# Line Capacitance

- A capacitor essentially consists of two conducting surfaces separated by a layer of insulating material.
- The insulating material is also known as dielectric.
- The conducting surfaces may be in the form of either circular (or rectangular) plates or spherical or cylindrical shape.
- Such a condition is fulfilled by an overhead transmission line.
- The conductors act as the conducting surfaces & they are separated by air dielectric.
- Suppose, one of the two surfaces has a charge of Q coulomb & V be the potential difference between the two surfaces.
- Then, capacitance between the two surfaces = C = $\frac{\mathrm{Q}}{\mathrm{V}} \frac{\mathrm{Coulomb}}{\mathrm{Volt}}$

$$= \frac{\mathrm{Q}}{\mathrm{V}}\ \mathrm{Farad}$$

- One Farad is actually too large for all practical purposes.
- Much smaller units like micro-Farad (μF) & pico-Farad (pF) are more commonly used.
- $1\ \mu F = 10^{-6}\ F$ & $1\ pF = 10^{-12}\ F$

## Electric Field Intensity or Field Strength

It may be defined in two ways

(i) At any point in an electric field, it is proportional to the flux passing normally through a unit area at that point.

If, E = electric field intensity,

q = charge (flux) in Coulomb,

A = area through which q flux pass &

$K = K_0 \, . \, K_r$ = permittivity of the medium where,

$K_0$ = permittivity of air = $\frac{1}{36.\ \pi.\ 10^{-9}}$ F/m = 8.84194. $10^{-12}$ F/m

$K_r$ = relative permittivity.

$E \propto \frac{q}{A}$ or, $E = \frac{1}{K} \cdot \frac{q}{A}$ ........................................ (1)

Again, flux density $\boxed{D = \frac{q}{A}}$..................................(2)

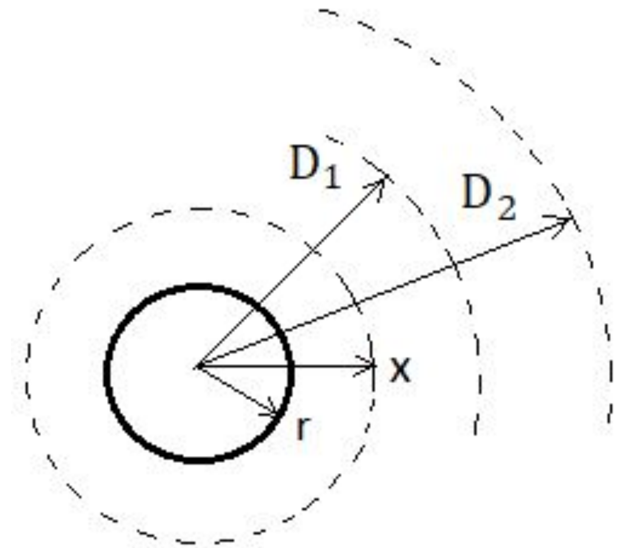
So, D = K. E ........................................(3)

Hence, D & E both are vectors.

(ii) Electric field intensity at any point in an electric field is also defined as the force experienced by a unit +Ve charge placed at that point.

## Potential at a Point

It is equal to the work done in bringing a unit +Ve charge from infinity to that point.




- Consider a cond. Carrying a charge of +q coulomb/m length.
- Electric flux density at a distance x =

$$D_x = \frac{q}{(2\pi x.1)} \text{ coulomb/m}^2$$

- Electric field intensity $\boxed{E_x = \frac{D_x}{K} = \frac{1}{K}\cdot\frac{q}{2\pi x} \text{ Volts/m}}$
- Potential at that point = $\int_{x=\infty}^{x=x} E_x.dx = \frac{q}{2\pi K}.\int_{x=\infty}^{x=x}\frac{dx}{x}$
- For any equipotential surfaces at distances $D_1$ & $D_2$,

- Potential difference $V_{D_1D_2} = \frac{q}{2\pi K} \cdot \int_{D_1}^{D_2} \frac{dx}{x} = \frac{q}{2\pi K} \cdot \text{Ln} \frac{D_2}{D_1}$

