CSE330: Numerical Methods

Assignment 5

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Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the pat of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

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1. A linear system is described by the following equations:

$$\begin{aligned} x_1 + 6x_2 + 2x_3 &= 10 \\ 3x_1 + 2x_2 + x_3 &= 6 \\ 4x_1 + 5x_2 + 2x_3 &= 9. \end{aligned}$$

   Based on these equations, answer the questions below.

   (a) **[1.5 marks]** From the given linear equations, identify the matrices A, x and b such the the linear system can be expressed as a matrix equation.

   (b) **[3 marks]** Construct the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ from this system.

   (c) **[1.5 marks]** Compute the unit lower triangular matrix L.

   (d) **[4 marks]** Now find the solution of the linear system using LU decomposition method. Use the unit lower triangular matrix found in the previous question.

2. A linear system is described by the following equations:

$$\begin{aligned} 6x_2 + 2x_3 &= 10 \\ 3x_1 + 2x_2 + x_3 &= 6 \\ 4x_1 + 5x_2 + 2x_3 &= 9. \end{aligned}$$

   Based on these equations, answer the questions below.

   (a) **[1.5 marks]** From the given linear equations, identify the matrices A, x and b such the the linear system can be expressed as a matrix equation.

   (b) **[1.5 marks]** Examine if the matrix A has any pivoting problem? Explain why or why not?

   (c) **[4 marks]** Write down the Augmented matrix, Aug(A), from the given linear system, and evaluate the upper triangular matrix U. Note that you have to show the row multipliers m_{ij} for each step as necessary.

   (d) **[3 marks]**Using the upper triangular matrix found in the previous question, compute the solution of the given linear system by Gaussian elimination method.

Motto: Mathematics is NOT difficult, but what is difficult is to believe that mathematics is NOT difficult.

Answer to the question -1

1(a)

Given,

$$x_1 + 6x_2 + 2x_3 = 10 \quad \text{--- (i)}$$

$$3x_1 + 2x_2 + x_3 = 6 \quad \text{--- (ii)}$$

$$4x_1 + 5x_2 + 2x_3 = 9 \quad \text{--- (iii)}$$

so,

$$A = \begin{pmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{pmatrix} \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$b = \begin{pmatrix} 10 \\ 6 \\ 9 \end{pmatrix}$$

1(b)

F_1 would be,

$$F_1 = \begin{pmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{pmatrix}$$

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{3}{1} = 3$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{4}{1} = 4$$

$$\therefore F_1 = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{pmatrix}$$

$$F_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{pmatrix}$$

$$A^{(2)} = F^{(1)} \times A$$

$$A^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{pmatrix}$$

$$F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 0 \end{pmatrix}$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{-19}{-16} = \frac{19}{16}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{19}{16} & 0 \end{pmatrix}$$

1(c)

$$L = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{10}{16} & 1 \end{pmatrix}$$

1(d)

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{10}{16} & 1 \end{pmatrix}$$

$$A^{(3)} = F^{(2)} \times A^{(2)}$$

$$A^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{19}{16} & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.0625 \end{pmatrix}$$

$$\therefore U = \begin{pmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.0625 \end{pmatrix}$$

$$Ly = b$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{10}{16} & 1 \end{pmatrix} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ 9 \end{pmatrix}$$

$$\therefore \quad y_1 = 10$$

$$3y_1 + y_2 = 6$$

$$\Rightarrow y_2 = 6 - 3y_1$$

$$= 6 - 3(10)$$

$$= -24$$

$$4y_1 + \frac{19}{16}y_2 + y_3 = 9$$

$$4 \times 10 + \frac{19}{16} \times (-24) + y_3 = 9$$

$$\Rightarrow y_3 = -2.5$$

$$\therefore y = \begin{pmatrix} 10 \\ -24 \\ -2.5 \end{pmatrix}$$

we know, $Ux = y$

$$\begin{pmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.0625 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ -24 \\ -2.5 \end{pmatrix}$$

$$-0.0625x_3 = -2.5$$

$$\Rightarrow x_3 = 40$$

$$-16x_2 - 5x_3 = -24$$

$$x_2 = \frac{5x_3 - 24}{-16}$$

$$= \frac{5(40) - 24}{-16}$$

$$= -11$$

$$x_1 + 6x_2 + 2x_3 = 10$$

$$\Rightarrow x_1 = -4$$

$$\therefore \quad x = \begin{pmatrix} -9 \\ -11 \\ 90 \end{pmatrix}$$

Answer to the question-2

2(a)

Given,

$$6x_2 + 2x_3 = 10$$
$$3x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 5x_2 + 2x_3 = 9$$

$$\therefore A = \begin{pmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad b = \begin{pmatrix} 10 \\ 6 \\ 9 \end{pmatrix}$$

2 (b)

Yes. The matrix A has pivoting problem.

Since the pivot element for the first row is zero,

2(c)

$$(A|b) \Rightarrow \left(\begin{array}{ccc|c} 0 & 6 & 2 & 10 \\ 3 & 2 & 1 & 6 \\ 4 & 5 & 2 & 9 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 4 & 5 & 2 & 9 \end{array}\right)$$

$R_1' = R_1/3$

$R_2' = \frac{R_2}{6}$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 2 \\ 0 & 1 & 1/3 & 5/3 \\ 4 & 5 & 2 & 9 \end{array}\right)$$

$$R_3' = R_3 - 4R_1 \Rightarrow \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 2 \\ 0 & 1 & 1/3 & 5/3 \\ 0 & \frac{7}{3} & \frac{2}{3} & 1 \end{array}\right)$$

$$R_3' = R_3 - \frac{7}{3} \times R_2 \Rightarrow \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 2 \\ 0 & 1 & 1/3 & 5/3 \\ 0 & 0 & -\frac{1}{9} & -\frac{26}{9} \end{array}\right)$$

$$R_3' = R_3 \times -9 \Rightarrow \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 2 \\ 0 & 1 & 1/3 & 5/3 \\ 0 & 0 & 1 & 26 \end{array}\right)$$

$$\therefore x_3 = 26$$

$$x_2 + \frac{1}{3}x_3 = 5/3$$

$$x_2 = \frac{5}{3} - \frac{1}{3} \times 26$$

$$= -7$$

$$x_1 + \frac{2}{3} \times x_2 + \frac{1}{3}x_3 = 2$$

$$x_1 = -2$$

$$x_1 = \begin{pmatrix} -2 \\ -7 \\ 26 \end{pmatrix}$$