6.4 Binomial Coefficients and Identities

19. Prove Pascal's identity, using the formula for $\binom{n}{r}$

Pascal's Identity

Let n and k be positive integers with $n \geq k$. Then $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

$$\binom{n}{k-1} + \binom{n}{k} = \frac{n!}{(k-1)!(n-(k-1))!} + \frac{n!}{k!(n-k)!} =$$

$$= \frac{n!k + n!(n-k+1)}{k!(n-k+1)!} = \frac{(n+1)\,n!}{k!((n+1)-k)!} =$$

$$= \frac{(n+1)!}{k!((n+1)-k)!} = \binom{n+1}{k}$$

33. a) Total length of this bit string is $m+n$.

b) The number of bit strings of length $m+n$ containing exactly n 1's is $\binom{m+n}{n}$, since one need only specify the positions of the 1's. This is the same as $\binom{m+n}{m}$