HEAVY BARYONS:
A COMBINED LARGE N_c AND HEAVY QUARK
EXPANSION FOR ELECTROWEAK CURRENTS

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The combined large N_c and heavy quark limit for baryons containing a single heavy quark is discussed. The combined large N_c and heavy quark expansion of the heavy quark bilinear operators is obtained. In the combined expansion the corrections proportional to m_N/m_Q are summed to all orders. In particular, the combined expansion can be used to determine semileptonic form factors of heavy baryons in the combined limit.

1 Introduction

The hadrons are strongly bound states of quarks and gluons. Their description directly from QCD—the underlying theory of strong interactions—is a daunting task significantly impeded by the highly non-perturbative nature of QCD at low energies. Lattice QCD is the only fully non-perturbative framework at the present time to calculate the low energy hadronic observables from QCD. Its present capabilities, however, are at a rudimentary level to perform fully dynamical (non-quenched) calculations of interesting QCD and electroweak observables. In the absence of the direct analytical methods, we turn to the approximations which lead to model independent predictions.

A popular approach to describe systems at low-energies in a model independent way is to use the methods of effective field theories (EFT) [1]. These methods are most successful when the spectrum of the underlying theory exhibits a large enough scale separation so that the low-energy interactions of relevant degrees of freedom can be described in terms of a small number of operators with the remainder being suppressed by higher powers of the ratio E/Λ, where E and Λ are typical low-energy and high-energy scales. A power counting scheme—a critical ingredient of EFT—selects all dominant interactions at a given order in the expansion in powers of E/Λ. A finite number of undermined constants at each order can be found from experiment and used to predict the outcome of others.

The symmetries of the system of interest greatly constrain model-independent power counting schemes. It is often the case that such symmetries are approximate and manifest themselves in a particular limit of QCD. An example is the

well-known chiral symmetry in the light-quark sector of QCD which appears in the zero quark mass limit. Chiral perturbation theory—an effective theory describing interactions of pions and nucleons—is based on power counting emerging from the underlying chiral symmetry.

Another example is heavy quark effective theory (HQET)[2] used to describe hadrons containing a single heavy quark—the *heavy mesons* and *heavy baryons*. HQET is based on the heavy quark spin-flavor symmetry which emerges in the heavy quark limit, *i.e.* when the heavy quark mass, m_Q, goes to infinity. The heavy quark effective Lagrangian has a form of an expansion in powers of k/m_Q, where k ($\mathcal{O}(\Lambda_{QCD})$) is a typical momentum of the light degrees of freedom inside the heavy hadron. To stress the non-perturbative nature of these degrees of freedom they are often collectively referred to as the *brown muck*.

The heavy quark spin-flavor symmetry relates the heavy hadrons moving at the same 4-velocity. The symmetry is described by the unitary transformations corresponding to $SU(2\,n_f)$ group, where n_f is the number of heavy flavors. This symmetry implies a number of important phenomenological consequences. In the heavy baryon sector of QCD, which is of the primary interest here, the heavy quark symmetry restricted to charm and bottom flavors predicts, for example, an existence of the degenerate heavy quark spin doublets. An approximately degenerate doublet of the excited charm baryons has been observed—$(\Lambda_{c1}, \Lambda_{c1}^*)$. The symmetry breaking is of order of $1/m_Q$ as expected. Heavy quark symmetry also leads to the relation between electroweak matrix elements that determine the semileptonic decay form factors of heavy baryons. In particular, the normalization of these form-factors at zero recoil is given by the conserving heavy quark number[3].

Here we will discuss a combined heavy quark and large N_c limit for the heavy baryon sector of QCD. It turns out, in this limit the spectrum of heavy baryons exhibits an approximate symmetry associated with a contracted $O(8)$ group[4,5]. This symmetry connects the low-energy excited states of heavy baryons to the ground state. An effective field theory can be derived to describe these low-energy excited states and their electroweak decays[6].

Large N_c QCD[7] is a very useful framework to obtain model independent predictions for baryons[8]. In particular, the baryons containing only u and d quarks in the large N_c exhibit light quark spin-flavor symmetry described by the contracted $SU(4)$ group[9]. An infinite dimensional irreducible representation of this symmetry is given by baryons with the spin-isospin quantum numbers $I = J = 1/2, 3/2, 5/2$, etc. The two lowest states $I = J = 1/2, 3/2$ correspond to the large N_c analogs of the nucleon and Δ. The baryons in the large N_c limit arise as solitons of the non-linear chiral Lagrangian[8].

