technische universität dortmund

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Classical scheduling algorithms for periodic systems

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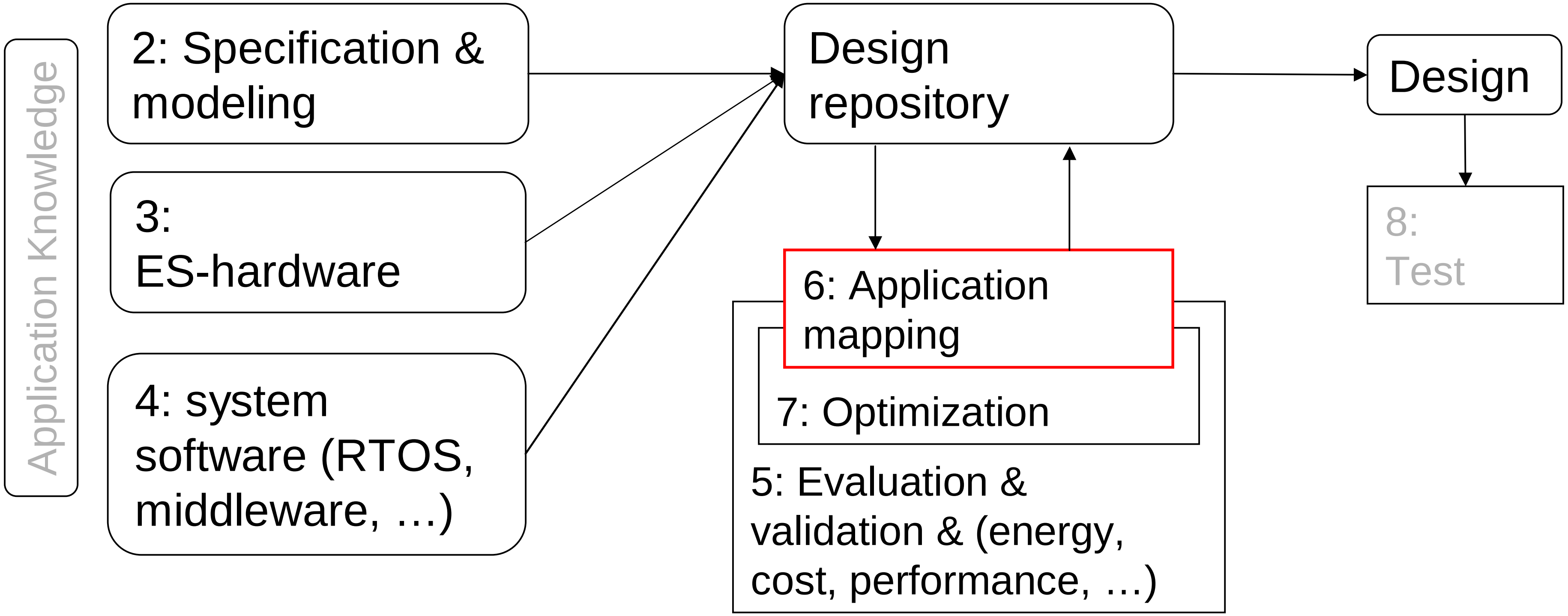
Structure of this course

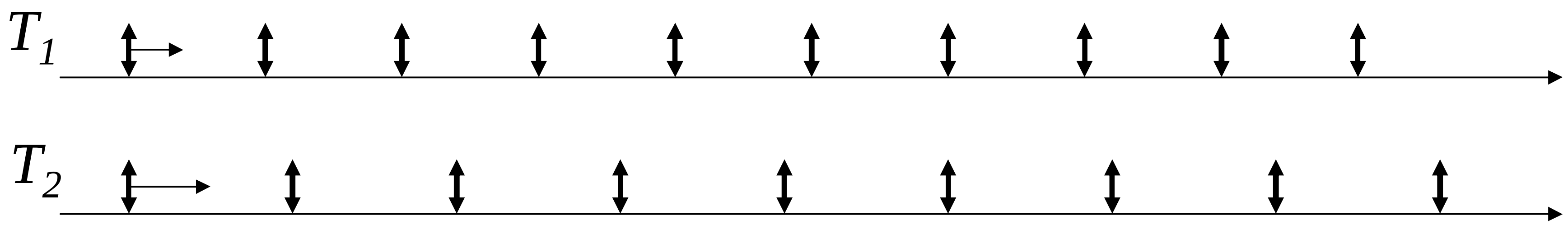


Numbers denote sequence of chapters

Classes of mapping algorithms considered in this course

- **Classical scheduling algorithms**
  Mostly for independent tasks & ignoring communication, mostly for mono- and homogeneous multiprocessors
- **Dependent tasks as considered in architectural synthesis**
  Initially designed in different context, but applicable
- **Hardware/software partitioning**
  Dependent tasks, heterogeneous systems, focus on resource assignment
- **Design space exploration using evolutionary algorithms;** Heterogeneous systems, incl. communication modeling

Periodic scheduling



Each execution instance of a task is called a **job**.

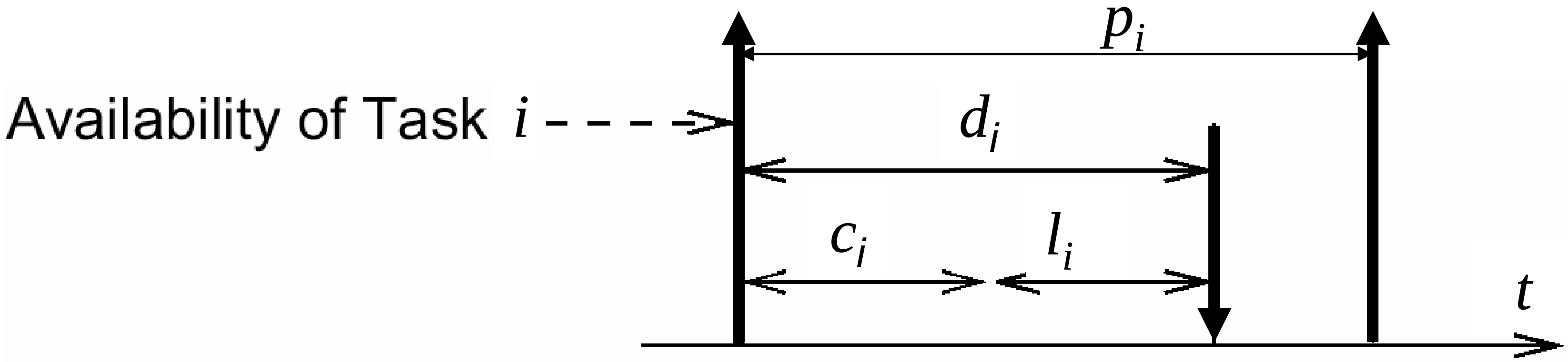
Notion of optimality for aperiodic scheduling does not make sense for periodic scheduling.

