Fuzzy C-means Clustering

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PATTERN RECOGNITION

Outline

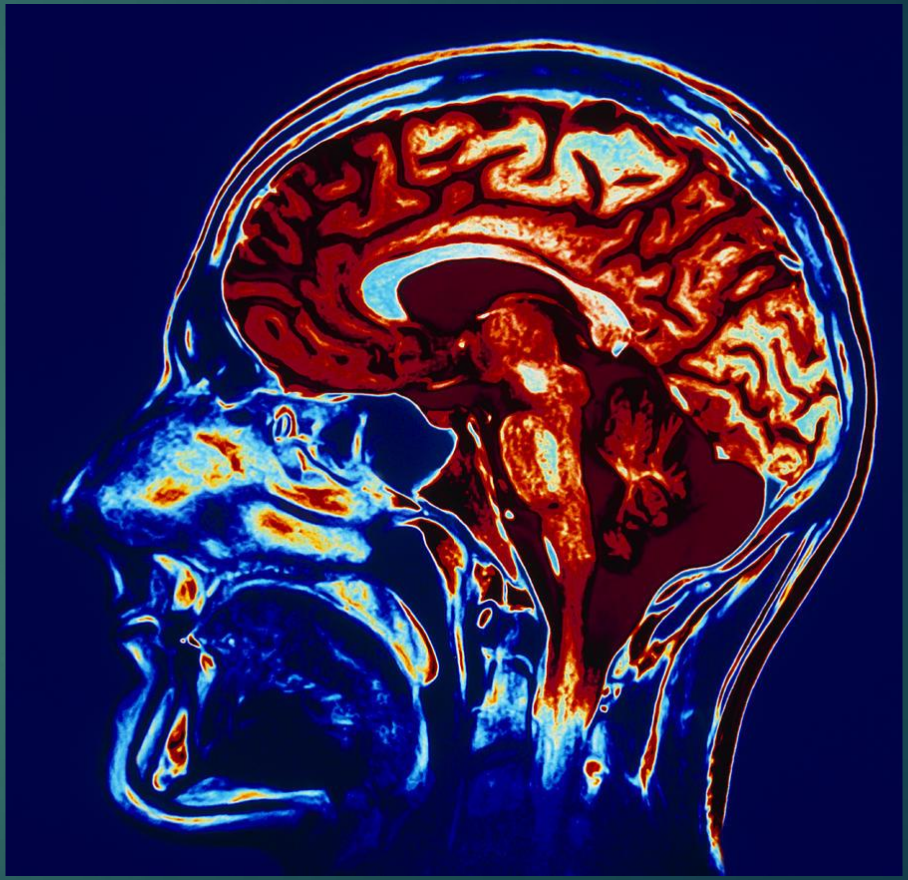
- Motivation and application
- Description of Fuzzy C-Means Clustering (FCM)
- Description of modified Fuzzy C-Means Clustering with spatial constraints (FCMS)
- Comparison of results of FCM and FCMS
- Conclusions

Primary Reference

- M.N. Ahmed, S. M. Yamay, N. Mohamed, A. A. Farag, and T. Moriarty. "A modified fuzzy C-means algorithm for bias field estimation and segmentation of MRI data." IEEE Trans. Med. Imaging, vol. 21, pp 193-199, Mar. 2002.
- S. Chen and D. Zhang, "Robust Image Segmentation Using FCM with Spatial Constraints Based on New Kernel-Induced Distance Measure," IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics, Vol. 34, No. 4, Aug. 2004.

