Choice of Voltage of Transmission

- In both a.c. & d.c. systems, power is proportional to product of voltage & current.
- So, to transmit a certain amount of power as system voltage increases, current reduces.

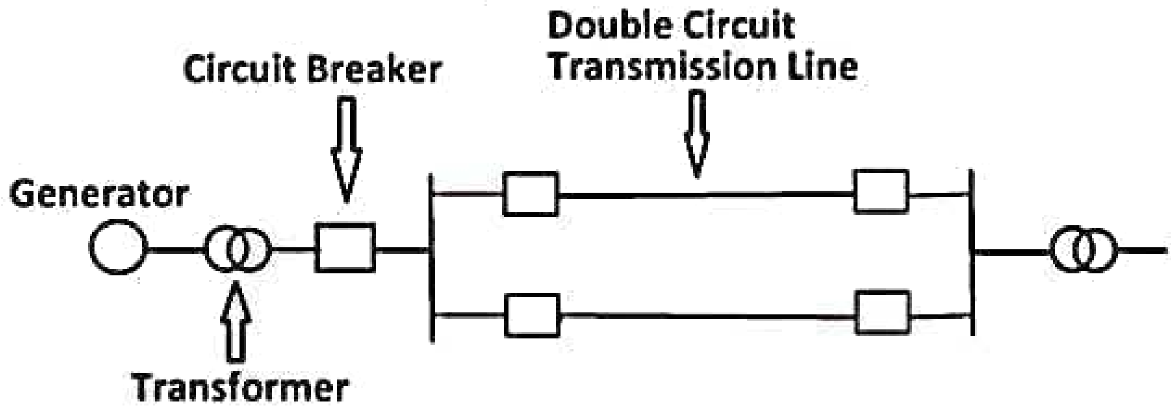
→ This, in turn allows to reduce conductor cross sectional area,

→ i.e. amount of conductor material requirement reduces for a specified voltage drop & line loss.

Further, for any existing system as the operating voltage increases → regulation & efficiency improves.

- But, a part of saving thus achieved is offset by the additional cost involved in the increased insulation level of the system.
- Also, there are some technical difficulties in designing insulation.
- With increased voltage level, spacing of conductors, ground clearance & charging current increase.

For the purpose of analysis consider a typical system:



- Cost of transmission line includes the following costs:

  (i) Transformer,

  (ii) Switchgear,

  (iii) Costs of insulators & line supports,

  (iv) Cost of conductor,

  (v) Costs of relays & protection systems and

  (vi) Sundry costs.

Costs of transformer & switchgear: May be represented as Rs.(A+BV)P, where A & B are constants and V & P being the system voltage & power respectively.

Costs of insulators & line supports: May be represented as Rs.(C+DV)L, where C & D are constants & L is length of the line.

Cost of conductor: A fixed current density (∂) is assumed.

$$\text{Now, } \partial \propto \frac{P}{V.a}$$

where, a is cross section of the conductor.

$$\text{So, } a \propto \frac{P}{\partial .V}$$

For a fixed current density, $a \propto \frac{P}{V}$

So, cost of conductor $\propto$ Volume of conductor (a.L).

Thus, cost of conductor = Rs. $E.\frac{PL}{V}$, E being a constant.

Costs of relays & protection systems and sundry costs: These are represented by a constant component of Rs. F.

So, total cost of the transmission line is

$$S = \text{Rs.}\left[(A+BV)P + (C+DV)L + E.\frac{PL}{V} + F\right]$$

To find out the voltage for minimum cost set $\frac{dS}{dV} = 0$.

i.e. $\frac{d}{dV}\left[(A+BV)P + (C+DV)L + E.\frac{PL}{V} + F\right] = 0.$

Or, $BP + DL - E\frac{PL}{V^2} = 0$

Or, $E\frac{PL}{V^2} = BP + DL$

$$\text{Or, } V = \sqrt{\frac{EPL}{(BP+DL)}}$$

While selecting the voltage, the following factors are to be considered:

1. The standard voltages,
2. Mechanical strength of the conductor,
3. Corona loss,
4. Voltage regulation &
5. Future growth.

Kelvin's Law

- Voltage drop is an important factor for distributor → since supply voltage to consumer must remain within ±5% for LV distribution & $\pm 12\frac{1}{2}\%$ for HV.
- Tappings in the form of service mains are taken from the distributor,
- So, a distributor has variable loading.
- Consumers situated near the feeding point enjoy better voltage compared to those situated near the far end.
- So, the distributor is designed on the basis of voltage drop.

On the contrary,

- Current loading of feeder is constant throughout whole length.
- So, feeder is designed on the basis of current carrying capacity & wherever practicable following maxm. financial economy as stated in Kelvin's Law.

Statement of Kelvin's law

The most economical cross section of conductor is that which makes annual value of interest & depreciation on the conductor equal to annual cost of energy wasted in conductor resistance.

➢ Proof

- Cost of conductor $\propto$ volume Conductor (a.l)

  $\propto$ cross 'a' → for given length 'l'

- Annual cost of interest & depreciation $\propto$ Cost of conductor

  $\propto$ cross section 'a'

So, annual cost of interest & depreciation = Rs. P.a .....(1)

where P is a constant.

