Effect of Earth on Capacitance

- In discussions so far, effect of earth has been neglected.
- It was assumed that conductors are situated in free space.
- Actually, the conductors run parallel to the ground.

- Earth is assumed to behave like an infinite, perfectly conducting plane.
- Its presence therefore, modifies the electric field of the line.
- This causes change in capacitance.

- Consider a circuit, comprising of a single overhead conductor with return path through the earth.
- If the conductor carries a charge of +q coul/m, then earth also has the same charge with opposite sign.

- Then, a potential difference exists between the conductor & earth.
- Electrical fluxes from the charges on the conductor enter the equipotential surface of earth perpendicularly.
- Since the earth's surface is assumed to be a perfect conductor.

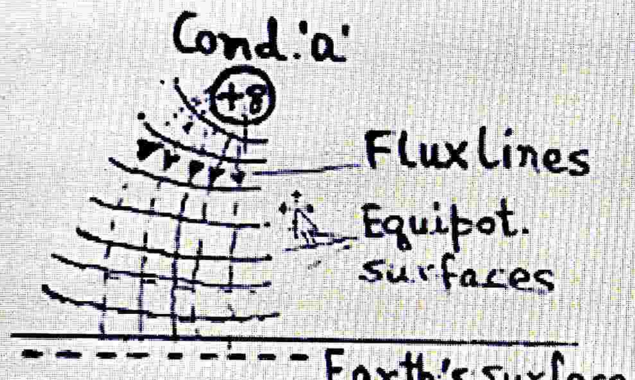


Fig.1

- Assume a fictitious cond. of the same size & shape as the overhead cond. lying directly below the original cond. at a distance 2h as shown.
- If the earth is removed and a charge of $-q$ coul/m is given to the fictitious cond. then,

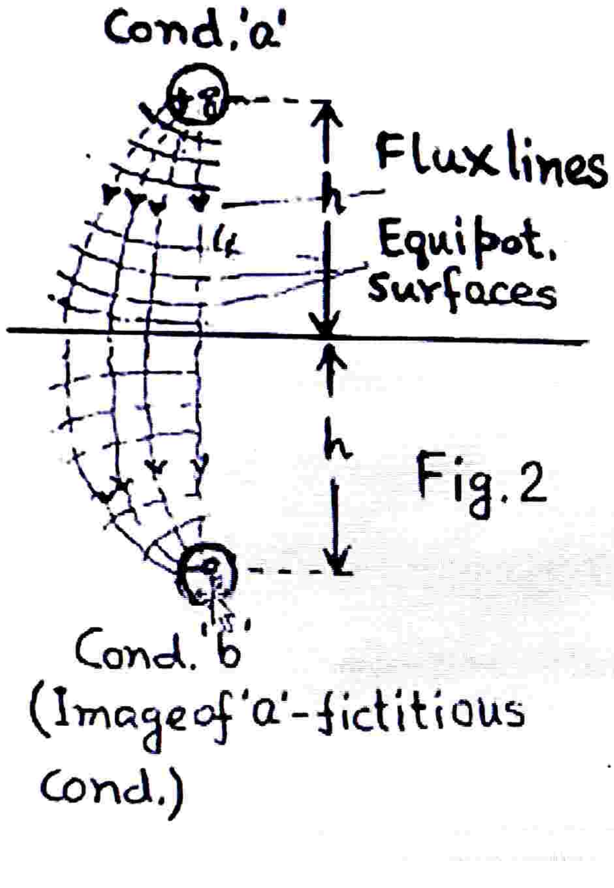


Fig. 2

- The plane midway between the two conds. is an equipotential surface & occupies the same position as the equipotential surface of the earth.
- Electric flux between the cond. & this equipotential surface is same as that existed between the cond. & the earth.

- So, for capacitance calculation fig.1 may be replaced by fig.2.
- Such a cond. is called the image conductor.
- This process may be extended for more than one cond.

Here, potential difference between conds. 'a' & 'b' (image of 'a') is

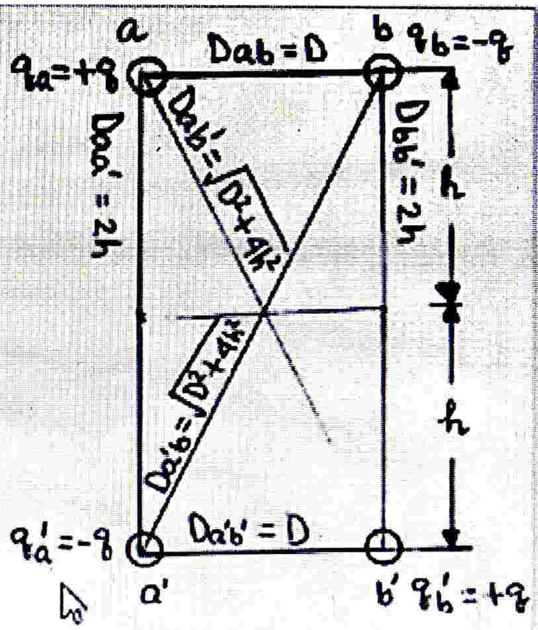
$$V_{ab} = 2V_{an} = \frac{1}{2\pi k}\left[q_a \ln\frac{D_{ab}}{D_{aa}} + q_b \ln\frac{D_{bb}}{D_{ba}}\right]$$

$$= \frac{q}{2\pi k}\left[\ln\frac{2h}{r} - \ln\frac{r}{2h}\right] = \frac{2q}{2\pi k}\cdot\ln\frac{2h}{r} \quad \ldots\ldots(1)$$

or, $V_{an} = \frac{q}{2\pi K} \ln \frac{2h}{r}$ Volts ..... (2)

So, Capacitance to ground $= C_{an} = \frac{q}{V_{an}} = \frac{2\pi K}{\ln \frac{2h}{r}}$ F/m ..... (3)

Effect of Earth on a Single Phase Line



Here, $q_a = +q$ coul./m ; $q_b = -q$ Coul./m ;

$q_a' = -q_a = -q$ coul./m;

$q_b' = -q_b = +q$ Coul/m.

