ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

INTRODUCTION TO QUANTITATIVE FINANCE AND FINANCIAL RISK

Triangular & One-way Arbitrage

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The Assignment

Triangular and One-way Arbitrage

Consider the attached Quotation Matrix. After downloading it, you will open it, and then save it locally on your computer. This way you will all have different tables.

Assignment 1 (Triangular Arbitrage)

Write a Python program that explores all possibilities of triangular arbitrage, produces the corresponding trades, and records the overall result in the chosen by the user currency.

Assignment 2 (One-Way Arbitrage)

Write a second program that explores all possibilities of one-way-arbitrage, after receiving intended currency trade from the user (e.g., I want to sell USD and buy GBP). The program must produce the corresponding trades.

The assignment is mandatory for the students taking the entire course and optional for the students of the half-course. The code should contain explanatory comments. An accompanying word file should briefly describe the structure of your program and your results. The assignment is due by May 31, 2023, for the half-course students and by June 30th for the full-course students. It will count towards 30% of the final grade. There will be a 10-minute presentation of the assignment in a skype session.

The Data

	USD	**EUR**	**JPY**	**GBP**	**CHF**	**CAD**	**AUD**
AUD	1,244	1,7441	0,0115	2,1829	1,0959	1,1634	
CAD	1,0693	1,4992	0,0099	1,8763	0,942		0,858948
CHF	1,1352	1,5915	0,0105	1,992		1,060511	0,911216
GBP	0,5699	0,799	0,0053		0,501707	0,532325	0,457969
JPY	107,86	151,22		188,3967	95,13345	100,9193	86,93913
EUR	0,7133		0,006604	1,250564	0,627648	0,666356	0,573247
USD		1,399975	0,009267	1,752416	0,88055	0,93463	0,802735

Arbitrage

Arbitrage refers to the practice of taking advantage of price discrepancies that exist between two or more financial assets or markets. The essence of arbitrage lies in exploiting temporary inefficiencies in prices to generate risk-free profits. In efficient markets, prices adjust instantaneously to reflect all available information, leaving no room for arbitrage opportunities. However, in real-world situations, market imperfections, transaction costs, and information asymmetry can create temporary price disparities, which arbitrageurs seek to exploit.

Arbitrage can be broadly classified into two main types: spatial arbitrage and temporal arbitrage. Spatial arbitrage involves taking advantage of price differences for the same asset across different markets, while temporal arbitrage exploits price variations of an asset over time. The goal of arbitrage is to eliminate price disparities by simultaneously buying and selling the same or similar assets, thereby ensuring market efficiency.

Triangular Arbitrage

Triangular arbitrage is a specific type of spatial arbitrage strategy that involves exploiting price inconsistencies between three different currencies in the foreign exchange market. This strategy takes advantage of the exchange rate relationships among three currency pairs to generate profits. Suppose we have three currencies: A, B, and C. By carefully executing a series of transactions, an arbitrageur can profit from discrepancies in the cross-exchange rates among these currencies.

For instance, if the exchange rate of A to B is overvalued compared to the exchange rates of B to C and C to A, an arbitrageur can engage in a series of trades to capitalize on this mispricing. The process typically involves converting currency A to B, then B to C, and finally C back to A, resulting in a riskless profit due to the pricing inefficiency. Triangular arbitrage relies on swift execution, as currency exchange rates can change rapidly, especially in highly liquid markets.

