6.2. 23. Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

Proof by contradiction:

Assume that no person has both neighbors as boys. This means each girl must have at least one girl and one boy as a neighbor.
Since there are 25 girls and 25 boys, if no girl has both neighbor as boys, then each girl must have at least one girl as a neighbor. This would imply that there are more than 25 girls, which contradicts the given condition. Therefore, there must be at least one person who has both neighbors as boys. ☒

31. Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily in the same year). Assume that everyone has three initials.

Solution:

Distinct combinations of initials and birthdays:
$26 \cdot 26 \cdot 26 \cdot 366 = 6\,432\,816$. By generalized ph P.:
$\lceil 37\,000\,000 / 6\,432\,816 \rceil = 6$
6 people with the same combinations.

41. An arm wrestler is the champion for a period of 75 hours. (Here, by an hour, we mean a period starting from an exact hour until the next hour). The arm wrestler had at least one match an hour, but no more than 125 total matches. Show that there is a period of consecutive hours during which the arm wrestler had exactly 24 matches.

Solution:

Let a_j be the number of matches held during or before the j^{th} hour. Then $a_1, a_2, \ldots a_{75}$ is an increasing sequence of distinct positive integers, since there was at least one match held every hour. Moreover, $a_1+24, a_2+24, \ldots a_{75}+24$ is also an increasing sequence of distinct positive integers, withe

$$25 \leq a_i + 24 \leq 149.$$

The 150 positive integers $a_1, a_2, \ldots, a_{75}, a_1+24, a_2+24, \ldots, a_{75}+24$ are all less than or equal to 150. Hence, by the pigeonhole principle two of these integers are equal. Because the integers $a_1, a_2, \ldots, a_{75}$ are all distinct and the integers $a_1+24, a_2+24, \ldots, a_{75}+24$ are all distinct, there must be indices i and j with $a_j = a_i + 24$. This means that exactly 24 matches were held from the beginning hour $i+1$ to the end of hour j.