Motivation and Applications

- Image Segmentation
  - Medical Imaging
    - X-Ray Computer Tomography (CT)
    - Magnetic Resonance Imaging (MRI)
    - Position Emission Tomography (PET)
- Several methods for segmenting images
  1. Discontinuity – Detect discontinuities between pixel values to identify segment boundaries
  2. Similarity – Utilize clustering methods to divide image pixels into different segments



Fuzzy C Means – Input Output

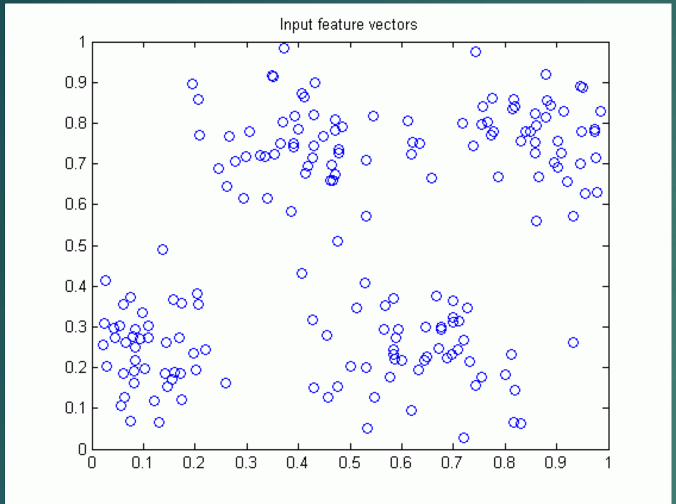
- Input: Takes in a set of data $X = \{x_1, x_2, \dots, x_n\}$
  - For the case of image segmentation, the data points consist of pixel values
- Output: A partition of c clusters of X represented as a $n\ x\ c$ matrix U
  - Each row of U is as follows:
    $Row\ i: [membership\ of\ x_i\ in\ cluster\ 1, membership\ of\ x_i\ in\ cluster\ 2, \dots]$
  - Allows data to have certain degree of membership to 2 or more clusters
- Additional Output: Set of cluster centers: $V = \{v_1, v_2, \dots, v_c\}$

