# National Institute of Technology, Durgapur
# Department of Electrical Engineering
# Network Analysis and Synthesis Laboratory
# (EES-451)

Experiment no: EES/451/01

**Title**: **Transient Response of RC and RL Circuit.**

**Objective:**

1. Study the transient response of a series RC circuit and understand the time constant concept using pulse waveforms.

**Theory:**

In this experiment, we apply a pulse waveform to the RC circuit to analyse the transient response of the circuit. The *pulse-width* relative to a circuit's *time constant* determines how it is affected by an RC circuit.

***Time Constant*** **(τ)**: A measure of time required for certain changes in voltages and currents in RC and RL circuits. Generally, when the elapsed time exceeds five time constants (5τ) after switching has occurred, the currents and voltages have reached their final value, which is also called steady-state response.

The time constant of an RC circuit is the product of equivalent capacitance and the Thévenin resistance as viewed from the terminals of the equivalent capacitor.

$\tau = RC$--------------- (1)

***A Pulse*** is a voltage or current that changes from one level to the other and back again. If a waveform's hight time equals its low time, as in figure, it is called a *square wave*. The length of each cycle of a pulse train is termed its *period* (**T**).

The *pulse width* ($t_p$) of an ideal square wave is equal to half the time period.

The relation between pulse width and frequency is then given by, f= $\frac{1}{2t_p}$

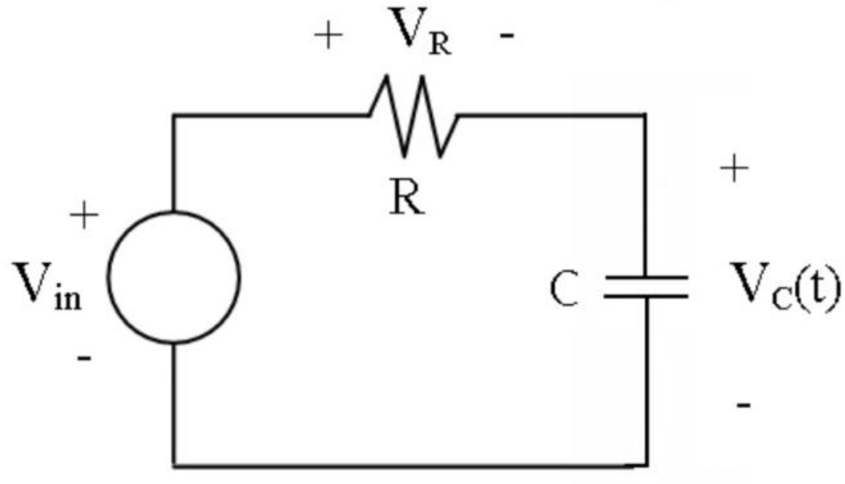



**Figure 1: Series RC circuit.**

From Kirchoff's laws, it can be shown that the charging voltage $\mathbf{V_C(t)}$ across the capacitor is given by: $V_C(t) = V(1 - e^{-t/RC})$ $t \geq 0$ (3) where, V is the applied source voltage to the circuit for $t \geq 0$. $RC = \tau$ is the time constant. The response curve is increasing and is shown in Figure 2.

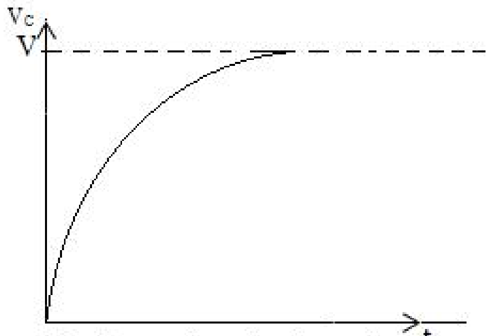


Fig -2 capacitor charging voltage

The discharge voltage for the capacitor is given by: $V_C(t) = V_o e^{-t/RC}$ $t \geq 0$ ------- (4), Where $V_o$ is the initial voltage stored in capacitor at t = 0, and $RC = \tau$ is time constant. The response curve is a decaying exponentials as shown in Figure 3.

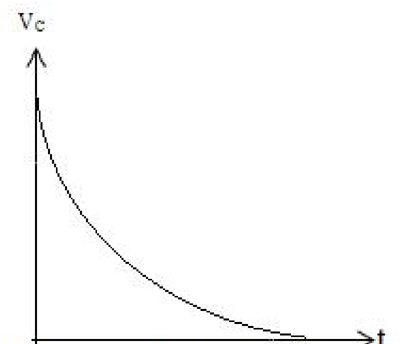


Fig-3 capacitor discharging voltage

**Procedure:**

1. Set up the circuit shown in **Figure 4** with the component values R = 2 KΩ and C = 1 μF and switch on the ELVIS board power supply.

2. Select the Function Generator from the NI - ELVIS Menu and apply a $4V_{p-p}$ square wave as input voltage to the circuit using the amplitude control on the FGEN.

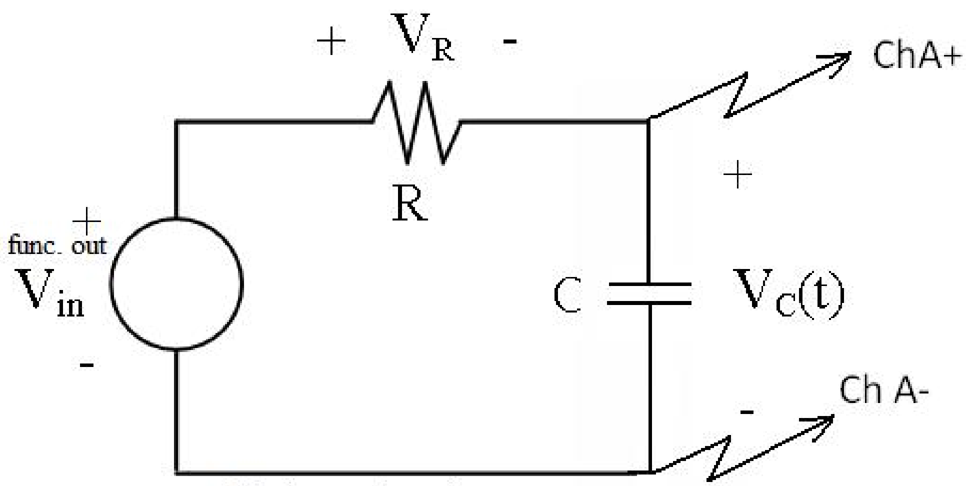



Fig-4. experimental setup

3. Open the Function Generator and Oscilloscope from the NI - ELVIS Menu. Set the Source on Channel A, Source on Channel B, Trigger and Time base input boxes as shown in figure 5 below

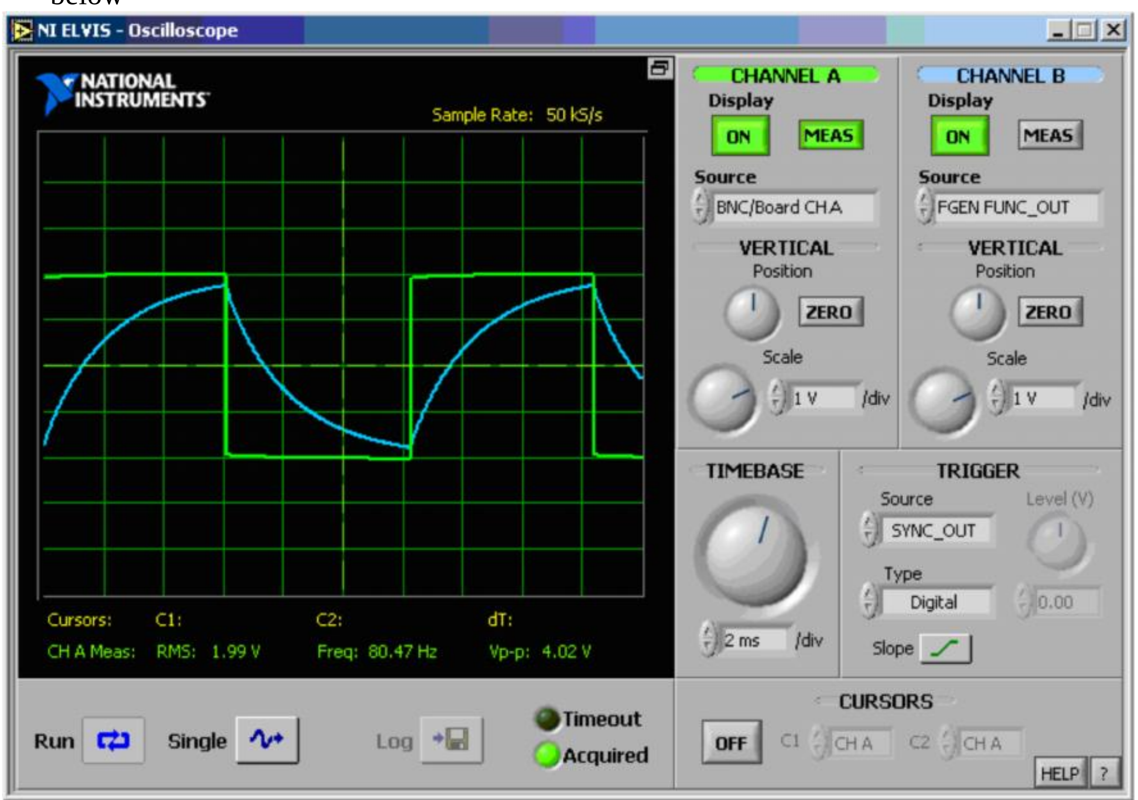



**Figure 5: Oscilloscope Configuration**

This configuration allows the oscilloscope to look at the output of the circuit on channel A, output of the function generator on channel B. Make sure you have clicked on the Run button of the FGEN panel and on the OSC panel. Any settings on the FGEN panel cause changes on the oscilloscope window.

4. Observe the response of the circuit for the following three cases and record the results.
   a. $\mathbf{t_p >> 5\tau}$ **:** Set the frequency of the FGEN output such that the capacitor has enough time to fully charge and discharge during each cycle of the square wave. So Let $t_p = 15\tau$ and accordingly set the FGEN frequency using equation (2). The value you have found should be approximately 17 Hz. Determine the time constant from the waveforms obtained on the OSC panel if you can. If you can not obtain the time constant easily, explain possible reasons.

b. $\mathbf{t_p = 5\tau}$ **:** Set the frequency such that $t_p = 5\tau$ (this should be 50 Hz)**.** Since the pulse width is exactly 5τ, the capacitor should just be able to fully charge and discharge during each pulse cycle. From the figure determine τ (see Figure 2 and Figure 6 below.)

