MAT215: Machine Learning & Signal Processing

Former Title: Complex variables & Laplace Transformations

Topic: Complex Number & related Properties

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MAT215
(Machine Learning & Signal Processing)

D- 21/1/24

Old title: Complex variables and Laplace Transformations

New title: Machine Learning & Signal Processing

PS: ML নিয়ে কোনো প্রশ্ন আসে না

$\boxed{\text{Course Books}}$

1) "Complex Variables"
 – Schaum's outlines series

2) "Differential Equations"

(only chapter 7) — Dennis a. Zill

Mark Distribution

Attendance	5
Assignments (n, best of all)	15
Quiz (n-1)	25
Mid	25
Final	30

Complex Number

$\mathbb{R} \rightarrow$ symbol for Real number system

$\mathbb{C} \rightarrow$ symbol for complex number system

Complex Number:

Any complex number can be written as a+ib where a and b are real numbers and $i = \sqrt{-1}$ or $i^2 = -1$. Here, a is called the real part and b (NOT bi) is called the imaginary part

e.g:

$z = 3+4i \Rightarrow$ Real part $= 3$, imaginary part $= 4$ (not $4i$)

$z = -2-3i \Rightarrow$ Real part $= -2$ imaginary part $= -3$

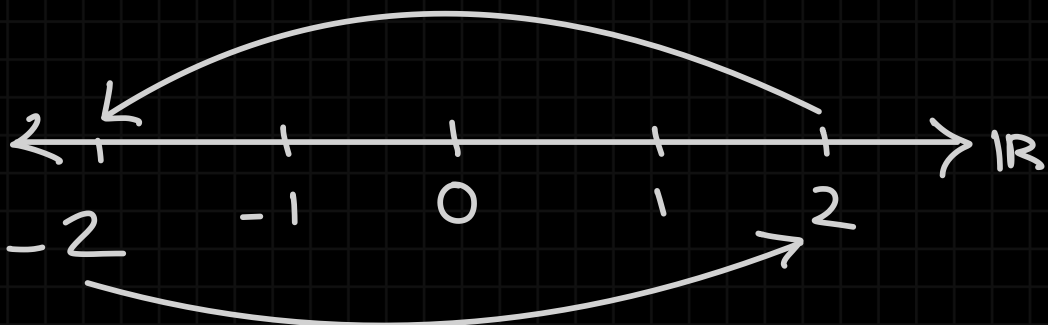
$z = 5i \Rightarrow$ Real part $= 0$, imaginary part $= 5$

$z = 2 = 2+0i \Rightarrow$ Real part $= 2$, imaginary part $= 0$

How the idea of imaginary number came to be:

$$x^2+1=0$$

$x \Rightarrow$ no solution in real number



$$2(-1) = -2$$

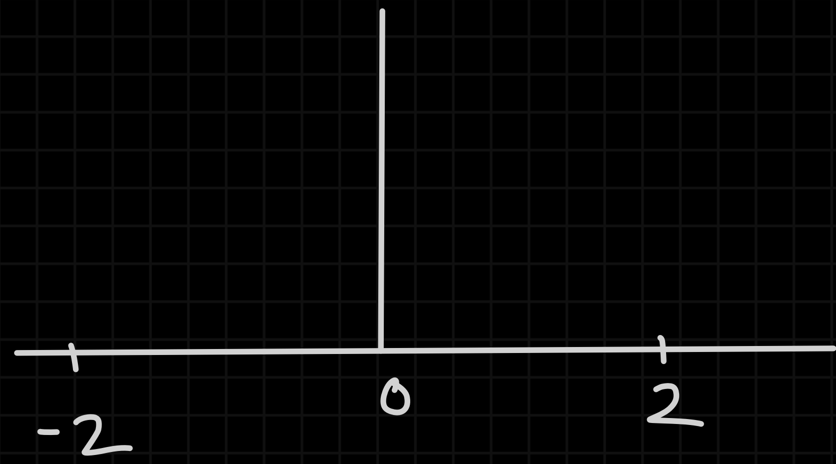
$\hookrightarrow$ 180° rotatoation

$$(-2)(-1) = 2$$

$\hookrightarrow$ 180° rotation

$\therefore$ multiplying by -1 means rotating by 180°

suppose there is a number that rotates the original number by exactly 90°, let the number be i.



