Joint Pilot Assignment and Pilot Power Allocation to Minimize Channel Estimation Error in Cell-free Massive MIMO Communications

Abstract—In cell free massive MIMO communications, optimal pilot assignment for reducing pilot contamination and normalized mean square error (NMSE) of channel estimation significantly depends on the pilot power allocation. However, most of the existing studies focus on the pilot assignment and pilot power allocation problem separately, which cannot provide the optimal performance. This study proposes joint pilot assignment and pilot power allocation to minimize the maximum NMSE of channel estimation in cell free massive MIMO communications. We consider a large number of access points (APs) to serve a much smaller number of users in TDD operation, where APs are grouped using a large scale fading based AP selection scheme for individual user. We formulate an optimization problem to minimize the maximum NMSE of channel estimation between the APs and users via joint pilot assignment and pilot power allocation. The problem is found to be non-convex mixed-integer nonlinear programming (MINLP). We solve the problem for several scenarios using the Knitro optimization tool with a low tolerance level. Our results show that the proposed joint scheme yields the lowest NMSE in channel estimation compared to systems that use full-power for pilot signal transmission or those where pilot assignment and pilot power allocation are performed separately.

Index Terms—Cell-free Massive MIMO, Channel Estimation, Pilot Contamination, Optimization, NMSE

I. INTRODUCTION

Conventional cellular communication encounters significant challenges at millimeter-wave (mmWave) and terahertz (THz) frequencies envisioned for 5G and 6G systems, primarily due to the severe attenuation experienced by non-line-of-sight (NLoS) propagation components. To overcome this, cell-free communication is proposed, enabling each user end (UE) to maintain at least one LoS link to a distributed access point (AP), ensuring reliable and seamless connectivity [1]. On the other hand, massive MIMO systems ensure higher energy efficiency and spectrum utilization [2]. Thus, cell-free massive MIMO is perfectly suited for providing ubiquitous access to high-rate data services, and so it has become one of the promising technologies for beyond 5G, i.e., 6G wireless communications.

In cell-free massive MIMO communications, channels between the APs and UEs need to be estimated as accurately as possible, since the estimated channels are used in decoding uplink data from users and precoding downlink data intended for users. This process occurs in the uplink training phase of each coherence interval, where all the UEs should be assigned mutually orthogonal pilot sequences. However, when the number of UEs becomes higher than the number of

orthogonal pilots, the UEs have to reuse the same pilots, i.e., they have to use non-orthogonal pilots [3]. Therefore, the transmitted pilot of a user at any AP will be interfered by the pilots of non-orthogonal users. This contamination severely affects the acquisition of channel state information (CSI) and consequently reduces overall system performance [4], [5].

To reduce pilot contamination, the pilot assignment and power control algorithms can be used [1], [6]-[11]. In [1], [6]-[8], the authors explored the effect of pilot assignment on pilot contamination where pilot sequences were transmitted with full power. In this process, a UE with poor channel can be strongly contaminated by UEs with stronger channels during pilot transmission. Pilot power control for cell-free massive MIMO has been studied in both [9] and [10]. [9] suggests pilot-power control to reduce pilot contamination and improve channel estimation. They report that this method achieves better spectral efficiency(SE) compared to full pilot power. A recent paper [10] addresses the same problem in a usercentric CF architecture and proposes a lightweight iterative update that minimizes the maximum normalized MMSE of channel estimation. This also results in improved SE while significantly lowering pilot transmit power. Both studies optimize pilot power with pilot assignments designed in advance. In literature, only [11] proposes a joint pilot and data power control and pilot assignment scheme with the objective to maximize the minimum user rate. Their joint approach results in an improved achievable network throughput and a minimum user rate; however, it does not address the channel quality improvement. The lack of literature in cell-free massive MIMO that addresses the issue of the channel estimation error motivates our work. Thus, the main objective of this research is to study the joint pilot assignment and power allocation in cellfree communications with the objective to minimize channel estimation error. We consider a communication system where a large number of access points serve a much smaller number of users in TDD operation. The main contributions in this paper are as follows.

- We formulate a mixed-integer nonlinear optimization problem to minimize the maximum NMSE of channel estimation between the APs and the users by joint pilot assignment and power allocation.
- We solve the optimization problem using a mathematical programming language (AMPL) with Knitro solver.
- For several network scenarios, we demonstrate that joint

pilot assignment and power allocation can significantly reduce the NMSE of channel estimation.

The remainder of the paper is organized as follows. Section II describes the cell-free massive MIMO communication system. In Section III, we formulate the joint pilot assignment and power allocation optimization problem and describe the solution approach of the problem. The numerical results for several scenarios are presented in Section IV. Section V concludes the paper.

