

Normal equations

$$\sum_{k=0}^P a_k \gamma_{xx}[n-k] = -\gamma_{xx}[n]$$

$$\Rightarrow a_1 \gamma_{xx}[0] + a_2 \gamma_{xx}[-1] = -\gamma_{xx}[1]$$

$$a_1 \gamma_{xx}[1] + a_2 \gamma_{xx}[0] = -\gamma_{xx}[2]$$

$$\gamma_{xx}[0] = \sigma_x \quad \gamma_{xx}[1] = \gamma_{xx}[-1] = a$$

$$\Rightarrow \gamma_{xx} \equiv 0 \text{ otherwise}$$

$$\text{I} \quad a_1 \sigma_x + a_2 a = -a$$

$$\text{II} \quad a_1 a + a_2 \sigma_x = 0$$

$$\text{II} : a_1 a = -a_2 \sigma_x$$

$$a_1 = \frac{-a_2 \sigma_x}{a}$$

$$\text{I} : -\frac{a_2 \sigma_x^2}{a} + a_2 a = -a$$

$$a_2 \left(-\frac{\sigma_x^2}{a} + a \right) = -a$$

$$a_2 = \frac{-a}{a - \frac{\sigma_x^2}{a}} = \frac{-a^2}{a^2 - \sigma_x^2} = \frac{a^2}{\sigma_x^2 - a^2}$$

$$a_1 = \frac{-a_2 \sigma_x}{a}$$

$$a_1 = -\frac{a^2}{\sigma_x^2 - a^2} \cdot \frac{\sigma_x}{a}$$

$$a_1 = -\frac{a}{\frac{\sigma_x^2}{a} - a} \cdot \frac{\sigma_x}{a}$$

$$a_1 = -\frac{a \sigma_x}{\sigma_x^2 - a^2} = \frac{a \sigma_x}{\sigma_x^2 - a^2}$$

$$\Rightarrow a_0 = 1, a_1 = \frac{a \sigma_x}{a^2 - \sigma_x^2} \text{ and } a_2 = \frac{a^2}{a^2 - \sigma_x^2}$$