or

$2i^2$

$$2i^2 = -2$$

$$i^2 = -1$$

$$i = \sqrt{-1}$$

$2x+3=9$

$x=3$

$x^2=4$

$x=-2,2$

$x(x-1)(x+2)=0$

$x=0,\ 1,\ -2$

$x^2=4$

$x=2i,-2i$

$x^3=1$

$x = 1,\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

Complex Number:

Any complex number can be written as a+ib where

$a, b \in \mathbb{R}$ and $i = \sqrt{-1}$

Operations of complex numbers:

addition, subtraction

multiplication, division

power

e.g: $(3+4i) + (1+2i)$

$= 4+6i$

e.g: $(3+4i) - (1+2i)$

$= 2-2i$

e.g: $(3+4i) \times (1+2i)$

$= 3 + 6i + 4i + 8i^2$

$= -5 + 10i$

e.g: $\dfrac{3+4i}{1+2i}$

multiply both numerator and denominator by the conjugate of denominator

$= \dfrac{(3+4i)(1-2i)}{(1+2i)(1-2i)}$

$= \dfrac{3 - 6i + 4i - 8i^2}{1-(2i)^2}$

$= \dfrac{11-2i}{1-(-4)}$

$$= \underbrace{\frac{11}{5}}_{a} + \underbrace{\frac{-2}{5}}_{b} i$$

Complex Conjugate:

The complex conjugate of a complex number z is denoted by $\bar{z}$

if $z = a + ib$, $\bar{z} = a - ib$

Properties of Complex Conjugate:

$z + \bar{z} =$ Real number

$z \cdot \bar{z} =$ Real number $= (\text{modulus})^2$

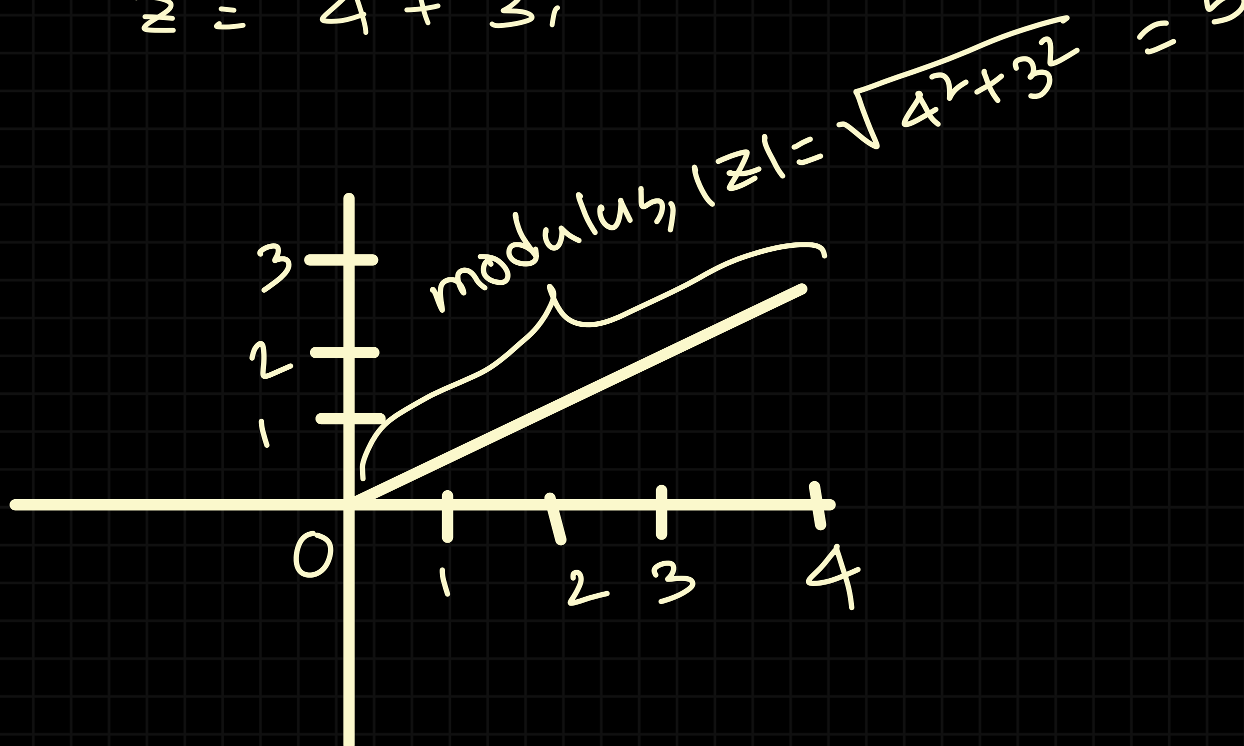
Modulus:

The modulus of a complex number z is denoted by $|z|$

if $z = a + ib$, $|z| = \sqrt{a^2 + b^2}$

Note: $|z|$ is the distance from the origin to the complex number

e.g.: $z = 4 + 3i$



Properties of Modulus:

① $|z_1 \cdot z_2| = |z_1|\,|z_2|$

② $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

③ $|z^m| = |z|^m$ where $m \in \mathbb{R}$

e.g.: $z_1 = 1+2i, \; z_2 = 3+2i$

$|z_1 \cdot z_2| = ?$

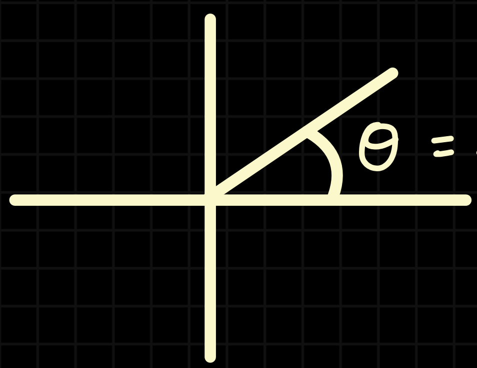
$\Rightarrow \because |z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$= \sqrt{1^2+2^2} \times \sqrt{3^2+2^2}$

$= \sqrt{65}$

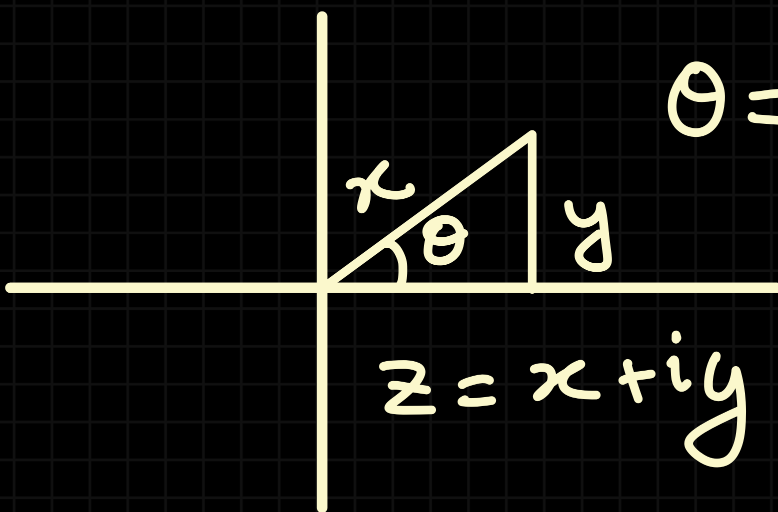
Argument(θ):

(The angle between the zero degree X-axis line and the line created by complex number)



$$\theta = \underbrace{30^\circ,\ 390^\circ,\ 750^\circ, \ldots\ldots}_{\text{same location for different values of angle}}$$