For periodic scheduling, the best that we can do is to design an algorithm which will always find a schedule if one exists.
☞ A scheduler is defined to be **optimal** iff it will find a schedule if one exists.

Periodic scheduling: Scheduling with no precedence constraints

Let $\{T_i\}$ be a set of tasks. Let:

- p_i be the period of task T_i,
- c_i be the execution time of T_i ,
- d_i be the **deadline interval**, that is, the time between T_i becoming available and the time until which T_i has to finish execution.
- l_i be the **laxity** or **slack**, defined as $l_i = d_i - c_i$
- f_i be the finishing time.



Average utilization: important charac-terization of scheduling problems:

Average utilization:

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

Necessary condition for schedulability (with m=number of processors):

$$\mu \leq m$$

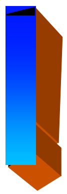
Independent tasks: Rate monotonic (RM) scheduling

Most well-known technique for scheduling independent periodic tasks [Liu, 1973].

Assumptions:

- All tasks that have hard deadlines are periodic.
- All tasks are independent.
- $d_i = p_i$, for all tasks.
- c_i is constant and is known for all tasks.
- The time required for context switching is negligible.
- For a single processor and for n tasks, the following equation holds for the average utilization μ:

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \leq n(2^{1/n} - 1)$$



Rate monotonic (RM) scheduling
- The policy -

RM policy: **The priority of a task is a monotonically decreasing function of its period.**

At any time, a highest priority task among all those that are ready for execution is allocated.

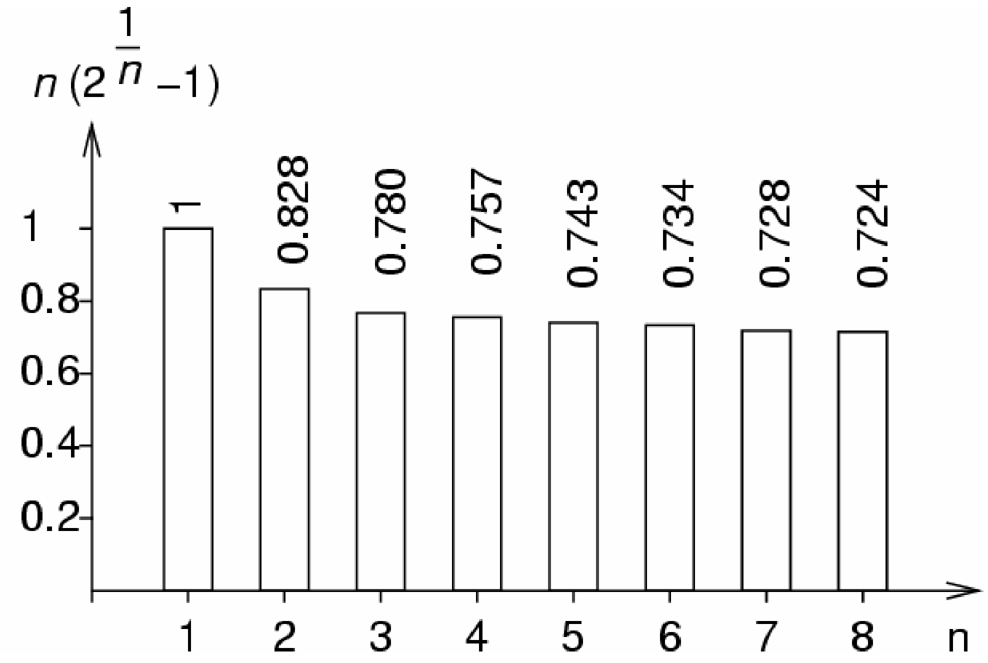
Theorem: If all RM assumptions are met, schedulability is guaranteed.

Maximum utilization for guaranteed schedulability

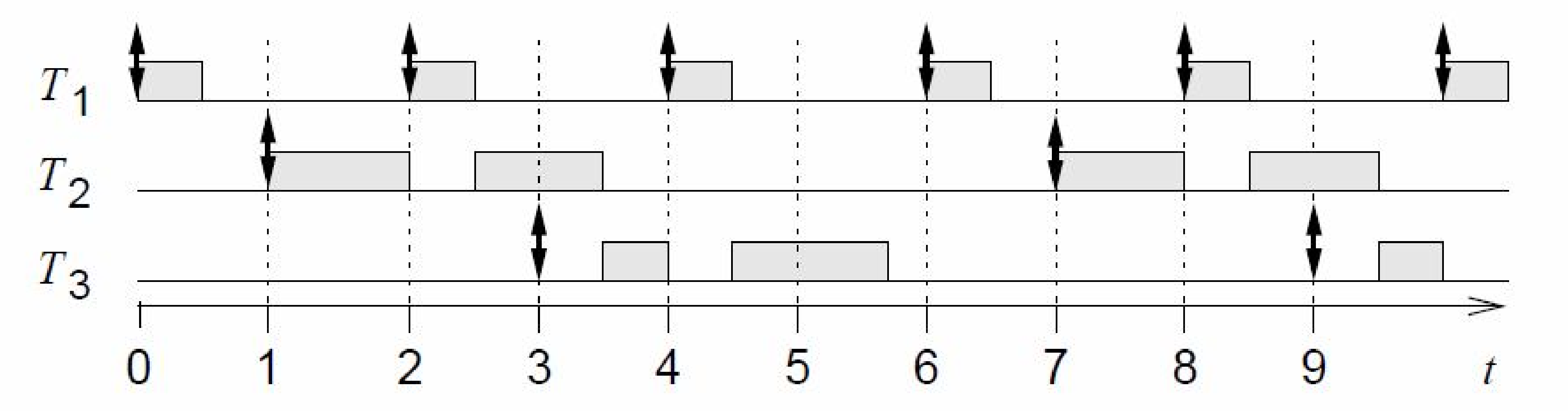
Maximum utilization as a function of the number of tasks:

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \le n(2^{1/n} - 1)$$

$$\lim_{n \to \infty} (n(2^{1/n} - 1)) = \ln(2)$$



Example of RM-generated schedule



T_1 preempts T_2 and T_3.
T_2 and T_3 do not preempt each other.