Now,

$$V_{ab} = \frac{1}{2\pi K}\left[q_a \ln \frac{D_{ab}}{D_{aa}} + q_b \ln \frac{D_{bb}}{D_{ba}} + q_a' \ln \frac{D_{a'b}}{D_{a'a}} + q_b' \ln \frac{D_{b'b}}{D_{b'a}}\right]$$

$$= \frac{1}{2\pi K}\left[q \ln \frac{D}{r} - q \ln \frac{r}{D} - q \ln \frac{\sqrt{D^2+4h^2}}{2h} + q \ln \frac{2h}{\sqrt{D^2+4h^2}}\right]$$

$$= \frac{2q}{2\pi K}\left[\ln \frac{D}{r} + \ln \frac{2h}{\sqrt{D^2+4h^2}}\right]$$

$$= \frac{2q}{2\pi K}\left[\ln \frac{D \cdot 2h}{r \cdot \sqrt{D^2+4h^2}}\right]$$

Therefore, line to line capacitance $C_{ab} = \dfrac{q}{V_{ab}}$

$$= \frac{q}{\frac{2q}{2\pi K}\left[\ln \frac{D.2h}{r.\sqrt{D^2+4h^2}}\right]}$$

$$= \frac{\pi K}{\ln\left(\frac{D}{r}\cdot\frac{1}{\sqrt{D^2/4h^2+1}}\right)} \text{ F/m} \ldots\ldots (1)$$

- To neglect effect of earth put $h \to \infty$. Then, $\sqrt{D^2/4h^2+1} \to 1$.

So, $C_{ab} = \dfrac{\pi K}{\ln \frac{D}{r}}$ F/m $\cdots\cdots$ (2)

$\therefore$ Line to neutral capacitance $= C_n = \dfrac{q}{\frac{1}{2}V_{ab}}$

$$= \frac{2\pi K}{\ln\left(\frac{D}{r}\cdot\frac{1}{\sqrt{D^2/4h^2+1}}\right)} \text{ F/m} \ldots\ldots (3)$$

- Due to presence of $\frac{1}{\sqrt{D^2/4h^2+1}}$ the total Value of the denominator reduces.
- Hence, the capacitance value increases with the presence of earth.

Performance of Transmission Lines

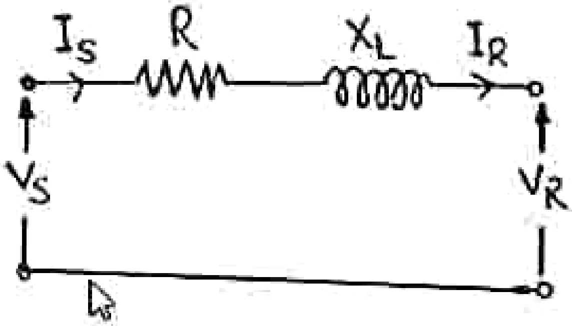
- Any transmission line has four distributed parameters R, L, C & G.
- These parameters together with load current & p.f. determine the electrical performance of the line.
- The term performance includes basically the calculation of sending end voltage, sending end current, sending end p.f., power loss in the line, efficiency of transmission, regulation etc.

- Usually, the values of receiving end voltage, current & p.f. are known.
- Prior performance calculations are useful in system planning.

- The predominance of one or more of the parameters of a line is governed by its length and cond. configuration.
- For overhead lines upto 80km the capacitance 'C' is negligibly small.
- All low voltage overhead lines having length upto 80 km are generally categorized as short lines.
- The lines ranging in length from 80km to 240km are termed as medium or moderately long lines.
- For such lines, 'C' is considered to be lumped at one or more points of the line. The leakage conductance 'G' is neglected.
- The term long line refers to a line having its length more than 240km.
- The long line treatment takes all the four parameters into account in completely distributed way.

- The above classification on the basis of length is not necessarily a perfect criterion to distinguish between short, medium & long lines.
- This classification has been done for the accuracy desired.
- The methods used for short & medium lines are approximate
- The method adopted for long line is rigorous.
- The approximate methods are simple without much appreciable error.

Short Transmission Line



- In this case effects of C & G are neglected.
- Only effects of R & L are considered.
- So, current entering the line (I_S) is equal to current leaving the line (I_R).
- Since same current flows through all sections, R & L are treated as lumped.

Here,

V_S = phase voltage at sending end,

V_R = phase voltage at receiving end,

I_S = phase current at sending end,

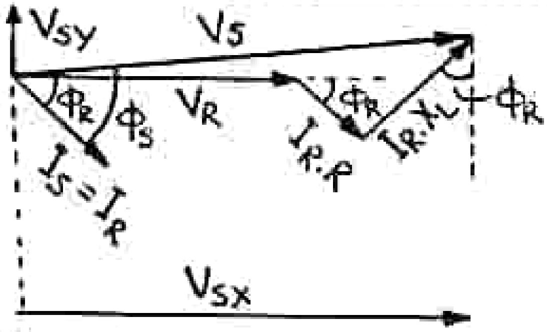
I_R = phase current at receiving end,

$\cos\phi_S$ = sending end p.f.,

$\cos\phi_R$ = receiving end p.f.,

R = total resistance per phase and

X_L = total inductive reactance per phase.



- Here, I_S & I_R are equal in magnitude but not in phase.

From the equivalent circuit,

$$|I_S| = |I_R| \cdots\cdots (1)$$

$$V_S = V_R + (R + jX_L).I_R$$

$$= V_R + Z.I_R \text{ --------- } (2)$$

where, $Z = R + jX_L$.

- Hence, if the receiving end conditions are known sending end voltage may be calculated.

