CSE330: Numerical Methods

Assignment 6

Email: sohan.academics@gmail.com
GitHub: https://github.com/saad-bin-sohan

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Answer only Question no 1. The last two questions are practice materials for final exam.
- Start answering a question (not the pat of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

---

1. A student has decided to sell the jerseys of football club as a relaxation after the stressful final exam. There are x_1 number of jerseys of PSG club and x_2 number of jerseys of Barcelona club in his shop. The total number of jersey is 30. On the first day, he sold each jersey of two clubs by 400 tk. and totally he earned 12000 tk by selling all the jerseys. But when Messi left Barcelona club and went to PSG club, each jersey of Barcelona costs 300 tk. and each jersey of PSG costs 500 tk. Then, he earned total 13000 tk. In the following, this overdetermined system will be solved by using the QR Decomposition Method by answering the following step by step:
   (a) **[1.5 marks]** Write down the linear equations that relate the variable x_1 and x_2.
   (b) **[1.5 marks]** Identify the matrices A, x and b so that the equations in the previous question can be expressed in the standard matrix equation form Ax = b.
   (c) **[5 marks]** From matrix A in the previous question, compute the matrices Q and R such that A = QR, where the symbols have their usual meanings.
   (d) **[2 marks]** Evaluate $Q^T b$, and finally solve the system by evaluating x (that is , evaluate x_1 and x_2).
2. Consider the following function: $f(x) = e^x - x$, which is continuous on the interval [1, 3]. Use this function to answer the following:
   (a) **[2 marks]** Find the actual integral value for this function.
   (b) **[5 marks]** Use Composite Newton-cotes formula to find the numerical integration for 4 segments.
   (c) **[3 marks]** Compute the error in percentage between the results obtained in the previous two parts. How can we decrease the error more?
   (d) **[3 marks]** Use the Simpson rule to find the numerical integration.
3. Consider the following function: $f(x) = 6x^2 - 4x - 9$.
   (a) **[3 marks]** Use the Trapezium rule to numerically integrate over the interval [-2, 2].
   (b) **[2 marks]** Compute the exact integrated value of the given function.
   (c) **[2 marks]** Calculate the relative error in percentage.

Motto: Mathematics is NOT difficult, but what is difficult is to believe that mathematics is NOT difficult.

3:15 PM Tue 19 Dec

CSE330_Assignment_06_Solution.pdf

Spring 2023
Assignment 06

01.

(a) $x_1 + x_2 = 30$

$400x_1 + 400x_2 = 12000$

$500x_1 + 300x_2 = 13000$

(b) $A = \begin{bmatrix} 1 & 1 \\ 400 & 400 \\ 500 & 300 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $b = \begin{bmatrix} 30 \\ 12000 \\ 13000 \end{bmatrix}$

(c) $v_1 = \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix}$

$p_1 = v_1 = \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix}$

$q_1 = \frac{1}{\sqrt{1^2+400^2+500^2}} \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix} = \frac{1}{640.31} \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix}$

$$p_2 = v_2 - (v_2^T q_1)\, q_1$$

$$= \begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix} - \left(\begin{bmatrix} 1 & 400 & 300 \end{bmatrix} \frac{1}{640.31} \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix} \right) \frac{1}{640.31} \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix} - \frac{310001}{640.31} \cdot \frac{1}{640.31} \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix} - 0.76 \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix}$$

$$= \begin{bmatrix} 0.24 \\ 96 \\ -80 \end{bmatrix}$$

$$q_2 = \frac{1}{\sqrt{0.24^2 + 96^2 + (-80)^2}} \begin{bmatrix} 0.24 \\ 96 \\ -80 \end{bmatrix} = \frac{1}{124.96} \begin{bmatrix} 0.24 \\ 96 \\ -80 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.0016 & 0.0019 \\ 0.625 & 0.768 \\ 0.781 & -0.640 \end{bmatrix}$$

3:15 PM Tue 19 Dec 75%
CSE330_Assignment_06_Solution.pdf

$$R = \begin{bmatrix} v_1^T a_1 & v_2^T a_1 \\ 0 & v_2^T a_2 \end{bmatrix}$$

$$= \begin{bmatrix} 640.31 & 484.14 \\ 0 & 115.24 \end{bmatrix}$$

(d) $Rx = Q^T b$

$$\Rightarrow \begin{bmatrix} 640.31x_1 + 484.14x_2 \\ 115.24x_2 \end{bmatrix} = \begin{bmatrix} 0.0016 & 0.625 & 0.781 \\ 0.0019 & 0.768 & -0.640 \end{bmatrix} \begin{bmatrix} 30 \\ 12000 \\ 13000 \end{bmatrix}$$

$$= \begin{bmatrix} 17653.048 \\ 896.057 \end{bmatrix}$$

$\therefore 115.24x_2 = 896.057 \qquad \therefore x_2 = 7.54 \approx x_2 = 8$

$640.31x_1 + 484.14x_2 = 17653.048 \qquad \therefore x_1 = 21.87 \approx x_1 = 22$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 22 \\ 8 \end{bmatrix}$$

02.

(a)

$$I = \int_1^3 e^x - x \, dx$$

$$= \int_1^3 e^x \, dx - \int_1^3 x \, dx$$

$$= \left[e^x\right]_1^3 - \left[\frac{x^2}{2}\right]_1^3$$

$$= e^3 - e - 4$$

$$= 13.3673$$

(b) $m = 4$, $a = 1$, $b = 3$

$$h = \frac{b-a}{m} = \frac{3-1}{4} = \frac{1}{2}$$

$a = x_0$, $x_1 = a + h$, $x_2 = x_1 + h$, $x_3 = x_2 + h$, $x_4 = b$

$\Rightarrow x_0 = 1$, $x_1 = \frac{3}{2}$, $x_2 = 2$, $x_3 = \frac{5}{2}$, $x_4 = 3$

$$C_{1,4} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$= \frac{1}{4}\left[1.7183 + 2\times 2.9817 + 2\times 5.3891 + 2\times 9.6825 + 17.0855\right]$$

$$= 13.7276$$

(c)

$$\text{Error} = \left|\frac{I - C_{1,4}}{I}\right| \times 100\%$$

$$= \left|\frac{13.3673 - 13.7276}{13.3673}\right| \times 100\%$$

$$= |-0.02695| \times 100\%$$

$$= 2.695\%$$

we can decrease the error more by increasing the value of m or the number of segments.

(d)

$$I_2(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$= \frac{3-1}{6}\left[1.7183 + 4 \times 5.3891 + 17.0855\right]$$

$$= 13.4534$$

03.

(a) $a = -2,\ b = 2$

$$I_1(f) = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$= \frac{2+2}{2}\left[23 + 7\right]$$

$$= 60$$

(b) $$I = \int_{-2}^{2} 6x^2 - 4x - 9$$

$$= \left[2x^3 - 2x^2 - 9x\right]_{-2}^{2}$$

$$= -10 + 6$$

$$= -4$$

(c) Relative Error $= \left| \frac{I - I_1}{I} \right| \times 100\%$

$= \left| \frac{-4 - 60}{-4} \right| \times 100\%$

$= 16 \times 100\%$

$= 1600\%$