Appendix A: The Markov Chain Monte Carlo algorithm

In this Appendix we supply some details concerning the MCMC method and its implementation.

Let N be the number of random draws and $\Theta = (\Theta_i)_{i=1}^{M}$ the vector of parameters. The series of latent jumps, jump sizes and variances, is $\left\{J_t, Z_t^Y, Z_t^V, V_t\right\}_{t=1}^{T}$. The MCMC algorithm can then be summarized as below.

ALGORITHM

For each $k \in \{1, \ldots, N\}$:

1. Draw parameters from $p(\Theta|\Theta_{-i}, J, Z^Y, Z^V, V)$, $i = 1, \ldots, M$ where Θ_{-i} denotes the vector Θ excluding element i.
2. Draw jump times from $p(J_t = 1|\Theta, Z^Y, Z^V, V)$, $t = 1, \ldots, T$.
3. Draw jump sizes from $p(Z_t^Y|\Theta, J_t = 1, Z^V, V)$ and $p(Z_t^V|\Theta, J_t = 1, Z^Y, V)$, $t = 1, \ldots, T$.
4. Draw volatilities from $p(V_t|\Theta, J, Z^Y, Z^V, V_{t+1}, V_t)$, $t = 1, \ldots, T$.

This procedure produces a sample of draws $\left\{\Theta^{(k)}, Z^{y,(k)}, Z^{v,(k)}, J^{(k)}, V^{(k)}\right\}_{k=1}^{N}$. Each parameter and state is estimated by it's sample mean, e.g. $\widehat{\Theta}_i = (1/N)\sum_{k=1}^{N}\Theta_i^{(k)}$ and $\widehat{V}_t = (1/N)\sum_{k=1}^{N} V_t^{(k)}$, and so on. Under mild regularity conditions these estimates converge to their true values as $N \to \infty$.

The posterior distributions for the parameters ρ and σ_V and the volatility states V_t are not known and we use the Metropolis-Hastings algorithm to estimate them. We use the random walk Metropolis-Hastings algorithm, with a student-t distribution for the random walk disturbances, for both the correlation parameter ρ and the volatility states V_t, $t = 1, \ldots, T$. For σ_V^2 we use the independence Metropolis-Hastings algorithm where the proposal draw is generated from an inverted gamma distribution. We refer to Johannes and Polson (2004) and Asgharian and Bengtsson (2006) for more background on the Metropolis-Hastings algorithm and the theory of MCMC estimation.

Finally, we give some details on the posterior distributions of the jump sizes Z^Y and Z^V. These posterior distributions were first derived in Eraker et al. (2003). Here we briefly review their arguments. First, from Bayes rule it follows that

$$p\left(Z_{t+1}^Y|Z_{t+1}^V, J_{t+1} = 1, \Theta, V_t, Y_{t+1}\right) \propto p\left(Y_{t+1}|Z_{t+1}^Y, Z_{t+1}^V, J_{t+1} = 1, \Theta, V_t\right) \times p\left(Z_{t+1}^Y|Z_{t+1}^V, \Theta\right).$$

Both the densities $p\left(Y_{t+1}|Z_{t+1}^Y, Z_{t+1}^V, J_{t+1} = 1, \Theta, V_t\right)$ and $p\left(Z_{t+1}^Y|Z_{t+1}^V, \Theta\right)$ are Gaussian. Since the product between two Gaussian densities gives another Gaussian density we conclude that the posterior is $Z^Y_{t+1} \sim N(a^*, A^*)$. A straightforward calculation shows that the mean and variance of this distribution are given by

$$a^* = A^* \left(\frac{e^Y_{t+1} - \frac{\rho}{\sigma_V} e^V_{t+1}}{(1-\rho^2)\, V_t} + \frac{\mu_Y + \rho_J Z^V_{t+1}}{\sigma^2_Y} \right) \quad ; \quad A^* = \left(\frac{1}{(1-\rho^2)\, V_t} + \frac{1}{\sigma^2_Y} \right)^{-1}.$$