- A more approximate method involving scalar quantities is as follows:

$$V_{sx} = V_R + (I_R.R)\cos\phi_R + (I_R.X_L)\sin\phi_R \text{ -----(3)}$$

$$V_{sy} = (I_R.X_L)\cos\phi_R - (I_R.R)\sin\phi_R \text{ ---------(4)}$$

$$\& \quad V_S^2 = V_{SX}^2 + V_{SY}^2 \text{ ----------------------(5)}$$

- However, $(I_R.R)$ & $(I_R.X_L)$ are very much less than V_R.
- Also, the small V_{sy} is in quadrature with much larger V_{sx}

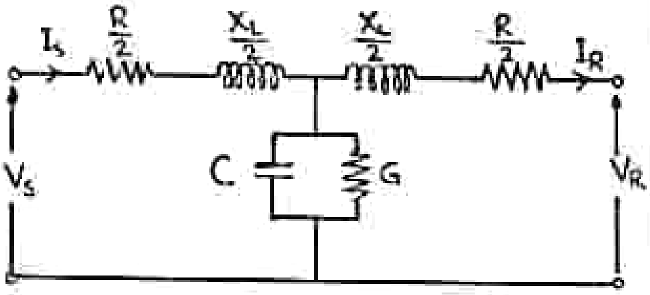
Hence, $V_s \approx V_{sx} = V_R + (I_R.R)\cos\phi_R + (I_R.X_L)\sin\phi_R$ ----- (6)

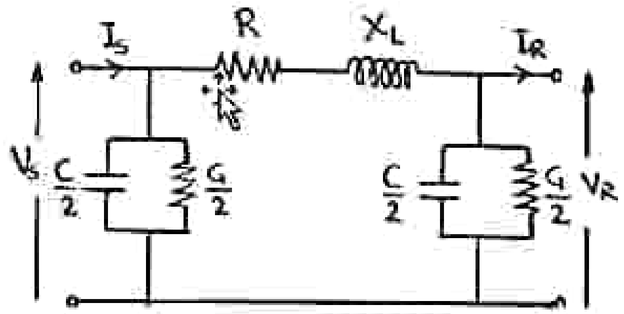
- Voltage regulation of the line is the rise in voltage when full load is removed.

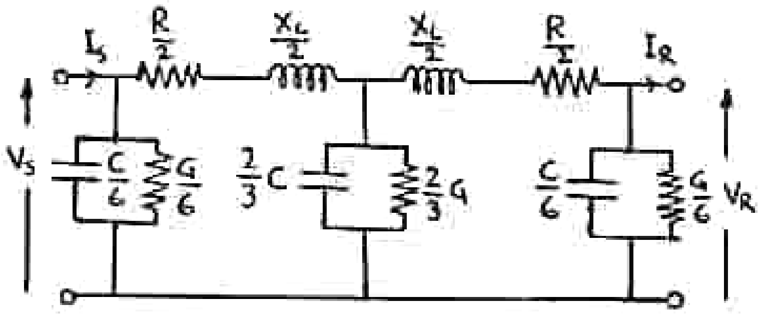
Hence, % Voltage regulation $= \frac{V_S - V_R}{V_R} \times 100 = \frac{(I_R.R)\cos\phi_R + (I_R.X_L)\sin\phi_R}{V_R} \times 100$ -----(7)

Moderately Long Transmission Line

- As the length & voltage of transmission line increases – effect of capacitance –hence, charging current becomes significant.
- For medium length lines, voltages up to about 100kV, it is sufficiently accurate to consider total capacitance to be lumped at some particular points.
- Some arrangements are shown.





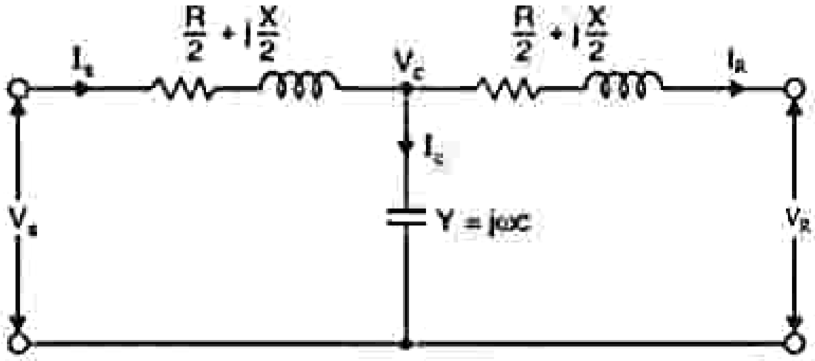


Nominal T – Representation

- Shunt conductance is neglected.

Here,

$V_C = V_R + I_R.\frac{Z}{2}$, where $Z = R + jX_L$



Nominal-T

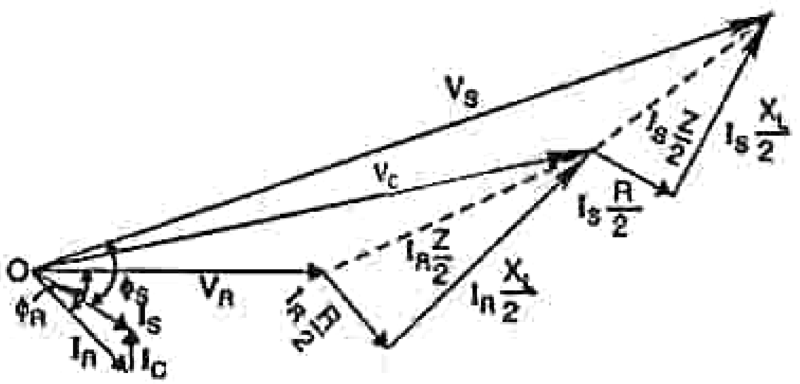
$I_C = V_C.Y$, where $Y = j\,\omega C$

$I_S = I_R + I_C$

$= I_R + V_C.Y$

$= I_R + (V_R + I_R.\frac{Z}{2}).Y$

$= V_R.Y + I_R.(1+\frac{YZ}{2})$.....(1)



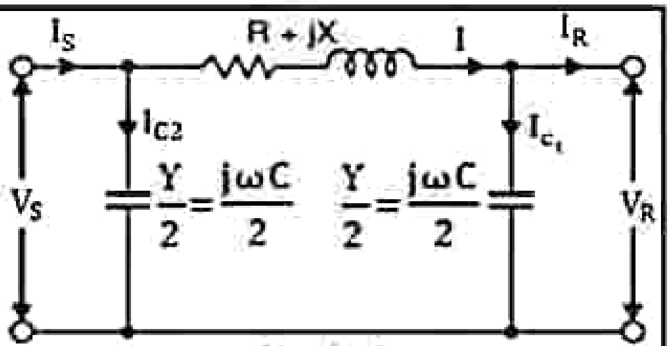
$V_S = V_C + I_S.\frac{Z}{2}$

$= V_R + I_R.\frac{Z}{2} + \{V_R.Y + I_R.(1+\frac{YZ}{2}).\frac{Z}{2}\}$

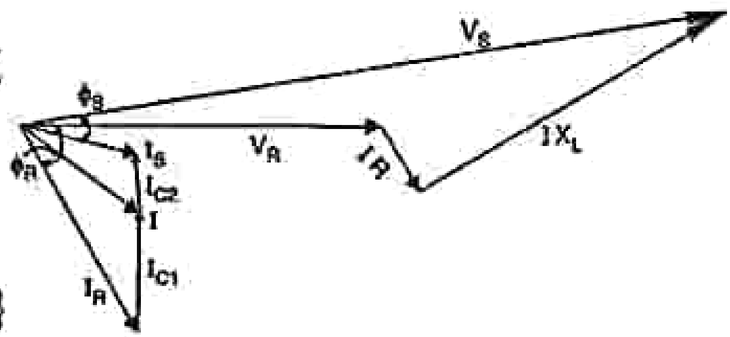
$= V_R.(1+\frac{YZ}{2}) + I_R.(1+\frac{YZ}{4}).Z$ ..... (2)