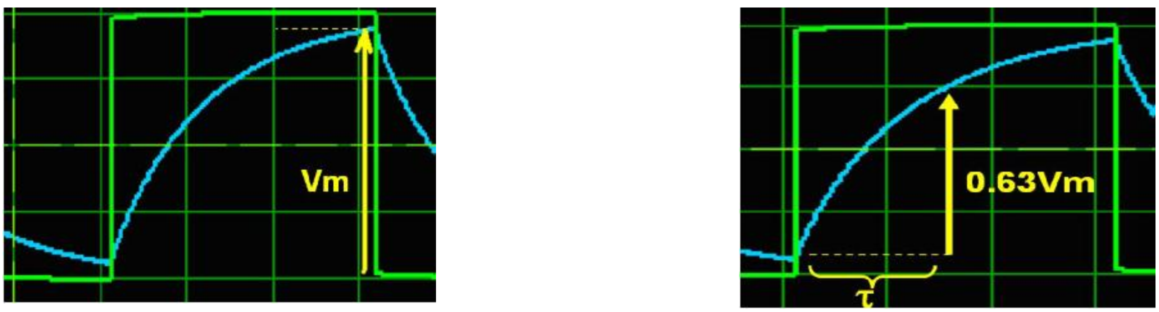



**Figure 6: Measuring the time constant τ approximately by counting the number of squares.**

c. $\mathbf{t_p << 5\tau}$ **:** In this case the capacitor does not have time to charge significantly before it is switched to discharge, and vice versa. Let $t_p = 0.5\tau$ in this case and set the frequency accordingly

5. Repeat the procedure using R = 100 KΩ and C = 0.01 μF and record the measurements

**Questions for Lab Report:**

1. Calculate the time constant using equation (1) and compare it to the measured value from 4b. Repeat this for other set of R and C values.

2. Discuss the effects of changing component values.

## Transient Response of an RL Circuit

**Objective:**

**2.** Study the transience due to inductors using a series RL circuit and understand the time constant concept.

**Theory:**

This lab is similar to the RC Circuit Lab except that the Capacitor is replaced by an Inductor. In this experiment, we apply a square waveform to the RL circuit to analyse the transient response of the circuit. The *pulse-width* relative to the circuit's *time constant* determines how it is affected by the RL circuit.

***Time Constant*** **(τ)**: It is a measure of time required for certain changes in voltages and currents in RC and RL circuits. Generally, when the elapsed time exceeds *five time constants* (5τ) after switching has occurred, the currents and voltages have reached their final value, which is also called steady-state response.
The time constant of an RL circuit is the equivalent inductance divided by the Thévenin resistance as viewed from the terminals of the equivalent inductor.
$\tau = L / R$ (1)
A ***Pulse*** is a voltage or current that changes from one level to the other and back again. If a waveform's hight time equals its low time, as in figure, it is called a *square wave*. The length of each cycle of a pulse train is termed its *period* (T). The *pulse width* ($t_p$) of an ideal square wave is equal to half the time period.
The relation between pulse width and frequency for the square wave is given by: $f = \frac{1}{2t_p}$

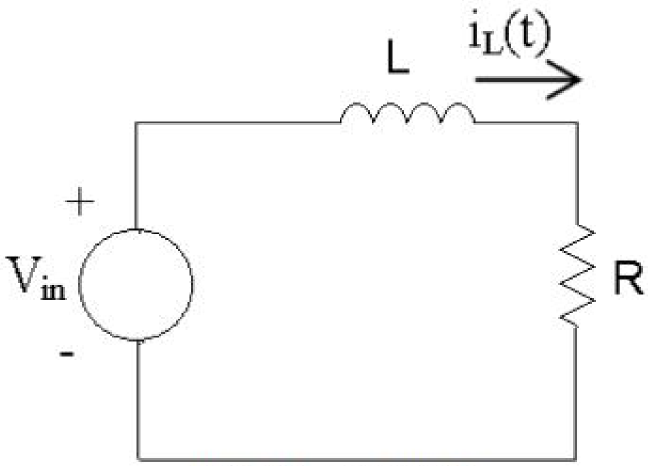


Fig-1, R-L series circuit

In an R-L circuit, voltage across the inductor decreases with time while in the *RC* circuit the voltage across the capacitor increased with time. Thus, *current* in an RL circuit has the same form as voltage in an RC circuit: they both rise to their final value exponentially according to $1 - e^{-t/\tau}$.
The expression for the current build-up across the Inductor is given by

$i_L(t) = RV(1 - e^{-(R/L)t})\ t \geq 0$-------- (3) ,where, *V* is the applied source voltage to the circuit for t ≥ 0. The response curve is increasing and is shown in figure 2.

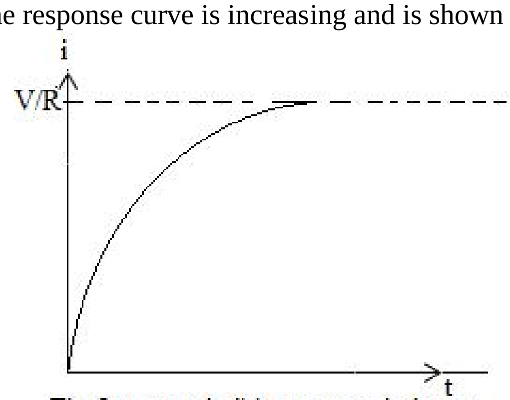


Fig-2,current build up across inductor

The expression for the current decay across the Inductor is given by:

$i_L(t) = i_0 e^{-(R/L)t}$ $t \geq 0$ ----------(4) ,where, $i_0$ is the initial current stored in the inductor at t = 0
L/R = τ is time constant.

The response curve is a decaying exponential and is shown in figure 3.

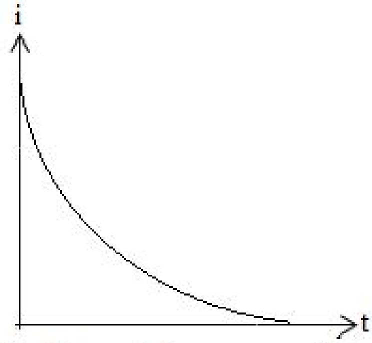


Fig-3,current decay across inductor

Since it is not possible to directly analyse the current through Inductor on a Scope, we will measure the output voltage across the Resistor. The resistor waveform should be similar to inductor current as $V_R = I_L R$. From the resistor voltage on the scope, we should be able to measure the time constant τ which should be equal to $\tau = L / R_{total}$ .
Here, $R_{total}$ is the total resistance and can be calculated from $R_{total} = R_{inductance} + R$.
$R_{inductance}$ is the measured value of inductor resistance and can be measured by connecting inductance to an ohm-meter prior to running the experiment.

**Procedure:**

1. Measure the inductor resistance by connecting its terminals to an Ohmmeter. You can use NI-Elvis Ohm-meter by connecting it into Current Hi and Current Low inputs on your board.
2. Set up the circuit shown in **Figure 4** with the component values R = 1kΩ and L = 33mH (or 150 mH if you are provided) and switch on the ELVIS board power supply.

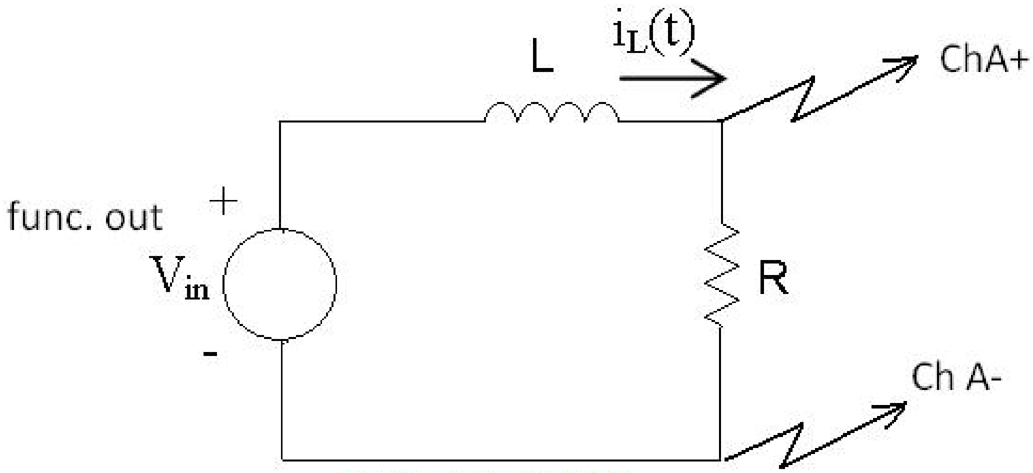



Fig-4, experimental setup

3. Select the Function Generator from the NI - ELVIS Menu and apply a $4V_{p\text{-}p}$ square wave as input voltage to the circuit using the amplitude control on the FGEN. Calculate the applied frequency using equation **(2)** for $\mathbf{t_p} = \mathbf{5}\tau$

4. Select the Oscilloscope from the NI - ELVIS Menu. Set the resistor voltage on Channel A, Source on Channel B, Trigger and Time base input boxes as shown in figure 4 below. (Note your output may not be different from the one in the figure- figure just for demonstration purposes.)

