CSE330: Numerical Methods

Topic:
Richard Extrapolation

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Richard Extro-polation

midterm ↳ question এসেছে must

in central difference, we use

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

but in Richard extrapolation we use,

$$D_h = \frac{f(x+h) - f(x-h)}{2h}$$

where

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)h^2}{2!} +$$

$$\frac{f'''(x)h^3}{3!} + \frac{f^{iv}(x)h^4}{4!} +$$

$$\frac{f^{v}(x)h^5}{5!} + \underbrace{O(h^6)}_{\text{order of 6}}$$

error term

যুলোড় প্রথম term। এখান থেকে error term গুলো ধরা

and

$$f(x-h) = f(x) - f'(x)h + f''(x)h^2 - \frac{f'''(x)h^3}{3!} + \frac{f^{iv}(x)h^4}{4!} - \frac{f^{v}(x)h^5}{5!} + O(h^6)$$

$$f(x+h) - f(x-h) = 2f'(x)h + \frac{2f'''(x)h^3}{3!} + \frac{2f^{v}(x)h^5}{5!} + O(h^7)$$

now,

$$\frac{f(x+h)-f(x-h)}{2h} = f'(x) + \frac{f'''(x)\,h^2}{3!} + \frac{f^{v}(x)}{5!}h^4 + O(h^6)$$

↳ error part যা বাদ দিতে হবে

$$D_h = f'(x) + \frac{f'''(x)\,h^2}{3!} + \frac{f^{v}(x)\,h^4}{5!}$$

$$+ O(h^6)$$

when $h = h/2$,

$$D_{h/2} = f'(x) + \frac{f'''(x)\ (h^2/4)}{3!} +$$

$$\frac{f^{v}(x)}{5!} \times \frac{h^4}{16} + O(h^6)$$

So,

$$4D_{H/2} - D_H = 3f'(x) + 0 + \left(\frac{1}{4} - 1\right) \times \frac{f^{v}(x)}{5!} h^4 + O(h^6)$$

$$= 3f'(x) - \frac{3}{4} \frac{f^{v}(x)\, h^4}{5!} + O(h^6)$$

$$\Rightarrow \frac{4D_{H/2} - D_H}{3} = f'(x) - \frac{1}{4} \frac{f^{v}(x)\, h^4}{5!} + O(h^6)$$

$$D_h^{(1)} = \frac{4D_{H/2} - D_H}{3}$$

this '1' means 1 তম term

বাদ দেওয়ার পর

expression যা আসবে সেটা।