II. SYSTEM MODEL

We consider a user-centric cell-free massive MIMO system as shown in Fig. 1, where M geographically distributed access points (APs) each with a single antenna jointly serves K users each with a single antenna at the same time-frequency. The APs are connected to a CPU. The set of APs and the set of users are denoted by \mathcal{M} and \mathcal{K} , respectively. Each user $k \in \mathcal{K}$ is served by a subset of APs, A_k , and a subset of users, \mathcal{U}_m is associated with each AP $m \in \mathcal{M}$. The channels are locally estimated at the APs can be used to decode the signals transmitted from the users in the uplink, and precode the downlink symbols. All the uplink and downlink transmissions take place in TDD mode that exploits channel reciprocity. Each AP is connected to the CPU through a fronthaul network that offers sufficient capacity. We assume an uncorrelated Rayleigh fading channel with channel coefficient $g_{m,k} \in \mathcal{CN}(0,\beta_{m,k})$ between the k-th user and the m-th AP which is modeled by

$$g_{m,k} = \beta_{m,k}^{\frac{1}{2}} h_{m,k} \quad \forall m \in \mathcal{M}, \forall k \in \mathcal{K}$$
 (1)

where, $\beta_{m,k}$ is the large-scale fading coefficient and $h_{m,k}$ is the small-scale fading coefficient. Each $h_{m,k}$ is i.i.d. random variable with $\mathcal{CN}(0,1)$, The CPU jointly determines the assignment of the pilot signal and the corresponding power control coefficients. The pilot signals, weighted by these coefficients, are then transmitted to the APs through dedicated channels.

In each coherence interval, the channel is assumed to be non-varying. Each coherence interval of length τ_c symbols is divided into three parts: (i) uplink training phase, (ii) uplink data transmission phase and (iii) downlink data transmission phase. During the training phase, AP selection, user clustering, pilot assignment, pilot power allocation and channel estimation are done. Within each coherence block of τ_c symbols, we reserve first τ_p symbols for training phase, therefore at most τ_p mutually orthogonal pilot sequences are available.

III. PROBLEM FORMULATION AND SOLUTION APPROACH

In this section, we first analyze the received signal for channel estimation and NMSE of channel estimation. Then we formulate the optimization problem to minimize the maximum NMSE between the APs and the users and describe its solution approach.

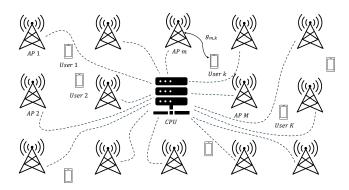


Fig. 1. A user-centric cell-free Massive MIMO Communication System.

A. Received Signal for Channel Estimation

Let P be the maximum power allocated to each pilot signal and $x_{k,i}$ be the binary variable to indicate the assigned i-th pilot signal to k-th user. Since only one pilot has to be assigned to a user $\sum_{i=1}^{\tau_p} x_{k,i} = 1$ for each user $k \in \mathcal{K}$. Further, the total number of pilots including reused pilots for all the users is K, thus, $\sum_{k=1}^K \sum_{i=1}^{\tau_p} x_{k,i} = K$. Let $\Psi_{\mathbf{i}} \in \mathcal{C}^{\tau_p \times 1}$ be the i-th pilot sequence of length τ_p and $\psi_{k,i} = \sum_{i=1}^{\tau_p} \Psi_i x_{k,i}$ for all $k \in \mathcal{K}, i \in \{1, 2, \dots, \tau_p\}$. Due to the orthogonality of pilot signals,

$$\psi_{k,i}^H \psi_{k',i'} = \begin{cases} 1, & \text{if } k' = k \text{ or } i = i' \\ 0, & \text{otherwise.} \end{cases}$$
 (2)

Let $\eta_{k,i}$, $0 \le \eta_{k,i} \le 1$, be the pilot power coefficient of the k-th user for assigning the i-th pilot to it. Note that $\eta_{k,i} = 0$ if i-th pilot is not assigned to k-th user.