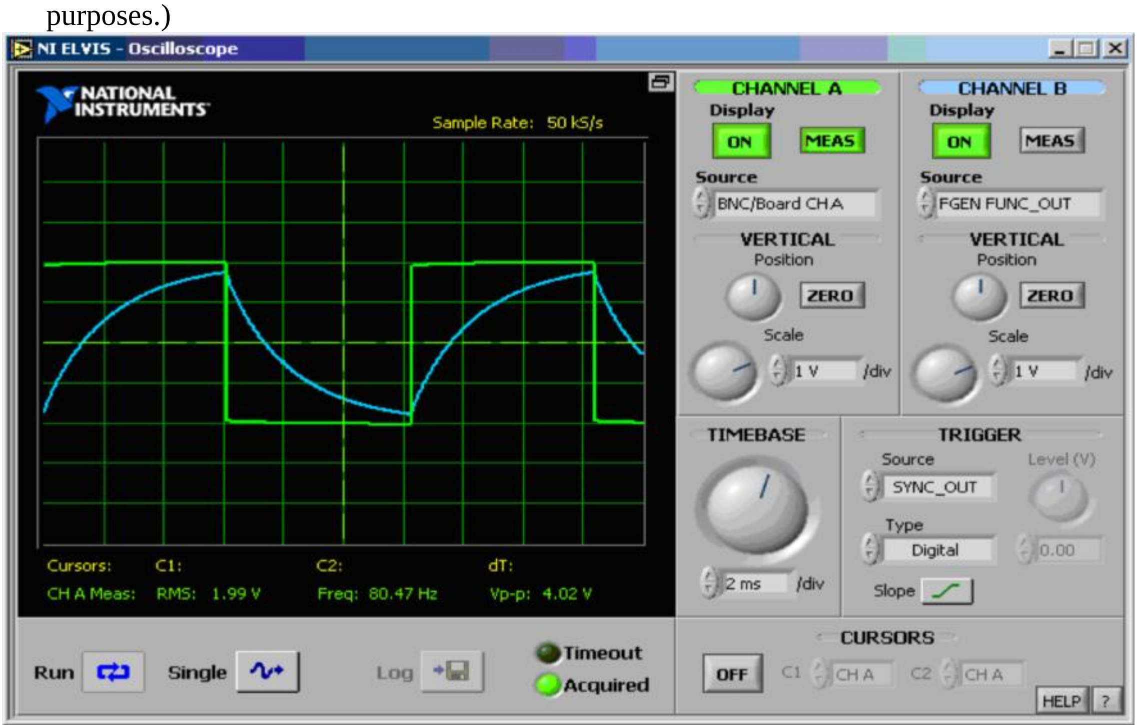



**Figure 5: Oscilloscope Configuration**

This configuration allows the oscilloscope to look at the output of the circuit on channel A, output of the function generator on channel B. Make sure you have clicked on the run button of the

FGEN panel and on the OSC panel. Any settings on the FGEN panel cause changes on the oscilloscope window

5. The $V_R$ waveform has the same shape as $i_L(t)$ waveform. From $V_R$ waveform measure time constant τ and compare with the one that you calculated from $L/R_{total}$.
(Hint: Find the time that corresponds to $0.63V_R$ value). See theory for more details.
6. Observe the response of the circuit and record the results again for $\mathbf{t_p}$ **= 25**τ, and $\mathbf{t_p}$ **= 0.5τ.**

**Questions for Lab Report:**

- Include plots of $V_R$ for different $t_p$ values given above in Procedure 4.

- A Capacitor stores charge. What do you think does an Inductor store? Answer in brief.

# National Institute of Technology, Durgapur
# Department of Electrical Engineering
# Network Analysis and Synthesis Laboratory
# (EES-451)

Experiment no: EES/451/02

**Title**: **Transient Response of RLC Series Circuit.**

**Objective:** To design over-damped, under-damped and critically-damped series *RLC* Circuit and observe their transient responses.

Theory:
A series RLC circuit can be modelled as a second order differential equation, having solution under the three conditions for its roots.
•When its roots are real and equal, the circuit response to a step input is called “Critically-damped”.
•When its roots are real but unequal the circuit response is “Over-damped”.
•When roots are a complex conjugate pair, the circuit response is labelled “Under-damped”.

We will analyse the circuit shown in fig.1 in order to determine its transient characteristics.

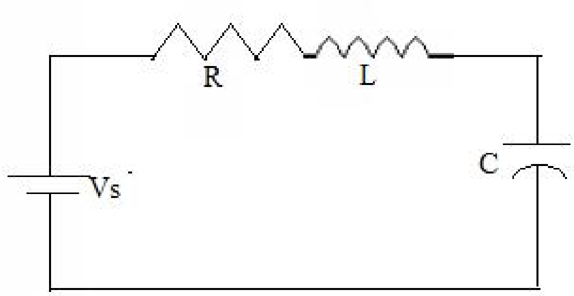



Fig-1

The equation that describes the response of the system is obtained by applying KVL around the mesh, i.e. $V_R+V_L+V_C = V_S$ --------------------- (1)

The current flowing in the circuit is I= $C\frac{d}{d}$ ----------- (2)

And thus the voltages $V_R$ and $V_L$ are given by $V_R$= I R = $RC\frac{d}{d}$ -------------- (3)
And $V_L$=L $\frac{d}{d}$ $=L \frac{d^2V}{d^2t}$ --------------- (4)

Substituting Equations (3) and (4) into Equation (1) we obtain

$$\frac{d^2V}{d^2t} + \frac{R}{L}\frac{d}{d} + \frac{1}{L}V \quad = \frac{1}{L}V \text{ -------------------------- (5)}$$

The solution to equation (5) is the linear combination of the homogeneous and the particular solution $V_C = V_{CP} + V_{Ch}$

The particular solution is $V_{CP} = V_S$ ------------------- (6)

And the homogeneous solution satisfies the equation $\frac{d^2V}{d^2t} + \frac{R}{L}\frac{d}{d} + \frac{1}{L}V \quad$ **= 0 ---------- (7)**

Assuming a homogeneous solution is of the form $Ae^{st}$ and by substituting into Equation (7) we obtain the characteristic equation $S^2+\frac{R}{L}S+\frac{1}{L}=0$ ---------------- (8)
By defining $\alpha = \frac{R}{2L}$ = Damping ratio------ (9) and $\omega_o = \frac{1}{\sqrt{L}}$ = natural frequency----- (10)

The characteristic equation becomes $S^2+2\alpha S+\omega_o^2=0$-------------- (11)

The roots of the characteristic equation are
$S_1 = -\alpha + \sqrt{\alpha^2 - \omega o^2}$ ----------- (12) and $S_2 = -\alpha - \sqrt{\alpha^2 - \omega o^2}$------------ (13)

And the homogeneous solution becomes $V_{Ch} = A_1e^{s1t} + A_2e^{s2t}$ ------------ (14)

The total solution now becomes $V_C = V_S + A_1e^{s1t} + A_2e^{s2t}$ ------------ (15)

The parameters $A_1$ and $A_2$ are constants and can be determined by the application of the initial conditions of the system.
The value of the term $\sqrt{\alpha^2 - \omega o^2}$ determines the behaviour of the response. Three types of responses are possible:
1. $\alpha = \omega_o$ then $S_1$ and $S_2$ are equal and real numbers: no oscillatory behaviour, i.e. **Critically Damped System.**
**2.** $\alpha > \omega_o$ then $S_1$ and $S_2$ are real numbers but are unequal : no oscillatory behaviour, i.e. **Over Damped System**
**3.** $\alpha < \omega_o$ , $i.e. \sqrt{\alpha^2 - \omega o^2} = j\sqrt{\omega o^2 - \alpha^2}$ In this case the roots $S_1$ and $S_2$ are complex numbers: System exhibits oscillatory behaviour, i.e. **Under Damped System.**

Some important observations:
As the resistance of the series circuit increases then the value of $\alpha$ increases and the system is driven towards an over damped response.
The frequency $\omega_o = \frac{1}{\sqrt{L}}$(rad/sec) is called the natural frequency of the system or the resonant frequency.
The parameter $\alpha = \frac{R}{2L}$ is called the damping rate and its value in relation to $\omega_0$ determines the behaviour of the response.
The quantity $\sqrt{\frac{L}{C}}$ has units of resistance
Figure 2 shows the response of the series RLC circuit and for three different values of R corresponding to the under damped, critically damped and over damped case. We will construct this circuit in the laboratory and examine its behaviour in more detail.

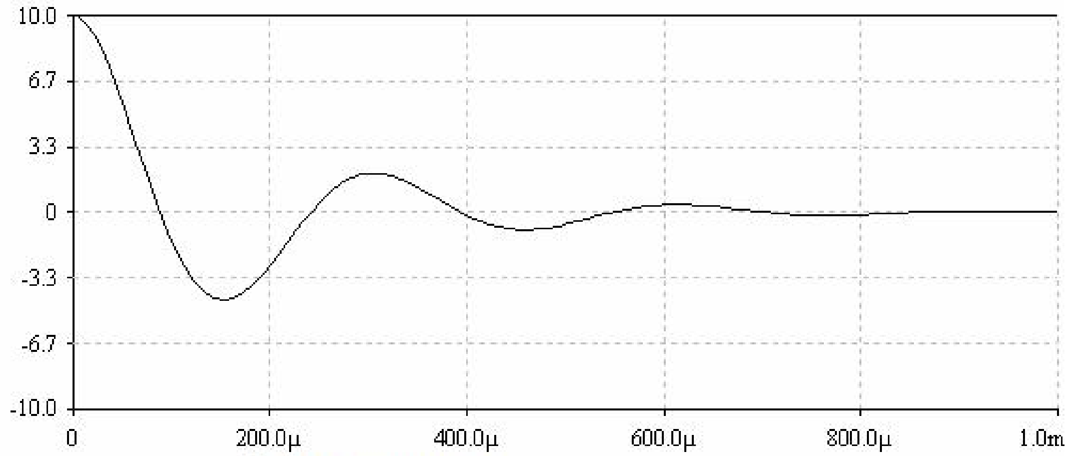



fig-2 Under-damped t (sec)

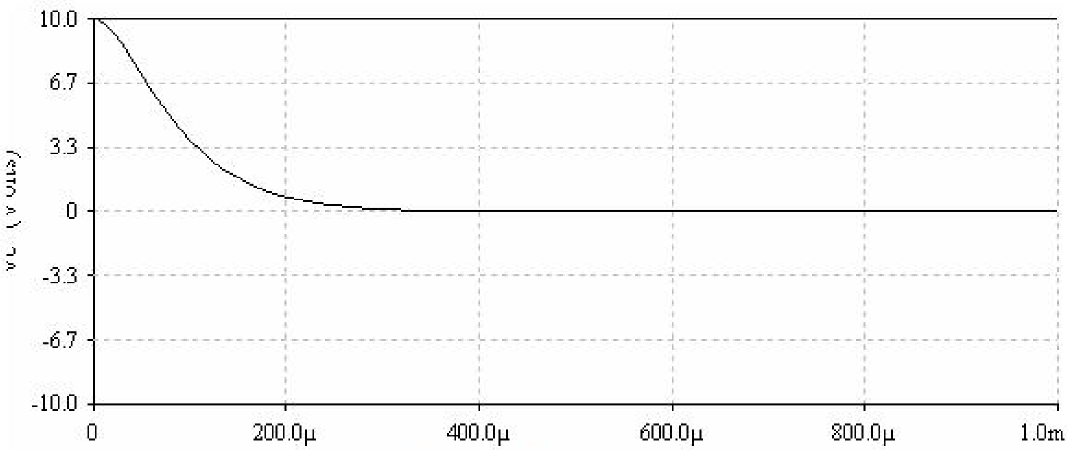



fig-3 critically damped t (sec)

