DESIGN AND ANALYSIS OF ALGORITHMS LAB

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BATCH: A

BRANCH: CSE DS

EXPT. NO.: 6

AIM: Experiment based on greedy graph algoritms: Djikstra's single source shortest path algorithm and Prim's minimum spanning tree algorithm

ALGORITHM:

Djikstra's algorithm:

ALGORITHM 1 Dijkstra's Algorithm.

procedure *Dijkstra*(G: weighted connected simple graph, with
    all weights positive)
{G has vertices $a = v_0, v_1, \ldots, v_n = z$ and lengths $w(v_i, v_j)$
    where $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge in G}
for $i := 1$ **to** n
    $L(v_i) := \infty$
$L(a) := 0$
$S := \emptyset$
{the labels are now initialized so that the label of a is 0 and all
    other labels are ∞, and S is the empty set}
while $z \notin S$
    $u :=$ a vertex not in S with $L(u)$ minimal
    $S := S \cup \{u\}$
    for all vertices v not in S
        if $L(u) + w(u, v) < L(v)$ **then** $L(v) := L(u) + w(u, v)$
        {this adds a vertex to S with minimal label and updates the
        labels of vertices not in S}
return $L(z)$ {$L(z) =$ length of a shortest path from a to z}

Prim's algorithm:

ALGORITHM 1 Prim's Algorithm.

procedure *Prim*(G: weighted connected undirected graph with n vertices)
$T :=$ a minimum-weight edge
for $i := 1$ **to** $n - 2$
    $e :=$ an edge of minimum weight incident to a vertex in T and not forming a
        simple circuit in T if added to T
    $T := T$ with e added
return T {T is a minimum spanning tree of G}

CODE:

