An Improvement of the Lucas-Kanade Optical-Flow Algorithm for every Circumstance

Lorenz Gerstmayr

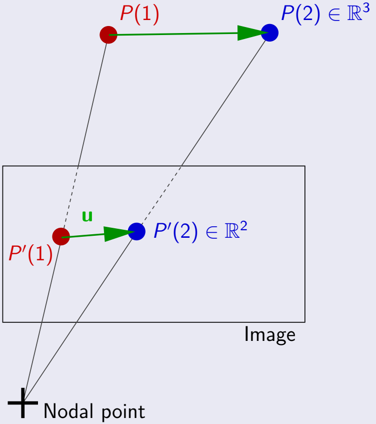
Computer Engineering Group
Faculty of Technology
University of Bielefeld

2008-06-11
Corrected version: 2008-08-05

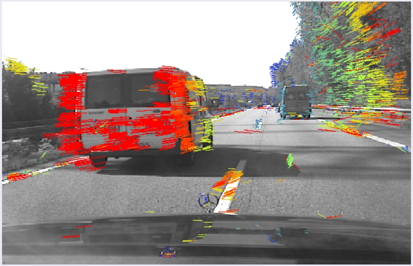
Today's talk

Definition of optical flow

Definiton



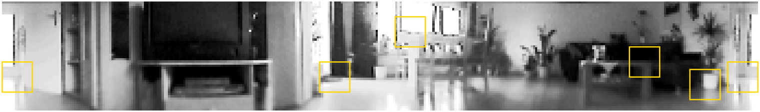
Example



http://www.ezls.fb12.uni-siegen.de/lehre/WebOne/index.htm

Image Brightness Constancy Constraint (IBCC)

B:



A:

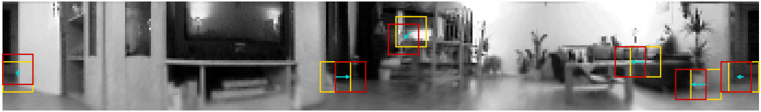
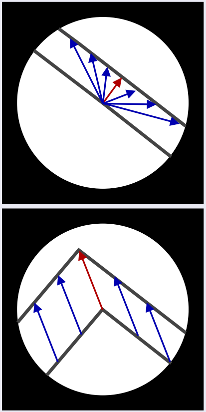


Image brightness constancy constraint:

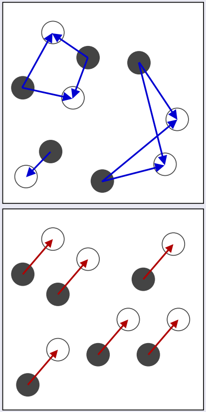
$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Common problems

Aperture problem



Correspondence problem



After: B. Jähne, Dig. Img. Proc., 2000

Optic flow equation

Image brightness constancy constraint:

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Optic flow equation:

First order Taylor approximation (with $\mathbf{u} = [u_x, u_y]^\top$):

$$A(\mathbf{p}) + A_x(\mathbf{p})u_x(\mathbf{p}) + A_y(\mathbf{p})u_y(\mathbf{p}) = B(\mathbf{p})$$

$$A + [A_x, A_y]\begin{pmatrix} u_x \\ u_y \end{pmatrix} = B$$

$$[A_x, A_y]\begin{pmatrix} u_x \\ u_y \end{pmatrix} + \underbrace{(A - B)}_{A_t} = 0$$

B. Jähne, Dig. Img. Proc., 2000; B.D. Lucas & T. Kanade, Proc. Im. Underst. Workshop, 1981

Derivation

Optic flow equation:

$$[A_x, A_y] \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A_t = 0$$

Underdetermined problem (aperture problem)

Approach:

Assuming optical flow to be locally constant
Weighting w.r.t. distance to center pixel **p** of image patch Ω:

$$e(\mathbf{u}) = \sum_{\mathbf{x} \in \Omega} w^2(\mathbf{x}) \left([A_x(\mathbf{x}), B_x(\mathbf{x})] \begin{pmatrix} u_x(\mathbf{x}) \\ u_y(\mathbf{x}) \end{pmatrix} + A_t(\mathbf{x}) \right)^2 \stackrel{!}{=} \min$$

B. Jähne, Dig. Img. Proc., 2000; B.D. Lucas & T. Kanade, Proc. Im. Underst. Workshop, 1981

Derivation

Weighted least squares approach

$$e(\mathbf{u}) = \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y] \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A_t \right)^2 \stackrel{!}{=} \min$$

Partial derivatives:

$$\frac{\partial e}{\partial u_x} = 2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y] \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A_t \right) A_x \stackrel{!}{=} \min$$

$$\frac{\partial e}{\partial u_y} = 2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y] \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A_t \right) A_y \stackrel{!}{=} \min$$

