brackets ([ ])
    - Each item in a list can be a different type
- Like strings, items in a list can be accessed by index
    - First item is index 0
- You can add lists together and repeat lists as well

Exercise

- biglist = [‘abcd’, 786, 2.23, “john”, 70.2]
- tinylist = [123, ‘john’]
- print(biglist)   # Prints complete list
- print(biglist[0])   # Prints first element of the list
- print(biglist[1:3])   # Prints elements starting from 2nd till 3rd
- print(biglist[2:])   # Prints elements starting from 3rd element
- print(tinylist * 2)   # Prints list two times
- print(biglist + tinylist) # Prints concatenated lists

Why does it look like Young Thug is a TA checking to see if the code compiles without any errors

when you had to call the teacher over because you picked the one computer that wasn't working properly

Python Day 1 Review

Sports and Coding

- https://www.youtube.com/watch?v=6XvmhE1J9PY
- https://www.youtube.com/watch?v=kx3DxBbweGY
- https://www.youtube.com/watch?v=NYzN0a1U8mI
- https://www.youtube.com/watch?v=aiU4xQ7ICM4
- https://www.youtube.com/watch?v=3unOyGsBuXc
- https://www.youtube.com/watch?v=n2UHWECRXrg

String Review

Define these two variables:

wall = ‘w’

cannon = ‘c’

Using wall and cannon, the + and * operators, create these strings:

- “wc”
- “wcw”
- “wwwcwww”
- “wccwccwccwcc”
- "wwwcwwwcwwwcwwwcw"
- "wwwwcwwwwcwwwwcwwwwcwwwwc"

String Review 2

Start with the following assignment statement:

s = "abcdefghijklmnopqrstuvwxyz"

Using only string concatenation and [ ] on s, get:

- “drip”
- “packs”

Try to create your own words

You can change variable types

a = 10

b = str(a)

c = “58”

d = int(c)

You can check the type of a variable by:

type(a)

What are these types:

“3”

4.0

“Bob”

4

“3.565465”

4.01232

9.

What are these types:

“3” str: has
quotes around it

4.0 float: has
decimal point

“Bob” str: has quotes
around it

4 int: no
decimal,no quotes

“3.565465” str: has quotes
around it

User Input

We will often want to get the input from a user!

Now that we have learned variables, what better way to collect it than to store it in a variable (and one with a meaningful name too!!)

input("prompt")

where **prompt** is the string you want to show the user (this can be a question, entry form (think usernames/passwords,etc.)

If not stored in a variable, the user's input won't be recorded or stored anywhere (so your program won't know what the user said!!)

User Input

user_input = input(“prompt”)

user_input = int(input(“prompt”)

```
hours = int(input('How many hours?'))
print('This many hours:', hours)
rate = float(input('How many dollars per hour?'))
print('This many dollars per hour:  ', rate)
print('Weekly salary:  ', hours * rate)
```

How would we produce this output?

Hello. What is your name? **Kobe Bryant**
Hello, Kobe Bryant
It's nice to meet you.
How old are you? **24**
Next year you will be 25 years old.
Good-bye!

Exercise:

Try creating your own program asking the user some questions

AI Videos

- https://www.youtube.com/watch?v=8nt3edWLgIg
- https://www.youtube.com/watch?v=USXoINPEhoA&t
- https://www.youtube.com/watch?v=JhHMJCUmq28
- https://www.youtube.com/watch?v=5jA8wYqQLBU
- https://www.youtube.com/watch?v=XIzimkcuEuk
- https://www.youtube.com/watch?v=kaJgt1uyiJ8

Libraries

- There are tons of coders all around the world that have written all types of code

import MODULE

Tkinter

Python comes with a library called tkinter that enables drawing and creating images

```
import tkinter              # Load the library; do this just once per program

my_window = tkinter.Tk()    # Create the graphics window

my_canvas = tkinter.Canvas(my_window, width=500, height=500)  # Create a 500x500 canvas to draw on
my_canvas.pack()            # Put the canvas into the window

my_canvas.create_line(100, 100, 300, 300, fill='orange') # Draw orange line
my_canvas.create_line(300, 100, 100, 300, fill='blue')   # Draw blue line

tkinter.mainloop()          # Combine all the elements and display the window
```

Tkinter Methods

my_canvas.create_line(x1, y1, x1, y2)

my_canvas.create_oval(x1, y1, x2, y2) # x = top left; y = bottom right

my_canvas.create_rectangle(x1, y1, x2, y2) x =top left; y = bottom right

my_canvas.create_polygon(x1, y1, x2, y2, z1, z2) # need at least 3 vertices

Exercise:

Try creating your own shapes and pictures

Functions

- A function is a block of organized, reusable code that is used to perform a single, related action
- Functions have three parts: function declaration, body, return
- Functions begin with the keyword **def** followed by the function name and parentheses ( )
- The arguments/parameters should be placed within these parentheses
- The code block within every function starts with a colon and is indented
- https://www.youtube.com/watch?v=8T5acEwfJbw

Syntax for Functions

```
def functionname(parameters):
    #Function body
    return something
```

Indentation

```
def functionname(parameter):
#Function body
return something
```

```
def functionname(parameter):
    #Function body
    return something
```

Function Example

```
def f(x):
    x = x + 5
    return x

def y():
    print "hello"
```

```
f(10)

y()
```

Exponent Function

```
def exponent(big, small):
        return big**small


exponent(2, 3)
```

Exercise:

How do we write a function to return the first three letters of a string parameter

Exercise:
How do we write a function to calculate the average of four test scores as the parameter

Indentation

```
x = 1
if x == 1:
    print(“x is 1.”)
```

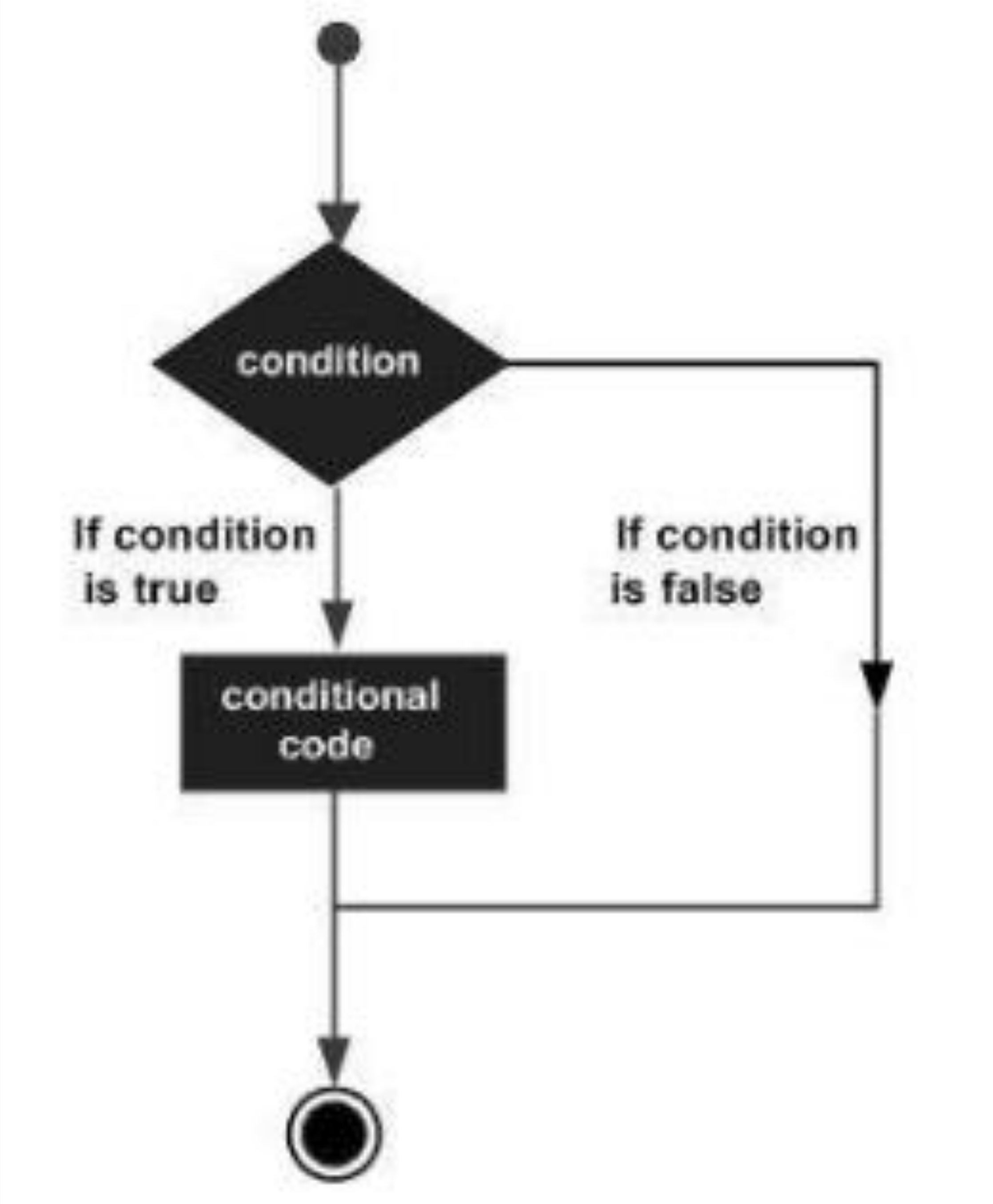
```
x = 1
if x == 1:
print(“x is 1.”)
```

Conditional Examples

```
x = 4 + 5
if x < 8:
   x = x + 2
elif x > 8:
   x = x + 3
else:
   x = x + 4
print x
```

```
x = 4 + 5
if x < 8:
   x = x + 2
if x > 8:
   x = x + 3
if x > 9:
   x = x + 4
print x
```

Equal to	==
Not equal to	!=
Less than	<
Less than or equal to	<=
Greater than	>
Greater than or equal to	>=



Conditional Example

```
name = "John"

age = 23

if name == "John" and age == 23:

    print("Your name is John, and you are also 23 years old.")

if name == "John" or name == "Rick":

    print("Your name is either John or Rick.")
```

Exercise

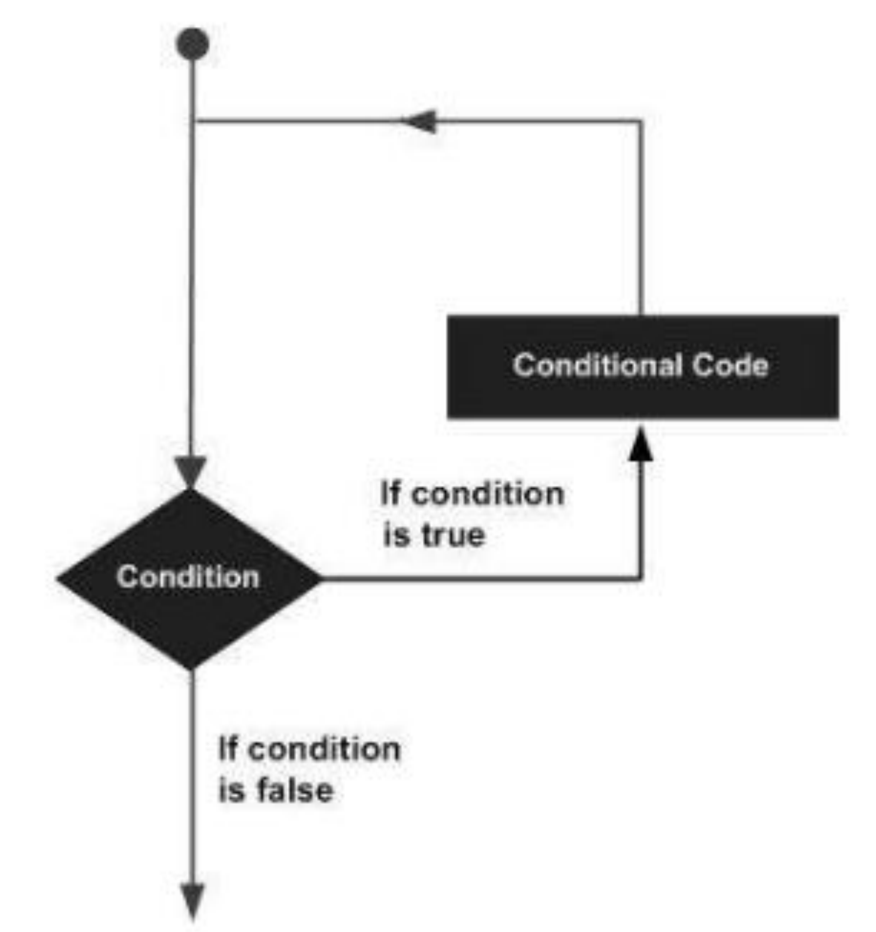
```
# change this code
number = 10
second_number = 10
first_array = [0, 3, 10]
second_array = [1, 2, 3]
```

```
if number > 15:
    print("1")
if len(second_array) == 2:
    print("3")
if len(first_array) + len(second_array) == 5:
    print("4")
if first_array[0] == 1:
    print("5")
```

Loops

- Used to handle repeated tasks
- For-loops
    - Loops a finite amount of times

```
for i in range(0, 3):

        print(i)
```



Loop Examples

```
primes = [2, 3, 5, 7]

for prime in primes:

    print(prime)
```

Loop Examples

```
for x in range(5):
    print(x)
for x in range(3, 6):
    print(x)
for x in range(3, 8, 2):
    print(x)
```

Recursion

- Recursion is used when you want to keep referring back to a function for a set amount of time
- Think of it like a for loop

Recursion (continued)

- If you wanted to calculate the sum of every number to a certain point you'd probably type
    - sum(1) = 1
    - sum(N) = N + sum(N-1) if N>1
- You'd input 4 and want to get something like
    - sum(4) = 4 + sum(3)
    - = 4 + 3 + sum(2)
    - = 4 + 3 + 2 + sum(1)
    - = 4 + 3 + 2 + 1
    - = 10

Recursion (continued)

```
sum = 0

def addin(x: int) -> int:

    global sum

    if x >= 1:

        sum = sum + x

        return addin(x - 1)

    else:

        return sum

print("The sum is " + str(addin(4)))
```

Let’s make a function!

```
def factorial (n: int) -> int:
    # Compute n! (n factorial)
    if n <= 0:
        return 1
    else:
        return n * factorial(n - 1)

print("10! is", factorial(10))
print("100! is", factorial(100))
```

Now try changing the print statement to print expressions

factorial(120)
factorial(50*10)

What happens when you do

factorial(factorial(50))

Let’s Change it up!

```
x = int(input("Enter a number:"))

def factorial(x):

    # Compute n! (n factorial)

    if x <= 0:

        return 1

    else:

        return x * factorial(x - 1)

print(str(x) + "! is ", str(factorial(x)))
```

Remember the adding problem!

Now write a recursion program that adds all of the numbers leading up to a number that you input

Name, Age, Year

- Have the code tell you your name, age, year it currently is, and year you'll be 100
- Hint: need input and data types

Odd or Even

- Have the code tell you whether a number is odd or even
- Hint: need a math symbol
- Whether a number is a multiple of 5, 6, and 7

Rock Paper Scissors

- Have two inputs that take in either rock, paper, or scissors and have it return who wins
- Hint: need inputs and if statements

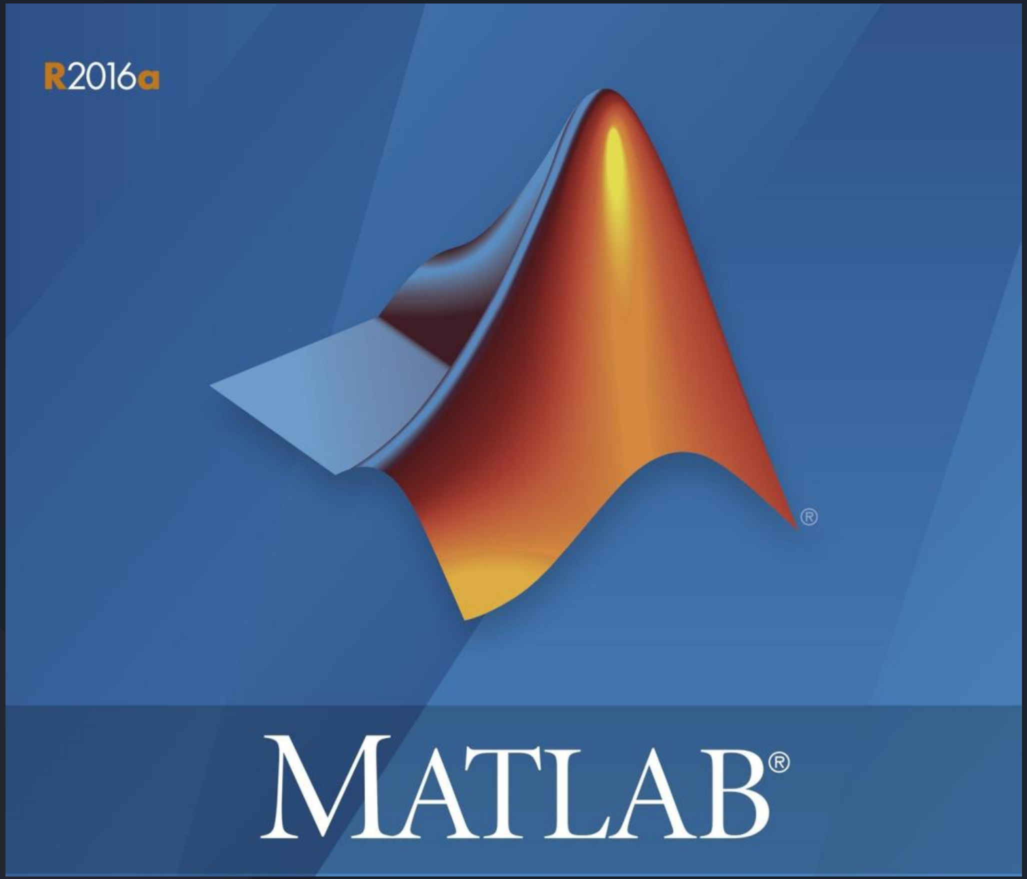
Fibonacci

- Tell the computer to give you a certain number of fibonacci numbers that you choose
- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,…..
- Hint: the pattern doesn’t follow for the first 2 numbers

Random Password Generator

- Have the code give you a random password that has lowercase letters, uppercase letters, and symbols
- Hint: need to use the random library and concatenation
- https://www.youtube.com/watch?v=RtUvMJFP_IE
- Ted Talk https://www.youtube.com/watch?v=hqKafI7Amd8
- https://medium.mybridge.co/30-amazing-python-projects-for-the-past-year-v-2018-9c310b04cdb3

MATLAB BOOTCAMP



With Jesus

Sample Syllabus

MATLAB Courses

- ENGRMAE 10
- ENGR 10
- EECS 10

Course Learning Outcomes. Students will:

1. Be introduced to Computing History.
3. Understand hierarchy of operations and command of various data types.
4. Understand and apply selective execution: Simple IF, nested IF, IF-THEN, and IF-THEN-ELSE structures.
5. Understand and use FORMATE input and output.
6. Understand and apply software flow control: DO loops, WHILE loops, and GOTO repetitive execution.
7. Understand and apply modular programming concepts: FUNCTIONS and SUBROUTINE.
8. Understand and apply single and multi-dimensional arrays: DIMENSION, vectors.

Prerequisites by Topic

Calculus

Lecture Topics:

- Introduction to Computing
- Basic UNIX and MATLAB commands
- Flow control - Selective execution, Repetitive execution and iterations
- Input and Output
- Modular Programming: Functions
- Arrays: one-dimensional and multi-dimensional
- Advanced data types
- Introduction to visualization and plotting
- Matlab as an engineering problem solving tool

Class Schedule: Meets for 3 hours of lecture and 1 hour of discussion each week for 10 weeks.

Computer Usage: Heavy computer usage. Students are required to develop computer programs for the solution of homework problems assigned on a weekly basis. The first week of the course introduces the students to basic navigation and file management commands used in the UNIX operating system. The MATLAB part of the course is executed on a Microsoft environment. No MATLAB toolkits are required.

http://plaza.eng.uci.edu/course/engrmae/10/outline/2017-2018

Day 1

The Basics

Fancy Calculator

5+5

10*7

3/2

15/2

3/2)

sin(2pi)

PEMDAS

7+8*5-

(7+8)*(5-

Layout

- Command Window
- Workspace
- Script
- Current Folder

Variables

Initialization

x=0 Always have to initialize before working with a variable

x=4

x='OAI'

You always need to have a variable equal something before you can start working with it

Elementary Math Functions

sin(pi) cos(3pi/4) tan(2pi) sqrt(4)

sind(90) cosd(75) tand(20) 5^7

log(21) exp(3)=e^4 abs(x)

mean(x)

median(x)

mode(x)

Maintenance

-clear

+variable

-clc

-close *everything is case sensitive*

+all

-format

+compact +default

Miscellaneous Functions

who -Tell you what variable you have

whos -Gives variables, sizes, space used, and type

real max -Gives the highest value matlab can work with

real min -Gives lowest value

eps -Gives smallest value of precious

size(x) -Returns rows and columns of matrix

length(x) -Returns largest array dimension

numel(x) -Returns total number of elements

find(x) -Returns location number of all non zero values

Printing to Command Window

disp(Variable): Displays the value of the variable

fprintf('This is a string %f1.8 %i ', x , y)

%f2.3 *2: Leading spaces* *3: Precision*

i% Displays an integer

x is the first variable displayed %f

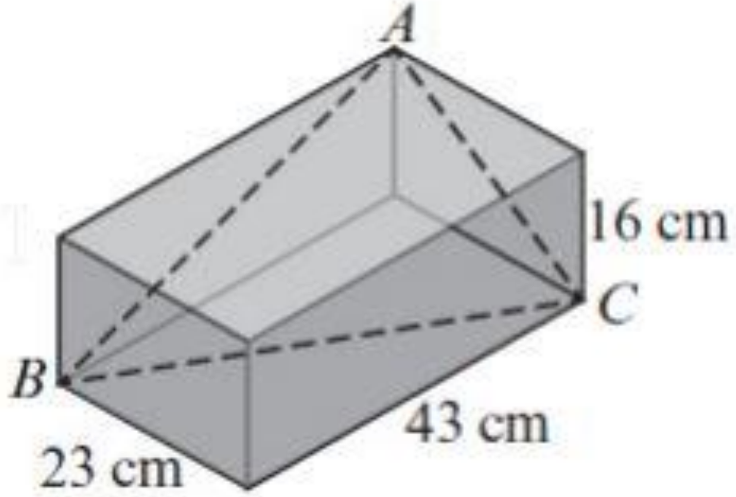
y is the second %i

HomeWork

1. A rectangular box has the dimensions shown.

   (a) Determine the angle BAC to the nearest degree.

   (b) Determine the area of the triangle ABC to the nearest tenth of a cm.



Useful relations:

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$

Heron's formula for triangular area: $A = \sqrt{(p(p-a)(p-b)(p-c)}$, where $p = \frac{a+b+c}{2}$

Day 2

With Jesus...again

Arrays

Introduction to One-Dim Vectors

Variables

x=4   b=7   OAI=18

Overwriting

x=4, x=7

Case Sensitive

x=4, X=7

Creating Arrays

x=[]

y=[5 3 2 5]   1x4 array

z=[5 3 2 5; 3 2 6 4; 3 5 7 4]   3x4 array

z'   the ' switches the rows and columns

Advanced Techniques

matrix=[1:100] - Makes a row of 1 to 100 with spacing of 1 (default)

matrix=[1:10:100] -Row 1 to 100 with spacing of 10

=[10,20,30,40,50,60,70,80,90,100]

matrix=[linspace(0,10)] creates 100 equal point (default) between 0 and 10

matrix=[linspace(0,10,5)] creates 5 equal points (default) between 0 and 10(can negative)

x=zeroes(x,y) creates an x by y matrix of zeroes

Manipulating Arrays

Indexing

x=[2 4 1 5 7]

x(4) Output=5

x(2:5) Output=[4 1 5 7]

Adding

x(2, :)= [1 2 3 4 5]   Output: x=[2 4 1 5 7; 1 2 3 4 5]

Deleting

x(1, :)=[]

String Arrays

f=’hello’

x=[‘hello’ ‘world’]

z=[‘hello’ ; ‘world’]

z=[‘hi’; ‘hello’]   Error: Dimensions do not match

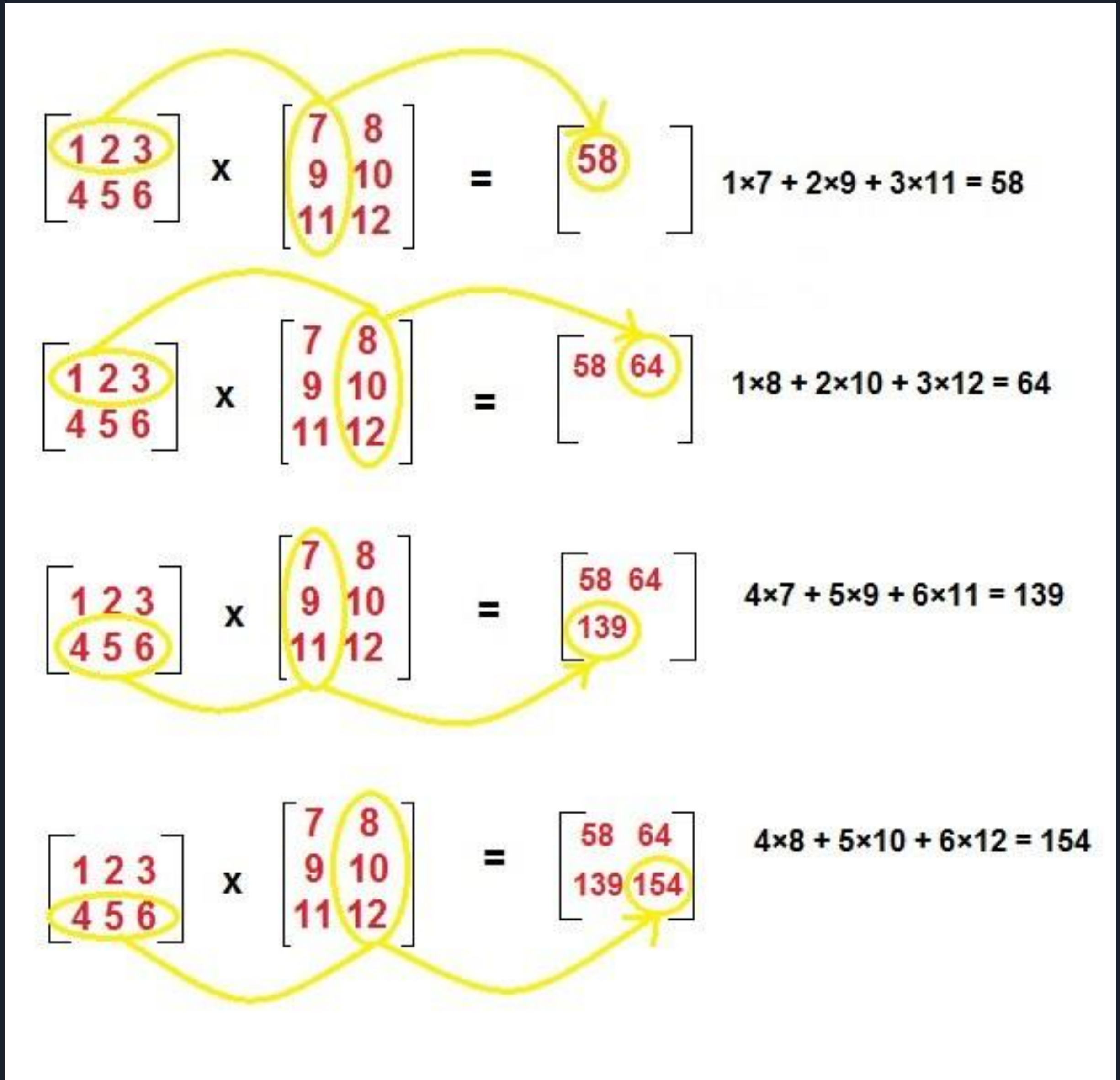
z(4) Output=e

x=345

num2str(x)   -Makes a number into a string

Mathematical Operations with Matrices

$$\begin{array}{ccccc} \mathbf{A} & + & \mathbf{B} & = & \mathbf{C} \\ \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -2 & 2 \end{bmatrix} & + & \begin{bmatrix} 1 & 1 \\ 4 & 2 \\ -2 & 1 \end{bmatrix} & = & \begin{bmatrix} 3 & 2 \\ 7 & 4 \\ -4 & 3 \end{bmatrix} \\ (3,2) & & (3,2) & & (3,2) \end{array}$$



Practice

Problem 3

Create a table of x, cos(**x**), and sin(**x**) where **x** goes from 0 to 2pi in increments of 0.1pi . The output should be in a three column table, the first column containing the values of **x**, the second column containing the values of cos(**x**), the third column containing the values of sin(**x**).

HomeWork

Problem 4

A cannon is fired on level ground. Allow the user to input the initial total velocity **v0** and initial elevation angle **theta** in degrees (0 degrees = cannon fired horizontally, 90 degrees = cannon fired straight up). Output the **x** and **y** position of the cannon ball from 0 seconds to 1 second in increments of 0.1 seconds in a nice table format with three columns. Your output should look similar to the following table:

time	x	y
0	0	0
0.1	1.21	2.28
0.2	2.42	4.18

(and so on until t = 1.0)
Remember that **theta** should be in radians for use in trigonometic functions, not degrees. Assume the initial position is **(x,y)** = (0,0). The vertical acceleration due to gravity is -9.8 m/s^2.

Day 3

Algorithm Development

Basic Programming Structures

Algorithms: Simple or Complex scripts to solve problems

If statments

For Loops

While Loops

Relational and Logical Operators

< Less Than

> Greater Than To be used with

== Equal to if
statements

>= Greater than or equal to While
loops

<=Less than or equal to

~= Not equal to

if , elseif , else

If: x==2

- Used to start the loop, only used at start
- Takes a conditional statement

Elseif: x>=2

- Follows if

Only one of any of

- Also takes a conditional statement these will happen
- If failed continues to next elseif in one loop

else:

- Goes at the end of if loop, only used at end
- Statements will happen for sure if all else fails

For Loops

For: x=[1:5]

- Loops over for a certain range
- Set a variable equal to a matrix of chosen values
- Goes through matrix top to bottom, then from left to right

end

While Loops

While x=<10

- Continues until the condition is false
- Have to integrate a variable into the loop
- Such as x=x+1
- Otherwise you get infinite loops

end

Pseudo Code

Initialize your variables

Check if k > p

If it is true	If false
End	Go through other statements

Increase by k by n

Restart loop

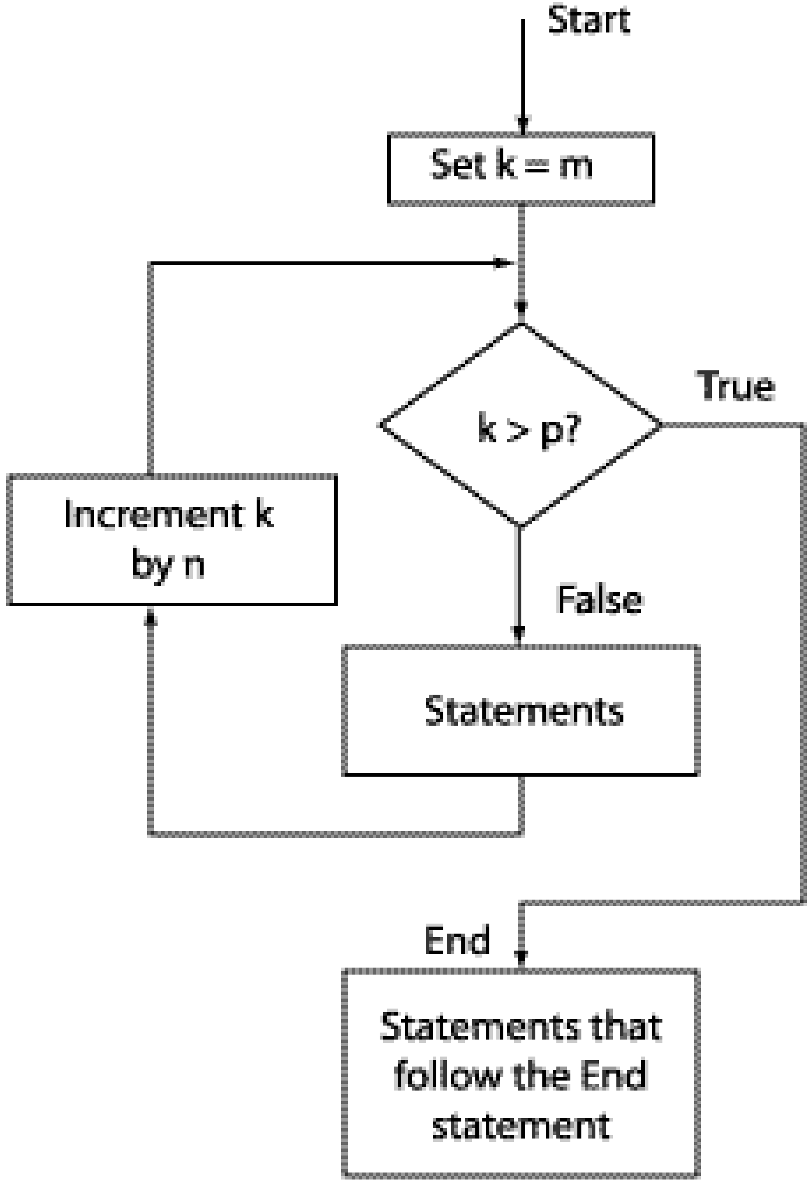
Flow Charts

Visualize the route of the code

Alternative to Pseudo Code

Use different shapes to differentiate

- Conditionals
- Statements



Flowchart of the for loop

Day 4

Practice Problems

Revisited Cannon Problem

Problem 2

Revisit the cannon problem from Homework 2. Do you remember how, depending on the input parameters, the trajectory could go negative? You can avoid this by writing a program with a loop.

A cannon is fired on level ground. Assume the user inputs an initial velocity (v0) of 100 m/s and an initial elevation angle of theta = 25 degrees. Using a **while** loop, calculate the x and y position of the cannon ball in time steps of 0.1 seconds until the cannon ball hits the ground. Assume the initial position is (x,y) = (0,0). The vertical acceleration due to gravity is -9.8 m/s^2. Display the time, x-position and y-position in a nice table format with three columns. (Your output should look similar to the table from the previous cannon problem.)

Revisited Cannon Problem

Problem 4

A cannon is fired on level ground. Allow the user to input the initial total velocity **v0** and initial elevation angle **theta** in degrees (0 degrees = cannon fired horizontally, 90 degrees = cannon fired straight up). Output the **x** and **y** position of the cannon ball from 0 seconds to 1 second in increments of 0.1 seconds in a nice table format with three columns. Your output should look similar to the following table:

time	x	y
0	0	0
0.1	1.21	2.28
0.2	2.42	4.18

(and so on until t = 1.0)
Remember that **theta** should be in radians for use in trigonometic functions, not degrees. Assume the initial position is **(x,y)** = (0,0). The vertical acceleration due to gravity is -9.8 m/s^2.

Fibonacci Sequence

Problem 4

The Fibonacci sequence is the following:

1, 1, 2, 3, 5, 8, 13, 21, ...

The sequence starts with 1 and 1, then all subsequent values are determined by summing the previous two numbers in the sequence.

Have your program display the first N Fibonacci numbers in a column, where N is a number you select. Choose a value of N that is greater than 2 (do not use the `input()` function, simply hard-code the value of N by typing N = some number greater than 2). For example, if you choose N=6 the program should display:

1
1
2
3
5
8

Calculate all Fibonacci numbers except the first two (1, 1) using a **for** loop and display them on the screen. Although you will assign a value to N in your program, make your algorithm general for any value of N > 2.

HomeWork

Problem 5

Make a program that calculates N! where,

N! = N * (N-1) * (N-2) * ... * 3 * 2 * 1

N is a value you choose (as in problem 2, do not use the `input()` function, simply state N = some number). For example, if you choose N=5 your program should calculate 120 and display the result to the screen.

Calculate N! using a **for** loop first, then recalculate N! using **while** loop. Did you get the same result? Check your answer with the `factorial()` function.

Estimate Pi

Problem 1

The value of π can be estimated from:

$$\frac{\pi^3}{32} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3}$$

Write a program (using a loop) that determines π for a given n. Run the program with $n = 10$, $n = 100$, and $n = 1{,}000$. Compare the result with pi (using 6 decimal places). Do you remember how to take a cube root? Hint: square root of 5 is 5^(1/2).

Stand for

Matrix

Laboratory

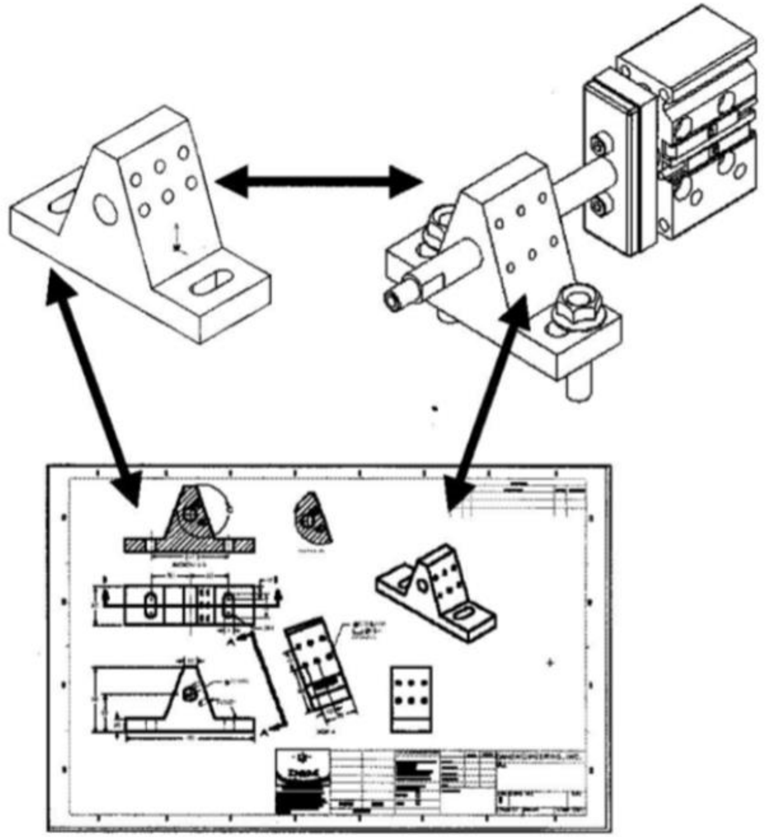
INTRO TO SOLIDWORKS

INTRO TO SOLIDWORKS

- About SolidWorks
- Basic Toolbars
    - Sketch
    - Feature
    - Assembly
- Keyboard/mouse tools
- Sketching Basics
    - Relations
- Feature Creation
    - Extrusions

WHAT IS SOLIDWORKS?

