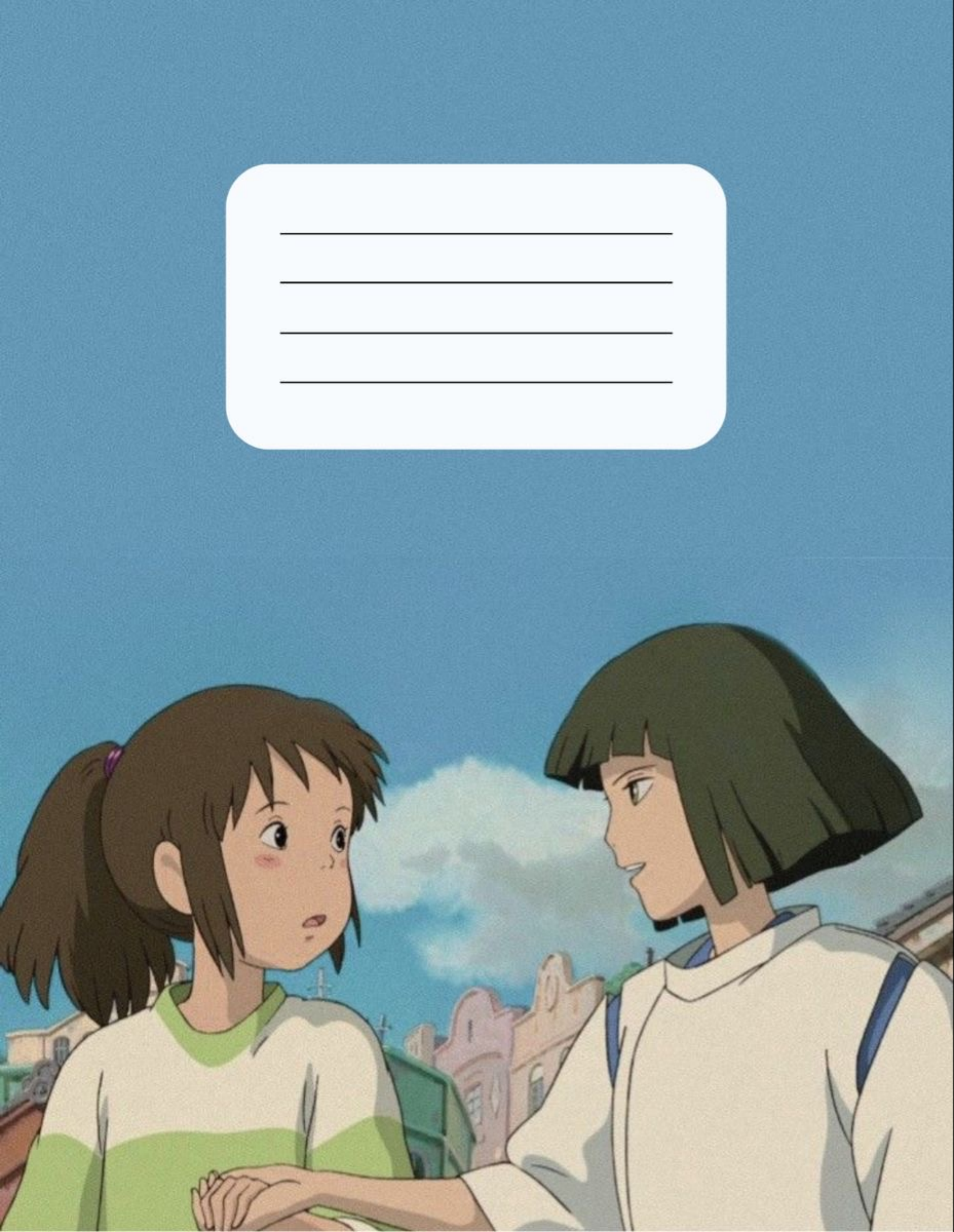

2 parts $\rightarrow$ complex (ch-4,5,6,7)
3set

$\rightarrow$ laplace
4set

Complex Variable

Ch-9 (2 parts) → complex integration (sir এর lecture এর সব math must)

↓

cauchy goursat theorem

(part 2 not important, skip)

*complex part theke proof ashbe na

Ch-5

→ proof লাগবে না

→ সব math হবে

partial fraction পড়বে না।

Residue দিয়ে হবে।

$$\oint_C \frac{f(z)}{(z-a)^{n+1}}\,dz = \frac{2\pi i}{n!}\cdot\left\{\frac{d^n}{dz^n}\{f(z)\}\right\}\Bigg|_{z=a}$$

Ch-6

2 parts → laurent series ([illegible])
(imp)

→ singularity ([illegible])

Ch-7

ch-5 এর pf → ch7 এর residue

Laplace Transform

→ a set

* using defn type
* derivative proof
* diff. eqn

$\boxed{\text{Imp math type}}$: Using definition type:

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt = F(s)$$

Q) show that using definition,

$$\mathcal{L}\{t^2\} = \frac{2}{s^3}$$

ans:

$$\mathcal{L}\{t^2\} = \int_0^\infty t^2 e^{-st}\, dt$$

L
I
A
T
E

$$= \left[t^2 \cdot \frac{-1}{s} e^{-st} \right]_0^\infty - \int_0^\infty -\frac{1}{s} e^{-st} \cdot 2t\,dt$$

$u = t^2$
$du = 2t\,dt$
$dv = e^{-st}\,dt$
$v = -\frac{1}{s} e^{-st}$

$$= \frac{2}{s} \int e^{-st}\, t\, dt$$

$u = t$
$du = dt$
$dv = e^{-st}\,dt$
$v = -\frac{1}{s} e^{-st}$

$$= \frac{2}{s} \left[\left[t \cdot -\frac{1}{s} e^{-st} \right]_0^\infty - \int -\frac{1}{s} e^{-st}\, dt \right]$$

$$= \frac{2}{s} \cdot \frac{1}{s} \cdot \int e^{-st}\, dt$$

$$= \frac{2}{s^2} \int e^{-st}\, dt$$

$$= \frac{2}{s^2} \cdot \left[\frac{-1}{s} e^{-st} \right]_0^\infty$$

$$= \frac{2}{s^3} \left[- e^{-st} \right]_0^\infty$$

$$= \frac{2}{s^3}$$

4) show that, $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

ans:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}\{\cos at\} = \int_0^\infty e^{-st} \cos at\,dt$$

$$I = \int_0^\infty e^{-st} \cos at\,dt$$

$$= \left[\cos at \cdot -\frac{1}{s} e^{-st}\right]_0^{\infty} -$$

$u = \cos at$

$dv = e^{-st} dt$

$v = -\frac{1}{s} e^{-st}$

$du = -a \sin at$

$$\int -\frac{1}{s} e^{-st} \cdot -a \sin at \; dt$$

$$= -1 \cdot \frac{-1}{s} - \frac{1}{s} \int e^{-st} a \sin at \, dt$$

$$= \frac{1}{s} - \frac{a}{s} \cdot \left[\cancel{\left[\sin at \cdot \frac{-1}{s} e^{-st}\right]_0^{\infty}}^{\,0} - \int -\frac{1}{s} e^{-st} \cdot a \cos at \, dt \right]$$

$u = \sin at$

$du = a \cos at \, dt$

$dv = e^{-st} dt$

$v = -\frac{1}{s} e^{-st}$

$$= \frac{1}{s} - \frac{a}{s} \left[\frac{a}{s} \int e^{-st} \cdot \cos at \, dt \right]$$

$$= \frac{1}{s} - \frac{a^2}{s^2} \int e^{-st} \cos at \, dt$$

$$I = \frac{1}{s} - \frac{a^2}{s^2} \cdot I$$

$$\Rightarrow \quad s^2 I = -a^2 I + s$$

$$\Rightarrow \quad I = \frac{s}{s^2 + a^2}$$

$$\therefore \mathcal{L}\{\cos at\} = \frac{s}{s^2 + a^2}$$

$$f(t) = \begin{cases} 0, & t \le \pi \\ \cos t, & t > \pi \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^\pi 0 \cdot e^{-st}\, dt + \int_\pi^\infty \cos t\, e^{-st}\, dt$$

$$= \int_\pi^\infty \cos t\, e^{-st}\, dt$$

$$I = \int_\pi^\infty \cos t \cdot e^{-st} \cdot dt$$

$$= \left[\cos t \cdot \frac{-1}{s} e^{-\pi s}\right]_\pi^\infty -$$

$u = \cos t$

$du = -\sin t\, dt$

$$\int \frac{-1}{s} e^{-st} \cdot -\sin t \, dt$$

$$dv = e^{-st}\, dt$$

$$dv = \frac{-1}{s} e^{-st}$$

$$= -(-1)\frac{-1}{s} \cdot e^{-\pi s} - \frac{1}{s}\int e^{-st} \sin t \, dt$$

$$= -\frac{1}{s} e^{-\pi s} - \frac{1}{s}\left[\left[\sin t \cdot \frac{-1}{s} e^{-st}\right]_{\pi}^{\infty} - \int -\frac{1}{s} e^{-st} \cos t \, dt \right]$$

$$u = \sin t$$

$$du = \cos t\, dt$$

$$dv = e^{-st}\, dt$$

$$v = -\frac{1}{s} e^{-st}$$

$$= \frac{-e^{-\pi s}}{s} - \frac{1}{s^2} \cdot \int e^{-\pi t} \cos t \, dt$$

$$\therefore I = \frac{-e^{-\pi s}}{s} - \frac{1}{s^2} I$$

$$\therefore \quad I\left(1+\frac{1}{s^2}\right) = \frac{-e^{-\pi s}}{s}$$

$$\therefore \quad I\left(\frac{s^2+1}{s^2}\right) = \frac{-e^{-\pi s}}{s}$$

$$I = \frac{-e^{-\pi s}}{s} \times \frac{s^2}{s^2+1}$$

$$= \frac{-se^{-\pi s}}{s^2+1}$$

$$\int_0^\infty t^2 e^{-st}\, dt$$

u → differentiate

v (integrate)