B. Jähne, Dig. Img. Proc., 2000; B.D. Lucas & T. Kanade, Proc. Im. Underst. Workshop, 1981

Derivation

Weighted least squares approach:

$$\frac{\partial e}{\partial u_x} = 2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y] \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A_t \right) A_x \stackrel{!}{=} \min$$

$$\frac{\partial e}{\partial u_y} = 2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y] \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A_t \right) A_y \stackrel{!}{=} \min$$

Sorting:

$$\underbrace{\begin{pmatrix} \langle A_x A_x \rangle & \langle A_x A_y \rangle \\ \langle A_x A_y \rangle & \langle A_y A_y \rangle \end{pmatrix}}_{\mathbf{G}} \begin{pmatrix} u_x \\ u_y \end{pmatrix} = \underbrace{- \begin{pmatrix} \langle A_x A_t \rangle \\ \langle A_y A_t \rangle \end{pmatrix}}_{\mathbf{b}}$$

$$\langle a \rangle = \sum_{\mathbf{x}\in\Omega} w^2(\mathbf{x}) a(\mathbf{x})$$

Solution

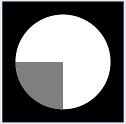
Weighted least squares approach:

$$\mathbf{G}\mathbf{u} = \mathbf{b}$$

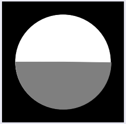
$$\mathbf{u} = \mathbf{G}^{-1}\mathbf{b}$$

Aperture problem

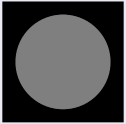
Eigenvalues λ_1, λ_2 of $\mathbf{G}$:



$\lambda_1, \lambda_2 > 0$



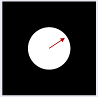
$\lambda_1 > 0, \lambda_2 \approx 0$



$\lambda_1, \lambda_2 \approx 0$

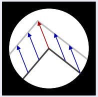
Limitations and possible improvements

Standard LK



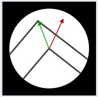
- baseline
- most efficient

Illumination Tolerance



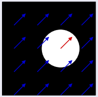
- illumination changes

Iterated LK



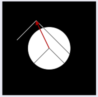
- more accurate

LK with flowfield constraints



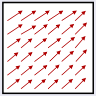
- types of flowfields

Multiscale LK



- large displacements

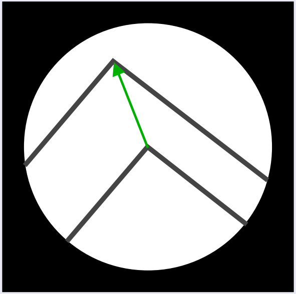
Global LK



- global flow computation

Motivation

Sketch



Motivation

- Deviations due to first order approximation
- Large displacements within aperture
- Iteratively refine the flow vector

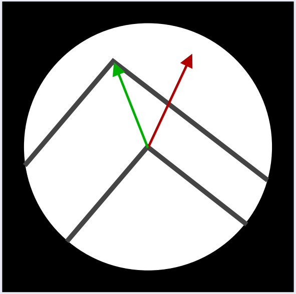
Improvement:

More accurate flow estimates

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation

- Deviations due to first order approximation
- Large displacements within aperture
- Iteratively refine the flow vector

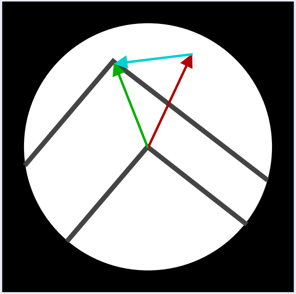
Improvement:

More accurate flow estimates

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation

- Deviations due to first order approximation
- Large displacements within aperture
- Iteratively refine the flow vector

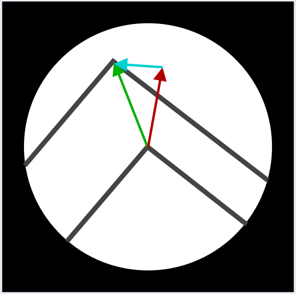
Improvement:

More accurate flow estimates

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation

- Deviations due to first order approximation
- Large displacements within aperture
- Iteratively refine the flow vector

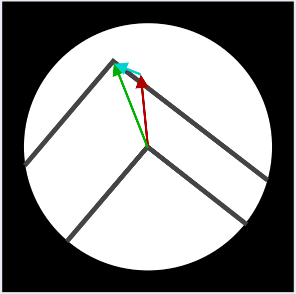
Improvement:

More accurate flow estimates

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation

- Deviations due to first order approximation
- Large displacements within aperture
- Iteratively refine the flow vector