FCM Illustration

$\mathbf{X}$

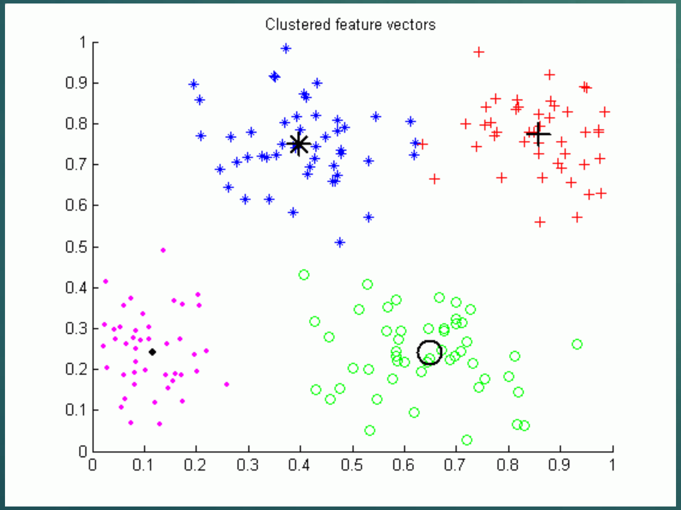
$n = 188$

$p = 2$

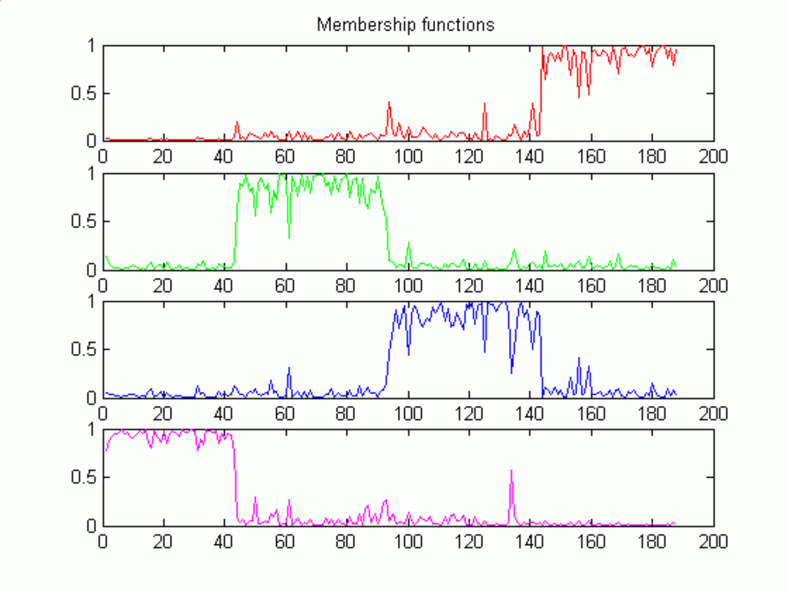


$\mathbf{U}$ and $\mathbf{V}$

$c = 4$



Columns of U (Membership Functions)



FCM – Objective Function

- The algorithm is based on minimizing an objective function:

$$\min_{(\mathbf{U},\mathbf{V})}\left\{ J_m(\mathbf{U},\mathbf{V}) = \sum_{i=1}^{c}\sum_{k=1}^{n} u_{ik}^{m} D_{ik}^{2} \right\}$$

- Constraint: The membership values of a point, u_{ik} must add up to 1:

$$\sum_{i=1}^{c} u_{ik} = 1, \forall k$$

- The distance between a point and cluster center is given by D_{ik}:

$$D_{ik}^{2} = \left\| \mathbf{x}_k - \mathbf{v}_i \right\|_{\mathbf{A}}^{2}$$

FCM - Minimize Objective Function

- Minimizing the objective function with respect to U and V yields the following result:

$$u_{ik} = \left[\sum_{j=1}^{c} \left(\frac{D_{ik}}{D_{jk}} \right)^{\frac{2}{m-1}} \right]^{-1}, \forall i, k$$

$$\mathbf{v}_i = \left(\sum_{k=1}^{n} u_{ik}^{m} \mathbf{x}_k \Big/ \sum_{k=1}^{n} u_{ik}^{m} \right), \forall i$$

FCM – Algorithm Steps

1. Choose the number of clusters (c), choose a fuzziness parameter (q), choose threshold to determine convergence (ϵ)
2. Initialize membership matrix to random values
3. Calculate the cluster centers with the expression for v_i
4. Calculate the new membership matrix with the expression for u_{ik}
5. Check for convergence: $||V_{new} - V_{old}|| < \epsilon$
6. If converged, finished, otherwise go to step 3

FCM Pros and Cons

- Advantages
  - Unsupervised
  - Always converges
- Disadvantages
  - Sensitive to initial guess
  - Sensitive to noise
  - Does not take into account spatial location of pixels

FCM with Spatial Constrains

- Modify the algorithm to take into account spatial information
- For images, take into account pixels that are within a chosen neighborhood of each pixel
- This is done by modifying the original objective function:

$$J(U,V) = \sum_{i=1}^{c} \sum_{k=1}^{n} u_{ik}^{m} D_{ik}^{2} \quad + \frac{\alpha}{N_R} \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{m} \sum_{r \,\epsilon\, N_k} D_{ik}^{2}$$

Optimizing new Objective Function

- Yields the following two expressions

$$u_{ik} = \frac{\left(D_{ik}^2 + \frac{\alpha}{N_R}\sum_{r\,\epsilon\,N_k} D_{ir}^2\right)^{-\frac{1}{m-1}}}{\sum_{j=1}^{c}\left(D_{jk}^2 + \frac{\alpha}{N_R}\sum_{r\,\epsilon\,N_k} D_{jr}^2\right)^{-\frac{1}{m-1}}}$$