Nominal π– Representation

- Shunt conductance is neglected.



Nominal-π



Here,

$$I_{C1} = V_R \cdot \frac{Y}{2}$$

$$I = I_{C1} + I_R = V_R \cdot \frac{Y}{2} + I_R$$

$$V_S = V_R + I.\,Z = V_R + \left(V_R \cdot \frac{Y}{2} + I_R \right).\,Z$$

$$= V_R.\left(1 + \frac{YZ}{2} \right) + I_R.\,Z \;\ldots\ldots (1)$$

$$I_{C2} = V_S \cdot \frac{Y}{2} = \frac{Y}{2} \cdot \left\{ V_R.\left(1 + \frac{YZ}{2} \right) + I_R.\,Z \right\}$$

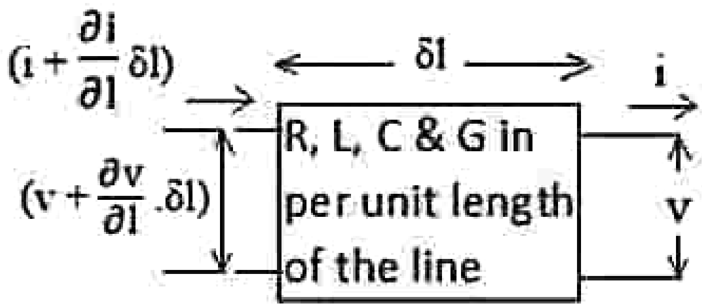
$$= V_R.\,Y.\left(\frac{1}{2} + \frac{YZ}{4} \right) + I_R.\,\frac{YZ}{2}$$

$$I_S = I + I_{C2} = \left(V_R \cdot \frac{Y}{2} + I_R \right) + \left\{ V_R.\,Y.\left(\frac{1}{2} + \frac{YZ}{4} \right) + I_R.\,\frac{YZ}{2} \right\}$$

$$= V_R.\,Y.\left(1 + \frac{YZ}{4} \right) + I_R.\left(1 + \frac{YZ}{2} \right) \;\ldots\ldots (2)$$

Long Transmission Line

- For long lines values of C & G are significant.
- All line parameters R, L, C & G are considered to be uniformly distributed.
- Consider, an elemental distance δl of a line at any distance 'l' from the receiving end.
- R, L, C & G are given per unit length of the line.
- Let, current & voltage leaving the section be i & v respectively.



- So, current & voltage entering the section are $(i + \frac{\partial i}{\partial l}\delta l)$ & $(v + \frac{\partial v}{\partial l}.\delta l)$ respectively.
- Electrostatic charge in elemental length δl is $(v.\ C.\ \delta l)$

- Current leaking through this capacitance is $\frac{\partial}{\partial t}(v. C. \delta l)$.
- Again, current leaking through conduction $(G. \delta l)$ is $(v. G. \delta l)$.

So,

$$(i + \frac{\partial i}{\partial l}\delta l) - i = v. G. \delta l + \frac{\partial}{\partial t}(v. C. \delta l)$$

$$\text{or, } \frac{\partial i}{\partial l} = v. G + C. \frac{\partial v}{\partial t} = (G + C. \frac{\partial}{\partial t}).v \text{ ..... (1)}$$

- Again, voltage drop in resistance $(R. \delta l)$ is $(i. R. \delta l)$.
- Similarly, voltage drop in inductance $(L. \delta l)$ is $(L. \delta l). \frac{\partial i}{\partial t}$.

So,

$$(v + \frac{\partial v}{\partial l}.\delta l) - v = i. R. \delta l + L. \delta l. \frac{\partial i}{\partial t}$$

$$\text{or, } \frac{\partial v}{\partial l} = i. R + L. \frac{\partial i}{\partial t} = (R + L. \frac{\partial}{\partial t}). i \text{ ..... (2)}$$

or, $\frac{\partial^2 v}{\partial l^2} = (R + L.\frac{\partial}{\partial t}).\frac{\partial i}{\partial l} = (R + L.\frac{\partial}{\partial t}).(G + C.\frac{\partial}{\partial t}).v$ ..... (3)

- When dealing with sinusoidal functions, the operator $\frac{\partial}{\partial t}$ may be replaced by $j\omega$, where $\omega = 2\pi f$. So,

  $\frac{\partial^2 v}{\partial l^2} = (R + j\omega L).(G + j\omega C).v = m^2.v$ ..... (4)

where, $m^2 = (R + j\omega L).(G + j\omega C)$ ..... (5)

The solution of eqn. (4) is of the form:

$v = C_1.e^{ml} + C_2.e^{-ml}$ ..... (6)

where, C_1 & C_2 are constants.