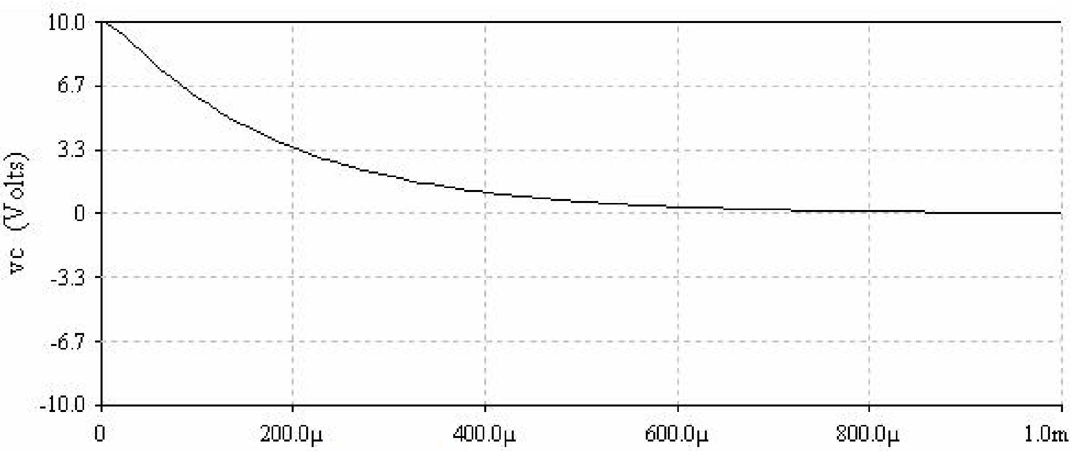



fig-4 over-damped t (sec)

**Over damped voltage transient response of capacitor in RLC circuit:**

Suppose the RLC circuit in Figure 1 has component values as displayed in the figure. Assume the function generator produces a square wave with peak-to-peak amplitude of -5 to + 5 volts, and a frequency of 50 Hz. Compute the damping factor, α, and the undamped natural frequency, $\omega_0$.

Determine the initial voltage across the capacitor, $v_C(0^+)$, and the final voltage across the capacitor, $v_C(\infty)$. Compute the characteristics roots $s_{1,2}$ , utilizing α and $\omega_0$ and state the type of damping.

The damping factor α should be greater than the undamped natural frequency $\omega_0$, thus the circuit is over damped and the capacitor's voltage transient can be expressed by the following equation with the two unknowns $A_1$ and $A_2$.

$V_C(t) = \quad V_C(\infty) + A_1 e^{S_1 t} + A_2 e^{S_2 t}$

Equating the time '0', the following relationship is formed.

$V_C(0) =$ ____ $= \quad V_C(\infty) + A_1 e^{0} + A_2 e^{0}$

Display all intermediate steps leading to a solution for $A_1$ and $A_2$.

Another equation is required to solve for both unknowns, thus the derivative of the capacitor voltage transient equation is taken.

$\frac{dv_c(t)}{d} = S_1\ A_1 e^{S_1 t} + S_2 A_2 e^{S_2 t}$

Again, evaluating at time 0, the following relationship is formed:

$\frac{dv_c(0^+)}{d} = \frac{i_L(0^+)}{C} = S_1\ A_1 e^{0} + S_2 A_2 e^{0}$

Display all intermediate steps leading to a solution for $A_1$ and $A_2$.

Having solved for two unknown variables, write the full equation for $V_c(t)$ and then evaluate $V_c(0.5\text{ ms})$, $V_c(1\text{ ms})$ and $V_c(2\text{ ms})$.

**Under damped voltage transient response of capacitor in RLC circuit:**

Compute the damping factor, α, and the undamped natural frequency, $\omega_0$. Determine the initial voltage across the capacitor, $v_C(0^+)$, and the final voltage across the capacitor, $v_C(\infty)$. Compute the characteristics roots $s_{1,2}$ , utilizing α and $\omega_0$ and state the type of damping.

The damping factor α should be less than the undamped natural frequency $\omega_0$, thus the circuit is under damped and the capacitor's voltage transient can be expressed by the following equation with the two unknowns $B_1$ and $B_2$.

$V_C(t) = \quad V_C(\infty) + (B_1 \cos\omega_d t + B_1 \sin\omega_d t) e^{-\alpha t}$

Equating the time '0', the following relationship is formed.

$V_C(0) =$ ____ $= \quad V_C(\infty) + [B_1 \cos(0) + B_1 \sin(0)\ ] e^{0}$

Display all intermediate steps leading to a solution for $B_1$.

Another equation is required to solve $B_2$. Thus, the derivative of the capacitor voltage transient-

$\frac{dv_c(t)}{d}$ = (B$_1$cos$\omega_d t$ + B$_1$sin$\omega_d$t)($-\alpha e^{-\alpha} t$) + ( $-\omega_d$B$_1$sin$\omega_d t$ + $\omega_d$B$_1$cos$\omega_d$t)$e^{-\alpha}t$

Again, evaluating at time 0, the following relationship is formed:

$\frac{dv_c(0^+)}{d}$ =$\frac{i_L(0^+)}{C}$ =_______= (B$_1$cos$\omega_d t$ + B$_1$sin$\omega_d$t)($-\alpha e^{-\alpha} t$) + ( $-\omega_d$B$_1$sin$\omega_d t$ + $\omega_d$B$_1$cos$\omega_d$t)$e^{-\alpha}t$

Display all intermediate steps leading to a solution for $B_1$ and $B_2$.

Having solved for two unknown variables, write the full equation for $V_c$(t) and then evaluate $V_c$(0.5 ms), $V_c$(1 ms) and $V_c$(2 ms).

**Critically damped voltage transient response of resistor in RLC circuit:**

Compute the damping factor, α, and the undamped natural frequency, $\omega_0$. Determine the initial voltage across the capacitor, $v_C(0^+)$, and the final voltage across the capacitor, $v_C(\infty)$. Compute the characteristics roots $s_{1,2}$ , utilizing α and $\omega_0$ and state the type of damping.

The damping factor α should be equal to the undamped natural frequency $\omega_0$, thus the circuit is critically damped and the capacitor's voltage transient can be expressed by the following equation with the two unknowns $C_1$ and $C_2$.

$V_C(t) = \quad V_C(\infty) + (C_1 + C_2 t)\, e^{-\alpha} t$

Equating the time '0', the following relationship is formed.

$V_C(0)$ = ____ = $V_C(\infty) + (C_1 + C_2.0)\, e^{0}$

Display all intermediate steps leading to a solution for $C_1$

Another equation is required to solve $C_2$. Thus, the derivative of the capacitor voltage transient-

$\frac{dv_c(t)}{d}$ = ($C_1$+ $C_2$t)($-\alpha e^{-\alpha} t$) + $C_2 e^{-\alpha} t$

Again, evaluating at time 0, the following relationship is formed:

$\frac{dv_c(0^+)}{d}$ =$\frac{i_L(0^+)}{C}$ = ($C_1$+ $C_2$.0)($-\alpha e^{0}$) + $C_2 e^{0}$

Display all intermediate steps leading to a solution for $C_1$ and $C_2$.

Having solved for two unknown variables, write the full equation for $V_c$(t) and then evaluate $V_c$(0.5 ms), $V_c$(1 ms) and $V_c$(2 ms).

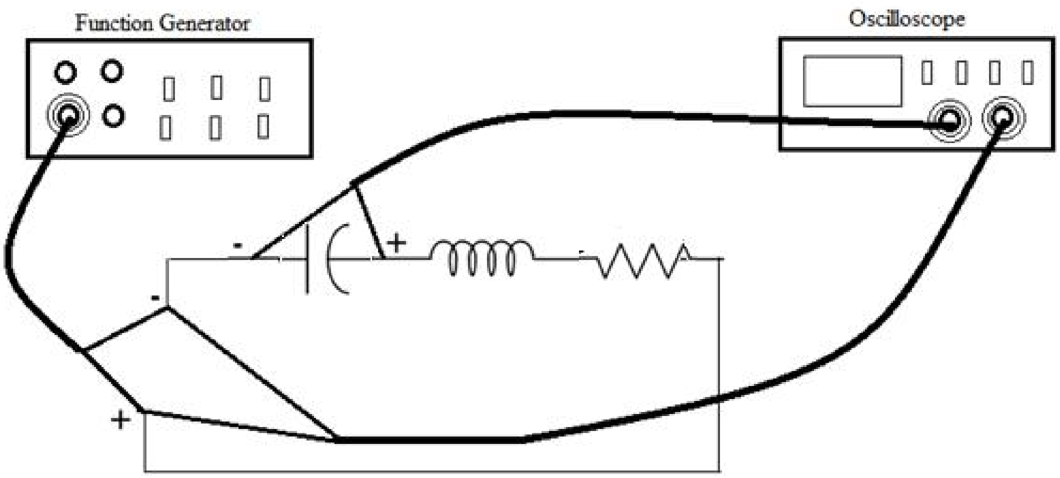


Fig-5, experimental circuit diagram

Procedure:

OVER-DAMPED RLC CAPACITOR VOLTAGE SETP RESPONSE:

With the RLC circuit disconnected, adjust the function generator to produce a repetitive pulse that is -5 volts for about 10 ms, then +5 volts for about 10 ms. (i.e. 10 Volts peak-to-peak, 0 Volts of DC offset, 20 ms Period or 50 Hz). For the circuit in Figure 5, calculate the output response, $\mathbf{V_C(t)}$, t > 0, to an input step, from -5 to +5 Volts. Connect the circuit in Figure 5. Measure the final value, $V_C(t=\infty)$, and the initial value, $V_C(t=0+)$, from the oscilloscope and record in the Data section. Also measure the voltages $V_C$(t=0.5 ms), $V_C$(t=1.0 ms), and $V_C$(t=2.0 ms) from the oscilloscope and record in the Data section.

First determine **$\alpha$ and $\omega_0$.** Calculate the roots of the characteristic equation, $\mathbf{S_{1,2}}$ and determine **Vc(0)**, and **Vc(∞),** and **d[Vc(0)]/dt**. Calculate $\mathbf{A_1}$ and $\mathbf{A_2}$, and fill in the Data Table-I below:

Table-I

**OVERDAMPED RLC**

| Sl No | **Quantity** | **Calculated Value** | **Measured Value** |
|---|---|---|---|
| R | | | |
| **$\alpha$** | | | |
| **$\omega_0$** | | | |
| **$S_{1,2}$** | | | |
| **$V_c(0)$** | | | |
| **$d[V_c(0)]/dt$** | | | |
| **$V_c(\infty)$** | | | |
| **$A_1$ and $A_2$,** | | | |
| **Equation for $V_c(t)$** | | | |
| **$V_c$(0.5ms)** | | | |
| **$V_c$(1ms)** | | | |
| **$V_c$(2ms** | | | |

UNDER-DAMPED RLC CAPACITOR VOLTAGE SETP RESPONSE:

Keep the function generator settings used in Part 1. For the circuit in Figure 5,change the resistor value for under-damped case and calculate the output response, $V_c(t)$, t > 0, to an input step, from -5 to +5 Volts.