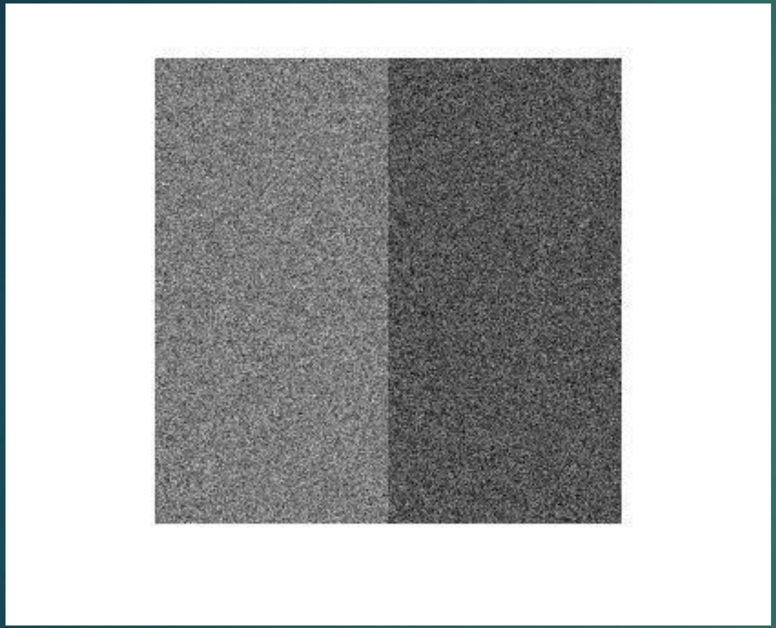
$$v_i = \frac{\sum_{k=1}^{n} u_{ik}^m \left(x_k + \frac{\alpha}{N_R}\sum_{r\,\epsilon\,N_k} x_R\right)}{1 + \alpha\sum_{k=1}^{n} u_{ik}^m}$$

FCMS – Algorithm Steps

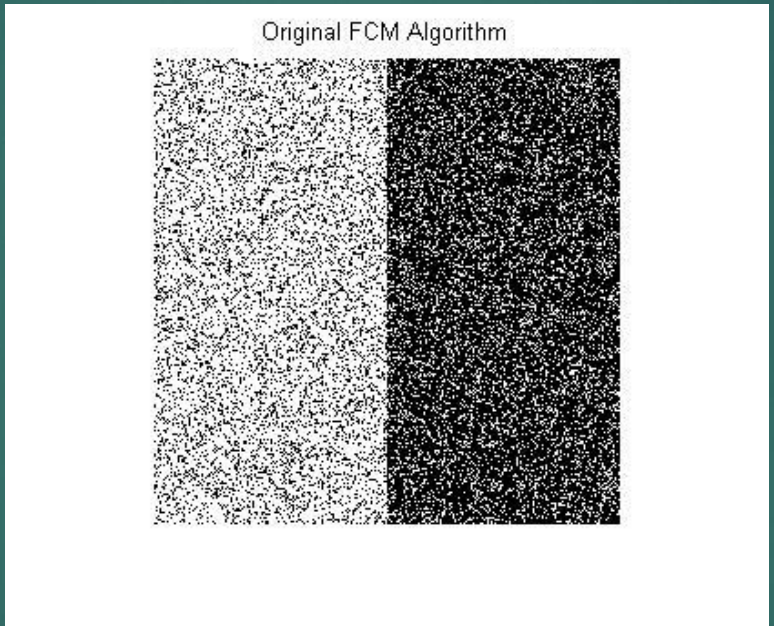
1. Choose the number of clusters (c), choose a fuzziness parameter (q), choose threshold to determine convergence (ϵ)
2. Initialize membership matrix to random values
3. Calculate the cluster centers with the new expression for v_i
4. Calculate the new membership matrix with the new expression for u_{ik}
5. Check for convergence: $||V_{new} - V_{old}|| < \epsilon$
6. If converged, finished, otherwise go to step 3