Argument (First Quadrate)



$$\theta = \tan^{-1}\left(\frac{y}{x}\right) + 2n\pi$$

where $n \in \mathbb{Z}$

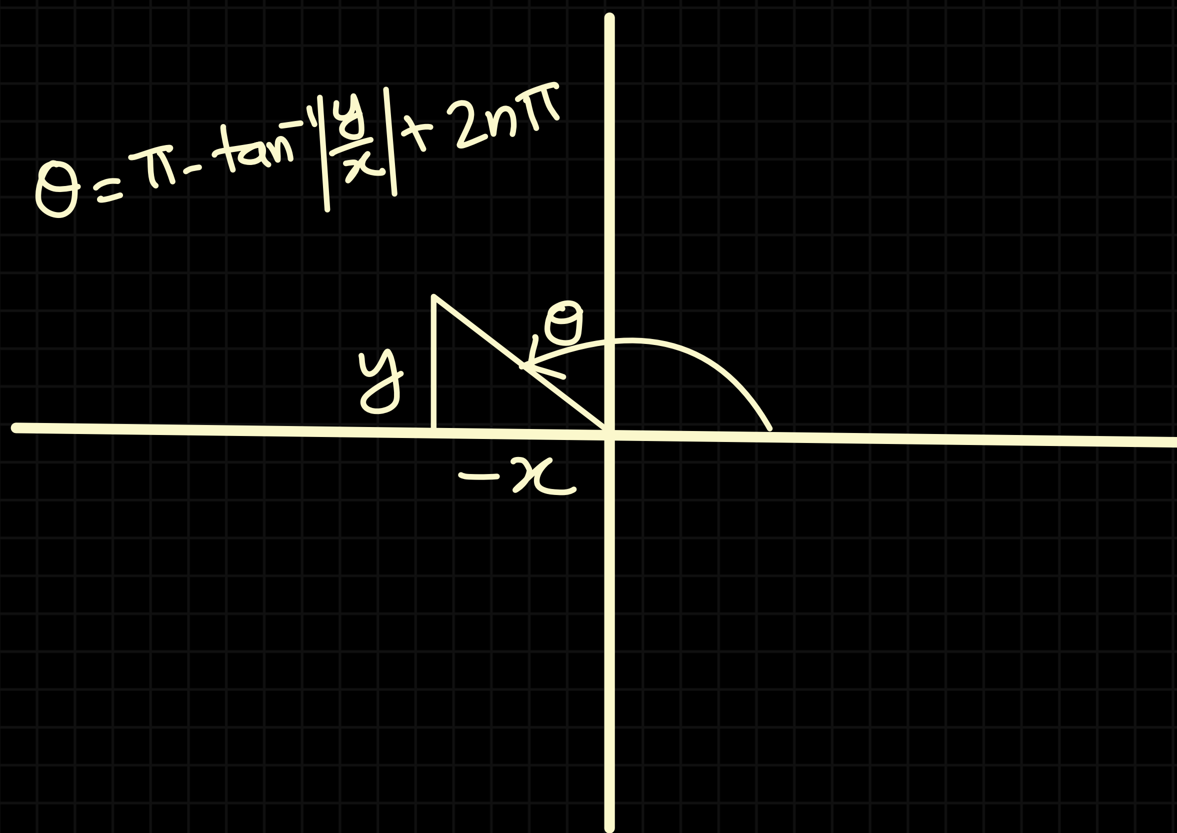
principal argument,
$= \tan^{-1}\frac{y}{x}$

Principal Argument: Shortest argument between the origin and z

Argument (second quadrate)