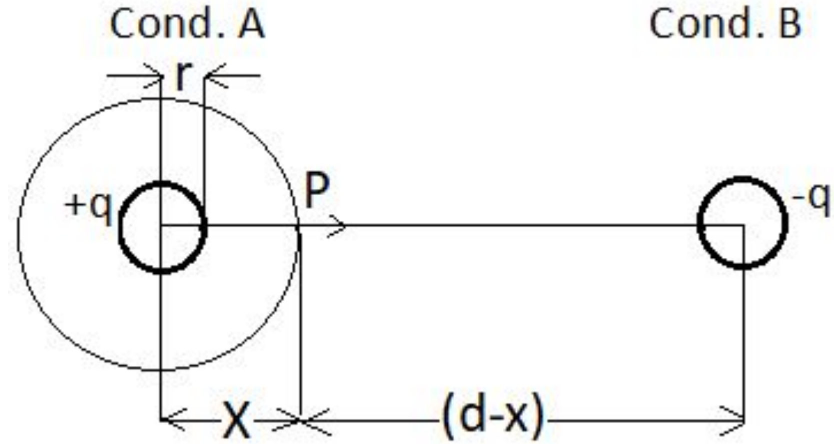
Capacitance between these equipotential surfaces $= \frac{q}{V_{D_1D_2}}$

$$= \frac{2\pi K}{\text{Ln} \frac{D_2}{D_1}} \text{ F/m}$$

## Capacitance of a single phase line

- Spacing of the conds. be d.
- Charge on cond.'A' is +q Coulomb/m & charge on cond. 'B' is –q Coulomb/m.
- Radius of conds. be r.
- P be any point at a distance x from cond. A.

- Electric field intensity at P, due to charge of +q on cond. A = $E_{xA} = \frac{q}{(2\pi x.1).K}$ V/m, acting in the direction of arrow ….. (1)




- Similarly, electric field intensity at P, due to charge of -q on cond. B = $E_{xB} = \frac{q}{\{2\pi(d-x).1\}.K}$ V/m, acting in the same direction ….. (2)
- Total electric field intensity at P due to both conds. A & B $=E_x = E_{xA} + E_{xB}$

$$= \frac{q}{(2\pi x.1).K} + \frac{q}{\{2\pi(d-x).1\}.K} = \frac{q}{2\pi K}\left[\frac{1}{x} + \frac{1}{d-x}\right] \text{ V/m} \quad ….. (3)$$

- Now, potential difference between conds. A & B= $V_{AB}$ =Work done in moving 1 coulomb of charge from one cond. to another against the electric field.

- If 1 coulomb is moved through a distance of dx, then work done is $E_x.dx$.
- So, $V_{AB} = \int_{X=r}^{X=d-r} E_x.dx = \frac{q}{2\pi K}.\int_{r}^{d-r} [\frac{1}{x} + \frac{1}{d-x}].dx$

$$= \frac{q}{\pi K}.\ln\frac{d-r}{r} \text{ Volts ..... (4)}$$

So, capacitance between A & B $C_{AB} = \frac{q}{V_{AB}} = \frac{\pi K}{\ln\frac{d-r}{r}}$ F/m ..... (5)

As $r<<d$, $d-r \approx d$.

Then, $C_{AB} = \frac{\pi K}{\ln\frac{d}{r}}$ F/m ..... (6)

- Since the two conds. are oppositely charged, potential of the points mid way between the conds. is zero.
- So, this is a zero potential (neutral) plane between A & B.
- So, potential of each cond. w.r.t. this neutral plane = $\frac{1}{2}.V_{AB}$ .
- Therefore, capacitance between each cond. & zero potential

plane = $C_n = \dfrac{q}{\frac{1}{2}.V_{AB}} = \dfrac{2\pi K}{\ln\frac{d}{r}}$ F/m = $2.C_{AB}$ ….. (7)

- $C_n$ is called the capacitance to neutral/ground.
- The term capacitance to neutral is more common in transmission line calculations.