where $e^Y_t = Y_t - \mu$ and $e^V_t = V_t - V_{t-1} - Z^V_t - \alpha - \beta V_{t-1}$. Again using Bayes rule it can be shown that the posterior distribution $p\left(Z^V_{t+1} | J_{t+1} = 1, \Theta, Z^Y, V, Y\right)$ is proportional to

$$p\left(Y_{t+1}, V_{t+1} | V_t, J_{t+1} = 1, \Theta, Z^Y_{t+1}, Z^V_{t+1}\right) \times p\left(Z^Y_{t+1} | J_{t+1} = 1, \Theta, Z^V_{t+1}\right) \times p\left(Z^V_{t+1} | J_{t+1} = 1, \Theta\right),$$

where the first density is a bivariate Gaussian, the second is also Gaussian and the third is an exponential. By gathering all the terms in the exponents of these three distributions as a function of Z^V_{t+1} leads to the truncated normal distribution $1\left\{Z^V_{t+1} > 0\right\} N(b^*, B^*)$ with

$$b^* = B^* \left(\frac{d^V_{t+1} - \rho \sigma_V d^Y_{t+1}}{(1-\rho^2)\, \sigma^2_V V_t} + \frac{\rho_J \left(Z^Y_{t+1} - \mu_Y\right)}{\sigma^2_Y} - \frac{1}{\mu_V} \right) \quad ; \quad B^* = \left(\frac{1}{(1-\rho^2)\, \sigma^2_V V_t} + \frac{\rho^2_J}{\sigma^2_Y} \right)^{-1}$$

where $d^V_t = V_t - V_{t-1} - \alpha - \beta V_{t-1}$ and $d^Y_t = Y_t - \mu - Z^Y_t$.

Appendix B: Fourier transform for the SVCJ model

In this appendix we present the closed form expression for the characteristic function for the SVCJ model and supply some details on option pricing using the associated Fourier transform.

The expression for the characteristic function $q(z) = E^{\mathbb{Q}}_0 \left[e^{z \log(S_T)}\right]$, $z \in \mathbb{C}$, is given in Duffie et al. (2000) and below we simply restate their result using our notation. Define

$$a = z(1-z) \quad ; \quad b = \sigma_V \rho z - \kappa \quad ; \quad c = 1 - \rho_J \mu_V z \quad ; \quad d = \sqrt{b^2 + \sigma^2_V a^2}.$$

and $\overline{\mu} = \exp\left(\mu_Y + \sigma^2_Y/2\right)/(1 - \rho_J \mu_V) - 1$. The characteristic function at $t = 0$ is given by the expression

$$\log q(z) = A(T, z) + B(T, z) V_0 + z \log(S_0)$$

where

$$B(T, z) = -\frac{a\left(1 - e^{-dT}\right)}{2d - (d+b)\left(1 - e^{-d}T\right)}$$

and

$$A(T, z) = (r - \delta) z T - \lambda T (1 + \overline{\mu} z) + A_0(T, z) + \lambda A_1(T, z).$$

The functions $A_0(T,z)$ and $A_1(T,z)$ are given by

$$A_0(T,z) = -\kappa\theta \left(\frac{d+b}{\sigma_V^2} T + \frac{2}{\sigma_V^2} \log \left[1 - \frac{d+b}{2d} \left(1 - e^{-dT}\right) \right] \right)$$

and

$$A_1(T,z) = \exp\left(\mu_Y z + \sigma_Y^2 \frac{z^2}{2} \right) \zeta(z)$$

where

$$\zeta(z) = \frac{d-b}{(d-b)c + \mu_V a} T - \frac{2\mu_V a}{(dc)^2 - (bc - \mu_V a)^2} \log \left[1 - \frac{(d+b)c - \mu_V a}{2dc} \left(1 - e^{-dT}\right) \right]$$

The characteristic function for the SVJ and SV models obtains as special cases of this general expression with the following parameter restrictions; $\mu_V = \rho_J = 0$ for the SVJ model and $\lambda = 0$ for the SV model. This result allows us to price a European option on S by the use of the method outlined e.g. in Carr and Madan (1998).