From eqn. (6), $\frac{\partial v}{\partial l} = C_1.m.e^{ml} - C_2.m.e^{-ml}$

or, $(R + j\omega L).i = C_1.m.e^{ml} - C_2.m.e^{-ml}$ {from eqn. (2)}

Or, $i = \frac{\sqrt{(R + j\omega L).(G + j\omega C)}}{(R + j\omega L)} . [C_1 . e^{ml} - C_2 . e^{-ml}]$

$$= \frac{1}{n} . [C_1 . e^{ml} - C_2 . e^{-ml}] \;.....\; (7)$$

Where, $\sqrt{\frac{(G + j\omega C)}{(R + j\omega L)}} = \frac{1}{n}$ ..... (8)

So, $n . i = [C_1 . e^{ml} - C_2 . e^{-ml}]$ ..... (9)

- Eqns. (6) & (9) give values of voltage & current at any distance 'l' from receiving end of the line, if values of C_1 & C_2 are known.
- At receiving end l=0, $v = V_R$ & $i = I_R$. Hence, from (6) & (9),

$V_R = C_1 + C_2$ ..... (10)

& $n.I_R = C_1 - C_2$ ..... (11)

So, $C_1 = \frac{V_R + n.I_R}{2}$ ..... (12)

& $C_2 = \frac{V_R - n.I_R}{2}$ ..... (13)

So, at any distance 'l' from receiving end, from eqn. (6),

$$v = \frac{V_R + n.I_R}{2}.\,e^{ml} + \frac{V_R - n.I_R}{2}.\,e^{-ml}$$

$$= V_R.\,[\frac{e^{ml} + e^{-ml}}{2}] + n.I_R.\,[\frac{e^{ml} - e^{-ml}}{2}]$$

$$= V_R.\,\text{Cosh}\ ml + n.I_R.\,\text{Sinh}\ ml \ ..... (14)$$

Also, from eqn. (9),

$$n.\,i = \frac{V_R + n.I_R}{2}.\,e^{ml} - \frac{V_R - n.I_R}{2}.\,e^{-ml}$$

$$= V_R.\,[\frac{e^{ml} - e^{-ml}}{2}] + n.I_R.\,[\frac{e^{ml} + e^{-ml}}{2}]$$

$$= V_R.\,\text{Sinh}\ ml + n.I_R.\,\text{Cosh}\ ml \ ..... (15)$$

or, $i = \frac{V_R}{n}. \text{Sinh } ml + I_R. \text{Cosh } ml$ ….. (16)

If 'l' be the distance from the receiving end to the sending end, then $v = V_S$ & $i = I_S$, the sending end values. Then,

$$\boxed{V_S = V_R. \text{Cosh } ml + n. I_R. \text{Sinh } ml} \quad ..... (17)$$

$$\boxed{I_S = \frac{V_R}{n}. \text{Sinh } ml + I_R. \text{Cosh } ml} \quad ..... (18)$$

Generalized Circuit Constants

- Eqns. of the sending end voltage & current for all categories of lines (i.e. short, medium & long) are of same type, namely,

$$\boxed{V_S = A.V_R. + B.I_R} \quad \ldots\ldots (19)$$

$$\boxed{I_S = C.V_R. + D.I_R} \quad \ldots\ldots (20)$$

- The terms A, B, C & D in eqns. (19) & (20) are known as the generalized circuit constants.
- Their values depend on line parameters & the type of representation chosen.

Generalized Circuit Constant	Short	Medium – T representation	Medium – π representation	Long
A	1	$(1+ YZ/2)$	$(1+ YZ/2)$	Cosh ml
B	Z	$(1+ YZ/4).Z$	Z	n. Sinh ml
C	0	Y	$(1+ YZ/4).Y$	$\frac{1}{n}$. Sinh ml
D	1	$(1+ YZ/2)$	$(1+ YZ/2)$	Cosh ml

- In all the cases A = D &
- A. D – B. C = 1
- The constants are complex numbers.
- A & D are dimension less.
- B has the dimension of impedance (ohm) & C has the dimension of admittance (mho).

Determination of A, B, C & D Constants

- It may be done in two ways.

(i) From the knowledge of line parameters R, L, C & G and the table given in last slide.

(ii) By direct measurement from the actual network.

It is known that,

$$V_S = A.V_R + B.I_R$$
$$I_S = C.V_R + D.I_R$$

- If the receiving end is open circuited & a voltage of V_S' be applied at sending end to give a voltage V_R' at receiving end, then receiving end current $I_R'=0$. If I_S' be the corresponding sending end current, then

$$A = \frac{V_S'}{V_R'} \quad \& \quad C = \frac{I_S'}{V_R'}$$

- If the receiving end of the line is now short circuited & a voltage of V_S'' be applied at sending end to give sending end & receiving end currents of I_S'' & I_R'' respectively, then $V_R''=0$.

Hence,

$$\boxed{B = \frac{V_S''}{I_R''} \quad \& \quad D = \frac{I_S''}{I_R''}}$$

Example: A single-phase 50 Hz generator supplies an inductive load of 5,000 kW at a power factor of 0.707 lagging by means of an over head transmission line 20 km long. The line resistance and inductance are 0.0195 Ω and 0.63 mH per km. The voltage at the receiving end is required to be kept constant at 10 kV. Find the sending end voltage and voltage regulation of the line.

Soln:

The line constants are

$$R = 0.0195 \times 20 = 0.39 \ \Omega$$

and $L = 0.63 \times 10^{-3} \times 20 = 0.0126$ H

$$X = 314.2857 \times 0.0126 = 3.96 \ \Omega$$

This is the case of a short line with $I = I_R = I_S$.

So, $|I| = \dfrac{5{,}000}{10 \times 0.707} = 707.2136$ A

Now, $|V_S| \simeq |V_R| + |I| (R \cos \phi_R + X \sin \phi_R)$

$= 10{,}000 + 707.2136 (0.39 \times 0.707 + 3.96 \times 0.707)$ V

$= 12.175$ kV

Voltage regulation $= \dfrac{12.175 - 10}{10} \times 100 = 21.75\%$

Example : Using the nominal-π method, find the sending-end voltage and voltage regulation of a 250 km, three-phase, 50 Hz, transmission line delivering 25 MVA at 0.8 lagging power factor to a balanced load at 132 kV. The line conductors are spaced equilaterally 3 m apart. The conductor resistance is 0.11 ohm/km and its effective diameter is 1.6 cm. Neglect leakance.