The m-th AP receives a pilot sequence from each user of length τ_p . The received signal by the m-th AP, $\mathbf{Y}_m \in \mathcal{C}^{1 \times \tau_p}$ can be written as

$$\mathbf{Y}_{m} = \sqrt{\tau_{p}P} \sum_{k=1}^{K} g_{m,k} \sum_{i=1}^{\tau_{p}} \eta_{k,i}^{\frac{1}{2}} \mathbf{\Psi}_{i}^{H} x_{k,i} + \mathbf{W}_{m}$$
 (3)

where, $\mathbf{W}_m \in \mathcal{C}^{1 \times \tau_p}$ is additive noise at receiver each element with i.i.d. $\mathcal{CN}(0, \sigma_n^2)$ random variables comprising noise power of σ_n^2 . After projecting the received pilot signal \mathbf{Y}_m to $\psi_{k,i}$, we obtain

$$\hat{y}_{m,k}^{p} = \mathbf{Y}_{m} \psi_{k,i}$$

$$= \sqrt{\tau_{p} P} g_{m,k} \eta_{k,i}^{\frac{1}{2}} x_{k,i} + \sqrt{\tau_{p} P} \sum_{k' \neq k}^{K} g_{m,k'} \eta_{k',i}^{\frac{1}{2}} x_{k',i}$$

$$+ \mathbf{W}_{m} \psi_{k,i} \tag{4}$$

where, the second term is the interference caused by all users $k' \neq k$ whose pilots are not orthogonal to the pilot of user k, and is termed pilot contamination. We assume that each AP in the coverage area individually estimates the channel, and there is no cooperation between APs on the channel estimation process.

B. Normalized Mean-square of Channel Estimation

The minimum mean square error (MMSE) of the original channel g_{mk} , can be formulated as follows [9].

$$\hat{g}_{m,k} = \mathbb{E}\left\{g_{m,k}\left(\hat{y}_{m,k}^p\right)^H\right\} \left(\mathbb{E}\left\{\hat{y}_{m,k}^p\left(\hat{y}_{m,k}^p\right)^H\right\}\right)^{-1} \cdot \hat{y}_{m,k}^p$$
(5)

where.

$$c_{m,k} = \mathbb{E}\left\{g_{m,k}\left(\hat{y}_{m,k}^{p}\right)^{H}\right\} \left(\mathbb{E}\left\{\hat{y}_{m,k}^{p}\left(\hat{y}_{m,k}^{p}\right)^{H}\right\}\right)^{-1}$$

$$= \frac{\sqrt{\tau_{p}P\eta_{k,i}}\beta_{m,k}x_{k,i}}}{\tau_{p}P\sum_{k'=1}^{K}\beta_{m,k'}\eta_{k',i}x_{k',i} + \sigma_{n}^{2}}.$$
(6)

The distribution of $\hat{g}_{m,k}$ can be expressed as $\hat{g}_{m,k} \in$ $\mathcal{CN}(0, \gamma_{m,k})$, where

$$\gamma_{m,k} = \mathbb{E}\left\{ |\hat{g}_{m,k}|^2 \right\}$$

$$= \sqrt{\tau_p P \eta_{k,i}} \beta_{m,k} x_{k,i} c_{m,k}$$
(7)

The distribution of channel estimation error, $\tilde{g}_{m,k} = g_{m,k}$ – $\hat{g}_{m,k}$ can be expressed as $\tilde{g}_{m,k} \in \mathcal{CN}(0,(\beta_{m,k}-\gamma_{m,k})).$ Finally, NMSE estimation of the channel between k-th user at m-th AP is given by

$$NMSE_{m,k} = \frac{\mathbb{E}\left\{ |g_{m,k} - \hat{g}_{m,k}|^2 \right\}}{\mathbb{E}\left\{ |g_{m,k}|^2 \right\}} = \frac{\beta_{m,k} - \gamma_{m,k}}{\beta_{m,k}}$$
$$= 1 - \frac{\tau_p P \beta_{m,k} \eta_{k,i} x_{k,i}}{\tau_p P \sum_{k'=1}^K \beta_{m,k'} \eta_{k',i} x_{k',i} + \sigma_n^2}. \quad (8)$$

C. Minimizing the Maximum NMSE Probl

Let the maximum value of NMSEs between the users and APs is ζ . Let x and η be the matrices for the variables $x_{k,i}$'s and $\eta_{k,i}$'s, respectively. The optimization problem for minimizing the maximum NMSE can be stated as follows:

$$\min_{\mathbf{x}, \boldsymbol{\eta}} \quad \zeta \tag{9}$$
s.t.
$$\zeta \ge \left(1 - \frac{\tau_p P \beta_{m,k} \eta_{k,i} x_{k,i}}{\tau_p P \sum_{k'=1}^K \beta_{m,k'} \eta_{k',i} x_{k',i} + \sigma_n^2}\right) x_{k,i}$$

$$\forall k \in \mathcal{K}, \forall i \in \{1, \dots, \tau_p\} \tag{10}$$

$$0 \le \eta_{k,i} \le 1 \qquad \forall k \in \mathcal{K}, \forall i \in \{1, \dots, \tau_p\} \tag{11}$$

$$\sum_{i=1}^{\tau_p} x_{k,i} = 1 \qquad \forall k \in \mathcal{K} \tag{12}$$

$$\sum_{k=1}^K \sum_{i=1}^{\tau_p} x_{k,i} = K \qquad \forall k \in \mathcal{K}, \forall i \in \{1, \dots, \tau_p\} \tag{13}$$

$$x_{k,i} \in \{0,1\} \qquad \forall k \in \mathcal{K}, \forall i \in \{1, \dots, \tau_p\} \tag{14}$$