2 The Bound State Picture

The attractiveness of the combined heavy quark and large N_c limit can be seen in a simple model, or more precisely, the type of models that have been considered in the past [10]. In these models the heavy baryon is thought of as the bound state of a heavy meson and an ordinary baryon. For example, the Λ_c baryon is thought of as bound state of D or D^* meson and a nucleon. As both m_Q and N_c tend to infinity, the difference between the heavy quark mass, m_Q, and the heavy meson mass, m_H, as well as the difference between the brown muck mass and the nucleon mass, m_N, are suppressed by powers of $1/N_c$ and $1/m_Q$.

The combined limit is taken in such a way as to hold the ratio $N_c\Lambda/m_Q$ constant but arbitrary (here Λ represents a typical hadronic scale of order of $1\,GeV$). In other words, the results are independent of the order in which two limits are taken. The corrections to the limiting values are given in powers of small dimensionless parameter λ,

$$\lambda \sim \frac{\Lambda}{m_Q}\,,\,\frac{1}{N_c}\,. \tag{1}$$

In the combined heavy quark and large N_c limit both m_Q and m_N go to infinity. On the other hand, as shown by Witten [8], the meson-baryon interaction is of order of N_c^0. As a result, the two heavy particles, the heavy meson and a nucleon, are bound by the potential of order unity. The lowest excitations of such a system correspond to excitations of the three dimensional harmonic oscillator with the *spring constant* κ of order λ^0 and mass $\mu = m_N m_Q/(m_N + m_Q) \sim \lambda^{-1}$. The energy splitting between the low-lying excited states is $(\kappa/\mu)^{1/2}$ which is of order $\lambda^{1/2}\Lambda_{QCD}$. Hence, as λ goes to infinity in the combined limit, the low-lying states become degenerate and exhibit a new symmetry. This symmetry is described by a contracted $O(8)$ group.

This symmetry has been shown to arise in a model-independent way in the heavy baryon sector in the combined limit [4,5]. This symmetry can be used to construct an effective theory in the combined limit which describes low-energy excited states of heavy baryons [6]. The dominant excitations in the combined limit are the harmonic modes of the collective motion of the brown muck as whole relative to the heavy quark. These excitations are of order $\lambda^{1/2}\Lambda_{QCD}$ while the excitations of the brown muck (*e.g.* the rotation in the isospin space) are of order $\lambda^0\Lambda_{QCD}$.

The effective Hamiltonian can be derived from QCD using the operators which excite the collective motions of the brown muck relative to the heavy

quark. It turns out that the effective Hamiltonian has the form of an expansion in powers of $\lambda^{1/2}$. The leading non-trivial terms exhibit a contracted $O(8)$ symmetry and contain one undetermined parameter corresponding to a *spring constant* κ. At next-to-leading order an additional parameter must be fit to experiment. Additional observables can be obtained from the effective expansion of the heavy baryon electroweak currents. This does not introduce any new constants at leading and next-to-leading order in $\lambda^{1/2}$.

The effective theory can be used to determine a number of heavy baryon observables, namely masses of the excited states, semileptonic form factors and electromagnetic decay rates. At leading order these predictions coincide with corresponding ones obtained from bound state models. The corrections at next-to-leading order are expected to be $\mathcal{O}(1/N_c)$.

Here we will describe a method to obtain a combined heavy quark and large N_c expansion of the heavy baryon electroweak current needed to calculate semileptonic form factors[6].

3   A Combined Expansion of the Heavy Quark Currents

In HQET the $1/m_Q$ corrections include those that are proportional to m_N/m_Q. These corrections are suppressed in HQET. However, in the combined limit m_N is formally of the same order as m_Q, so that the convergence of the heavy quark expansion is spoiled. Note, that the ratio m_N/m_Q is approximately $1/2$ for charmed meson, which is not much smaller unity. This may have some phenomenological consequences. In fact, the effective theory in the combined limit predicts the ratio of the charm to bottom electromagnetic decay rates to be approximately 0.2 at leading order while the leading order HQET value is unity[11,12]. It is desirable, therefore, to improve the convergence of the effective expansion in the combined limit.

The effective $1/m_Q$ expansion in HQET is obtained by integrating out the types of the heavy quark interactions that are $1/m_Q$ suppressed relative to the leading terms. One effect corresponds to the heavy quark spin interactions with the chromomagnetic field. The suppression is made explicit by splitting the heavy quark field in two parts using projection operators $P_\pm = (1\pm\not{v})/2$. The lower components of the heavy quark field are $1/m_Q$ suppressed. In the combined heavy quark and large N_c limit such suppression holds as well.

