$$z = f(x, y)$$

σ_x = Error in variable x

σ_y = Error in variable y

$$\sigma_z^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2$$

Error in variable z

(1) $D_L = D_A (1+z)^2$ with σ_{D_A}, σ_z

$D_L = f(D_A, z)$

$\rightarrow$

$$\sigma_{D_L}^2 = \left(\frac{\partial f}{\partial D_A}\right)^2 \sigma_{D_A}^2 + \left(\frac{\partial f}{\partial z}\right)^2 \sigma_z^2$$

$$= (1+z)^2 \sigma_{D_A}^2 + 2 D_A (1+z) \sigma_z^2$$

(2) $L = 4\pi D_L^2 P_{bolo}$ $\quad L = f(D_L, P_{bolo})$

$\rightarrow$

$$\sigma_L^2 = \left(\frac{\partial f}{\partial D_L}\right)^2 \sigma_{D_L}^2 + \left(\frac{\partial f}{\partial P_{bolo}}\right)^2 \sigma_{P_{bolo}}^2$$

$$= 8\pi D_L P_{bolo}\, \sigma_{D_L}^2 + 4\pi D_L^2\, \sigma_{P_{bolo}}^2$$

(3) $$E_{ISO} = \frac{4\pi D_L^2 S_{bolo}}{(1+z)}$$

$$E_{ISO} = f(D_L, S_{bolo}, z)$$

$$\sigma_{E_{ISO}}^2 = \left(\frac{\partial f}{\partial D_L}\right)^2 \sigma_{D_L}^2 + \left(\frac{\partial f}{\partial S_{bolo}}\right)^2 \sigma_{S_{bolo}}^2 + \left(\frac{\partial f}{\partial z}\right)^2 \sigma_z^2$$

$$= \frac{8\pi D_L S_{bolo}}{(1+z)} \sigma_{D_L}^2 + \frac{4\pi D_L^2}{(1+z)} \sigma_{S_{bolo}}^2 - \frac{4\pi D_L^2 S_{bolo}}{(1+z)^2} \sigma_z^2$$

(4)

$$E_\gamma = E_{iso} \times F_{beam}$$

$$E_\gamma = f\left(E_{iso}, F_{beam}\right)$$

$\rightarrow$

$$\sigma_{E_\gamma}^2 = \left(\frac{\partial f}{\partial E_{iso}}\right)^2 \sigma_{E_{iso}}^2 + \left(\frac{\partial f}{\partial_{F_{beam}}}\right)^2 \sigma_{F_{beam}}^2$$

$$= F_{beam}\, \sigma_{E_{ISO}}^2 + E_{iso}\, \sigma_{F_{beam}}^2$$