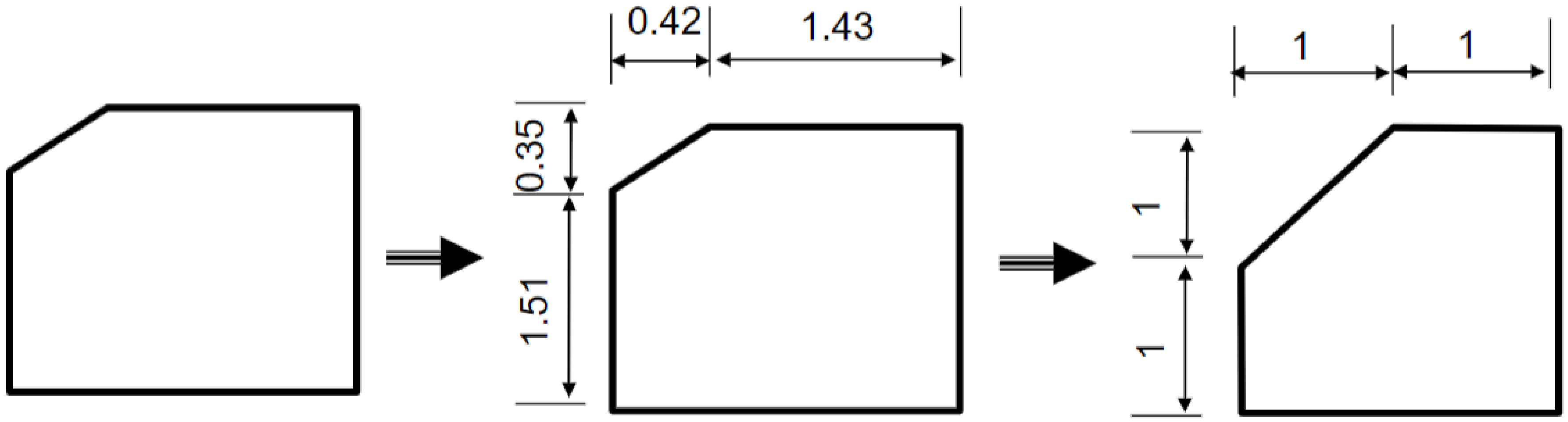
- Computer Aided Design (CAD)
  - Design Automation Software package used to produce
    - Parts
    - Assemblies
    - Drawings



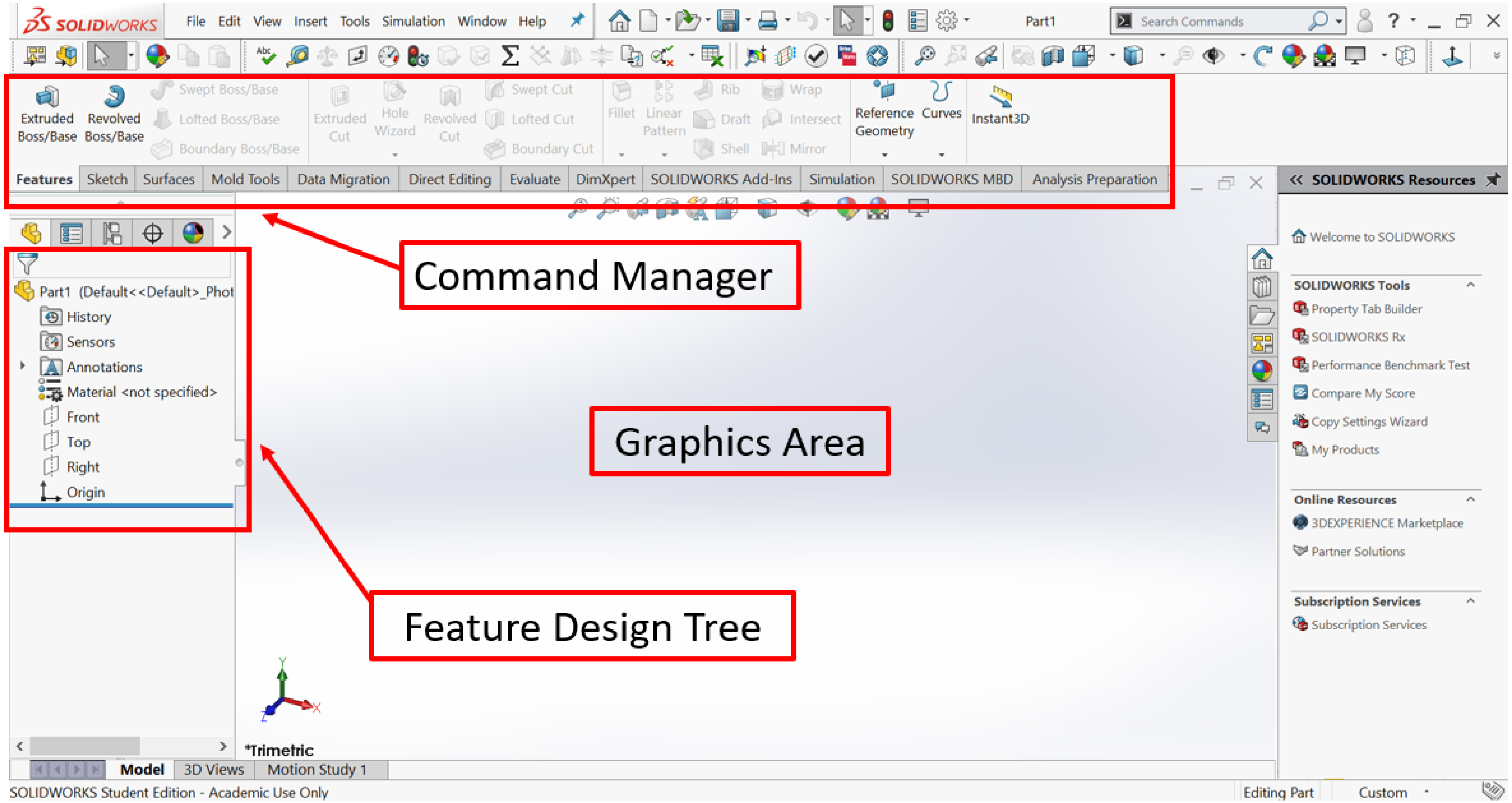
(From Planchard and Planchard 2003)

2D-SKETCHING PROCEDURE

1. Sketch the geometry
2. Dimension the geometry
3. Modify the dimension values



SOLIDWORKS: USER INTERFACE



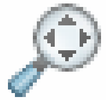
KEYBOARD: IMPORTANT KEY COMMANDS

- Control Z is your best friend (the undo option) and you'll use it a lot in the beginning!

- Escape is how you get "out" of any operation you are "in"

MOUSE TOOLS

- Zoom
  - Zoom out: Scroll up
  - Zoom in: Scroll down
- Bear in mind that where you zoom in depends on where your cursor is on the screen
  - Move cursor to the left/right: zooms to left/right
  - Move cursor to the top/bottom: zooms top/bottom
- If you don't like that, just use:
  - Zoom to Fit (fits whole part on screen)

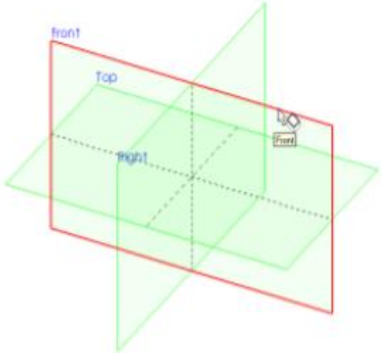
    

  - Zoom to Area

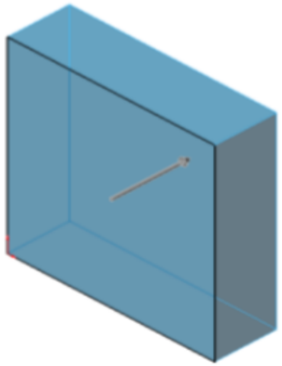
GENERAL PROCEDURE

1. Select a sketch plane. Choose a flat 2D surface to define your plane.
2. Sketch a 2D profile. Sketching make use of various geometric relationships and dimensions to define a profile. Think of this as your "cookie cutter."
3. Add a Feature. Features are the building blocks of the part. Features are the shapes and operations that construct the part. In this example, we Extrude the sketch perpendicular to sketch plane to get a base feature.



Select the sketch plane

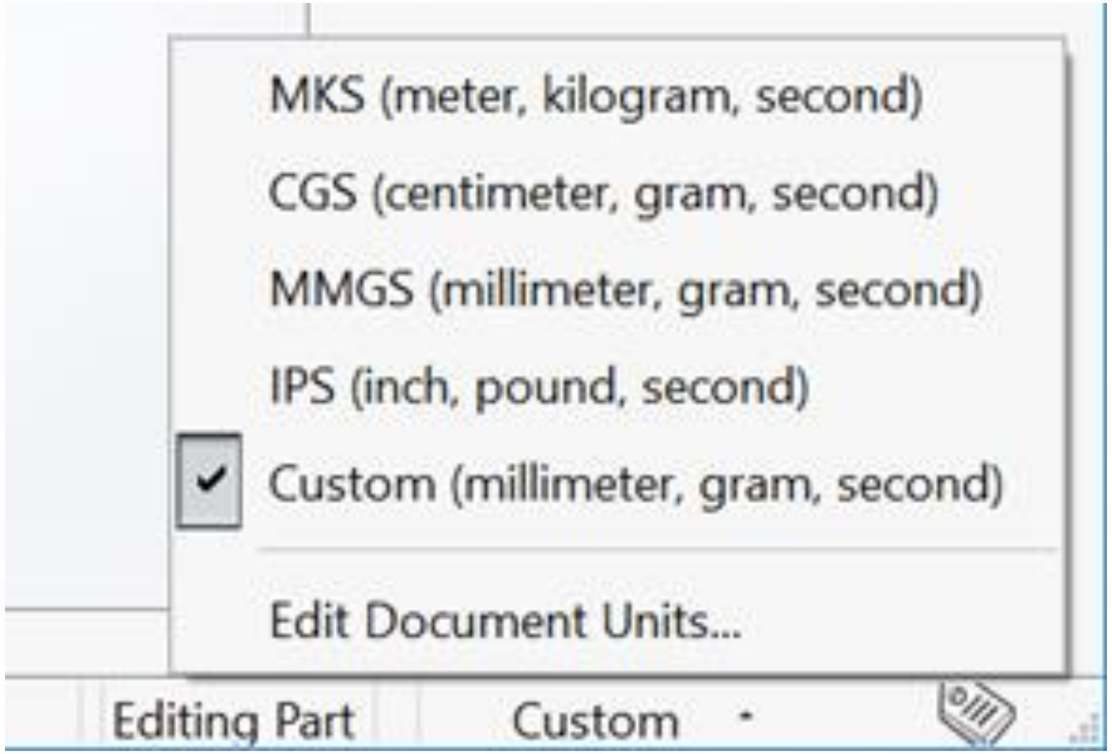
Sketch the 2D profile



Extrude the sketch

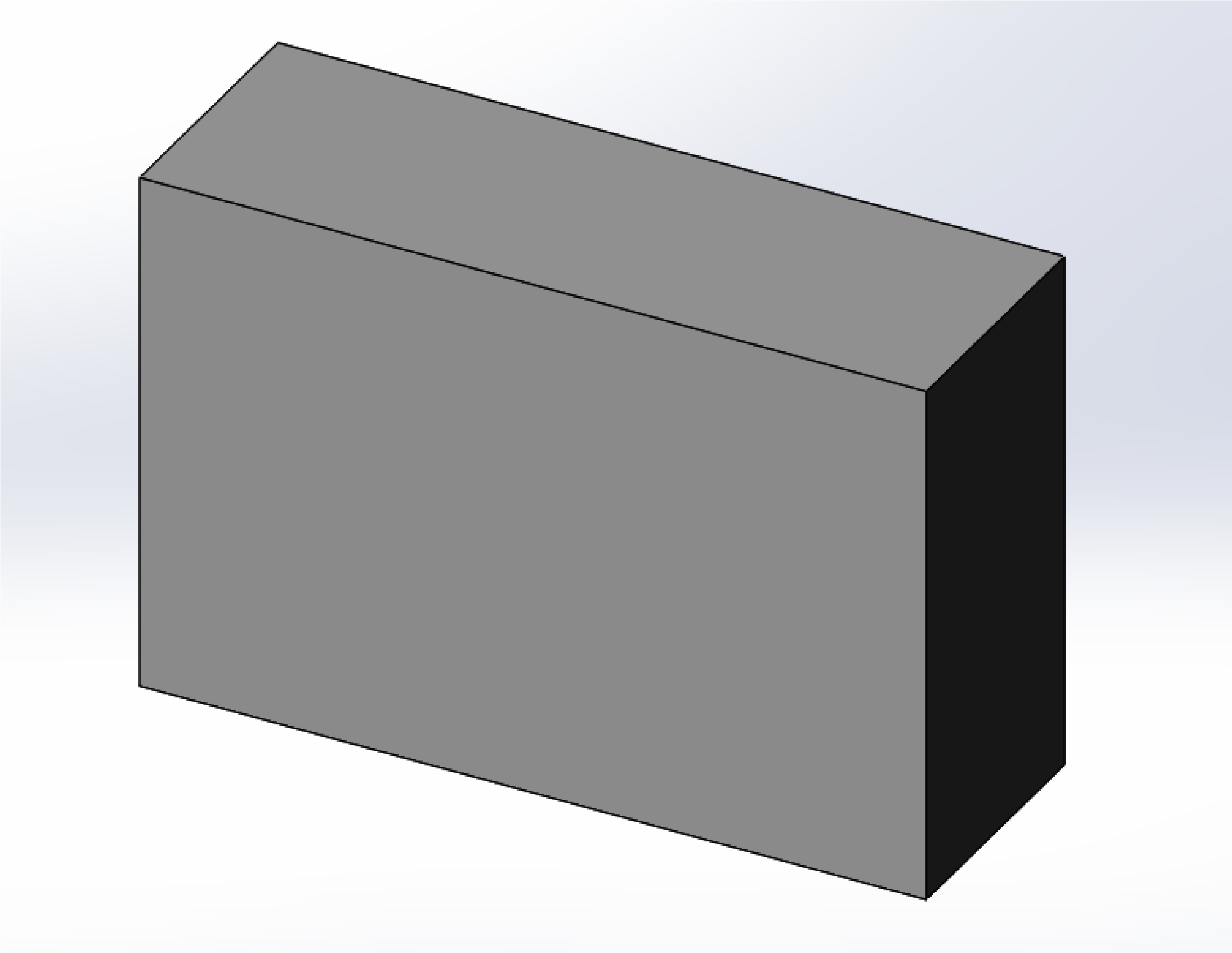
SOLIDWORKS: DOCUMENT PROPERTIES

Unit system is displayed at the bottom right corner for easy reference.

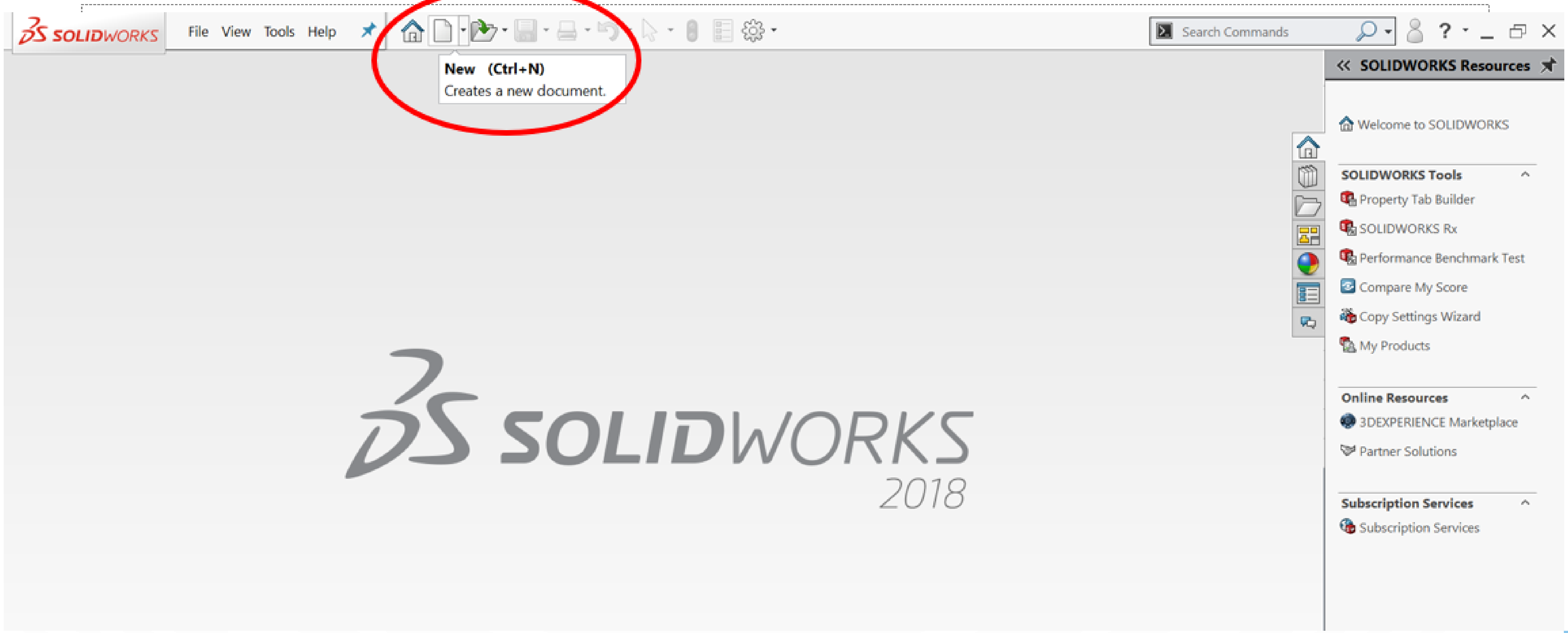


NOW, LET'S DO SOME 3D MODELING!

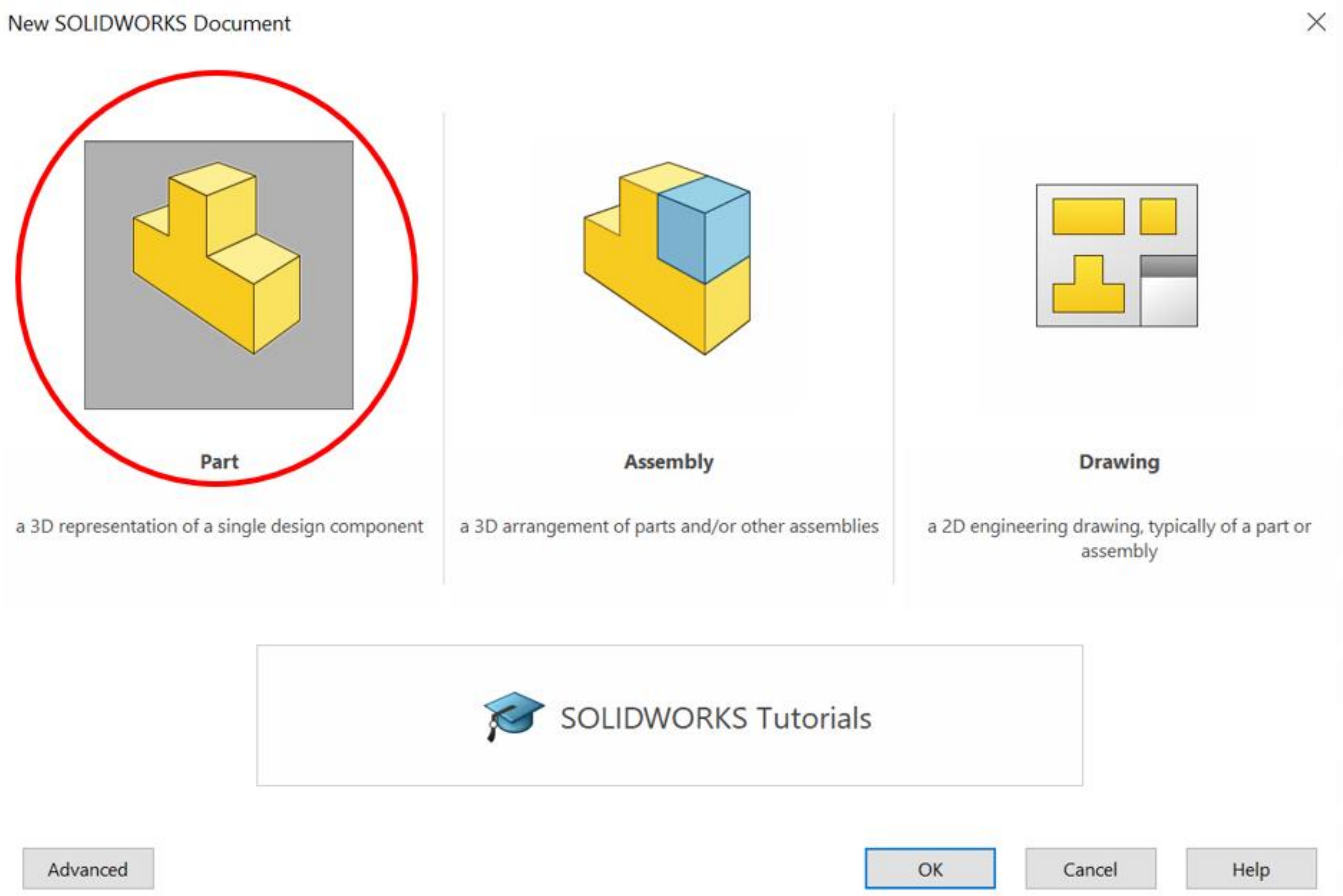
- Create a new part
- Sketches
- Extrude
- Extrude Cut



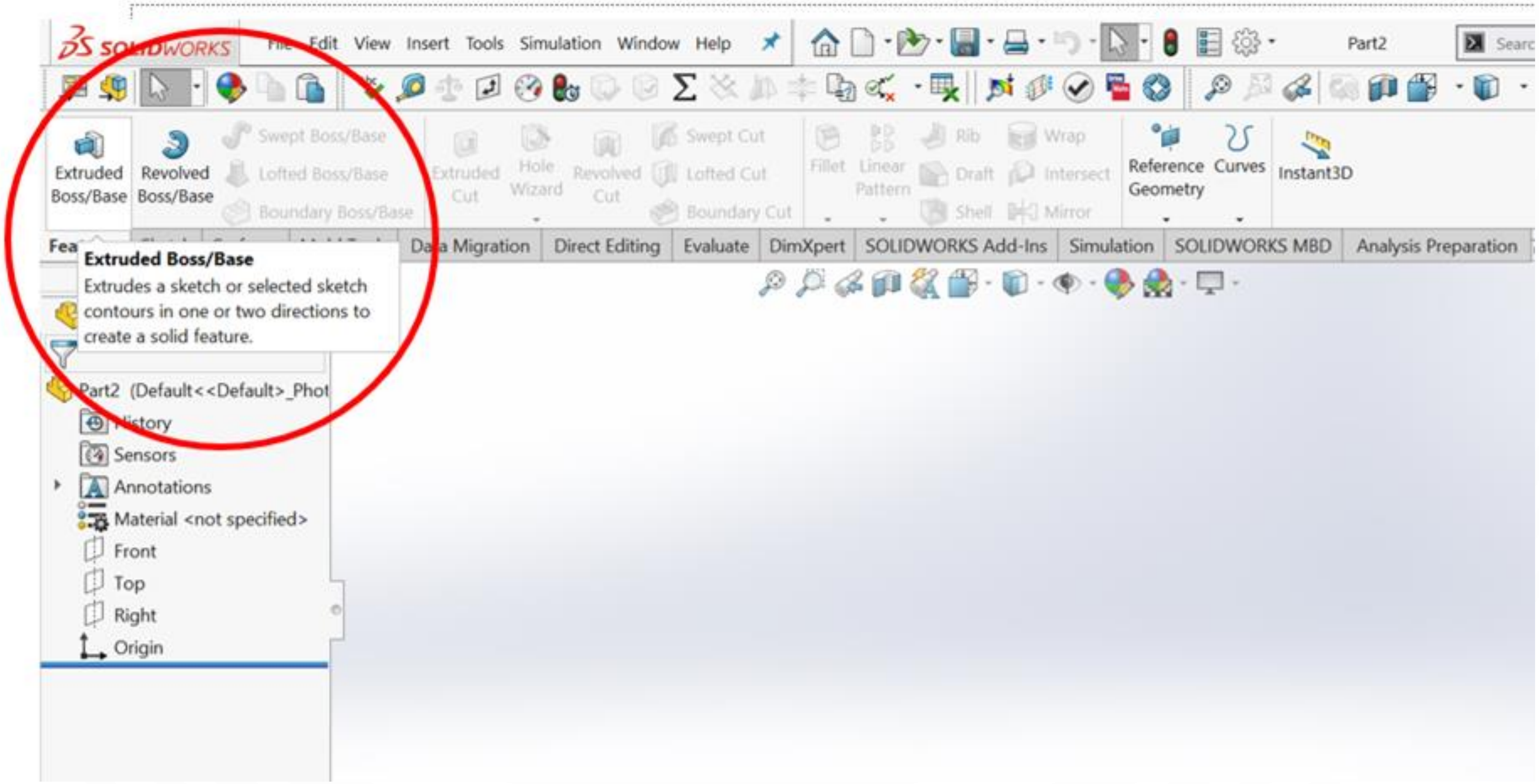
SOLIDWORKS BASICS



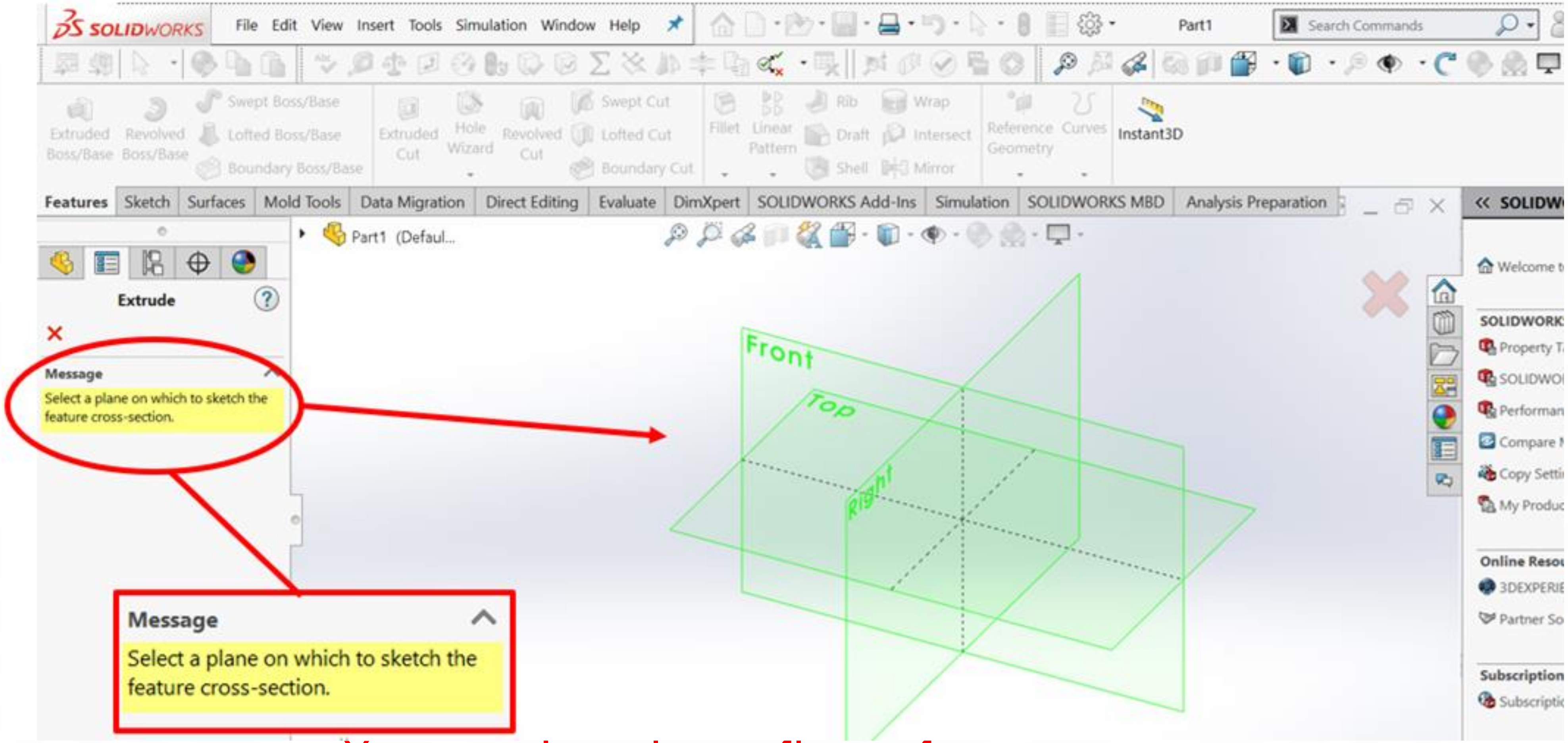
SOLIDWORKS BASICS



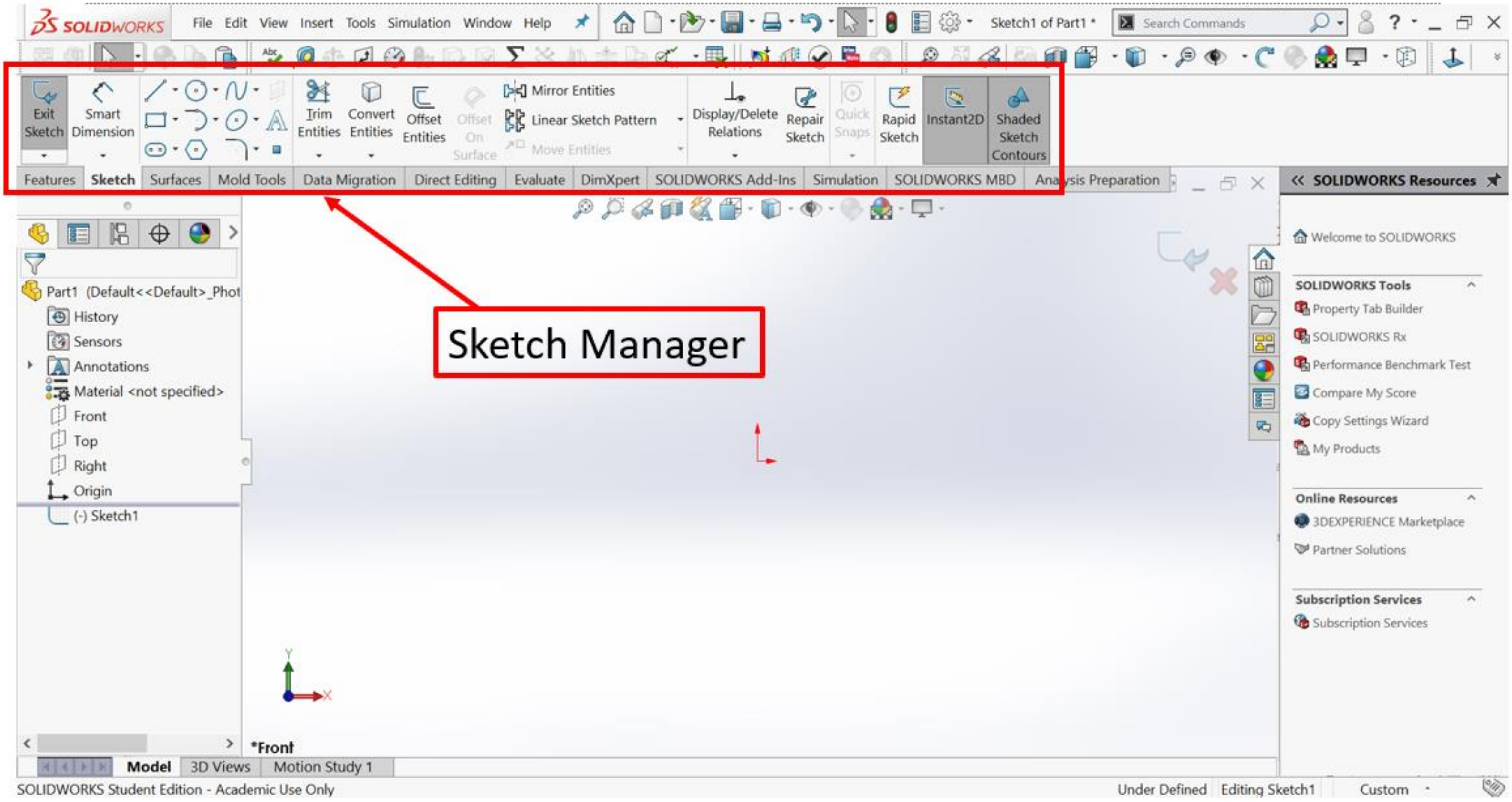
SOLIDWORKS BASICS



SOLIDWORKS BASICS



You can also select a flat surface on a part.


Sketch Manager
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File Edit View Insert Tools Simulation Window Help
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Mirror Entities
Linear Sketch Pattern
Move Entities
Display/Delete Relations
Repair Sketch
Quick Snaps
Rapid Sketch
Instant2D
Shaded Sketch Contours
Features Sketch Surfaces Mold Tools Data Migration Direct Editing Evaluate DimXpert SOLIDWORKS Add-Ins Simulation SOLIDWORKS MBD Analysis Preparation
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Performance Benchmark Test
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Copy Settings Wizard
My Products
Online Resources
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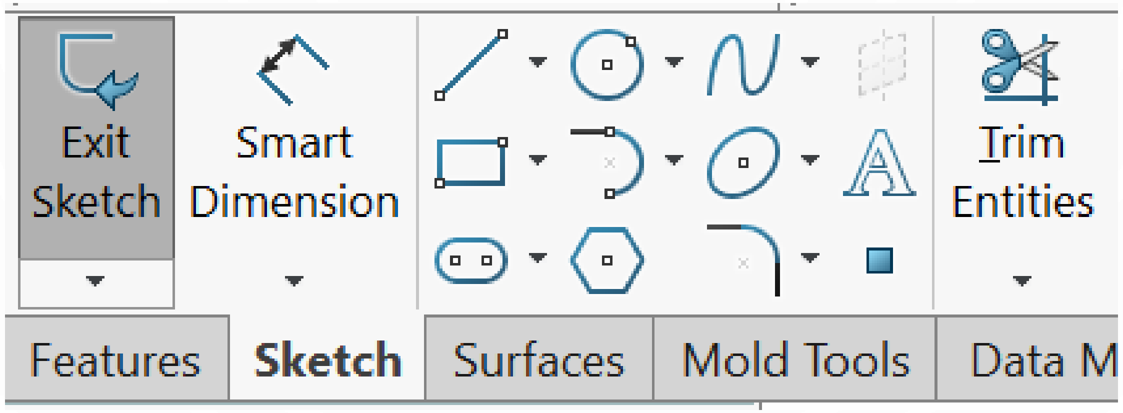
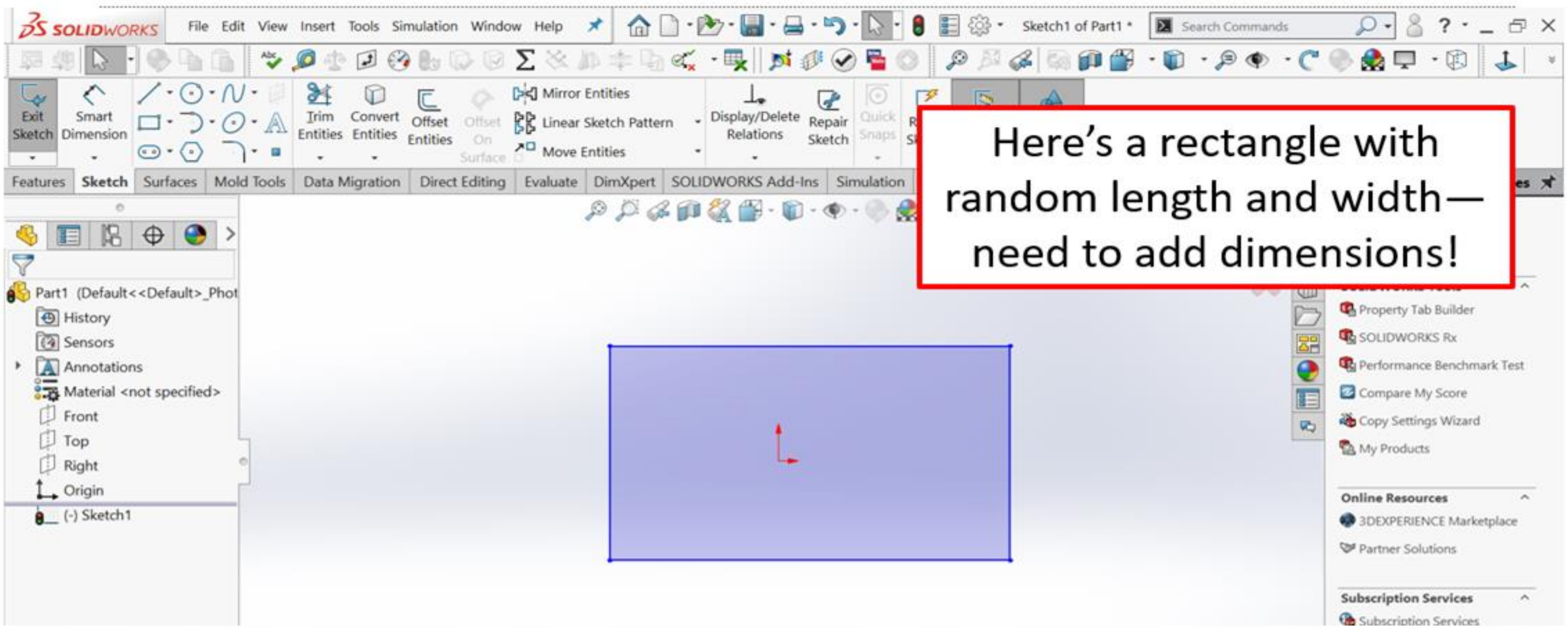


Almost all the sketch tools you need are here

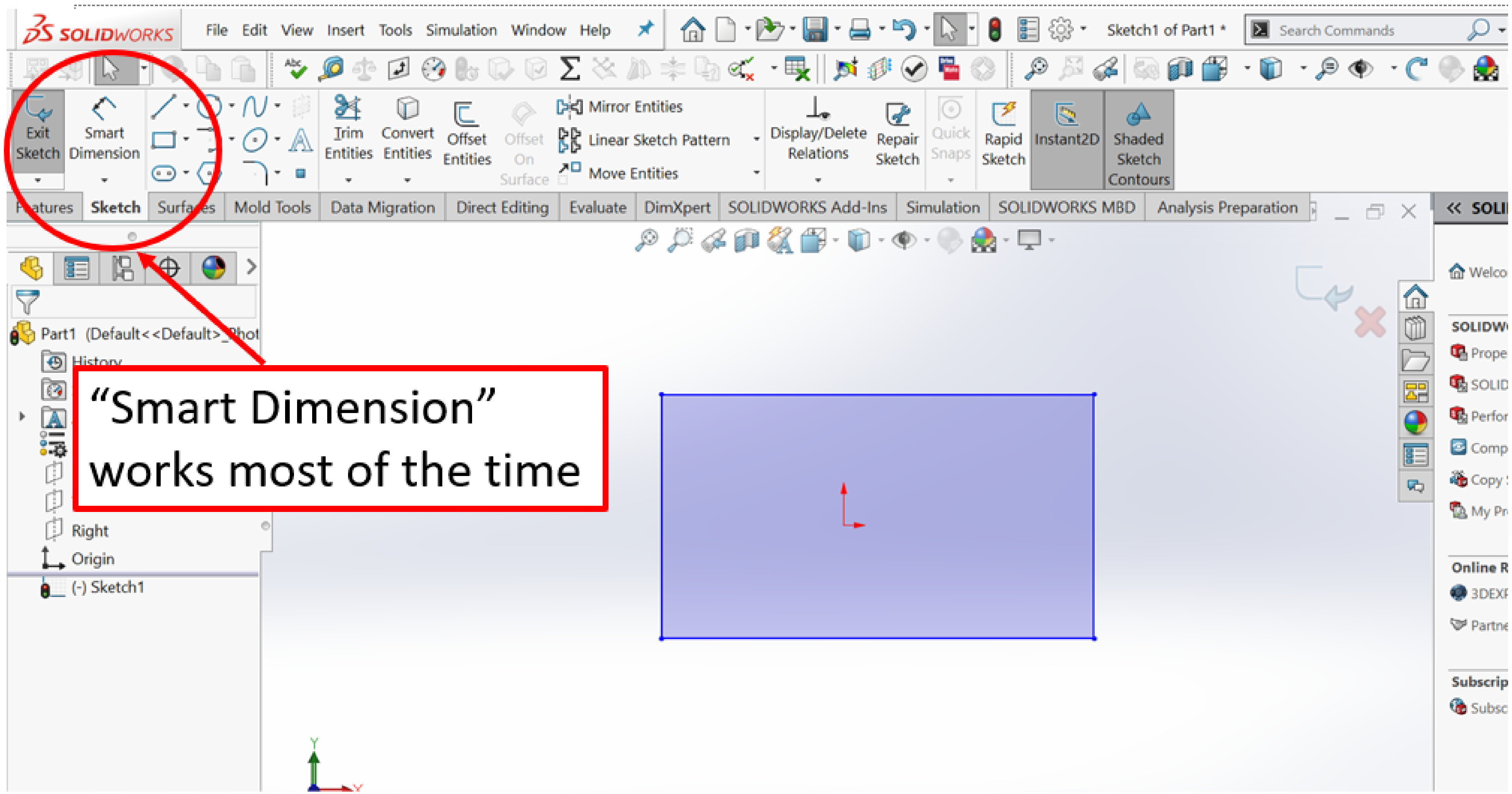
- Lines
- Rectangles
- Slot
- Circle
- Arc
- Polygons
- Spline
- Ellipse
- Fillet/Chamfer
- Trim
- Dimension

SOLIDWORKS BASICS



Use the sketch tools to make "cookie cutters" to grow or cut material from your part.