The problem (9)-(14) is a mixed integer nonlinear programming (MINLP) which is a non-convex problem. The constraints in (10) ensure that ζ is the maximum NMSE and the constraints in (11) limit the transmit power between 0

 $x_{k,i} \in \{0,1\}$

and P. The other constraints in (12)-(14) are set as required for pilot allocation. We use AMPL to model the optimization problem and solve it using the Artelys Knitro 14.2.0 solver of the NEOS server.

IV. RESULTS AND DISCUSSION

In this section, we present results by numerically solving the optimization problem for several network scenarios. We also compare the performance of the proposed joint scheme with the following schemes:

- I. Full Power: When min-max optimization is done without the power control i.e. pilot is transmitted with full power. It is obtained by solving the our optimization problem with fixed value of $\eta = 1$ (i.e., $\eta_{k,i} = 1, \forall k \& \forall i$).
- II. RA (Full) RA Random pilot assignment with full pilot power (i.e. $\eta_{k,i} = 1, \forall k \& \forall i$).
- III. RA (Pwr Ctrl)RA with pilot power control (i.e. $0 \le$ $\eta_{k,i} \leq 1$).
- IV. GCPA(Full): GCPA [7] Graph Coloring Based Pilot Assignment with full power
- V. GCPA (Pwr Ctrl): GCPA is done along with pilot power control.
- VI. GMAPA(Full): GMAPA [8] Granting massive access by adaptive pilot assignment scheme with full pilot power.
- VII. GMAPA (Pwr Ctrl): GMAPA is done along with pilot power control.

A. Model and Parameters Setup

We consider a square area of size 1×1 km^2 , where APs are systematically placed at fixed intervals rather than randomly to ensure uniform coverage throughout the area. However, the users are randomly distributed over the region. The set of serving APs for the k-th user, A_k , is selected using the largest-large-scale-fading-based selection method proposed in [12], with a predefined threshold of 95\%, i.e., the selected APs contribute at least 95% of the total received power of the desired signal at the k-th user. The noise model $\sigma_n^2[dBm] = -174 + 10 \log_{10}(B \text{ [Hz]})$ is used, where B is the bandwidth of the system. The large-scale fading coefficient $\beta_{m,k}$ models path loss and shadow fading. The model and parameters resemble those of [1], i.e., if the distance between the m-th AP and the k-th user is denoted by $d_{m,k}$, then the path loss in dB is as follows:

$$\beta_{m,k} = PL_{m,k} \cdot 10^{\frac{\sigma_{\text{sh}} z_{m,k}}{10}} \tag{15}$$

where, $10^{\frac{\sigma_{\rm sh}z_{m,k}}{10}}$ denotes shadow fading with standard deviation $\sigma_{\rm sh}$ in dB and $z_{m,k} \sim \mathcal{N}(0,1)$. We consider $\sigma_{\rm sh} = 8$ dB. $PL_{m,k}$ represents the path loss in dB. We adopt the three slope models for path loss as in [1].

$$PL_{m,k} = \begin{cases} -L - 35 \log_{10}(d_{m,k}); & \text{if } d_{m,k} > d_1 \\ -L - 15 \log_{10}(d_1) - 20 \log_{10}(d_{m,k}); & \text{if } d_0 < d_{m,k} \le d_1 \\ -L - 15 \log_{10}(d_1) - 20 \log_{10}(d_0); & \text{if } d_{m,k} \le d_0 \end{cases}$$

$$(1)$$

where, d_0 is the small distance threshold, d_1 is the large distance threshold, and

$$L \triangleq 46.3 + 33.9 \log_{10}(f) - 13.82 \log_{10}(H_b) - (1.1 \log_{10}(f) - 0.7) H_m + (1.56 \log_{10}(f) - 0.8)$$
 (17)

with carrier frequency f, AP height H_b and user height H_m . The values of the different system parameters are summarized in Table I.