Soln: $L = 2.\,10^{-7}.\ln\dfrac{D}{r'}\ \text{H/m} = 2.\,10^{-7}.\ln\dfrac{300}{0.7788 \times 0.8}$

$$= 1.24\ \text{mH/km}$$

$$C = \frac{2\pi K}{\ln\frac{D}{r}}\ \text{F/m} = \frac{0.0556}{\ln\frac{300}{0.8}} = 0.0094\ \mu\text{F/km}$$

$$R = 0.11 \times 250 = 27.5\ \Omega$$

$$X = 2\pi f L = 2\pi \times 50 \times 1.24 \times 10^{-3} \times 250 = 97.4\ \Omega$$

$$Z = R + jX = 27.5 + j97.4 = 101.2\ \angle 74.2^\circ\ \Omega$$

$$Y = j\omega Cl = 2\pi \times 50 \times 0.0094 \times 10^{-6} \times 250\ \angle 90^\circ$$

$$= 7.38 \times 10^{-4}\ \angle 90^\circ\ \mho$$

$$I_R = \frac{25 \times 1,000}{\sqrt{3} \times 132} \angle -36.9^\circ = 109.3 \angle -36.9^\circ \text{ A}$$

V_R (per phase) = $(132/(\sqrt{3}) \angle 0^\circ = 76.2 \angle 0^\circ$ kV

$$V_S = \left(1 + \frac{1}{2} YZ\right) V_R + ZI_R$$

$$= \left(1 + \frac{1}{2} \times 7.38 \times 10^{-4} \angle 90^\circ \times 101.2 \angle 74.2^\circ\right) \times 76.2$$

$$+ 101.2 \angle 74.2^\circ \times 109.3 \times 10^{-3} \angle -36.9^\circ$$

$$= 76.2 + 2.85 < 164.2^\circ + 11.06 < 37.3^\circ$$

$$= 82.26 + j7.48 = 82.6 < 5.2^\circ$$

$\therefore |V_S|$ (line) $= 82.6 \times \sqrt{3} = 143$ kV

Now, $A = 1 + \frac{1}{2}YZ = 1 + 0.0374\ \angle 164.2° = 0.964 + j0.01$

$$|V_{R0}| \text{ (line no load)} = \frac{143}{\left|1 + \frac{1}{2}YZ\right|} = \frac{143}{0.964} = 148.3 \text{ kV}$$

$$\therefore \text{Voltage regulation} = \frac{148.3 - 132}{132} \times 100 = 12.3\%$$

Example: The sending end voltage per phase of a long transmission line is given by the expression

$$V_s = (0.986 \angle 0.32^o).\, V_R + (70.3 \angle 69.2^o).\, I_R.$$

Determine the capacity of a phase modifier to be installed at the receiving end so that when a load of 50MVA is delivered at 132kV and power factor 0.707 lagging, the sending end voltage can also be 132kV.

Soln: It is known that

$$V_s = A.\, V_R. + B.I_R$$

By the problem

$$A = 0.986 \angle 0.32^o$$
$$= (0.986 + j\, 0.0055)\ \&$$
$$B = 70.3 \angle 69.2^o$$
$$= (24.96 + j\, 65.72)$$

Load current magnitude $|I_L| = \frac{50,000}{\sqrt{3}.132} = 218.6933$ A

Taking receiving end voltage as reference phasor,

$$V_R = \frac{132}{I\sqrt{3}} = 76.2102 \angle 0^o \text{ kV/phase} = (76.2102 + j\,0.0) \text{ kV/phase}$$

Load current $I_L = 218.6933 \angle -45^o$ A (as p.f. is 0.707 lagging)

$$= (154.6395 - j\,154.6395) \text{ A}$$

- Problem is to determine the capacity of a phase modifier to be installed at the receiving end.
- Let, modifier current be $j\,I_m$ (assuming losses negligible).

So, receiving end current $= I_R = I_L + j\,I_m$ (as two are in parallel).

$$= (154.6395 - j\,154.6395) + j\,I_m = \{154.6395 - j\,(154.6395 - I_m)\} \text{ A}$$

Then, sending end voltage is

$$V_s = A.V_R. + B.I_R$$

$=(0.986 + j\,0.0055).(76210.2 + j\,0.0) + (24.96 + j\,65.72).$

$$\{154.6395 - j\,(154.6395 - I_m)\}$$

$= (89152.33 - 65.72.\,I_m) + j\,(6728.21 + 24.96\,I_m)$

$$\text{Given that, } |V_s| = |V_R| = 76210.2\text{ V/phase}$$

Hence,

$$(76210.2)^2 = (89152.33 - 65.72.\,I_m)^2 + (6728.21 + 24.96\,I_m)^2$$

Solving, I_m = 2093 A or 212 A

- Although both values of I_m seem to be valid, but they are mathematical results only.
- In practice, higher value is invalid as it lies outside the region of stable operation.

- So, the accepted value of I_m is 212 A.
- Hence, phase modifier capacity to meet the specification would be

$$\frac{\sqrt{3}.\ 132.\ 212}{1000}\ \text{MVAR} = 48.47\ \text{MVAR}$$

- In practice a power line may be erected in close proximity to telephone lines or communication circuits.
- The communication circuits may be the property of the power company → used for protection or communication link.
- Power lines give rise to electromagnetic & electrostatic fields of sufficient magnitudes.
- These induce currents & voltages respectively in the neighbouring communication lines.
- Induced currents are superimposed on true speech currents and thereby setup distortion.
- Induced voltages raise potential of communication circuits with consequent risk to both equipment & users.

Electromagnetic Effect

- Let, currents I_a, I_b & I_c be flowing through power line conductors a, b & c respectively.
- They form a circuit, i.e.

$$I_a + I_b + I_c = 0 \quad \ldots\ldots (1)$$

- Flux linkage of cond. 'd' up to infinity.

due to $I_a = \Psi_{ad} = 2.10^{-7}.I_a.\ln\frac{\infty}{D_{ad}}$ wbT/m

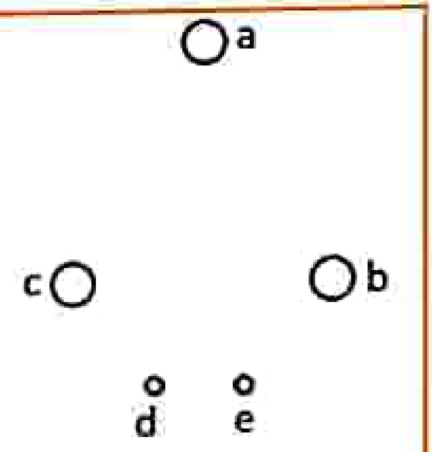


Fig. a, b & c: power line conductors;
d & e: communication line conductors.