## System of n-conductors

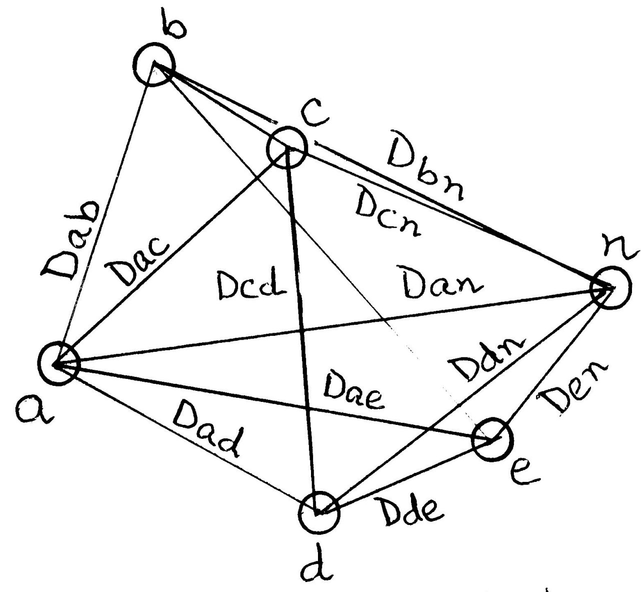


Fig. n-parallel conductors

- Consider, a system with ‘n’ long, straight and parallel conds.
- Conds. have a radius of ‘r’.
- Charges on conds. be $q_a, q_b, q_c, \ldots, q_n$.
- Conds. are forming a circuit, i.e.

  $q_a + q_b + q_c + \ldots + q_n = 0$ ….. (1)
- Spacing between conds. $D_{ab}, D_{bc}, D_{cd}$ etc. are very large compared to radius ‘r’.
- Conds. are far removed from ground.

➢ Principle of superposition is applied to find out the potential difference between any two conds.

• **Principle:** Potential difference between any two conds.=

Potential difference due to charge on the 1st cond. alone +

Potential difference due to charge on the 2nd cond. alone +

Potential difference due to other charged conds. present individually. This means

Potential difference between conds. 'a' & 'b' = $V_{ab}$ =

Potential diff. between conds. 'a' & 'b' due to charge $q_a$ on 'a' +

Potential diff. between conds. 'a' & 'b' due to charge $q_b$ on 'b' +

Potential diff. between conds. 'a' & 'b' due to charge $q_c$ on 'c' +

……………+

Potential diff. between conds. 'a' & 'b' due to charge $q_n$ on 'n'

$$= \frac{q_a}{2\pi K}.\ln\frac{D_{ab}}{r} + \frac{q_b}{2\pi K}.\ln\frac{r}{D_{ba}} + \frac{q_c}{2\pi K}.\ln\frac{D_{cb}}{D_{ca}} + \ldots\ldots + \frac{q_n}{2\pi K}.\ln\frac{D_{nb}}{D_{na}}$$

[Taking help of the relation $\rightarrow V_{D_1D_2} = \frac{q}{2\pi K}.\ln\frac{D_2}{D_1}$] Volts

- $= \frac{1}{2\pi K} [q_a . \ln \frac{D_{ab}}{r} + q_b . \ln \frac{r}{D_{ba}} + q_c . \ln \frac{D_{cb}}{D_{ca}} + \ldots\ldots + q_n . \ln \frac{D_{nb}}{D_{na}} ] \text{ V}$

..... (2)

For symmetrical results 'r' may be written as $D_{aa}$, $D_{bb}$, $D_{cc}$ etc.