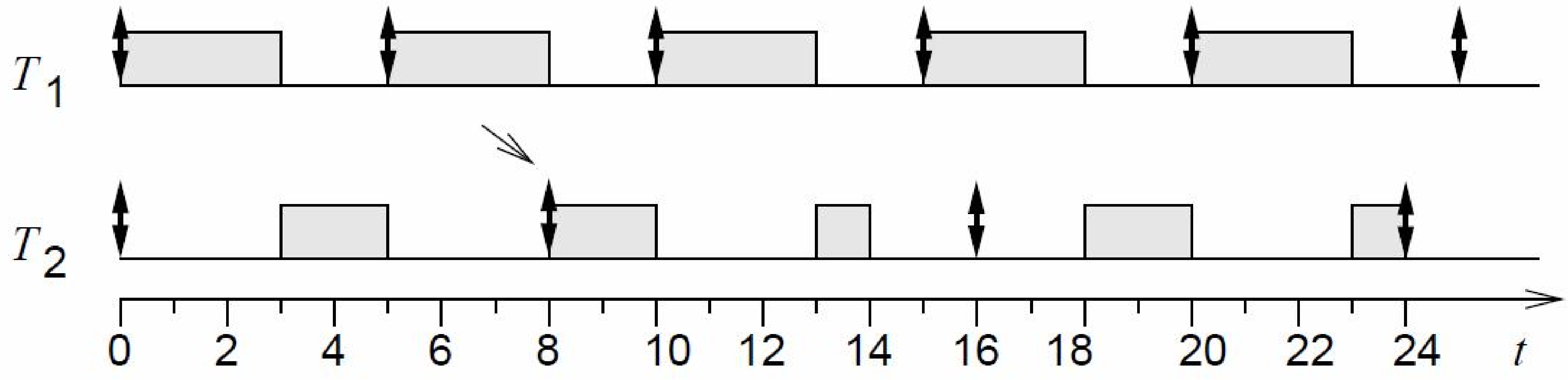
Failing RMS

Task 1: period 5, execution time 3

Task 2: period 8, execution time 3

μ=3/5+3/8=24/40+15/40=39/40 $\approx$ 0.975

$2(2^{1/2}-1) \approx 0.828$



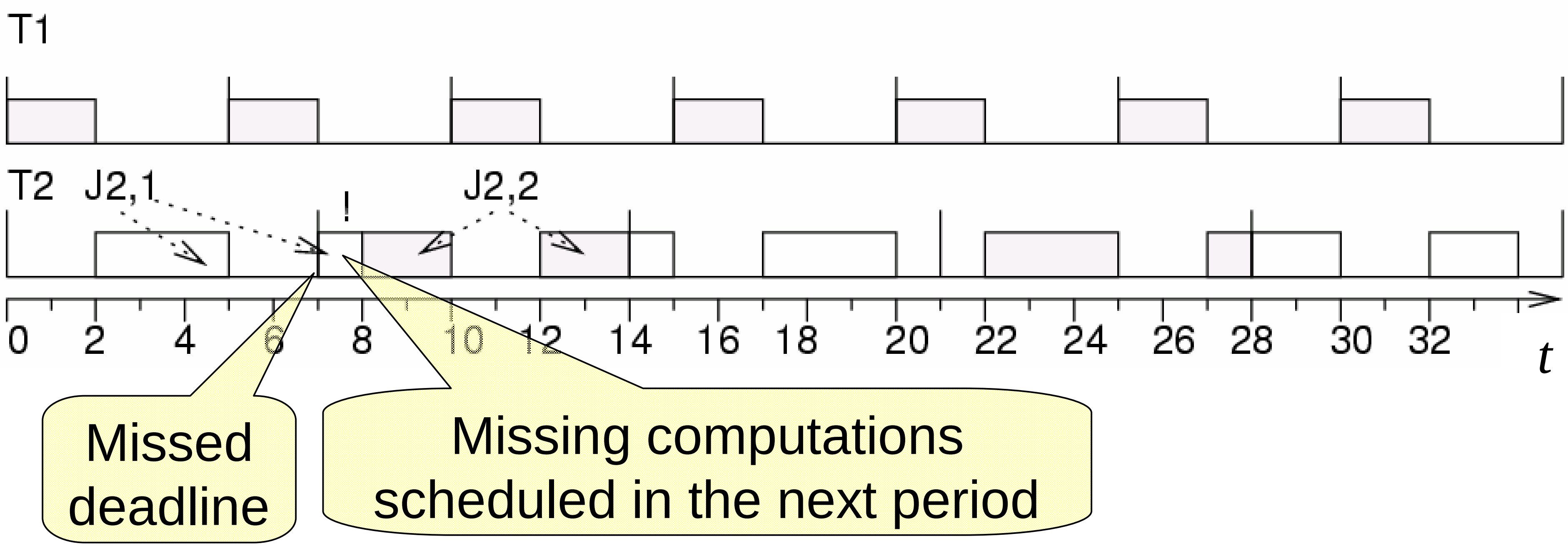
Case of failing RM scheduling

Task 1: period 5, execution time 2

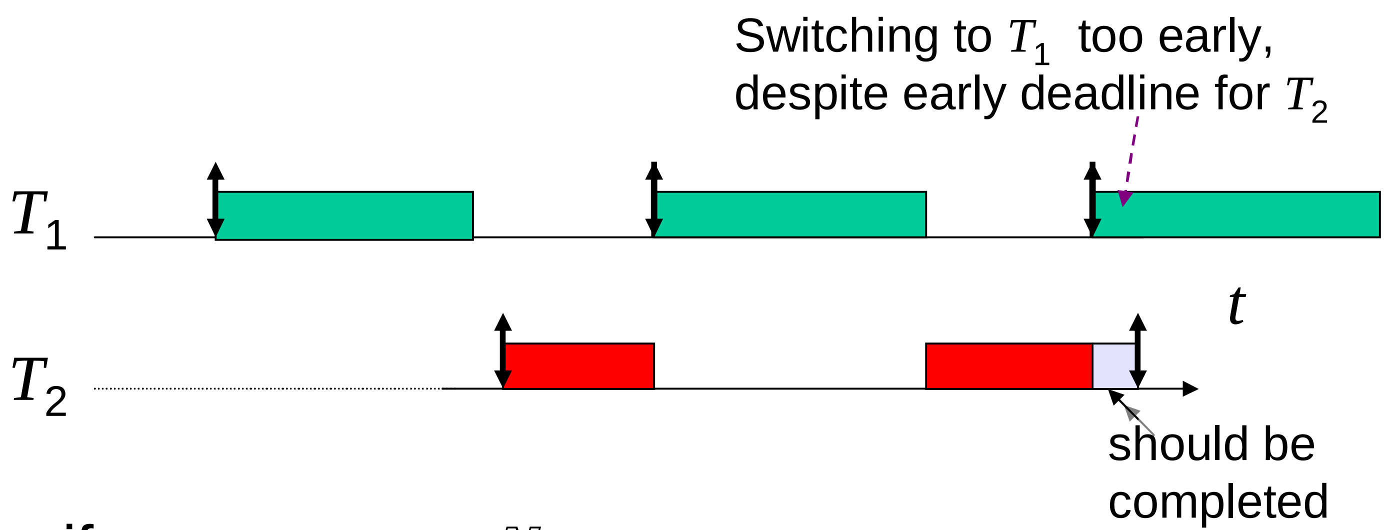
Task 2: period 7, execution time 4

$\mu = 2/5 + 4/7 = 34/35 \approx 0.97$

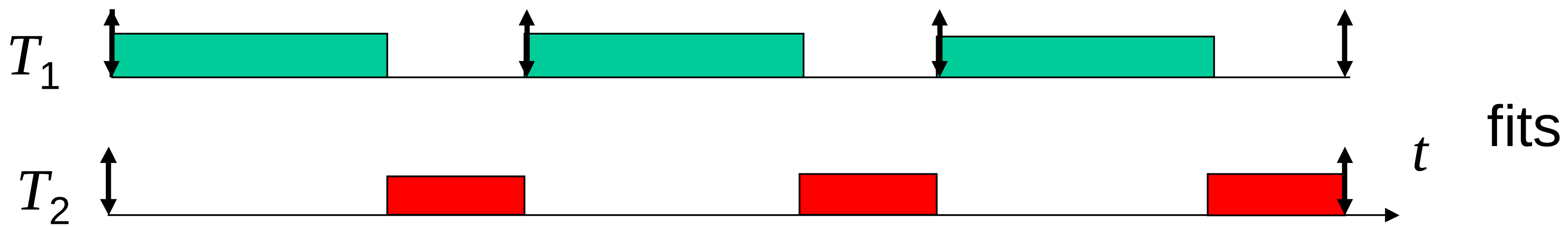
$2(2^{1/2}-1) \approx 0.828$



Intuitively: Why does RM fail ?



No problem if $p_2 = m\, p_1,\ m \in \mathbb{N}$:



Critical instants

