CSE330: Numerical Methods

Topic: Gaussian Elimination Method

Prepared by:

Saad Bin Sohan

BRAC University

Email: sohan.academics@gmail.com
GitHub: https://github.com/saad-bin-sohan

NB: Gaussian / LU decomposition either way তে math এর ক্ষেত্রে must check two things

(i) A এর determinant can't be zero

(ii) pivoting issue থাকলে solve করে নিতে হবে।

Gaussian Elimination Method

কোনো linear system কে upper / lower triangular matrix এ represent করতে পারলে system টার unique solution বের করতে পারতো।

এই upper/lower triangular matrix এ convertion এর একটা way হলো Gaussian Elimination method.

e.g.

$$x_1 + 2x_2 + x_3 = 0$$

$$x_1 - 2x_2 + 2x_3 = 0$$

$$2x_1 + 12x_2 - 2x_3 = 4$$

Process:

1) Matrix A, x, b তৈরি করো, $Ax = b$

coefficient matrix, $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{pmatrix}$

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $\quad b = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix}$

Augmented matrix $\rightarrow$ A, b একসাথে seperate -ly লেখো

Augmented matrix,

$$\operatorname{Aug}(A) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 2 & 12 & -2 & 4 \end{array}\right)$$

ii) Aug(A) কে lower/upper triangular matrix বানানো

upper triangular → কর্ণের নিচে সবাই zero
lower triangular → কর্ণের উপরে সবাই zero

how pc approaches solving → যে element গুলো zero বানানোর জন্য তাদের top to bottom row by row (each row এ column by column) zero বানাবো

and the rule it follows is,

$$m_{ik} = \frac{a_{ik}}{a_{kk}}$$

NB: যে position(i,k) zero করতে হবে m_{ik} তার row multiplier.

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{1} = 1$$

$$r_2' = r_2 - r_1 \times m_{21} \Rightarrow r_2' = r_2 - r_1$$

$$\therefore \text{Aug}(A) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 2 & 12 & -2 & 4 \end{array}\right)$$

$$\xrightarrow{r_2' = r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 2 & 12 & -2 & 4 \end{array}\right)$$

$$m_{31} = \frac{a_{31}}{a_{11}}$$ updated latest matrix থেকে

$$= \frac{2}{1} = 2$$

$$r_3' = r_3 - r_1 \times m_{31} = r_3 - 2r_1$$

$$\therefore \text{Aug}(A) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 2 & 12 & -2 & 4 \end{array}\right)$$

$$\xrightarrow{r_3' = r_3 - 2r_1} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 0 & 8 & -4 & 4 \end{array}\right)$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{8}{-4} = -2$$

$$r_3' = r_3 - r_2 \times m_{32} = r_3 + 2r_2$$

$$\therefore \text{Aug}(A) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 0 & 8 & -4 & 4 \end{array}\right)$$

$$\xrightarrow{r_3' = r_3 + 2r_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 0 & 0 & -2 & 12 \end{array}\right)$$

iii) আলাদা matrix, A, x, b তে ভাগালো।

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 12 \end{pmatrix}$$

iv) now applying the $\underbrace{\text{backward}}_{\hookrightarrow \text{(forward / lower)}}$ substitution,

$$-2x_3 = 12$$

$$\therefore \boxed{x_3 = -6}$$

$$-4x_2 + x_3 = 4$$

$$x_2 = \frac{4 - x_3}{-4} = \frac{4+6}{-4}$$

$$\boxed{x_2 = -2.5}$$

$$x_1 + 2x_2 + x_3 = 0$$

$$x_1 = -2x_2 - x_3$$

$$= -2(-2.5) - (-6)$$

$$= 11$$