Now, resistance of conductor $R = \rho \frac{l}{a}$, ρ = specific resistance.

So, a given length, $R \propto \frac{1}{a}$

Annual energy loss $\propto R \propto \frac{1}{a}$

Annual cost of energy loss = Rs. $\frac{Q}{a}$ ….. (2)

where Q is a constant.

Therefore, total annual cost 'S' = Rs. $(Pa + \frac{Q}{a})$ …..(3)

For min. annual cost, $\frac{ds}{da} = 0$

$$\text{Or,}\quad \frac{ds}{da} = P - \frac{Q}{a^2} = 0$$

$$\text{Or, } Pa = \frac{Q}{a} \quad \text{.....(4)}$$

Thus, for min. total annual cost → cost of interest & depreciation = cost of energy loss → This is Kelvin's law.

$$\text{Or, } a = \sqrt{\frac{Q}{P}} \quad \text{.....(5)}$$

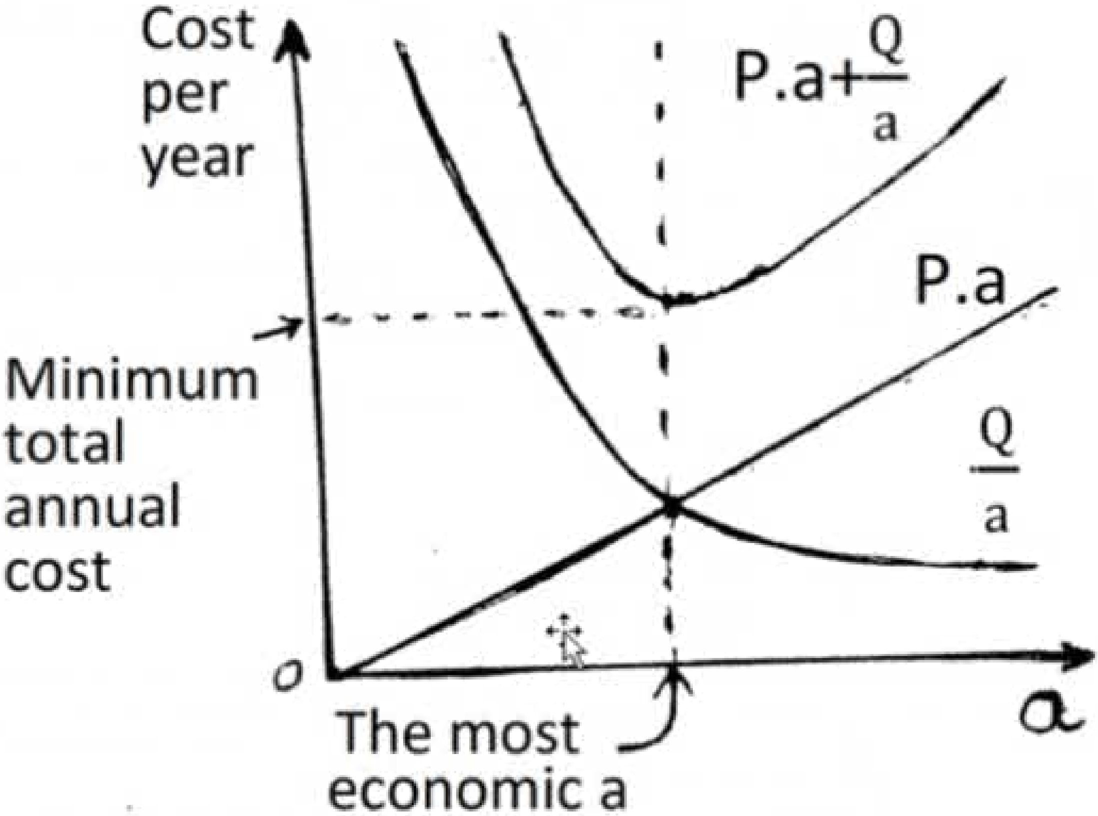


Fig. Illustration of Kelvin's law

Kelvin's law considering cost of insulation

- If cost of insulation is considered → as for u.g. cables, then
- It is independent of cross section 'a' &
- Insulation (i.e. cost of it) depends mainly on voltage.

Then, annual cost of interest & depreciation = Rs. (P.a + T).....(6)
where T is a constant.

Therefore, total annual cost 'S' = Rs. $(Pa + T + \frac{Q}{a})$ .....(7)

$$\frac{ds}{da} = \frac{d}{da}(Pa + T + \frac{Q}{a}) = 0 \rightarrow P - \frac{Q}{a^2} = 0$$

$$\text{Or, } Pa = \frac{Q}{a} \text{ .....(8)}$$

$$\text{Or, } a = \sqrt{\frac{Q}{P}} \text{ .....(9)}$$

So, cost of insulation has no influence on the most economic cross section.

