6.1 The Basics of Counting

41. A palindrome is string whose reversal is identical to the string. How many bit strings of length n are palindromes?

Answer: $2^{\frac{n+1}{2}}$ for odd and $2^{\frac{n}{2}}$ for even

57. The name of a variable in the Java programming language is a string of between 1 and 63,535 characters, inclusive, where each character can be an uppercase or a lower case letter, a dollar sign, an underscore, or a digit, exept that the first character must not be a digit. Determine the number of different variable names in Java.

Answer: $\sum_{k=1}^{63535} 54 \cdot 64^{k-1}$

$1 \leq k \leq 63535$; $26+26+1+1+10=64$ choices for each character, except first which cannot be a digit $\Rightarrow$ $64-10=54$ for 1^{st} character. By product rule $54 \cdot 64^{k-1}$ choices for such a string. For final answer we need to sum this over all lengths.

63. Use the principle of inclusion-exclusion to find the number of of positive integers less than 1,000,000 that are not divisible by either 4 or by 6.

Answer: 666,667.

Numbers divisible by 4: $999{,}999 : 4 = 249{,}999$

Numbers divisible by 6: $999{,}999 : 6 = 166{,}000$

Numbers divisible by 4 And 6 (12) $999{,}999 : 12 = 83{,}333$

$$999{,}999 - 249{,}999 - 166{,}000 + 83{,}333 = 666{,}667.$$

65. How many ways are there to arrange the letters a, b, c and d such that a is not followed immediately by b?

We assume that what is intended is that each of the 4 letters is to be used exactly once. There are at least 2 ways to do this problem:

1) a comes at the end of string: $3 \cdot 2 \cdot 1 = 6$

2) a doesnot come at the end of string: there are 3 places, the there are only 2 places to put the b, 2 pos in which the c can go, 1 pos for d.

By product raule: $3 \cdot 2 \cdot 2 \cdot 1 = 12$

By the sum rule the answer is:

$12 + 6 = 18$.

71. Use mathematical induction to prove the sum rule for m tasks from the sum rule for two tasks.

$P(m)$ the sum rule for m tasks, which says that if tasks $T_1, T_2, T_3, \ldots T_m$ can be done in $n_1, n_2, n_3, \ldots, n_m$ ways, and no two of them can be done at the same time, then there are $n_1 + n_2 + \ldots + n_m$ ways to do one of the tasks.

Basis step: $m=2$ that has been given.

Inductive step: assume $P(m)$ is true, We need to prove $P(m+1)$. Either we choose one from among the first m, or we choose the task T_{m+1}. By the sum rule for 2 tasks, the number of ways we can do this in $n + n_{m+1}$, where n is the number of ways we can do one of the tasks among the first m. But by the inductive hypothesis $n = n_1 + n_2 + \ldots + n_m$. There fore the number ways we can do one of the m+1 tasks is $n_1 + n_2 + \ldots + n_m + n_{m+1}$.