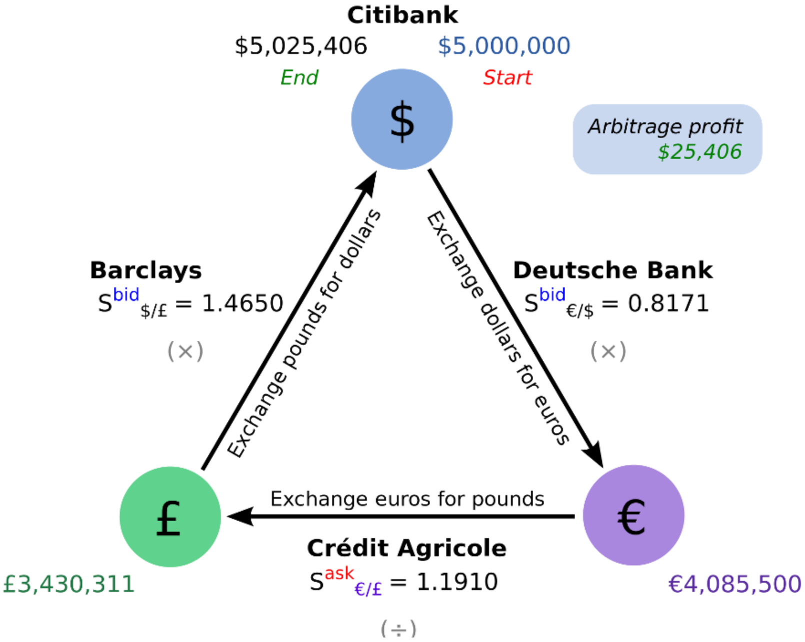

Fig.1: Example of Triangular arbitrage, by John Sandy

One Way Arbitrage

One-way arbitrage, also known as directional arbitrage, is a strategy that takes advantage of pricing discrepancies within a single financial asset across different markets or trading venues. Unlike spatial arbitrage, which involves exploiting price differences between multiple assets or markets, one-way arbitrage focuses on the same asset but across different locations or platforms.

In this strategy, an arbitrageur identifies instances where the price of an asset is temporarily mispriced in one market compared to another. They would then buy the undervalued asset in the market where it is cheaper and sell it in the market where it is relatively more expensive. This process continues until the price differential narrows or disappears, allowing the arbitrageur to lock in a profit.

One-way arbitrage can occur due to factors such as market fragmentation, trading restrictions, regulatory differences, or even technological limitations. However, it is important to note that one-way arbitrage opportunities are often short-lived and can quickly diminish as market participants spot and exploit the pricing discrepancies.

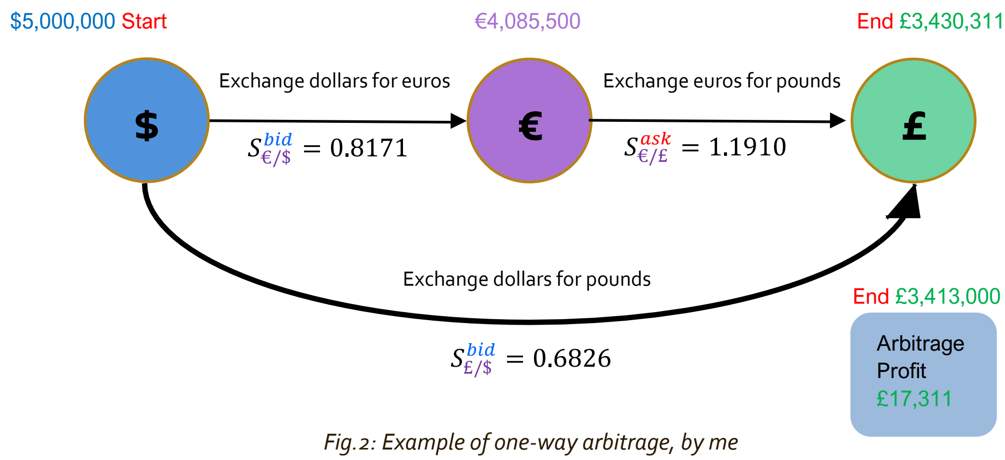

Fig.2: Example of one-way arbitrage, by me

Methodology

For the purposes of this assignment, we consider the problem to be a Spot-Spot inter-market FOREX Arbitrage. Instead of calculating possible mispricing's between triplets of currencies I instead approached the problem for a completely different perspective. To begin with if we consider 6 possible options to transition for each currency independently, we produce a number of possible paths the same process is repeated for the second transition. It is important to avoid having repeating currencies in the possible paths. No matter the currency the last jump is brings us back us back to our base currency, so the number of paths remain the same. The size of possible pair combinations without repetitions is:

$$N = 7, \quad \frac{N!}{(N-2)!} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7*6*5!}{5!} = 7*6 = 42$$