First determine **α and $\omega_0$.** Calculate the roots of the characteristic equation, **$S_{1,2}$** and determine **Vc(0)**, and **Vc(∞),** and **d[Vc(0)]/dt**. Calculate **$B_1$** and **$B_2$**, and fill in the Data Table II below:

Table-II

**UNDERERDAMPED RLC**

| Sl No | **Quantity** | **Calculated Value** | **Measured Value** |
|---|---|---|---|
| R | | | |
| **α** | | | |
| **$\omega_0$** | | | |
| **$S_{1,2}$** | | | |
| **$V_c(0)$** | | | |
| **$d[V_c(0)]/dt$** | | | |
| **$V_c(\infty)$** | | | |
| **$A_1$ and $A_2$,** | | | |
| **Equation for $V_c(t)$** | | | |
| **$V_c(0.5ms)$** | | | |
| **$V_c(1ms)$** | | | |
| **$V_c(2ms$** | | | |

CRITICALLY-DAMPED RLC CAPACITOR VOLTAGE SETP RESPONSE:

Keep the function generator settings used in Part 1. For the circuit in Figure 5,change the resistor value for critical-damped case and calculate the output response, VC(t), t > 0, to an input step, from -5 to +5 Volts.

First determine **α and $\omega_0$.** Calculate the roots of the characteristic equation, $S_{1,2}$ and determine **Vc(0)**, and **Vc(∞),** and **d[Vc(0)]/dt**. Calculate **$C_1$** and **$C_2$**, and fill in the Data Table III below:

Table-II

**CRITICALLY-DAMPED RLC**

| Sl No | **Quantity** | **Calculated Value** | **Measured Value** |
|---|---|---|---|
| R | | | |
| **α** | | | |
| **$\omega_0$** | | | |
| **$S_{1,2}$** | | | |
| **$V_c(0)$** | | | |
| **$d[V_c(0)]/dt$** | | | |
| **$V_c(\infty)$** | | | |
| **$A_1$ and $A_2$,** | | | |
| **Equation for $V_c(t)$** | | | |
| **$V_c(0.5ms)$** | | | |
| **$V_c(1ms)$** | | | |
| **$V_c(2ms$** | | | |

# National Institute of Technology, Durgapur
# Department of Electrical Engineering
# Network Analysis and Synthesis Laboratory
# (EES-451)

Experiment no: EES/451/03
Title: Determination of frequency response of current in RLC circuit with sinusoidal ac input.

Objective: To observe amplitude and phase change in an RLC circuit under a sinusoidal forcing function.

Theory: A resonant circuit, also called a tuned circuit consists of an inductor and a capacitor together with a voltage or current source. It is one of the most important circuits used in electronics. For example, a resonant circuit, in one of its many forms, allows us to select a desired radio or television signal from the vast number of signals that are around us at any time. A network is in resonance when the voltage and current at the network input terminals are in phase and the input impedance of the network is purely resistive.

**Series Resonant Circuit:** In a series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. The point at which this occurs is called the **Resonant Frequency**, ($f$r ) and as we are analysing a series RLC circuit this resonance frequency produces a **Series Resonance** circuit.

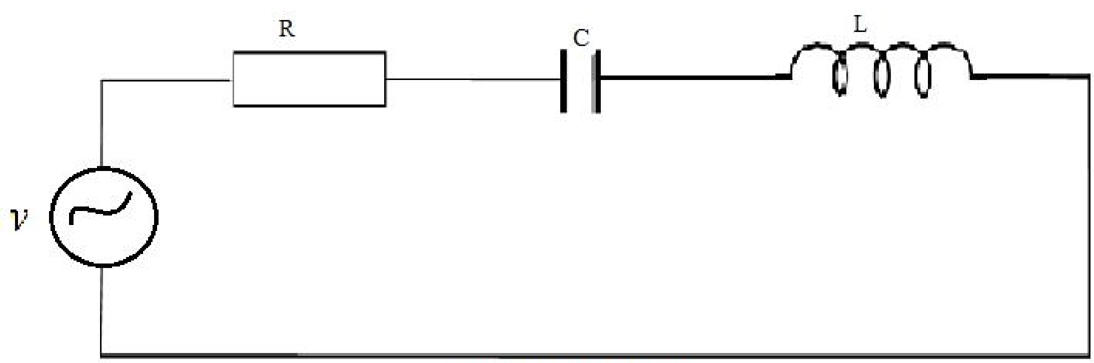



Fig-1 Series RLC Circuit

For the above circuit,

$I = \frac{V}{R+j\omega L+\frac{1}{j\omega C}}$ , If we let V = $V_m\angle 0^0$, we get the following characteristics; the amplitude of the current depends on several variables:

1. It is directly related to the amplitude of the forcing signal $V_m$.
2. It is inversely related to the value of the **resistor R**.
3. It is inversely related to the value of the **inductor L**.
4. It is directly related to the value of the **capacitor C**.

In a series resonant circuit, the resonant frequency, $f$r point can be calculated $f_r = \frac{1}{2\pi\sqrt{L}}$

As starts ω from a value of zero, the current starts to increase from zero up to the point where, $\omega = \frac{1}{\sqrt{L}}$ , At this point, the current will have maximum amplitude of $\frac{V_m}{R}$ . As ω is increased further, the current will decrease towards zero as goes to ω infinity (∞ ).

## The phase angle of the current:

1. The source voltage will be leading by 90 at a frequency very close to zero. As the frequency is increased, this leading angle will start to decrease up to the point $\omega = \frac{1}{\sqrt{L}}$ at which there will be no phase shift. As the frequency is increased further, the angle of the current starts to lag the source voltage. It will approach $90^o$lagging as the frequency ω approaches infinity.
2. A decrease in the value of R serves to increase the sharpness of the curve and compress it at the point of inversion.
3. A change in the value of L or C will move the point of inversion along the frequency axis.

**Parallel Resonant Circuit**:A parallel circuit containing a resistance, R, an inductance, L and a capacitance, C will produce aparallel resonance (also called anti-resonance) circuit when the resultant current through the parallel combination is in phase with the supply voltage. At resonance there will be a large circulating current between the inductor and the capacitor due to the energy of the oscillations. A parallel resonant circuit stores the circuit energy in the magnetic field of the inductor and the electric field of the capacitor. This energy is constantly being transferred back and forth between the inductor and the capacitor which results in zero current and energy being drawn from the supply. This is because the corresponding instantaneous values of $I_L$ and $I_C$ will always be equal and opposite and therefore the current drawn from the supply is the vector addition of these two currents and the current flowing in $I_R$.

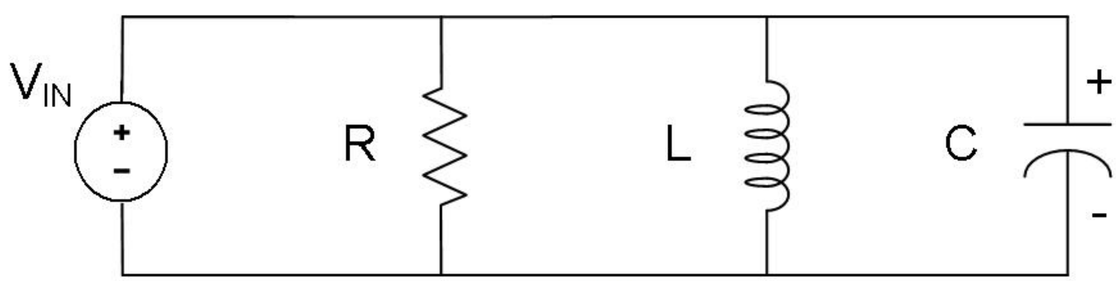



Fig-2, parallel RLC circuit

Consider the Parallel RLC circuit of figure 2. The steady-state admittance offered by the circuit is Y = 1/R + j(ωC – 1/ωL) . Resonance occurs when the voltage and current at the input terminals are in phase. This corresponds to a purely real admittance, so that the necessary condition is given by ωC – 1/ωL = 0. The resonant condition may be achieved by adjusting L, C, or ω. Keeping L and C constant, the resonant frequency $\omega_o$ is given by: $\omega = \frac{1}{\sqrt{L}}$

The forced response of a parallel RCL circuit is given by the form: $I = (\frac{1}{R} + j\omega C + \frac{1}{j})$ V. If we let $V = V_m\angle 0^0$, we get the following characteristics; the amplitude of the current depends on several variables:

1. It is directly related to the amplitude of the forcing signal $V_m$ and value of the capacitor C.
2. It is inversely related to the value of the **resistor R** and **inductor L**.

As far as is concerned, we note that as ω starts from a value of zero the current will start to decrease from∞ up to the point where $\omega=\frac{1}{\sqrt{L}}$at which the current will have minimum amplitude of$\frac{V_m}{R}$ , as ω is increased further, the current will increase back to∞ as ω goes to infinity.

## The phase angle of the current:

1. The source voltage will be lagging by 90 at a frequency very close to zero. As the frequency is increased, this lagging angle will start to decrease up to the point $\omega=\frac{1}{\sqrt{L}}$at which there will be no phase shift. As the frequency is increased further, the angle of the current starts to lead the source voltage. It will approach 90$^o$lagging as the frequency ω approaches infinity.
2. A decrease in the value of R serves to decrease the sharpness of the curve and expands the point of inversion.
3. A change in the value of L or C will move the point of inversion along the frequency axis.

**Frequency Response:** It is a plot of the magnitude of output Voltage of a resonance circuit as function of frequency. The response of course starts at zero, reaches a maximum value in the vicinity of the natural resonant frequency, and then drops again to zero as ω becomes infinite. The frequency response is shown in figure 3.