B. Simulation Result and Analysis

The network scenario generation, large-scale fading coefficients of different links and AP grouping, and the data file for the Knitro solver are completed using the MATLAB tool. We select Mixed-Integer Sequential Quadratic Programming (MISQP), $mip_heuristic_strategy = -1$, $xtol = 10^{-12}$ and $xtol_iters = 3$ in the Knitro solver as options. Since the optimization problem is non-convex, it does not guarantee global optimality. However, the optimization process demonstrated numerically stable convergence. The solver stopped after satisfying the stopping condition with a final feasibility error of less than 10^{-12} , indicating the satisfaction of the tight constraint.

For any setting of M, K, and τ_p , we have taken 100 realizations for random locations of users each time and the maximum NMSE is determined by taking the average over 100 realizations. In Fig. 2, we present the maximum NMSE with $M=30,\,K=15,\,$ and $\tau_p=5$ under various design schemes. The results show that the proposed joint design achieves the lowest maximum NMSE across all channels compared to the other schemes. The results show that RA and GCPA pilot allocation schemes with full power provide worst NMSE, i.e., 0.99 for channel estimation. The optimal pilot assignment with full power allocation provides a maximum of NMSE of 0.61. GCPA with power control shows the closest NMSE to the proposed joint design, which is 0.56. The maximum NMSE

TABLE I SYSTEM PARAMETERS FOR THE SIMULATION

Symbol	Value
H_b	15 m
H_m	1.65 m
f	1.9 GHz
d_0	10 m
d_1	50 m
B	20 MHz
P	100 mW
	$ \begin{array}{c} H_b \\ H_m \\ f \\ d_0 \\ d_1 \\ B \end{array} $

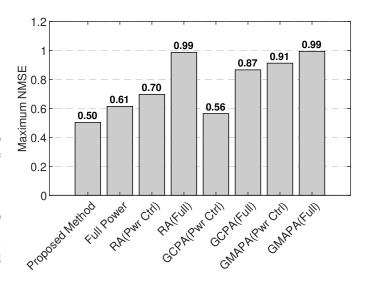


Fig. 2. Comparison of NMSE of channel estimation for different design schemes with $M=30,\,K=15,$ and $\tau_p=5.$

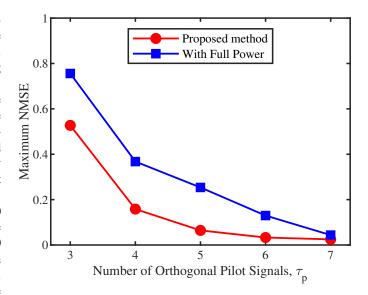


Fig. 3. Maximum NMSE by varying the number of orthogonal pilots with M=30 and K=15.

under the proposed joint design is found to be 0.5, which is high for a communication system. It is due to the fact that for a fixed number of serving APs per user, a smaller value of τ_p causes more pilot reuse among users, causing high interchannel interference and resulting in channel estimation error. The effects of changing the value of τ_p and K are presented in Fig. 3 and Fig. 4.

We determine the values of maximum NMSEs by varying the number of orthogonal pilots under the proposed joint design and the design with optimal pilot assignment and full power allocation considering M=30 and K=15. The results are presented in Fig. 3. The results show that the maximum channel estimation error reduces due to the increment of orthogonal pilots for the both design schemes as pilot reuse among users as well as inter-channel interference

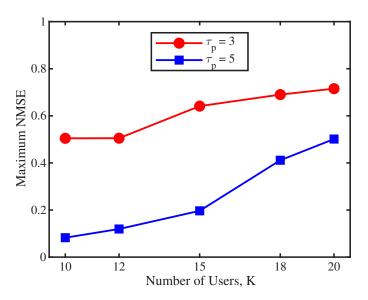


Fig. 4. Maximum NMSE by vaying the number of users for different values of τ_p with M=30.

reduces with the increment of the number of orthogonal pilots. However, the performance of joint design is superior compared to the design with full power. At the higher values of orthogonal pilots, performance of both design schemes become closer. We attribute to this fact that the distance of the users with same pilot increases with the increment of the number of orthogonal pilot sequences which results in lower inter-channel interference.

We also determine the maximum NMSE under the proposed design by varying the number of users and the results are presented in Fig. 4. The results show that the increase in users in the system leads to more interference by more reusing the smae pilots by the users, and thus, the channel estimation error increases.

V. CONCLUSION

We have proposed joint pilot assignment and pilot power allocation for cell free massive MIMO communications to minimize the channel estimation error. By formulating an optimization problem and solving the problem numerically with optimization tools for several scenarios, we have demonstrated that joint design can provide better NMSE for channel estimation. However, there are some limitations in practice in implementing the optimization tool-based joint design due to the requirement of high computing resources and large computation time. Thus, heuristic solution approach taking the insights from the optimal joint design could be the future work relevant to this research.

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