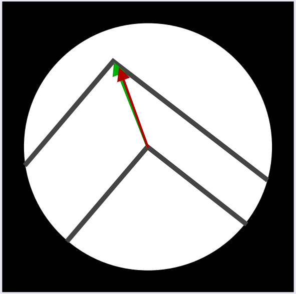
Improvement:

More accurate flow estimates

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation

- Deviations due to first order approximation
- Large displacements within aperture
- Iteratively refine the flow vector

Improvement:

More accurate flow estimates

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Derivation

Image brightness constancy constraint:

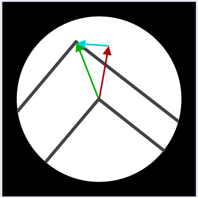
$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Refined IBCC:

$$A(\mathbf{p} + \mathbf{u}^{(\kappa-1)}(\mathbf{p}) + \mathbf{t}^{(\kappa)}(\mathbf{p})) = B(\mathbf{p})$$

Iterations $1 \leq \kappa \leq k$
Recursion: $\mathbf{u}^{(\kappa)}$ from $\mathbf{u}^{(\kappa-1)}$ and $\mathbf{t}^{(\kappa)}$
Initial estimate: $\mathbf{u}^0 = \mathbf{0}$

Sketch



J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Derivation

Weighted least squares approach

$$e^{(\kappa)}\left(\mathbf{t}^{(\kappa)}\right) = \sum_{\mathbf{x}\in\Omega} w^2(\mathbf{x})\left(A\left(\underbrace{\mathbf{x}+\mathbf{u}^{(\kappa-1)}}_{\eta}+\mathbf{t}^{(\kappa)}\right) - B(\mathbf{x})\right)^2 \stackrel{!}{=} 0$$

$$e(\mathbf{t}) = \sum_{\mathbf{x}\in\Omega} w^2\left(A\left(\eta+\mathbf{t}\right) - B\right)^2$$

Taylor-approximation of $A(\eta+\mathbf{t})$ (omitting $\mathbf{x}$):

$$= \sum_{\mathbf{x}\in\Omega} w^2\left(\underbrace{A(\eta) - B}_{A_t(\eta)} + [A_x(\eta), A_y(\eta)]\begin{pmatrix} t_x \\ t_y \end{pmatrix}\right)^2$$

Derivation

Weighted least squares approach

$$e(\mathbf{t}) = \sum_{\mathbf{x}\in\Omega} w^2 \left(A_t(\eta) + [A_x(\eta), A_y(\eta)] \begin{pmatrix} t_x \\ t_y \end{pmatrix} \right)^2$$

Partial derivatives:

$$\left[\frac{\partial e}{\partial t_x}, \frac{\partial e}{\partial t_y} \right]^\top = 2 \sum_{\mathbf{x}\in\Omega} w^2 \left(A_t(\eta) + [A_x(\eta), A_y(\eta)] \begin{pmatrix} t_x \\ t_y \end{pmatrix} \right) \begin{pmatrix} A_x(\eta) \\ A_y(\eta) \end{pmatrix}$$

Solution (with $\eta = \mathbf{x} + \mathbf{u}^{(\kappa-1)}$):

$$\begin{pmatrix} \langle A_x(\eta)A_x(\eta)\rangle & \langle A_x(\eta)A_y(\eta)\rangle \\ \langle A_x(\eta)A_y(\eta)\rangle & \langle A_y(\eta)A_y(\eta)\rangle \end{pmatrix} \begin{pmatrix} t_x \\ t_y \end{pmatrix} = - \begin{pmatrix} \langle A_x(\eta)A_t(\eta)\rangle \\ \langle A_y(\eta)A_t(\eta)\rangle \end{pmatrix}$$

Simplified solution

Solution

$$\underbrace{\begin{pmatrix} \langle A_x(\eta)A_x(\eta)\rangle & \langle A_x(\eta)A_y(\eta)\rangle \\ \langle A_x(\eta)A_y(\eta)\rangle & \langle A_y(\eta)A_y(\eta)\rangle \end{pmatrix}}_{=\mathbf{G}(\eta)} \begin{pmatrix} t_x \\ t_y \end{pmatrix} = -\underbrace{\begin{pmatrix} \langle A_x(\eta)A_t(\eta)\rangle \\ \langle A_y(\eta)A_t(\eta)\rangle \end{pmatrix}}_{=\mathbf{b}(\eta)}$$

Problem

- $\mathbf{G}$ depends on $\eta = \mathbf{x} + \mathbf{u}^{(\kappa-1)}$
  $\Rightarrow$ Recomputation of $\mathbf{G}^{-1}$ needed for each iteration
- A and B contain identical gradient information
- Compute B_x and B_y instead of A_x and A_y