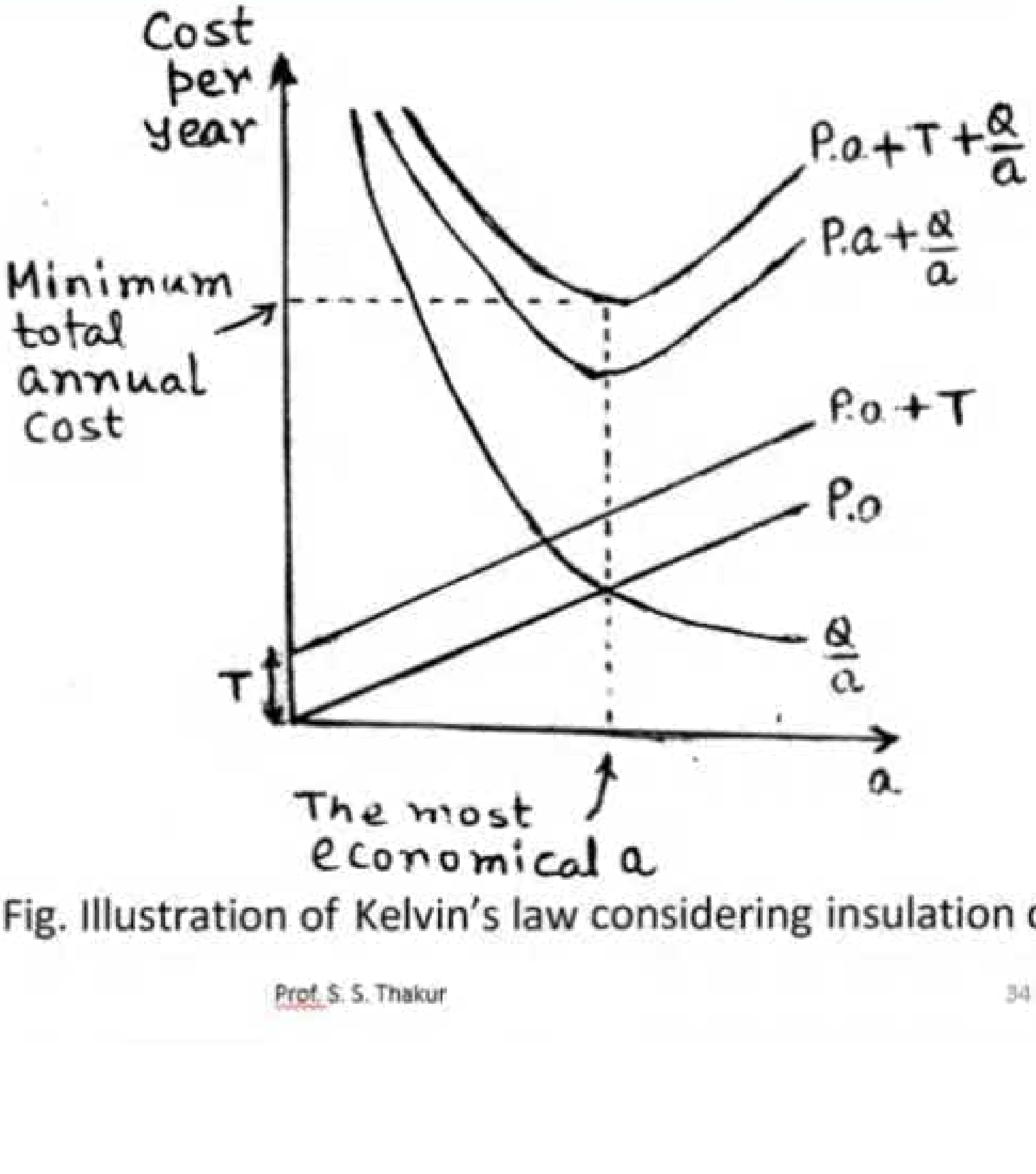


Fig. Illustration of Kelvin's law considering insulation cost

Prob. 20 (PS, pp 233): Determine the most economical cross section for a 3Φ line, 6km long to supply at a constant voltage of 30kV, following the daily load cycle: 10hr. at 3MW, 0.8p.f. ; 6hr. at 1.5MW, 0.9p.f. ; 8hr. at 0.5MW, 0.9p.f. The line is in use 365 days yearly. The cost per km of line, completely erected = £(1500 + 1200a), where a = cross section in cm^2 of each conductor. Interest = 8% of capital cost. Energy costs = 1.4d per kWh. The resistance per km of conductor of cross section a is $\frac{0.185}{a}\,\Omega$.

Daily energy loss

$$= 3\left[\left(\frac{3.10^6}{\sqrt{3}.30.10^3.0\cdot8}\right)^2.10+\left(\frac{1\cdot5.10^6}{\sqrt{3}.30.10^3.0\cdot9}\right)^2.6+\left(\frac{0.5.10^6}{\sqrt{3}.30.10^3.0\cdot9}\right)^2.8\right].R$$

$$= 177512\cdot033R$$

$$R = \text{Resistance per conductor for 6 km} = \frac{0\cdot185}{a}.6$$

$$\text{Energy loss per year} = 365.177512\cdot033.\frac{0\cdot185}{a}.6$$

$$= \frac{71918987\cdot90}{a} \text{ watt-hour}$$

$$= \frac{71918\cdot98790}{a} \text{ kWh}$$

Cost of annual energy loss

$$= \frac{1.4}{240} \cdot \frac{71918.98790}{a}$$

[Note: 12 Pence = 1 shilling
20 shilling = 1 Pound]