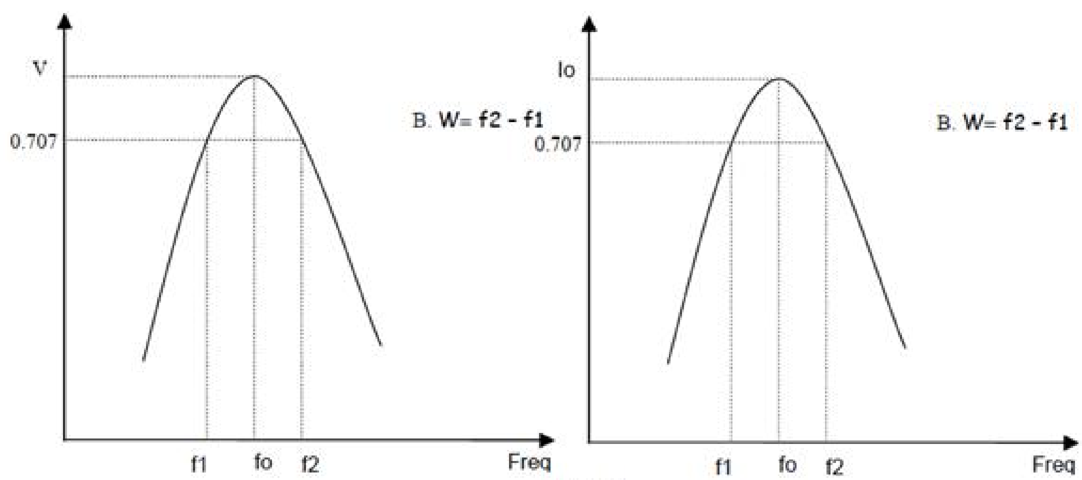



Fig-3

The two additional frequencies $f_1$ and $f_2$ are also indicated which are called *half-power frequencies*. These frequencies locate those points on the curve at which the voltage response is $1/\sqrt{2}$ or 0.707 times the maximum value. They are used to measure the band-width of the response curve. This is called the *half-power bandwidth* of the resonant circuit and is defined as:$\beta = f_2 - f_1$

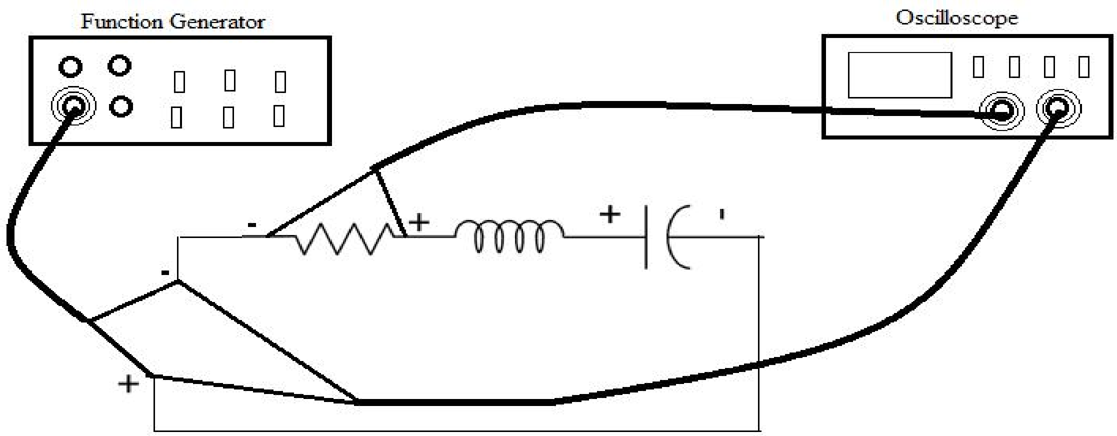



Fig-4, experimental circuit diagram

Procedure:

1. Connect the circuit as shown in fig-4.
2. Set the function generator output voltage to a particular value.
3. Cheque the phase angle current behaviour for RLC series and parallel circuit both separately.
4. Increase the function generator output signal frequency from minimum to a maximum signal frequency in decade steps.
5. For applied each signal frequency measure current or voltage and record it.
6. Calculate theoretical resonance frequency using $f_r=\frac{1}{2\pi\sqrt{L}}$.
7. Plot the graph of frequency v/s current for series circuit and frequency v/s voltage for parallel circuit, find the frequency on the graph at which Current and voltage is maximum, this frequency is known as Resonant frequency and this should be approximately to the theoretical frequency calculated in previous step.

## Observation Table:

### Table-I: for series circuit

| Sl No | Frequency | Voltage drop across R ($V_R$) | Current |
|---|---|---|---|
| | | | |

**Table-II: for parallel circuit**

| Sl No | Frequency | Voltage drop across R ($V_R$) | Current |
|---|---|---|---|
| | | | |
| | | | |

Table-III

| | Series Resonance | | Parallel Resonance | |
|---|---|---|---|---|
| Resonant frequency | Theoretical Practical | Theoretical Practical | Theoretical Practical | Theoretical Practical |
| Bandwidth | | | | |
| Q-factor | | | | |

# National Institute of Technology, Durgapur
# Department of Electrical Engineering
# Network Analysis and Synthesis Laboratory (EE-451)

Experiment no: EE/451/04

Title: Determination of frequency response characteristics of a low pass and high pass passive filters.

Objectives: Study the characteristics of passive filters by obtaining the frequency response ofLow Pass RC filter and High Pass RL filter.

**Theory:**

The impedance of an inductor is proportional to frequency and the impedance of a capacitor is inversely proportional to frequency. These characteristics can be used to selector reject certain frequencies of an input signal. This selection and rejection of frequencies is called filtering, and a circuit which does this is called a *filter*. If a filter passes high frequencies and rejects low frequencies, then it is a high-pass filter. Conversely, if it passes low frequencies and rejects high ones, it is a low-pass filter. Filters, like most things, aren't perfect. They don't absolutely pass some frequencies and absolutely reject others. A frequency is considered passed if its magnitude (voltage amplitude) is within70%(or1/√2) of the maximum amplitude passed and rejected otherwise. The 70% frequency is called corner frequency, roll-off frequency or half- power frequency.

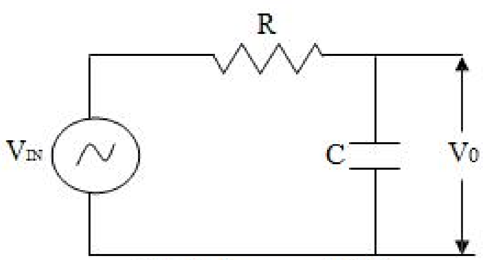


Fig-1, low pass RC filter

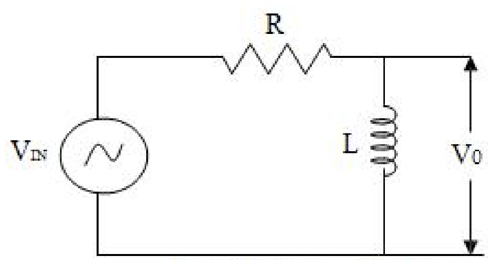


Fig-2, high pass RL filter

The corner frequencies for RC filter and RL filter are as follows:

For RC filter, $f_C = \frac{1}{2\pi}$ and for RL filter, $f_C = \frac{1}{2\pi}$

**Frequency Response:** It is a graph of magnitude of the output voltage of the filter as a function of the frequency. It is generally used to characterize the range of frequencies in which the filter is designed to operate within

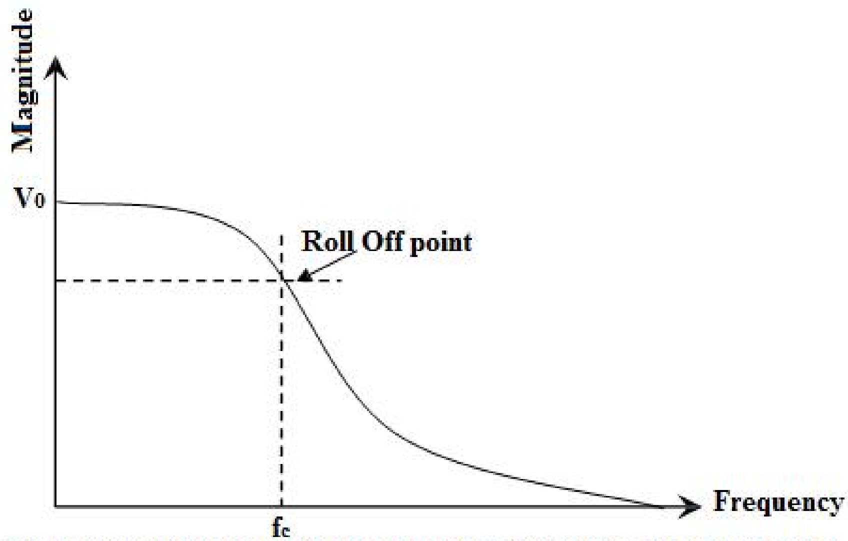


**Figure 3: Frequency Response of a typical Low Pass Filter with a cut-off frequency $f_c$**

**Procedure:**

**A. Low Pass RC Filter:**

1. SetupthecircuitshownintheFigure1withthecomponentvaluesR=1.1kΩ, C = 1µF. Switch on the Elvis Power Supply.

2. SelecttheFunctionGeneratorfromtheNI-ELVISMenuandapplya4Vpeak-peak

Sinusoidal wave as a input voltage to the circuit.

3. SelecttheOscilloscopefromtheNI-ELVISMenu.MakesuretheSourceonChannel

A, function out on Channel B, Trigger and Time base input boxes are properly set.

4. Start with a low frequency, i.e.50Hz and measure output voltage peak to peak from the scope screen. It should be same as the signal generator output. Vary the frequency of the FGEN panel until you see roughly $0.7V_{fgen}$peak to peak

5. Compute the70% of Vp-p and obtain the frequency at which this occurs on the oscilloscope. This gives the cut-off (roll-off) frequency for the constructed Low Pass RC filter.

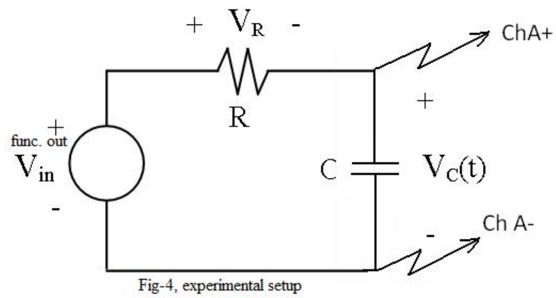


Fig-4, experimental setup

**B. High Pass RL Filter:**

1. Setupthecircuitshowninthe**Figure2**withthecomponentvaluesR=1.1KΩ, L = 33mHand Switch on the Elvis Power Supply.

2. Repeat steps 2 and 3 as in Part A to obtain the frequency response on the Oscilloscope.

3. Start with a high frequency (20 KHz) and measure the output voltage peak to peak from the scope screen. It should be same as the signal generator output. Adjust the time base of the scope accordingly to observe this.

4. Compute the70% of Vp-p and obtain the frequency at which this occurs on the Oscilloscope. This gives the cut-off (roll-off) frequency for the constructed High Pass RL filter.

