CSE330: Numerical Methods

Topic: Significant Figure, Loss of Figures, Polynomial Interpolation, Taylor Series

Prepared by:

Saad Bin Sohan

BRAC University

Email: sohan.academics@gmail.com
GitHub: https://github.com/saad-bin-sohan

Significant Figure

☆ leftmost non zero digit হলো first significant bit

computer's configurations are designed like that যেন they can represent numbers to a specific significant figure

(যেমন: 4 বা 5 bit significant figure পর্যন্ত represent করতে পারে।

first significant bit

যেমন: $\underbrace{10.3491}$

4 bit sf হলে 10.35

first significant bit

0.2352

4 bit sf হলে → 0.2352

0.2352

3 bit sf হলে → 0.235 (rounding করে)

again,

first significant bit

0.023015

4 digit sf digit হলে → 0.02302

rounding করে

4 digit decimal place হলে 0.0230

☆ significant place ≠ decimal place

Loss of Significance

\# $x^2 - 56 + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

cal
cul
ator
এ
চিপলাম

$$\left\{ \begin{aligned} x_1 &= 28 + \sqrt{783} = 55.98 \\ x_2 &= 28 - \sqrt{783} = 0.01786 \end{aligned} \right.$$

[এই x_1, x_2 হিসাবের জন্য computer আগে root part calculate করে। then যোগ/বিয়োগ করে।]

$$so, \sqrt{783} = 27.98$$

$$x_1 = 28 + 27.98 = 55.98$$

$$x_2 = 28 - 27.98 = 0.02$$ (we saw it was 0.01786 but now, by definition, the computer solves it, we should get 0.02. so to avoid this loss of significance, the following process shows how the pc actually solves it.)

→ আগে actual value বের করতে হবে

→ then PC এর হিসাব দেখাবো

we know,

$$ax^2 + bx + c = 0 \text{ এর root}$$

α, β হলে $\quad \alpha\beta = \dfrac{c}{a}$

now, for $x^2 - 56x + 1 = 0$,

$$\alpha\beta = \frac{1}{1} = 1$$

$$\alpha = 55.98$$

$$\because \alpha\beta = 1, \Rightarrow \beta = \frac{1}{\alpha}$$

$$\beta = \frac{1}{55.98} = 0.01786$$

this is how a computer deals with loss of significance while also maintaining its limit of significant bits.

[NB: এই loss gulo (-ve) root এর
জন্যই হচ্ছে। plus গুলো ঠিক আছে]

Polynomial Interpolation

$2x + 5x^3$

degree $= 3 = n$

$$P_n(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \dots + a_n x^n$$

$a_0, a_1, a_2 \dots a_n \rightarrow$ constant/coefficient

$P_n(x)$ has $(n+1)$ সংখ্যক coefficients

Taylor Swift

jk

xD

Taylor Series

we represent a function as a sum of infinite number of terms in Taylor series. The greater the number of terms, the greater the accuracy.

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$$

$$+ \frac{f'''(x_0)}{3!}(x-x_0)^3 + \cdots$$

Que: $f(x) = \sin x$, $x_0 = 0$

Taylor expansion বের করো।

Solⁿ:

$$f(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2$$

$$+ \frac{f'''(0)}{3!}(x-0)^3 + \cdots$$

$$= 0 + x + \frac{(-1)}{3!}x^3 + \cdots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

so,

$$\sin(x) \rightarrow x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

now,

$$\sin(x) \rightarrow \underbrace{x - \frac{x^3}{3!} + \frac{x^5}{5!}}$$

$P_5(x) \rightarrow 5/6$ degree এর

polynomial কে করতে হবে

যেহেতু $f(x)$ কে limited সংখ্যক term দিয়ে $P_n(x)$ দিয়ে represent করছি there is some loss of value and we call it error.

within $[a, b]$ range

$$|f(x) - P(x)| \leq \epsilon$$

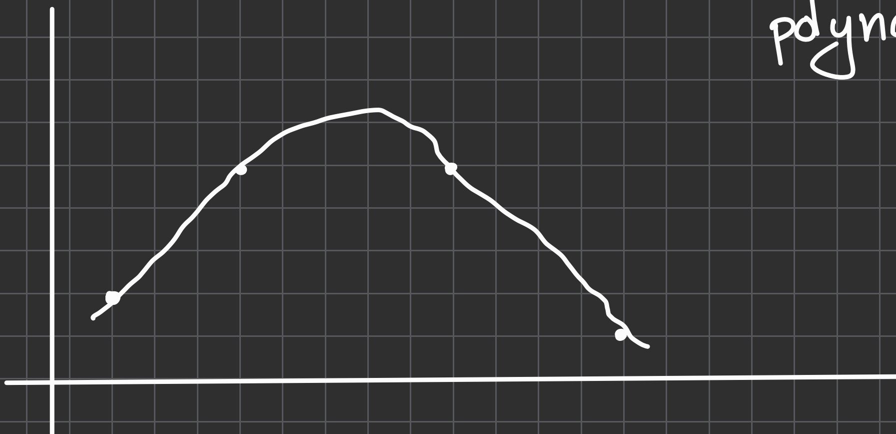
ϵ → max error

হল weinstrass approximation theorem

interpolate → guess করা / ধাব ঘটিয়ে নেয়া

বিন্দু will be called node. যেমন: (9,3)

interpolated polynomial



number of nodes ↑ accuracy of ↑ the interpolated function

Taylor series ⊲

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \cdots$$

$$= \sum_{k=0}^{n} \left\{ \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k \right\} +$$

জাই / Xi symbol

$$\underbrace{\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}}$$

its called error part of Taylor series / Lagrange form of remainder

$X_i \rightarrow$ unknown variable

within $[a, b]$ range

Que: $f(x) = \sin x$ এর (n=5 বা 5th degree expansion) হতে within the interval $[0, 0.1]$ max error বের কর।

Sol^n:

$$f(x) = \sin x$$

$$P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

since $n = 5$.

$x_0 = 0$

$x = 0.1$

now,

$$\left| f(x) - P_5(x) \right| \le \left| \frac{f^6(\xi)}{6!}(0.1-0)^6 \right|$$

$$\le \left| \frac{-\sin(\xi)}{6!}(0.1)^6 \right|$$

$$\le \left| \frac{-\sin(0.1)}{6!}(0.1)^6 \right|$$ radian mode

$$\left| f(x) - P_5(x) \right| \le \left| \left(-1.3866 \times 10^{-10}\right) \right|$$

$$\left| f(x) - P_5(x) \right| \leq 1.3866 \times 10^{-10}$$

—— X ——

finished