Definition: A **critical instant** of a task is the time at which the release of a task will produce the largest response time.

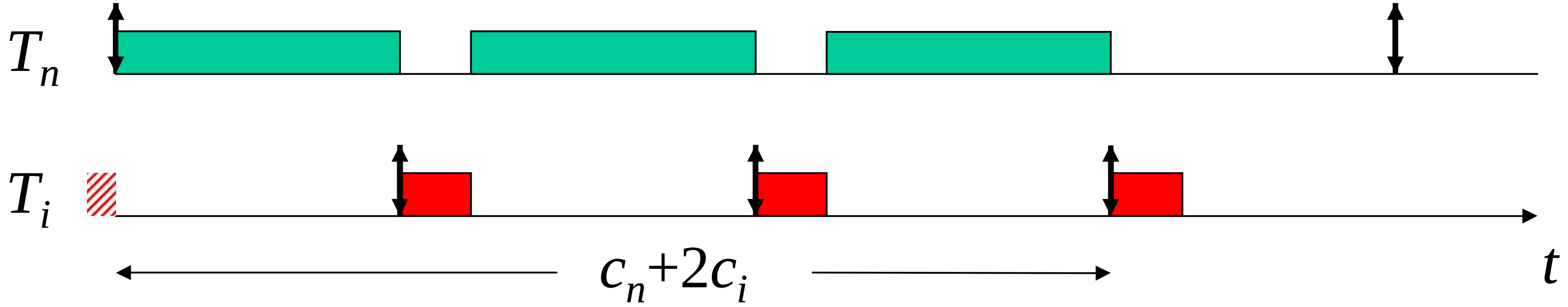
Lemma: For any task, the **critical instant** occurs if that task is simultaneously released with all higher priority tasks.

Proof: Let $T=\{T_1, \ldots, T_n\}$: periodic tasks with $\forall i$: $p_i \leqq p_{i+1}$.

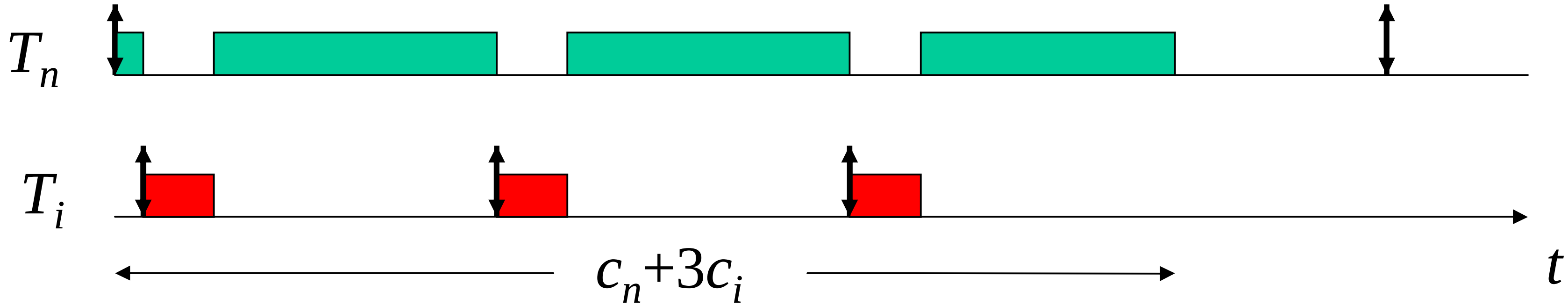
Source: G. Buttazzo, Hard Real-time Computing Systems, Kluwer, 2002

Critical instances (1)

Response time of T_n is delayed by tasks T_i of higher priority:



Delay may increase if T_i starts earlier



Maximum delay achieved if T_n and T_i start simultaneously.

