CSE330: Numerical Methods

Topic: Nweton's Polynomial, Chobysher Nodes

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CSE330

Newton's Polynomial

$$P_n(x) = \sum_{k=0}^{n} a_k n_k(x)$$

$n_k(x)$ → newton's basis

a_k → coeffficient

so,

$$P_n(x) = a_0 n_0(x) + a_1 n_1(x) + a_2 n_2(x) + \dots + a_n r_n(x)$$

where,

$$n_0(x) = 1$$

$$n_1(x) = (x - x_0)$$

$$n_2(x) = (x - x_0)(x - x_1)$$

$$n_3(x) = (x - x_0)(x - x_1)(x - x_2)$$

$$\vdots$$

and

$$a_0 = f[x_0]$$

$$a_1 = f[x_0, x_1]$$

$$a_2 = f[x_0, x_1, x_2]$$

$$\vdots$$

so,

$$P_n(x) = f[x_0] \times 1 + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

Que

$x_0 = -1$ $\quad x_1 = 0$ $\quad x_2 = 1$ $\quad x_3 = 2$

$f(x_0) = 5$ $\quad f(x_1) = 1$ $\quad f(x_2) = 1$ $\quad f(x_3) = 11$

interpolate the polynomial using newton's polynomial method.

soln $\quad n = 3$

$$P_3(x) = a_0 n_0(x) + a_1 n_1(x) + a_2 n_2(x) + a_3 n_3(x)$$

$$= f[x_0] \times 1 + f[x_0, x_1](x - x_0) +$$

$$f[x_0, x_1, x_2](x-x_0)(x-x_1)(x-x_2) +$$

$$f[x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

after inserting the values we acheive from later calculation,

$$P_3(x) = 5 - 4(x+1) + 2(x+1)x + 1(x+1)x(x-1)$$

(# $P_3(1) = ?$)

$$\frac{x_{-1}, x_2}{x_0} = \frac{5-2}{2+1} = 1$$

$$\frac{- f[x_0, x_1]}{- x_0} = 2$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0,}{x_3 -}$$

$$\frac{- f[x_1, x_2]}{x_1} = 5$$

$x_0 = -1 \quad f[x_0] = 5$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = -4$$

$x_1 = 0 \quad f[x_1] = 1$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2]}{x_2 -}$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = 0$$

$x_2 = 1 \quad f[x_2] = 1$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] -}{x_3 -}$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = 10$$

$x_3 = 2 \quad f[x_3] = 11$

Chebysher Nodes

Runge function

let $f(x) = \dfrac{1}{1+25x^2}$

$$x_j = A\cos\left\{\frac{(2j+1)\pi}{2(n+1)}\right\} + \text{center}$$

chebysher node

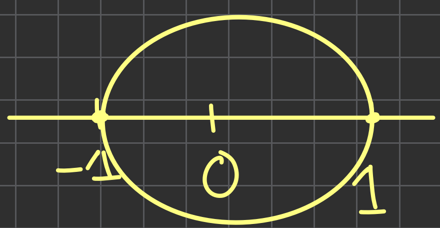
where A = interval এর radius

j = j^{th} node

Que 4 টি chebyshev nodes বের করো where interval range is $[-1, 1]$

soln: 4 nodes, so $n=3$

$$x_j = A \cos \frac{(2j+1)\pi}{2(n+1)} + \text{center}$$



Center will be at middle. so center = 0

so A = interval এর radius = 1

$$x_0 = 1\cos\frac{(2\times 0+1)\pi}{2(3+1)} + 0 = \cos\frac{\pi}{8}$$

$$x_1 = 1\cos\frac{(2\times 1+1)\pi}{2(3+1)} + 0 = \cos\frac{3\pi}{8}$$

$$x_2 = 1\cos\frac{(2\times 2+1)\pi}{2\times(3+1)} + 0 = \cos\frac{5\pi}{8}$$

$$x_3 = 1\cos\frac{(2\times 3+1)\pi}{2(3+1)} + 0 = \cos\frac{7\pi}{8}$$