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Simplified solution

Solution

$$\underbrace{\begin{pmatrix} \langle A_x(\eta)A_x(\eta)\rangle & \langle A_x(\eta)A_y(\eta)\rangle \\ \langle A_x(\eta)A_y(\eta)\rangle & \langle A_y(\eta)A_y(\eta)\rangle \end{pmatrix}}_{=\mathbf{G}(\eta)} \begin{pmatrix} t_x \\ t_y \end{pmatrix} = -\underbrace{\begin{pmatrix} \langle A_x(\eta)A_t(\eta)\rangle \\ \langle A_y(\eta)A_t(\eta)\rangle \end{pmatrix}}_{=\mathbf{b}(\eta)}$$

Simplified solution

$$\underbrace{\begin{pmatrix} \langle B_x B_x\rangle & \langle B_x B_y\rangle \\ \langle B_x B_y\rangle & \langle B_y B_y\rangle \end{pmatrix}}_{=\mathbf{G}} \begin{pmatrix} t_x \\ t_y \end{pmatrix} = -\underbrace{\begin{pmatrix} \langle B_x A_t(\eta)\rangle \\ \langle B_y A_t(\eta)\rangle \end{pmatrix}}_{=\mathbf{b}(\eta)}$$

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Building blocks

Refinement vector

$$\mathbf{t}^{(\kappa)} = \mathbf{G}^{-1}\mathbf{b}^{(\kappa)}$$

Image mismatch vector

$$\mathbf{b}^{(\kappa)} = -\left[\langle B_x A_t^{(\kappa)}\rangle, \langle B_y A_t^{(\kappa)}\rangle\right]^\top$$

Temporal derivative

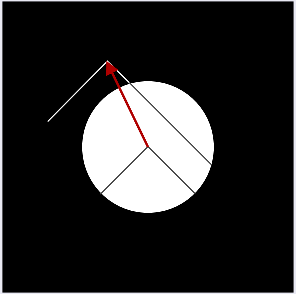
$$A_t^{(\kappa)} = B(\mathbf{p}) - A\left(\mathbf{p} + \mathbf{u}^{(\kappa-1)}\right)$$

Recursion

$$\mathbf{u}^{(\kappa)} = \mathbf{u}^{(\kappa-1)} + \mathbf{t}^{(\kappa)}$$

Motivation

Sketch



Motivation and approach

- IBCC and Taylor only valid for small movements
- Larger displacements are common

- Multi-scale approach
- Coarse-to-fine propagation
- Identical aperture sizes but shifted center position

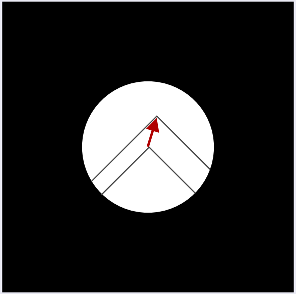
Improvement

Applicable for large image displacements

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation and approach

- IBCC and Taylor only valid for small movements
- Larger displacements are common

- Multi-scale approach
- Coarse-to-fine propagation
- Identical aperture sizes but shifted center position

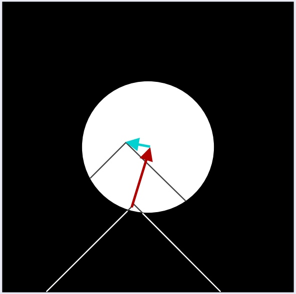
Improvement

Applicable for large image displacements

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation and approach

- IBCC and Taylor only valid for small movements
- Larger displacements are common

- Multi-scale approach
- Coarse-to-fine propagation
- Identical aperture sizes but shifted center position

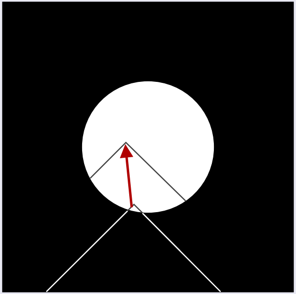
Improvement

Applicable for large image displacements

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation and approach

- IBCC and Taylor only valid for small movements
- Larger displacements are common

- Multi-scale approach
- Coarse-to-fine propagation
- Identical aperture sizes but shifted center position

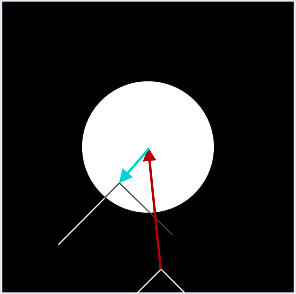
Improvement

Applicable for large image displacements

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation and approach

- IBCC and Taylor only valid for small movements
- Larger displacements are common

