Comment on : C,PT,CPT invariance of pseudo-Hermitian Hamiltonians [Z.Ahmed arXiv:quant-ph/0302141v1]

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We notice that the un-normalised wave functions reflected in the paper fail to satisfy the appropriate eigevalue relations . Further the C-symmetry operator presented in the paper also fails to satisfy the appropriate commutation relationi.e $[H, C] \neq 0$. However we correctely represent above points.

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Standard quantum mechanics based on Hermitian operator took a new turn after the work of Bender,Brody and Jones [1] ,who introduced the concept of C in non-Hermitian quantum mechanics . Even though the model looks simple but is very thought provoking . In fact its correct use can give many new concepts. Similarly it is also very difficult to visualise its incorrect approach[2]. However , its[1] iso-spectral nature is also very interesting ,which can be presented in a different way using a (2x2) matrix as [3,4]

$$H^{ZPT} = \begin{bmatrix} a-c & bi \\ bi & a+c \end{bmatrix} \tag{1}$$

where it understood that a,b,c are real numbers/variables . The un-normalised wave functions discussed earlier by Zafar [4] are the following Let us consider two simple cases as follows

$$\psi_- = \begin{bmatrix} 1 \\ -ir \end{bmatrix} \tag{2}$$

and

$$\psi_+ = \begin{bmatrix} 1 \\ -i/r \end{bmatrix} \tag{3}$$

where $r = \frac{c+\sqrt{c^2-b^2}}{b}$[3] The eigenvalues of this non-hermian operator are

$$E_+ = a + br - c = a + \sqrt{c^2 - b^2} \tag{4}$$

and

$$E_- = a - \sqrt{c^2 - b^2} \tag{5}$$

Interested reader will notice that the above wave functions do not satisfy the eigenvalue relation as

$$H|\psi_- > \neq E_-|\psi_- > \tag{6}$$

More explicitly , this is due to

$$\frac{b}{r} + c \neq \sqrt{c^2 - b^2} \tag{7}$$

Similarly

$$H|\psi_- > \neq E_-|\psi_- > \tag{8}$$

More explicitly , this is due to

$$\frac{b}{r} - c \neq -\sqrt{c^2 - b^2} \tag{9}$$

However we find the two un-normalised wave functions as

$$\phi_- = \begin{bmatrix} \sqrt{R_+} \\ -i\sqrt{R_-} \end{bmatrix} \tag{10}$$

and

$$\phi_+ = \begin{bmatrix} i\sqrt{R_-} \\ \sqrt{R_+} \end{bmatrix} \tag{11}$$

where $R_\pm = \frac{c\pm\sqrt{c^2-b^2}}{2c}$ Interested reader will notice that the present un-normalised wave functions satisfy the eigenvalue relation as

$$H|\phi_\mp >= E_\mp|\phi_\mp > \tag{12}$$

Similarly we also notice that C-symmetry operator (presented earlier [4])

$$C^Z = \begin{bmatrix} 0 & -i/r \\ ir & 0 \end{bmatrix} \tag{13}$$

does not commute with the hamiltonian matrix as

$$[H, C^Z] =\neq 0 \tag{14}$$

However , we present the correct form of C-symmetry operator[4,5]

$$C^B = \frac{1}{\sqrt{c^2-b^2}} \begin{bmatrix} c & -ib \\ -ib & -c \end{bmatrix} \tag{15}$$

which commutes with the Hamiltonian matrix as

$$[H, C^B] = 0 \tag{16}$$

Further it is easy to check the following $\lambda_{C^B} = \pm 1$ and $(C^B)^2 = 1$. In conclusion we rectify the iso-spectral PT-symmetry operator discussed earlier by Zafar [4] some time back in view of C-symmetry importance in quantum mechanics of non-hermitian operators [5,6].

References

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