Critical instants (2)

Repeating the argument for all $i = 1, \ldots n$-1*:*

- ☞ The worst case response time of a task occurs when it is released simultaneously with all higher-priority tasks. q.e.d.

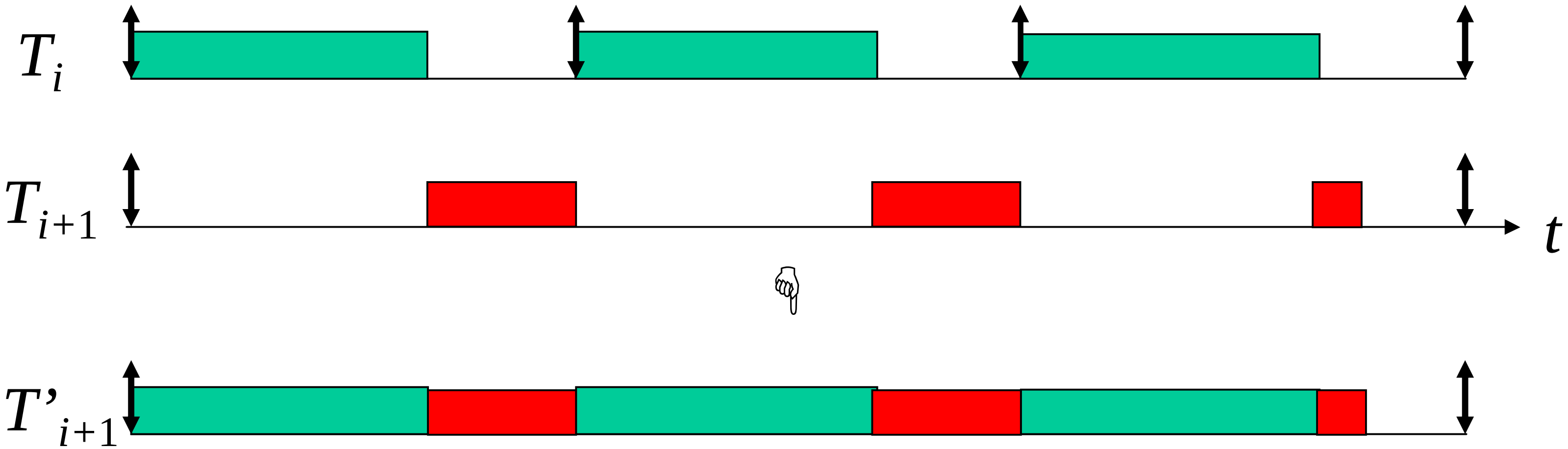
- ☞ Schedulability is checked at the critical instants.
- ☞ If all tasks of a task set are schedulable at their critical instants, they are schedulable at all release times.
- ☞ Observation helps designing examples

The case $\forall i$: $p_{i+1} = m_i \, p_i$

Lemma*: **If each task period is a multiple of the period of the next higher priority task**, then schedulability is also guaranteed if $\mu \leq 1$.

Proof: Assume schedule of T_i is given. Incorporate T_{i+1}:

T_{i+1} fills idle times of T_i; T_{i+1} completes in time, if $\mu \leq 1$.



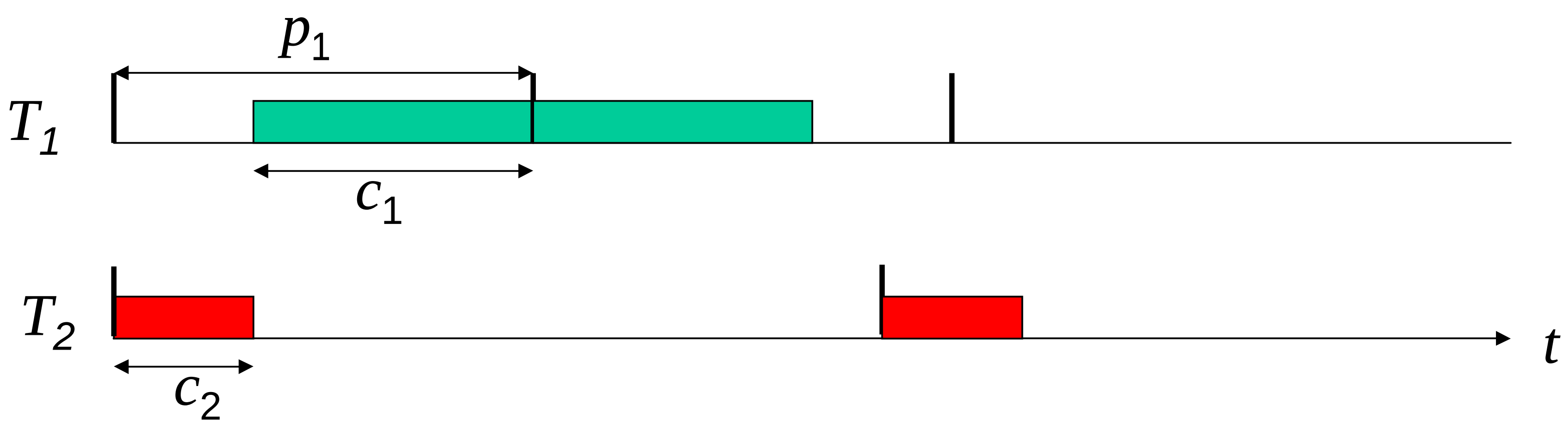
Used as the higher priority task at the next iteration.

More in-depth:

Proof of the RM theorem

Let $T=\{T_1, T_2\}$ with $p_1 < p_2$.