- Multi-scale approach
- Coarse-to-fine propagation
- Identical aperture sizes but shifted center position

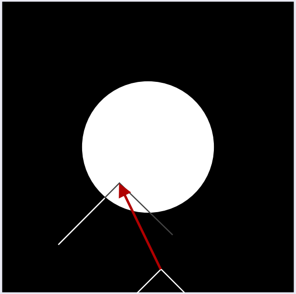
Improvement

Applicable for large image displacements

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation and approach

- IBCC and Taylor only valid for small movements
- Larger displacements are common

- Multi-scale approach
- Coarse-to-fine propagation
- Identical aperture sizes but shifted center position

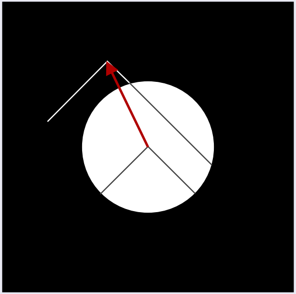
Improvement

Applicable for large image displacements

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Motivation and approach

- IBCC and Taylor only valid for small movements
- Larger displacements are common

- Multi-scale approach
- Coarse-to-fine propagation
- Identical aperture sizes but shifted center position

Improvement

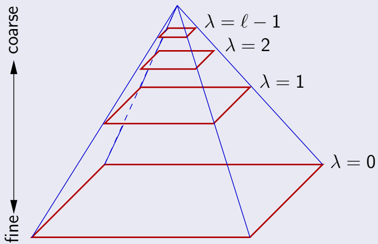
Applicable for large image displacements

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Gaussian image pyramid

Theory

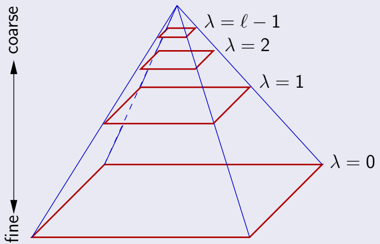
Pyramidal images: $I^{(\kappa)}$
Pixel positions: $\mathbf{p}^{(\kappa)} = \frac{1}{2^\kappa}\mathbf{p}$



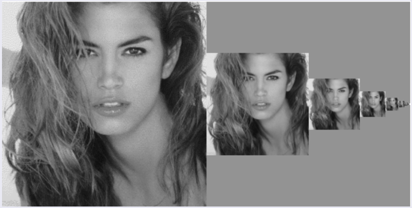
Gaussian image pyramid

Theory

Pyramidal images: $I^{(\kappa)}$
Pixel positions: $\mathbf{p}^{(\kappa)} = \frac{1}{2^\kappa}\mathbf{p}$



Example



fine $\leftrightarrow$ coarse

http://www.prip.tuwien.ac.at/research/research-areas/image-pyramids/

Algorithm

Input:

Two image pyramides $A^{(\lambda)}$ and $B^{(\lambda)}$, $0 \leq \lambda \leq \ell - 1$

Coarse-to-fine recursion (at level λ)

- Flow estimate $\mathbf{u}^{(\lambda)}$ is given (initial estimate: $\mathbf{u}^{(\ell-1)} = \mathbf{0}$)
- Compute refinement vector $\mathbf{t}^{(\lambda)}$:

$$e\left(\mathbf{t}^{(\lambda)}\right) = \sum_{\mathbf{x}\in\Omega} w(\mathbf{x})^2 \left(A\left(\mathbf{x} + \mathbf{u}^{(\lambda)} + \mathbf{t}^{(\lambda)}\right) - B(\mathbf{x})\right)^2$$

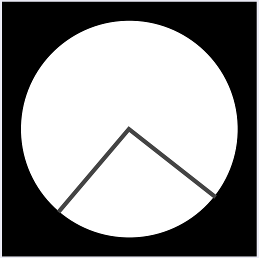
- Propagation to finer level $\lambda - 1$:

$$\mathbf{u}^{(\lambda-1)} = 2\left(\mathbf{u}^{(\lambda)} + \mathbf{t}^{(\lambda)}\right)$$

J.Y. Bouguet, Tech. Report, Intel Cooperation, 2000

Motivation

Sketch



Improvement

Tolerance against illumination changes

IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Refined IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = \alpha(\mathbf{p})B(\mathbf{p}) + \beta(\mathbf{p})$$

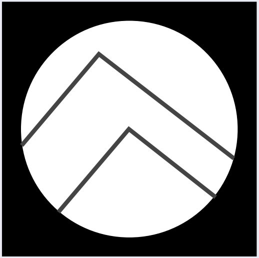
Brigthness change model

- Linear gray value changes
- α and β locally constant
- Compute $\mathbf{t} = [u_x, u_y, \alpha, \beta]^\top$

