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Section: 19

Course: MAT216

1.

(i) $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\det\begin{pmatrix} 2-\lambda & 0 & 0 \\ 0 & 4-\lambda & 5 \\ 0 & 4 & 3-\lambda \end{pmatrix} = 0$

Expanding,

$(2-\lambda)\left[(4-\lambda)(3-\lambda) - 20\right] = 0$

$(2-\lambda)\left[\lambda^2 - 7\lambda + 12 - 20\right] = 0$

$(2-\lambda)(\lambda^2 - 7\lambda - 8) = 0$

For λ_1, $2-\lambda = 0$, $\lambda_1 \Rightarrow 2$

For λ_2 and λ_3, $\lambda^2 - 7\lambda - 8 = 0$

$\lambda_2 = -1$, $\lambda_3 = 8$

So, for l_1

$$(A - 2I)v = 0$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

From the second row, we get $2y + 5z = 0$

choosing $y = 1$, $x = -\frac{5}{2}$.

so, eigen vector $[0, 1, -\frac{5}{2}]$

For $l_2 = -1$,

$$(A + I)v = 0$$

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & 4 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

From the second row, we get $5y + 5z = 0$. choosing

$y = 1$ and $z = -1$, we got an eigenvector $[0, 1, -1]$

For $l_3 = 8$

$$(A - 8I)v = 0$$

$$\begin{bmatrix} -6 & 0 & 0 \\ 0 & -4 & 5 \\ 0 & 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

From the second row, we got $-4y + 5z = 0$

choosing $y = 5$, $z = 4$,

So, the eigen values and vectors are

$l_1 = 2$, $l_2 = -1$, $l_3 = 8$

(ii) $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

now, $\det(A - lI) = 0$

$$\det \begin{bmatrix} 3-l & 1 & 1 \\ 2 & 4-l & 2 \\ 1 & 1 & 3-l \end{bmatrix} = 0$$

expanding,

$$(3-l)[(4-l)(3-l)-2] - (1)[(2)(3-l)-2] + (1)[(2)(1)-4] = 0$$

$$(3-l)[l^2 - 7l + 10] - (2l-2) + (2-4) = 0$$

$$(3-l)(l^2 - 7l + 10) - 2l = 0$$

$$(3-l)(l^2 - 7l + 10) - 2l = 0$$

$$(3-l)(l-2)(l-5) - 2l = 0$$

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So, eigen values will be,

for l_1, $3 - l = 0 \Rightarrow l_1 = 3$

l_2, $l - 2 = 0 \Rightarrow l_2 = 2$

l_3, $l - 5 = 0 \Rightarrow l_3 = 5$

for $l_1 = 3$

$$(A - 3I)V = 0$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

From the third row we get $x + y = 0$

and from the first row, $y + z = 0$. choosing $x = 1$, $y = -1$, $z = 1$, we got $= [1, -1, 1]$

$l_2 = 2$

$$(A-2I)\,V = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From the second row we get. $x+y+z = 0$

choosing $x = 1$, $y = -1$, $z = 1$

we get an eigen vector $[1, -1, 1]$

For $l_3 = 5$

$$(A-5I)\,V = 0$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

from the second row we get $-2x+y+z = 0$

we can solve and find $x=1, y=1, z=-1$

2.5.

a) $v_1 = (1,-1,1)$, $v_2 = (3,2,-2)$,

$v_3 = (0,-1,1)$

1. Normalize v_1.

$$u_1 = \frac{v_1}{\|v_1\|}$$

2. orthogonalize the second vector.

$$v_2 = v_2 - \mathrm{Proj}_{u_1}(v$$

3. Normalize the orthogonalized vector $u_2 = \frac{v_2}{\|v_2\|}$

a) $v_1 = (1, -1, 1)$

$v_2 = (3, 2, -2)$

$v_3 = (0, -1, 1)$

$$u_1 = \frac{v_1}{\|v_1\|} = \frac{(1, -1, 1)}{\sqrt{1^2 + (-1)^2 + 1^2}} = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}},$$

$$u_2 = v_2 - \text{proj}_{u_1}(v_2)$$

$$\text{proj}_{u_1}(v_2) = (u_2 \cdot u_1)\, u_1$$

$$= \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \cdot (3, 2, -2) \cdot \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{3}\right)$$

$$= \frac{3}{\sqrt{3}} \cdot \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{3}\right)$$

$$= (1, -1, 1)$$

$$u_2 = (3, 2, -1) - (1, -1, 1)$$

$$= (2, 3, -3)$$

$$u_2 = \frac{2, 3, -3}{\sqrt{2^2 + 3^2 + (-3)^2}}$$

$$= \frac{2}{\sqrt{22}}, \frac{3}{\sqrt{22}}, -\frac{3}{\sqrt{22}}.$$

$$v_3 = (0, -1, 1)$$

b) $v_1 = (1, 0, 1)$, $v_2 = (3, 1, -1)$, $v_3 = (1, 1, -2$

$$u_1 = \frac{v_1}{\|v_1\|}$$

$$= \frac{(1,0,1)}{\sqrt{1^2+0^2+1^2}}$$

$$= \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$u_2 = v_2 - \mathrm{proj}_{u_1}(v_2)$$

$$= \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \cdot (3, 1, -1) \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}} \cdot \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = (1, 0, 1)$$

$$u_2 = (3, 1, -1) - (1, 0, 1) = (2, 1, -2)$$

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3.

$$u_2 = \frac{2, -1, -2}{\sqrt{2^2+1^2+(-2)^2}}$$

$$= \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$$ m

4.

a) span $= \left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \right\}$

let $v = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ be a vector orthagonal to span.

$$\begin{vmatrix} -1 \\ 1 \\ 0 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 \\ -2 \\ 0 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = 0$$

$-a+b=0 \quad \ldots \text{(i)}$

$2a - 2b = 0 \quad \ldots \text{(ii)}$

from i and ii.

The equation is satisfied for any value of a.

$$u_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\\ 0\end{pmatrix}$$

b) $\text{Span} = \left\{ \begin{pmatrix}1\\-2\\2\end{pmatrix}, \begin{pmatrix}2\\-2\\-1\end{pmatrix}, \begin{pmatrix}3\\-4\\1\end{pmatrix} \right\}$

let, $v_2 \begin{pmatrix}a\\b\\c\end{pmatrix}$ be a vector orthogonal to Span.

$$\begin{vmatrix}1\\-2\\2\end{vmatrix} \begin{pmatrix}a\\b\\c\end{pmatrix} = 0$$

$$\begin{vmatrix}2\\-2\\-1\end{vmatrix} \begin{vmatrix}a\\b\\c\end{vmatrix} = 0$$

$$\begin{vmatrix}3\\-4\\1\end{vmatrix} \begin{vmatrix}a\\b\\c\end{vmatrix} = 0$$

solving

$a - 2b + 2c = 0 \quad \ldots \ldots$ (i)

$2a - 2b - c = 0 \quad \ldots \ldots$ (ii)

$3a - 4b + c = 0 \quad \ldots \ldots$ (iii)

from The (i), (ii), (iii) equation, we get

$a = b \ldots$

$$u_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$$

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3.

1.

$$A = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{vmatrix}$$

$$\det A = \begin{vmatrix} 2-l & 0 & 0 \\ 1 & 2-l & 1 \\ -1 & 0 & 1-l \end{vmatrix}$$

$$= (2-l) * ((2-l) - (-1))$$

$$= (2-l) * (3-l)$$

$$l = 2, \quad l = 3$$

2.