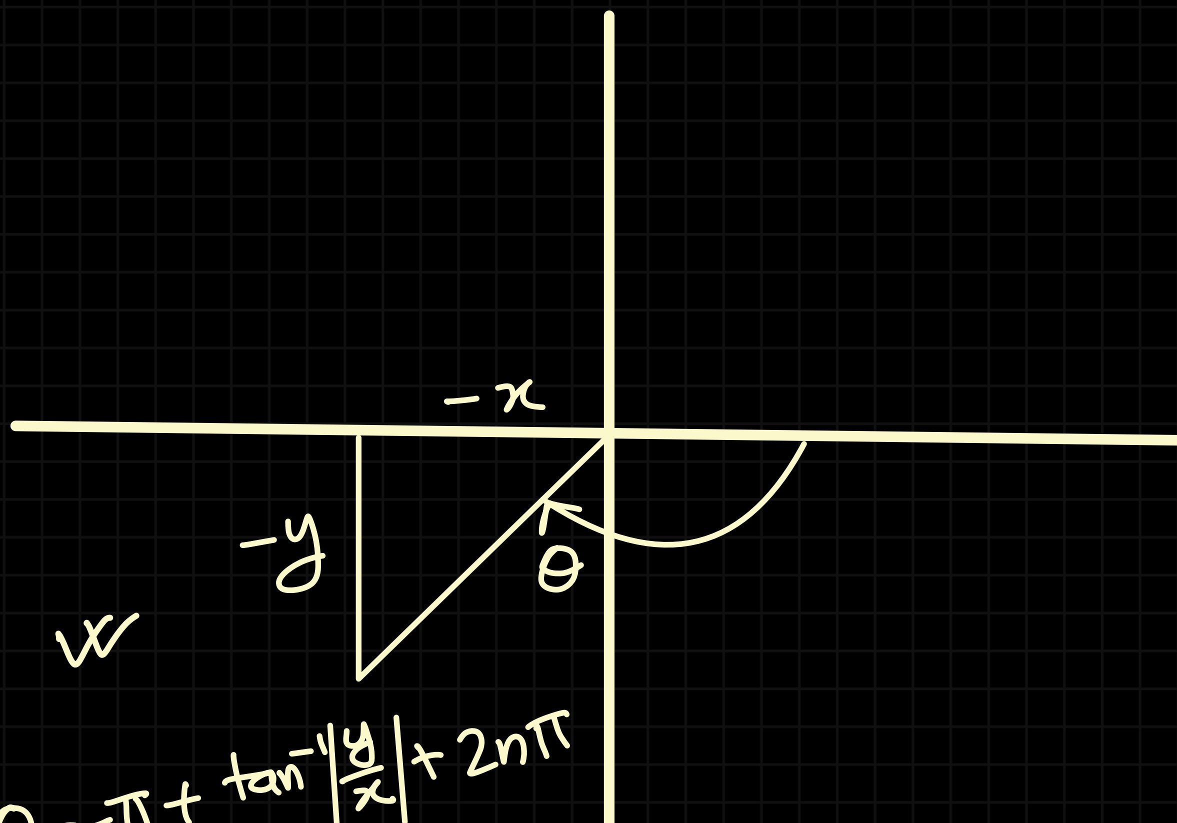
iz

argument, $\theta = \pi - \tan^{-1}\left|\frac{y}{x}\right| + 2n\pi$



principal argument $= \pi - \tan^{-1}\left|\frac{y}{x}\right|$

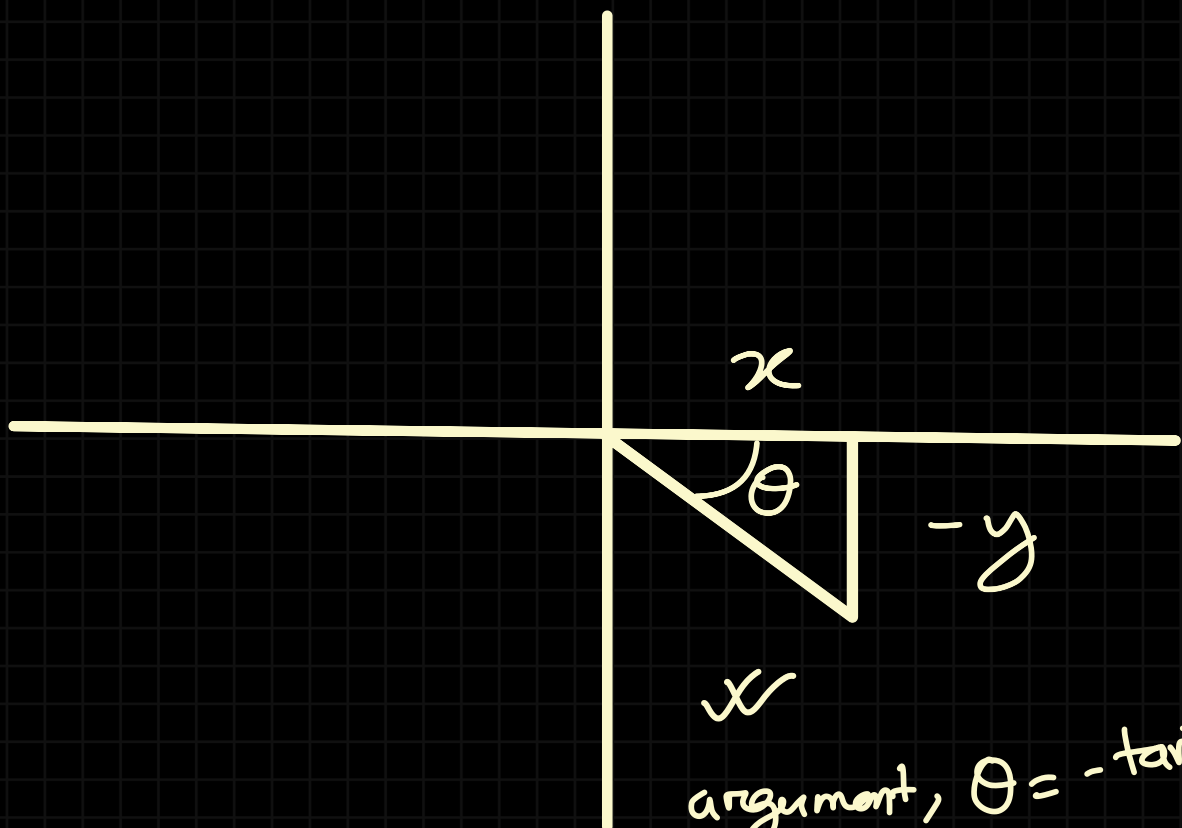
Argument (third quadrate)



argument, $\theta = -\pi + \tan^{-1}\left|\frac{y}{x}\right| + 2n\pi$

principal argument $= -\pi + \tan^{-1}\left|\frac{y}{x}\right|$

Argument (fourth quadrate)



argument, $\theta = -\tan^{-1}\left|\frac{y}{x}\right| + 2n\pi$

$z = 1 + \sqrt{3}\, i$; argument = ?

modulus = ? principal argument = ?

$\Rightarrow$ $x, y \Rightarrow$ both positive $\therefore$ first quadrate

$\therefore$ argument $= \tan^{-1}\left|\frac{y}{x}\right| + 2n\pi$

$$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| + 2n\pi$$

$$= \frac{\pi}{3} + 2n\pi$$

modulus, $|z| = \sqrt{1^2 + (\sqrt{3})^2}$

$$= 2$$

principal argument $= \pi/3$

