BRAC University

Department of Mathematics and Natural Sciences

MAT 216: Linear Algebra & Fourier Transformations

Assignment 3

Deadline : SPRING 2024 Total Marks: 50

Make a Front Page by yourself, mentioning your #name, #ID, and #section. (Compulsory)

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1. Find the eigenvalues and corresponding basis of the eigenspace for each of the following matrices, **(10)**

$$(i) \quad A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix} \qquad (ii) \quad A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

2. Find which of the following matrices are diagonalizable and which one is not. If A is dioagonalizable, find A^5 using $A = PDP^{-1}$. **(10)**

$$(i) \quad A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad (ii) \quad A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & 2 & 1 \\ 4 & 0 & 1 & 2 \end{bmatrix}$$

3. (a) Let P_2 have the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$. If $p = x^2 + 2x + 5$ and $q = x^2 - 5$, find $\langle p, q \rangle$. Also compute the norm $\|p\|, \|q\|$. **(5)**

   (b) Let the inner product defined by $\langle p, q \rangle = \int_{0}^{\pi/4} p(x)q(x)dx$. If $p = \sin^2 x$ and $q = \cos^3 x$, find $\langle p, q \rangle$. Also compute the norm $\|p\|, \|q\|$. **(5)**

4. Find the orthogonal complement $(W^\perp)$ and basis of $W^\perp$ of the subspace W, where, **(10)**

$$(a) \quad W = span\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\} \qquad (b) \quad W = span\left\{ \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \right\}$$

5. Use the Gram–Schmidt process to transform the basis (v_1, v_2, v_3) into an orthonormal basis. **(10)**

   (a) $v_1 = (1, -1, 1), v_2 = (3, 2, -2), v_3 = (0, -1, 1)$

   (b) $v_1 = (1, 0, 1), v_2 = (3, 1, -1), v_3 = (1, 1, -2)$