Cost of interest $= \frac{8}{100} \cdot (1200a \cdot 6)$

According to Kelvin's law:

$$\frac{8}{100} \cdot (1200a \cdot 6) = \frac{1.4}{240} \cdot \frac{71918.98790}{a}$$

or, $a = 0.8534\ cm^2$

Limitations of Kelvin's law

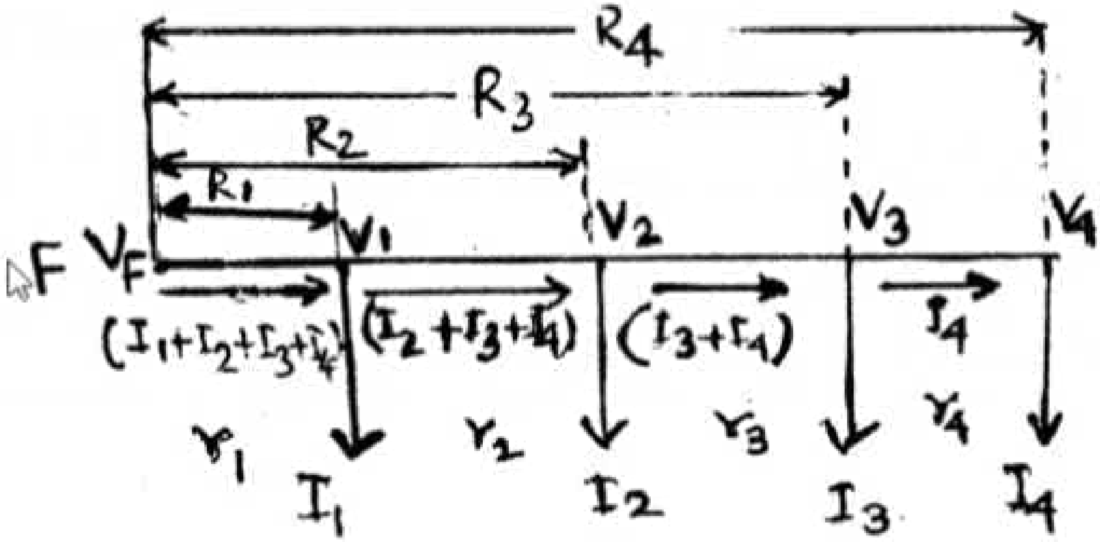
1. It is difficult to calculate the energy loss correctly as the load varies continuously.
2. This law is artificial and has no relation to the physical aspects like temperature rise, voltage drop etc.
3. The size of conductor obtained from Kelvin's law might be too small so as to have adequate mechanical strength.

Distribution Systems Calculations

- Voltage drop is an important factor for distributor.
- Supply voltage to consumer must remain within $\pm 5\%$ for LV distribution & $\pm 12\frac{1}{2}\%$ for HV.
- Consumers situated near the feeding point enjoy better voltage compared to those situated near the far end.
- So, it is necessary to calculate the voltage at the consumer's terminals.
- Accordingly, distribution systems calculations are important.

D.C. Radial Distributor Fed at One End (for concentrated loads)

- Consider the distributor with four load currents I_1, I_2, I_3 & I_4.
- Voltages of the four load points be V_1, V_2, V_3 & V_4.
- Resistance of each conductor of sections be r_1, r_2, r_3 & r_4.
- Feeding point be F & voltage at feeding point voltage be V_F.



Total voltage drop $= 2(I_1+I_2+I_3+I_4)r_1$
$+ 2(I_2+I_3+I_4)r_2 + 2(I_3+I_4)r_3$
$+ 2(I_4).r_4$
$= 2.I_1 r_1 + 2I_2[r_1+r_2] + 2I_3[r_1+r_2+r_3]$
$+ 2I_4[r_1+r_2+r_3+r_4]$
$= 2.I_1R_1 + 2I_2R_2 + 2I_3R_3 + 2I_4R_4$
$= 2.\sum I.R$

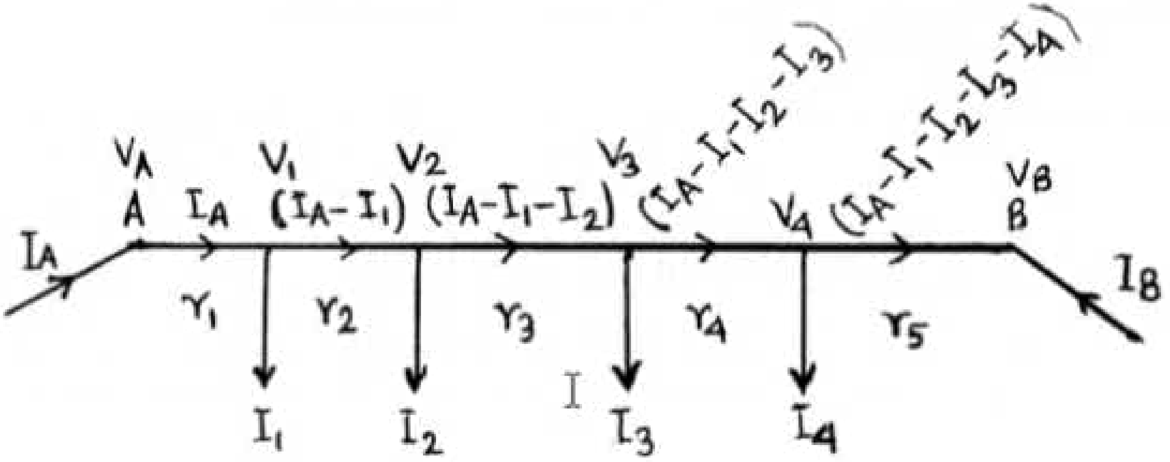
In the above '2' comes for go & return conductors.