Assume RM is **not** used → prio(T_2) is highest:



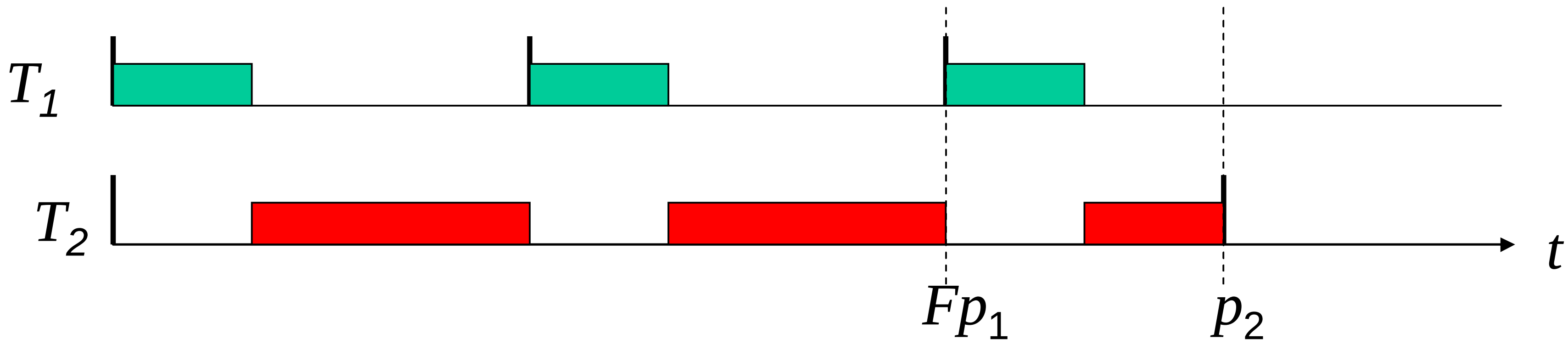
Schedule is feasible if $c_1+c_2 \leqq p_1$ (1)

Define $F=\lfloor p_2/p_1 \rfloor$: # of periods of T_1 fully contained in T_2

Case 1: $c_1 \leq p_2 - Fp_1$

Assume RM is used → prio(T_1) is highest:

Case 1*: $c_1 \leq p_2 - F\,p_1$
(c_1 small enough to be finished before 2nd instance of T_2)



Schedulable if $(F+1)\,c_1 + c_2 \leq p_2$ (2)

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* Typos in [Buttazzo 2002]: < and ≤ mixed up]

Proof of the RM theorem (3)

Not RM: schedule is feasible if $c_1+c_2 \le p_1$ (1)

RM: schedulable if $(F+1)c_1 + c_2 \le p_2$ (2)

From (1): $Fc_1+Fc_2 \le Fp_1$

Since $F \ge 1$: $Fc_1+c_2 \le Fc_1+Fc_2 \le Fp_1$

Adding c_1: $(F+1)\, c_1+c_2 \le Fp_1 + c_1$

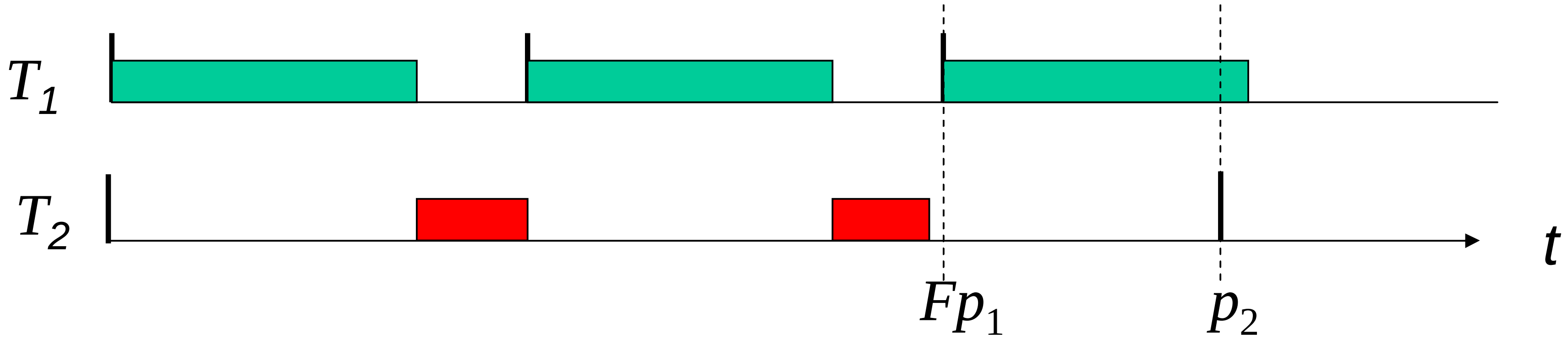
Since $c_1 \le p_2 - Fp_1$: $(F+1)\, c_1+c_2 \le Fp_1 + c_1 \le p_2$

Hence: if (1) holds, (2) holds as well

☞ For case 1: Given tasks T_1 and T_2 with $p_1 < p_2$, then if the schedule is feasible by an arbitrary (but fixed) priority assignment, it is also feasible by RM.

Case 2: $c_1 > p_2 - Fp_1$

Case 2: $c_1 > p_2 - Fp_1$
(c_1 large enough not to finish before 2nd instance of T_2)



Schedulable if $\qquad F\,c_1 + c_2 \le F\,p_1 \qquad (3)$

$c_1 + c_2 \le p_1 \qquad (1)$

Multiplying (1) by *F* yields $\qquad F\,c_1 + F\,c_2 \le F\,p_1$

Since $F \ge 1$: $\qquad F\,c_1 + c_2 \le F\,c_1 + Fc_2 \le F\,p_1$

☞ Same statement as for case 1.

Calculation of the least upper utilization bound

Let $T=\{T_1, T_2\}$ with $p_1 < p_2$.

