H/W 1.8: 25.

$1, 2, \ldots, 2n$; n- odd integer

j, k , $|j-k|$.

Prove that the last integer must be odd.

Solution: j is odd, k is even $\rightarrow |j-k|$ is odd

j is odd, k is odd $\rightarrow |j-k|$ is even

j is even, k is even $\rightarrow |j-k|$ is even

$$1+2+3+4+\ldots+2n = \frac{\cancel{2}n\cdot(2n+1)}{\cancel{2}} = \underbrace{n}_{odd} \times \underbrace{(2n+1)}_{odd} \rightarrow odd$$

1) j: odd, k: even ;

$$\underbrace{\underbrace{sum-j}_{even}-k}_{even} + \underbrace{|j-k|}_{odd} = odd$$

2) j: odd, k: odd ;

$$\underbrace{\underbrace{sum-j}_{even}-k}_{odd} + \underbrace{|j-k|}_{even} = odd$$

3) j: even, k: even ;

$$\underbrace{\underbrace{sum-j}_{odd}-k}_{odd} + \underbrace{|j-k|}_{even} = odd$$

Proved ■

27. n - integer

$n = 10k + l$, where k - a quotient

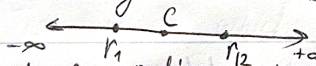
l - remainder

$l = \{0, 1, \dots 9\}$

1) $(10k+0)^4 = 10000k^4 + 0$
2) $(10k+1)^4 = 10000k^4 + \dots + 1$
3) $(10k+2)^4 = 10000k^4 + \dots + 16$
4) $(10k+3)^4 = 10000k^4 + \dots + 81$
5) $(10k+4)^4 = 10000k^4 + \dots + 256$
6) $(10k+5)^4 = 10000k^4 + \dots + 625$
7) $(10k+6)^4 = 10000k^4 + \dots + 1296$
8) $(10k+7)^4 = 10000k^4 + \dots + 2401$
9) $(10k+8)^4 = 10000k^4 + \dots + 4096$
10) $(10k+9)^4 = 10000k^4 + \dots + 6561$

The decimal digits 0, 1, 5, 6 always appears as the final decimal digit of the fourth power of an integer. Proved. ∎

35.



r_1, r_2 - rational numbers, c - irrational number

$r_1 = 0$, $r_2 = 1$

$c = \frac{1}{\sqrt{2}} \approx 0.7071\dots$

$0 < \frac{1}{\sqrt{2}} < 1 \qquad | \cdot (r_2 - r_1) > 0$

$0 < \frac{1}{\sqrt{2}}(r_2 - r_1) < r_2 - r_1 \qquad | + r_1$

$r_1 < \underbrace{r_1 + \frac{1}{\sqrt{2}}(r_2 - r_1)}_{\text{irrational}} < r_2$

Proved. ∎