$z = -1 + i$

argument = ?

$\Rightarrow$ $x = -ve$, $y = +ve$

$\therefore$ 2nd quadrate

argument $= \pi - \tan^{-1}\left|\frac{1}{-1}\right| + 2n\pi$

$= \pi - \tan^{-1}(1) + 2n\pi$

$= \frac{3\pi}{4} + 2n\pi$

$z = -\sqrt{3} - i$

argument = ?

$\Rightarrow$ $x, y \Rightarrow -ve$ $\therefore$ 3rd quadrate

$$\arg = -\pi + \tan^{-1}\left|\frac{-1}{\sqrt{3}}\right| + 2n\pi$$

$$= -\pi + \tan^{-1}(1/\sqrt{3}) + 2n\pi$$

$$= -\pi + \frac{\pi}{6} + 2n\pi$$

$$= \frac{-5\pi}{6} + 2n\pi$$

$z = 1 - 2i$, argument = ?

$\Rightarrow$ $x \Rightarrow +ve$, $y = -ve$ $\therefore$ 4th quadrate

$$\text{argument} = -\tan^{-1}\left|\frac{-2}{1}\right| + 2n\pi$$

$= -\tan^{-1}(2) + 2n\pi$

Ans.

$z = -3i$, argument $= ?$

$\Rightarrow$ $x \Rightarrow +ve$, $y = -ve$ $\therefore$ 4th quadrate

$\text{argument} = -\tan^{-1}\left(\frac{-3}{0}\right)$

$= -\pi/2$