SOLIDWORKS BASICS



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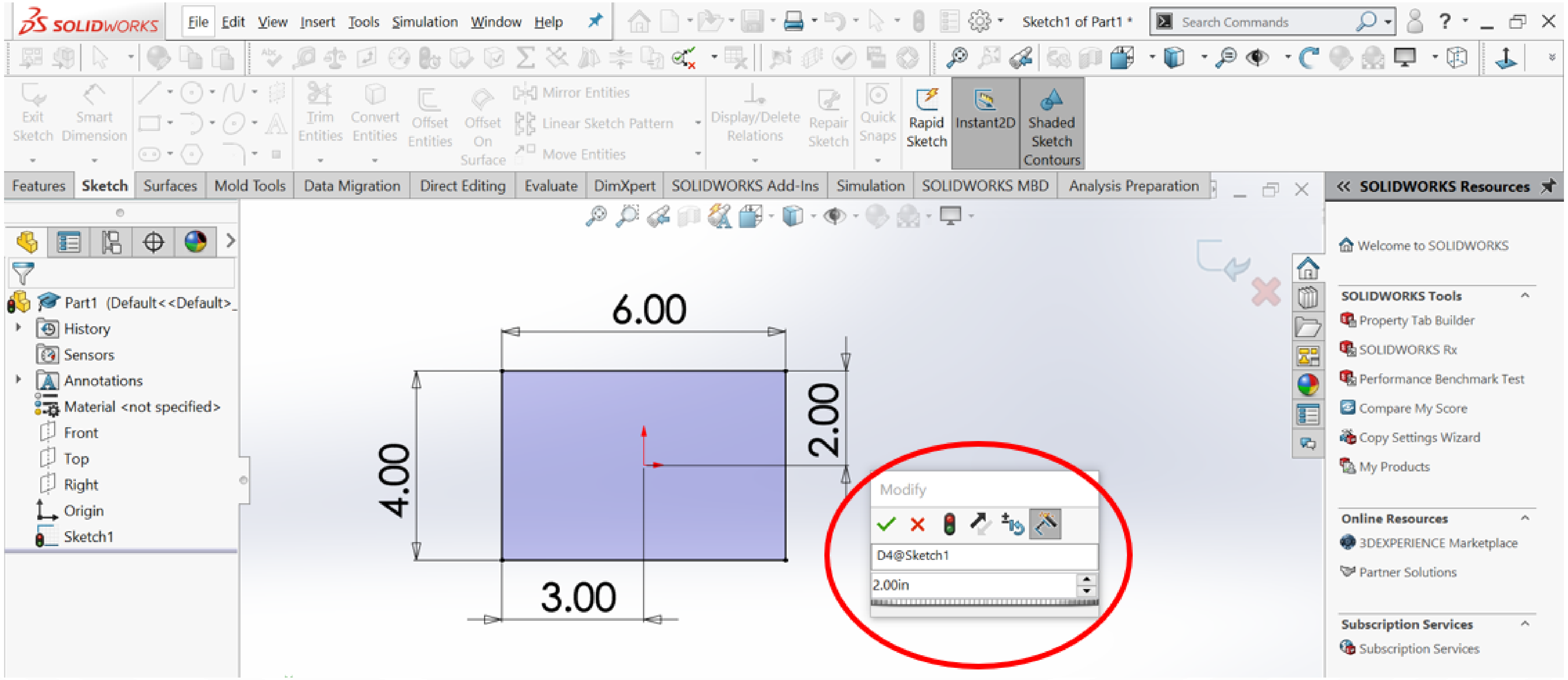
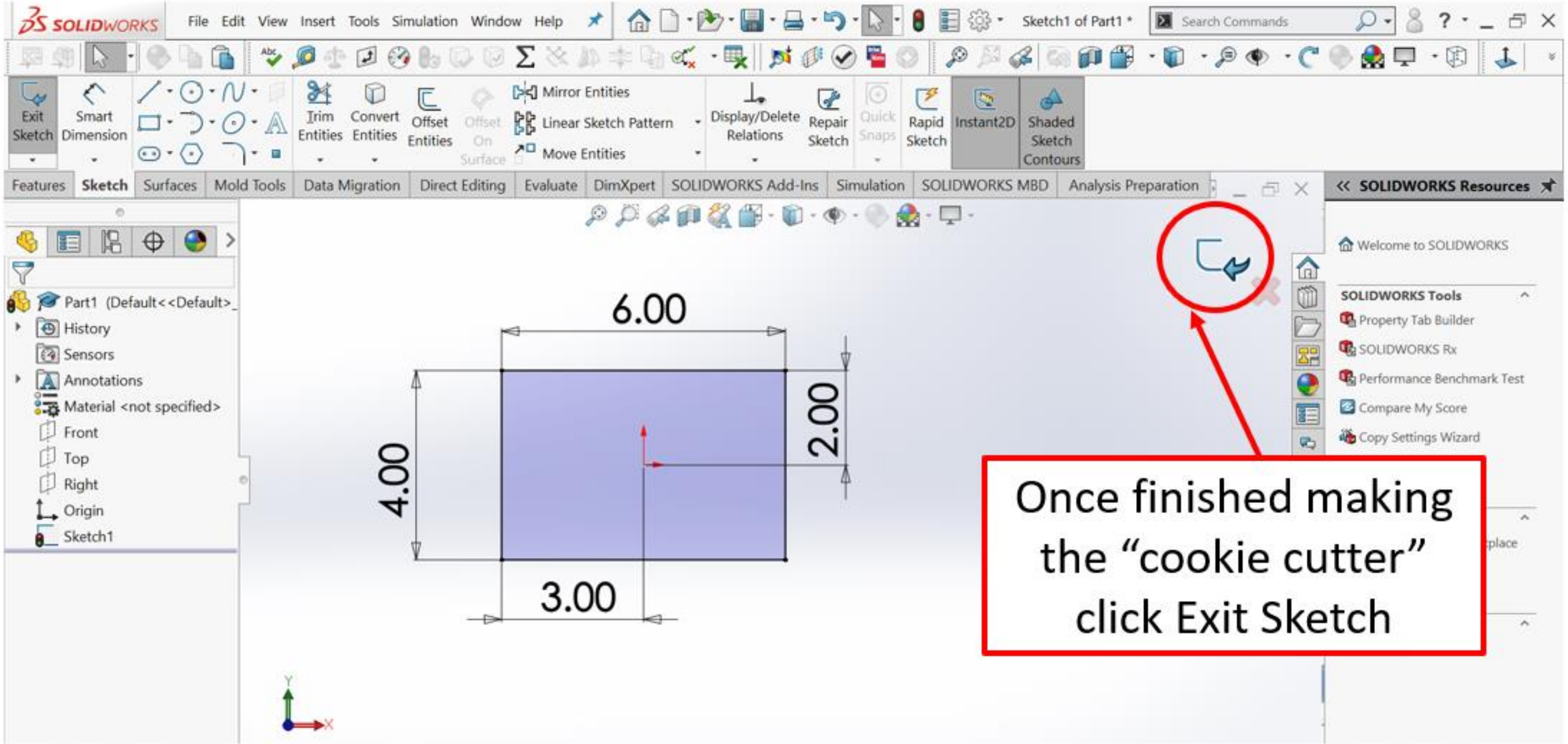


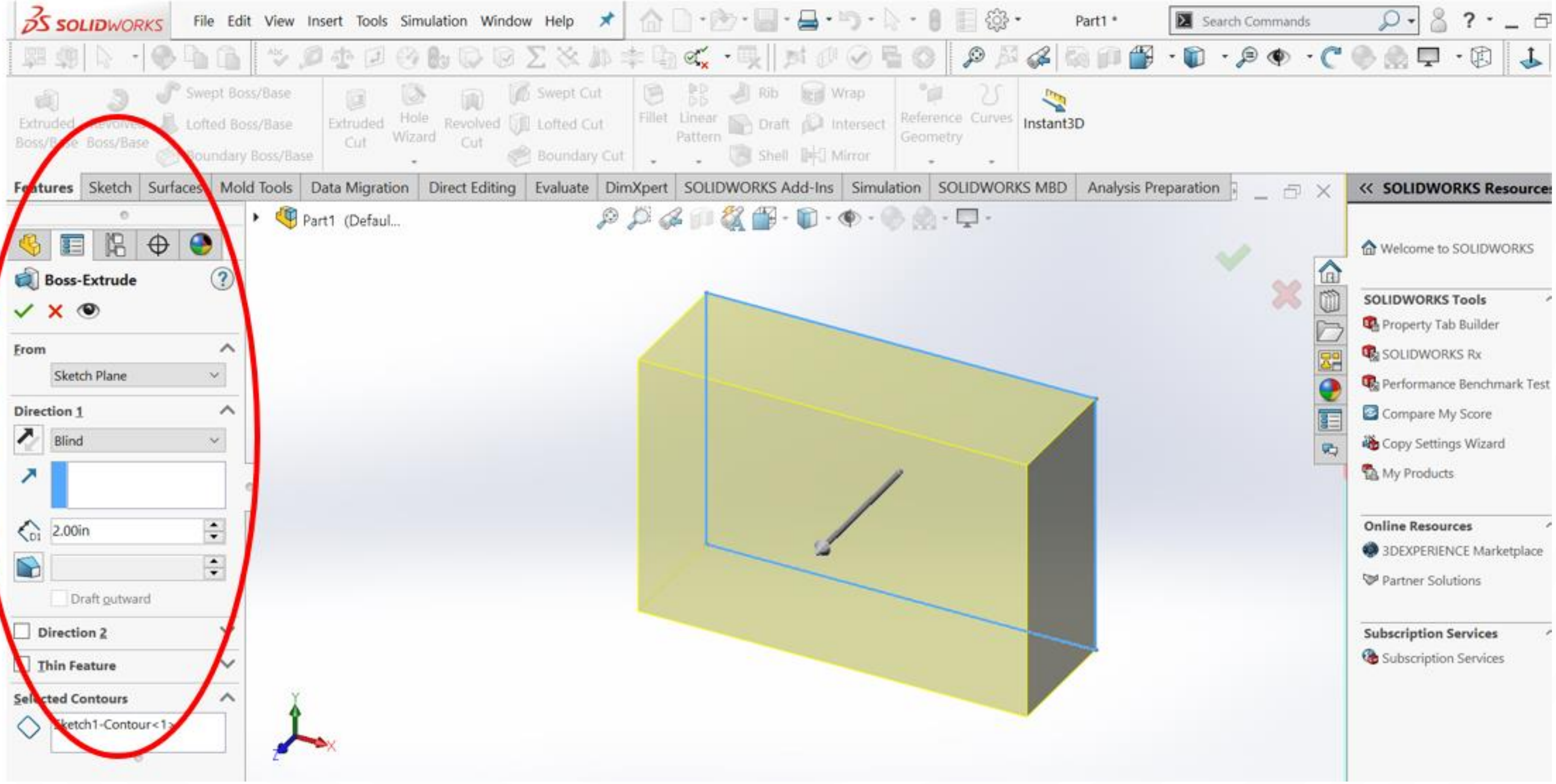
Click a line, two points, or two lines to create dimensions. Then change the value in the Modify Box.

SOLIDWORKS BASICS



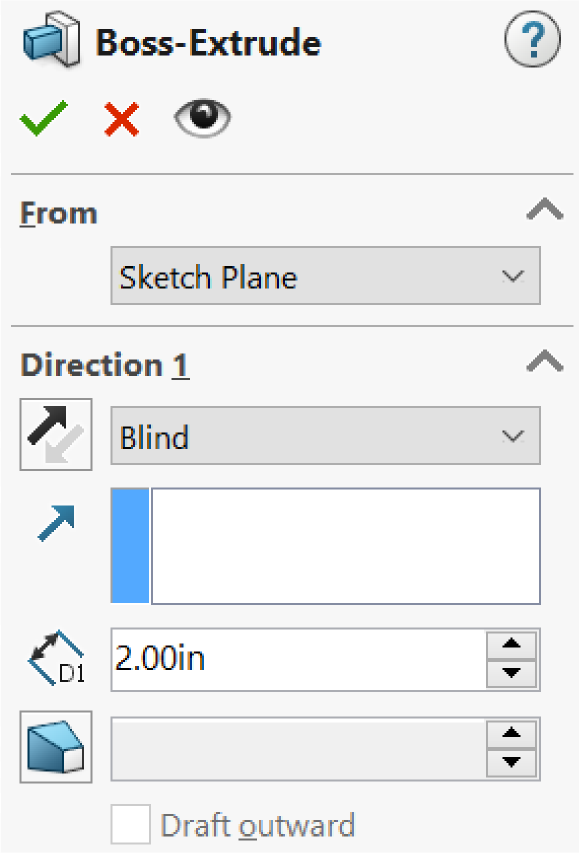
Black lines are good--that means they are fully defined!

SOLIDWORKS BASICS



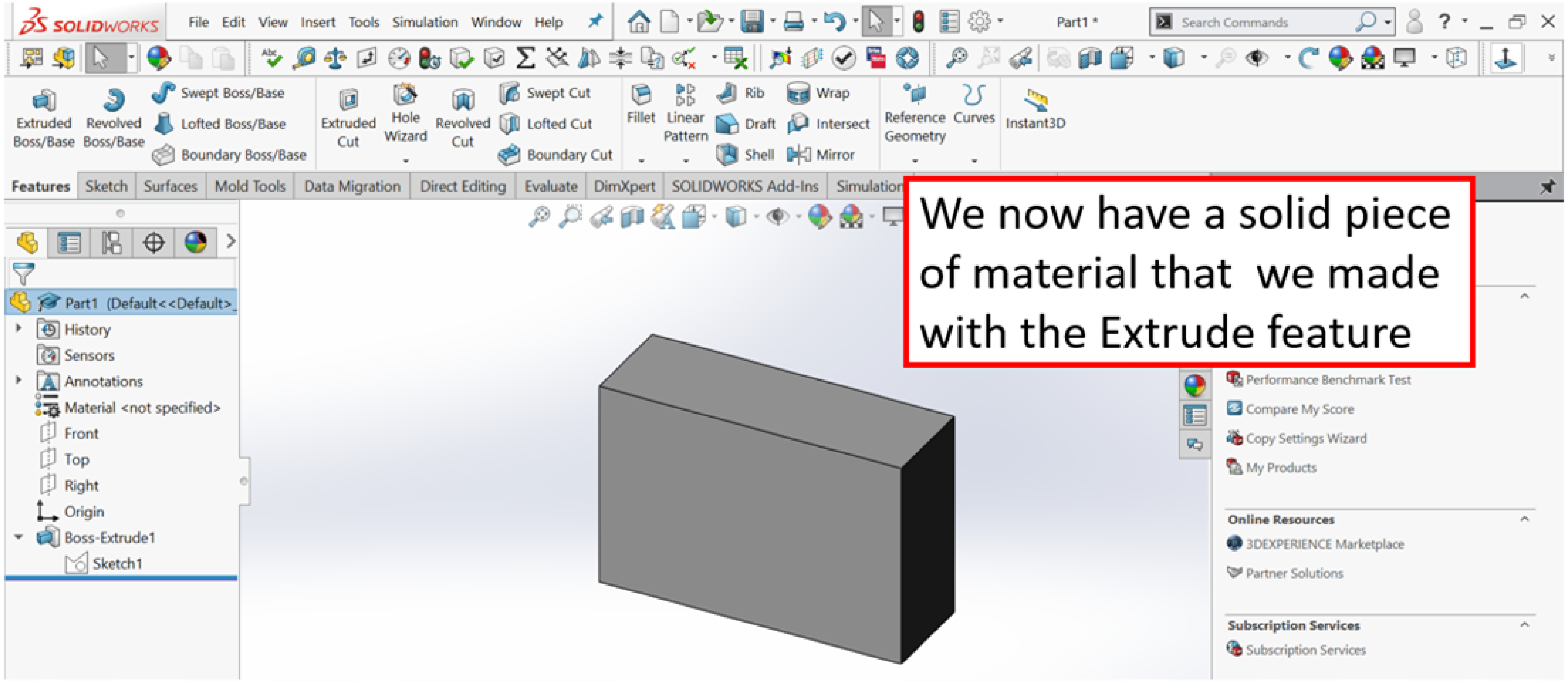
Change the size and properties of the feature in the dialog box on the left.

SOLIDWORKS BASICS

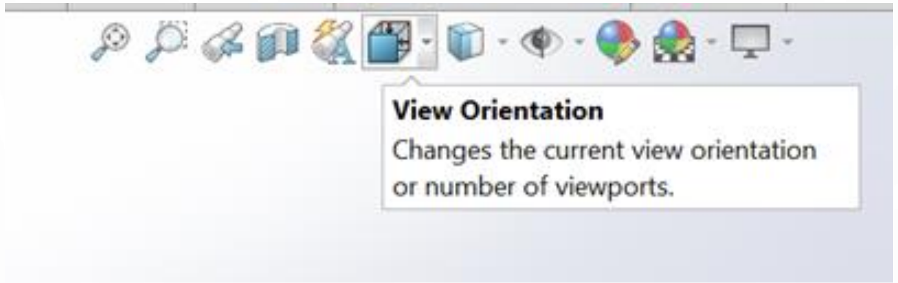


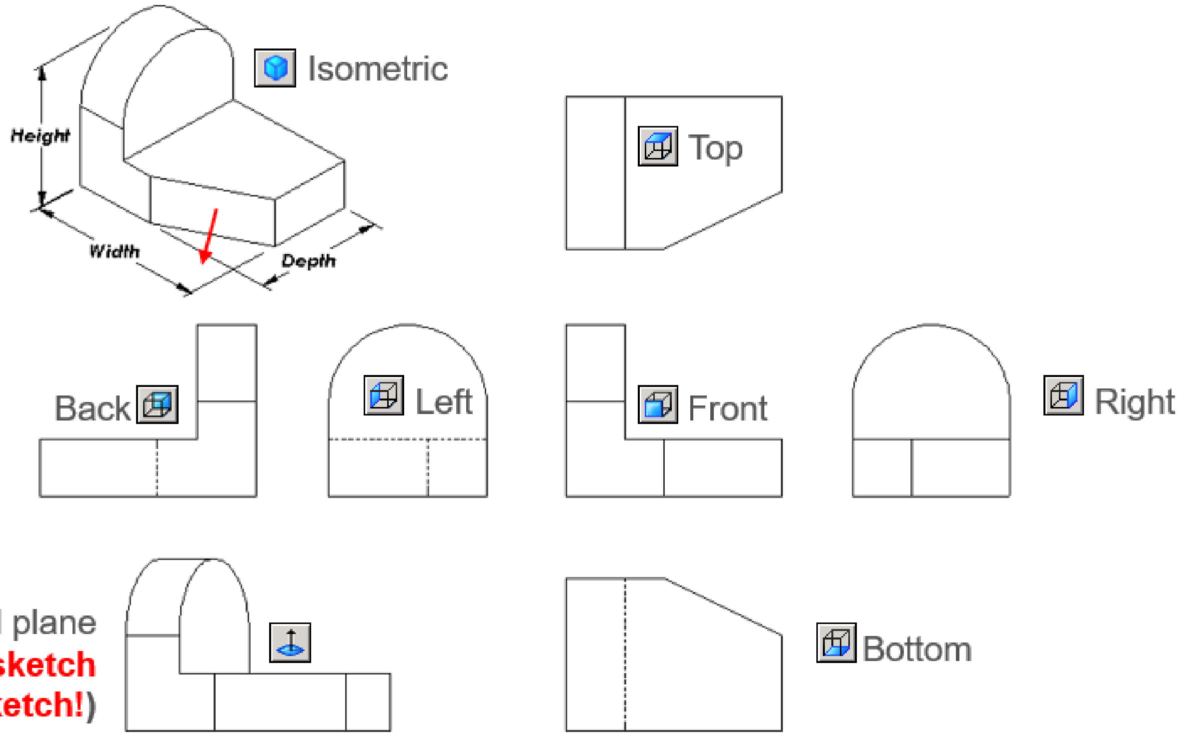
When finished, click OK (the green check mark)

SOLIDWORKS BASICS



SOLIDWORKS: VIEW ORIENTATION

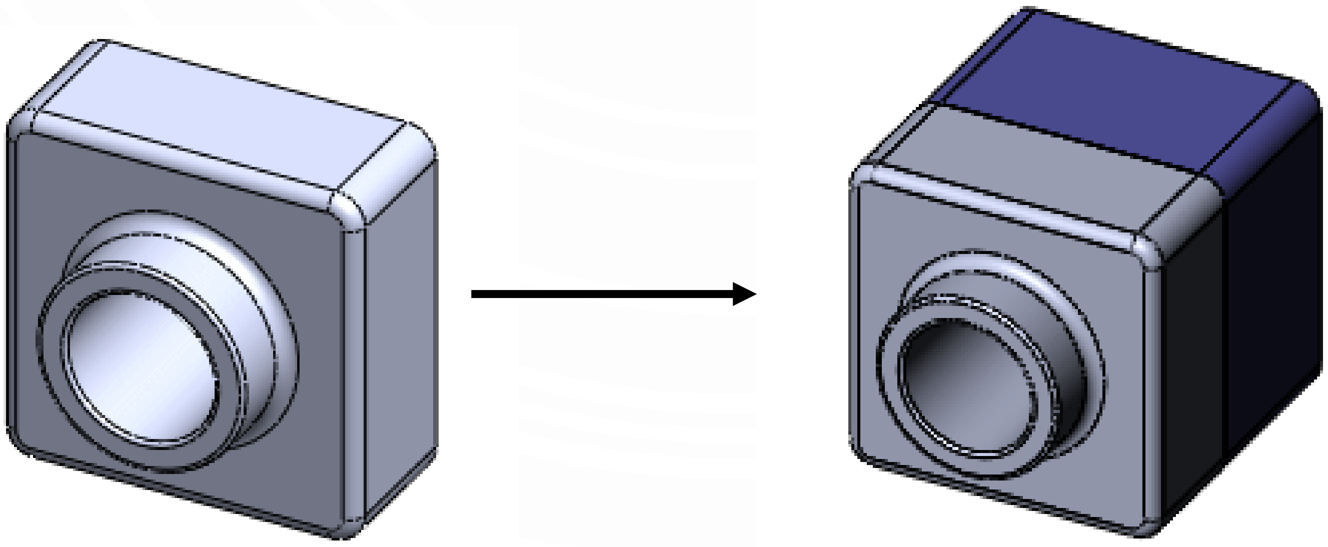




Normal To (selected plane or surface, **or sketch plane within a sketch!**)

PRACTICE SOLIDWORKS

- Go on “Tutorials”
  - Lesson 1: Parts
  - Lesson 2: Assembly



Calculus: Single Variable

Preparation for Math 2A

Revised and Edited by Rosetta Pendleton

Student Learning Outcomes

Students are expected to have an understanding in

- Calculating limits using the limit laws
- Calculating limits at infinity and horizontal asymptotes
- Derivatives and Rates of Change
- Techniques of Derivatives
- Applications of Derivatives

Math 2A Syllabus Sample

	Lecture	Section	Topic
Week 1	1	1.1, 1.2, 1.3	Four ways to represent a function, Mathematical models, Transformations of graphs
	2	1.4	Exponential Functions. Start 1.5.
	3	1.5	Inverse Functions and logarithms
Week 2	4	2.1, 2.2	Tangent and Velocity Problems, Limit of a Function
	5	2.3	Calculating Limits from Limit Laws
	6	2.5	Continuity
	7	2.6	Limits at Infinity, Horizontal Asymptotes
Week 3	8	2.7	Derivative and Rate of change
	9	2.8	Derivative as a function
	10		Review
Week 4	11		**Midterm # 1**
	12	3.1, 3.2	Derivative of a Polynomial and Exponential Function, Product and Quotient Rules
Week 5	13	3.2, 3.3	Derivative of Trigonometric Functions
	14	3.4	Chain Rule
	15	3.5	Implicit Differentiation
Week 6	16	3.6, 3.8	Derivative of Logarithm Functions
	17	3.8, 3.9	Exponential Growth and Decay
	18	3.9	Related Rates
Week 7?	19	3.10	Linear Approximation and Differentials, Review

Review

Domain and Range

Domain - the set of all possible x-values which will make the function "work", and will output real y-values.

- To find domain always check for
  - The denominator (bottom) of a fraction cannot be zero
  - The number under a square root sign must be ≥ 0

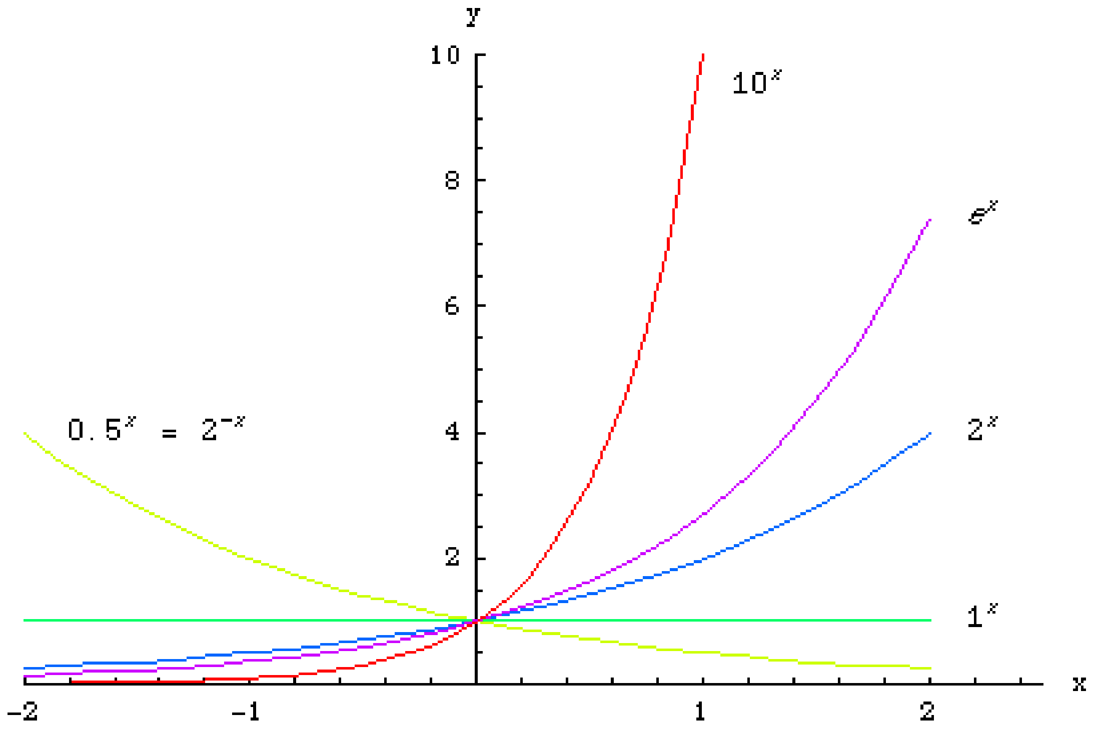
Range - the complete set of all possible resulting values of the dependent variable (y, usually), after we have substituted the domain.

Examples: $f(x) = x^2$, $f(x) = 1/x-2$, $f(x) = \sqrt{x}$

Exponential Functions

$f(x) = b^x$

- b is the base
- X is the exponent
- What happens when x >0?
- What happens when x < 0?
- What happens when x = 0?
- What does it mean when 0 < x < 1?



Laws of Exponents

Laws of Exponents		
product	$a^m \cdot a^n = a^{m+n}$ Add exponents	$2^2 \cdot 2^3 = (2\cdot2)(2\cdot2\cdot2) = 2^5$
quotient	$\frac{a^m}{a^n} = a^{m-n}$ Subtract exponents	$\frac{2^3}{2^2} = \frac{2\cdot\cancel{2}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}} = 2^{3-1} = 2$
power	$(a^m)^n = a^{m\cdot n}$ Multiply exponents	$(2^2)^3 = (2\cdot2\cdot2)(2\cdot2\cdot2)(2\cdot2\cdot2) = 2^9$
inverse	$a^{-1} = \frac{1}{a}$ Take the reciprocal	$2^{-1} = \frac{1}{2}$ (this is a definition)
zero exponent	$a^0 = 1$	$2^0 = 1$ Anything raised to the zero power is one.

The natural exponential function

$f(x) = e^x$

$e \simeq 2.71828$

What makes *e* so special?

the slope tangent to e^x when $x = 0$ is exactly 1

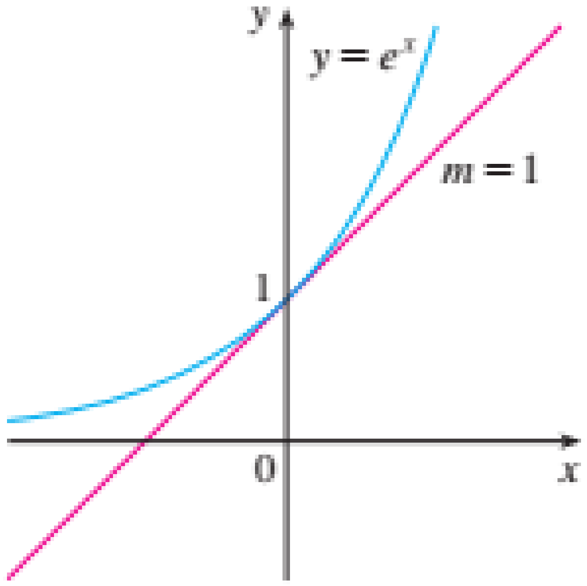


FIGURE 15
The natural exponential function crosses the *y*-axis with a slope of 1.

Logarithms

$\log_b x = y \Leftrightarrow b^y = x$

if $f(x) = \log_b x$ then,

- imagine b to be the base of an exponential function
- This function **returns the exponent** the base (b) must be raised to give x.

Examples: $\log_2 16$, $\log_{10} 1000$

What is the domain?

The range?

Logarithm Rules

Laws of logarithms

Common logarithms	Natural logarithms
$\log(uw) = \log u + \log w$	$\ln(uw) = \ln u + \ln w$
$\log\left(\frac{u}{w}\right) = \log u - \log w$	$\ln\left(\frac{u}{w}\right) = \ln u - \ln w$
$\log(u^c) = c \log u$	$\ln(u^c) = c \ln u$

WARNING!!

$$\log_a(u + w) \neq \log_a u + \log_a w$$

$$\log_a(u - w) \neq \log_a u - \log_a w$$

Inverse Functions

Let f be a **one-to-one** function with domain A and range B. Then its **inverse** function is denoted as f^{-1} and has domain B and range A.

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

Horizontal line test: a function is one-to-one iff no horizontal line intersects its graph more than once. [$f(x_1) \neq f(x_2)$ wherever $x_1 \neq x_2$]

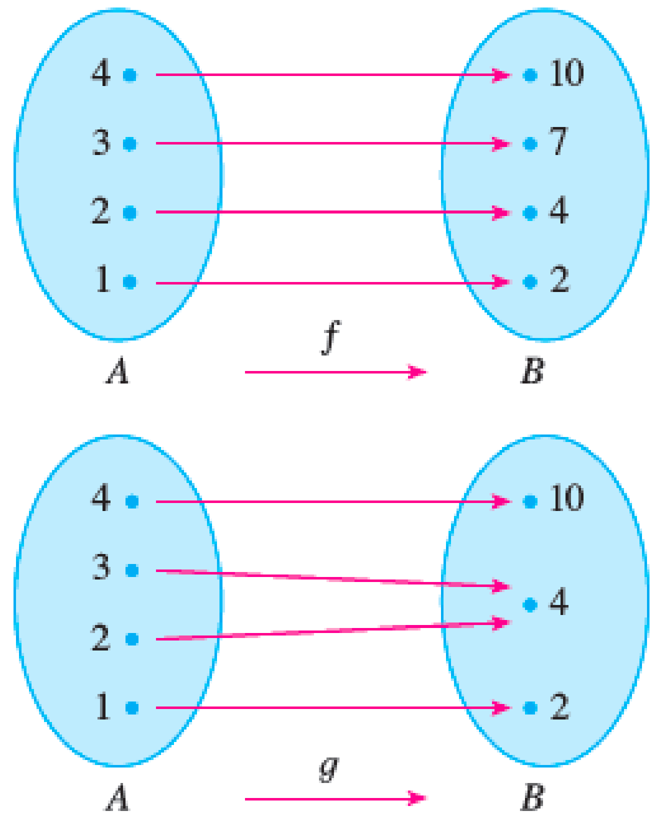


FIGURE 1
f is one-to-one; *g* is not.

The Natural Logarithm

$\log_e x = \ln x$

The logarithm of base *e* has a special notation and is called the natural logarithm

$\ln e^x = x$ $\qquad\qquad$ $e^{\ln x} = x$

so, if $f(x) = e^x$ then $f^{-1}(x) = \ln x$

How to Find the Inverse Function

1. Write $y = f(x)$
2. Solve the equation for x in terms of y (if possible)
3. Express the resulting equation as $f^{-1}(x)$ by replacing the y in the function with x.

ex. $f(x) = 2x$

$y = 2x \rightarrow x = y/2 \rightarrow f^{-1}(x) = x/2$

Limits

Limit Definition

Limits are one of basic concepts of calculus.

They help to describe the behavior of the output (L) for a function, f(x), when the input (x) is approaching a particular value a.

$$\lim_{x \to a} f(x) = L$$

$$f(x) \longrightarrow L \qquad \text{as} \qquad x \longrightarrow a$$

f(x) DOES NOT NEED to be defined when x = a. The only thing that matters is how f(x) is defined near a.

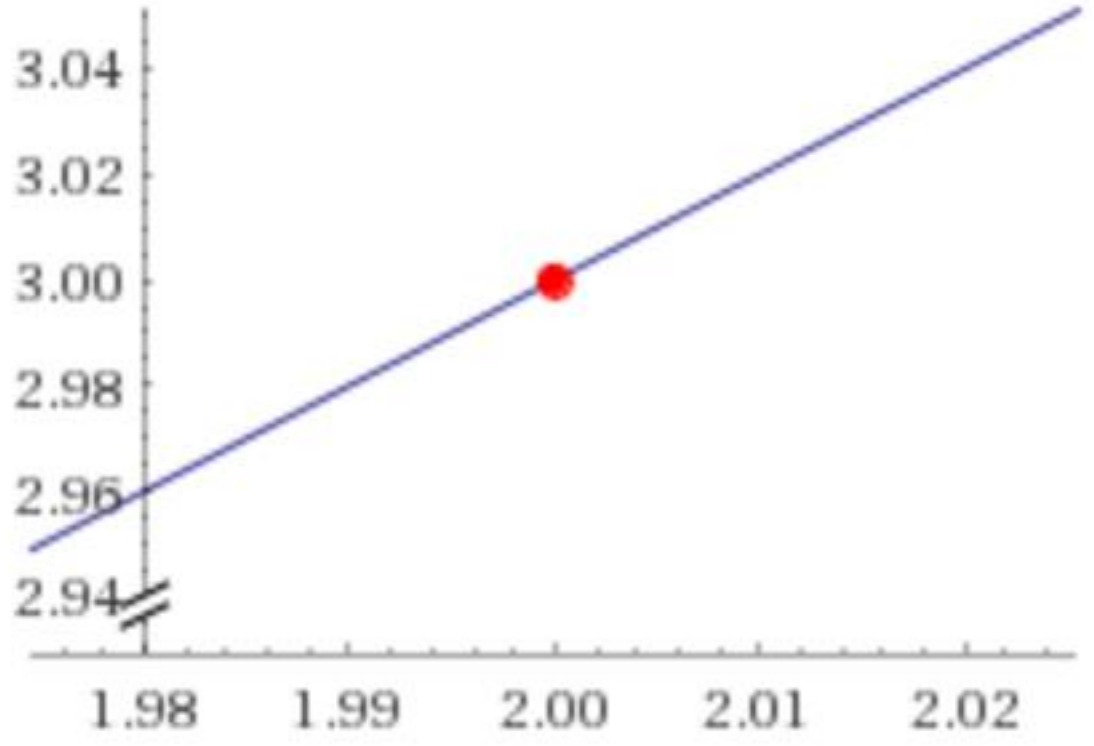
Example 1: Using a graph

f(x) = 2x - 1 find the limit as x approaches to 2

Limit:

$$\lim_{x \to 2} (2x - 1) = 3$$

Plot:



Example 2: Using a table of Values

What is the value of $\lim_{x \to 1} \dfrac{x - 1}{x^2 - 1}$

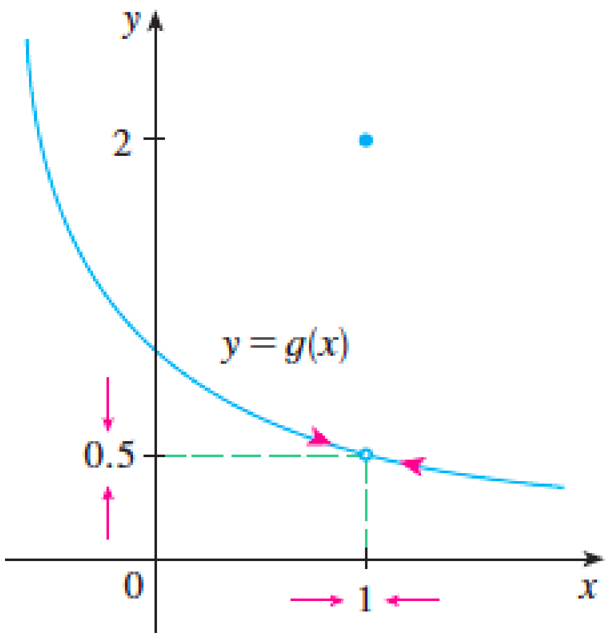
$x < 1$	$f(x)$
0.5	0.666667
0.9	0.526316
0.99	0.502513
0.999	0.500250
0.9999	0.500025

$x > 1$	$f(x)$
1.5	0.400000
1.1	0.476190
1.01	0.497512
1.001	0.499750
1.0001	0.499975

$\downarrow$	$\downarrow$
1	0.5

Caution: The limit is the value the function approaches

$$g(x) = \begin{cases} \dfrac{x-1}{x^2-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$



Even though g(1) exists and it is defined with the value of 2, the limit is still 0.5 because that is what the function approaches on both sides.

Does this limit exi $\lim_{x \to 2} \frac{x-2}{|x-2|}$

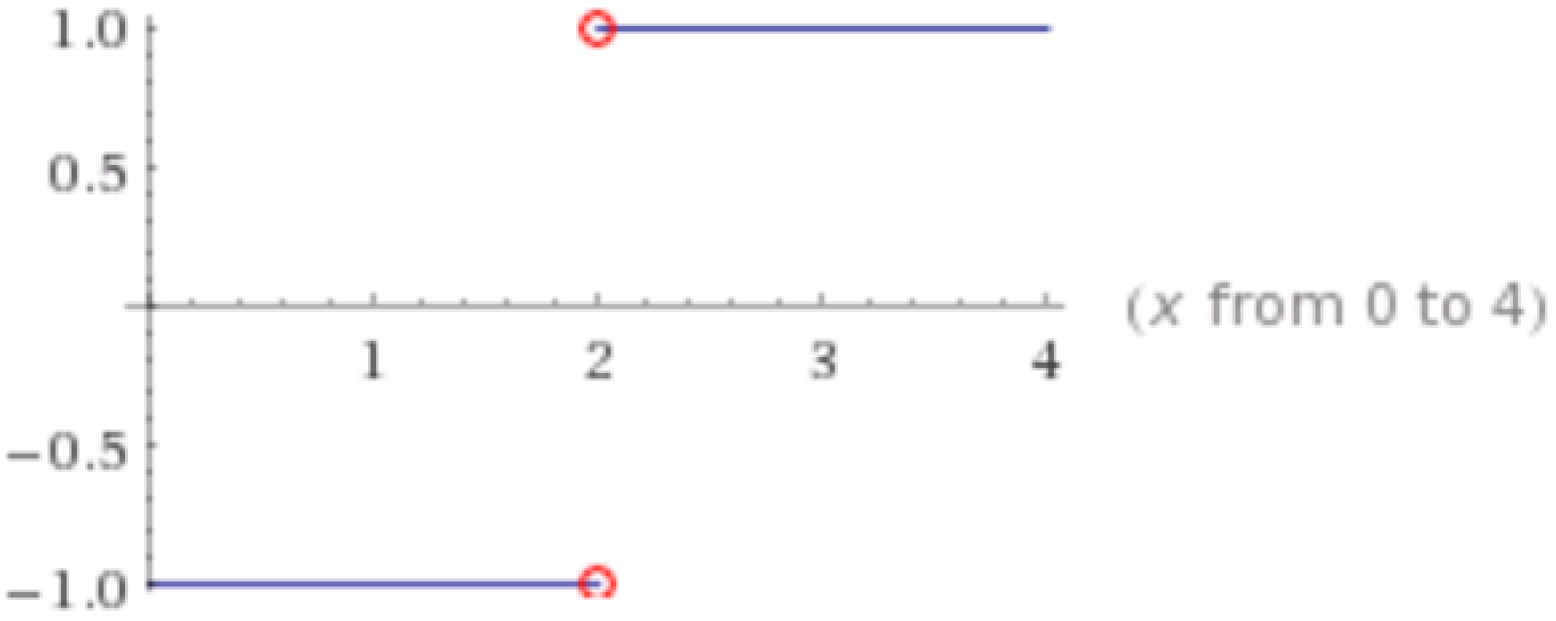
Limit from the left:

$$\lim_{x \to 2^-} \frac{x-2}{|x-2|} = -1$$

Limit from the right:

$$\lim_{x \to 2^+} \frac{x-2}{|x-2|} = 1$$

Plot:



Two sided limits

$$\lim_{x \to a^-} f(x) = L$$

We call L the **limit from the left** (or **left-hand limit**) if f(x) is close to L whenever x is close to a, but to the left of a on the real number line.

$$\lim_{x \to a^+} f(x) = L$$

We can L the **limit from the right** (or **right-hand limit**) if f(x) is close to L whenever x is close to a, but to the right of a on the real number line.

For the limit to exist, the limit must exist on both sides and be equal.

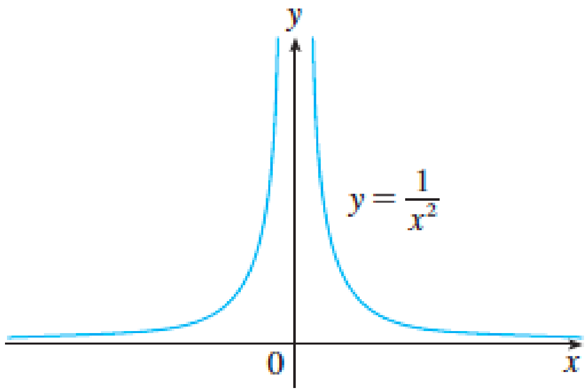
Infinite Limits

x	$\frac{1}{x^2}$
± 1	1
± 0.5	4
± 0.2	25
± 0.1	100
± 0.05	400
± 0.01	10,000
± 0.001	1,000,000

4 Intuitive Definition of an Infinite Limit Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

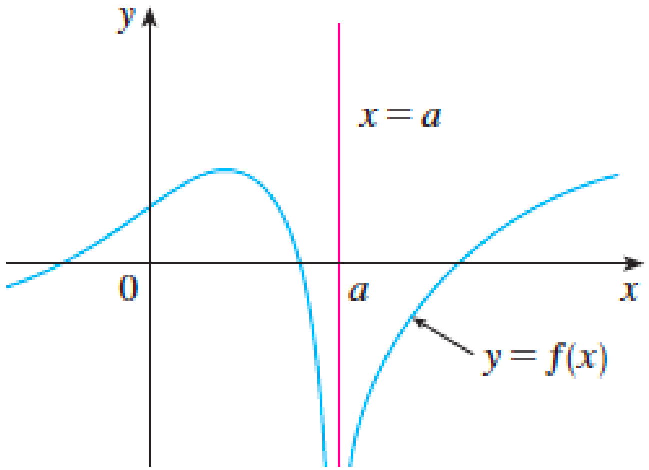
means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.