utilities.c:

```c
#include <stdio.h>
#include <stdlib.h>
#include <limits.h>
// linked list implementation:

typedef struct edge
{
    int dest;
    int source; // for prims
    int weight;
    int shortest_distance;
    struct edge *next;
} *p_edge;
typedef struct linked_list
{
    p_edge head;
    p_edge tail;
} *p_linked_list;

p_edge create_edge(int dest, int weight)
{
    p_edge new = malloc(sizeof(struct edge));
    new->next = NULL;
    new->dest = dest;
    new->weight = weight;
    return new;
}

p_linked_list create_linked_list()
{
    p_linked_list ll = malloc(sizeof(struct linked_list));
    ll->head = NULL;
    ll->tail = NULL;
}

void insert_edge(p_linked_list ll, int dest, int weight)
{
    if (ll->tail == NULL)
    {
        ll->head = create_edge(dest, weight);
        ll->tail = ll->head;
    }
    else
    {
        ll->tail->next = create_edge(dest, weight);
        ll->tail = ll->tail->next;
```

```
    }
}

// priority queue implementation:

typedef struct priority_queue
{
    int arr_size;
    int heap_size;
    p_edge *arr;
    int type; // 0 -> min heap based on weights, 1 -> min heap based
on shortest distance
} *p_priority_queue;

p_priority_queue create_priority_queue(int max_size, int type)
{
    p_priority_queue ppq = malloc(sizeof(struct priority_queue));
    ppq->arr = malloc(sizeof(p_edge) * max_size);
    ppq->arr_size = max_size;
    ppq->heap_size = 0;
    ppq->type = type;
    return ppq;
}

void enqueue(p_priority_queue pq, p_edge edge)
{
    if (pq->heap_size >= pq->arr_size)
    {
        printf("Queue is full. Element cannot be inserted.\n");
        return;
    }
    // heap insertion:
    pq->arr[pq->heap_size] = edge;
    int parent = ((pq->heap_size + 1) / 2) - 1;
    int child = pq->heap_size;
    pq->heap_size++;
    p_edge temp;
    if (pq->type == 1)
    {
        while (parent >= 0)
        {
            if (pq->arr[child]->shortest_distance < pq->arr[parent]-
>shortest_distance)
            {
                temp = pq->arr[child];
                pq->arr[child] = pq->arr[parent];
                pq->arr[parent] = temp;
                child = parent;
                parent = ((parent + 1) / 2) - 1;
```

```
            }
            else
                return;
        }
    }
    else
    {
        while (parent >= 0)
        {
            if (pq->arr[child]->weight < pq->arr[parent]->weight)
            {
                temp = pq->arr[child];
                pq->arr[child] = pq->arr[parent];
                pq->arr[parent] = temp;
                child = parent;
                parent = ((parent + 1) / 2) - 1;
            }
            else
                return;
        }
    }
}

p_edge dequeue(p_priority_queue pq)
{
    if (pq->heap_size == 0)
        return NULL;
    p_edge ret_val = pq->arr[0];
    pq->heap_size--;
    pq->arr[0] = pq->arr[pq->heap_size];
    // heapify:
    int parent = 0, smallest, left, right;
    p_edge temp;
    if (pq->type == 1)
    {
        while (parent < pq->heap_size)
        {
            left = parent * 2 + 1;
            right = left + 1;
            smallest = parent;
            if (left < pq->heap_size && pq->arr[left]-
>shortest_distance < pq->arr[smallest]->shortest_distance)
                smallest = left;
            if (right < pq->heap_size && pq->arr[right]-
>shortest_distance < pq->arr[smallest]->shortest_distance)
                smallest = right;
            if (smallest != parent)
            {
                temp = pq->arr[smallest];
```

```
                pq->arr[smallest] = pq->arr[parent];
                pq->arr[parent] = temp;
                parent = smallest;
            }
            else
                break;
        }
    }
    else
    {
        while (parent < pq->heap_size)
        {
            left = parent * 2 + 1;
            right = left + 1;
            smallest = parent;
            if (left < pq->heap_size && pq->arr[left]->weight < pq-
>arr[smallest]->weight)
                smallest = left;
            if (right < pq->heap_size && pq->arr[right]->weight <
pq->arr[smallest]->weight)
                smallest = right;
            if (smallest != parent)
            {
                temp = pq->arr[smallest];
                pq->arr[smallest] = pq->arr[parent];
                pq->arr[parent] = temp;
                parent = smallest;
            }
            else
                break;
        }
    }
    return ret_val;
}

int is_empty(p_priority_queue pq)
{
    return pq->heap_size == 0;
}

// graphs:
typedef struct graph
{
    int directed;
    int V;                  // no. of vertices
    int E;                  // no. of edges
    p_linked_list *edges; // array of pointers to linked lists of
edges...index of the array represents source vertex and each linked
list containes all outgoing edges of that source vertex
```

```c
} *p_graph;

p_graph take_directed_graph_input()
{
    p_graph g = malloc(sizeof(struct graph));
    g->directed = 1;
    printf("Enter number of vertices: ");
    scanf("%d", &g->V);
    printf("Enter number of edges: ");
    scanf("%d", &g->E);
    g->edges = malloc(sizeof(p_linked_list) * g->V);
    for (int i = 0; i < g->V; i++)
        g->edges[i] = create_linked_list();
    int source, dest, weight;
    for (int i = 0; i < g->E; i++)
    {
        printf("For edge %d:\nEnter source vertex: ", i + 1);
        scanf("%d", &source);
        printf("Enter destination vertex: ");
        scanf("%d", &dest);
        printf("Enter weight: ");
        scanf("%d", &weight);
        insert_edge(g->edges[source], dest, weight);
    }
    return g;
}

void add_edge_to_undirected_graph(p_graph g, int source, int dest,
int weight)
{
    insert_edge(g->edges[source], dest, weight);
    g->edges[source]->tail->source = source;
    insert_edge(g->edges[dest], source, weight);
    g->edges[dest]->tail->source = dest;
}

p_graph take_undirected_graph_input()
{
    p_graph g = malloc(sizeof(struct graph));
    g->directed = 0;
    printf("Enter number of vertices: ");
    scanf("%d", &g->V);
    printf("Enter number of edges: ");
    scanf("%d", &g->E);
    g->edges = malloc(sizeof(p_linked_list) * g->V);
    for (int i = 0; i < g->V; i++)
        g->edges[i] = create_linked_list();
    int source, dest, weight;
    for (int i = 0; i < g->E; i++)
```

```
    {
        printf("For edge %d:\nEnter source vertex: ", i + 1);
        scanf("%d", &source);
        printf("Enter destination vertex: ");
        scanf("%d", &dest);
        printf("Enter weight: ");
        scanf("%d", &weight);
        add_edge_to_undirected_graph(g, source, dest, weight);
    }
    return g;
}

void display_graph(p_graph g)
{
    for (int i = 0; i < g->V; i++)
    {
        printf("%d : ", i);
        if (g->edges[i]->head != NULL)
        {
            printf("(%d,%d) ", g->edges[i]->head->dest, g->edges[i]-
>head->weight);
            for (p_edge e = g->edges[i]->head->next; e != NULL; e =
e->next)
                printf(", (%d,%d) ", e->dest, e->weight);
        }
        printf("\n");
    }
}
```

