4.2 7. Convert the hexadecimal expansion of each of this integers to a binary expansion.

a) $(80E)_{16} = (1000\ 0000\ 1110)_2$

b) $(135AB)_{16} = (0001\ 0011\ 0101\ 1010\ 1011)_2$

c) $(ABBA)_{16} = (1010\ 1011\ 1011\ 1010)_2$

d) $(DEFACED)_{16} =$

$= (1101\ 1110\ 1111\ 1010\ 1100\ 1110\ 1101)_2$

19. Give a procedure for converting from the octal expasion of an integer to its hexadecimal expasion using binary notation as an intermediate step.

Convert from the octal to binary, then convert from binary hexadecimal.

27. Use Algorithm 5 to find $3^{2003} \bmod 99$

Algorithm 5

procedure modular exponentiation (b: int, $n = (a_{k-1} \ldots a_1 a_0)_2$)

$x = 1$

power $= b \bmod m$

for $i = 0$ to $k-1$

    if $a_i = 1$ then $x = (x \cdot \text{power}) \bmod m$

    power $= (\text{power} \cdot \text{power}) \bmod m$

return x {x equals $b^n \bmod m$}

$3^{2003} \bmod 99$, $2003 = (\underset{10}{1}\underset{9}{1}\underset{8}{1}\underset{7}{1}\underset{6}{1}\underset{5}{0}\underset{4}{1}\underset{3}{0}\underset{2}{0}\underset{1}{1}\underset{0}{1})_2$

$i=0,\ a_0=1,\ x=1\cdot 3 \bmod 99 = 3,\ power = 3^2 \bmod 99 = 9$

$i=1,\ a_1=1,\ x=3\cdot 9 \bmod 99 = 27,\ power = 9^2 \bmod 99 = 81$

$i=2,\ a_2=0,\ x=27,\ power = 81^2 \bmod 99 = 27$

$i=3,\ a_3=0,\ x=27,\ power = 27^2 \bmod 99 = 36$

$i=4,\ a_4=1,\ x=27\cdot 36 \bmod 99 = 81,\ power = 36^2 \bmod 99 = 9$

$i=5,\ a_5=0,\ x=81,\ power = 9^2 \bmod 99 = 81$

$i=6,\ a_6=1,\ x=81\cdot 81 \bmod 99 = 27,\ power = 81^2 \bmod 99 = 27$

$i=7,\ a_7=1,\ x=27\cdot 27 \bmod 99 = 36,\ power = 27^2 \bmod 99 = 36$

$i=8,\ a_8=1,\ x=36\cdot 36 \bmod 99 = 9,\ power = 36^2 \bmod 99 = 9$

$i=9,\ a_9=1,\ x=9\cdot 9 \bmod 99 = 81,\ power = 9^2 \bmod 99 = 81$

$i=10,\ a_{10}=1,\ x=81\cdot 81 \bmod 99 = 27,\ power = 81^2 \bmod 99 = 27$

$$\boxed{3^{2003} \bmod 99 = 27.}$$

37\. How is the one's complement representation of the sum of two integers obtained from the one's complement representations of these integers?

Assume that n bits are being used, so the range of that numbers is between -2^{n-1} and 2^{n-1}.

To obtain the one's complement representation of the sum of two numbers, we add the two strings representing these numbers using algorithm. After perfoming this operation, there may be a carry out of the left-most column, in such case we then add 1 more to the answer.

43\. Answer Ex. 37 for two's complement expansions.

To obtain the two's complement representation of the sum of two integers given in two's complement representation, add them as if they were binary integers, and ignore any carry out of the left-most column. However, if the left-most digits of the two addends agree and the left-most digit of the answer is different from their common value, then an overflow has occured, and the answer is not valid.