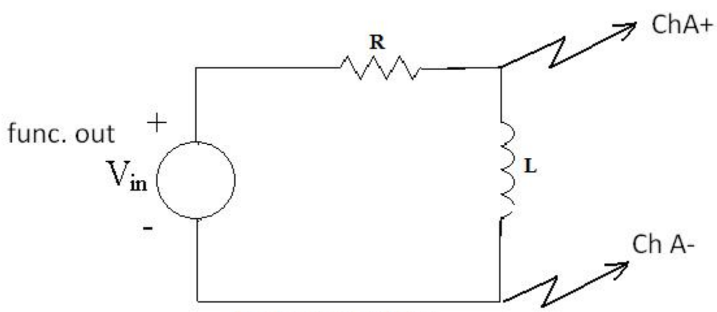



Fig-4, experimental setup

**Questions for Lab Report:**

1. Compute the Cut-off frequencies for the RC and RL filter using the formulae in equations (1) and (2). Compare these theoretical values to the ones obtained from the experiment and provide suitable explanation for any differences

# National Institute of Technology, Durgapur
# Department of Electrical Engineering
# Network Analysis and Synthesis Laboratory
# (EE-451)

Experiment no: EE/451/05

Title: Determination of Z and Y parameters (dc only) for two port network.

Objective:

1. To calculate and verify Z parameters of two-port network.
2. To calculate and verify Z parameters of two-port network.

Brief Theory: In a Z parameter of two-port network, the input and output voltages $V_1$ and $V_2$ can be expressed in terms of input and output current $I_1$ and $I_2$. Out of four variables (i.e. $V_1$, V2, $I_1$ and $I_2$) $V_1$ and $V_2$ are dependent variables and $I_1$ and $I_2$ are independent variables. Thus

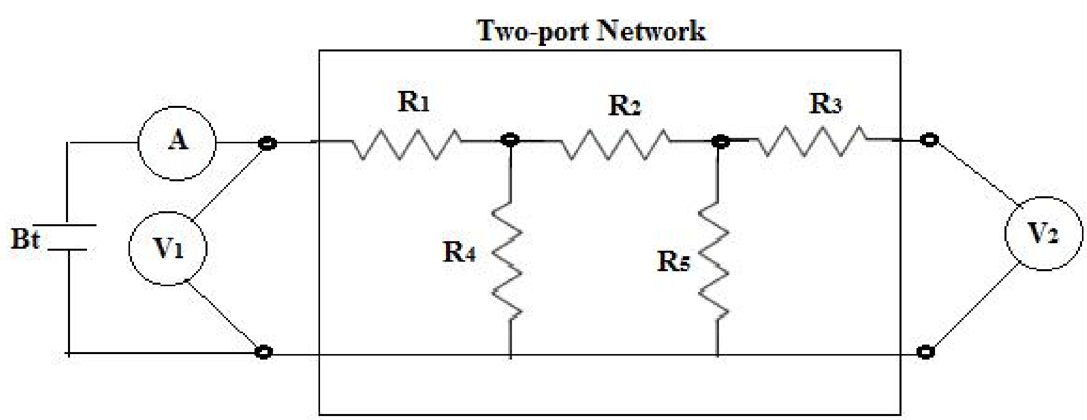
$$V_1 = Z_{11}I_1 + Z_{12}I \text{---------------------------------- (i)}$$

$$V_2 = Z_{21}\, I_1 + Z_{22}I_2 \text{------------------------------- (ii)}$$

Here $Z_{11}$ and $Z_{22}$ are the input and output driving point impedances while $Z_{12}$ and $Z_{21}$ are the reverse and forward transfer impedances.

Now, $Z_{11} = \frac{V_1}{I_1}$ at $I_2=0$; $Z_{21} = \frac{V_2}{I_1}$ at $I_2=0$ and $Z_{12} = \frac{V_1}{I_2}$ at $I_1=0$; $Z_{22} = \frac{V_2}{I_2}$ at $I_1 = 0$.

Circuit Diagram:



Procedure:

1. Connect the circuit as shown in the above figure.
2. First open the output port and supply some voltage to input port. Measure the input and output voltages and also measure the input current.
3. Secondly open the input port and supply some voltage to output port. Measure the input and output voltages and also measure the output current.
4. Calculate the values of Z parameters.
5. Switch off the supply after taking the readings.

Observation Table:

| Sl No | When input port is open circuited | | | When output port is open circuited | | |
|---|---|---|---|---|---|---|
| | $V_1$ | $V_2$ | $I_2$ | $V_1$ | $V_2$ | $I_1$ |
| | | | | | | |
| | | | | | | |

Verification Table:

| Parameters | Practical Values | Theoretical Values |
|---|---|---|
| $Z_{11}$ | | |
| $Z_{12}$ | | |
| $Z_{21}$ | | |
| $Z_{22}$ | | |

Brief Theory: In a Y parameter of two-port network, the input and output currents $I_1$ and $I_2$ can be expressed in terms of input and output voltages $I_1$ and $I_2$. Out of four variables (i.e. $V_1$, V2, $I_1$ and $I_2$) $I_1$ and $I_2$ are dependent variables and $V_1$ and $V_2$ are independent variables. Thus

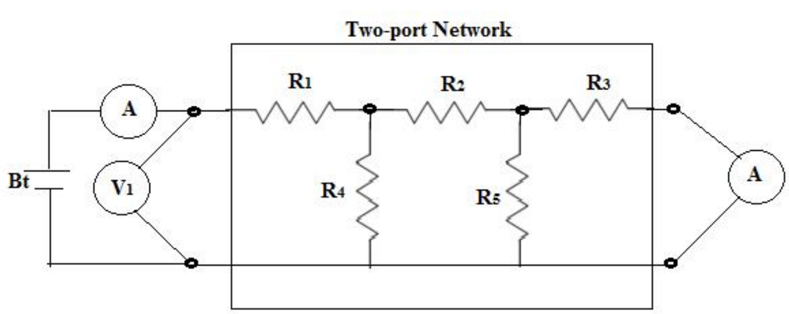
$$I_1 = Y_{11} V_1 + Y_{12} V_2 \text{---------------------------------} \text{(i)}$$

$$I_2 = Y_{21} I_1 + Y_{22} I_2 \text{-------------------------------} \text{(ii)}$$

Here $Y_{11}$ and $Y_{22}$ are the input and output driving point admittances while $Y_{12}$ and $Y_{21}$ are the reverse and forward transfer admittances.

Now, $Y_{11}=\frac{I_1}{V_1}$at $V_2=0$; $Y_{21}=\frac{I_2}{V_1}$at $V_2=0$ and $Y_{12}=\frac{I_1}{V_2}$at $V_1=0$; $Y_{22}=\frac{I_2}{V_2}$at $V_1=0$.

Circuit Diagram:



Procedure:

1. Connect the circuit as shown in the above figure.
2. First short the output port through ammeter and supply some voltage to input port. Measure the input and output current and also measure the input voltage.
3. Secondly short the output port through ammeter and supply some voltage to output port. Measure the input and output currents and also measure the output voltages.
4. Calculate the values of Z parameters.
5. Switch off the supply after taking the readings.

Observation Table:

| Sl No | When input port is short circuited | | | When output port is short circuited | | |
|---|---|---|---|---|---|---|
| | $V_2$ | $I_1$ | $I_2$ | $V_1$ | $I_1$ | $I_2$ |
| | | | | | | |
| | | | | | | |

Verification Table:

| Parameters | Practical Values | Theoretical Values |
|---|---|---|
| $Y_{11}$ | | |
| $Y_{12}$ | | |
| $Y_{21}$ | | |
| $Y_{22}$ | | |

# National Institute of Technology, Durgapur
# Department of Electrical Engineering
# Network Analysis and Synthesis Laboratory (EE-451)

Experiment no: EE/451/06

Title: Magnetically coupled circuit.

Objective:

1. Determination of self-inductance, mutual-inductance and coefficient of coupling for a magnetic coupled circuit.
2. Determination of driving point and transfer impedance of a coupled circuit.

Brief Theory:

**Self-Inductance:**Consider a single coil wound around a magnetic core as shown in fig.1 below.

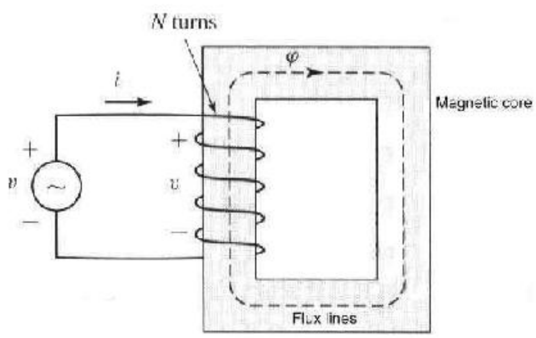



Fig. 1: A single coil wound around a magnetic core.

A current i flowing in the coil will produce a magnetic flux ϕ in the magnetic core. If the current is varying with time, the produced flux is also varying with time and then according to Faraday's law of electro-magnetic induction, an induced voltage v will be generate across the coil terminals. V is given by the relation, v = N$\frac{d}{d}$---------------------------------------------- (1).The self-inductance of the coil is defined as the ratio between the flux linkage, N ϕ, linking the coil and the current, I, producing the flux. So L = $\frac{N}{i}$-------------------------------------------- (2). Substituting from equation (2) into (1) gives v = L$\frac{d}{d}$-------------------------------------------- (3).

Mutual inductance: If a second coil is wound around the magnetic core of fig.1 as shown in fig.2, the two coils become magnetically coupled. The flux produced by the first links the second. The mutual inductance, $M_{12}$, between coils 1 & 2 in defined as the ratio between the fluxlinkage, $N_2\,\phi$, linking the coil 2 and the current $i_1$ producing this flux.

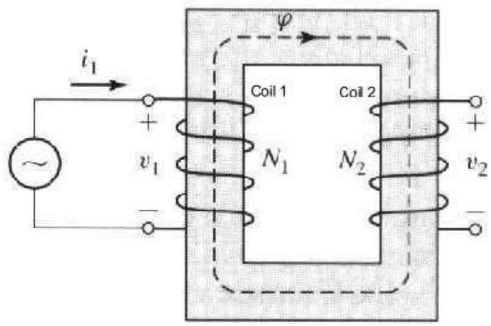



Fig. 2: Two magnetically-coupled coils.