Results of Noisy Image

Original Noisy Image

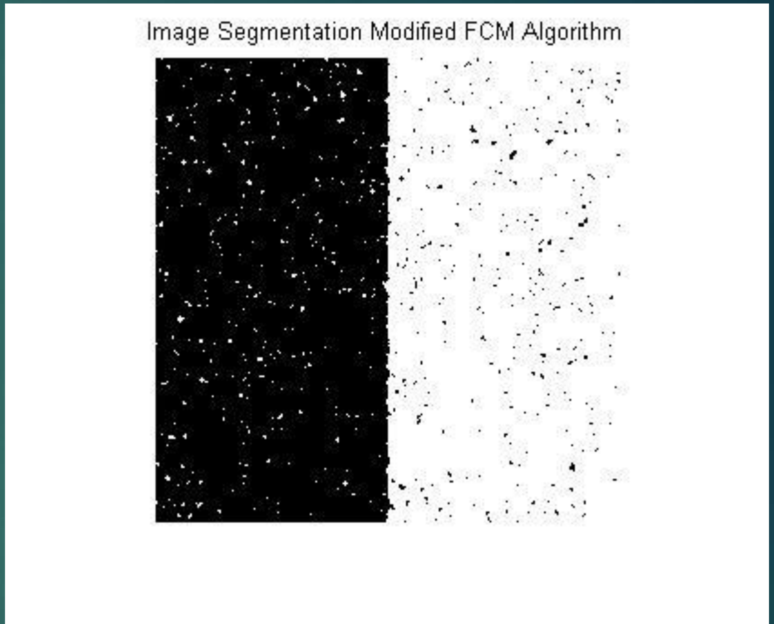


Result with FCM Algorithm



78.67% Accuracy

Result with FCMS Algorithm

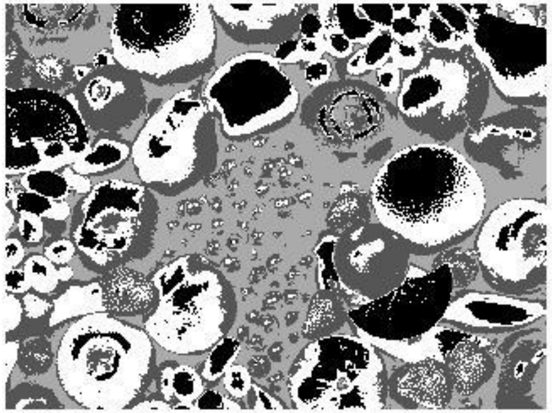


98.16% Accuracy

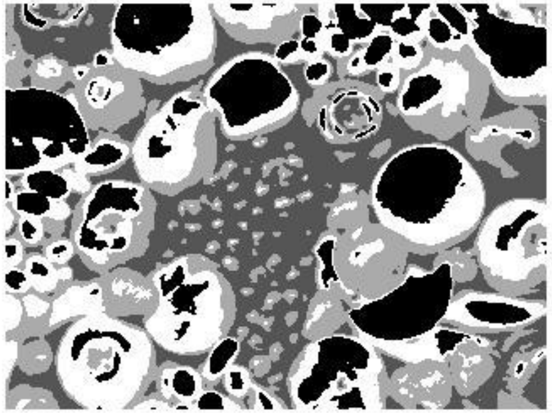
Result of Image without Noise

Original Image

Result with FCM Algorithm



Result with FCMS Algorithm



Executed with $c = 4, m = 2, \alpha = 3, \varepsilon = 0.001$

Conclusions

- FCMS algorithm yields better results in the presence of noise
- FCMS has more uniform image segmentation
- Cons of the FCM with spatial constraints:
  - Adds more computation to an algorithm that already takes very long to execute
  - Requires more memory resources to store neighborhood information
  - Still sensitive to the random initialization of the membership matrix
- Improved results have been shown by using kernel distance functions instead of Euclidean distance
- Change penalty term added to the objective function

Questions?