S. Negahdaripour, IEEE PAMI, 1998; H.W. Haussecker & D.J. Fleet, Proc. CVPR, 2000; Y. Altunbasak et.al., IEEE TIP, 2003; Y.H. Kim et.al., Img. Vis. Comp., 2005

Motivation

Sketch



Improvement

Tolerance against illumination changes

IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Refined IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = \alpha(\mathbf{p})B(\mathbf{p}) + \beta(\mathbf{p})$$

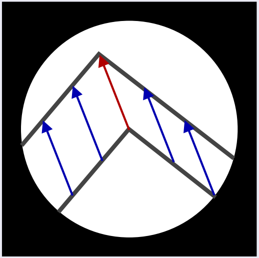
Brigthness change model

- Linear gray value changes
- α and β locally constant
- Compute $\mathbf{t} = [u_x, u_y, \alpha, \beta]^\top$

S. Negahdaripour, IEEE PAMI, 1998; H.W. Haussecker & D.J. Fleet, Proc. CVPR, 2000; Y. Altunbasak et.al., IEEE TIP, 2003; Y.H. Kim et.al., Img. Vis. Comp., 2005

Motivation

Sketch



Improvement

Tolerance against illumination changes

IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Refined IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = \alpha(\mathbf{p})B(\mathbf{p}) + \beta(\mathbf{p})$$

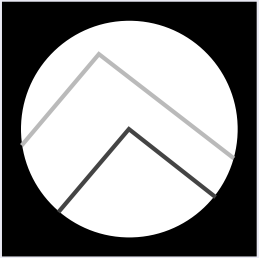
Brigthness change model

- Linear gray value changes
- α and β locally constant
- Compute $\mathbf{t} = [u_x, u_y, \alpha, \beta]^\top$

S. Negahdaripour, IEEE PAMI, 1998; H.W. Haussecker & D.J. Fleet, Proc. CVPR, 2000; Y. Altunbasak et.al., IEEE TIP, 2003; Y.H. Kim et.al., Img. Vis. Comp., 2005

Motivation

Sketch



Improvement

Tolerance against illumination changes

IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Refined IBCC

$$A(\mathbf{p} + \mathbf{u}(\mathbf{p})) = \alpha(\mathbf{p})B(\mathbf{p}) + \beta(\mathbf{p})$$

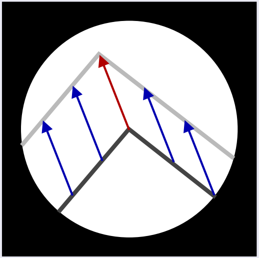
Brigthness change model

- Linear gray value changes
- α and β locally constant
- Compute $\mathbf{t} = [u_x, u_y, \alpha, \beta]^\top$

S. Negahdaripour, IEEE PAMI, 1998; H.W. Haussecker & D.J. Fleet, Proc. CVPR, 2000; Y. Altunbasak et.al., IEEE TIP, 2003; Y.H. Kim et.al., Img. Vis. Comp., 2005

Motivation

Sketch



Improvement

Tolerance against illumination changes

IBCC

$$A(\mathbf{p}+\mathbf{u}(\mathbf{p})) = B(\mathbf{p})$$

Refined IBCC

$$A(\mathbf{p}+\mathbf{u}(\mathbf{p})) = \alpha(\mathbf{p})B(\mathbf{p}) + \beta(\mathbf{p})$$

Brigthness change model

- Linear gray value changes
- α and β locally constant
- Compute $\mathbf{t} = [u_x, u_y, \alpha, \beta]^\top$

S. Negahdaripour, IEEE PAMI, 1998; H.W. Haussecker & D.J. Fleet, Proc. CVPR, 2000; Y. Altunbasak et.al., IEEE TIP, 2003; Y.H. Kim et.al., Img. Vis. Comp., 2005

Derivation

Error function (omitting x)

$$e(\mathbf{t}) = \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y]^\top \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A - \alpha B - \beta \right)^2$$

S. Negahdaripour, IEEE PAMI, 1998

Derivation

Partial derivatives (omitting x)

$$\frac{\partial e}{\partial u_x} = 2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y]^\top \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A - \alpha B - \beta \right) A_x$$

$$\frac{\partial e}{\partial u_y} = 2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y]^\top \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A - \alpha B - \beta \right) A_y$$

$$\frac{\partial e}{\partial \alpha} = -2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y]^\top \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A - \alpha B - \beta \right) B$$

$$\frac{\partial e}{\partial \beta} = -2 \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y]^\top \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A - \alpha B - \beta \right)$$