$f(x) = 1/x^2$

The limit of f(x) as x approaches 0 is infinity.

More on Infinite Limits



5 Definition Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to a, but not equal to a.

6 Definition The vertical line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

Practice Two-sided Limits

EXAMPLE 9 Find $\lim_{x \to 3^+} \frac{2x}{x-3}$ and $\lim_{x \to 3^-} \frac{2x}{x-3}$.

Calculating limits using limit laws

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \qquad \text{and} \qquad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer

7. $\lim_{x \to a} c = c$

8. $\lim_{x \to a} x = a$

9. $\lim_{x \to a} x^n = a^n$ where n is a positive integer

10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer

(If n is even, we assume that $a > 0$.)

And

11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer

$\left[\text{If } n \text{ is even, we assume that } \lim_{x \to a} f(x) > 0.\right]$

Useful Limit Theorems

1 Theorem $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

2 Theorem If $f(x) \leqslant g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\lim_{x \to a} f(x) \leqslant \lim_{x \to a} g(x)$$

3 The Squeeze Theorem If $f(x) \leqslant g(x) \leqslant h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$

Evaluate

$\lim_{x \to 5} (2x^2 - 3x + 4)$.

$\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$

$\lim_{h \to 0} \frac{(3 + h)^2 - 9}{h}$.

$\lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$

Continuity

A function f(x) is said to be **continuous** on an interval if for all values (a) in the interval:

1. f(a) exists
2. the limit from the left exists and the limit from the right exists
3. the left limit = the right limit = f(a)

Some functions may have **Jump discontinuities** where the graph has a break in it but it is only continuous if f(a) = limit at a.

Continuity Theorems

$\boxed{4}$ **Theorem** If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$ **2.** $f - g$ **3.** cf

4. fg **5.** $\dfrac{f}{g}$ if $g(a) \neq 0$

$\boxed{7}$ **Theorem** The following types of functions are continuous at every number in their domains:

- polynomials
- rational functions
- root functions
- trigonometric functions
- inverse trigonometric functions
- exponential functions
- logarithmic functions

Continuity Problems

EXAMPLE 2 Where are each of the following functions discontinuous?

(a) $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

(b) $f(x) = \begin{cases} \dfrac{1}{x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

(c) $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

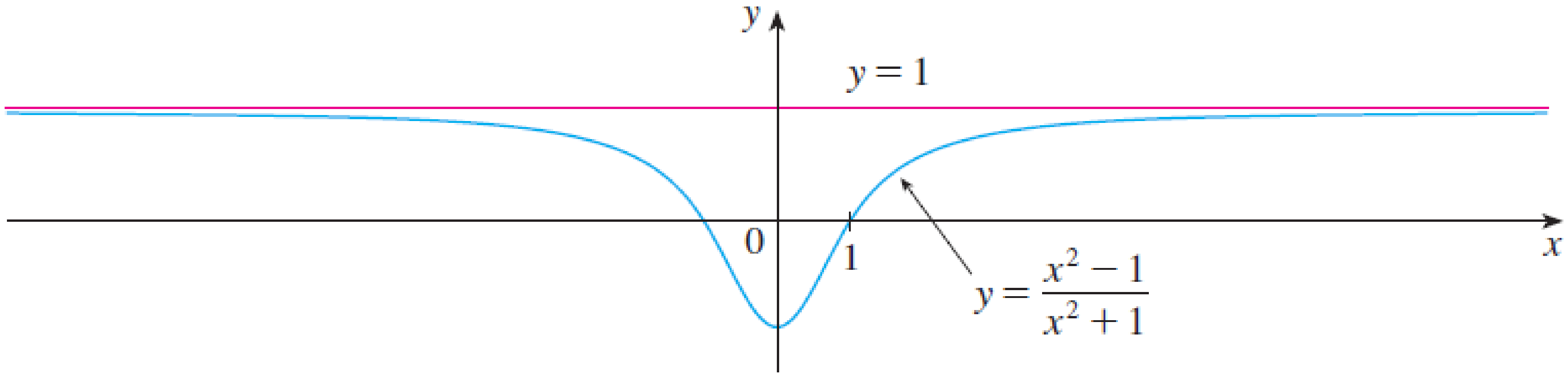
(d) $f(x) = [\![x]\!]$

Limits at Infinity; Horizontal Asymptotes

We will investigate f(x) by letting the input become arbitrarily large (+ or -).

x	$f(x)$
0	-1
± 1	0
± 2	0.600000
± 3	0.800000
± 4	0.882353
± 5	0.923077
± 10	0.980198
± 50	0.999200
± 100	0.999800
± 1000	0.999998

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$



Limits at Infinity; Horizontal Asymptotes

$$\lim_{x \to \infty} \frac{x^2 - 1}{x^2 + 1} = 1$$

$\boxed{1}$ **Definition** Let f be a function defined on some interval (a, ∞). Then

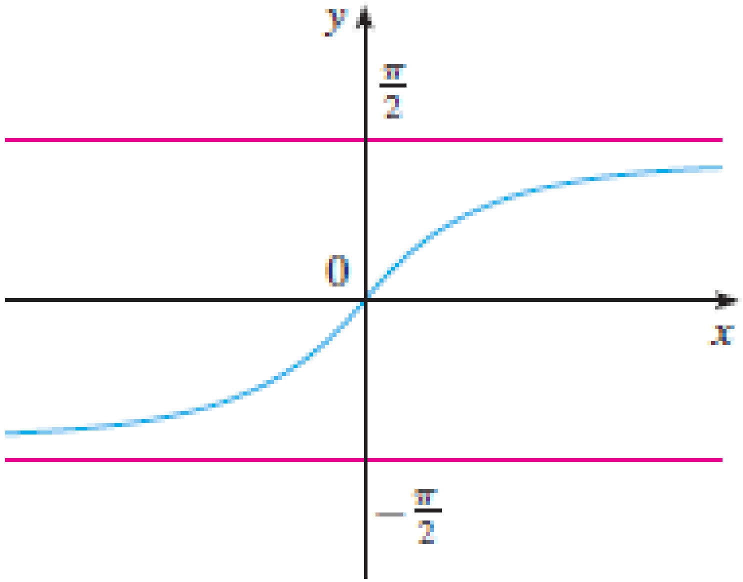
$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

Limits at Infinity; Horizontal

$\boxed{3}$ **Definition** The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L \qquad \text{or} \qquad \lim_{x \to -\infty} f(x) = L$$



$$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2} \qquad \lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$

Horizontal Asymptote Tricks

$$\lim_{x \to \infty} \frac{ax^m}{bx^n}$$

1. if m > n then the limit goes to infinity.
2. if n > m then the limit goes to 0.
3. if m = n then the limit is a/b

Problems

Find $\lim_{x\to\infty} \dfrac{1}{x}$ and $\lim_{x\to-\infty} \dfrac{1}{x}$.

$$\lim_{x\to\infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$$

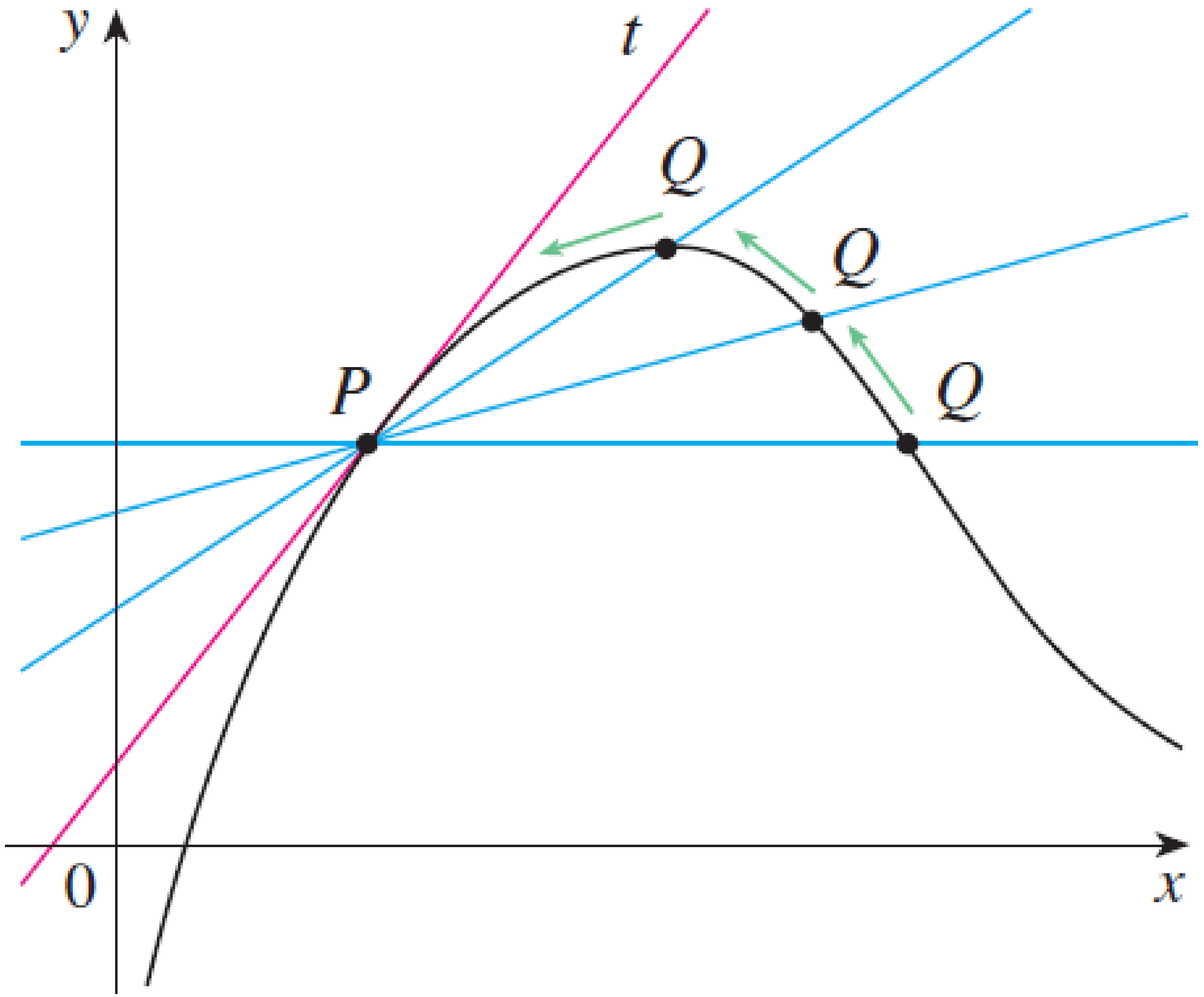
Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$$

Derivatives

What is a derivative?

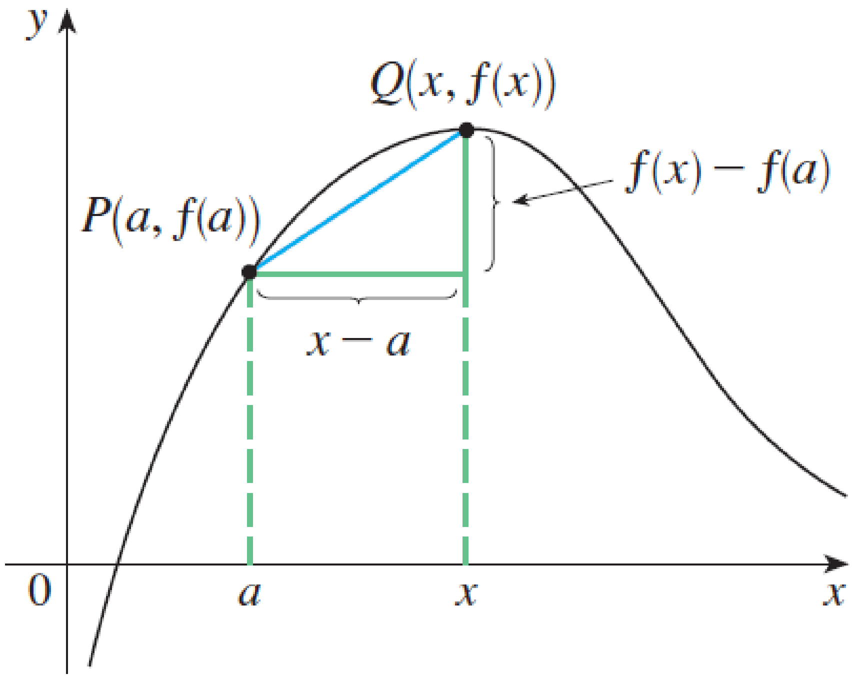
It is the slope of the tangent line at a point.

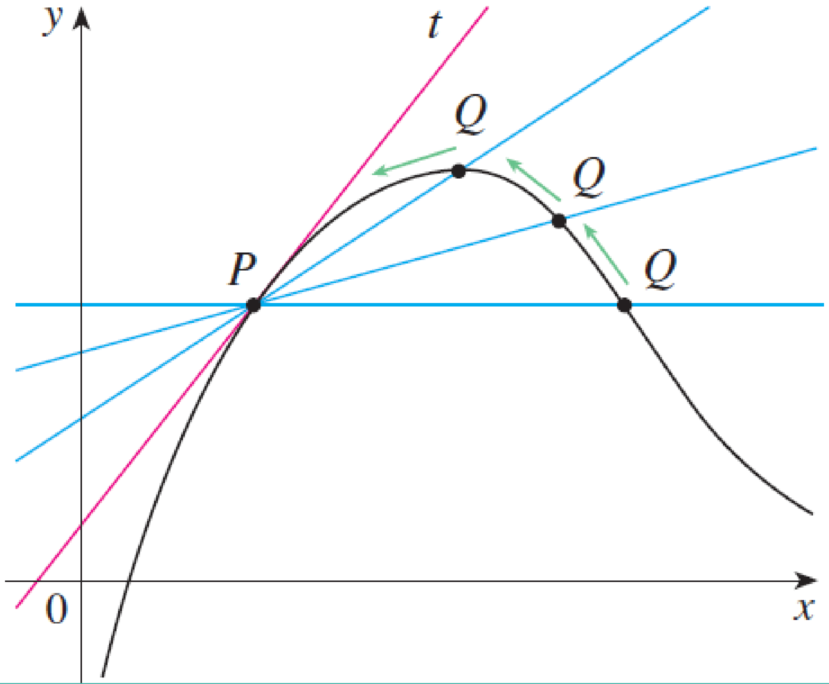


What is a derivative?

To get the **slope** of the tangent line (or a derivative) of a function we let x approach to a as described below.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$





Limit Definition of a Derivative

$\boxed{4}$ **Definition** The **derivative of a function** f **at a number** a, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

Using the limit definition

EXAMPLE 4

Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number a.

SOLUTION From Definition 4 we have

$$
\begin{aligned}
f'(a) &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h \to 0} \frac{[(a+h)^2 - 8(a+h) + 9] - [a^2 - 8a + 9]}{h} \\
&= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - 8a - 8h + 9 - a^2 + 8a - 9}{h} \\
&= \lim_{h \to 0} \frac{2ah + h^2 - 8h}{h} = \lim_{h \to 0} (2a + h - 8) \\
&= 2a - 8
\end{aligned}
$$

Derivatives Basics

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

The Power Rule: If n is any real number then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

The Constant Multiple Rule: If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Sum Rule and Difference Rule

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Differentiate:

$$f(x) = \frac{1}{x^2}$$

$$y = \sqrt[3]{x^2}$$

$$\frac{d}{dx}(3x^4)$$

Example

V **EXAMPLE 6** Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

SOLUTION Horizontal tangents occur where the derivative is zero. We have

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) - 6\frac{d}{dx}(x^2) + \frac{d}{dx}(4)$$

$$= 4x^3 - 12x + 0 = 4x(x^2 - 3)$$

Thus $dy/dx = 0$ if $x = 0$ or $x^2 - 3 = 0$, that is, $x = \pm\sqrt{3}$. So the given curve has horizontal tangents when $x = 0$, $\sqrt{3}$, and $-\sqrt{3}$. The corresponding points are $(0, 4)$, $(\sqrt{3}, -5)$, and $(-\sqrt{3}, -5)$. (See Figure 5.) ■

Rates of Change

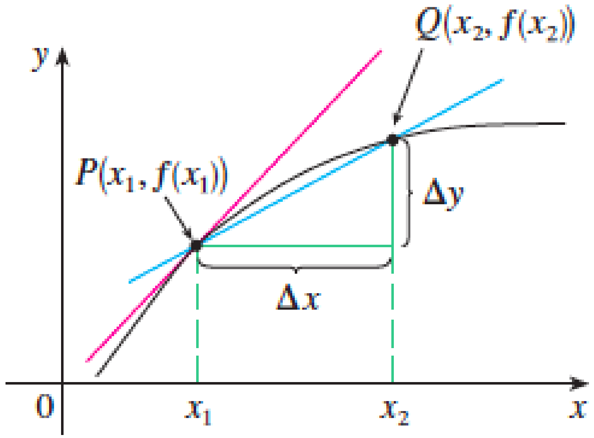
The difference quotient

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

is called the **average rate of change of y with respect to x** over the interval $[x_1, x_2]$ and can be interpreted as the slope of the secant line PQ in Figure 8.

We know that one interpretation of the derivative $f'(a)$ is as the slope of the tangent line to the curve $y = f(x)$ when $x = a$. We now have a second interpretation:

The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$.



average rate of change $= m_{PQ}$

instantaneous rate of change $=$ slope of tangent at P

FIGURE 8

EXAMPLE 6 A manufacturer produces bolts of a fabric with a fixed width. The cost of producing x yards of this fabric is $C = f(x)$ dollars.

(a) What is the meaning of the derivative $f'(x)$? What are its units?

(b) In practical terms, what does it mean to say that $f'(1000) = 9$?

(c) Which do you think is greater, $f'(50)$ or $f'(500)$? What about $f'(5000)$?

SOLUTION

(a) The derivative $f'(x)$ is the instantaneous rate of change of C with respect to x; that is, $f'(x)$ means the rate of change of the production cost with respect to the number of yards produced. (Economists call this rate of change the *marginal cost*. This idea is discussed in more detail in Sections 3.7 and 4.7.)

Because

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta C}{\Delta x}$$

the units for $f'(x)$ are the same as the units for the difference quotient $\Delta C/\Delta x$. Since ΔC is measured in dollars and Δx in yards, it follows that the units for $f'(x)$ are dollars per yard.

(b) The statement that $f'(1000) = 9$ means that, after 1000 yards of fabric have been manufactured, the rate at which the production cost is increasing is \$9/yard. (When $x = 1000$, C is increasing 9 times as fast as x.)

Since $\Delta x = 1$ is small compared with $x = 1000$, we could use the approximation

$$f'(1000) \approx \frac{\Delta C}{\Delta x} = \frac{\Delta C}{1} = \Delta C$$

and say that the cost of manufacturing the 1000th yard (or the 1001st) is about \$9.

Natural Exponential Function

Definition of the Number *e*

$$e \text{ is the number such that } \lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

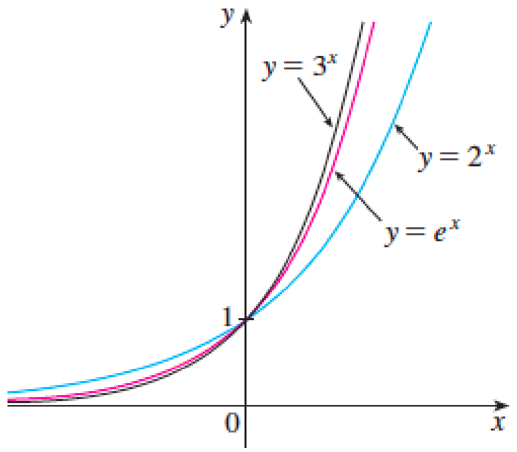


FIGURE 6

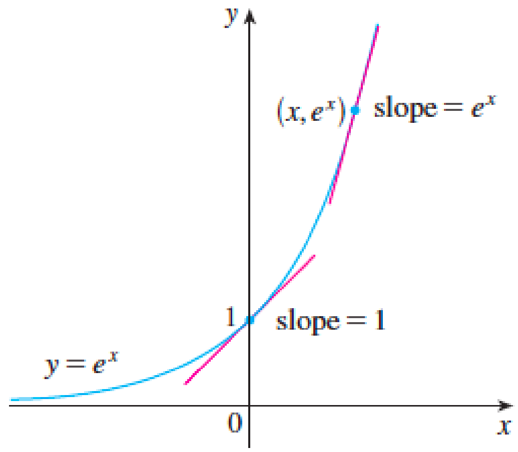


FIGURE 7

If we put $b = e$ and, therefore, $f'(0) = 1$ in Equation 4, it becomes the following important differentiation formula.

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Product Rule And Quotient Rule

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\dfrac{d}{dx}[f(x)] - f(x)\dfrac{d}{dx}[g(x)]}{[g(x)]^2}$$

Problems

$$g(x) = \sqrt{x}\, e^x$$

$$y = \frac{e^x}{1 - e^x}$$

Find equations of the tangent lines to the curve

$$y = \frac{x - 1}{x + 1}$$

that are parallel to the line $x - 2y = 2$.

Derivatives of Trigonometric Fu

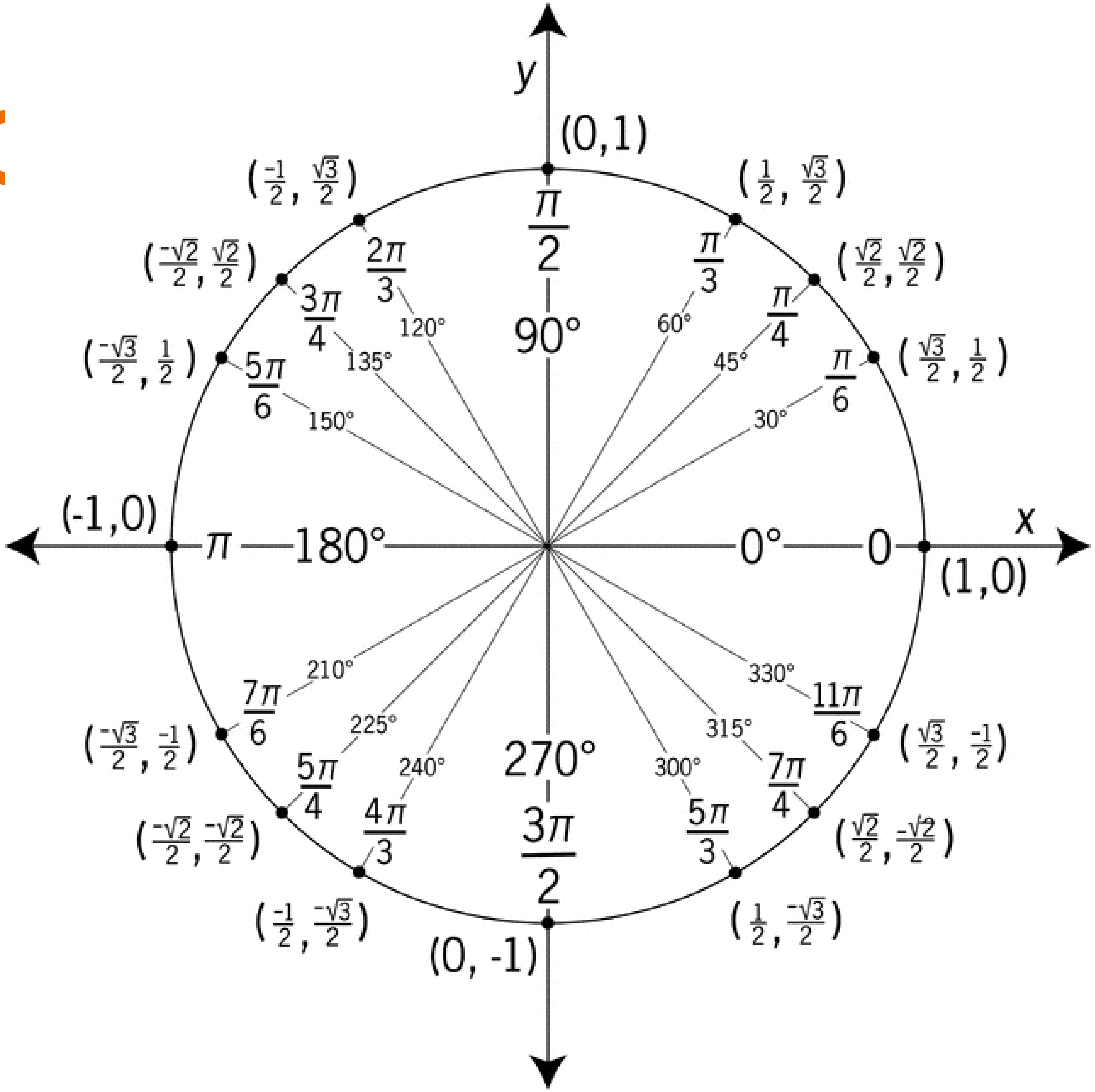
Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

Review Trig Unit C



Reviewing Trig Identities

Reciprocal and Quotient Identities	$\sin\theta = \dfrac{1}{\csc\theta}$ $\quad \csc\theta = \dfrac{1}{\sin\theta}$ $\quad \cos\theta = \dfrac{1}{\sec\theta}$ $\quad \sec\theta = \dfrac{1}{\cos\theta}$ $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$
Pythagorean Identities	$\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad \cot^2\theta + 1 = \csc^2\theta$
Sum and Difference Identities	$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad \cos(A+B) = \cos A\cos B - \sin A\sin B$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\quad \cos(A-B) = \cos A\cos B + \sin A\sin B$ $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad \tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
Double Angle Identities	$\sin(2A) = 2\sin A\cos A$ $\quad \cos(2A) = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$ $\quad \tan(2A) = \dfrac{2\tan A}{1-\tan^2 A}$
Half Angle Identities	$\sin\left(\dfrac{A}{2}\right) = \pm\sqrt{\dfrac{1-\cos A}{2}}$ $\quad \cos\left(\dfrac{A}{2}\right) = \pm\sqrt{\dfrac{1+\cos A}{2}}$ $\quad \tan\left(\dfrac{A}{2}\right) = \dfrac{\sin A}{1+\cos A} = \dfrac{1-\cos A}{\sin A}$

Evaluate

Find the derivatives of the following:

1. $y = x^2 \sin(x)$
2. $y = x/(2-\tan(x))$
3. $f(x) = \sec(x) / 1+\tan x$

Chain Rule

The Chain Rule If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Chain Rule Example

EXAMPLE 1 Find $F'(x)$ if $F(x) = \sqrt{x^2 + 1}$.

SOLUTION 1 (using Equation 2): At the beginning of this section we expressed F as $F(x) = (f \circ g)(x) = f(g(x))$ where $f(u) = \sqrt{u}$ and $g(x) = x^2 + 1$. Since

$$f'(u) = \tfrac{1}{2}u^{-1/2} = \frac{1}{2\sqrt{u}} \qquad \text{and} \qquad g'(x) = 2x$$

we have

$$\begin{aligned} F'(x) &= f'(g(x)) \cdot g'(x) \\ &= \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x = \frac{x}{\sqrt{x^2 + 1}} \end{aligned}$$

L’Hospital’s Rule

L’Hospital’s Rule Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = 0$$

or that

$$\lim_{x \to a} f(x) = \pm\infty \qquad \text{and} \qquad \lim_{x \to a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or ∞/∞.) Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

39–48 Find the limit. **(Using L'Hospitals Rule)**

39. $\lim_{x \to 0} \frac{\sin 3x}{x}$

41. $\lim_{t \to 0} \frac{\tan 6t}{\sin 2t}$

43. $\lim_{x \to 0} \frac{\sin 3x}{5x^3 - 4x}$

Implicit Differentiation

The functions that we have met so far can be described by expressing one variable explicitly in terms of another variable—for example,

$$y = \sqrt{x^3 + 1} \qquad \text{or} \qquad y = x \sin x$$

or, in general, $y = f(x)$. Some functions, however, are defined implicitly by a relation between x and y such as

$$x^2 + y^2 = 25 \tag{1}$$

or

$$x^3 + y^3 = 6xy \tag{2}$$

Fortunately, we don't need to solve an equation for y in terms of x in order to find the derivative of y . Instead we can use the method of **implicit differentiation.**

Implicit Differentiation

Implicit differentiation - consists of differentiating both sides of the equation with respect to x and then solving the resulting equation for y′.

SOLUTION 1

(a) Differentiate both sides of the equation $x^2 + y^2 = 25$:

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

Remembering that y is a function of x and using the Chain Rule, we have

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2)\frac{dy}{dx} = 2y\frac{dy}{dx}$$

Thus $$2x + 2y\frac{dy}{dx} = 0$$

Now we solve this equation for dy/dx:

$$\frac{dy}{dx} = -\frac{x}{y}$$

(b) At the point (3, 4) we have $x = 3$ and $y = 4$, so

$$\frac{dy}{dx} = -\frac{3}{4}$$

An equation of the tangent to the circle at (3, 4) is therefore

$$y - 4 = -\tfrac{3}{4}(x - 3) \qquad \text{or} \qquad 3x + 4y = 25$$

Derivatives of Inverse Trig Functions

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

Practice

Differentiate the following

1. $y = (\tan^{-1}(x))^2$
2. $y = \sin^{-1}(2x + 1)$

Differentiating Logarithms

1 $$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

2 $$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Logarithmic Differentiation - The calculation of derivatives of complicated functions involving products, quotients, or powers can often be simplified by taking logarithms.

Steps in Logarithmic Differentiation

1. Take natural logarithms of both sides of an equation $y = f(x)$ and use the Laws of Logarithms to simplify.
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y'.

Practice

Differentiate the following

1. $f(x) = \sqrt{\ln(x)}$
2. $f(x) = \log_{10}(2 + \sin(x))$
3. $y = x^{\sqrt{x}}$

Curve Sketching

Guidelines for sketching a curve

1. Domain
2. Intercepts
3. Symmetry
4. Asymptotes
5. Intervals of increase and decrease
6. local maxima and minima
7. Concavity and points of inflection

Applications of Derivatives

Higher Derivatives

Higher Derivatives

If f is a differentiable function, then its derivative f' is also a function, so f' may have a derivative of its own, denoted by $(f')' = f''$. This new function f'' is called the **second derivative** of f because it is the derivative of the derivative of f. Using Leibniz notation, we write the second derivative of $y = f(x)$ as

$$\underbrace{\frac{d}{dx}}_{\text{derivative of}} \underbrace{\left(\frac{dy}{dx}\right)}_{\text{first derivative}} = \underbrace{\frac{d^2y}{dx^2}}_{\text{second derivative}}$$

Velocity and Acceleration

The position function of a particle moving in a straight line gives the position of the particle at some time (t).

$s = f(t)$

The derivative of the position function represents the rate the position changes, giving us the **velocity** at some time (t).

$v = f(t) = s'(t)$

The derivative of the velocity function represents the rate the velocity changes, giving us the **acceleration** at some time (t)

$a = f(t) = v'(t) = s''(t)$

Practice Velocity and Acceleration

EXAMPLE 1 The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

(a) Find the velocity at time t.
(b) What is the velocity after 2 s? After 4 s?
(c) When is the particle at rest?
(d) When is the particle moving forward (that is, in the positive direction)?
(e) Draw a diagram to represent the motion of the particle.
(f) Find the total distance traveled by the particle during the first five seconds.
(g) Find the acceleration at time t and after 4 s.

Exponential Growth and Decay

In many natural phenomena, quantities grow or decay at a rate proportional to their size. For instance, if y = f(t) is the number of individuals in a population at time t, then it seems reasonable to expect that the rate of growth f′(t) is proportional to the population.

1

$$\frac{dy}{dt} = ky$$

Law of natural growth when k>0

Law of natural decay when k<0

2 Theorem The only solutions of the differential equation $dy/dt = ky$ are the exponential functions

$$y(t) = y(0)e^{kt}$$

Practice Exponential Growth and Decay

EXAMPLE 1 Use the fact that the world population was 2560 million in 1950 and 3040 million in 1960 to model the population of the world in the second half of the 20th century. (Assume that the growth rate is proportional to the population size.) What is the relative growth rate? Use the model to estimate the world population in 1993 and to predict the population in the year 2020.

Related Rates

In a related rates problem the idea is to compute the rate of change of one quantity in terms of the rate of change of another quantity (which may be more easily measured).

1. Draw out the problem and label relevant quantities
2. Find an equation that relates the two changing quantities
   a. ex. if height (h) and volume (v) of a cube is changing, write $V = h^3$
3. Differentiate **each side** of the equation with respect to time (t).
   a. ex. $(dV/dh)(dh/dt) = 3h^2 (dh/dt)$
4. Solve for the unknown rate

Related Rates Example

EXAMPLE 1 Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

EXAMPLE 2 A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

Resources

UCI MATH 2A/2B practice midterm, syllabi, suggested hwk etc.

https://www.math.uci.edu/undergraduate/courses/calculus-2a2b-resources

UCI MATH Tutoring Center Information

https://www.math.uci.edu/undergrad-courses/tutoring-assistance

OAI Tutoring Center

http://tech.uci.edu/access/students-current.php

Additional Resources

https://online.math.uh.edu/HoustonACT/videocalculus/index.html

Last Tips

1. Read before coming to class
2. Take notes as appropriate
3. Do the homework AND extra problems if you need it
4. Ask questions to TA/professor/ tutors etc.
5. Aim for excellence

Single Variable Math 2B

All problems covered are from Stewart Calculus Early Transcendentals 7th Edition Textbook

Sample Syllabus

Lecture	Section	Topic
1	4.9	Antiderivatives (Review)
2	5.1	Areas and Distances
3	5.2	Definite Integral
4	5.3	Fundamental theorem of calculus
5	5.4	Indefinite integrals and Net change theorem
6	5.5	Substitution rule
7	6.1, 6.2	Area between curves; Start 6.2
8	6.2	Volumes (continued)
9	6.5	Average value of a function; Review
10		**Midterm # 1**
11	7.1	Integration by Parts
12	7.2	Trigonometric Integrals
13	7.3	Trigonometric Substitution
14	7.4	Integration by Partial Fractions
15	7.5	Strategy for Integration
16	7.8	Improper Integrals
17	8.1	Review of integration techniques; Arc Length
18	11.1	Sequences
19		Review
20		**Midterm # 2**

Section	Topic
5.2	Definite Integral
5.3	Fundamental Theorem of Calculus
5.4	Indefinite integrals and Net change theorem
5.5	Substitution rule
6.1, 6.2	Area, Volume
7.1	Integration by Parts
7.3	Trig Sub
7.4	Partial Fractions

Section	Topic
7.5	Strategy for Integration
7.8	Improper Integrals
8.1	Arc Length/Review

5.2 Definite Integral

$\boxed{2}$ **Definition of a Definite Integral** If f is a function defined for $a \leqslant x \leqslant b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0 \,(= a), x_1, x_2, \ldots, x_n \,(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \ldots, x_n^*$ be any **sample points** in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

provided that this limit exists and gives the same value for all possible choices of sample points. If it does exist, we say that f is **integrable** on $[a, b]$.

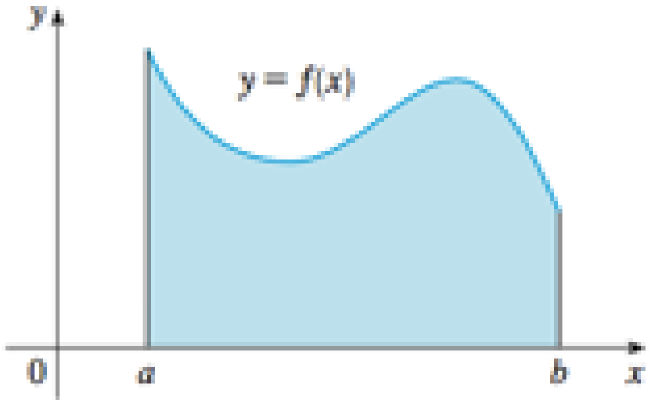


FIGURE 2
If $f(x) \geq 0$, the integral $\int_a^b f(x)\,dx$ is the area under the curve $y = f(x)$ from a to b.

Properties of the Definite Integral

When we defined the definite integral $\int_a^b f(x)\,dx$, we implicitly assumed that $a < b$. But the definition as a limit of Riemann sums makes sense even if $a > b$. Notice that if we reverse a and b, then Δx changes from $(b - a)/n$ to $(a - b)/n$. Therefore

$$\boxed{\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx}$$

If $a = b$, then $\Delta x = 0$ and so

$$\boxed{\int_a^a f(x)\,dx = 0}$$

Properties of the Integral

1. $\int_a^b c\,dx = c(b - a)$, where c is any constant

2. $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, where c is any constant

4. $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

5. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

5.3 Fundamental Theorem of Calculus

Where F is continuous function on [a,b] and x varies between a and b.

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

$$F(x) = \int_a^x f(t)\,dt,$$

$$F'(x) = f(x)$$

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \leqslant x \leqslant b$$

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

5.4 Indefinite Integrals Net Change Theorem

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

The integral of a rate of change is the net change

1 Table of Indefinite Integrals

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

5.5 The Substitution Rule

$\boxed{4}$ **The Substitution Rule** If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

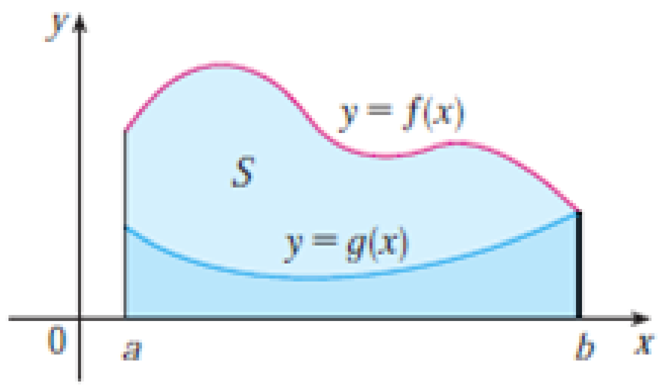
$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

EXAMPLE 1 Find $\int x^3 \cos(x^4 + 2)\, dx$.

SOLUTION We make the substitution $u = x^4 + 2$ because its differential is $du = 4x^3\, dx$, which, apart from the constant factor 4, occurs in the integral. Thus, using $x^3\, dx = \frac{1}{4}\, du$ and the Substitution Rule, we have

$$
\begin{aligned}
\int x^3 \cos(x^4 + 2)\, dx &= \int \cos u \cdot \tfrac{1}{4}\, du = \tfrac{1}{4} \int \cos u\, du \\
&= \tfrac{1}{4} \sin u + C \\
&= \tfrac{1}{4} \sin(x^4 + 2) + C
\end{aligned}
$$

6.1 Area between Curves



$\boxed{2}$ The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$, is

$$A = \int_a^b [f(x) - g(x)]\, dx$$

6.2 Volumes

Definition of Volume Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i^*)\, \Delta x = \int_a^b A(x)\, dx$$

6.5 Average Value of a Function

Therefore we define the **average value of f** on the interval $[a, b]$ as

$$\boxed{f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

V EXAMPLE 1 Find the average value of the function $f(x) = 1 + x^2$ on the interval $[-1, 2]$.

SOLUTION With $a = -1$ and $b = 2$ we have

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-(-1)}\int_{-1}^{2} (1 + x^2)\,dx \\ &= \frac{1}{3}\left[x + \frac{x^3}{3}\right]_{-1}^{2} = 2 \end{aligned}$$

Mean Value Theorem

The Mean Value Theorem for Integrals If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f(c) = f_{\text{ave}} = \frac{1}{b - a} \int_a^b f(x)\, dx$$

that is,

$$\int_a^b f(x)\, dx = f(c)(b - a)$$

7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

EXAMPLE 2 Evaluate $\int \ln x\,dx$.