Another suppressed effect comes from the off-shell momentum of the heavy quark due to its interaction with the brown muck, $k = p - m_Q v \sim \Lambda_{QCD}$, where p and v are the heavy quark 4-momentum and 4-velocity. In the heavy quark limit p and v are almost equal to the total 4-momentum and 4-velocity of the heavy hadron. In the combined limit, however, the brown muck contribution

to the total 4-momentum is formally of the same order as that of the heavy quark since $m_N \sim m_Q \sim \lambda^{-1}$. Thus, it is more appropriate to define the off-shell momentum of the heavy quark as $k = P - (m_Q + m_N)v \sim \Lambda_{QCD}$, where P is the total 4-momentum of the heavy hadron.

The suppression of the heavy quark off-shell fluctuations is made explicit in HQET by redefining a phase of the heavy quark field:

$$h_Q(x) = e^{im_Q v \cdot x} P_+ Q(x), \quad H_Q(x) = e^{im_Q v \cdot x} P_- Q(x) \tag{2}$$

where $Q(x)$ is the total heavy quark field and v is the 4-velocity of the heavy hadron. The operators $P_\pm = (1 \pm \slashed{v})/2$ project the total heavy quark field in two parts. The $H_Q(x)$ component is suppressed by $1/m_Q$ relative to $h_Q(x)$ which can be seen from equations of motions written in terms of these fields.

The typical 4-momentum carried by the field $h_Q(x)$ in the heavy hadron state is $k \sim M - m_Q$, where M is the total mass of the hadron, so that in HQET k is $\mathcal{O}(\Lambda_{QCD}/m_Q)$. However, in the combined limit $k \sim m_N \sim \lambda^{-1}$ and is not suppressed. In order to remove the brown muck contribution to the off-shell momentum one can redefine the effective fields $h_Q(x)$ and $H_Q(x)$ by changing their phases:

$$h_Q(x) \to h_Q^\lambda(x) = e^{i(m_Q + m_N)v \cdot x} P_+ Q(x),$$
$$H_Q(x) \to H_Q^\lambda(x) = e^{i(m_Q + m_N)v \cdot x} P_- Q(x), \tag{3}$$

where a superscript λ indicates that effective fields $h_Q^\lambda(x)$ and $H_Q^\lambda(x)$ are used in the combined expansion. The field $h_Q^\lambda(x)$ defined in Eq. (3) carries a typical off-shell 4-momentum $k \sim M - (m_Q + m_N) \sim \lambda^0 \Lambda_{QCD}$ which is suppressed in the combined limit.

The phase redefinition in Eq. (3) limits the off-shell fluctuations of the effective heavy quark fields $h_Q^\lambda(x)$ and $H_Q^\lambda(x)$. However, it can potentially spoil relative scaling of the fields $h_Q^\lambda(x)$ and $H_Q^\lambda(x)$. Indeed, using the equations of motion that follow from the heavy quark Lagrangian, $\mathcal{L}_Q = \bar{Q}(x)(i\slashed{D} - m_Q)Q(x)$, one sees that $H_Q(x) \sim (1/m_Q) h_Q(x)$ while $H_Q^\lambda(x) \sim (m_N/m_Q) h_Q^\lambda(x)$. Thus, it seems that new definition, Eq. (3), prevents the consistent elimination of $H_Q^\lambda(x)$ in terms of $h_Q^\lambda(x)$. This seeming inconsistency is resolved, however, by noting that in the combined limit the equations of motions for $h_Q^\lambda(x)$ and $H_Q^\lambda(x)$ should be obtained not from $\mathcal{L}_Q$ but from an effective Lagrangian that takes into account the large contribution of the brown muck.

What is this Lagrangian? The total Lagrangian has the form:

$$\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_q + \mathcal{L}_{YM}, \tag{4}$$

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where $\mathcal{L}_q$ is the light quark Lagrangian and $\mathcal{L}_{YM}$ is the Yang-Mills Lagrangian for gluon fields. In the combined limit $\mathcal{L}_q + \mathcal{L}_{YM}$ contains a large piece corresponding to the brown muck mass m_N. This contribution can be made explicit by adding and subtracting from the total Lagrangian an effective term $m_N \bar{Q} \slashed{v} Q$, which contains only the heavy quark fields:

$$\mathcal{L} = (\mathcal{L}_Q - m_N \bar{Q} \slashed{v} Q) + (\mathcal{L}_q + \mathcal{L}_{YM} + m_N \bar{Q} \slashed{v} Q). \tag{5}$$

The operator $\bar{Q} \slashed{v} Q$ in the heavy baryon reference frame counts the number of heavy quarks.