- Similarly, flux linkage of conductor 'e' up to infinity, due to $I_a = \Psi_{ae} = 2.10^{-7}.I_a.\ln\frac{\infty}{D_{ae}}$ wbT/m
- So, mutual flux linkage of the loop formed by conductors 'd' & 'e' due to $I_a = \Psi_{de} = \Psi_{ad} - \Psi_{ae} = 2.10^{-7}.I_a.\ln\frac{D_{ae}}{D_{ad}}$ wbT/m

- So, mutual inductance between cond. 'a' & the communication circuit consisting of conds. 'd' & 'e' is

$$M_a = \frac{\Psi_{de}}{I_a} = 2.10^{-7}.\ln\frac{D_{ae}}{D_{ad}} \;\; H/m \,\ldots..\,(2)$$

- Likewise, M_b & M_c are mutual inductances between cond. 'b' & the loop 'de' and between cond. 'c' & the loop 'de' respectively.
- These are given by

$$M_b = 2.10^{-7}.\ln\frac{D_{be}}{D_{bd}} \;\; H/m \,\ldots..\,(3)$$

$$M_c = 2.10^{-7}.\ln\frac{D_{ce}}{D_{cd}} \;\; H/m \,\ldots..\,(4)$$

- Here, M_a, M_b & M_c are due to fluxes which have mutual phase displacement of 120°.
- So, net effect of the magnetic field is

$$\vec{M} = \vec{M}_a + \vec{M}_b + \vec{M}_c \quad ..... (5)$$

- Here $\vec{M}$ is net mutual inductance → phasor sum of three inductances.
- If 'I' be the current in power conds. & 'f' is the frequency, then the induced voltage in the communication line consisting of conds. 'd' & 'e' is

$$V = 2\pi f . M . I \text{ Volts/m} \quad ..... (6)$$

- There is a partial cancellation of the induced voltages due to power line currents.
- This cancellation is almost complete for balanced 3ϕ line.
- However, greater the distance between two circuits lesser is the mutual effect (see expressions of M_a, M_b & M_c → all distances D_{ae}, D_{ad}, D_{be}, D_{bd}, D_{ce} & D_{cd} tend to become equal).

Electrostatic Effect

- Let, cond. 'a' be carrying a charge of +q coulomb/m.
- So, potential difference 'a' and its image 'a'' is $V_{aa'} = 2V_{an}$

$$= \frac{1}{2\pi K_0}[q_a.\ln\frac{D_{aa'}}{D_{aa}} + q_{a'}.\ln\frac{D_{a'a'}}{D_{a'a}}]$$

$$= \frac{q}{2\pi K_0}[\ln\frac{2.h_a}{r} - \ln\frac{r}{2.h_a}] \text{ ..... (1)}$$

or, $$V_{an} = \frac{q}{2\pi K_0}.\ln\frac{2.h_a}{r} \text{ ..... (2)}$$

or, $$\frac{q}{2\pi K_0} = \frac{V_{an}}{\ln\frac{2.h_a}{r}} \text{ ..... (3)}$$

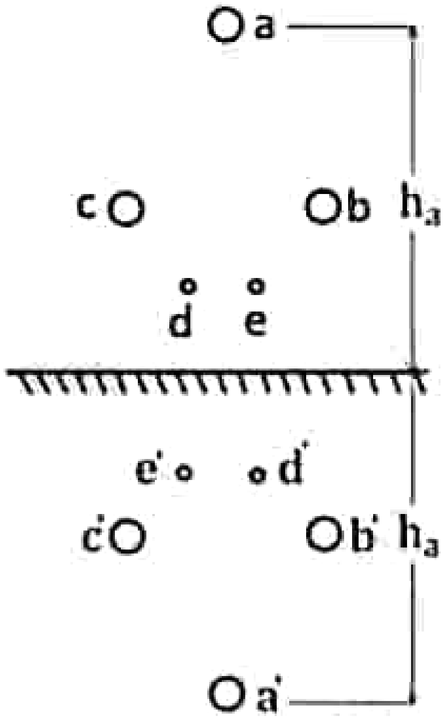


Fig. 3ϕ line, communication Line & their images

- Similarly, potential difference $V_{dd'a}$ between communication line conds. 'd' & its image 'd'' due to a charge '+q' on cond. 'a' & a charge of '-q' on image 'a'' is

$$V_{dd'a} = \frac{1}{2\pi K_o}[q_a . \ln \frac{D_{ad'}}{D_{ad}} + q_{a'} . \ln \frac{D_{a'd'}}{D_{a'd}}]$$

$$= \frac{q}{2\pi K_o}[\ln \frac{D_{ad'}}{D_{ad}} - \ln \frac{D_{a'd'}}{D_{a'd}}] = \frac{q}{2\pi K_o}[\ln \frac{(2.h_a - D_{ad})}{D_{ad}} - \ln \frac{D_{ad}}{(2.h_a - D_{ad})}]$$

$$= \frac{q}{2\pi K_o} . 2. \ln \frac{(2.h_a - D_{ad})}{D_{ad}} \quad ..... (4)$$

- So, potential of cond. 'd' w.r.t. neutral due to charge q_a & $q_{a'}$ is

$$V_{dn_a} = \frac{q}{2\pi K_o} . \ln \frac{(2.h_a - D_{ad})}{D_{ad}} \quad .....(5)$$

Substituting the value of $\frac{q}{2\pi K_o}$ from eqn. (3),

$$V_{dn_a} = \frac{V_{an}}{\ln\frac{2.h_a}{r}} . \ln\frac{(2.h_a - D_{ad})}{D_{ad}} = V_{an} . \frac{\ln\frac{(2.h_a - D_{ad})}{D_{ad}}}{\ln\frac{2.h_a}{r}} \; ..... (6)$$

- Similarly, we can obtain potential of cond. 'd' w.r.t. neutral as

$$V_{dn_b} = V_{bn} . \frac{\ln\frac{(2.h_b - D_{bd})}{D_{bd}}}{\ln\frac{2.h_b}{r}} \; ..... (7)$$

and

$$V_{dn_c} = V_{cn} . \frac{\ln\frac{(2.h_c - D_{cd})}{D_{cd}}}{\ln\frac{2.h_c}{r}} \; ..... (8)$$

- So, the potential of cond. 'd' due to all the conds. a, b & c is

$$\overrightarrow{V_{d_n}} = \overrightarrow{V_{d_{na}}} + \overrightarrow{V_{d_{nb}}} + \overrightarrow{V_{d_{nc}}} \quad ..... (9)$$

- Similarly, potential of cond. 'e' may be calculated.

