5.1.37. Prove that if n is a positive integer, then 133 divides $11^{n+1} + 12^{2n-1}$.

Basis step: $n=1$: $11^{1+1} + 12^{2\cdot 1-1} = 121 + 12 = 133 \mid 133$ true.

Inductive step:

The inductive hypothesis is the statement that $P(k)$ is true, that is $133 \mid 11^{k+1} + 12^{2k-1}$, where k is an arbitrary nonnegative integer. We must show that if $P(k)$ is true, then $P(k+1)$ is also true. So assuming the inductive hypothesis, it follows that.

$$P(k+1) = 11^{(k+1)+1} + 12^{2(k+1)-1} = 11^{k+1+1} + 12^{2k+2-1} =$$

$$= 11 \cdot 11^{k+1} + 12^2 \cdot 12^{2k-1} = 11 \cdot 11^{k+1} + 144 \cdot 12^{2k-1}$$

$$= 11 \cdot 11^{k+1} + (11+133) \cdot 12^{2k-1} = 11 \cdot 11^{k+1} + 11 \cdot 12^{2k-1} + 133 \cdot 12^{2k-1}$$

$$= 11 \cdot \underbrace{\left(11^{k+1} + 12^{2k-1}\right)}_{\text{133 divides by inductive hypothesis}} + \underbrace{133 \cdot 12^{2k-1}}_{\text{133 divides.}}$$

55. We use induction on $i+j$ to show that every square can be reached by the knight.
There are six base cases:
- $i+j \leq 2$: The knight is already at $(0;0)$ to start, so the empty sequence of moves reaches that square.
- To reach $(1;0)$, the knight moves from $(0;0)$ to $(2;1) \rightarrow (0;2) \rightarrow (1;0)$.
- to reach $(0;1)$, the knight moves from $(0;0) \rightarrow (1;2) \rightarrow (2;0) \rightarrow (0;1)$
- to reach $(1;1)$: $(0;0) \rightarrow (1;2) \rightarrow (2;0) \rightarrow (0;1) \rightarrow (2;2) \rightarrow (0;3) \rightarrow (1;1)$.

Now we assume the inductive hypothesis, that the knight can reach any square $(i;j)$ for which $i+j=k$, where k is an integer greater than 1, and we wanto show how the knight can reach each square $(i;j)$, when $i+j=k+1$. Since $k+1 \geq 3$, at least one of i and j is at least 2. If $i>2$, then by the inductive hypothesis, there is a sequance of moves ending at $(i-2, j+1)$, since $i-2+j+1=i+j-1=k$; from there it is just one step to (i,j).
Similarly, then by the inductive hypothesis, there is a sequence of moves ending at $(i+1; j-2)$ since $i+1+j-2=i+j-1=k$; from there it is again just one step to (i,j).