shortcut method

L
I
A
T
E

u	v
t^2	e^{-st}
$2t$	$-\dfrac{e^{-st}}{s}$
2	$\dfrac{e^{-st}}{s^2}$
0	$-\dfrac{e^{-st}}{s^3}$

$$= \left[t^2 \cdot \frac{e^{-st}}{-s} - 2t \cdot \frac{e^{-st}}{s^2} + 2 \cdot \frac{-e^{-st}}{-s^3} \right]_0^\infty$$

$$= 0 - \left[2 \cdot \frac{1}{-s^3} \right] = \frac{2}{s^3}$$

shortcut

$$\textcircled{1} \int \cos at \, e^{-st} \, dt$$

u	v
$\cos at$	e^{-st}
$-a \sin at$	$\frac{e^{-st}}{-s}$
$-a^2 \cos at$	$\frac{e^{-st}}{s^2}$
$\vdots$	

$$= \left[\cos at \cdot \frac{e^{-st}}{-s} - \left\{ -a \sin at \cdot \frac{e^{-st}}{s^2} \right\} \right]_0^{\infty} + \int_0^{\infty} -a^2 \cos at \cdot \frac{e^{-st}}{s^2} \, dt$$

2nd type: derivitive type proof

e.g: prove,

$$\mathcal{L}\{y''\} = s^2 Y(s) - s \cdot y(0) - y'(0)$$

formula চালানো কিন্তু দেওয়া
হবে না:

$$\mathcal{L}\{t\cdot f(t)\} = -\frac{d}{ds}\left(F(s)\right)$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a) \quad \text{বা}$$

$$\mathcal{L}\{f(t)\}\Big|_{s \to s-a}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{F(s+a)\}e^{at}$$

Sample Problems

1st transl.

$\boxed{1}$ $\mathcal{L}\{e^{2t}\sin 3t\}$

$\boxed{2}$ $\mathcal{L}\{e^{2t}\cdot\sin 5t\}$

2nd transl.

$\boxed{1}$ $\mathcal{L}\{\sin 5t \cdot u(t-\pi)\}$

inverse-$\mathcal{L}$

1st transl.

$\boxed{1}$ $\mathcal{L}^{-1}\left\{\dfrac{s}{(s-3)^2+5^2}\right\}$

2nd

$$\mathcal{L}^{-1}\left\{ \frac{s-6}{s^2+5^2} e^{-\pi s} \right\}$$

$$\mathcal{L}^{-1}\left\{ \frac{s-2}{(s-3)^2+5^2} e^{-\pi s} \right\}$$

also:

$$\mathcal{L}^{-1}\left\{ \frac{1}{(s-1)^2(s-2)^2} \right\}$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{(s^2+1)(s^2+9)} e^{-\pi s} \right\}$$

Differential Eqn

→ normal (third order)

→ with unit step

$y' + y = f(t)$

$y(0) = 5$

$$f(t) = \begin{cases} 0, & 0 \le t < \pi \\ \cos t, & t \ge \pi \end{cases}$$

BVP লাগবে না।

eg: $y'' + 9y = \cos 2t$, $y(0) = 1$
$y(\frac{\pi}{2}) = 1$

(B) Evaluation of improper integral

লাগবে না

$\mathcal{L}\{t\, e^{-3t} \cos 2t \cos 4t\}$

$$\boxed{2\cos A \cos B = \cos(A+B) + \cos(A-B}$$

<u>ans:</u>

$$\Rightarrow \mathcal{L}\{\cos 2t \cos 4t\}$$

$$= \frac{1}{2} \mathcal{L}\{2\cos 2t \cos 4t\}$$

$$= \frac{1}{2} \mathcal{L}\{(\cos 6t) + (\cos 2t)\}$$

$$= \frac{1}{2} \frac{s^2}{s^2+36} + \frac{1}{2} \frac{s^2}{s^2+4}$$

now, $\mathcal{L}\left\{t \cdot \cos 2t \cos 4t\right\}$

$$= -\frac{d}{ds}\left(\frac{\frac{1}{2}s}{s^2+36} + \frac{\frac{1}{2}s}{s^2+4}\right)$$

$$= \ldots\ldots$$

$$\mathcal{L}\left\{e^{-3t} \cdot t \cdot \cos 2t \cdot \cos 4t\right\}$$

$$= \left(\quad \right)\Bigg|_{s \to s+3}$$

Laplace (9 sets)

1 set → using definition

1 set → basic math ($\mathcal{L}$ ও $\mathcal{L}^{-1}$)

e.g: $\mathcal{L}\{t \cdot e^{2t} \sin 7t\}$

a, b

2 sets → differential eqn

→ i) normal

a, b আসবে
না।

→ ii) with unit step

laplace → 4 set ans kora better

7 que

→ ans 5