CSE330: Numerical Methods

Topic: Hermite Polynomial, Numerical Differentiation, Central Difference

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☆ so far we have learned three methods of polynomial interpolation.

<u>method-1:</u> Vandermondej method

reason: PC does not save function as function. It saves them as polynomial. So we started guessing the function.

<u>method-2:</u> Lagrange polynomial

reason: In method-1, inversion of matrix needs to be done. But it is an inefficient operation for

PC. so we learn lagrange, where no inversion of matrix is involved.

method-3: Newton's Polynomial

extra node add করলে newton's polynomial এর reusability এর জন্য reuse করা যায়, কিন্তু Lagrange method পুরো math আবার করা লাগে।

Today we learn method-4 (Hermite Interpolation) because it gives more accurate polynomial in same degree since it adds more terms.

Hermite Interpolation

$(n+1)$ সংখ্যক nodes $\longrightarrow$ degree n

$(n+1)$ সংখ্যক nodes $\longrightarrow$ $(n+1)$ equation/conditions

$(n+1)$ " nodes $\longrightarrow$ $(n+1)$ equations

↳ reason! derivative

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total number of equations $= 2n+2$

$(2n+2)-1$ degree

$= 2n+1$ degree

so we will get P_{2n+1} and not P_n

so,

$$P_{2n+1}(x) = \sum_{k=0}^{n} \left\{ f(x_k)\, \underbrace{h_k(x)} + f'(x_k)\, \underbrace{\hat{h}_k(x)} \right\}$$

hermite basis 2টি

where,

$$h_k(x) = \left[1 - 2(x - x_k)\, l_k'(x_k) \right] l_k^2(x)$$

and

$$\hat{h}_k(x) = (x - x_k)\, l_k^2(x)$$

$$P_{2n+1}(x) = \sum_{k=0}^{n} \left\{ f(x_k)\, h_k(x) + f'(x_k)\, \hat{h}_k(x) \right\}$$

\# $f(x) = \sin x$

$x_0 = 0 \qquad x_1 = \frac{\pi}{2}$

$f(x_0) = 0 \qquad f(x_1) = 1$

interpolate the polynomial using hermite polynomial.

<u>soln:</u>

$$P_3(x) = f(x_0)\,h_0(x) + f'(x_0)\,\hat{h}_0(x) + f(x_1)\,h_1(x) + f'(x_1)\,\hat{h}_1(x)$$

2n+1

line টাঁকা যাবে এঃ →

$$= 0 + 1 \times \hat{h}_0(x) + 1 \times h_1(x) + 0$$

$$= \hat{h}_0(x) + h_1(x)$$

so, $P_3(x) = x\left(1 - \frac{2x}{\pi}\right)^2 + \frac{4x^2}{\pi^2} \times \left(3 - \frac{4x}{\pi}\right)$

now,

$$h_1(x) = \left[1 - 2(x - x_1)\, l_1'(x_1)\right] l_1^2(x)$$

here $\quad l_1(x) = \dfrac{x - x_0}{x_1 - x_0}$

$$= \frac{x - 0}{\frac{\pi}{2} - 0}$$

$$= \frac{2x}{\pi}$$

$l_1'(x) = \dfrac{2}{\pi} \quad ; \quad l_1'(\pi/2) = \dfrac{2}{\pi}$

$$h_1(x) = \left[1 - 2\left(x - \frac{\pi}{2}\right) \times \frac{2}{\pi}\right] \times \frac{4x^2}{\pi^2}$$

not necessary $\hookrightarrow$ $h_1(x) = \frac{4}{\pi^2} x^2 \left(3 - \frac{4x}{4}\right)$

now, $\hat{h}_0(x) = (x - x_k)\, l_k^2(x)$

again, $L_0(x) = \frac{x - x_1}{x_0 - x_1}$

$$= \frac{x - \pi/2}{0 - \frac{\pi}{2}}$$

$$= 1 - \frac{2x}{\pi}$$

$$\hat{h}_0(x) = (x - 0)\left(1 - \frac{2x}{\pi}\right)^2$$

Ans:

$$P_3(x) = x\left(1 - \frac{2x}{\pi}\right)^2 + \frac{4x^2}{\pi^2}\left(3 - \frac{4x}{\pi}\right)$$

previous question ↳ $P_3(10) = ?$

— × —

End of chapter 2

Ch-3: Numerical Differentiation

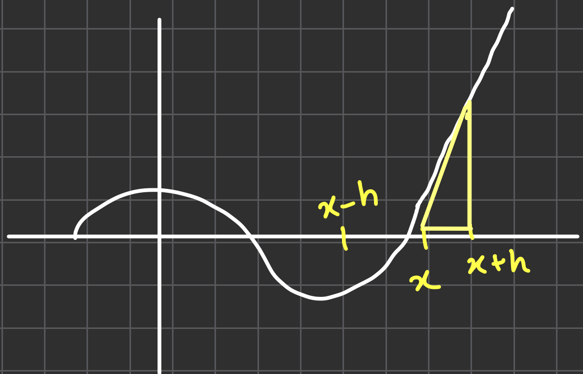
easy marks

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

forward difference

$$y = mx + c$$

ঢাল / derivative



Backward difference

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

$\boxed{\text{Que:}}$ $f(x) = x^3 - 4x + 1$;

$f'(x)$ বের করো using backward difference.

Sol^n:

$$f'(x) = \frac{f(2) - f(2 - 0.1)}{0.1}$$

numeric derivative

$$= 7.41$$

error কত?

$\Rightarrow f'(x) = 3x^2 - 4$

$f'(2) = 3(2)^2 - 4$

$= 8$

$\therefore$ error $= 8 - 7.41$

$= 0.59$

relative error কত?

Central Difference

$x, x+h, x-h$ ৩টা point use করে

derivative এবং Accuracy দেখি।

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$f(x) = \ln(x)$, $x = 2$

$h = 1, 0.1, 0.01, 0.001$

forward difference ব্যবহার করে

derivative এবং error বের করো।

$f'(x) = \frac{1}{x}$

Soln: $f'(2) = 0.5$

also called truncation error

h	$f'(x)$	error
1	0.405465	$\lvert 0.5 - 0.4055 \rvert$ $= 0.09453$
0.1	0.48709	0.01209
0.01	0.4987	0.001245
0.001	0.499875	0.00012

forward / backward $\rightarrow$ Error $\propto h$

central $\rightarrow$ Error $\propto h^2$