So, eqn. (2) can be written as

$$V_{ab} = \frac{1}{2\pi K} [q_a . \ln \frac{D_{ab}}{D_{aa}} + q_b . \ln \frac{D_{bb}}{D_{ba}} + q_c . \ln \frac{D_{cb}}{D_{ca}} + \ldots\ldots + q_n . \ln \frac{D_{nb}}{D_{na}} ] \text{ V}$$

..... (3)

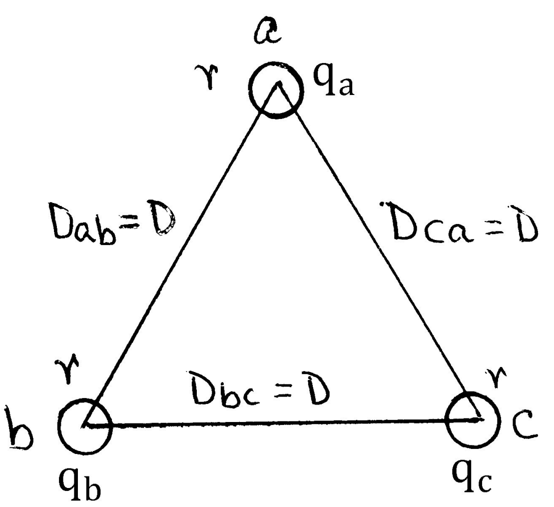
Similarly, potential diff. between conds. 'a' & 'n' = $V_{an}$

$$= \frac{1}{2\pi K} [q_a . \ln \frac{D_{an}}{D_{aa}} + q_b . \ln \frac{D_{bn}}{D_{ba}} + q_c . \ln \frac{D_{cn}}{D_{ca}} + \ldots\ldots + q_n . \ln \frac{D_{nn}}{D_{na}} ] \text{ V}$$

$$= \frac{1}{2\pi K} . \sum_{x=a}^{x=n} q_x . \ln \frac{D_{xn}}{D_{xa}} \text{ Volts} \quad \ldots\ldots (4)$$

# Capacitance of Symmetrical Three Phase Line

- Let, a balance voltage be applied.
- Charges on the conductors be $q_a$, $q_b$ and $q_c$ Coulomb/m.




$$V_{ab}=\frac{1}{2\pi K}[q_a.\ln\frac{D_{ab}}{D_{aa}}+q_b.\ln\frac{D_{bb}}{D_{ba}}+q_c.\ln\frac{D_{cb}}{D_{ca}}]$$

$$=\frac{1}{2\pi K}[\ q_a.\ln\frac{D}{r}+q_b.\ln\frac{r}{D}+q_c.\ln\frac{D}{D}\ ]$$

$$=\frac{1}{2\pi K}[\ q_a.\ln\frac{D}{r}+q_b.\ln\frac{r}{D}\ ]\ .....\ (1)$$

Similarly, $V_{ac}=\frac{1}{2\pi K}[\ q_a.\ln\frac{D_{ac}}{D_{aa}}+q_b.\ln\frac{D_{bc}}{D_{ba}}+q_c.\ln\frac{D_{cc}}{D_{ca}}\ ]$

$$=\frac{1}{2\pi K}[\ q_a.\ln\frac{D}{r}+q_b.\ln\frac{D}{D}+q_c.\ln\frac{r}{D}\ ]=\frac{1}{2\pi K}[\ q_a.\ln\frac{D}{r}+q_c.\ln\frac{r}{D}\ ]$$

$$.....\ (2)$$

- $V_{ab}+V_{ac}=\frac{1}{2\pi K}[\,2q_a.\ln\frac{D}{r}+(q_b+q_c).\ln\frac{r}{D}\,]$ ….. (3)

For a balanced system $q_a+q_b+q_c=0$ Or, $q_b+q_c=-q_a$

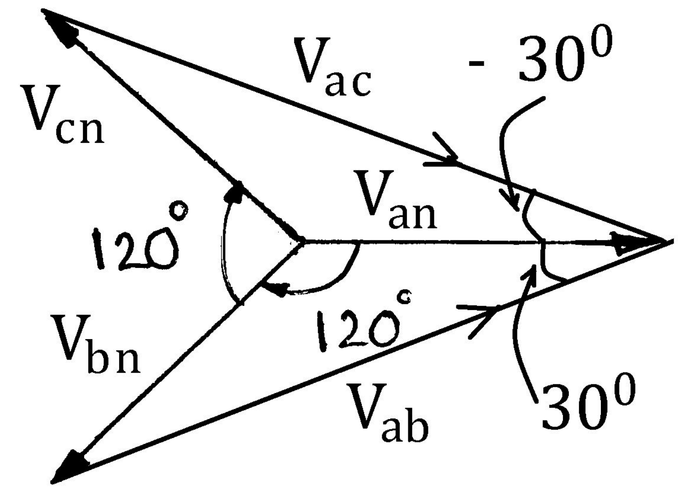
So, $V_{ab}+V_{ac}=\frac{1}{2\pi K}[\,2q_a.\ln\frac{D}{r}-q_a.\ln\frac{r}{D}\,]=\frac{3q_a}{2\pi K}.\ln\frac{D}{r}$ ….. (4)