Or as:

$$N = 7, \ N \cdot (N-1) = 7*6 = 42$$

While the size of possible triplets' combinations without repetitions is:

$$N = 7, \quad \frac{N!}{(N-3)!} = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7*6*5*4!}{4!} = 7*6*5 = 210$$

Or as:

$$N = 7, \ N \cdot (N-1) \cdot (N-2) = 7*6*5 = 210$$

The size of the created dictionary that contains all possible transactions from one currency to another has 42 rows. The dictionary that was created to calculate all resulting transactions on the pyramid contains 210 rows.

By calculating the result value of each row and evaluating the difference from 1, in other words the starting base value, we can conclude if the trip was profitable or not. Values larger than 1 are and lower than 1 the costs were greater than the profit.

For the one-way arbitrage the procedure is the same minus the last trip, the problem was that the comparison between different currencies, my solution was to normalize based on the value of currency that would be produced if instead of arbitrage we chose a direct exchange. Lastly the returns were presented on a percentage basis.

Results

The produced currency pair dictionary allows for easy access to the transaction rates for the second step.

```
{'AUD|USD': 1.244,
 'AUD|EUR': 1.7441,
 'AUD|JPY': 0.0115,
 'AUD|GBP': 2.1829,
 'AUD|CHF': 1.0959,
 'AUD|CAD': 1.1634,
 'CAD|USD': 1.0693,
 'CAD|EUR': 1.4992,
 'CAD|JPY': 0.0099,
 'CAD|GBP': 1.8763,
 'CAD|CHF': 0.942,
 'CAD|AUD': 0.8589483321791644,
 'CHF|USD': 1.1352,
 'CHF|EUR': 1.5915,
 'CHF|JPY': 0.0105,
 'CHF|GBP': 1.992,
 'CHF|CAD': 1.0605106146507421,
 'CHF|AUD': 0.9112163128568629,
 'GBP|USD': 0.5699,
 'GBP|EUR': 0.799,
 'GBP|JPY': 0.0053,
 'GBP|CHF': 0.5017070079237598,
 'GBP|CAD': 0.5323250217311043,
 'GBP|AUD': 0.45796879837510107,
 'JPY|USD': 107.86,
 'JPY|EUR': 151.22,
 'JPY|GBP': 188.3966503075575,
 'JPY|CHF': 95.13344844480594,
 'JPY|CAD': 100.9192736638036,
 'JPY|AUD': 86.93913391234797,
 'EUR|USD': 0.7133,
 'EUR|JPY': 0.006604296308728027,
 'EUR|GBP': 1.2505640043659692,
 'EUR|CHF': 0.6276476334718583,
 'EUR|CAD': 0.6663560558971445,
 'EUR|AUD': 0.5732469697792567,
 'USD|EUR': 1.3999747052570253,
 'USD|JPY': 0.009266644259920846,
 'USD|GBP': 1.7524156655656298,
 'USD|CHF': 0.8805498237632362,
 'USD|CAD': 0.9346304683289345,
 'USD|AUD': 0.802734692331058}
```

By calling the dictionary keys is very efficient to calculate the possible profits of a round trip. Each row as mentioned before represents a transition from a node to another. By calling "EUR|USD" we immediately know the cost of 1$ in euros, "0.7133", and have the base for our next calculation. For example, changing the euros to pounds, we would require buying pounds using euros. This could be easily found by using the dictionary key "EUR|GBP", "1,2505…", Finally we would need to return to our base currency, so we find the value of "USD|GBP", "1,7524…". To accurately find the true exchange rates we can achieve I have calculated to buy at the ask price and sell at the bid.

The created path "EUR|USD, EUR|GBP, USD|GBP" is only one of the 210 keys of the second dictionary and is value calculated as:

$$\frac{bv \cdot 0.7133 \cdot 1.7524}{1.2505} = bv \cdot 0.9995$$

We consider the base value to be equal to 1 then if the result is less than 1, means that this "path" presents no Triangular arbitrage opportunities. While if it larger than 1 then is profitable and an arbitrage opportunity.