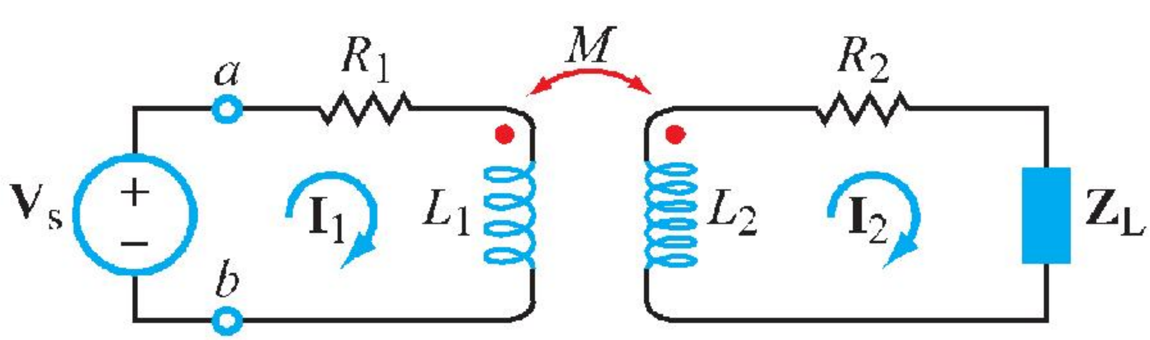
Therefore,

$M_{12} = \frac{N_2\phi}{i_1}$-------- (4). When the terminals of coil 2 are open circuited, ie, no current following in the coil 2, then the following equations can be written.

$V_1 = \frac{d}{d}(N_1\phi) = L_1\frac{di_1}{d}$---------- (6) and $V_2 = \frac{d}{d}(N_2\phi) = \pm\, M_{12}\frac{di_1}{d}$---------------- (6), where $V_2$ is the induced voltage in coil 2 due to current $i_1$ in coil 1.It can be shown that $M_{12}=M_{21}$.

**Coefficient of coupling:** Since all the fluxare not linking a coil flow inside the magnetic core then a small percentage of the flux produced by a coil leaks away from the core. This is known as leakage flux. The measurement of the leakage flux is the coupling coefficient k and can be expressed as $k = \frac{M}{\sqrt{L1L2}}$----------- (7)

**Input impedance**: In addition to the two coupled coils, a realistic transformer circuit should be includes two resistors $R_1$ and $R_2$, to account for ohmic losses in the coils. The circuit shown in fig below reflects this reality by including resistor $R_1$on the side of the coil 1 and resistance $R_2$ on the side of coil 2. The circuit is driven by the voltage source $V_S$ on the primary side and terminated in the complex load $Z_L$ on the secondary side.



In terms of the designated mesh current $I_1$ and $I_2$, the KVL mesh equations are

$V_S = I_1R_1 + j\omega L_1I_1 - j\omega MI_2$ ------------------------------------------------ (i)

And $0 = I_2R_2 + j\omega L_2I_2 - j\omega MI_1 + I_2Z_L$ ------------------------------------- (ii)

The input driving point is given by $Z_D = \frac{V_S}{I_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + J\omega L_2 + Z_L}$

$= R_1 + j\omega L_1 + Z_R$ ---------------- (iii), where $Z_R$ defined as reflected impedance $Z_R = \frac{\omega^2 M^2}{R_2 + J\omega L_2 + Z_L}$

In a similar manner, we can derive the expression of transfer impedance,

$Z_r = \frac{V_1}{I_2} = j\omega M + \frac{(R_2+R_L+j\ {}_2+j\ {}_L)(R_1+j\ {}_1)}{\omega}$-------------------- (iv)

**Procedure I: Determination of Self Inductance $L_1$ and $L_2$.**

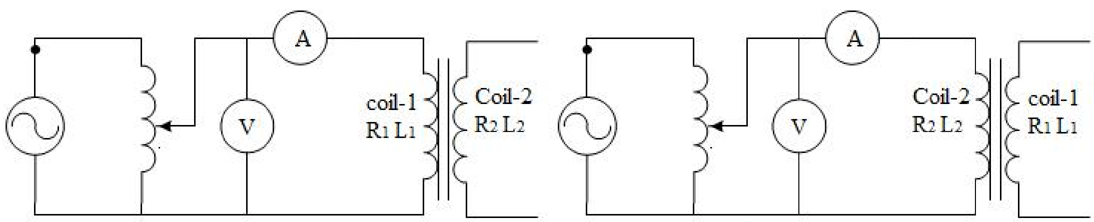



Fig-4 circuit diagram for measurement of $L_1$ and $L_2$

1. Measure the resistance $R_{dc}$ of coil 1 with the help of volt-ampere method. This is dc resistance of coil 1.
2. Determine the coil 1 ac resistance $R_1$by allowing 15% increase due to skin effect, such that

   $R_1 = 1.15\ R_{dc}$
3. Similarly find out $R_2$,the ac resistance of coil 2.
4. Connect coil 1 to a variable 50 Hz voltage source as shown in Fig. 4.
5. Adjust the magnitude of the applied voltage to a particular value and then take their readings of both the ammeter and voltmeter.
6. Calculate the coil (self) inductance as follows:

   $Z_1 = \frac{V_1}{I_1} = \sqrt{R_1^2 + X_1^2}$ and $X_1 = \sqrt{Z_1^2 - X_1^2}$

   So $L_1 = \frac{X_1}{\omega}$
7. Repeat the procedure 4 to 6 for coil 2 and calculate the self-inductance, $L_2$.

**Procedure II: Dete**

**rmination of mutual Inductance.**

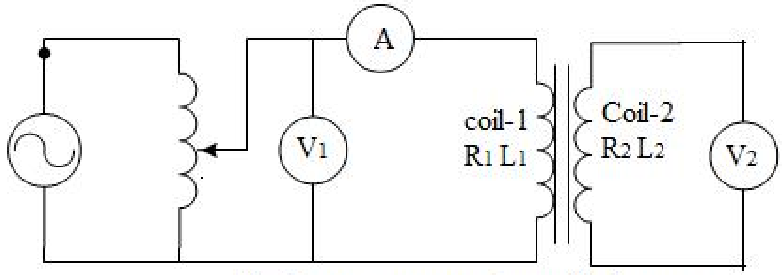


Fig-5, measurement of mutual indactance

1. Connect the circuit as shown in Fig. 5, with coil 2 magnetically coupled to coil 1.
2. Adjust the magnitude of the applied voltage to a particular value and then take their readings of both the voltmeters and ammeter.
3. Calculate the mutual inductance as follows:

$X_M = \frac{V_2}{I_1}$ *and* $M = \frac{X_M}{\omega}$

**Procedure III: Determination of Polarity ofMagnetically Coupled Coils**

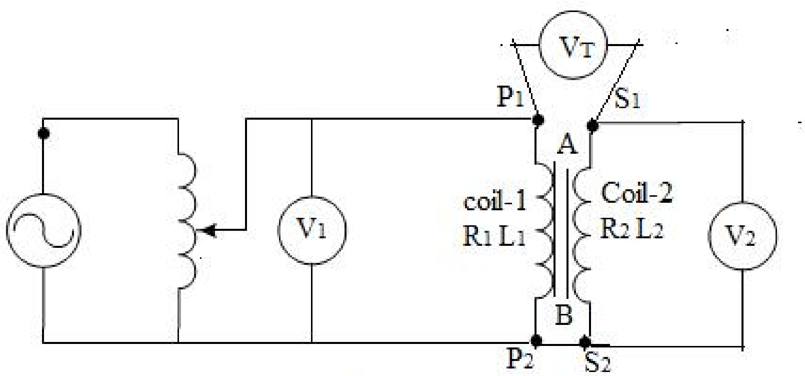


fig-6,Determination of polarirty

1. Connect the electric circuit as shown in Fig. 6
2. With specified voltage applied to coil 1, take the reading of $V_T$.
3. If $V_T$ is the difference between $V_1$and $V_2$, the dot is to be put at point A, otherwise It is to be put at point B.
.

**Procedure IV- Series Connected Magnetically Coupled Coils**

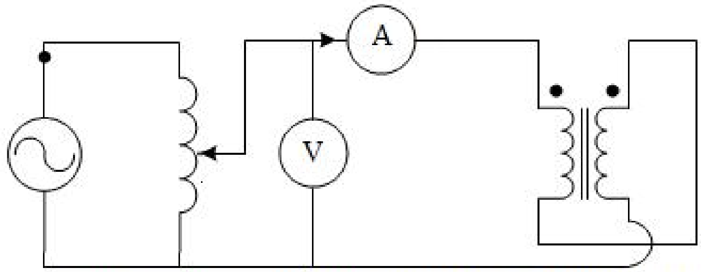



Fig. 7 : Series-connected magnetically -coupled coils (adding).

1. According to the polarity determined above, connect the electric circuit as shown infig.7.
2. Adjust the magnitude of the applied voltage to a particular value and then take the readings of both the ammeter and voltmeter.
3. Calculate the equivalent impedance in this case as follows:

$$X_{eq1} = \sqrt{(V/I)^2 + (R1 + R2)^2} \;\; = \omega(L_1+L_2+2M)$$

4. Repeat steps1 and 2 but with the two coils connected in opposition

$$X_{eq2} = \sqrt{(V/I)^2 + (R1 + R2)^2} \;\; = \omega\ (L_1+L_2\text{-}2M)$$

5. Determine the mutual inductance $M_{12}$using the relation, $M_{12}= (Xeq1 - Xeq2)/4\omega$
6. Check with the value obtained before andcomment on any discrepancy between these results.
7. Determine the coupling coefficient k. Where,k = $\frac{M}{\sqrt{L_1 L_2}}$

**Procedure V–Determination of driving point and transfer impedanceof Magnetically Coupled Coils.**

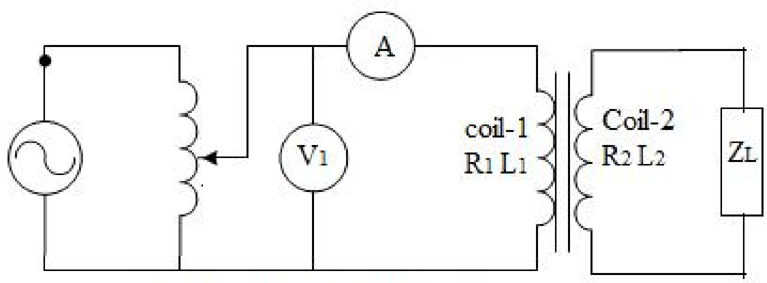



Fig-8,determination of D.P. & Transfer impedance.

1. Connect the electric circuit as shown in Fig. 8.
2. Adjust the magnitude of the applied voltage to a particular value and then take the readings of both the voltmeters and ammeter.
3. Calculate the driving point and transfer impedance.