6.3 Permutations and Combinations

23. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?

Solution:

$P(8,8) = 8!$ for men

$P(9,5) = \frac{9!}{4!}$ for women

$P(8,8) \cdot P(9,5) = 40,320 \cdot 15,120 = 609,638,400.$

25. One hundred tickets, numbered 1, 2, 3, ..., 100, are sold to 100 different people to drawing. Four different prizes are awarded, including a grand prize. How many ways are there to award the prizes if

a) there are no restrictions?

$P(100,4) = \frac{100!}{(100-4)!} = 94,109,400.$

b) the person holding ticket 47 wins the grand prize?

$P(99,3) = \frac{99!}{(99-3)!} = 941,094.$

c) the person holding ticket 47 wins one of the prizes?

4 ways to determine which prize ticket 47 won. $P(99,3)$

$4 \cdot P(99,3) = 3,764,376$

d) the person holding ticket 47 does not win a prize?

$$P(99,4) = \frac{99!}{(99-4)!} = 99 \cdot 98 \cdot 97 \cdot 96 = 90,345,024$$

e) the people holding tickets 19 and 47 both win prizes?

$4 \cdot 3 = 12$ way to determine which prizes these people will win. $P(98,2) = \frac{98!}{(98-21!} = \frac{98!}{96!} = 98 \cdot 97 = 9,506$.

$12 \cdot 9,506 = 114,072$

f) the people holding tickets 19, 47 and 73 all win prizes?

$4 \cdot 3 \cdot 2 = 24$; $P(97,1)$

$24 \cdot 97 = 2,328$

g) the people holding tickets 19, 47, 73 and 97 all win prizes?

$P(4,4) = 24$

h) none of the people holding tickets 19, 47, 73, 97 wins a prize!

$$P(96,4) = \frac{96!}{(96-4)!} = \frac{96!}{92!} = 96 \cdot 95 \cdot 94 \cdot 93 = 79,727,040.$$

i) the grand prize winner is a person holding ticket 19, 47, 73, 97?

$4 \cdot P(99,3) = 3,764,376$

j) the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?

$4 \cdot 3 = 12$ for t. 19 and 47; $P(96,2) = \frac{96!}{94!} = 96 \cdot 95 =$

$= 9,120$; $12 \cdot 9,120 = 109,440$

29. How many 4-permutations of the positive integer not exceeding 100 contain three consecutive integers k, $k+1$, $k+2$, in the correct order

a) where these consecutive integers can perhaps be separated by other integers in the permutation?

$$98 \cdot 4 \cdot 97 - 97 = 37,927.$$

b) where they are in consecutive positions in the permutation?

$$98 \cdot 2 \cdot 97 - 97 = 18,915.$$

31. The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

a) exactly one vowel?

$$6 \cdot 5 \cdot 21^5 = 122,523,030.$$

b) exactly two vowels?

$$C(6,2) \cdot 5^2 \cdot 21^4 = 72,930,375$$

c) at least one vowel?

$$26^6 - 21^6 = 223,149,655$$

d) at least two vowels?

$$26^6 - 21^6 - 6 \cdot 5 \cdot 21^5 = 100,626,625$$

41. Find a formula for the number of circular r-permutations of n people

$$\frac{P(n,r)}{r} = \frac{n!}{r(n-r)!}$$

43. How many ways are there for a horse race with three horses to finish if ties are possible?

$6+6+1=13$