MAT215: Machine Learning & Signal Processing

Former Title: Complex variables & Laplace Transformations

Topic: Complex Integration

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MAT215
(Machine Learning & Signal Processing)

→here begins ch-4

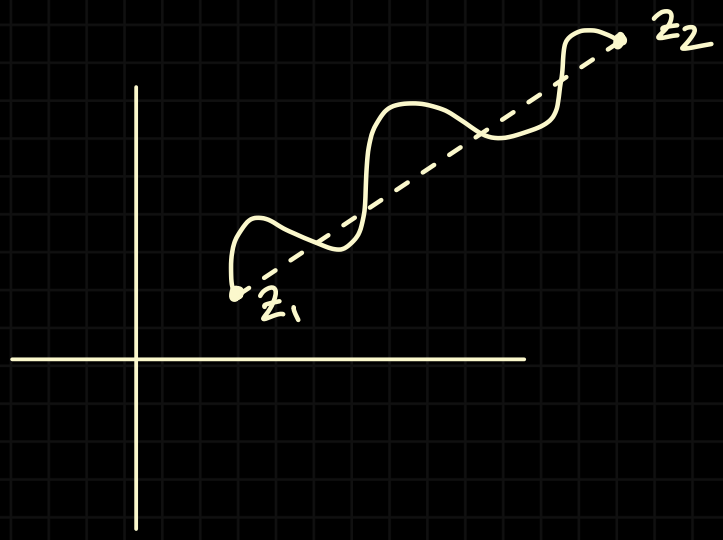
Complex Integration

in real number system,

$$\int_a^b f(x)\,dx = \text{area}$$

in complex number system,

$$\int_{z_1}^{z_2} f(z)\,dz$$



$f(z)$ non analytic $\Rightarrow$ path dependant (straight line or curve)

Path

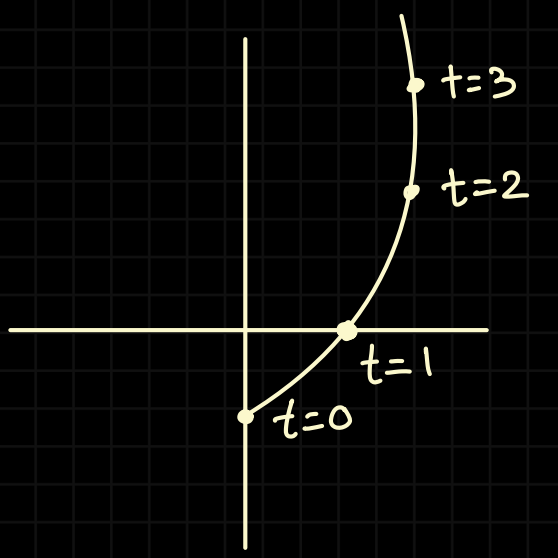
Suppose the **continuous** real valued functions $x(t)$ and $y(t)$, $a \leq t \leq b$, are parametric equations of a curve C on the complex plane. If we use these equations as the real and imaginary parts in $z = x + iy$, then the set of points of z is a path

let $x(t) = t$

$y(t) = t^2 - 1$

$0 \leq t \leq 3$

$\therefore x + iy = t + i(t^2 - 1)$



$t = 0 \Rightarrow 0 - i$

$t = 1 \Rightarrow 1 + 0i$

$t = 2 \Rightarrow 2 + 3i$

$t = 3 \Rightarrow 3 + 8i$

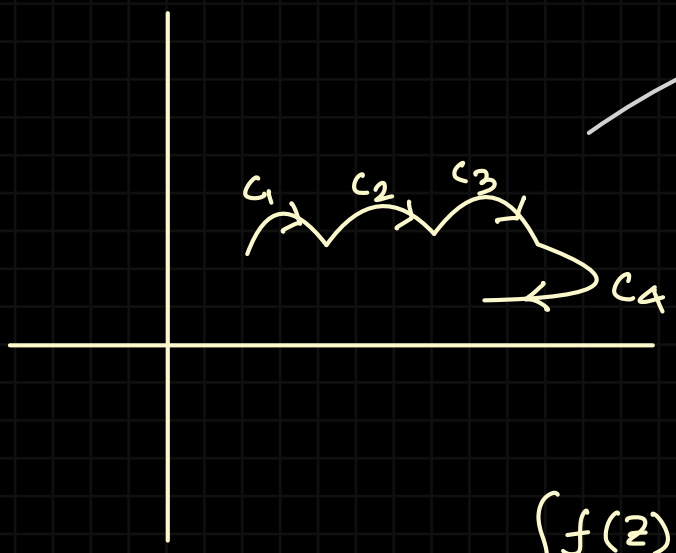
means → differentiable

Smooth Curve :

A path $z(t)$, $a \leq t \leq b$ is called a smooth curve if $z'(t)$ is continuous and $z'(t) \neq 0$ for all $a < t < b$

Contour :

Piecewise connected smooth curves is called a contour.