So, $V_1 = V_F - 2(I_1+I_2+I_3+I_4)r_1$
$V_2 = V_1 - 2(I_2+I_3+I_4)r_2$
$V_3 = V_2 - 2(I_3+I_4)r_3$
$V_4 = V_3 - 2(I_4)r_4$

D.C. Radial Distributor Fed at Both Ends (for concentrated loads)

- For long distributors carrying heavy loads voltage drop may be considerably high → resulting voltage of consumers at far end beyond tolerable limits.
- Under such situation distributors are fed at both ends.
- Then, point of min. potential occurs at some point where currents meet from both ends.
- Voltage drop up to point of min. potential is much less compared to the distributor fed at one end only.

- Consider the distributor with four load currents I_1, I_2, I_3 & I_4.
- Voltages of the four load points be V_1, V_2, V_3 & V_4.
- Resistance of each conductor of sections be r_1, r_2, r_3, r_4 & r_5.
- Two feeding point voltages be V_A & V_B.
- I_A & I_B be the currents fed from the two ends.



Using Kirchhoff's Current law,

$$I_A + I_B = I_1 + I_2 + I_3 + I_4 \quad ---(1)$$

If r_1, r_2, r_3, r_4 and r_5 be the resistances of each conductor for the five sections, then voltage drop from A to B,

$$V_A - V_B = 2 I_A . r_1 + 2(I_A - I_1).r_2 + 2(I_A - I_1 - I_2).r_3$$
$$+ 2(I_A - I_1 - I_2 - I_3) r_4 + (I_A - I_1 - I_2 - I_3 - I_4).r_5 \quad ---(2)$$

In eqn.(2) known values are

- Two feeding point voltages V_A and V_B.
- Load currents I_1, I_2, I_3, I_4 and I_5.
- Sectional resistances r_1, r_2, r_3, r_4 and r_5.

So, I_A can be calculated.

- Then I_B can be calculated from eqn. (1)

Once I_A is known, one may find

- Currents in all the sections
- Voltage at any loading point.

So,

$$V_1 = V_A - 2.I_A.r_1$$

$$V_2 = V_1 - 2.(I_A - I_1).r_2$$

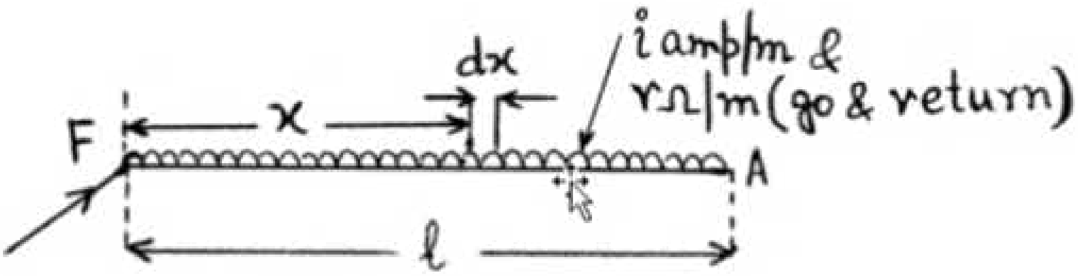
$$V_3 = V_2 - 2(I_A - I_1 - I_2).r_3$$

$$V_4 = V_3 - 2(I_A - I_1 - I_2 - I_3).r_4$$

The point of minimum potential is that where currents meet from two sides.

Uniformly Loaded Distributor Fed At One End

- When the distributor supplies similar houses or establishments on a long street &
- When approximately equal loads are tapped off at regular & brief intervals.
- Loads can be considered to be uniformly distributed.
- Let, the uniformly loaded distributor be fed at one end only.



- Here, current loading is i a/m & r= resistance/m (go & return).

Current through the small section $dx = (l.i - x.i) = (l - x).i$ amp

Voltage drop dv in the section $dx = (l-x).i.(r.dx) = (l-x).ir\,dx$

So, the total voltage drop from the feeding point F to far end A

$$= \int_0^l dv = \int_0^l (l-x).ir\,dx = \int_0^l l.ir.dx - \int_0^l i.r.x\,dx = l^2.ir - ir.\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{1}{2} l^2.ir = \frac{1}{2}(l.i).(l.r) = \frac{1}{2} I.R \text{ volts}$$