S. Negahdaripour, IEEE PAMI, 1998

Solution

Error function (omitting $\mathbf{x}$)

$$e(\mathbf{t}) = \sum_{\mathbf{x}\in\Omega} w^2 \left([A_x, A_y]^\top \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A - \alpha B - \beta \right)^2$$

Solution

$$\mathbf{t} = \mathbf{G}^{-1}\mathbf{b}$$

$$\begin{pmatrix} u_x \\ u_y \\ \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \langle A_x A_x \rangle & \langle A_x A_y \rangle & \langle -A_x B \rangle & \langle -A_x \rangle \\ \langle A_x A_y \rangle & \langle A_y A_y \rangle & \langle -A_y B \rangle & \langle -A_y \rangle \\ \langle -A_x B \rangle & \langle -A_y B \rangle & \langle BB \rangle & \langle B \rangle \\ \langle -A_x \rangle & \langle -A_y \rangle & \langle B \rangle & \langle -1 \rangle \end{pmatrix}^{-1} \begin{pmatrix} \langle -A_x A \rangle \\ \langle -A_y A \rangle \\ \langle AB \rangle \\ \langle -A \rangle \end{pmatrix}$$

S. Negahdaripour, IEEE PAMI, 1998

Discussion

Properties of **G**

- Full rank $\longrightarrow$ invertible
- Ill-conditioned
  - Areas of weak/regular texture
  - Multiple interpretations of motion and illuminance

Radiometric cues α and β

- Discontinuous at motion boundaries or occlusions
  $\Rightarrow$ errouneous estimates
  $\Rightarrow$ large residuals
- Physical interpretations possible:
  e.g. irradiance, surface normals, reflectance

S. Negahdaripour, IEEE PAMI, 1998; H.W. Haussecker & D.J. Fleet, Proc. CVPR, 2000

Motivation

Motivation

- Known camera motion
- Known object motion
- Flowfield “patterns”

Approach

Parameterize the flowfields:

$$\mathbf{u}(\mathbf{x}) = \mathbf{f}(\mathbf{x}, \mathbf{a})$$

Solve for **a**

Improvements

- More accurate flow fields
- Uses available knowledge about flow fields
- Derive motion parameters
- Flowfield segmentation

Motivation

Motivation

- Known camera motion
- Known object motion
- Flowfield “patterns”

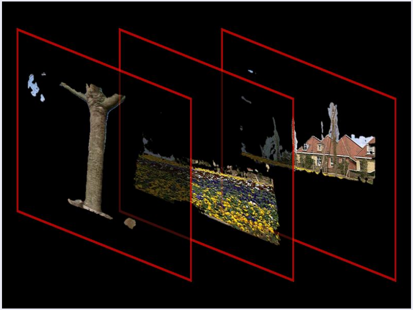
Approach

Parameterize the flowfields:

$$\mathbf{u}(\mathbf{x}) = \mathbf{f}(\mathbf{x}, \mathbf{a})$$

Solve for **a**

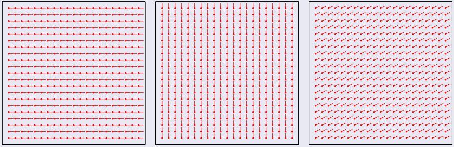
Example: motion segmentation



http://www-bcs.mit.edu/people/jyawang/demos/garden-layer/

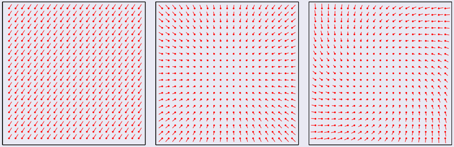
Motion models

Translational model



$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Affine model



$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} a_1 x + a_2 y + a_3 \\ a_4 x + a_5 y + a_6 \end{pmatrix}$$

Y. Altunbasak et.al., IEEE TIP, 2003

Solution

Refined image brightness constancy constraint

$$A(\mathbf{p} + \mathbf{f}(\mathbf{p}, \mathbf{a})) = B(\mathbf{p})$$

Example: Taylor approximation for affine model

$$\begin{aligned} A(\mathbf{p}) + A_x(\mathbf{p})(a_1 p_x + a_2 p_y + a_3) \\ + A_y(\mathbf{p})(a_4 p_x + a_5 p_y + a_6) = B(\mathbf{p}) \end{aligned}$$

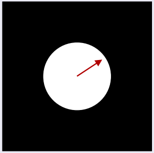
Properties and discussion

No interesting properties mentioned

Y. Altunbasak et.al., IEEE TIP, 2003

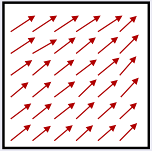
Local vs. global optical flow

Local optical flow



- "Lucas-Kanade"
- Sparse flow fields
- Analytical solution
- Parsimonious
- Robust against noise