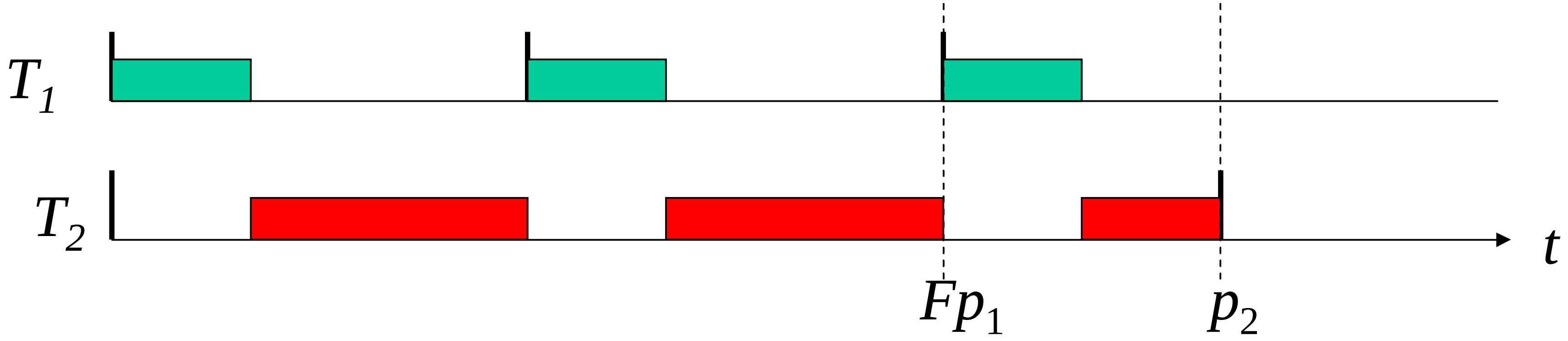
Proof procedure: compute least upper bound U_{lup} as follows

- Assign priorities according to RM
- Compute upper bound U_{up} by setting computation times to fully utilize processor
- Minimize upper bound with respect to other task parameters

As before: $F=\lfloor p_2/p_1 \rfloor$

c_2 adjusted to fully utilize processor.

Case 1: $c_1 \le p_2 - Fp_1$



Largest possible value of c_2 is $\quad c_2 = p_2 - c_1\,(F+1)$

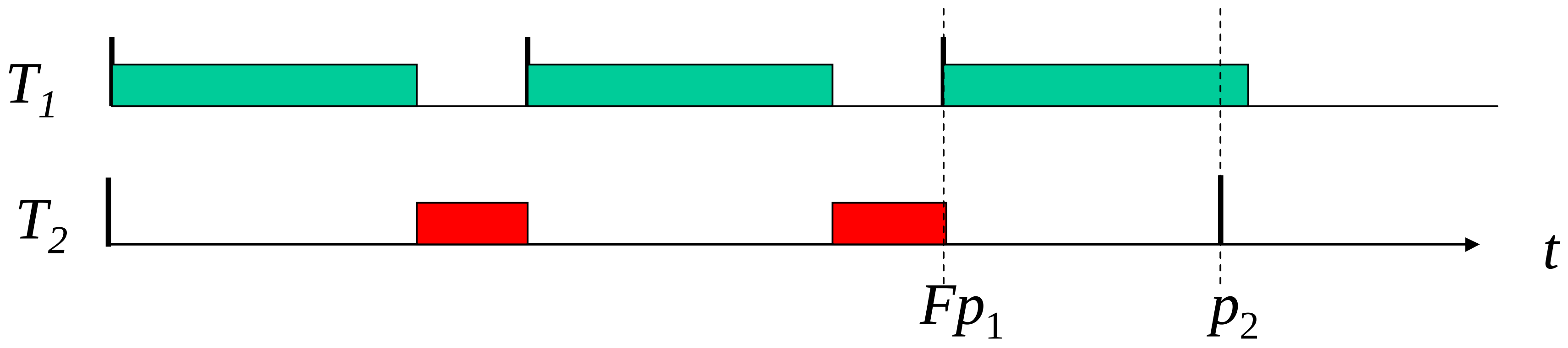
Corresponding upper bound is

$$U_{ub} = \frac{c_1}{p_1} + \frac{c_2}{p_2} = \frac{c_1}{p_1} + \frac{p_2 - c_1\,(F+1)}{p_2} = 1 + \frac{c_1}{p_1} - \frac{c_1\,(F+1)}{p_2} = 1 + \frac{c_1}{p_2}\left\{\frac{p_2}{p_1} - (F+1)\right\}$$

{ } is <0 → U_{ub} monotonically decreasing in c_1

Minimum occurs for $c_1 = p_2 - Fp_1$

Case 2: $c_1 \geq p_2 - Fp_1$



Largest possible value of c_2 is $c_2 = (p_1 - c_1)F$
Corresponding upper bound is:

$$U_{ub} = \frac{c_1}{p_1} + \frac{c_2}{p_2} = \frac{c_1}{p_1} + \frac{(p_1 - c_1)\,F}{p_2} = \frac{p_1}{p_2}F + \frac{c_1}{p_1} - \frac{c_1}{p_2}F = \frac{p_1}{p_2}F + \frac{c_1}{p_2}\left\{\frac{p_2}{p_1} - F\right\}$$

{ } is ≥ 0 → U_{ub} monotonically increasing in c_1 (independent of c_1 if {}=0)

Minimum occurs for $c_1 = p_2 - Fp_1$, as before.

Utilization as a function of $G=p_2/p_1-F$

For minimum value of c_1:

$$U_{ub}=\frac{p_1}{p_2}F+\frac{c_1}{p_2}\left(\frac{p_2}{p_1}-F\right)=\frac{p_1}{p_2}F+\frac{\left(p_2-p_1F\right)}{p_2}\left(\frac{p_2}{p_1}-F\right)=\frac{p_1}{p_2}\left\{F+\left(\frac{p_2}{p_1}-F\right)\left(\frac{p_2}{p_1}-F\right)\right\}$$

Let $G=\frac{p_2}{p_1}-F;\qquad \Rightarrow$

$$U_{ub}=\frac{p_1}{p_2}\left(F+G^2\right)=\frac{\left(F+G^2\right)}{p_2/p_1}=\frac{\left(F+G^2\right)}{\left(p_2/p_1-F\right)+F}=\frac{\left(F+G^2\right)}{F+G}=\frac{\left(F+G\right)-(G-G^2)}{F+G}$$

$$=1-\frac{G\left(1-G\right)}{F+G}$$

Since $0 \le G < 1$: $G(1-G) \ge 0$ → U_{ub} increasing in F →
Minimum of U_{ub} for min(F): F=1 →

$$U_{ub}=\frac{1+G^2}{1+G}$$

Proving the RM theorem for *n*=2

end

$$U_{ub} = \frac{1+G^2}{1+G}$$

Using derivative to find minimum of U_{ub} :

$$\frac{dU_{ub}}{dG} = \frac{2G(1+G)-(1+G^2)}{(1+G)^2} = \frac{G^2+2G-1}{(1+G)^2} = 0$$

$$G_1 = -1-\sqrt{2}; \qquad G_2 = -1+\sqrt{2};$$

Considering only G_2, since $0 \leq G < 1$:

$$U_{lub} = \frac{1+(\sqrt{2}-1)^2}{1+(\sqrt{2}-1)} = \frac{4-2\sqrt{2}}{\sqrt{2}} = 2(\sqrt{2}-1) = 2(2^{\frac{1}{2}}-1) \cong 0.83$$