Where, $I = (l.i)$ = Total consumption in the distributor

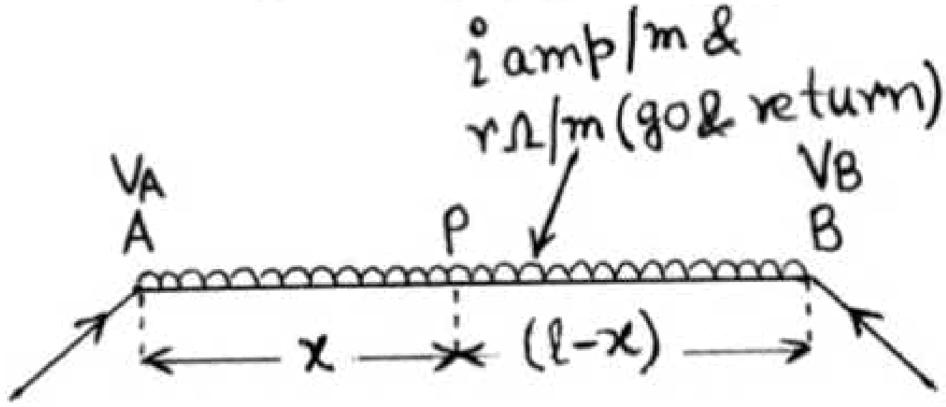
$R = (l.r)$ = Total go & return resistance of the distributor.

Upto any distance x, voltage drop is given by

$$v_x = \left(l.ir.x - ir\frac{x^2}{2}\right) \text{ volts}$$

Uniformly Loaded Distributor Fed At Both Ends

- For long uniformly distributors (fed at one end) voltage drop may be considerably high → resulting voltage of consumers at far end beyond tolerable limits.
- Under such situation Uniformly loaded distributors are fed at both ends.
- Then, point of min. potential occurs at some point where currents meet from both ends.
- Voltage drop up to point of min. potential is much less compared to the case when distributor fed at one end only.



- Consider, P be the point of min. potential in the system situated at a distance x from end A.
- So, current fed from end A is just enough to meet the load from A to P.
- Likewise, current fed from end B is just enough to meet the load from B to P.

Voltage drop from A to P $= v_1$

$$= ir\frac{x^2}{2}$$

Voltage drop from B to P $= v_2$

$$= ir\frac{(l-x)^2}{2}$$

Voltage drop from A to B $= V_A - V_B = v_1 - v_2 = v$ (Say)

$$\text{So, } v = ir.\frac{x^2}{2} - ir\frac{(l-x)^2}{2} = \frac{ir}{2}\left[x^2 - (l-x)^2\right]$$

$$= -\frac{ir}{2}.l^2 + irlx$$

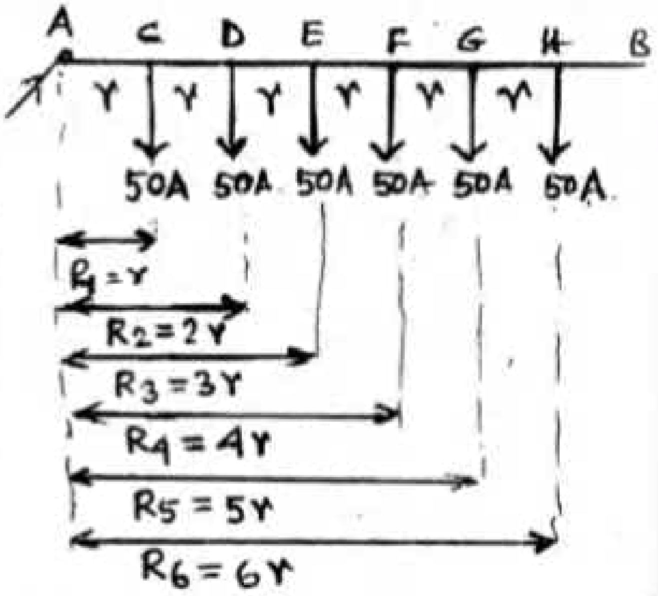
Therefore, point of minimum potential

$$x = \left[\frac{l}{2} + \frac{v}{irl}\right]$$

Knowing x, voltage drop upto P may be calculated.

If $V_A = V_B$, $v = 0$ and $x = \frac{l}{2}$, i.e. at the mid point.

Problem: A 2-wire distributor AB is fed at A & supplies six concentrated loads, each of 50A at C, D, E, F, G & H, as shown in the fig. What must be the resistance of each section so that the max. voltage drop for ant consumer does not exceed 7V? Also calculate the power loss with this resistance. Assume, AC=CD=DE=EF=FG=GH=HB.



Soln:

Since all the segments are of equal length, consider resistance of each segment is r (go and return). So, the resistances from the feeding point to the different loads are as shown.