Here, $V_{ab}=|V_{ab}|\angle 30^0$

$=\sqrt{3}.V_{an}\angle 30^0=\sqrt{3}.V_{an}(\frac{\sqrt{3}}{2}+j.\frac{1}{2})$

Also, $V_{ac}=|V_{ac}|\angle -30^0$

$=\sqrt{3}.V_{an}(\frac{\sqrt{3}}{2}-j.\frac{1}{2})$




Therefore, $V_{ab}+V_{ac}=3.V_{an}$ ….. (5)

From eqns. (4) & (5)

$$3.V_{an}=\frac{3q_a}{2\pi K}.\ln\frac{D}{r}$$

- Or, $V_{an} = \frac{q_a}{2\pi K}.\ln\frac{D}{r}$ Volts

Hence, line to neutral capacitance$= C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi K}{\ln\frac{D}{r}}$ F/m…..(6)

$= C_{bn} = C_{cn}$ (due to symmetrical spacing)

- So, for same cond. spacing and cond. radius capacitance to neutral is same for single phase line & three phase symmetrical line.

## Capacitance of Unsymmetrical Three Phase Line

$$V_{ab}=\frac{1}{2\pi K}[q_a.\ln\frac{D_{ab}}{D_{aa}}+q_b.\ln\frac{D_{bb}}{D_{ba}}+q_c.\ln\frac{D_{cb}}{D_{ca}}]$$

$$=\frac{1}{2\pi K}[q_a.\ln\frac{D_{ab}}{r}+q_b.\ln\frac{r}{D_{ba}}+q_c.\ln\frac{D_{cb}}{D_{ca}}]\text{ Volts ..... (1)}$$

$$V_{ac}=\frac{1}{2\pi K}[q_a.\ln\frac{D_{ac}}{D_{aa}}+q_b.\ln\frac{D_{bc}}{D_{ba}}+q_c.\ln\frac{D_{cc}}{D_{ca}}]$$

$$=\frac{1}{2\pi K}[q_a.\ln\frac{D_{ac}}{r}+q_b.\ln\frac{D_{bc}}{D_{ba}}+q_c.\ln\frac{r}{D_{ca}}]\text{ Volts ..... (2)}$$

$$V_{ab}+V_{ac}=\frac{1}{2\pi K}[q_a.\ln\frac{D_{ab}.D_{ac}}{r^2}+q_b.\ln\frac{r.D_{bc}}{D_{ba}^2}+q_c.\ln\frac{D_{cb}.r}{D_{ca}^2}]\text{ Volts}$$

..... (3)

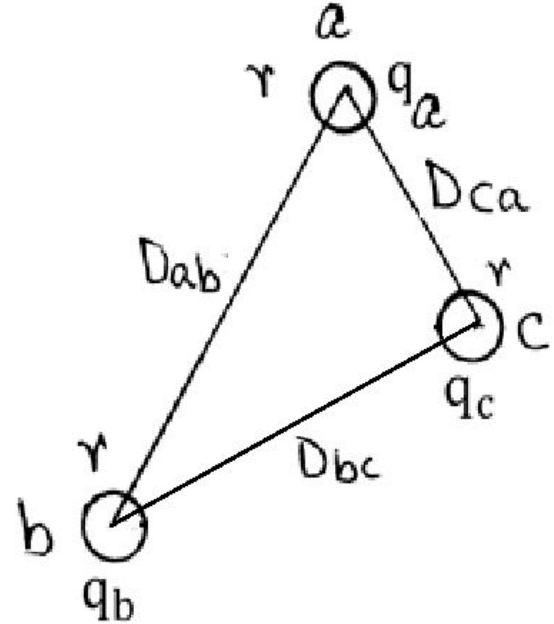
For a complete circuit $q_a+q_b+q_c=0$.