Global optical flow



- "Horn-Schunck"
- Dense flow fields (fill-in)
- Iterative solution
- Computationally cheap
- Less robust

B.D. Lucas & T. Kanade, Proc. Im. Underst. Workshop, 1981; B.K.P. Horn & B.G. Schunck, AI, 1981; A. Bruhn et.al., Proc. DAGM, 2002; A. Bruhn et.al., IJCV, 2005

Horn-Schunck in a nutshell

Global error functional (omitting $\mathbf{x}$)

$$e(\mathbf{u}) = \sum_{\mathbf{x}\in\Gamma} \left(\underbrace{(A_x u_x + A_y u_y + A_t)^2}_{\text{IBCC}} + \varphi \underbrace{\left(|\nabla u_x|^2 + |\nabla u_y|^2 \right)}_{\text{Divergence}} \right)$$

Smoothness weight $\varphi > 0$, Γ is the whole image

Solution

Compute optimal $\mathbf{u}$ which minimizes $e(\mathbf{u})$

1. Euler-Lagrange mechanism $\Rightarrow$ system of equations
2. Numerical solution (Gauss-Seidel, SOR, ...)

A. Bruhn et.al., Proc. DAGM, 2002; A. Bruhn et.al., IJCV, 2005

Lucas-Kanade meets Horn-Schunck

Horn-Schunck error functional

$$e(\mathbf{u}) = \sum_{\mathbf{x}\in\Gamma} \left((A_x u_x + A_y u_y + A_t)^2 + \varphi\left(|\nabla u_x|^2 + |\nabla u_y|^2\right) \right)$$

Combined error functional

$$e(\mathbf{u}) = \sum_{\mathbf{x}\in\Gamma} \left(\sum_{\mathbf{x}'\in\Omega} w^2 \left([A_x, A_y] \begin{pmatrix} u_x \\ u_y \end{pmatrix} + A_t \right)^2 + \dots \right.$$
$$\left. \varphi\left(|\nabla u_x|^2 + |\nabla u_y|^2\right) \right)$$

A. Bruhn et.al., Proc. DAGM, 2002; A. Bruhn et.al., IJCV, 2005

Lucas-Kanade meets Horn-Schunck

Combined error functional

$$e(\mathbf{u}) = e_{\mathrm{LKHS}}(\mathbf{u}) + \varphi e_{\mathrm{R}}(\mathbf{u})$$

Euler-Lagrange solutions

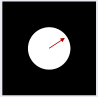
$$0 = \Delta u_x - \frac{1}{\varphi}\left(\langle A_x A_x \rangle u_x + \langle A_x A_y \rangle u_y + \langle A_x A_t \rangle\right)$$
$$0 = \Delta u_y - \frac{1}{\varphi}\left(\langle A_x A_y \rangle u_x + \langle A_y A_y \rangle u_y + \langle A_y A_t \rangle\right)$$

Laplaceans Δu_x, Δu_y
Discrete approximation
Standard solvers for sparse sets of linear equations (SOR)

A. Bruhn et.al., Proc. DAGM, 2002; A. Bruhn et.al., IJCV, 2005

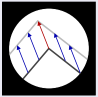
Limitations and possible improvements

Standard LK



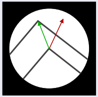
- baseline
- most efficient

Illumination Tolerance



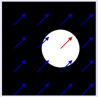
- illumination changes

Iterated LK



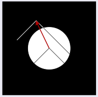
- more accurate

LK with flowfield constraints



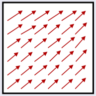
- types of flowfields

Multiscale LK



- large displacements

Global LK



- global flow computation

Results?!?

Unifying review

- Does not exist

Reviewing myself

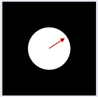
- Different data sets
- Unclear parameters
- Missing visualizations for parameters
- From some papers only a small subset was presented
- Quality of figures

Benchmarking myself

- Availability of implementations
- Costs and benefits?

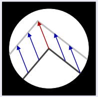
Relevance for my work

Standard LK



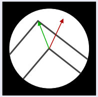
- tested
- 12° to 15°

Illumination Tolerance



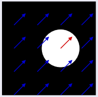
- TODO

Iterated LK



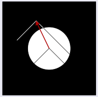
- tested
- 8° to 11°

LK with flowfield constraints



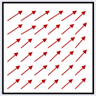
- motion seg.?
- parameters?

Multiscale LK



- tested
- 5° to 8°

Global LK



- role of fill-in effects?

A. Vardy & R. Möller, Conn. Sci., 2005; L. Gerstmayr, Tech. Report, Uni Bielefeld, 2007