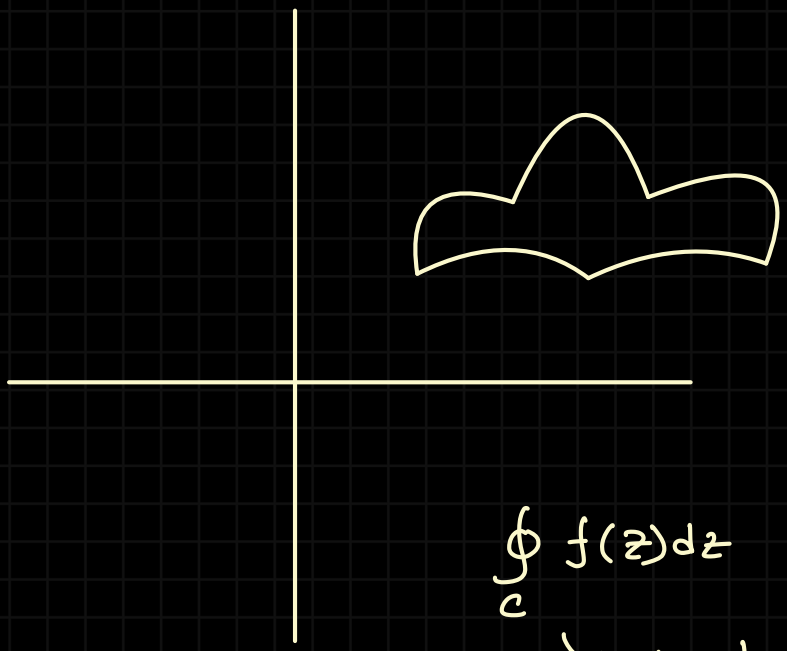


as a whole part its not differentiable

$$\int_C f(z)\,dz$$

↳ contour

Closed Contour: if the start and endpoint of a contour are same, its called a closed contour

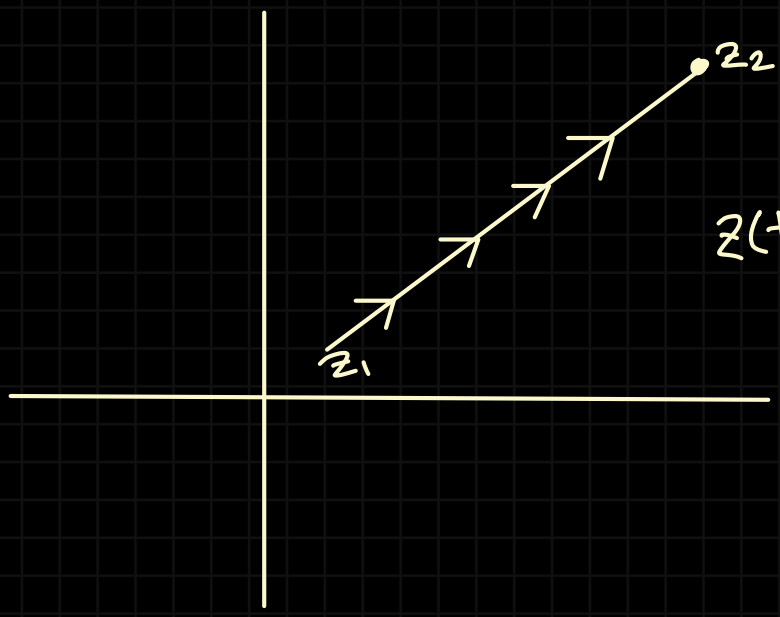


$$\oint_C f(z)\,dz$$

↳ closed contour integral

Formula:

find the contour $z(t)$, the straight line that connects the points z_0 and z_1.



eg: find the straight line contour from $(1,2)$ to $(4,-5)$