To obtain the correct scaling of $h_Q^\lambda(x)$ and $H_Q^\lambda(x)$ one needs to use equations of motion obtained from an effective Largangian, $\mathcal{L}_{eff} = \mathcal{L}_Q - m_N \bar{Q} \slashed{v} Q$. Indeed, it can be shown that in this case H_Q^λ is $\mathcal{O}(\lambda h_Q^\lambda)$ in the combined limit. As a result, $H_Q^\lambda(x)$ can be consistently eliminated from the effective Lagrangian, $\mathcal{L}_{eff}$, and heavy quark operators at each order in the combined expansion. In fact, the resulting expansion of the heavy quark current has the same form as the corresponding expansion in HQET:

$$\bar{c} \, \Gamma \, b = \bar{h}_c^\lambda \, \Gamma \, h_b^\lambda + \frac{1}{2m_c} \, \bar{h}_c^\lambda \, \Gamma \, (i \slashed{D}) \, h_b^\lambda - \frac{1}{2m_b} \, \bar{h}_c^\lambda \, \Gamma \, (i \overleftarrow{\slashed{D}}) \, h_b^\lambda + \mathcal{O}(\lambda^2), \tag{6}$$

where Γ gives a Dirac structure of the current.

The first term in Eq. (6) contributes $\mathcal{O}(\lambda^0)$ to the heavy baryon matrix elements while the two other terms give $\mathcal{O}(\lambda)$ contributions. This happens because the covariant derivative brings down one power of k which is $\mathcal{O}(\lambda^0)$ if the field $h_Q^\lambda(x)$ is defined as in Eq. (3). In other words, the corrections proportional to m_N/m_Q are summed to all orders.

The expansion in Eq. (6) can be used to determine the semileptonic form factors of the heavy baryons in the combined limit. The effective Hamiltonian in the combined limit has a form of expansion in powers of $\lambda^{1/2}$. Hence, at leading and next-to-leading order only the first term in Eq. (6) contributes to the electroweak matrix elements. The matrix elements of this leading operator can be determined in the combined limit. They give the leading contributions to the semileptonic form factors Θ and Ξ defined by [11,12]:

$$< \Lambda_c(\vec{v}') \, | \, \bar{c} \, \gamma^\mu (1 - \gamma_5) \, b \, | \, \Lambda_b(\vec{v}) > \, = \, \Theta \, \bar{u}_c \, \gamma^\mu (1 - \gamma_5) \, u_b \, (1 + \mathcal{O}(\lambda^{3/2})), \tag{7}$$

$$< \Lambda_{c1}(\vec{v}') \, | \, \bar{c} \, \gamma^\mu (1 - \gamma_5) \, b \, | \, \Lambda_b(\vec{v}) > \, = \, \sqrt{3} \, \Xi \, \bar{u}_c \, \gamma^\mu (1 - \gamma_5) \, u_b \, (1 + \mathcal{O}(\lambda^{3/2})),$$

$$< \Lambda_{c1}^*(\vec{v}') \, | \, \bar{c} \, \gamma^\mu (1 - \gamma_5) \, b \, | \, \Lambda_b(\vec{v}) > \, = \, \Xi \, \bar{u}_{c\nu} \, (\sigma^{\nu\mu} \gamma_5 - g^{\mu\nu}) \, u_b \, (1 + \mathcal{O}(\lambda^{3/2})),$$

where $(\Lambda_{c1}, \Lambda_{c1}^*)$ is a doublet of the first excited state of Λ_c with $J = 1/2$ and $J = 3/2$; $u_{c\nu}$ is a Rarita-Schwinger spinor normalized by $\bar{u}_{c\nu} \, u_c^\nu = -1$. Equations (8) are valid for the velocity transfers $|\vec{v}' - \vec{v}|^2$ of order $\lambda^{3/2}$.

6

4 Conclusion

We have discussed the way to obtain the combined heavy quark and large N_c expansion for the heavy baryon electroweak currents. The definition of the effective heavy quark fields differs from the corresponding definition in HQET by a phase factor (Eq. (3)) proportional to the brown muck mass. The latter is suppressed in HQET but formally is of the same order as the heavy quark mass in the combined limit. In the combined expansion the m_N/m_Q corrections are summed to all orders.

Acknowledgments

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