The produced "paths" dictionary has the following form:

```
('EUR|USD', 'EUR|JPY', 'USD|JPY'): 0.9995486773579649,
('EUR|USD', 'EUR|GBP', 'USD|GBP'): 0.998748477304123,
('EUR|USD', 'EUR|CHF', 'USD|CHF'): 0.9996150852555385,
('EUR|USD', 'EUR|CAD', 'USD|CAD'): 0.9994745320580963,
('EUR|USD', 'EUR|AUD', 'USD|AUD'): 0.9986553631989169,
('EUR|JPY', 'EUR|USD', 'JPY|USD'): 0.9972571413611366,
('EUR|JPY', 'EUR|GBP', 'JPY|GBP'): 0.9941376144601385,
('EUR|JPY', 'EUR|CHF', 'JPY|CHF'): 0.9999227112405547,
```

By shorting we are able to find the values with the greater distance from 1. Also as requested the user can enter a base currency and find the greatest opportunities. Here are presented the best possible triangular arbitrage trips for JPY as base currency.

```
Best Trades:
('GBP|JPY', 'GBP|AUD', 'JPY|AUD'): 1.0058310077115014
('GBP|JPY', 'GBP|CHF', 'JPY|CHF'): 1.0043808953008833
('GBP|JPY', 'GBP|CAD', 'JPY|CAD'): 1.003580615829592
('GBP|JPY', 'GBP|EUR', 'JPY|EUR'): 1.002284530323176
('USD|JPY', 'USD|AUD', 'JPY|AUD'): 1.0022087286317982
('GBP|JPY', 'GBP|USD', 'JPY|USD'): 1.0017824345459168
('EUR|JPY', 'EUR|AUD', 'JPY|AUD'): 1.00141303844036
('CAD|JPY', 'CAD|AUD', 'JPY|AUD'): 1.001335850968938
('USD|JPY', 'USD|CHF', 'JPY|CHF'): 1.0007557937565956
('CHF|JPY', 'CHF|AUD', 'JPY|AUD'): 1.0004042669726927
--------------Unfavorable Trades-----------------
('USD|JPY', 'USD|CAD', 'JPY|CAD'): 0.9999911904673027
('EUR|JPY', 'EUR|CHF', 'JPY|CHF'): 0.9999227112405547
('USD|JPY', 'USD|EUR', 'JPY|EUR'): 0.9995486773579648
('EUR|JPY', 'EUR|CAD', 'JPY|CAD'): 0.9992179791768276
('CAD|JPY', 'CAD|CHF', 'JPY|CHF'): 0.9988113156520538
('CHF|JPY', 'CHF|CAD', 'JPY|CAD'): 0.9981925358086814
('CAD|JPY', 'CAD|USD', 'JPY|USD'): 0.998011498908193
('CAD|JPY', 'CAD|EUR', 'JPY|EUR'): 0.9975869914503854
('EUR|JPY', 'EUR|USD', 'JPY|USD'): 0.9972571413611366
('CHF|JPY', 'CHF|USD', 'JPY|USD'): 0.9972490919065778
```

The one-way arbitrage mirror that process, but with the “paths” dictionary having one less trip but the same number of rows and the user being able to enter both the base currency and the target currency:

```
Best Trades:
('JPY|CHF', 'JPY|USD'): 100.07557937565956 % return
--------------Unfavorable Trades------------------
('CAD|CHF', 'CAD|USD'): 99.94549422034801 % return
('AUD|CHF', 'AUD|USD'): 99.86546808744285 % return
('GBP|CHF', 'GBP|USD'): 99.80669547820017 % return
('EUR|CHF', 'EUR|USD'): 99.74898082782988 % return
```

As stated previously the returns are calculated as a percentage to avoid the conundrum of comparison between different starting and finishing currency.

To verify the validity of the results I calculated the product of JPY|CHF JPY|USD and comparing it with the CHF|USD.

$$\frac{95.1334}{0.0115} = 0.882 > 0.5699$$

Indeed, an arbitrage opportunity has been found as the $S_{EUR|JPY}^{ask}$ is lower than it should have been.