SOLUTION USING FORMULA 2 Let

$$u = x \qquad dv = \sin x\,dx$$

Then $$du = dx \qquad v = -\cos x$$

and so

$$\int x\sin x\,dx = \int \overbrace{x}^{u}\ \overbrace{\sin x\,dx}^{dv} = \overbrace{x}^{u}\ \overbrace{(-\cos x)}^{v} - \int \overbrace{(-\cos x)}^{v}\ \overbrace{dx}^{du}$$

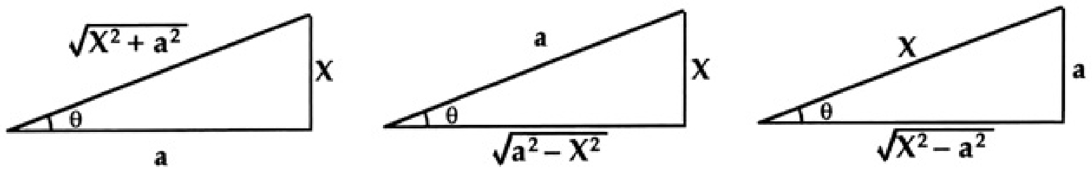
$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

7. 3 Trig Substitution

EXAMPLE 3 Find $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx.$

EXAMPLE 4 Find $\displaystyle\int \frac{x}{\sqrt{x^2+4}}\,dx.$



$\sin\theta = \dfrac{X}{\sqrt{X^2+a^2}}$	$\sin\theta = \dfrac{X}{a}$	$\sin\theta = \dfrac{a}{X}$
$\cos\theta = \dfrac{a}{\sqrt{X^2+a^2}}$	$\cos\theta = \dfrac{\sqrt{a^2-X^2}}{a}$	$\cos\theta = \dfrac{\sqrt{X^2-a^2}}{X}$
$\tan\theta = \dfrac{X}{a}$	$\tan\theta = \dfrac{X}{\sqrt{a^2-X^2}}$	$\tan\theta = \dfrac{a}{\sqrt{X^2-a^2}}$
$\csc\theta = \dfrac{\sqrt{X^2+a^2}}{X}$	$\csc\theta = \dfrac{a}{X}$	$\csc\theta = \dfrac{X}{a}$
$\sec\theta = \dfrac{\sqrt{X^2+a^2}}{a}$	$\sec\theta = \dfrac{a}{\sqrt{a^2-X^2}}$	$\sec\theta = \dfrac{X}{\sqrt{X^2-a^2}}$
$\cot\theta = \dfrac{a}{X}$	$\cot\theta = \dfrac{\sqrt{a^2-X^2}}{X}$	$\cot\theta = \dfrac{\sqrt{X^2-a^2}}{a}$

7.4 Partial Fractions

Factor in denominator	Term in partial fraction decomposition
$ax+b$	$\dfrac{A}{ax+b}$
$(ax+b)^k$	$\dfrac{A_1}{ax+b}+\dfrac{A_2}{(ax+b)^2}+\cdots+\dfrac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\dfrac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\dfrac{A_1x+B_1}{ax^2+bx+c}+\dfrac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\dfrac{A_kx+B_k}{(ax^2+bx+c)^k}$

7.5 Strategies for Integration

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \dfrac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int a^x\,dx = \dfrac{a^x}{\ln a}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
15. $\int \sinh x\,dx = \cosh x$
16. $\int \cosh x\,dx = \sinh x$
17. $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
18. $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right), \quad a > 0$

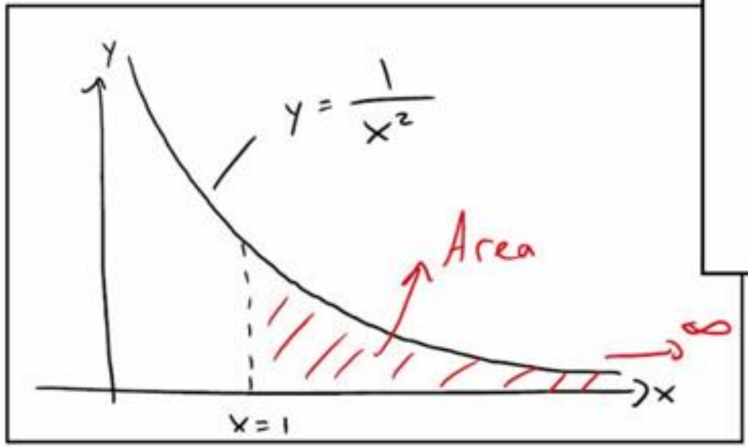
*19. $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right|$

*20. $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

7.8 Improper Integrals

Improper Integarls:
Type 1: Infinite Intervals

Type 1: Infinite Intervals



Definition of an Improper Integral of Type 1

(a) If $\int_a^t f(x)dx$ exists for every number t ≥ a, then:

$$\int_a^\infty f(x)\,dx = \lim_{t \to \infty} \int_a^t f(x)\,dx$$

Assuming this limit exists as a finite number.

8.1 Arc Length and Review

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Find the derivatives

1. $f(x) = \frac{x^2}{3} - \frac{3}{x^2}$

2. $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$

3. $f(x) = \frac{x^2 - 2}{x^2}$ + $\frac{x^2}{x^2 - 2}$

4. $f(x) = \sqrt{x}\left(x^2 + 1\right)$

5. $f(x) = \frac{e^x}{e^x - 1}$

6. $y = 2x^{\left(\frac{1}{2} - e\right)}$

Resources

Tutoring Center - Free Tutoring!

http://www.math.uci.edu/undergrad-courses/tutoring-assistance

LARC Tutoring - $100/per quarter

http://www.larc.uci.edu/

Student Support Services - Qualify for LARC tutoring scholarship?

http://sss.uci.edu/resources-scholarships/scholarships/

UCI Math Calculus Resources page which includes review materials.

http://www.math.uci.edu/undergraduate/courses/calculus-2a2b-resources

Khan Academy

https://www.khanacademy.org/

Multivariable Calculus Math 2D

Textbook: Stewart Calculus Early Transcendentals 8th Edition

Sample Syllabus

Lecture	Section	Topic
1	10.1	Curves defined by Parametric Equations
2	10.2	Calculus with Parametric Curves
3	10.3	Polar Coordinates
4	12.1, 12.2	Three-Dimensional Coordinate Systems, Vectors
5	12.3, 12.4	The Dot Product, The Cross Product
6	12.5	Equations of Lines and Planes
7	12.6	Cylinders and Quadric Surfaces
8	13.1	Vector Functions and Space Curves
9	13.2, 13.3	Derivatives and Integrals of Vector Functions, Arc Length
10	13.4	Motion in Space: Velocity and Acceleration
11		Review
12		**Midterm # 1**
13	14.1	Functions of Several Variables
14	14.2	Limits and Continuity
15	14.3	Partial Derivatives
16	14.4	Tangent Planes and Linear Approximations
17	14.5	The Chain Rule
18	14.6	Directional Derivatives and the Gradient Vector
19	14.6, 14.7	Finish 14.6, Begin 14.7
20	14.7	Maximum and Minimum Values
21	14.8	Lagrange Multipliers
22		Review
23		**Midterm # 2**
24	15.1	Double Integrals over Rectangles
25	15.2	Iterated Integrals
26	15.3	Double Integrals over General Regions
27	15.4	Double Integrals in Polar Coordinates
28	15.7	Triple Integrals
29		Review

Parametric Equations And Curves

Another way to represent functions:

Suppose that x and y are both given as functions of a third variable t (called a **parameter**) by the equations

$$x = f(t) \qquad y = g(t)$$

(called **parametric equations**). Each value of t determines a point (x, y), which we can plot in a coordinate plane. As t varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve C, which we call a **parametric curve**.

Parametric Equations And Curves

Make it's possible to represent functions that fail the Vertical Line Test.

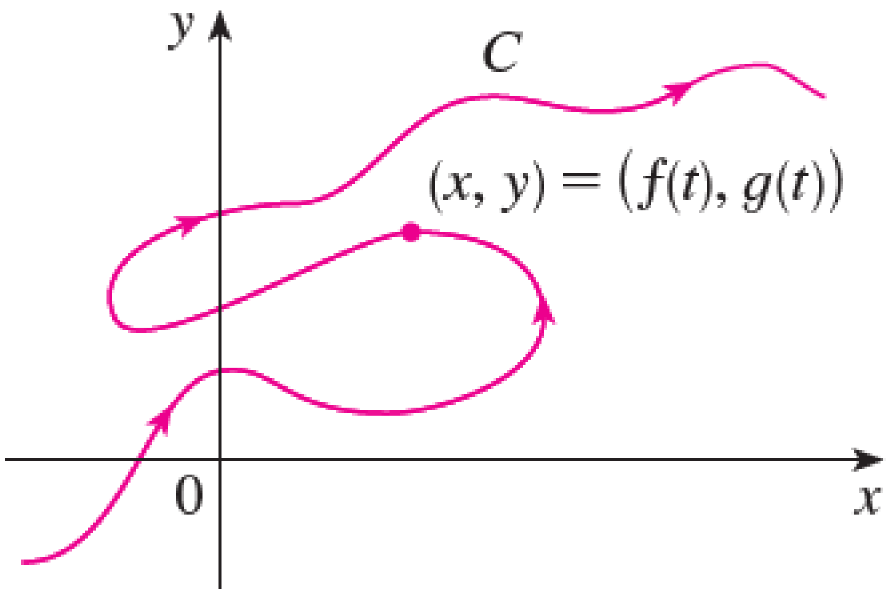


FIGURE 1

Parametric Equations And Curves

Example:

EXAMPLE 2 What curve is represented by the following parametric equations?

$$x = \cos t \qquad y = \sin t \qquad 0 \leqslant t \leqslant 2\pi$$

EXAMPLE 4 Find parametric equations for the circle with center (h, k) and radius r.

Solution:

EXAMPLE 2 What curve is represented by the following parametric equations?

$$x = \cos t \qquad y = \sin t \qquad 0 \leqslant t \leqslant 2\pi$$

SOLUTION If we plot points, it appears that the curve is a circle. We can confirm this impression by eliminating t. Observe that

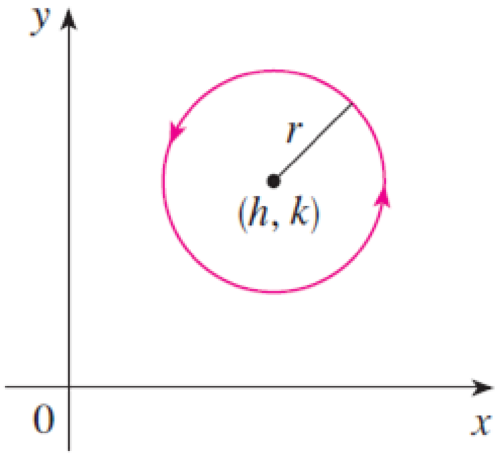
$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

Solution:

EXAMPLE 4 Find parametric equations for the circle with center (h, k) and radius r.

SOLUTION If we take the equations of the unit circle in Example 2 and multiply the expressions for x and y by r, we get $x = r\cos t$, $y = r\sin t$. You can verify that these equations represent a circle with radius r and center the origin traced counterclockwise. We now shift h units in the x-direction and k units in the y-direction and obtain parametric equations of the circle (Figure 6) with center (h, k) and radius r:

$$x = h + r\cos t \qquad y = k + r\sin t \qquad 0 \leqslant t \leqslant 2\pi$$



Derivative with Parametric Curve

Suppose f and g are differentiable functions and we want to find the tangent line at a point on the parametric curve $x = f(t)$, $y = g(t)$, where y is also a differentiable function of x. Then the Chain Rule gives

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

If $dx/dt \neq 0$, we can solve for dy/dx:

1

$$\boxed{\frac{dy}{dx} = \frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} \qquad \text{if} \quad \frac{dx}{dt} \neq 0}$$

Second Derivative with Parametric Curve

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{\dfrac{dx}{dt}}$$

Integral with Parametric Curve

According to the Chain Rule

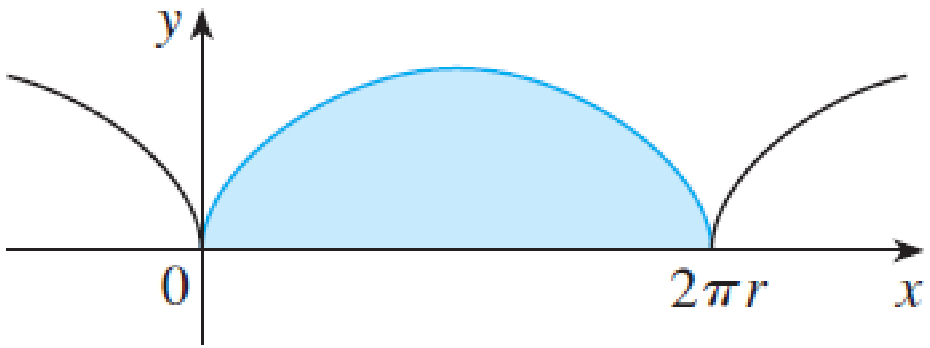
$$A = \int_a^b y\,dx = \int_\alpha^\beta g(t)f'(t)\,dt \qquad \left[\text{or} \quad \int_\beta^\alpha g(t)f'(t)\,dt\right]$$

Integral with Parametric Curve

Examples:

EXAMPLE 3 Find the area under one arch of the cycloid

$$x = r(\theta - \sin\theta) \qquad y = r(1 - \cos\theta)$$



Solution:

SOLUTION One arch of the cycloid is given by $0 \leqslant \theta \leqslant 2\pi$. Using the Substitution Rule with $y = r(1 - \cos\theta)$ and $dx = r(1 - \cos\theta)\,d\theta$, we have

$$A = \int_0^{2\pi r} y\,dx = \int_0^{2\pi} r(1 - \cos\theta)\,r(1 - \cos\theta)\,d\theta$$

$$= r^2 \int_0^{2\pi} (1 - \cos\theta)^2\,d\theta = r^2 \int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\,d\theta$$

$$= r^2 \int_0^{2\pi} \left[1 - 2\cos\theta + \tfrac{1}{2}(1 + \cos 2\theta)\right] d\theta$$

$$= r^2\left[\tfrac{3}{2}\theta - 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{2\pi}$$

$$= r^2\left(\tfrac{3}{2}\cdot 2\pi\right) = 3\pi r^2$$

$$\sin^2(x) = \tfrac{1}{2}[1 - \cos(2x)]$$

$$\cos^2(x) = \tfrac{1}{2}[1 + \cos(2x)]$$

$$\tan^2(x) = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

Arc Length with Parametric Curve

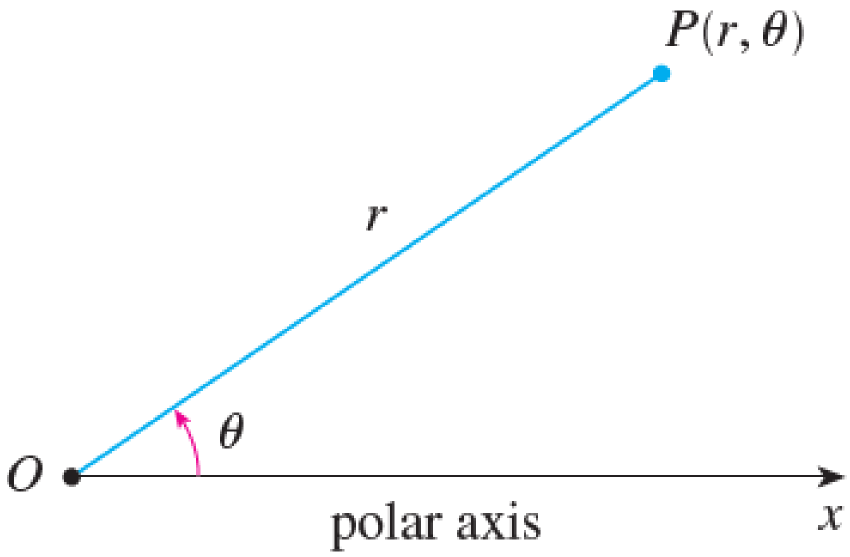
$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Surface Area with Parametric Curve

$$S = \int_{\alpha}^{\beta} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Polar Coordinates

Another coordinate system



Conversion

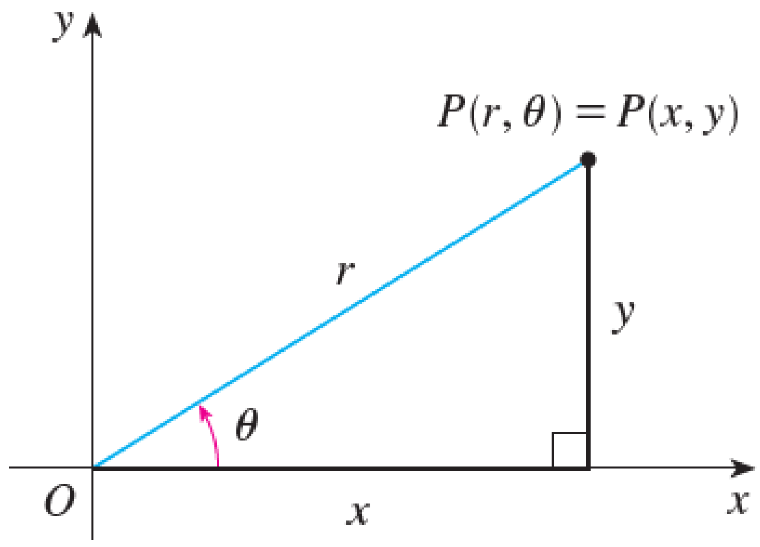
The connection between polar and Cartesian coordinates can be seen from Figure 5, in which the pole corresponds to the origin and the polar axis coincides with the positive x-axis. If the point P has Cartesian coordinates (x, y) and polar coordinates (r, θ), then, from the figure, we have

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$$

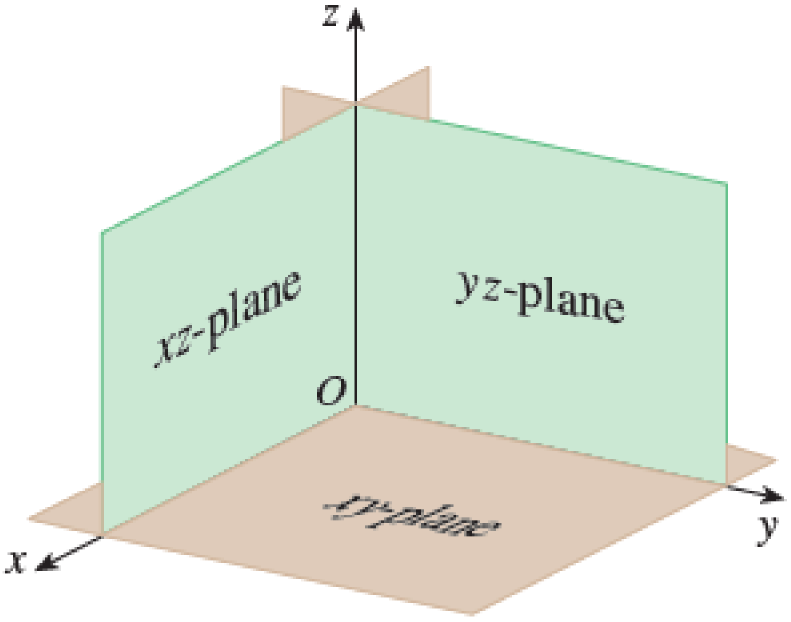
and so

$$\boxed{x = r\cos\theta \qquad y = r\sin\theta}$$

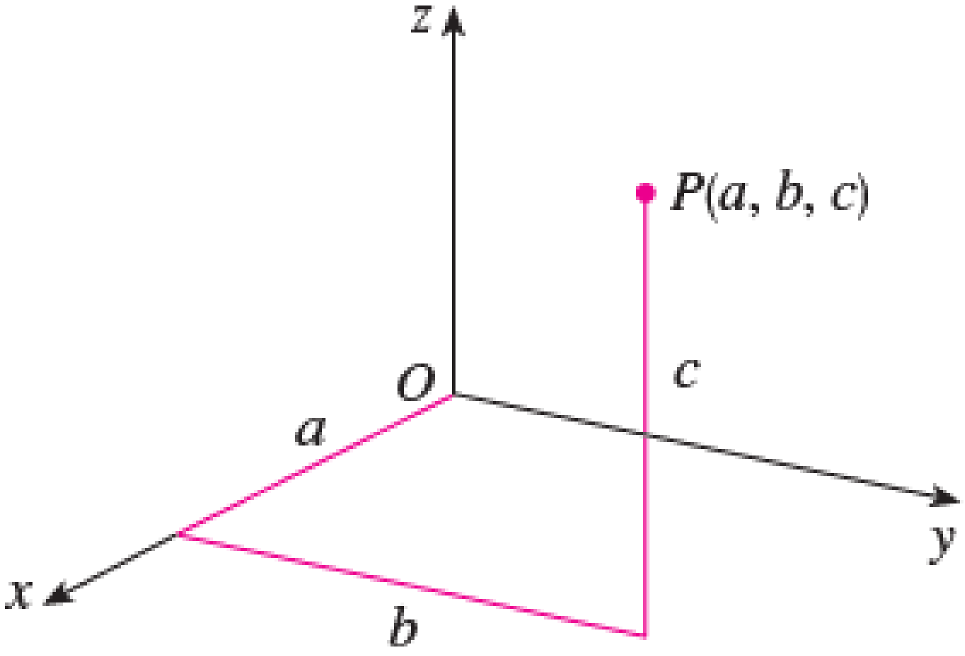
$$\boxed{r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}}$$



Three-Dimensional Coordinate Systems



(a) Coordinate planes



Vectors

A quantity that has both magnitude and direction.

For instance, suppose a particle moves along a line segment from point A to point B. The corresponding **displacement vector** $\mathbf{v}$, shown in Figure 1, has **initial point** A (the tail) and **terminal point** B (the tip) and we indicate this by writing $\mathbf{v} = \overrightarrow{AB}$. Notice that the vector $\mathbf{u} = \overrightarrow{CD}$ has the same length and the same direction as $\mathbf{v}$ even though it is in a different position. We say that $\mathbf{u}$ and $\mathbf{v}$ are **equivalent** (or **equal**) and we write $\mathbf{u} = \mathbf{v}$. The **zero vector**, denoted by $\mathbf{0}$, has length 0. It is the only vector with no specific direction.

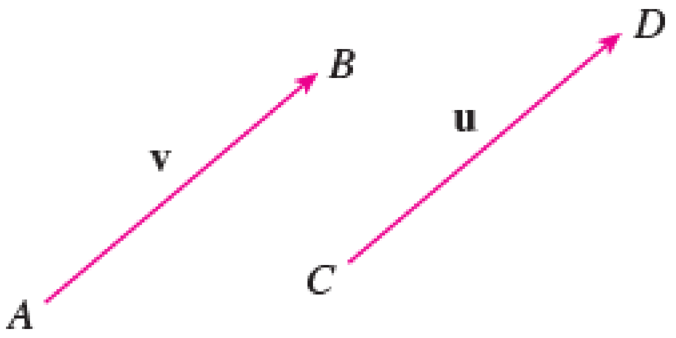


FIGURE 1
Equivalent vectors

Vector Addition

Definition of Vector Addition If **u** and **v** are vectors positioned so the initial point of **v** is at the terminal point of **u**, then the **sum u + v** is the vector from the initial point of **u** to the terminal point of **v**.

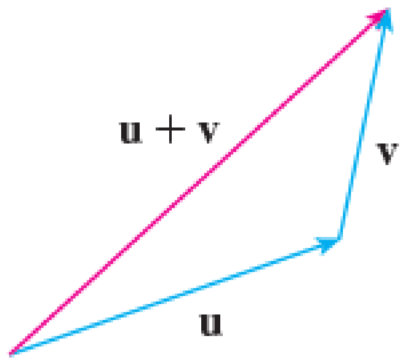


FIGURE 3
The Triangle Law

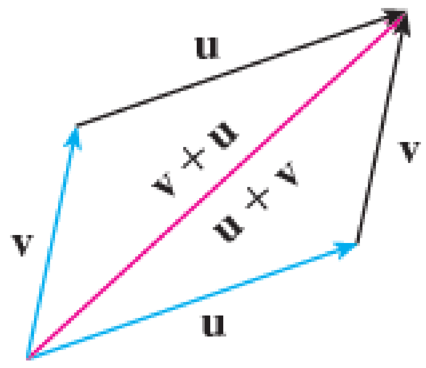
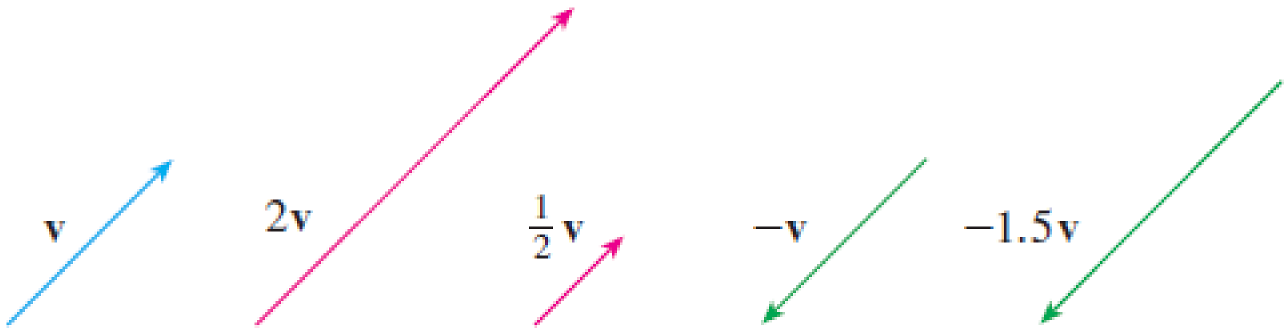


FIGURE 4
The Parallelogram Law

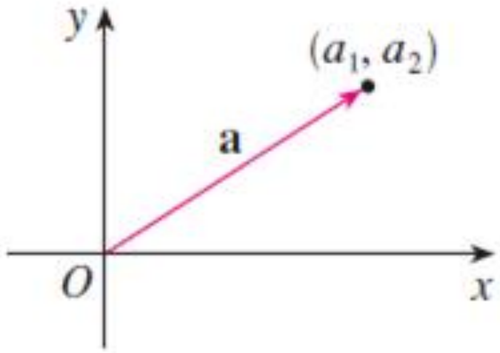
Scalar Multiplication

Definition of Scalar Multiplication If c is a scalar and $\mathbf{v}$ is a vector, then the **scalar multiple** $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose direction is the same as $\mathbf{v}$ if $c > 0$ and is opposite to $\mathbf{v}$ if $c < 0$. If $c = 0$ or $\mathbf{v} = \mathbf{0}$, then $c\mathbf{v} = \mathbf{0}$.

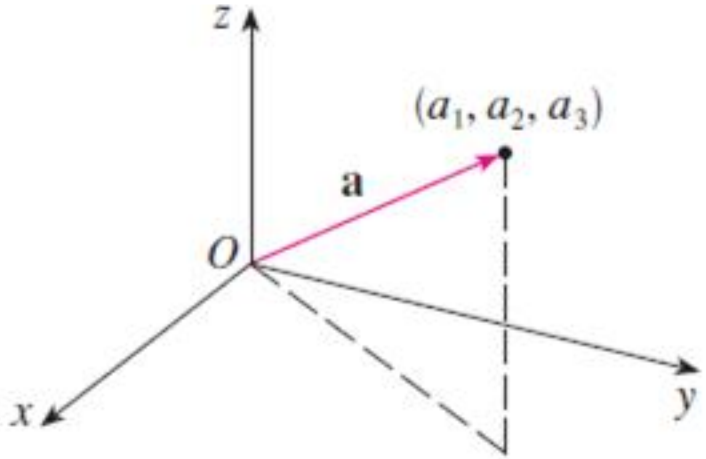


Components of Vector

AKA the coordinates of the vector



$\mathbf{a} = \langle a_1, a_2 \rangle$



$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

1 Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

Unit Vectors

$$\mathbf{i} = \langle 1, 0, 0 \rangle \qquad \mathbf{j} = \langle 0, 1, 0 \rangle \qquad \mathbf{k} = \langle 0, 0, 1 \rangle$$

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, then we can write

$$\begin{aligned} \mathbf{a} = \langle a_1, a_2, a_3 \rangle &= \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0, 0, a_3 \rangle \\ &= a_1 \langle 1, 0, 0 \rangle + a_2 \langle 0, 1, 0 \rangle + a_3 \langle 0, 0, 1 \rangle \end{aligned}$$

$$\boxed{2} \qquad \mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$$

Questions:

1. What's the magnitude of vector *a* where *a* = <*x*, *y*, *z*>?
2. If a = <a1, a2 > and b = <b1, b2>, then
   a. What's the component of a+b
   b. What about a-b?
   c. If *c* is a scalar, what's *c*a?

Properties of Vector Addition And Scalar Multiplication

Properties of Vectors If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in V_n and c and d are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$

Dot Product

1 Definition If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of $\mathbf{a}$ and $\mathbf{b}$ is the number $\mathbf{a} \cdot \mathbf{b}$ given by

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Also called: Scalar product inner product

Properties of Dot Product

2 Properties of the Dot Product If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in V_3 and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $\mathbf{0} \cdot \mathbf{a} = 0$

3 Theorem If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

7 Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Projection

Scalar projection of **b** onto **a**: $\quad \text{comp}_{\mathbf{a}}\, \mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$

Vector projection of **b** onto **a**: $\quad \text{proj}_{\mathbf{a}}\, \mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}$

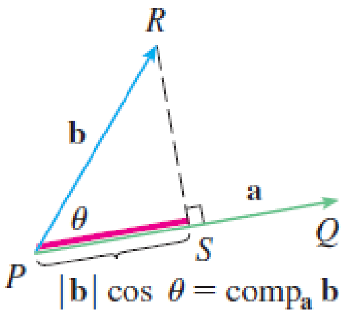


FIGURE 5
Scalar projection

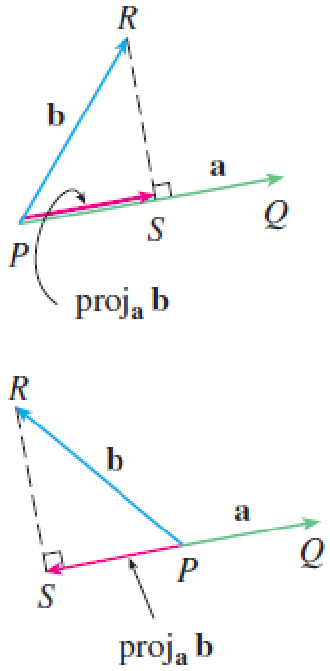


FIGURE 4
Vector projections

Cross Product

4 Definition If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of $\mathbf{a}$ and $\mathbf{b}$ is the vector

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1 \rangle$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$\mathbf{i} = \langle 1, 0, 0 \rangle$ $\quad$ $\mathbf{j} = \langle 0, 1, 0 \rangle$ $\quad$ $\mathbf{k} = \langle 0, 0, 1 \rangle$

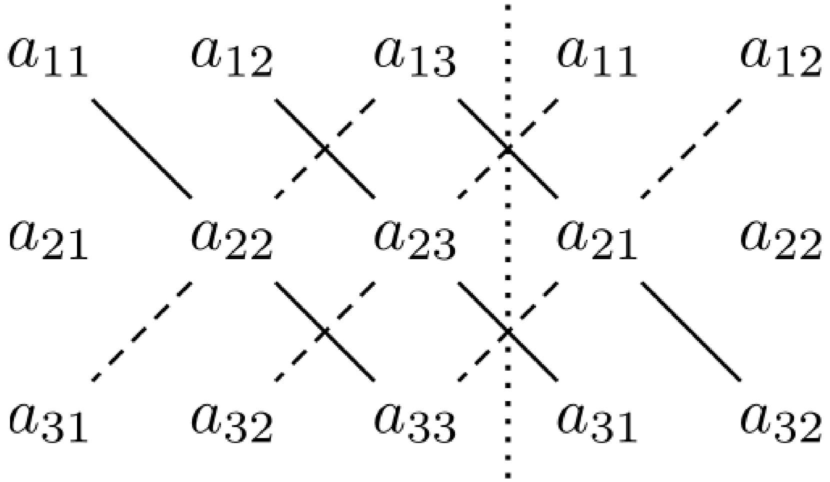
Determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$= aei + bfg + cdh - ceg - bdi - afh.$$



Property of Cross Product

8 Theorem The vector $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

9 Theorem If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0 \leqslant \theta \leqslant \pi$), then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$$

10 Corollary Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if

$$\mathbf{a} \times \mathbf{b} = \mathbf{0}$$

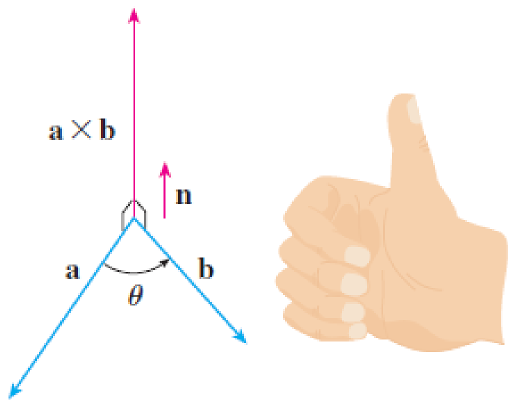


FIGURE 1
The right-hand rule gives the direction of $\mathbf{a} \times \mathbf{b}$.

Property of Cross Product

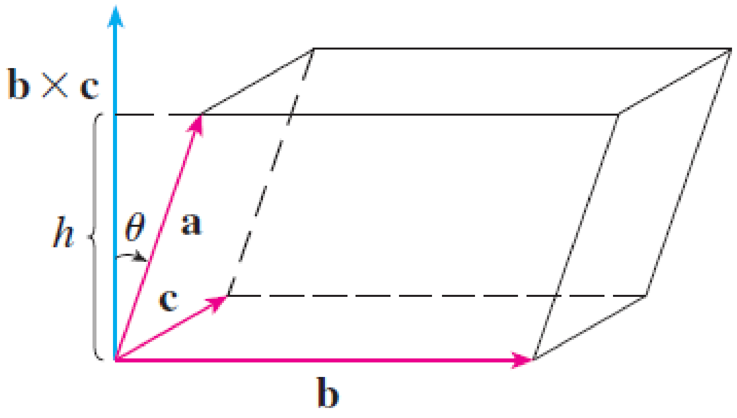
$\boxed{11}$ **Properties of the Cross Product** If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Volume of the Parallelepiped

14 The volume of the parallelepiped determined by the vectors **a**, **b**, and **c** is the magnitude of their scalar triple product:

$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$



Vector Equations of Lines

$$\mathbf{r} = \mathbf{r}_0 + t\,\mathbf{v}$$

2 Parametric equations for a line through the point (x_0, y_0, z_0) and parallel to the direction vector $\langle a, b, c \rangle$ are

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

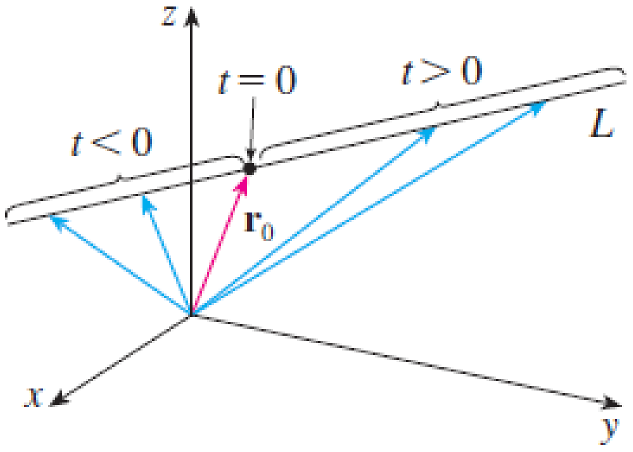


FIGURE 2

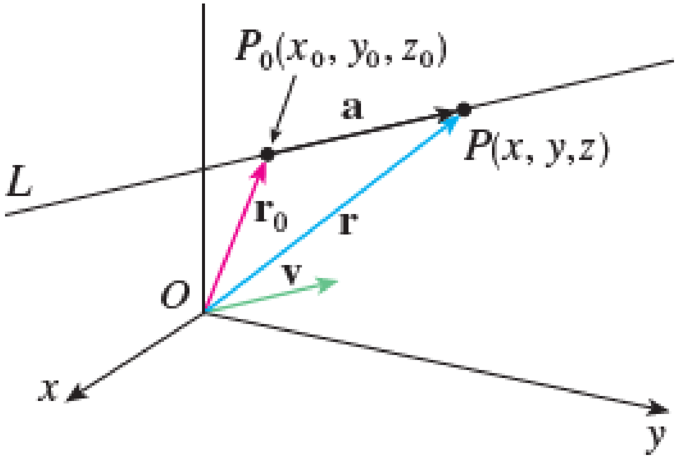


FIGURE 1

Example

EXAMPLE 1

(a) Find a vector equation and parametric equations for the line that passes through the point $(5, 1, 3)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

Solution

SOLUTION

(a) Here $\mathbf{r}_0 = \langle 5, 1, 3 \rangle = 5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, so the vector equation (1) becomes

$$\mathbf{r} = (5\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + t(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

or

$$\mathbf{r} = (5 + t)\,\mathbf{i} + (1 + 4t)\,\mathbf{j} + (3 - 2t)\,\mathbf{k}$$

Parametric equations are

$$x = 5 + t \qquad y = 1 + 4t \qquad z = 3 - 2t$$

Vector Equation of Plane

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

$\boxed{7}$ **A scalar equation of the plane** through point $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

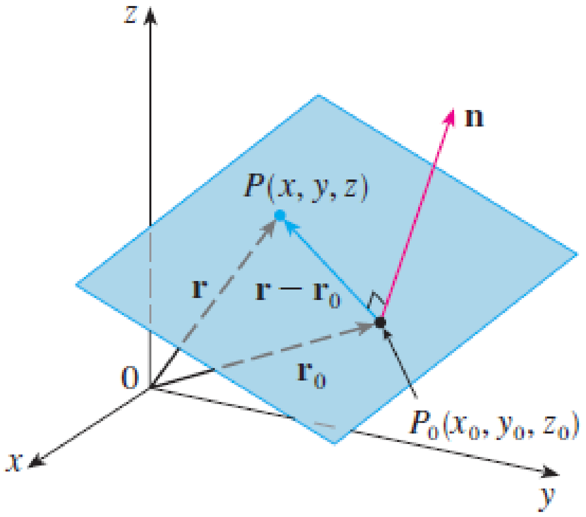


FIGURE 6

Example

EXAMPLE 4 Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $\mathbf{n} = \langle 2, 3, 4 \rangle$. Find the intercepts and sketch the plane.

EXAMPLE 5 Find an equation of the plane that passes through the points $P(1, 3, 2)$, $Q(3, -1, 6)$, and $R(5, 2, 0)$.

Solution

SOLUTION Putting $a = 2$, $b = 3$, $c = 4$, $x_0 = 2$, $y_0 = 4$, and $z_0 = -1$ in Equation 7, we see that an equation of the plane is

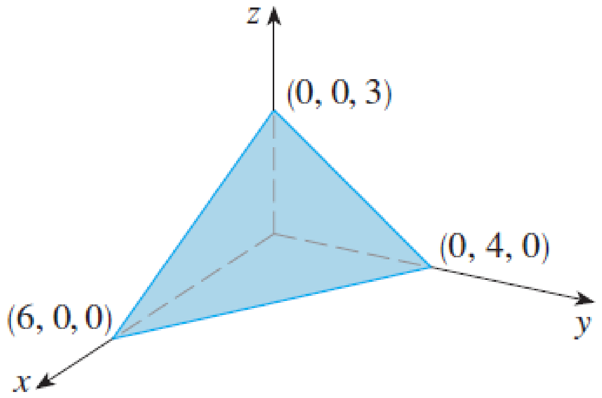
$$2(x - 2) + 3(y - 4) + 4(z + 1) = 0$$

or

$$2x + 3y + 4z = 12$$

When x=z=0, y = 4;

When y=z=0, x = 6.



Solution

SOLUTION The vectors **a** and **b** corresponding to $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are

$$\mathbf{a} = \langle 2, -4, 4 \rangle \qquad \mathbf{b} = \langle 4, -1, -2 \rangle$$

Since both **a** and **b** lie in the plane, their cross product $\mathbf{a} \times \mathbf{b}$ is orthogonal to the plane and can be taken as the normal vector. Thus

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 4 \\ 4 & -1 & -2 \end{vmatrix} = 12\,\mathbf{i} + 20\,\mathbf{j} + 14\,\mathbf{k}$$

With the point $P(1, 3, 2)$ and the normal vector **n**, an equation of the plane is

$$12(x - 1) + 20(y - 3) + 14(z - 2) = 0$$

or

$$6x + 10y + 7z = 50$$

■

Vector Functions

A function whose domain is real numbers and whose range is vectors

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$$

Space Curves

Curve that traced by a moving particle whose position at time *t* is *(f(t), g(t), h(t))*.

Limits of Vector Functions

$\boxed{1}$ If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \to a} \mathbf{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

provided the limits of the component functions exist.

Derivative of Vector Functions

2 Theorem If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$, where f, g, and h are differentiable functions, then

$$\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle = f'(t)\,\mathbf{i} + g'(t)\,\mathbf{j} + h'(t)\,\mathbf{k}$$

Integral of Vector Functions

$$\int_a^b \mathbf{r}(t)\,dt = \left(\int_a^b f(t)\,dt\right)\mathbf{i} + \left(\int_a^b g(t)\,dt\right)\mathbf{j} + \left(\int_a^b h(t)\,dt\right)\mathbf{k}$$

Property of Derivatives

3 Theorem Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Then

1. $\dfrac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t)$

2. $\dfrac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t)$

3. $\dfrac{d}{dt}[f(t)\,\mathbf{u}(t)] = f'(t)\,\mathbf{u}(t) + f(t)\,\mathbf{u}'(t)$

4. $\dfrac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$

5. $\dfrac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$

6. $\dfrac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\,\mathbf{u}'(f(t))$ (Chain Rule)

Arc Length

$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\, dt$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

Arc Length Function

$$s(t) = \int_a^t |\mathbf{r}'(u)|\, du = \int_a^t \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2}\, du \qquad \frac{ds}{dt} = |\mathbf{r}'(t)|$$

Curvature

A measure of how quickly the curve changes direction at one point

Unit tangent vectors of *r(t)*:

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

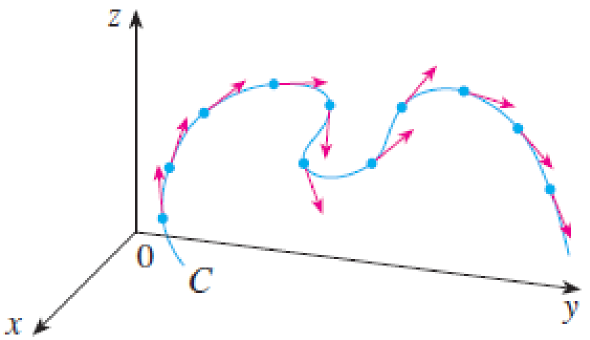


FIGURE 4
Unit tangent vectors at equally spaced points on C

Curvature:

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right|$$

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$$

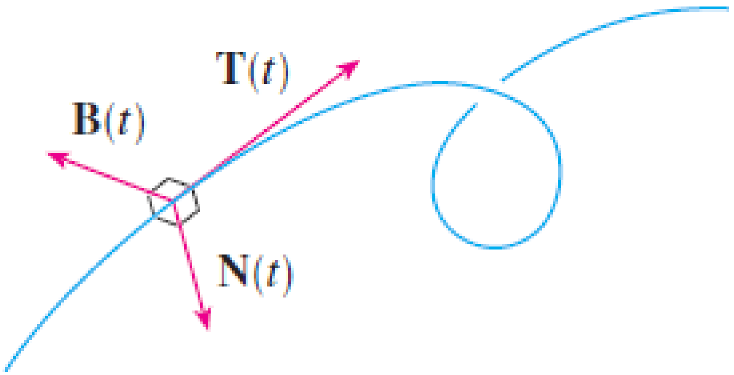
$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

Unit Normal Vector

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$$

Binormal Vector

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$



Questions

EXAMPLE 1

(a) Find the derivative of $\mathbf{r}(t) = (1 + t^3)\,\mathbf{i} + te^{-t}\,\mathbf{j} + \sin 2t\,\mathbf{k}$.