In order to obtain the implied volatilities displayed in figure .6 we calculated European option prices using the relation

$$e^{-rT} E_0^{\mathbb{Q}} \left[(S_T - K)^+ \right] = e^{-rT} \frac{1}{\pi} \int_0^\infty \mathrm{Re} \left(\frac{e^{-iuk - \eta k} e^{\log(S_0)(1+\eta+iu)} q(1+\eta+iu)}{(\eta + iu)(1+\eta+iu)} \right) du$$

where $k = \log(K)$, $u \in \mathbb{R}$ and $i = \sqrt{-1}$. We refer to Carr and Madan (1998) for a derivation of this result. The integral needs to be calculated by a numerical integration routine and we used a Gauss-Laguerre quadrature with 500 evaluation points. The parameter η is introduced in the above representation in order to achieve sufficient integrability. Any η chosen in the set $A^+ = \{\eta > 0 : E_0^{\mathbb{Q}} \left[S_T^{1+\eta} \right] < \infty\}$ can be used without altering the theoretical option price. However, since the option price is obtained by integrating numerically the parameter η should be chosen optimally. As explained in Lee (2004) a rational thing to do is to minimize the function

$$g(\eta) = q(1+\eta) \exp(\log(S_0)(1+\eta) - \eta k)/\eta(\eta+1)$$

over the set A^+. This optimization can be performed using standard numerical methods.

Appendix C: The Least Squares Monte Carlo algorithm

In this Appendix we give a brief account of the Least Squares algorithm used to evaluate the oil field option. Let X_t be the state vector at time t, including all state variables. Let $X_{t_i}^j$, $\widehat{\Pi}_{t_i}^j$ and $\Phi_{t_i}^j$ be the value of the state vector, the estimated option value and the option payoff (intrinsic value), respectively at the exercise date t_i and sample path j. Following Longstaff and Schwartz (2000) and Glasserman (2003) the algorithm can be described as follows.

ALGORITHM

1. Simulate M paths of X. Let $\mathcal{K}_{t_i}$ be the subset of paths where the option is in-the-money at time t_i.
2. At the terminal time t_N set $\widehat{\Pi}^m_{t_N} = \Phi^m_{t_N}$
3. Loop backwards for $i = N-1$ to $i = 1$. At t_i run the least squares regression

$$e^{-r(t_{i+1}-t_i)}\widehat{\Pi}^m_{t_{i+1}} = \sum_{l=1}^{L} \beta_l h_l(X^m_{t_i}) + \varepsilon^m \qquad \text{for} \qquad m \in \mathcal{K}_{t_i},$$

   where $h_l(x)$, $l = 1, \ldots, L$, are suitably chosen functions (discussed below).
   In order to determine to exercise or not the value of continuation is estimated as $\widehat{C}^m_{t_i} = \sum_{l=1}^{L} \widehat{\beta}_l \psi_l(X^m_{t_i})$. Defining the set $A = \{\Phi^m_{t_i} \geq \widehat{C}^m_{t_i}\} \cap \left\{m \in \mathcal{K}_{t_i}\right\}$ we update the estimated option value according to

$$\widehat{\Pi}_{t_i} = 1\left\{A\right\}\Phi^m_{t_i} + 1\left\{A^c\right\} e^{-r(t_{i+1}-t_i)}\widehat{\Pi}_{t_{i+1}}$$

   where A^c is the complement of the set A.
4. Finally compute the estimated option value at time $t = 0$ as

$$\widehat{\Pi}_0 = e^{-rt_1}\frac{1}{M}\sum_{m=1}^{M}\widehat{\Pi}^m_{t_1}.$$

The choice of basis functions is important for the algorithm to be accurate. We follow the literature and choose the following functions: 1, S, S^2, V, V^2, $S \times V$. We refer to Longstaff and Schwartz (2001) and Glasserman (2004) for further details.

We use the Euler discretization (3) to simulate all models. The estimation of the option value only requires draws of the underlying process at the exercise times. To minimize any bias that may occur from sampling at distant times we simulate all models on a fine time grid that includes the exercise times.