Taking phase sequence as a-b-c

$q_a = q(1 + j.0)$

$q_b = q(-\frac{1}{2} - j.\frac{\sqrt{3}}{2})$

$q_c = q(-\frac{1}{2} + j.\frac{\sqrt{3}}{2})$




Substituting in eqn. (3)

$$V_{ab}+V_{ac} = \frac{q}{2\pi K}\left[\ln\frac{D_{ab}.D_{ac}}{r^2}+\left(-\frac{1}{2}-j.\frac{\sqrt{3}}{2}\right).\ln\frac{r.D_{bc}}{D_{ba}^{\;2}}+\left(-\frac{1}{2}+j.\frac{\sqrt{3}}{2}\right).\ln\frac{D_{cb}.r}{D_{ca}^{\;2}}\right]$$

Volts ….. (4)

Simplifying and rearranging the real and imaginary parts,

$$V_{ab}+V_{ac} = \frac{q}{2\pi K}\left[\ln\frac{(D_{ab}.D_{ac})^2}{r^3.D_{bc}} + j.\sqrt{3}.\ln\frac{D_{ab}}{D_{ac}}\right] = 3.V_{an}$$

- $$V_{an} = \frac{q}{6\pi K}\left[\ln \frac{(D_{ab}.D_{ac})^2}{r^3.D_{bc}} + j.\sqrt{3}.\ln\frac{D_{ab}}{D_{ac}}\right] \text{ Volts} \ldots\ldots (5)$$

$$C_{an} = \frac{q_a}{V_{an}} = \frac{6\pi K}{\left[\ln\frac{(D_{ab}.D_{ac})^2}{r^3.D_{bc}} + j.\sqrt{3}.\ln\frac{D_{ab}}{D_{ac}}\right]} \text{ F/m} \ldots\ldots (6)$$

Similarly,

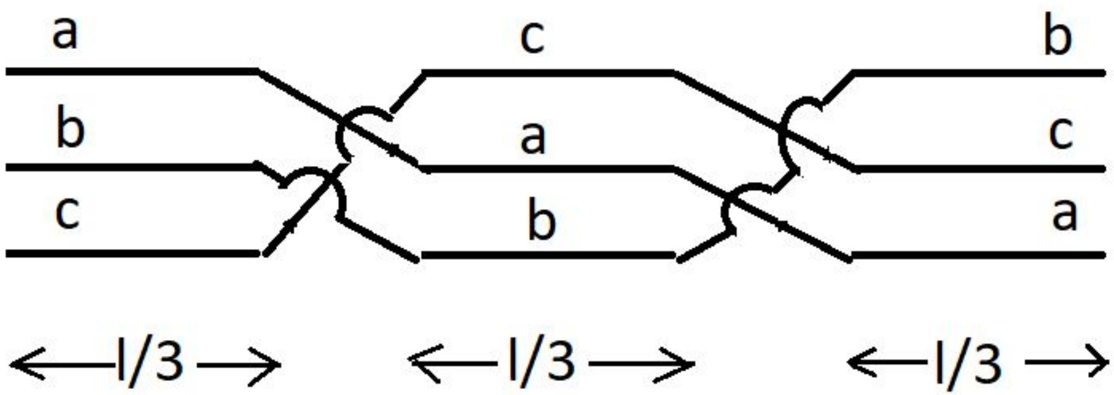
$$C_{bn} = \frac{6\pi K}{\left[\ln\frac{(D_{bc}.D_{ba})^2}{r^3.D_{ac}} + j.\sqrt{3}.\ln\frac{D_{bc}}{D_{ba}}\right]} \text{ F/m} \ldots\ldots (7)$$

$$C_{bn} = \frac{6\pi K}{\left[\ln\frac{(D_{ca}.D_{cb})^2}{r^3.D_{ab}} + j.\sqrt{3}.\ln\frac{D_{ca}}{D_{cb}}\right]} \text{ F/m} \ldots\ldots (8)$$