(b) Find the unit tangent vector at the point where $t = 0$.

Solution

SOLUTION

(a) According to Theorem 2, we differentiate each component of $\mathbf{r}$:

$$\mathbf{r}'(t) = 3t^2\,\mathbf{i} + (1 - t)e^{-t}\,\mathbf{j} + 2\cos 2t\,\mathbf{k}$$

(b) Since $\mathbf{r}(0) = \mathbf{i}$ and $\mathbf{r}'(0) = \mathbf{j} + 2\mathbf{k}$, the unit tangent vector at the point $(1, 0, 0)$ is

$$\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \frac{\mathbf{j} + 2\mathbf{k}}{\sqrt{1 + 4}} = \frac{1}{\sqrt{5}}\,\mathbf{j} + \frac{2}{\sqrt{5}}\,\mathbf{k} \qquad \blacksquare$$

Question

Evaluate the integral

35. $\int_0^2 (t\,\mathbf{i} - t^3\,\mathbf{j} + 3t^5\,\mathbf{k})\,dt$

Solution

35. $2\,\mathbf{i} - 4\,\mathbf{j} + 32\,\mathbf{k}$

Example

EXAMPLE 1 Find the length of the arc of the circular helix with vector equation $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$ from the point $(1, 0, 0)$ to the point $(1, 0, 2\pi)$.

Solution

SOLUTION Since $\mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \mathbf{k}$, we have

$$|\mathbf{r}'(t)| = \sqrt{(-\sin t)^2 + \cos^2 t + 1} = \sqrt{2}$$

The arc from $(1, 0, 0)$ to $(1, 0, 2\pi)$ is described by the parameter interval $0 \leqslant t \leqslant 2\pi$ and so, from Formula 3, we have

$$L = \int_0^{2\pi} |\mathbf{r}'(t)|\,dt = \int_0^{2\pi} \sqrt{2}\,dt = 2\sqrt{2}\pi \qquad \blacksquare$$

Example

EXAMPLE 4 Find the curvature of the twisted cubic $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at a general point and at $(0, 0, 0)$.

Solution

SOLUTION We first compute the required ingredients:

$$\mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle \qquad\qquad \mathbf{r}''(t) = \langle 0, 2, 6t \rangle$$

$$|\mathbf{r}'(t)| = \sqrt{1 + 4t^2 + 9t^4}$$

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = 6t^2\,\mathbf{i} - 6t\,\mathbf{j} + 2\,\mathbf{k}$$

$$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{36t^4 + 36t^2 + 4} = 2\sqrt{9t^4 + 9t^2 + 1}$$

Theorem 10 then gives

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{2\sqrt{1 + 9t^2 + 9t^4}}{(1 + 4t^2 + 9t^4)^{3/2}}$$

At the origin, where $t = 0$, the curvature is $\kappa(0) = 2$.

Functions of Two Variables

Definition A **function** f **of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

Partial Derivatives

partial derivative of *f* with respect to *x*:

Notations for Partial Derivatives If $z = f(x, y)$, we write

$$f_x(x, y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x, y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$

Regard the other variables (*y*) as constant when differentiate the function with respect of *x*.

Partial Derivative

Examples:

EXAMPLE 1 If $f(x, y) = x^3 + x^2y^3 - 2y^2$, find $f_x(2, 1)$ and $f_y(2, 1)$.

EXAMPLE 2 If $f(x, y) = 4 - x^2 - 2y^2$, find $f_x(1, 1)$ and $f_y(1, 1)$ and interpret these numbers as slopes.

Solution

SOLUTION Holding y constant and differentiating with respect to x, we get

$$f_x(x, y) = 3x^2 + 2xy^3$$

and so

$$f_x(2, 1) = 3 \cdot 2^2 + 2 \cdot 2 \cdot 1^3 = 16$$

Holding x constant and differentiating with respect to y, we get

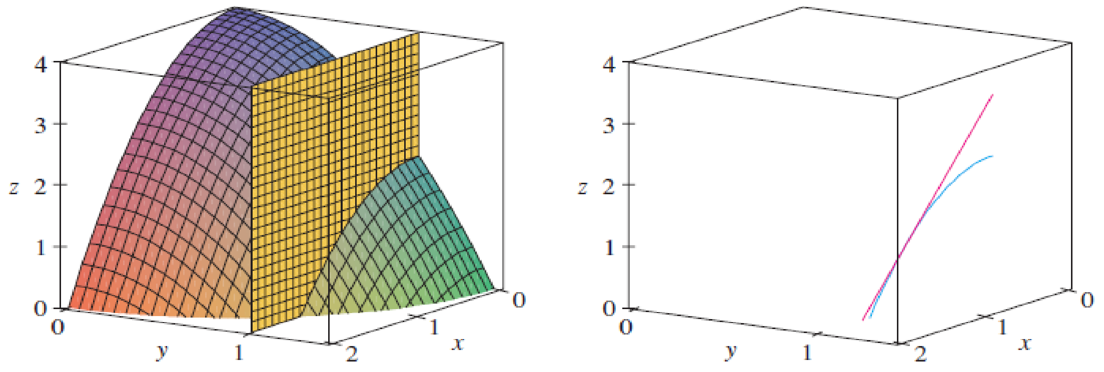
$$f_y(x, y) = 3x^2y^2 - 4y$$

$$f_y(2, 1) = 3 \cdot 2^2 \cdot 1^2 - 4 \cdot 1 = 8$$

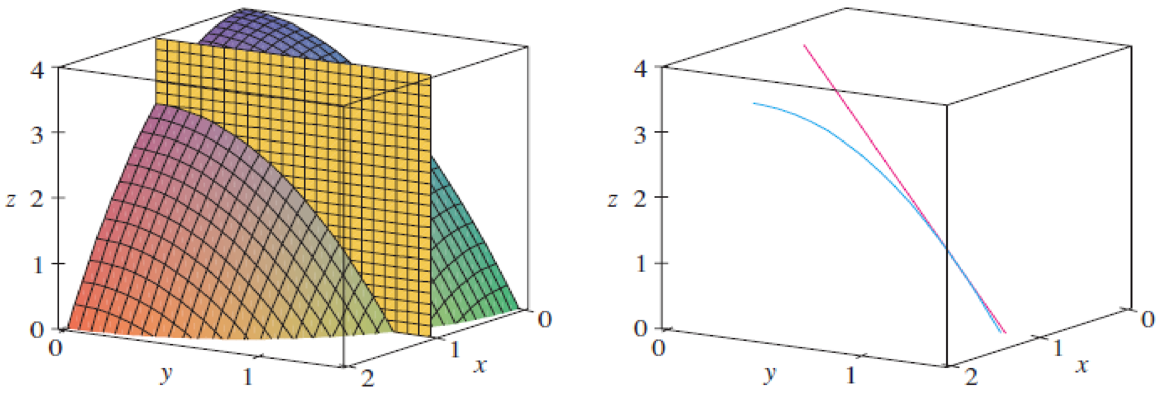
SOLUTION We have

$$f_x(x, y) = -2x \qquad\qquad f_y(x, y) = -4y$$

$$f_x(1, 1) = -2 \qquad\qquad f_y(1, 1) = -4$$



(a) (b)



Tangent Planes

2 Suppose f has continuous partial derivatives. An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

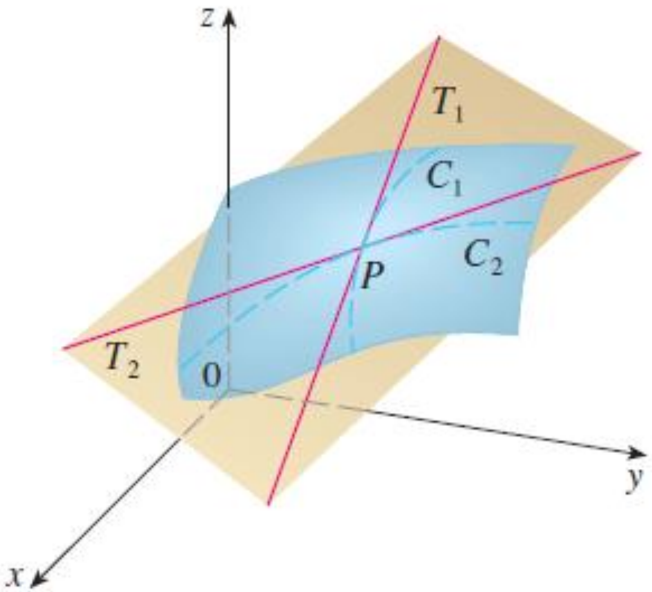
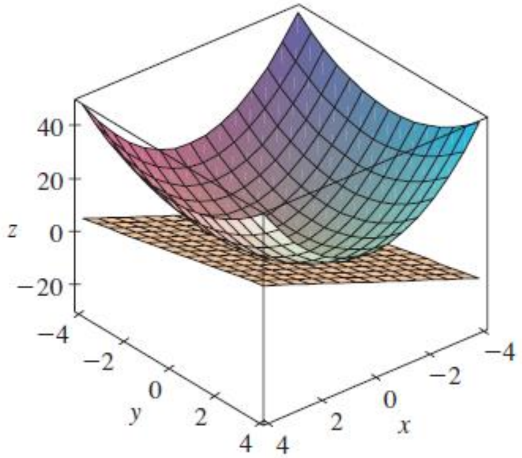


FIGURE 1
The tangent plane contains the tangent lines T_1 and T_2.



Linear Approximations

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Differential

$$dz = f_x(x, y)\,dx + f_y(x, y)\,dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

If we take $dx = \Delta x = x - a$ and $dy = \Delta y = y - b$:

$$dz = f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

The Chain Rule

2 The Chain Rule (Case 1) Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t. Then z is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

3 The Chain Rule (Case 2) Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t. Then

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Differential

Example:

EXAMPLE 1 If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt when $t = 0$.

Solution

SOLUTION The Chain Rule gives

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= (2xy + 3y^4)(2\cos 2t) + (x^2 + 12xy^3)(-\sin t)$$

When $t=0$, $y = \cos(2t) = 1$, $x = \sin(2t) = 0$

$$\left.\frac{dz}{dt}\right|_{t=0} = (0 + 3)(2\cos 0) + (0 + 0)(-\sin 0) = 6$$

Implicit Differentiation

$$\frac{dy}{dx} = -\frac{\dfrac{\partial F}{\partial x}}{\dfrac{\partial F}{\partial y}} = -\frac{F_x}{F_y}$$

$$\frac{\partial z}{\partial x} = -\frac{\dfrac{\partial F}{\partial x}}{\dfrac{\partial F}{\partial z}} \qquad \frac{\partial z}{\partial y} = -\frac{\dfrac{\partial F}{\partial y}}{\dfrac{\partial F}{\partial z}}$$

Directional Derivatives

3 Theorem If f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $\mathbf{u} = \langle a, b \rangle$ and

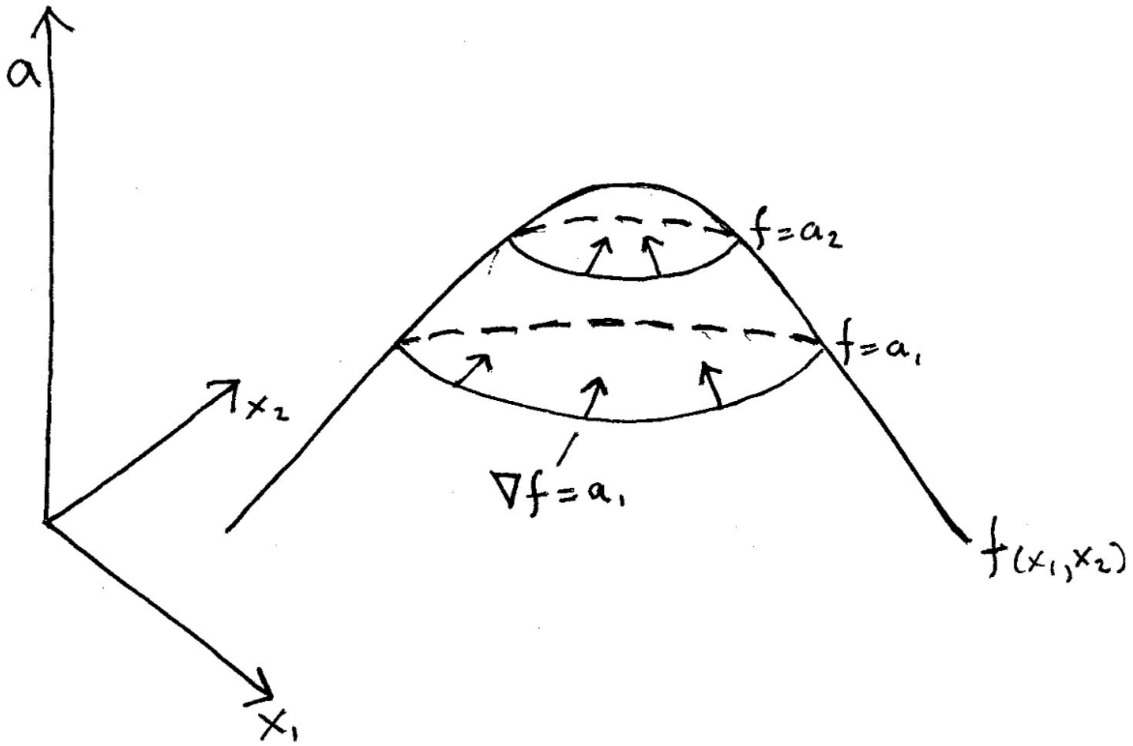
$$D_{\mathbf{u}} f(x, y) = f_x(x, y)\, a + f_y(x, y)\, b$$

Gradient Vector

8 Definition If f is a function of two variables x and y, then the **gradient** of f is the vector function ∇f defined by

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

Illustrations of Gradient Vector



Maximum and Minimum Values

2 Theorem If f has a local maximum or minimum at (a, b) and the first-order partial derivatives of f exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

3 Second Derivatives Test Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ [that is, (a, b) is a critical point of f]. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

(a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

(b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

(c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

Lagrange Multipliers

maximizing or minimizing a general function $f(x, y, z)$ subject to a constraint of the form $g(x, y, z) = k$.

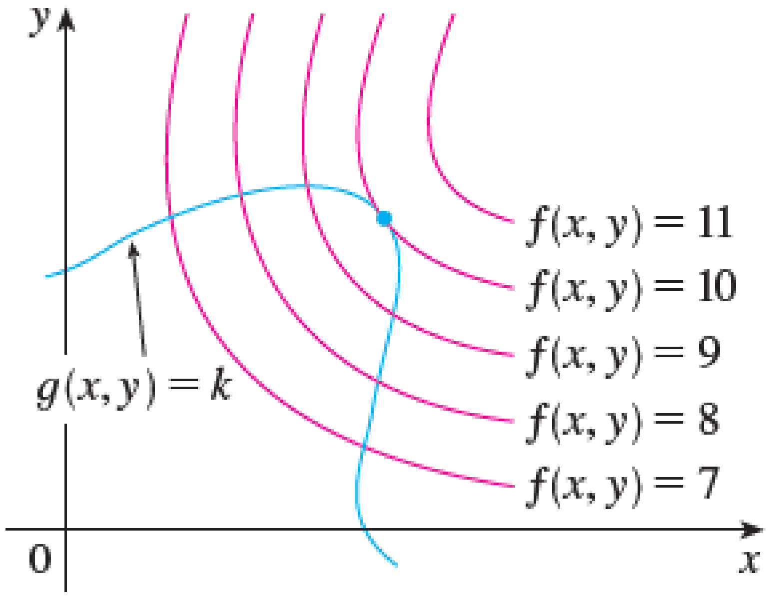


FIGURE 1

Lagrange Multiplier

Therefore, if $\nabla g(x_0, y_0, z_0) \neq \mathbf{0}$, there is a number λ such that

$$\boxed{1} \qquad \boxed{\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0)}$$

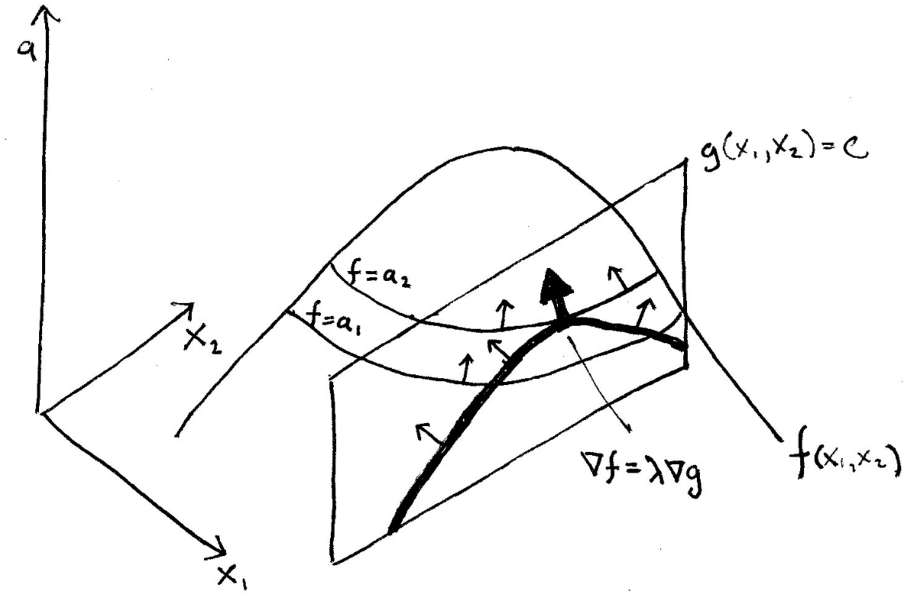
The number λ in Equation 1 is called a **Lagrange multiplier**. The

To find the extreme value of *f(x, y, z)*, solve equations:

or

$$\nabla f(x, y) = \lambda \nabla g(x, y) \qquad \text{and} \qquad g(x, y) = k$$

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = k$$



Double Integrals

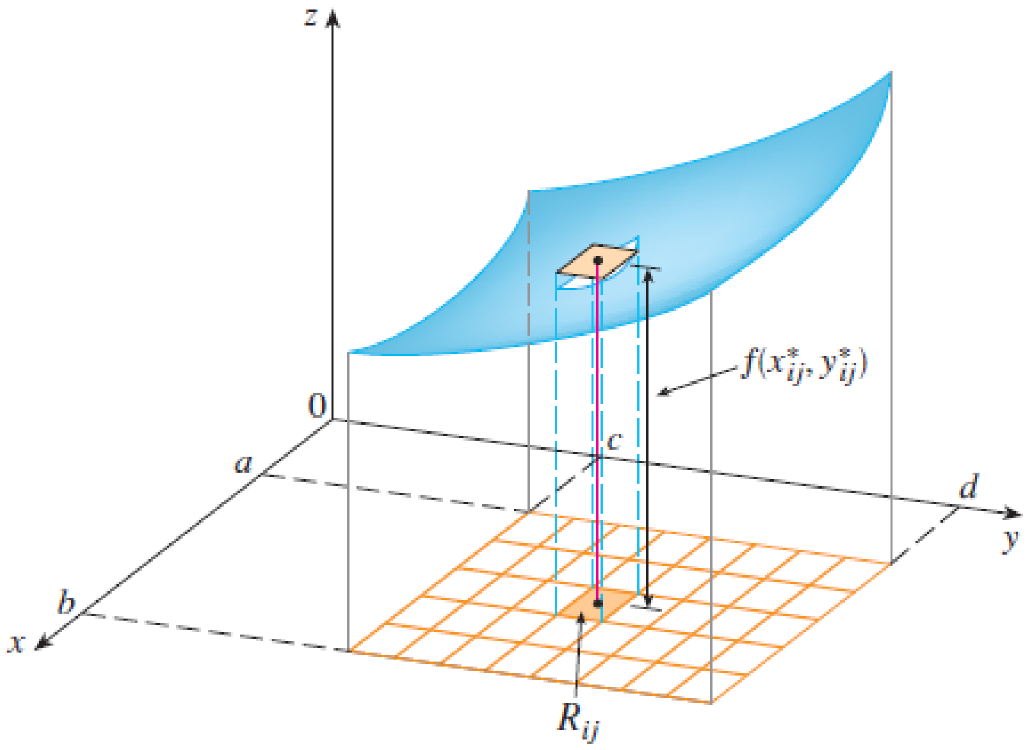


FIGURE 4

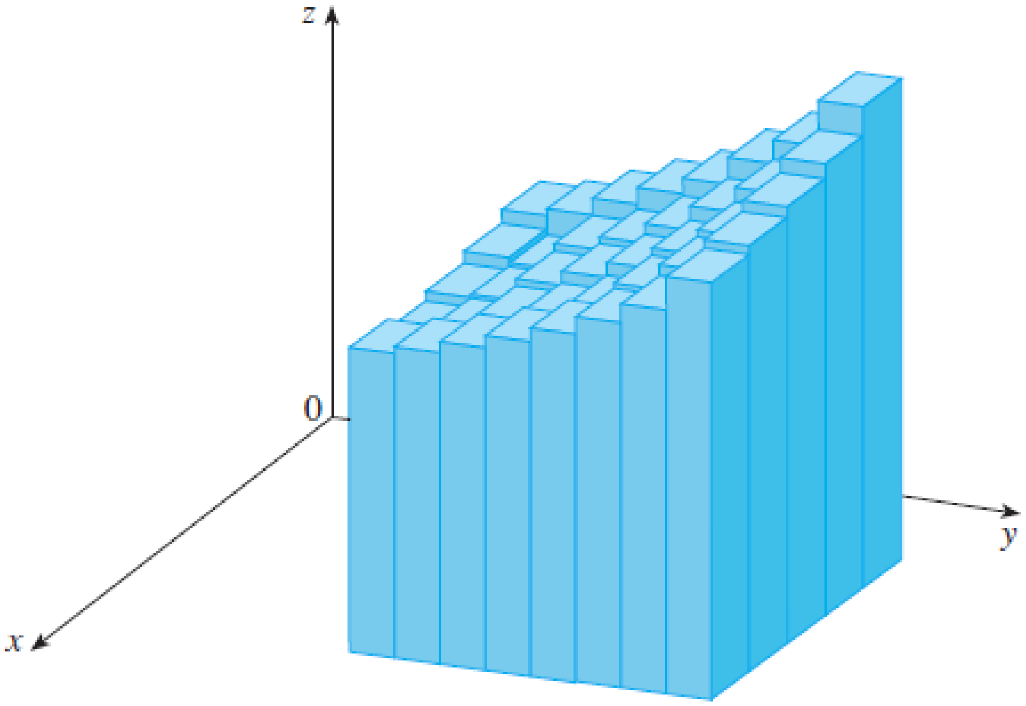


FIGURE 5

Double Integral

10 **Fubini's Theorem** If f is continuous on the rectangle $R = \{(x, y) \mid a \leqslant x \leqslant b, c \leqslant y \leqslant d\}$, then

$$\iint_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy$$

More generally, this is true if we assume that f is bounded on R, f is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

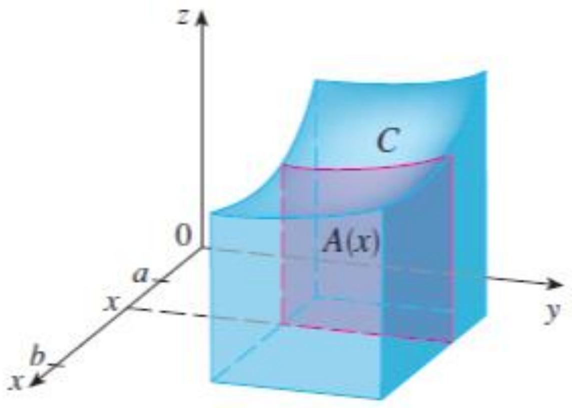


FIGURE 11

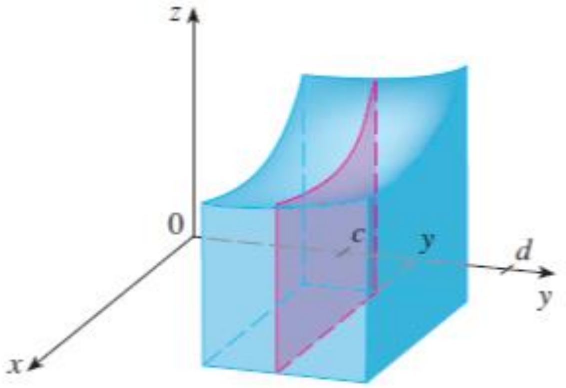


FIGURE 12

Double Integral over General Regions

3 If f is continuous on a type I region D such that

$$D = \{(x, y) \mid a \leqslant x \leqslant b,\ g_1(x) \leqslant y \leqslant g_2(x)\}$$

then
$$\iint_D f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$

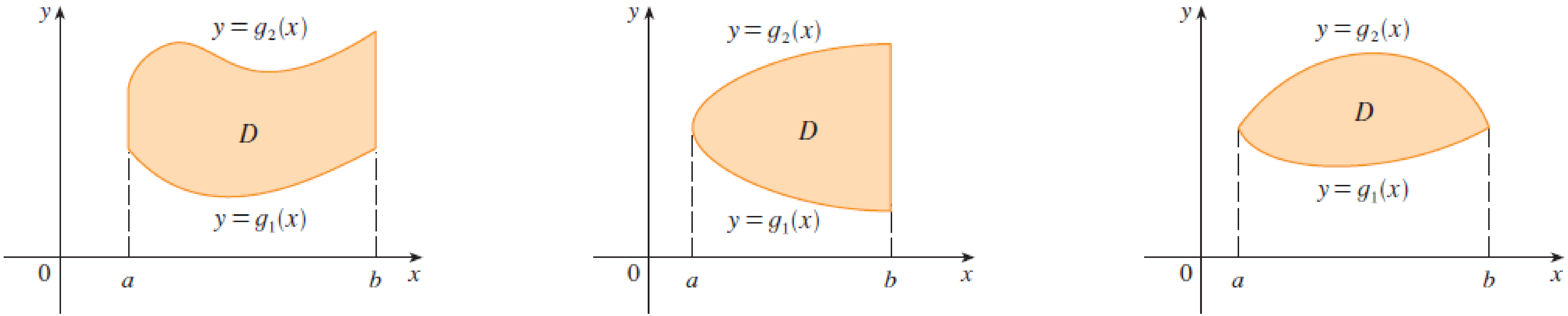


FIGURE 5
Some type I regions

Double Integral over General Regions

$$D = \{(x, y) \mid c \leqslant y \leqslant d,\ h_1(y) \leqslant x \leqslant h_2(y)\}$$

$\boxed{5}$

$$\iint_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy$$

where D is a type II region given by Equation 4.

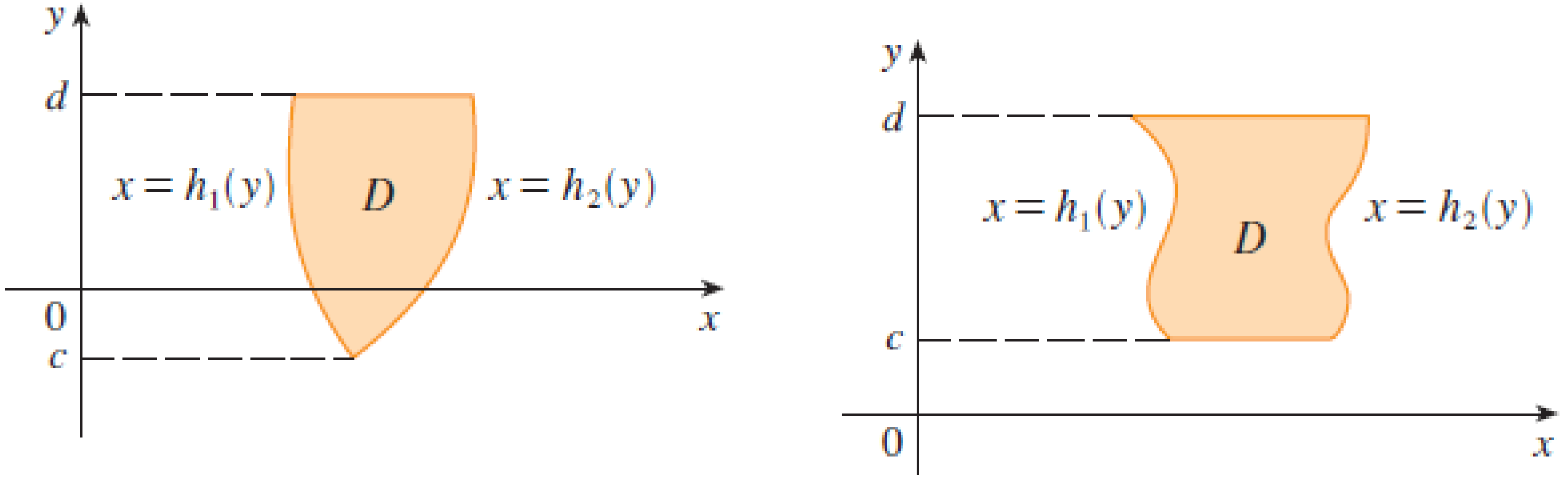


FIGURE 7
Some type II regions

Resources

LARC

http://www.larc.uci.edu/students/tutoring/larc-tutorial-schedules/

OAI Tutoring Center

http://tech.uci.edu/access/students-current.php

General Chemistry
Chem 1A

General Syllabus

CHEM 1A GENERAL CHEMISTRY

Catalog Data:
CHEM 1A General Chemistry (Credit Units: 4) Atomic structure; general properties of the elements; covalent, ionic, and metallic bonding; intermolecular forces; mass relationships. Corequisite: Concurrent enrollment in the corresponding laboratory courses. Prerequisite: One year of high school chemistry is strongly recommended.

Required Textbook:
Loretta Jones, *Chemical Principles.* Fifth Edition Edition, W. H. Freeman, 2009, ISBN-13 978-1429219556 .

Recommended Textbook:
Carl Hoeger, et al., *Student Study Guide and Solutions Manual for Chemical Principles: The Quest for Insight.* 5thth Edition, W. H. Freeman, 2010, ISBN-13 978-1429231350.

Course Learning Outcomes. Students will:

Lecture Topics:
Atomic Structure and Quantum Mechanics
The Periodic Table
Periodic trends in Chemical properties
Lewis dot symbols and ionic bonds
Covalent bonds, Lewis Structures
Resonance, expanded octets, bond enthalpy
Molecular geometry
Chemical bonding
Molecular orbital theory

Class Schedule:
Meets for 3 hours of lecture and 1 hour of discussion each week for 10 weeks.

Significant Figures

1. Nonzero numbers are always significant
2. Zero digits (special cases)
    a. Leading zeros are never significant
    b. Zeros between nonzero numbers are always significant
    c. Zeros at the end of numbers are significant ONLY if the number contains a decimal point

Examples

1. 54
2. 504
3. 0.00504
4. 2.000
5. 4200
6. 4200.
7. 4.20 *10^3

Solutions

1. 54 (2)
2. 504 (3)
3. 0.00504 (3)
4. 2.000 (4)
5. 4200 (2)
6. 4200. (4)
7. 4.20 *10^3 (3)

Significant Figures in Calculations

1. Rules for addition and subtractions

   Use the lowest decimal places

   Ex: $0.24 + 0.345 = 0.5\underline{9}$, $3233 + 1300 = 4\underline{5}00$

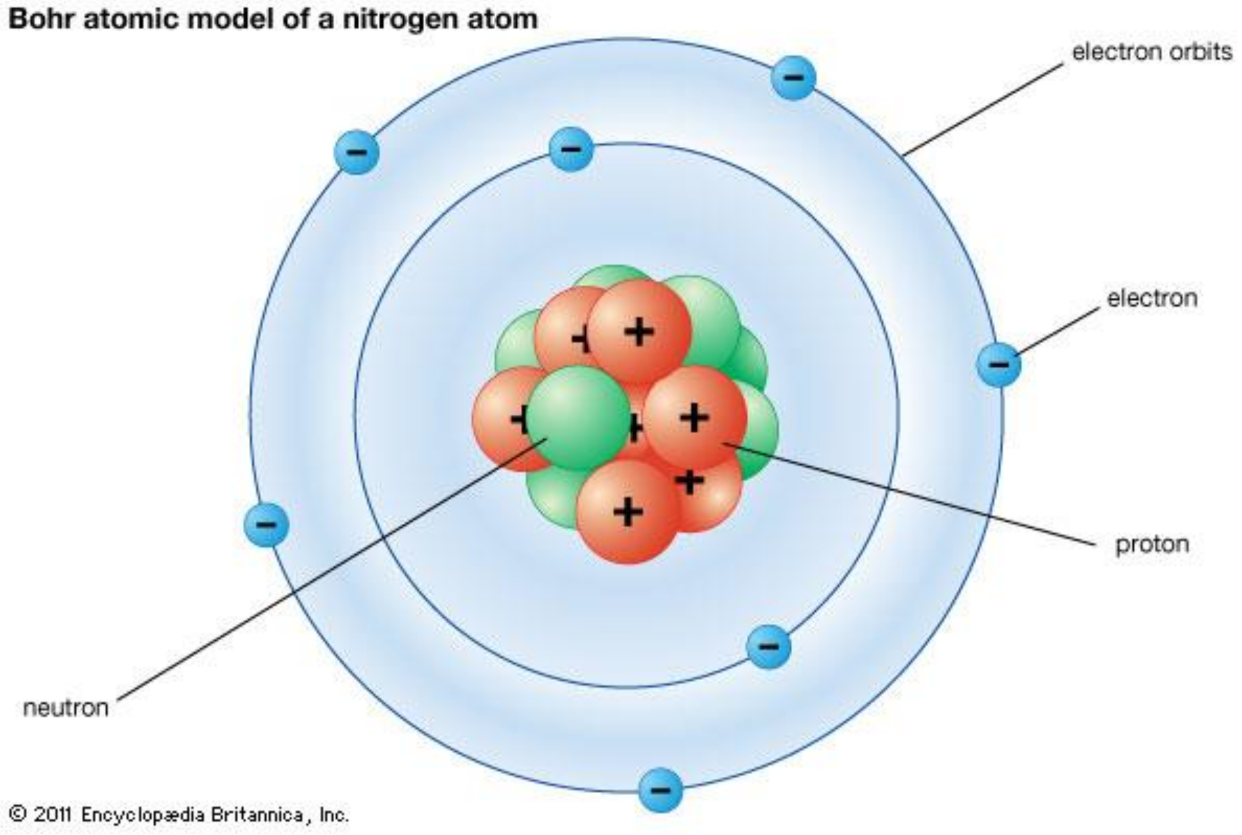
2. Rules for multiplication and division

   Use lowest number of significant figures

   Ex: $23 * 436 = 1.\underline{0} *10\text{^}4$ $453 / 3.2 = 1\underline{4}0$

Structure of an Atom

Bohr Model (Not accurate, but good starting point)

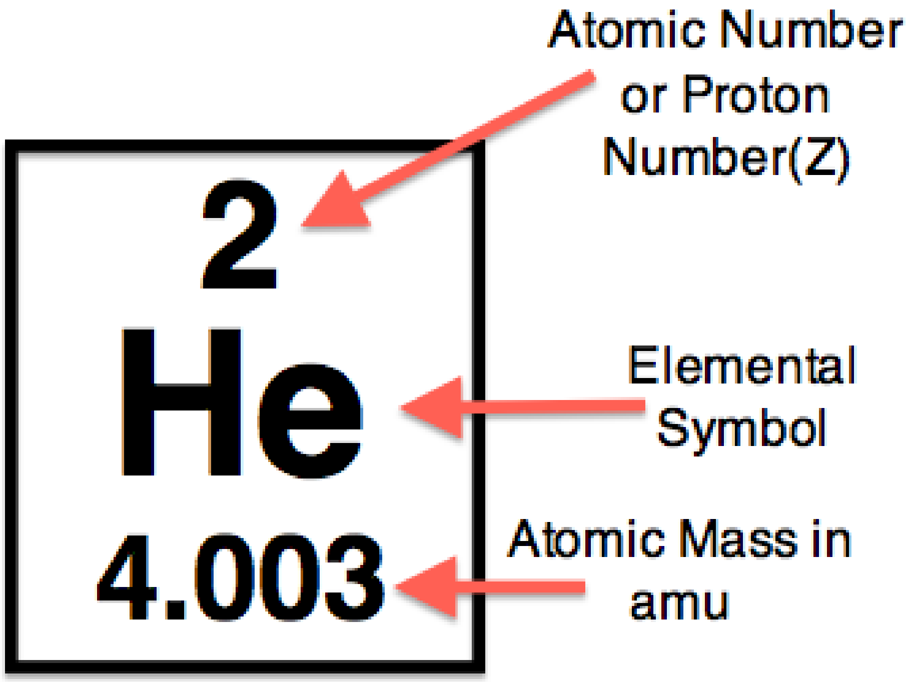


Structure of Atoms

of Protons - # of electrons = charge

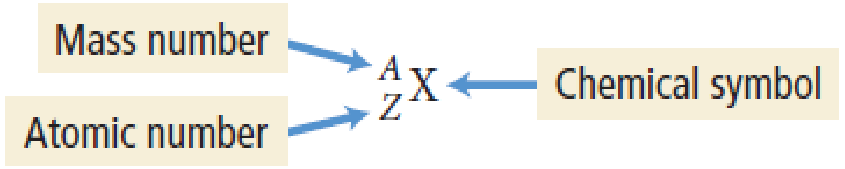
# of Protons + # of Neutrons = Atomic Mass

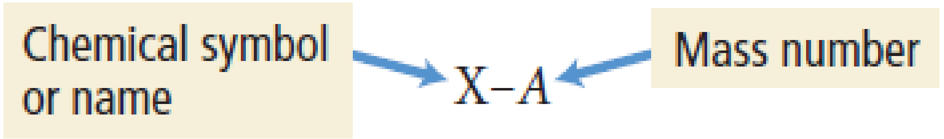
# of Protons = Atomic number



Isotopes

Same number of protons (atomic number) different number of neutrons.

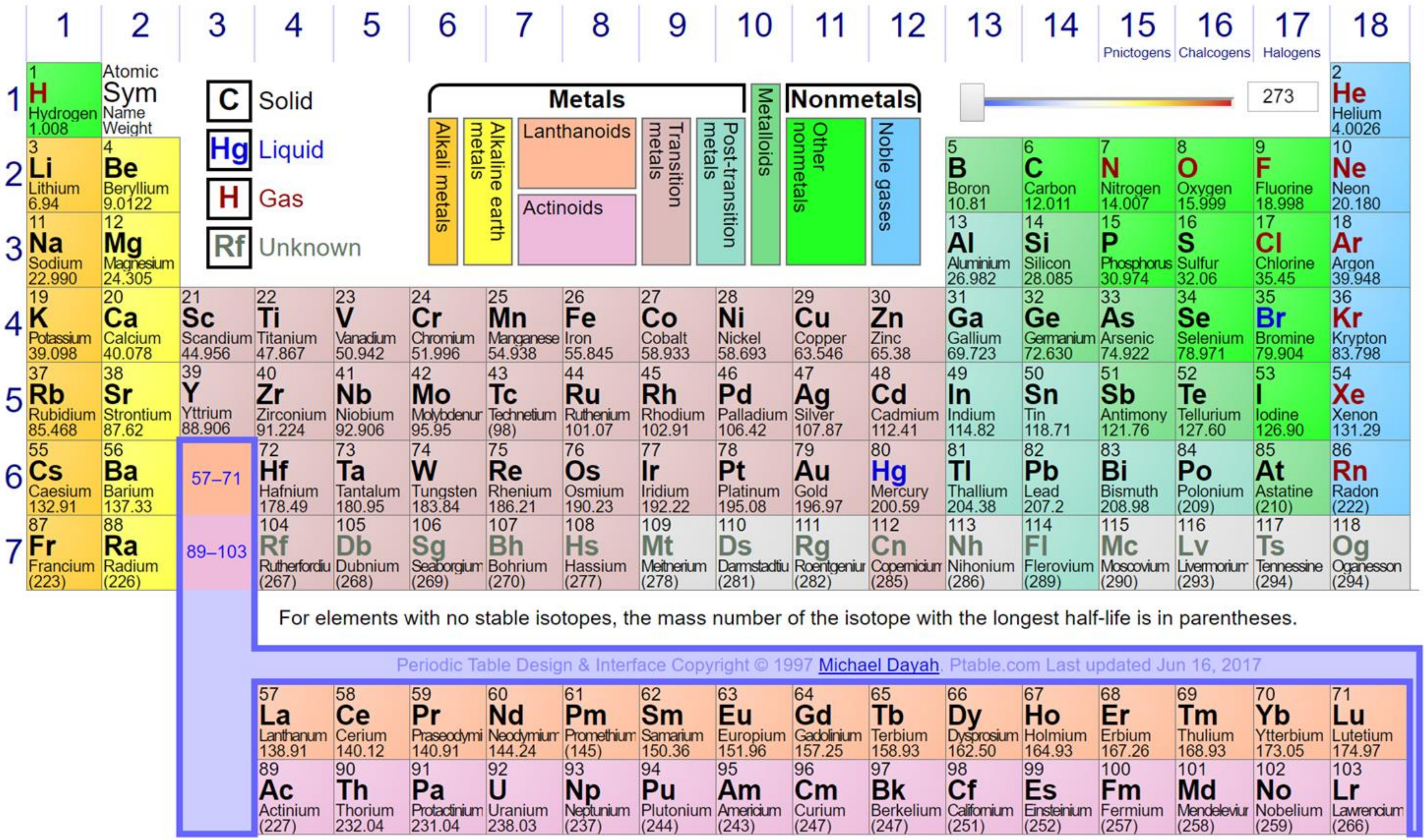




Atomic mass of an element is the weighted average of mass of all its isotope

$$\text{Atomic mass} = \sum_{n} (\text{fraction of isotope } n) \times (\text{mass of isotope } n)$$

Periodic Table



Bonding

‘Complete an octet’

Try to get 8 electrons in its outside shell.