$\therefore$ Total voltage drop from A to B

$= $ voltage drop from A to H

$= \sum I.R = 50[r + 2r + 3r + 4r + 5r + 6r] = 50.21r$

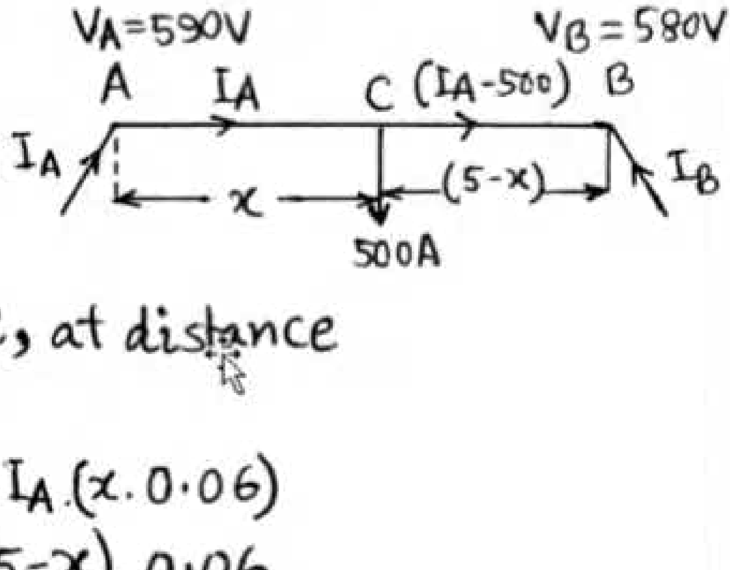
By the problem, $50.21r = 7$

or, $r = \dfrac{7}{50.21} = \dfrac{1}{150}\,\Omega$

Total power loss $= r.(300)^2 + r(250)^2 + r(200)^2 + r(150)^2 + r(100)^2 + r(50)^2$

$= r.\ 227500 = \dfrac{1}{150}.\ 227500 = 1516.\dot{6}\dot{6}\ W$

Problem: An electric train moving in a section of line between two substations, takes a current of 500A. The substations are 5km apart & are maintained at 590V and 580V respectively. The track resistance is 0.06Ω/km go and return. Show graphically the variation of current received from either substation & find the point of min. potential along the track. Also, calculate the current taken from each substation at point of min. potential.



Soln:

Let, the train is at some point C, at distance x km from the feeding point A.

Then, the potential at $C = 590 - I_A (x \cdot 0.06)$

$$= 580 + (I_A - 500)(5 - x) \cdot 0.06$$

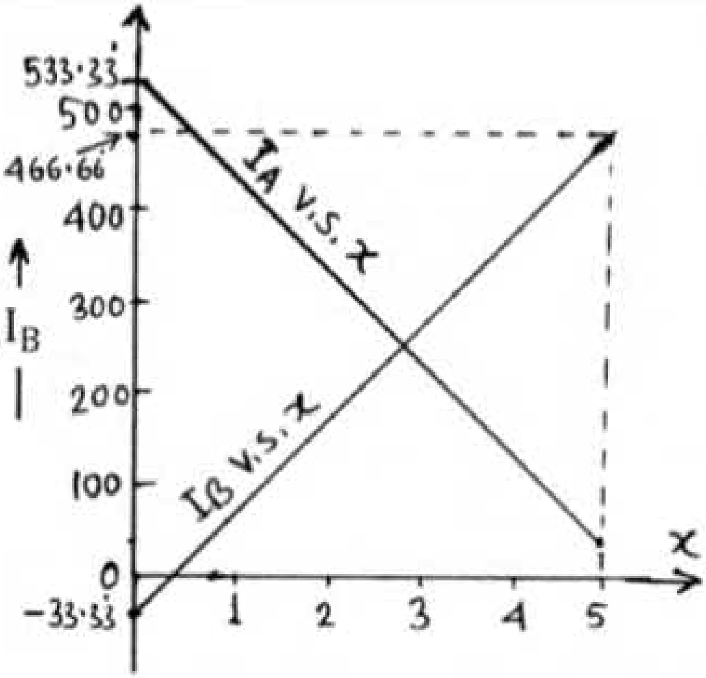
Simplifying,

$$I_A = 533.3\dot{3} - 100x \text{ ----(1)}$$

Current supplied by substation B,

$$I_B = -(I_A - 500)$$

$$= -33.3\dot{3} + 100x \text{ ---(2)}$$



So, potential at $C = 590 - I_A.(x.0.06)$

$$= 590 - (533.3\dot{3} - 100x)(x.0.06)$$

$$= 590 - 31.9\dot{9}x + 6x^2 \text{ ----(3)}$$

$$= V_C \text{ (say)}$$

For point of minimum potential,

$$\frac{dV_c}{dx} = -31.9\dot{9} + 12x = 0$$

or, $x = 2.66$ km (from A)

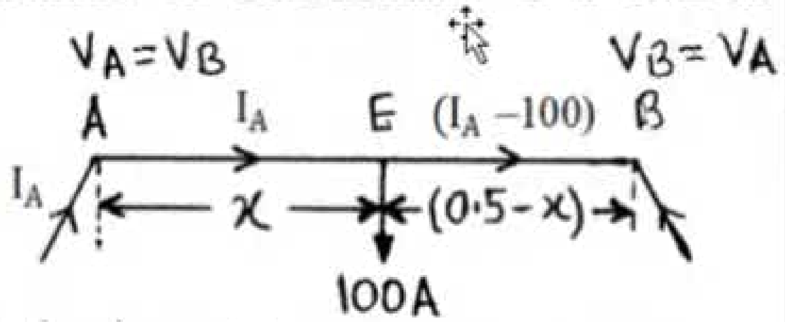
At point of min. potential $V_C = 590 - 31.9\dot{9} \cdot 2.6\dot{6} + 6 \cdot (2.6\dot{6})^2$
$= 511.77$ V

For point of min. potential

$$I_A = 533.3\dot{3} - 100 \cdot 2.66 = 266.6\dot{6} \text{ A}$$

$$I_B = -33.3\dot{3} + 100x = 233.3\dot{3} \text{ A}$$

Problem: A bus running at 30miles/hour takes 100A from a system which has a loop resistance of 0.25Ω/mile & is fed at equipotential points 0.5 mile apart. Find the rate of change of voltage w.r.t. time at the bus when it is (i) 220 yards & (ii) 440 yards from a feeding point.