Reduction of Interference

- Thoroughly transposition of both the (i) power line & (ii) communication line conds.

→ It has the effect of splitting the induced emf into a series of mutually opposed emfs.

- Use of screened cables for communication lines overcomes the trouble due to electrostatic interference.
- Same effect is obtained by the use of an earth wire between the power line & the communication line.
- Further, electromagnetic interference may be reduced by splitting the telephone line into short lengths each being separated from the adjacent section by 1:1 isolating transformer.

→ Such a method can't be used if d.c. signal needs to be sent.

Prob: A load of 30MW is delivered at a distance of 160km at a voltage of 132kV & at a frequency of 50Hz, the p.f. being 0.9. The line conds. which have a radius of 5.5mm are situated at the corners of an equilateral triangle with a side of 3.5m and arranged as shown in the fig. The height of the lowest cond. is 15m from the ground. A telephone line runs on the same supporting towers, the distances between the telephone line & the power line conds. being as shown. Find magnitudes of (i) the electromagnetic & (ii) the electrostatic emf induced in the telephone line.

Soln: Electromagnetic Effect

From the Δabo,

$$(3.5)^2 = D_{ao}{}^2 + \left(\frac{3.5}{2}\right)^2$$

or, $D_{ao} = 3.0311m$

Now, $D_{ad} = D_{ao} + 3 = 6.0311m$

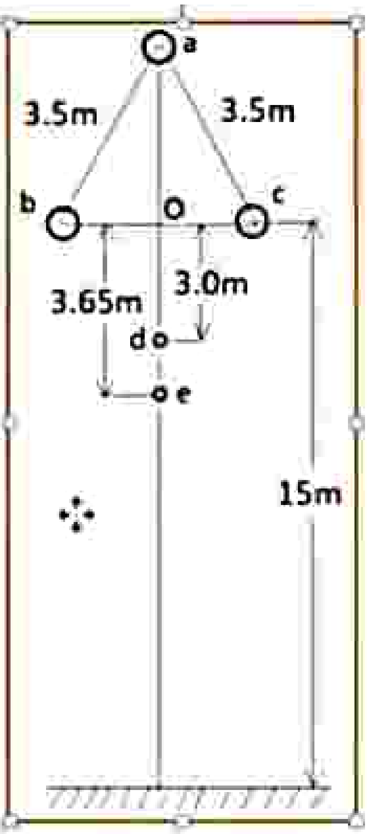
$D_{ae} = D_{ao} + 3.65 = 6.6811m$

Again, $(D_{bd})^2 = (3)^2 + \left(\frac{3.5}{2}\right)^2$

or, $D_{bd} = 3.4731m = D_{cd}$

$$(D_{be})^2 = (3.65)^2 + \left(\frac{3.5}{2}\right)^2$$

or, $D_{be} = 4.0478m = D_{ce}$



$$M_a = 2.10^{-7}.\ln\frac{D_{ae}}{D_{ad}} = 2.10^{-7}.\ln\frac{6.6811}{6.0311} = 0.2047.\,10^{-7}\,H/m$$

$$= 0.2047.\,10^{-4}\,H/km$$

$$M_b = M_c = 2.10^{-7}.\ln\frac{D_{be}}{D_{bd}} = 2.10^{-7}.\ln\frac{4.0478}{3.4731} = 0.3063.\,10^{-7}\,H/m$$

$$= 0.3063.\,10^{-4}\,H/km$$

- Net mutual inductance is equal to vector sum of M_a, M_b & M_c.
- So, $\vec{M} = \vec{M}_a + \vec{M}_b + \vec{M}_c = M_a + M_b\angle 120^o + M_c\angle 240^o$ (ph seq. a-b-c)

or, $|M|^2 = (M_a + M_b.\text{Cos}120^o + M_c.\text{Cos}240^o)^2 +$

$$(M_b.\text{Sin}120^o + M_c.\text{Sin}240^o)^2$$

or, $|M| = 0.1016.\,10^{-4}\,H/km$

Now, current $I = \dfrac{30.10^6}{\sqrt{3}.132.\,10^3.0.9} = 145.7955\ A$

Electromagnetically induced emf = $2\pi f.\ |M|.I = 0.4654\ V/km$

Total electromagnetically induced emf = $0.4654.\ 160 = 74.464\ V$

Electrostatic Effect

- Height of cond. 'a' above ground=15 + D_{ao} = 18.0311m
- Height of cond. 'b' & 'c' above ground = 15m
- So, magnitude of potential of cond. 'd' w.r.t. neutral = $|V_{dn_a}|$

$$= |V_{an}|.\frac{\ln\frac{(2.h_a - D_{ad})}{D_{ad}}}{\ln\frac{2.h_a}{r}} = \frac{132.10^3}{\sqrt{3}.}.\frac{\ln\frac{(2.18.0311-6.0311)}{6.0311}}{\ln\frac{2.18.0311}{5.5.\,10^{-3}}}$$

$$= 13920.8142\ V$$

Likewise, magnitude of potential of cond. 'd' w.r.t. neutral due to cond. 'b' = magnitude of potential of cond. 'd' w.r.t. neutral due to cond. 'c' (due to symmetry) = $|V_{dn_b}| = |V_{dn_c}|$

$$=|V_{bn}|.\frac{\ln\frac{(2.h_b-D_{bd})}{D_{bd}}}{\ln\frac{2.h_b}{r}}=\frac{132.10^3}{\sqrt{3}.}.\frac{\ln\frac{(2.15-3.4731)}{3.4731}}{\ln\frac{2.15}{5.5.10^{-3}}}$$

$$=18007.8369\text{ V}$$

- So, total potential of cond. 'd' w.r.t. neutral = $\overrightarrow{V_{d_n}}$
- $=\overrightarrow{V_{d_{na}}}+\overrightarrow{V_{d_{nb}}}+\overrightarrow{V_{d_{nc}}}=|V_{dn_a}|+|V_{dn_b}|\angle 120^o+|V_{dn_c}|\angle 240^o$

or, $|V_{dn}| = 4087.0227\text{ V}$

Similarly, the potential of cond. 'e' can be determined.