Types of Bonds

Covalent Bond

-Hydrogen forces

-Dipole-dipole forces (polar)

-Van der Waals forces

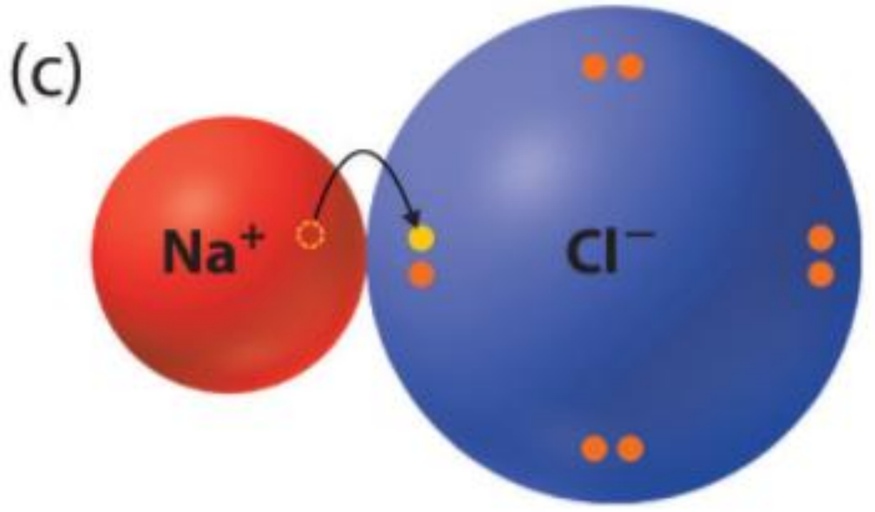
Ionic Bond

Metallic Bond

Ionic Bond

'Trade' electrons, usually between metal and a non-metal.

Ex: NaCl



Ionic bond

Complete transfer of one or more valence electrons.
Full charges on resulting ions.

Covalent Bond

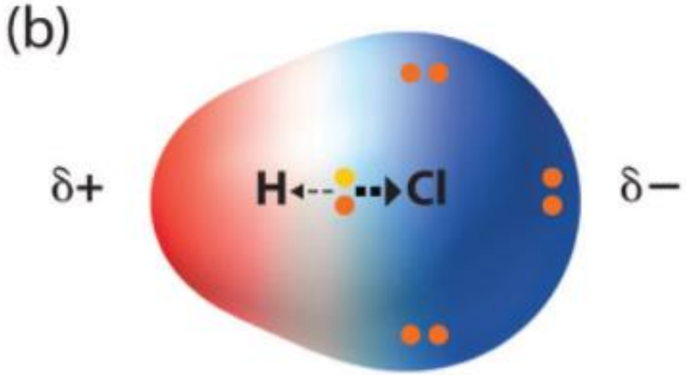
'Share' electrons, usually between non-metals.

Nonpolar:

Polar:

Nonpolar covalent bond

Bonding electrons shared equally between two atoms. No charges on atoms.

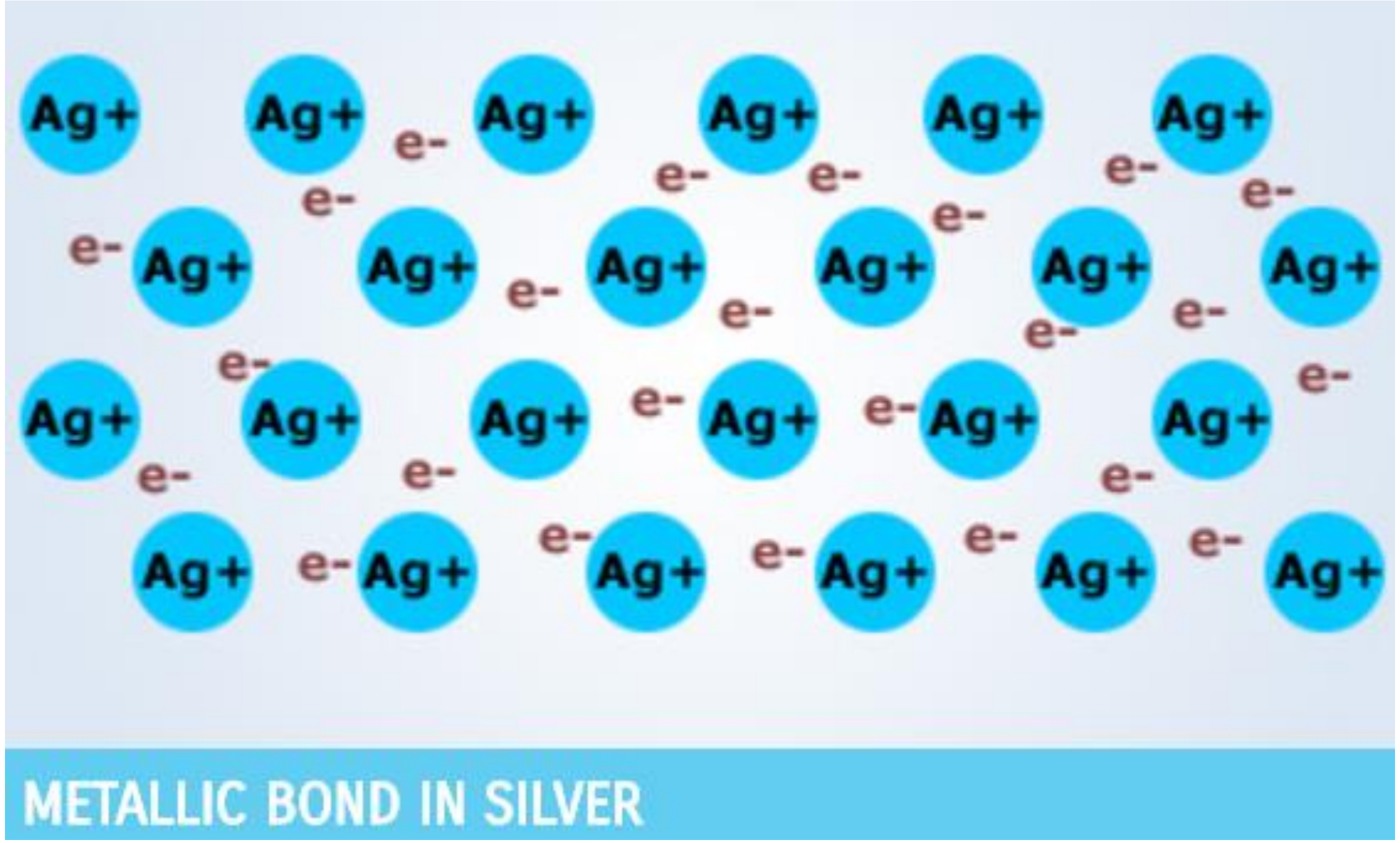


Polar covalent bond

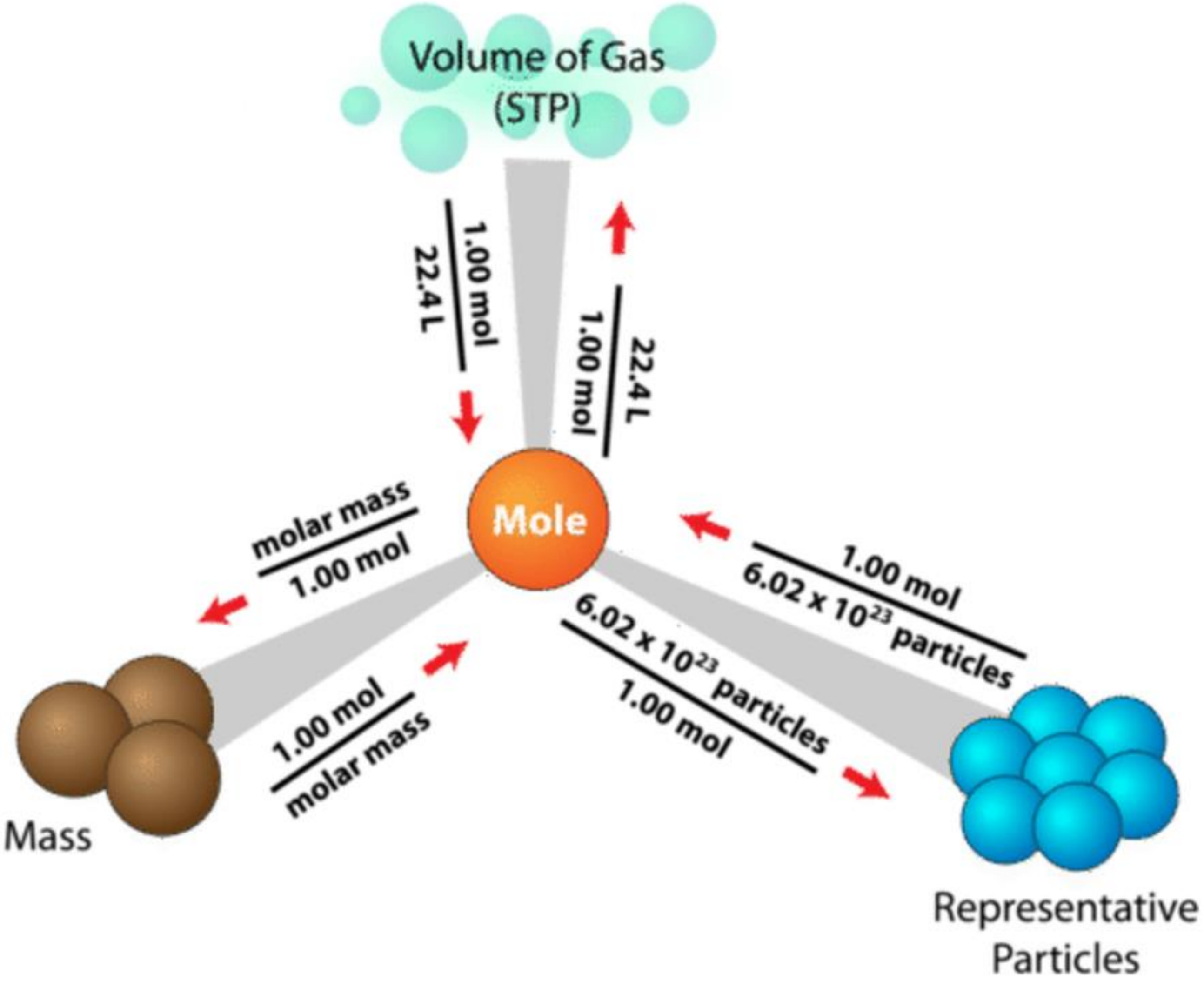
Bonding electrons shared unequally between two atoms. Partial charges on atoms.

Metallic Bond

Delocalized electrons, only metals. Ex: Ag



Moles, Mass, Particles, and Volumes



Avogadro's number:

$$1 \text{ mol} = 6.02214 \times 10^{23} \text{ particles}$$

Example 1

How many copper atoms are in a copper penny with a mass of 3.10 g? (Assume that the penny is composed of pure copper.)

Given:

63.55 g Cu = 1 mol Cu (molar mass of copper)

6.022 * 10^23 = 1 mol (Avogadro's number)

Solution

$$3.10\ \cancel{\text{g Cu}} \times \frac{1\ \cancel{\text{mol Cu}}}{63.55\ \cancel{\text{g Cu}}} \times \frac{6.022 \times 10^{23}\ \text{Cu atoms}}{1\ \cancel{\text{mol Cu}}} = 2.94 \times 10^{22}\ \text{Cu atoms}$$

Example 2

An aluminum sphere contains 8.55 * 10^22 aluminum atoms. What is the sphere's radius in centimeters? The density of aluminum is 2.70 g/cm3.

Given:

$$6.022 \times 10^{23} = 1 \text{ mol (Avogadro's number)}$$

$$26.98 \text{ g Al} = 1 \text{ mol Al (molar mass of aluminum)}$$

$$2.70 \text{ g/cm}^3 \text{ (density of aluminum)}$$

$$V = \frac{4}{3}\pi r^3 \quad \text{(volume of a sphere)}$$

Solution

$$8.55 \times 10^{22}\ \cancel{\text{Al atoms}} \times \frac{1\ \cancel{\text{mol Al}}}{6.022 \times 10^{23}\ \cancel{\text{Al atoms}}} \times \frac{26.98\ \cancel{\text{g Al}}}{1\ \cancel{\text{mol Al}}} \times \frac{1\ \text{cm}^3}{2.70\ \cancel{\text{g Al}}} = 1.41\underline{8}7\ \text{cm}^3$$

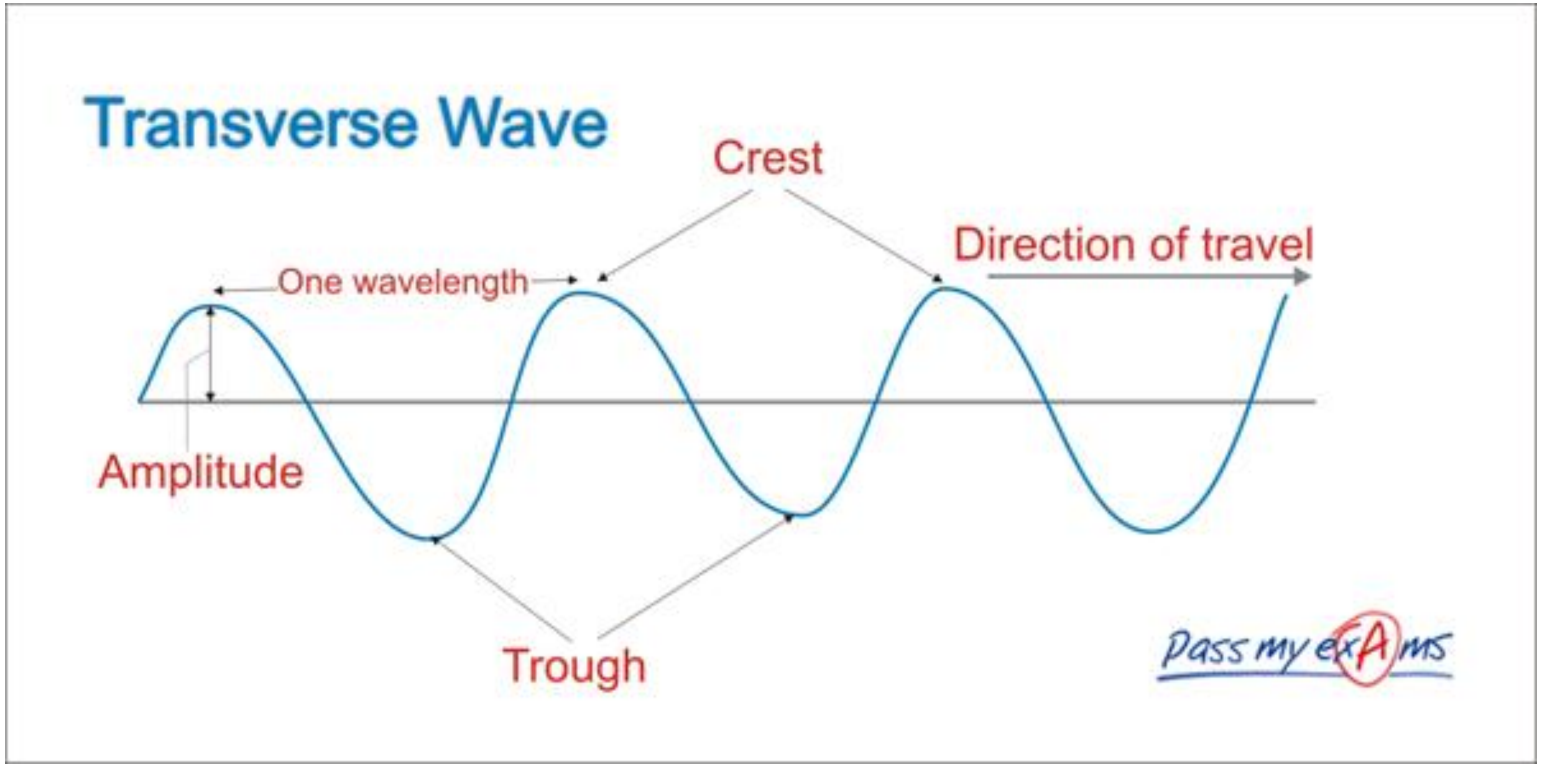
$$V = \frac{4}{3}\pi r^3$$

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(1.41\underline{8}7\ \text{cm}^3)}{4\pi}} = 0.697\ \text{cm}$$

General Chemistry Chem 1A

Intro to Quantum Mechanics

Light as Waves



ν (nu)

Frequency in Hz

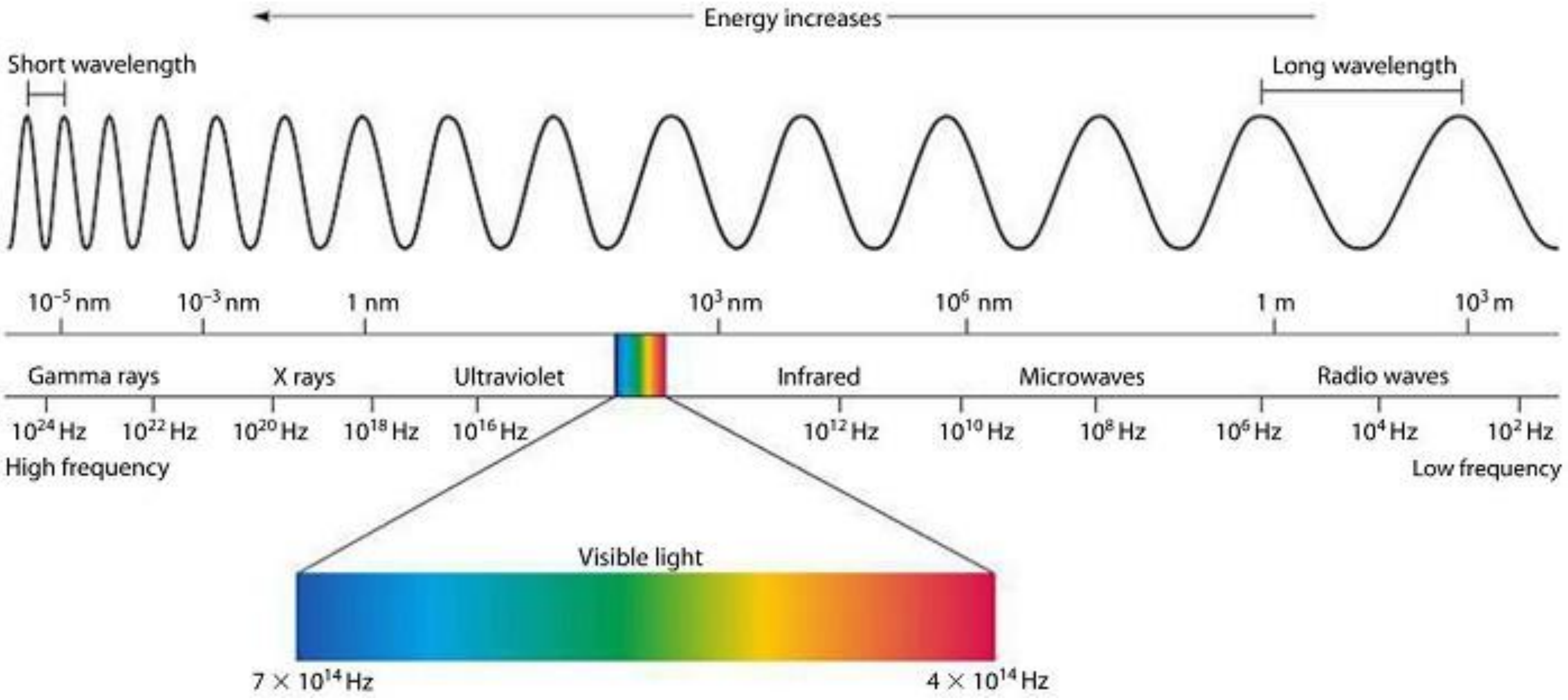
λ (lambda)

Wavelength in m

c speed of light

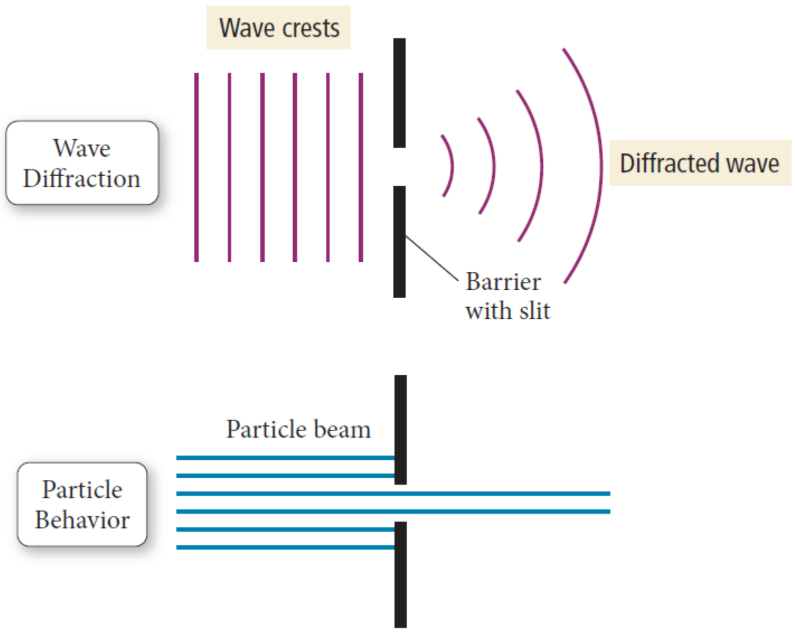
$$\nu = \frac{c}{\lambda}$$

Electromagnetic Spectrum



Light behaves like a wave

- It propagates as an electromagnetic wave.
- It spreads out (diffraction)
- It interference with other waves, constructively or deconstructively



Light behaves like a particle

- Photons
- It carries certain amount of energy
- The amount of energy is discrete (instead of continuous), no less than that.

Planck's Quantum Theory

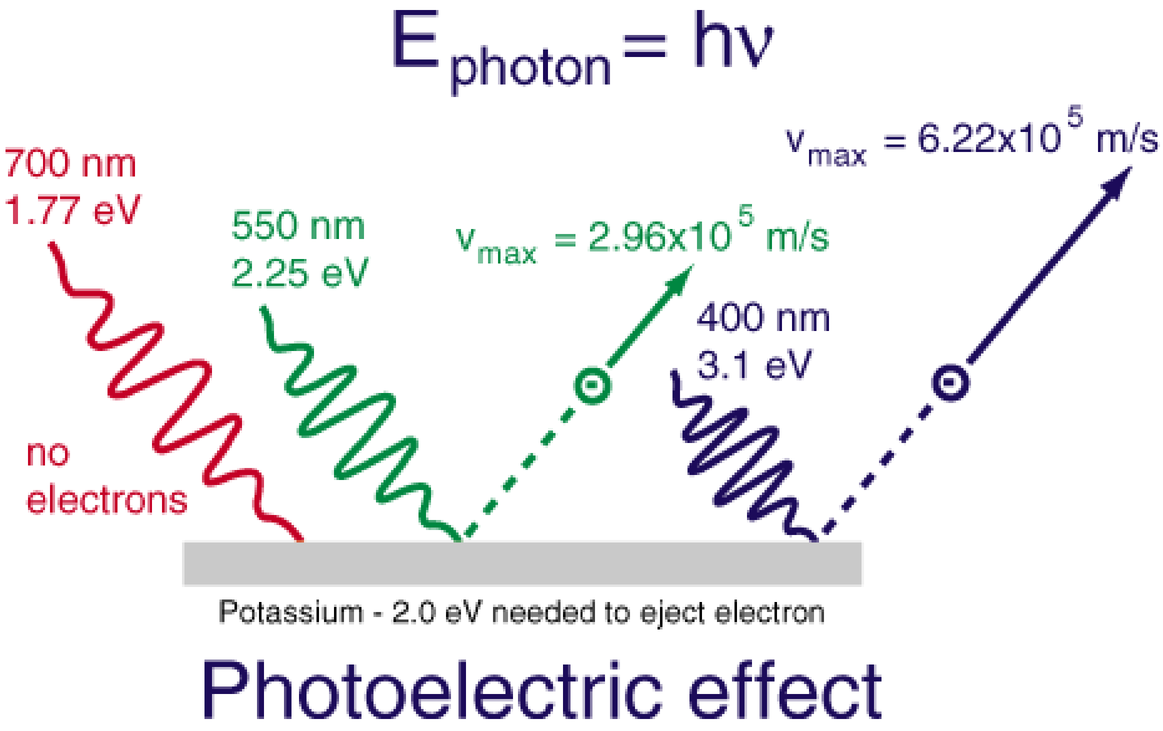
- Molecules can emit photon with certain amount energy (quantum)
- Quantum: smallest quantity (unit) of energy

Planck's Equation

- Energy of one photon given its frequency
- $$E = h\nu = \frac{hc}{\lambda}$$ h = 6.626*10^-34 Joules/ Hz.

Photoelectric Effect

- Ejection of electrons from the surface of a metal when light shines on it
- The light has to pass the certain <u>frequency</u>,i.e. the energy of individual photons has to higher than a threshold (work function)
- If the energy of the photon is higher, the electron will acquire kinetic energy



$$KE = h\nu - W$$

$$W:\ \text{work function}$$

De Broglie Wavelength

- Wave particle duality applies to electrons.
- Proven by experiments
- Shows Bohr’s atomic model works

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

h: Planck’s constant

p: momentum, m: mass, v: velocity

Lambda: wavelength

Heisenberg Uncertainty Principle

- It’s impossible to know simultaneously both the momentum and the position of a particles with certainty
- Certainty:

$$\Delta x \ \Delta p \geq \frac{h}{4\pi}$$

Δx = Uncertainty of Position

Δp = Uncertainty of Momentum

Wavefunction

- Describe the movement of a particle
- Different particles have different wavefunctions
- $\Psi(x,t)$ (Psi) x: position, t: time

Probability Density

- Probability of finding the particle in a region
- Equals to the square of wavefunction

$$|\Psi(x,t)|^2$$

Node

- Point where $\Psi = 0 \quad \Psi^2 = 0$
- The probability of finding the particle there is 0

Particle in a Box

- A single particle (wave) in a box
- Only certain wavelengths are allowed
- The energy of particles is ‘quantized’

Resources

OAI Tutoring Center

http://tech.uci.edu/access/students-current.php

LARC

http://www.larc.uci.edu/students/tutoring/larc-tutorial-schedules/

Molecular and Electron Geometry

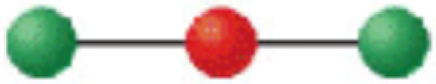
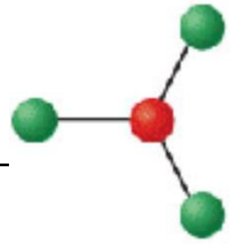
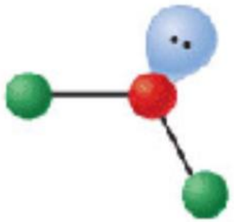
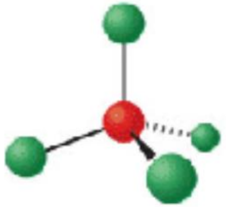
By: Alyssa Pronovost

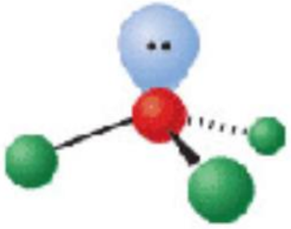
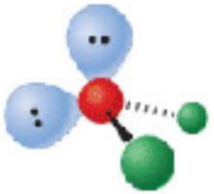
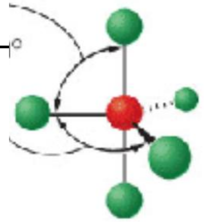
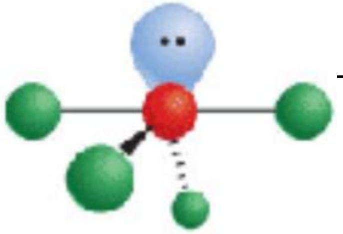
Electron Geometry

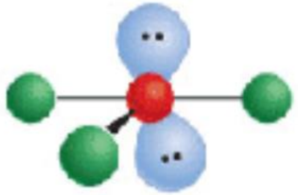
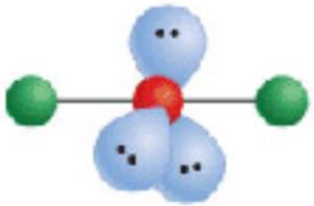
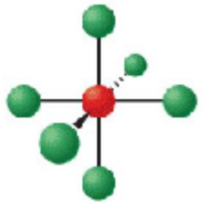
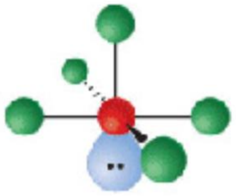
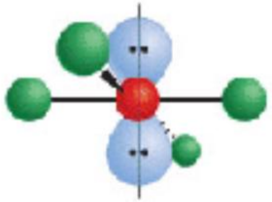
- Steric number: number of atoms bonded to the central atom of a molecule plus the number of lone pairs attached to the central atom (in other words, how many groups of electrons are around the central atom)
- Electron geometry is the prediction of a molecule’s shape based on the steric number
  - It acts as if each electron pair around the central atom were a bonding pair

Molecular Geometry (VSEPR Model)

- VSEPR stands for Valence-Shell Electron-Pair Repulsion
- This is used to predict the shape of molecules and polyatomic ions
    - Differentiates lone pairs and bonding pairs
- The electrons in bonds and lone pairs repel each other so the VSEPER model predicts the structure that will place electrons as far apart as possible
- Lone pairs repel more than bonding pairs

Steric Number	# of bonding groups	# of lone pairs	Electron Geometry	Molecular Geometry	Angle(s)	Picture
2	2	0	linear	linear	180	
3	3	0	trigonal planar	trigonal planar	120	
3	2	1	trigonal planar	bent	<120	
4	4	0	tetrahedral	tetrahedr	109.5	

Steric Number	# of bonding groups	# of lone pairs	Electron-Pair Geometry	Molecular Geometry	Angle(s)	Picture
4	3	1	tetrahedral	trigonal pyramidal	<109.5	
4	2	2	tetrahedral	bent	<109.5	
5	5	0	trigonal bipyramidal	trigonal bipyramidal	90, 120, 180	
5	4	1	trigonal bipyramidal	seesaw	90, 120, 180	

Steric Number	# of bonding groups	# of lone pairs	Electron Pair Geometry	Molecular Geometry	Angle(s)	Picture
5	3	2	trigonal bipyramidal	T-shaped	90, 180	
5	2	3	trigonal bipyramidal	linear	180	
6	6	0	octahedral	octahedral	90, 180	
6	5	1	octahedral	square pyramidal	90, 180	
6	4	2	octahedra	square	90, 180	

Notice a pattern?

What would you predict the molecular and electron-pair geometry to be for a molecule that has 3 bonding pairs and 3 lone pairs around its central atom?

For a molecule with 2 bonding pairs and 4 lone pairs around the central atom?

Physics 7c Intro

Images and Examples taken from the University Physics and Physics for Scientists and Engineers Textbook

Sample Schedule

Date	Topics	Chapter
Week 1	Motion of mass points, Newton's 1^{st}+2^{nd} law	1-3 (review), 4.1-4.4
Week 2	Applying Newton's laws	4.5-4.6, 5.1-5.4
Week 3	Work and Kinetic Energy	6.1-6.4
Week 4	Potential Energy and Energy Conservation	7.1-7.5
	Midterm 1	
Week 5	Momentum, Impulse, and Collisions	8.1-8.4
Week 6	Rotation of rigid bodies	8.5-8.6, 9.1-9.5
Week 7	Rotational motion	10.1-10.4
	Midterm 2	
Week 8	Rotational motion II	10.5-10.7
Week 9	Equilibrium and Elasticity	11.1-11.5
Week 10	Gravitation	13.1-13.8
	Final	

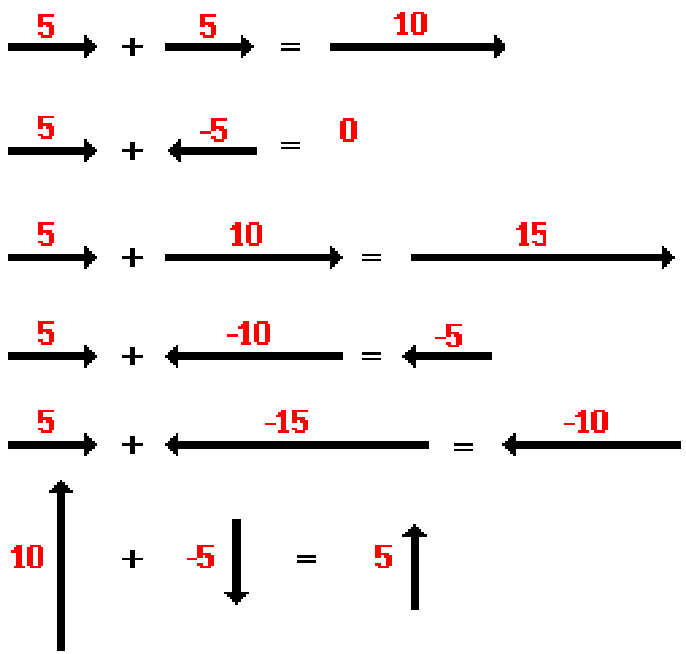
Fundamental Forces

- Gravitational force - attraction between masses
- Electromagnetic force - attraction and repulsion of atoms and molecules
- Strong nuclear force - binding of the nuclear particles
- Weak nuclear force - underlies nuclear reactions and decay

Vectors

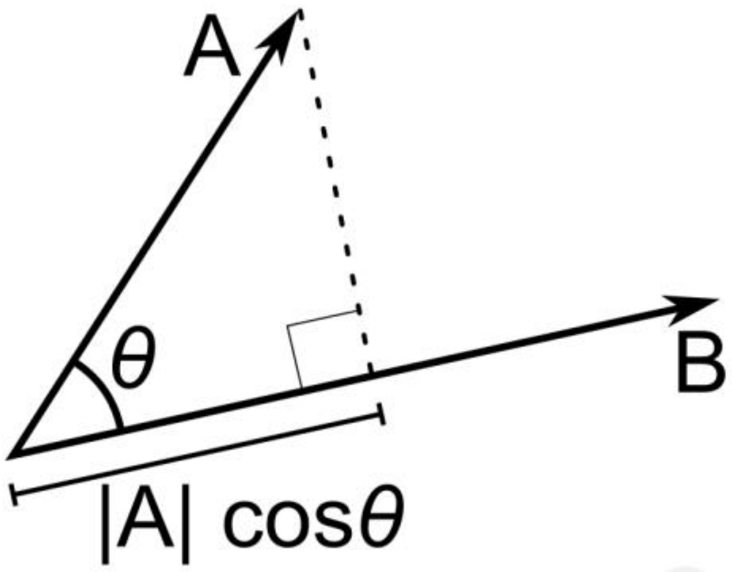
- Scalars- measurement described by magnitude
- Examples: Speed, Temperature, Time, Volume
- Vectors-measurement described by magnitude and direction
- Examples: Velocity, Increase/Decrease in Temperature

Vector Addition:



Dot Product

- Multiply two vectors that result in scalar
- If there is an angle between the vectors, it becomes
    - A*B = ABcosθ

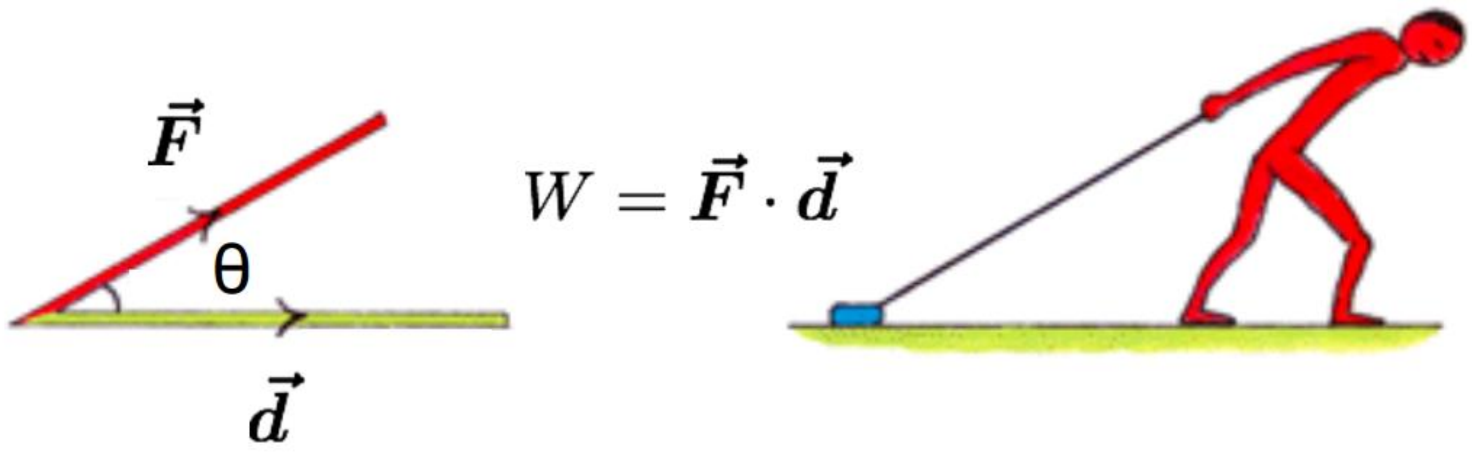


$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\cos\theta$$

Use of Dot Product- Work

Work= (Force vector) * (displacement vector)

W= F*d



$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$$

$$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$$

Dot Product- Components

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$$

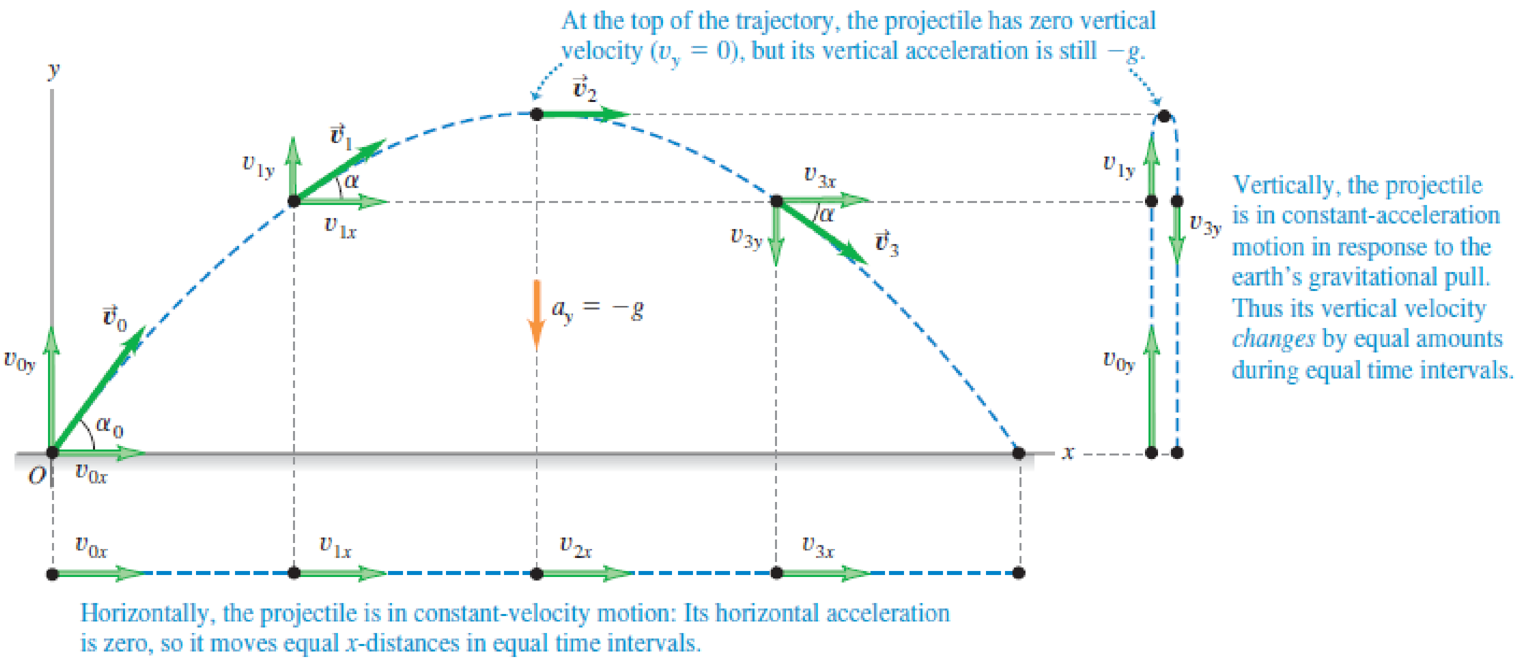
Kinematics Equations

$$x = x_0 + v_0 t + 1/2at^2$$

$$v = v_0 + at$$

$$v_1 = v_0{}^2 + 2a\,(x - x_0)$$

3.17 If air resistance is negligible, the trajectory of a projectile is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.