This proves the RM theorem for the special case of *n*=2

Properties of RM scheduling

- RM scheduling is based on **static** priorities. This allows RM scheduling to be used in an OS with static priorities, such as Windows NT.
- No idle capacity is needed if $\forall i$: $p_{i+1}=F\ p_\mathrm{i}$: i.e. if the **period of each task is a multiple of the period of the next higher priority task**, schedulability is then also guaranteed if $\mu \le 1$.
- A huge number of variations of RM scheduling exists.
- In the context of RM scheduling, many formal proofs exist.

EDF

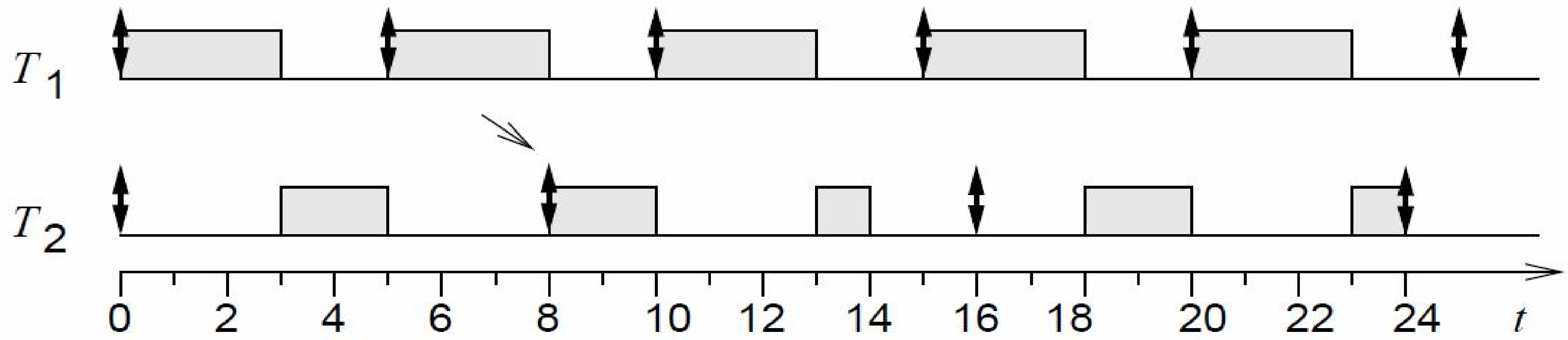
EDF can also be applied to periodic scheduling.

EDF optimal for every **hyper-period**
(= least common multiple of all periods)

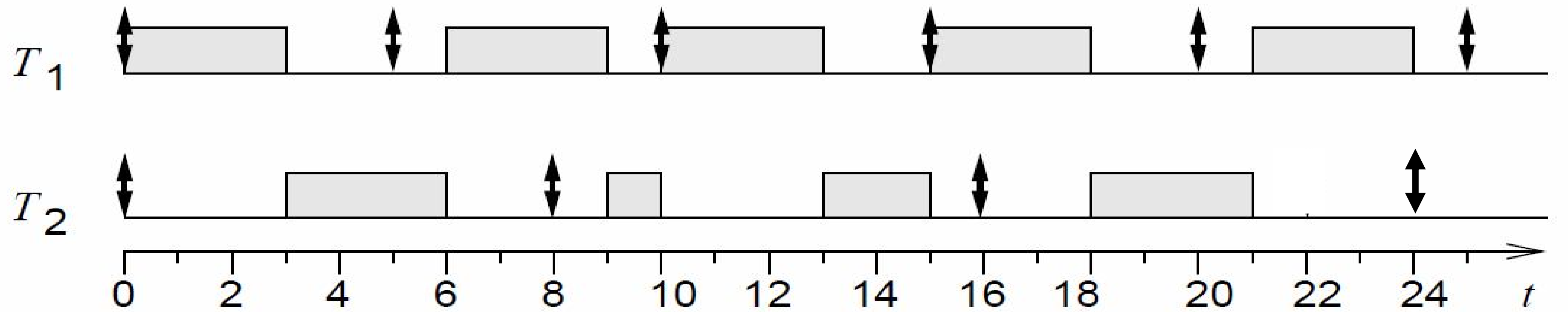
- ☞ Optimal for periodic scheduling
- ☞ EDF must be able to schedule the example in which RMS failed.

Comparison EDF/RMS

RMS:



EDF:



T_2 not preempted, due to its earlier deadline.

EDF: Properties

EDF requires dynamic priorities

☞ EDF cannot be used with an operating system just providing static priorities.

However, a recent paper (by Margull and Slomka) at DATE 2008 demonstrates how an OS with static priorities can be extended with a plug-in providing EDF scheduling
(key idea: delay tasks becoming ready if they shouldn't be executed under EDF scheduling.

Comparison RMS/EDF

	RMS	EDF
Priorities	Static	Dynamic
Works with OS with fixed priorities	**Yes**	**No***
Uses full computational power of processor	No, just up till $\mu = n(2^{1/n}-1)$	Yes
Possible to exploit full computational power of processor without provisioning for slack	**No**	**Yes**

* Unless the plug-in by Slomka et al. is added.

Sporadic tasks

If sporadic tasks were connected to interrupts, the execution time of other tasks would become very unpredictable.

☞ Introduction of a sporadic task server, periodically checking for ready sporadic tasks;

☞ Sporadic tasks are essentially turned into periodic tasks.

Dependent tasks

The problem of deciding whether or not a schedule exists for a set of dependent tasks and a given deadline is NP-complete in general [Garey/Johnson].

Strategies:

1. Add resources, so that scheduling becomes easier
2. Split problem into static and dynamic part so that only a minimum of decisions need to be taken at run-time.
3. Use scheduling algorithms from high-level synthesis

Summary

Periodic scheduling

- Rate monotonic scheduling
- EDF
- Dependent and sporadic tasks (briefly)