Loop resistance = go & return resistance = $r = 0.25\ \Omega$/mile

Soln:

Consider, the bus is at a distance x mile from the feeding point A.

Voltage drop from A to B $= V_A - V_B$

$$= I_A.(r.x) + (I_A - 100)\{r.(0.5 - x)\} = 0$$

Simplifying, $I_A = 100 - 200x$ ...... (1)

Then, voltage at point E $= V_E = V_A - I_A.(r.x)$

$= V_A - (100 - 200x).(r.x)$

$= V_A - 100r.x + 200.r.x^2$

Differentiating w.r.t. time,

$$\frac{dV_E}{dt} = 0 - 100.r.\frac{dx}{dt} + 400.r.x.\frac{dx}{dt}$$

$$= [400.r.x - 100r].\frac{dx}{dt} \quad \text{.....} (2)$$

Now, speed of the bus $= \frac{dx}{dt} = 30$ miles/hour

$= \frac{30}{60.60}$ miles/sec $= \frac{1}{120}$ miles/sec

$$\therefore \frac{dV_E}{dt} = [400.0.25.x - 100.0.25].\frac{1}{120}$$

$$= [100x - 25].\frac{1}{120} \quad \text{......} (3)$$

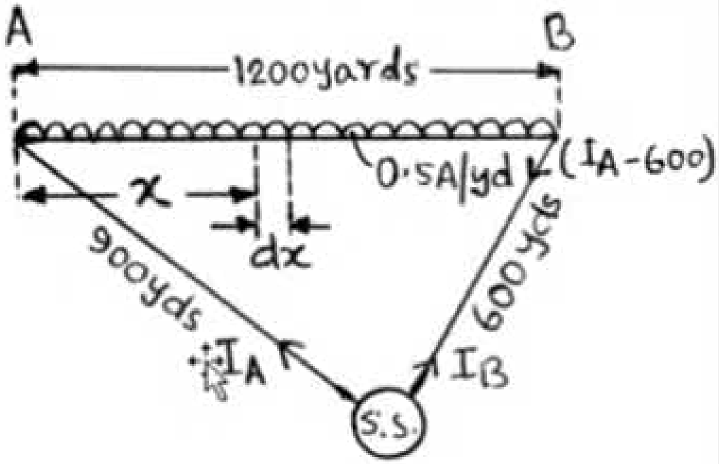
(a) 220 yards $= \frac{220}{1760} = \frac{1}{8}$ mile

$$\therefore \left.\frac{dV_E}{dt}\right|_{220\text{yards}} = \left[100 \cdot \frac{1}{8} - 25\right] \cdot \frac{1}{120} = -0.10416 \text{ V/sec}$$

(b) 440 yards $= \frac{440}{1760} = \frac{1}{4}$ mile

$$\therefore \left.\frac{dV_E}{dt}\right|_{440\text{yards}} = \left[100 \cdot \frac{1}{4} - 25\right] \cdot \frac{1}{120} = 0 \text{ V/sec}$$

Prob: A distributor 1200yds long, carries a uniformly distributed load of 0.5A/yd. It is supplied at both ends from a substation by feeders, one 900yds & the other 600yds long. The feeders have a cross sectional area 50% greater than that of distributor. Find the point where the consumer's voltage is the lowest & the current supplied by each feeder.



Soln: Let, the resistance of the feeder for one conductor be $r\,\Omega$/yd. Then distributor resistance is $1.5r\,\Omega$/yd.

$$\left[r_{dist.} = \rho \frac{l}{a} ; r_{feeder} = \rho \frac{l}{1.5a} = \frac{1}{1.5} r_{dist.} \right.$$

or, $r_{dist.} = 1.5\, r_{feeder}$.

$$\left. \text{If, } r_{feeder} = r\ \Omega/yd \text{, then } r_{dist.} = 1.5 r\ \Omega/yd \right]$$

Equating voltage drop from s.s. through A & B and back to s.s.

$$I_A.(2r.900) + \int_0^{1200} (I_A - 0.5x)(2.1.5r\,dx) + (I_A - 600).(2r.600) = 0$$

Solving, $I_A = 272.7272\text{ A}$

Current supplied by the other feeder $= I_B = -(I_A - 600)$

$$= 327.2727\text{A}$$

For point of minimum potential, $\frac{dv}{dx} = 0$

Now, the voltage drop in the elemental distance dx

$= dv = (I_A - 0.5.x)(2.1.5r\,dx)$

or, $\frac{dv}{dx} = (I_A - 0.5x)(2.1.5r) = 0$

or, x = distance of point of minimum potential from A

$= 545.4545$ yds.