Kinematic equations are used in instances of constant acceleration, such as projectile motion.

More Kinematic Equations

TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions

x component (horizontal)		*y* component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–11a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(Eq. 2–11b)	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify Eqs. 2–11 to use for projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes y is positive upward, so $a_y = -g = -9.80\,\mathrm{m/s^2}$.

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\,\mathrm{m/s^2}$)

Horizontal Motion ($a_x = 0$, v_x = constant)		**Vertical Motion**† ($a_y = -g$ = constant)
$v_x = v_{x0}$	(Eq. 2–11a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0} t$	(Eq. 2–11b)	$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$
	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

† If y is taken positive downward, the minus (−) signs in front of g become + signs.

If the projection angle θ_0 is chosen relative to the $+x$ axis (Fig. 3–20), then

$$v_{x0} = v_0 \cos\theta_0, \qquad \text{and} \qquad v_{y0} = v_0 \sin\theta_0 .$$

Problem Solving Skills

1. Draw a careful diagram showing what is happening to the object.
2. Choose an origin and an x-y coordinate system.
3. Decide on the time interval, which for projectile motion can only include motion under the effect of gravity alone, not throwing or landing. The time interval must be the same for the x and y analyses. The x and y motions are connected by the common time, t
4. Examine the horizontal (x) and vertical (y) motions separately. If you are given the initial velocity, you may want to resolve it into its x and y components.

Problem Solving Skills

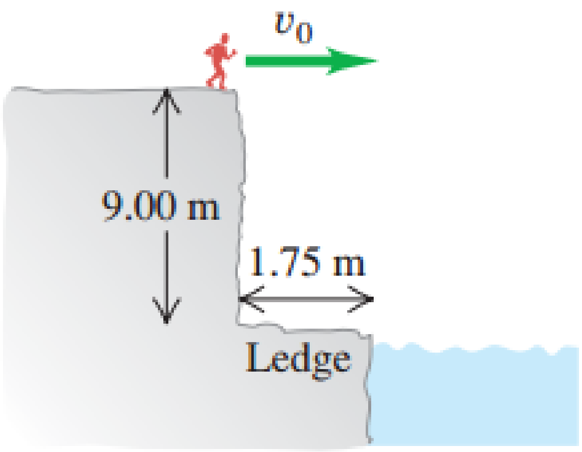
5.List the known and unknown quantities, choosing and or where and using the or sign, depending on whether you choose y positive up or down. Remember that never changes throughout the trajectory, and that at the highest point of any trajectory that returns downward. The velocity just before landing is generally not zero.

6. Think for a minute before jumping into the equations. A little planning goes a long way. Apply the relevant equations, combining equations if necessary. You may need to combine components of a vector to get magnitude and direction

Example

3.10 •• A daring 510-N swimmer dives off a cliff with a running horizontal leap, as shown in Fig. E3.10. What must her minimum speed be just as she leaves the top of the cliff so that she will miss the ledge at the bottom, which is 1.75 m wide and 9.00 m below the top of the cliff?

Figure **E3.10**



Example

3.9 • A physics book slides off a horizontal tabletop with a speed of 1.10 m/s. It strikes the floor in 0.350 s. Ignore air resistance. Find (a) the height of the tabletop above the floor; (b) the horizontal distance from the edge of the table to the point where the book strikes the floor; (c) the horizontal and vertical components of the book's velocity, and the magnitude and direction of its velocity, just before the book reaches the floor. (d) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

Example

3.12 • A rookie quarterback throws a football with an initial upward velocity component of 12.0 m/s and a horizontal velocity component of 20.0 m/s. Ignore air resistance. (a) How much time is required for the football to reach the highest point of the trajectory? (b) How high is this point? (c) How much time (after it is thrown) is required for the football to return to its original level? How does this compare with the time calculated in part (a)? (d) How far has the football traveled horizontally during this time?

Newton's Laws

I: An object in motion will stay in motion until acted upon an outside object. (Law of Inertia)

II: The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

III: Whenever one object exerts a force on a second object, the second object exerts an equal force in the opposite direction on the first. $F_{AB} = -F_{BA}$

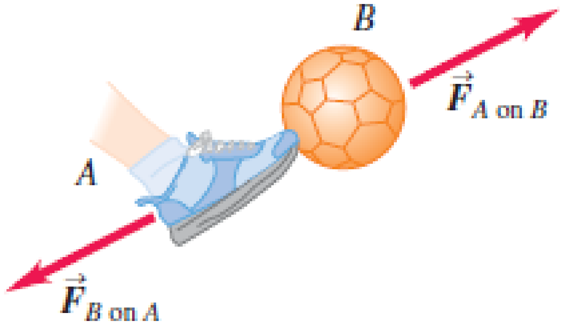


TABLE 4–1
Units for Mass and Force

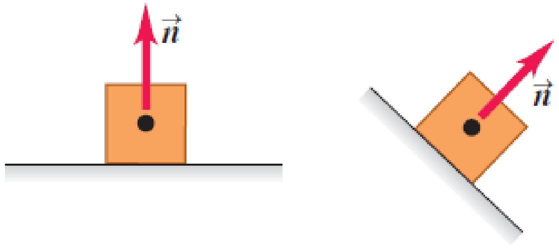
System	Mass	Force
SI	kilogram (kg)	newton (N) ($= kg \cdot m/s^2$)
cgs	gram (g)	dyne ($= g \cdot cm/s^2$)
British	slug	pound (lb)

Conversion factors: 1 dyne $= 10^{-5}$ N;
1 lb $\approx$ 4.45 N;
1 slug $\approx$ 14.6 kg.

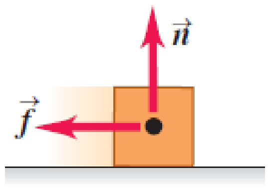
Free Body Diagrams

Draw all forces that act ON the object, common forces include:

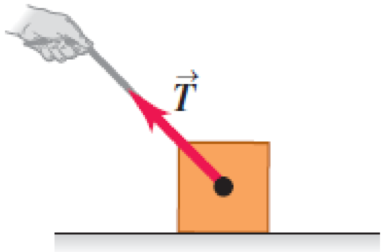
(a) **Normal force $\vec{n}$:** When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



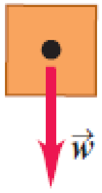
(b) **Friction force $\vec{f}$:** In addition to the normal force, a surface may exert a frictional force on an object, directed parallel to the surface.



(c) **Tension force $\vec{T}$:** A pulling force exerted on an object by a rope, cord, etc.



(d) **Weight $\vec{w}$:** The pull of gravity on an object is a long-range force (a force that acts over a distance).



CONCEPTUAL EXAMPLE 4–1 **Newton's first law.** A school bus comes to a sudden stop, and all of the backpacks on the floor start to slide forward. What force causes them to do that?

EXAMPLE 4–2 **ESTIMATE** **Force to accelerate a fast car.** Estimate the net force needed to accelerate (*a*) a 1000-kg car at $\frac{1}{2}g$; (*b*) a 200-gram apple at the same rate.

EXAMPLE 4–3 **Force to stop a car.** What average net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

APPROACH We use Newton's second law, $\Sigma F = ma$, to determine the force, but first we need to calculate the acceleration a. We assume the acceleration is constant so that we can use the kinematic equations, Eqs. 2–11, to calculate it.

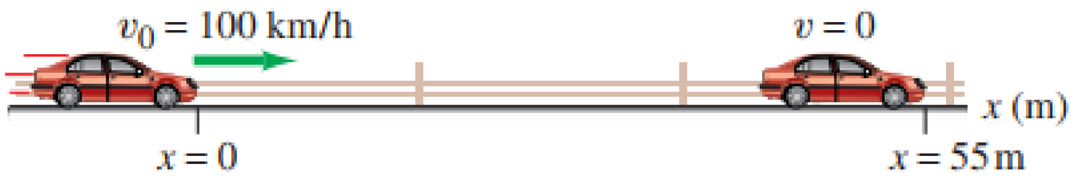


FIGURE 4–6 Example 4–3.

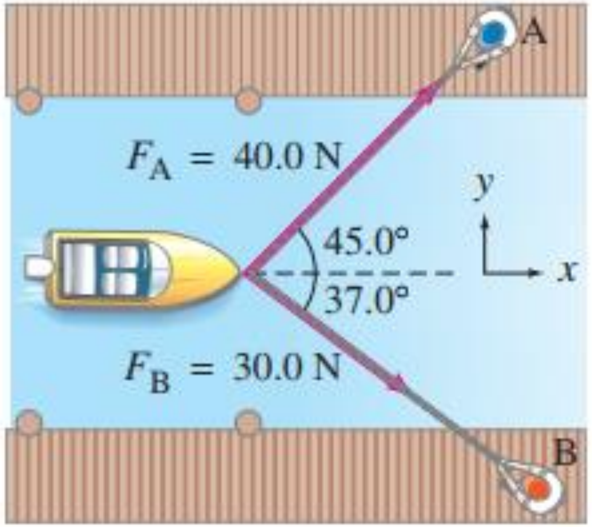
EXAMPLE 4–6 **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 4–15a). (*a*) Determine the weight of the box and the normal force exerted on it by the table. (*b*) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 4–15b. Again determine the normal force exerted on the box by the table. (*c*) If your friend pulls upward on the box with a force of 40.0 N (Fig. 4–15c), what now is the normal force exerted on the box by the table?

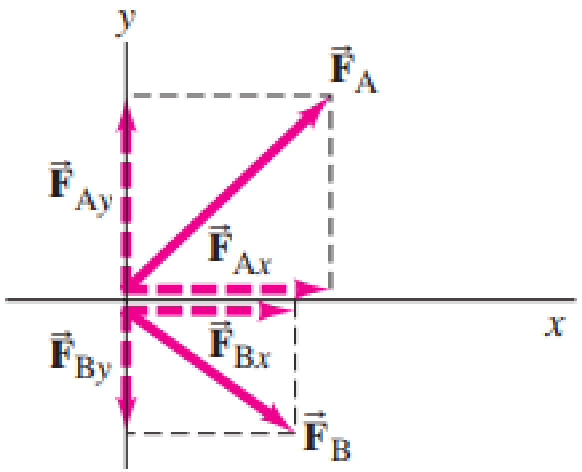
Example

EXAMPLE 4–7 **Accelerating the box.** What happens when a person pulls upward on the box in Example 4–6*c* with a force equal to, or greater than, the box's weight? For example, let $F_P = 100.0\,\mathrm{N}$ (Fig. 4–16) rather than the 40.0 N shown in Fig. 4–15c.

EXAMPLE 4–9 **Adding force vectors.** Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 4–19a.

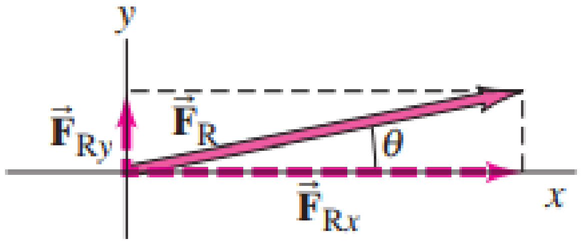
FIGURE 4–19 Example 4–9: Two force vectors act on a boat.





y
$\vec{F}_A$
$\vec{F}_{Ay}$
$\vec{F}_{Ax}$
x
$\vec{F}_{By}$
$\vec{F}_{Bx}$
$\vec{F}_B$

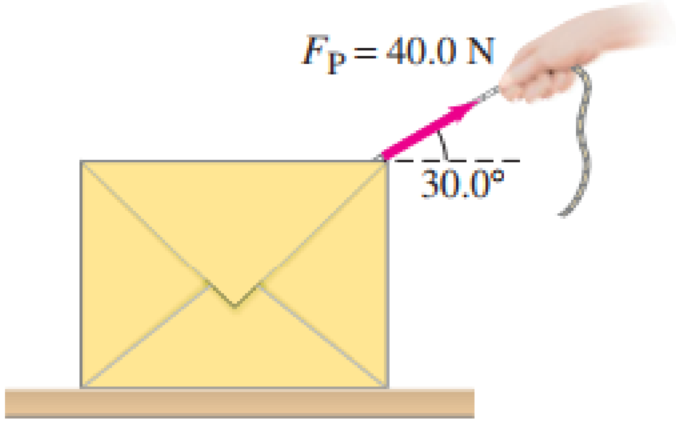
(b)



y
$\vec{F}_{Ry}$
$\vec{F}_R$
θ
x
$\vec{F}_{Rx}$

(c)

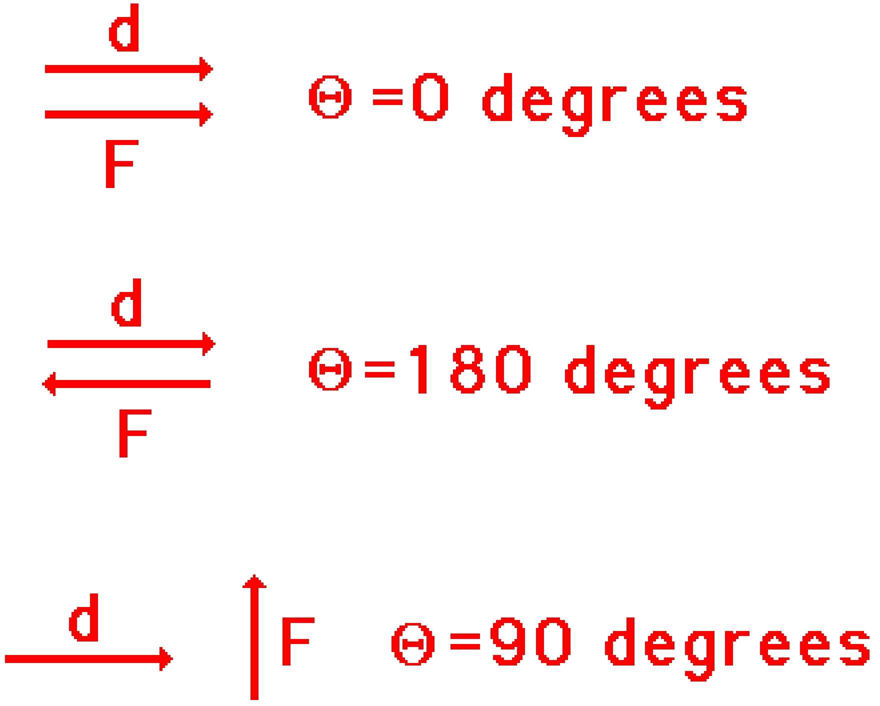
EXAMPLE 4–11 **Pulling the mystery box.** Suppose a friend asks to examine the 10.0-kg box you were given (Example 4–6, Fig. 4–15), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached cord, as shown in Fig. 4–21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0\ \text{N}$, and it is exerted at a $30.0°$ angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.



Work

Work is done when a force acts upon an object and causes a displacement

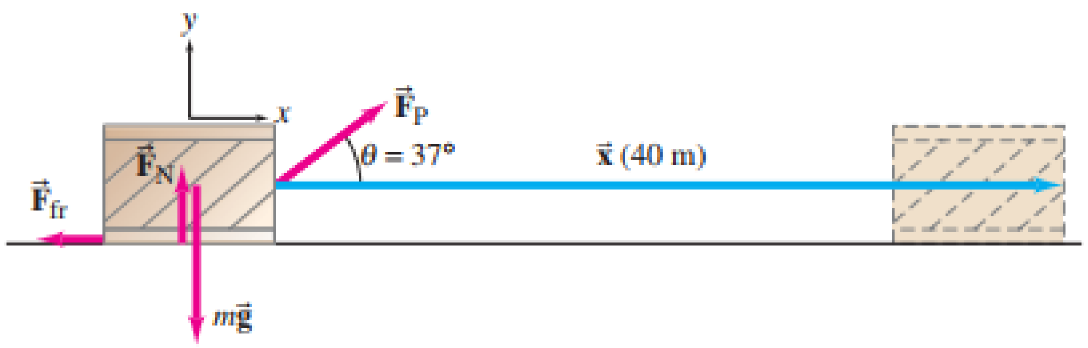
W = F*d*cosΘ



The units of work are measured in jc

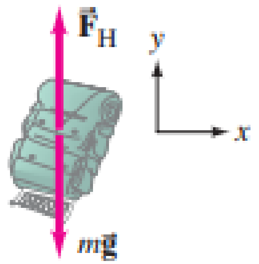
Work done by a constant force

EXAMPLE 6–1 **Work done on a crate.** A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_P = 100\ \mathrm{N}$, which acts at a 37° angle as shown in Fig. 6–3. The floor is rough and exerts a friction force $\vec{F}_{fr} = 50\ \mathrm{N}$. Determine (*a*) the work done by each force acting on the crate, and (*b*) the net work done on the crate.



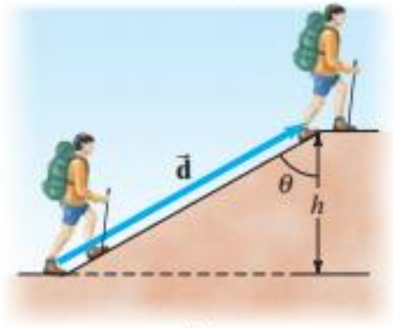
Example

EXAMPLE 6–2 **Work on a backpack.** (*a*) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0\,\text{m}$, as shown in Fig. 6–4a. Determine also (*b*) the work done by gravity on the backpack, and (*c*) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is zero).

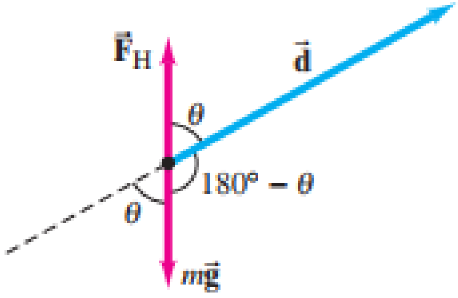


(b)

FIGURE 6–4 Example 6–2.



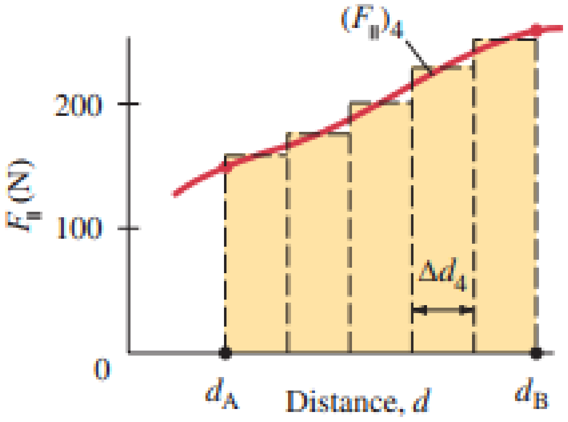
(a)



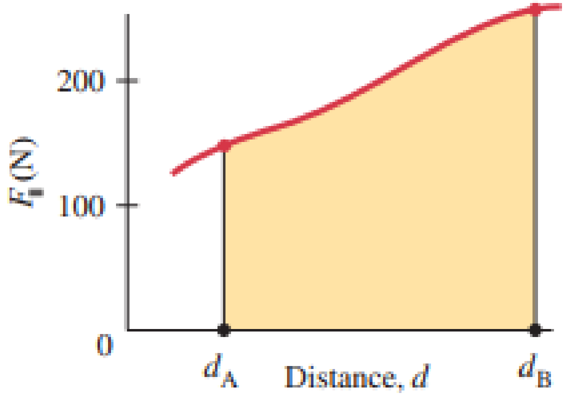
(c)

Work done by varying force

The work done by a variable force in moving an object between two points is equal to the area under the vs. d curve between those two points



(a)



(b)

Energy

- Total energy is the same after any process as it was before: that is, energy is a *conserved* quantity.
- An object in motion has the ability to do work and thus can be said to have energy or *kinetic energy*
- The net work done on an object is equal to the change in the object's kinetic energy.

  (*Work-energy principle*)
- Potential Energy: the energy associated with forces that depend on the position or configuration of an object (or objects) relative to the surroundings such as *Gravitational Potential Energy, Potential Energy of Elastic Spring, Potential Energy as Stored Energy.*

$$W_{\text{net}} = \Delta \text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 .$$

Energy

Gravitational Potential Energy = mass*g*height

Kinetic Energy = $1/2m*v^2$

Elastic Potential Energy = $1/2k*x^2$

Units of energy are measured in joules. 1 J = 1 $kg*m^2/s^2$

Total Mechanical Energy = $PE_{gravitational} + KE + PE_{elastic}$

Example

EXAMPLE 6–6 **Potential energy changes for a roller coaster.** A 1000-kg roller-coaster car moves from point 1, Fig. 6–12, to point 2 and then to point 3. (*a*) What is the gravitational potential energy at points 2 and 3 relative to point 1? That is, take $y = 0$ at point 1. (*b*) What is the change in potential energy when the car goes from point 2 to point 3? (*c*) Repeat parts (*a*) and (*b*), but take the reference point $(y = 0)$ to be at point 3.

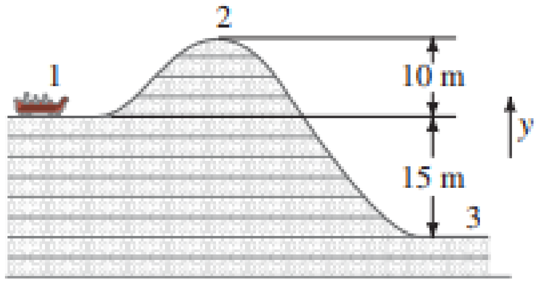


FIGURE 6–12 Example 6–6.

Momentum & Impulse

Momentum = mass*velocity

Impulse = Force*time = mass*Δvelocity

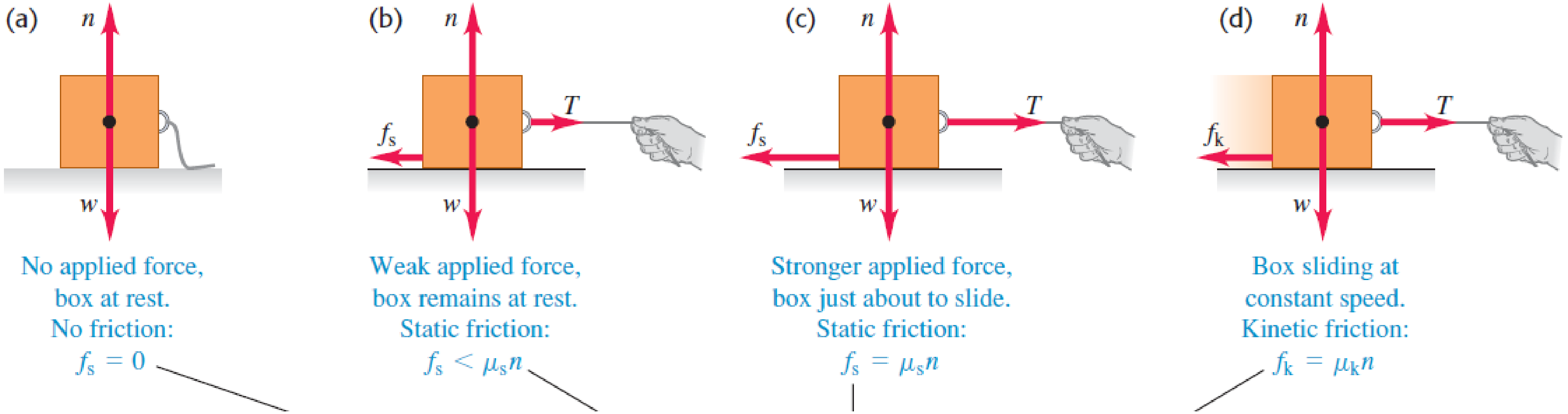
Impulse is the change in momentum.

Friction

Ƒ = μ*N where μ is the coefficient of friction and N is the normal force

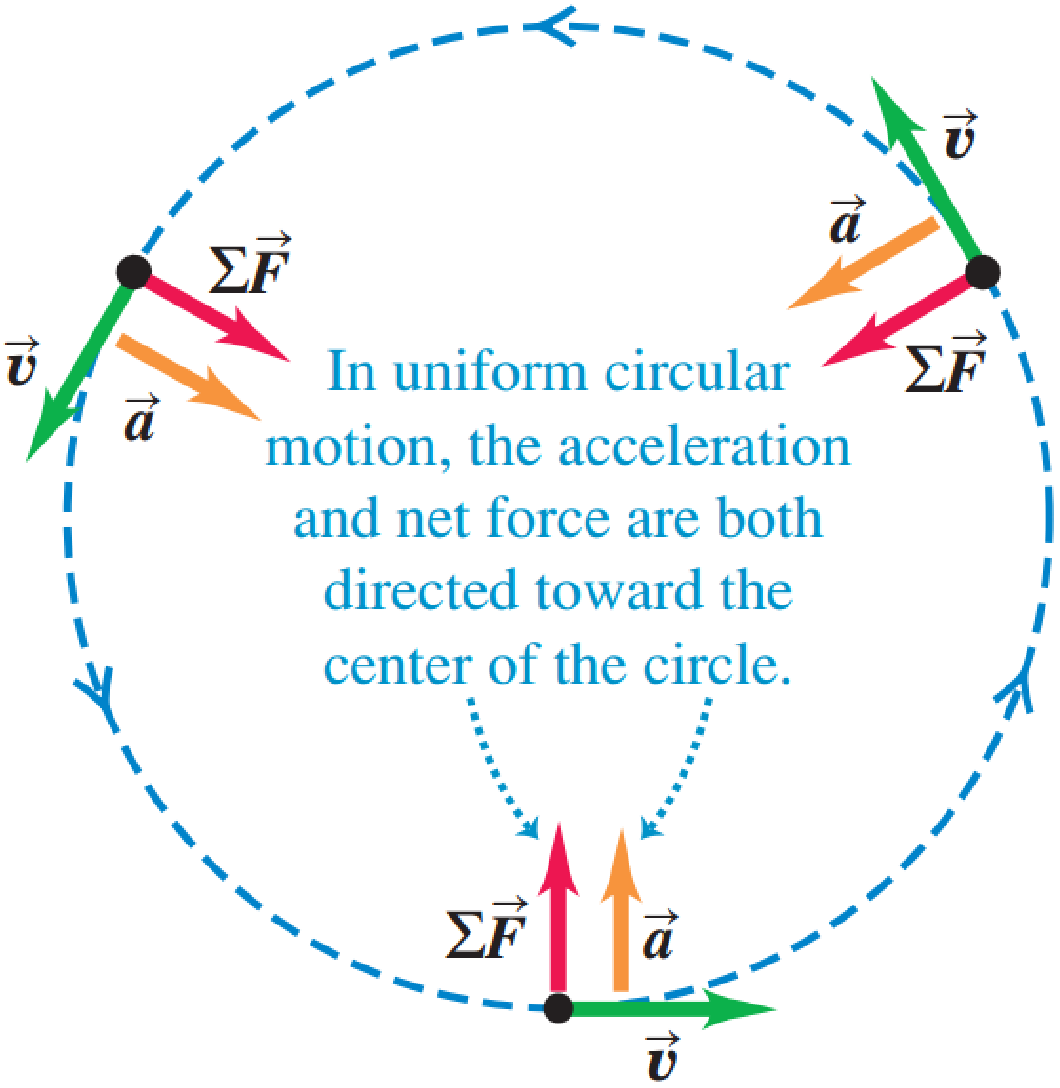
Before an object is in motion, static friction works to keep the object from moving. While moving, kinetic friction resists the movement of the object.

5.19 (a), (b), (c) When there is no relative motion, the magnitude of the static friction force f_s is less than or equal to $\mu_s n$. (d) When there is relative motion, the magnitude of the kinetic friction force f_k equals $\mu_k n$. (e) A graph of the friction force magnitude f as a function of the magnitude T of the applied force. The kinetic friction force varies somewhat as intermolecular bonds form and break.



Circular Motion

For circular motion, there is radial acceleration that points toward the center of the circle and is defined as $a_{rad} = v^2/R$



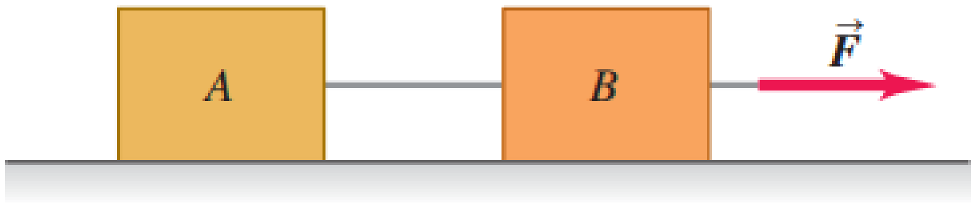
Example

5.13 •• **CP** ***Genesis* Crash.** On September 8, 2004, the *Genesis* spacecraft crashed in the Utah desert because its parachute did not open. The 210-kg capsule hit the ground at 311 km/h and penetrated the soil to a depth of 81.0 cm. (a) Assuming it to be constant, what was its acceleration (in m/s^2 and in g's) during the crash? (b) What force did the ground exert on the capsule during the crash? Express the force in newtons and as a multiple of the capsule's weight. (c) For how long did this force last?

Example

5.35 • Two crates connected by a rope lie on a horizontal surface (Fig. E5.35). Crate A has mass m_A and crate B has mass m_B. The coefficient of kinetic friction between each crate and the surface is μ_k. The crates are pulled to the right at constant velocity by a horizontal force $\vec{F}$. In terms of m_A, m_B, and μ_k, calculate (a) the magnitude of the force $\vec{F}$ and (b) the tension in the rope connecting the blocks. Include the free-body diagram or diagrams you used to determine each answer.

Figure **E5.35**



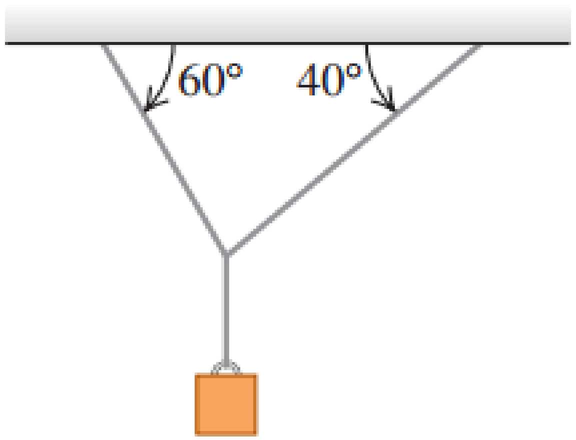
Example

5.44 • A flat (unbanked) curve on a highway has a radius of 220.0 m. A car rounds the curve at a speed of 25.0 m/s. (a) What is the minimum coefficient of friction that will prevent sliding? (b) Suppose the highway is icy and the coefficient of friction between the tires and pavement is only one-third what you found in part (a). What should be the maximum speed of the car so it can round the curve safely?

Example

5.57 ••• Two ropes are connected to a steel cable that supports a hanging weight as shown in Fig. P5.57. (a) Draw a free-body diagram showing all of the forces acting at the knot that connects the two ropes to the steel cable. Based on your force diagram, which of the two ropes will have the greater tension? (b) If the maximum tension either rope can sustain without breaking is 5000 N, determine the maximum value of the hanging weight that these ropes can safely support. You can ignore the weight of the ropes and the steel cable.

Figure **P5.57**



Rotational Kinematics

$$x = x_0 + v_0 t + 1/2at^2$$

$$v = v_0 + at$$

$$v_1 = v_0{}^2 + 2a\,(x - x_0)$$

$$\theta = \theta_0 + \omega_0 t + 1/2\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega_0 = \omega_0 + 2\alpha\,(\theta - \theta_0)$$

$$s = r\theta$$

$$v = r\omega$$

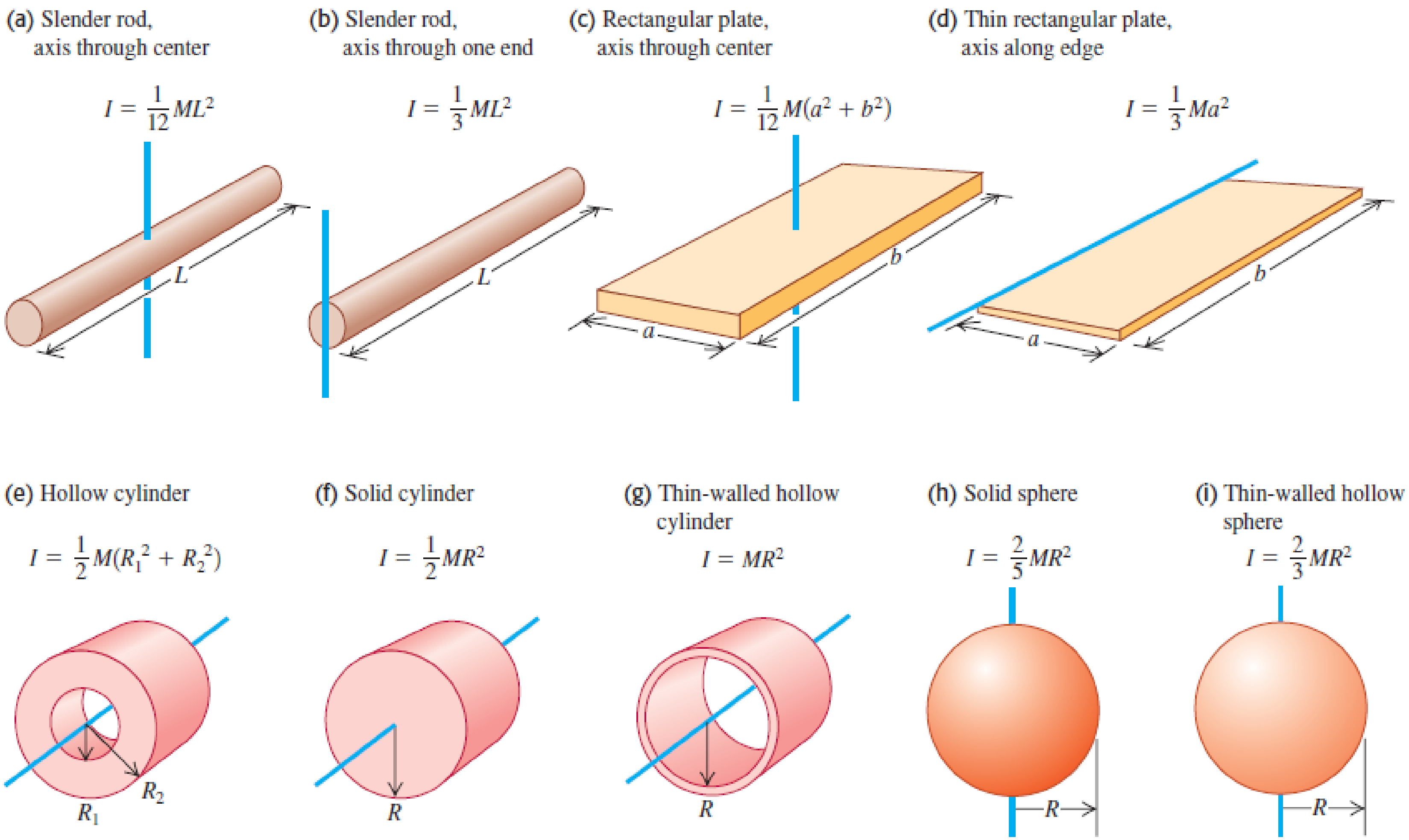
$$a = r\alpha$$

Inertia

$K = 1/2 I\omega^2$

I is the moment of inertia and depends on the object.

Table 9.2 Moments of Inertia of Various Bodies



Example

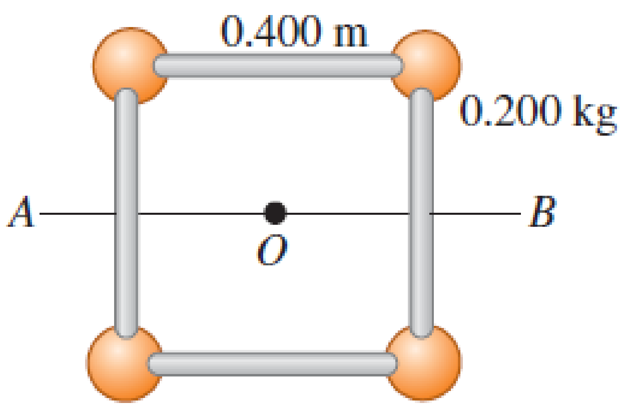
9.14 • A circular saw blade 0.200 m in diameter starts from rest. In 6.00 s it accelerates with constant angular acceleration to an angular velocity of 140 rad/s. Find the angular acceleration and the angle through which the blade has turned.

Example

Section 9.4 Energy in Rotational Motion

9.30 • Four small spheres, each of which you can regard as a point of mass 0.200 kg, are arranged in a square 0.400 m on a side and connected by extremely light rods (Fig. E9.30). Find the moment of inertia of the system about an axis (a) through the center of the square, perpendicular to its plane (an axis through point O in the figure); (b) bisecting two opposite sides of the square (an axis along the line AB in the figure); (c) that passes through the centers of the upper left and lower right spheres and through point O.

Figure **E9.30**



Kinematics & Newtonian Homework

3.16 • On level ground a shell is fired with an initial velocity of 50.0 m/s at 60.0° above the horizontal and feels no appreciable air resistance. (a) Find the horizontal and vertical components of the shell's initial velocity. (b) How long does it take the shell to reach its highest point? (c) Find its maximum height above the ground. (d) How far from its firing point does the shell land? (e) At its highest point, find the horizontal and vertical components of its acceleration and velocity.

4.10 •• A dockworker applies a constant horizontal force of 80.0 N to a block of ice on a smooth horizontal floor. The frictional force is negligible. The block starts from rest and moves 11.0 m in 5.00 s. (a) What is the mass of the block of ice? (b) If the worker stops pushing at the end of 5.00 s, how far does the block move in the next 5.00 s?

Challenge Problem:

4.47 •• **CP** A 6.50-kg instrument is hanging by a vertical wire inside a space ship that is blasting off at the surface of the earth. This ship starts from rest and reaches an altitude of 276 m in 15.0 s with constant acceleration. (a) Draw a free-body diagram for the instrument during this time. Indicate which force is greater. (b) Find the force that the wire exerts on the instrument.

Work & Energy Homework

6.2 • A tow truck pulls a car 5.00 km along a horizontal roadway using a cable having a tension of 850 N. (a) How much work does the cable do on the car if it pulls horizontally? If it pulls at 35.0° above the horizontal? (b) How much work does the cable do on the tow truck in both cases of part (a)? (c) How much work does gravity do on the car in part (a)?

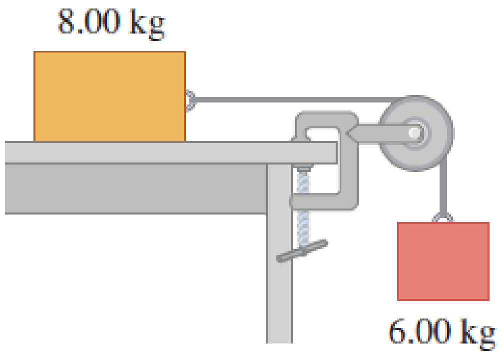
6.20 •• You throw a 20-N rock vertically into the air from ground level. You observe that when it is 15.0 m above the ground, it is traveling at 25.0 m/s upward. Use the work–energy theorem to find (a) the rock's speed just as it left the ground and (b) its maximum height.

7.14 •• An ideal spring of negligible mass is 12.00 cm long when nothing is attached to it. When you hang a 3.15-kg weight from it, you measure its length to be 13.40 cm. If you wanted to store 10.0 J of potential energy in this spring, what would be its *total* length? Assume that it continues to obey Hooke's law.

Challenge Problem:

6.86 •• Consider the system shown in Fig. P6.86. The rope and pulley have negligible mass, and the pulley is frictionless. The coefficient of kinetic friction between the 8.00-kg block and the tabletop is $\mu_k = 0.250$. The blocks are released from rest. Use energy methods to calculate the speed of the 6.00-kg block after it has descended 1.50 m.

Figure **P6.86**



Fricton & Stuff

6.10 •• An 8.00-kg package in a mail-sorting room slides 2.00 m down a chute that is inclined at 53.0° below the horizontal. The coefficient of kinetic friction between the package and the chute's surface is 0.40. Calculate the work done on the package by (a) friction, (b) gravity, and (c) the normal force. (d) What is the net work done on the package?

7.11 •• You are testing a new amusement park roller coaster with an empty car of mass 120 kg. One part of the track is a vertical loop with radius 12.0 m. At the bottom of the loop (point A) the car has speed 25.0 m/s, and at the top of the loop (point B) it has speed 8.0 m/s. As the car rolls from point A to point B, how much work is done by friction?

7.52 •• **Ski Jump Ramp.** You are designing a ski jump ramp for the next Winter Olympics. You need to calculate the vertical height h from the starting gate to the bottom of the ramp. The skiers push off hard with their ski poles at the start, just above the starting gate, so they typically have a speed of 2.0 m/s as they reach the gate. For safety, the skiers should have a speed no higher than 30.0 m/s when they reach the bottom of the ramp. You determine that for a 85.0-kg skier with good form, friction and air resistance will do total work of magnitude 4000 J on him during his run down the ramp. What is the maximum height h for which the maximum safe speed will not be exceeded?

Resources

Physics Department Tutoring

Open 6-9:30 pm Monday-Friday in Rowland Hall

LARC Tutoring

Sign up on WebReg (100$ for a whole quarter)

Professor Office Hours!!!!

Online Videos

Khan Acadamey: https://www.khanacademy.org/science/physics

UCI OpenCourseWare: http://ocw.uci.edu/courses/physics_7c_classical_physics.html http://fcis.aisdhaka.org/personal/chendricks/IB/Giancoli/Giancoli%20Chapters.html