- Capacitance of different phases are different.
- It is due to unsymmetrical spacing of conductors.
- So, different charging currents for different phases.
- This causes unbalanced receiving end voltages although sending end voltages are balanced.
- Capacitances are complex numbers as $q_a$, $q_b$ & $q_c$ are not in phase with $V_{an}$, $V_{bn}$ & $V_{cn}$ respectively.
- Due to presence of the imaginary part power transfer takes place between various phases, although at any instant of time total power transfer is zero.

# Capacitance of a Transposed 3Φ Unsymmetrical Line

- Here, average capacitance of 3Φ are equal.




For the 1st 1/3 distance,

$$(V_{ab}+V_{ac})_1=\frac{q}{2\pi K}\left[\ln\frac{(D_{ab}.D_{ac})^2}{r^3.D_{bc}}+j.\sqrt{3}.\ln\frac{D_{ab}}{D_{ac}}\right]\text{ Volts}$$

For the 2nd 1/3 distance,

$$(V_{ab}+V_{ac})_2=\frac{q}{2\pi K}\left[\ln\frac{(D_{bc}.D_{ba})^2}{r^3.D_{ac}}+j.\sqrt{3}.\ln\frac{D_{bc}}{D_{ba}}\right]\text{ Volts}$$

For the 3rd 1/3 distance,

$$(V_{ab}+V_{ac})_3=\frac{q}{2\pi K}\left[\ln\frac{(D_{ca}.D_{cb})^2}{r^3.D_{ab}}+j.\sqrt{3}.\ln\frac{D_{ca}}{D_{cb}}\right]\text{ Volts}$$

$$(V_{ab} + V_{ac}) = \frac{1}{3} \cdot [(V_{ab} + V_{ac})_1 + (V_{ab} + V_{ac})_2 + (V_{ab} + V_{ac})_3]$$

$$= \frac{q}{6\pi K} \left[ \ln \left\{ \frac{(D_{ab} . D_{bc} . D_{ca})^4}{(r^9 . D_{ab} . D_{bc} . D_{ca})} \right\} + j . \sqrt{3} . \ln \left\{ \frac{(D_{ab} . D_{bc} . D_{ca})}{(D_{ab} . D_{bc} . D_{ca})} \right\} \right]$$

$$= \frac{q}{6\pi K} . \ln \left\{ \frac{(D_{ab} . D_{bc} . D_{ca})^3}{r^9} \right\}$$

$$= \frac{9q}{6\pi K} . \ln \left\{ \frac{\sqrt[3]{D_{ab} . D_{bc} . D_{ca}}}{r} \right\} \text{ Volts } \ldots\ldots (1)$$

Again, $V_{ab} + V_{ac} = 3 . V_{an}$ Volts ….. (2)

So, from eqns. (1) & (2),

$$3 . V_{an} = \frac{9q}{6\pi K} . \ln \left\{ \frac{\sqrt[3]{D_{ab} . D_{bc} . D_{ca}}}{r} \right\} \ldots\ldots (3)$$

- 

Or, $$V_{an} = \frac{q}{2\pi K} . \ln \left\{\frac{\sqrt[3]{D_{ab}.D_{bc}.\,D_{ca}}}{r}\right\} \text{Volts} \ldots\ldots (4)$$

Hence, line to neutral capacitance= $C_{an} = \dfrac{q}{V_{an}}$

$$= \frac{2\pi K}{\ln \left\{\frac{\sqrt[3]{D_{ab}.D_{bc}.\,D_{ca}}}{r}\right\}} = \frac{2\pi K}{\ln \frac{D_{eq}}{r}} = C_{bn} = C_{cn} \text{ F/m} \ldots\ldots(5)$$

Where, $D_{eq} = \sqrt[3]{D_{ab}.D_{bc}\,.\,D_{ca}}$

# Thank you.