djikstras.c:

```
#include <stdio.h>
#include <stdlib.h>
#include <limits.h>
#include "utilities.h"

struct djikstra_result
{
    int *parent;
    int *shortest_distance;
    int size;
    int source;
};

struct djikstra_result *djikstras(p_graph g, int source)
{
    p_priority_queue pq = create_priority_queue(g->V, 1); // min
heap of shortest distance
    int visited[g->V];
    struct djikstra_result *dr = malloc(sizeof(struct
djikstra_result));
    dr->parent = malloc(sizeof(int) * g->V);
    dr->shortest_distance = malloc(sizeof(int) * g->V);
    dr->parent[source] = -1;
    dr->size = g->V;
    dr->source = source;
    for (int i = 0; i < g->V; i++)
    {
        visited[i] = 0;
        dr->shortest_distance[i] = INT_MAX;
    }
    dr->shortest_distance[source] = 0;
    struct edge self = {.dest = 0, .weight = 0, .shortest_distance =
0, .next = g->edges[source]->head};
    enqueue(pq, &self);
    while (!is_empty(pq))
    {
        // extracting the edge with minimum weight
        int curr_vtx = dequeue(pq)->dest;
        // traversing neighbours of current vertex
        for (p_edge i = g->edges[curr_vtx]->head; i != NULL; i = i-
>next)
        {
            if (!visited[i->dest])
            {
                if (dr->shortest_distance[curr_vtx] + i->weight <
dr->shortest_distance[i->dest])
                {
```

```
                    dr->shortest_distance[i->dest] = dr-
>shortest_distance[curr_vtx] + i->weight;
                    dr->parent[i->dest] = curr_vtx;
                }
                i->shortest_distance = dr->shortest_distance[i-
>dest];
                enqueue(pq, i);
            }
        }
        visited[curr_vtx] = 1;
    }
    return dr;
}

void print_path(int *parents, int dest)
{
    if (parents[dest] == -1)
    {
        printf("%d", dest);
        return;
    }
    print_path(parents, parents[dest]);
    printf(" -> %d", dest);
}

void print_all_shortest_paths(struct djikstra_result *dr)
{
    for (int i = 0; i < dr->size; i++)
    {
        printf("To %d : ", i);
        print_path(dr->parent, i);
        printf("  :  %d\n", dr->shortest_distance[i]);
    }
}

int main()
{
    freopen("input.txt", "r", stdin);
    p_graph g = take_directed_graph_input();
    printf("Enter source vertex: ");
    int source;
    scanf("%d", &source);
    struct djikstra_result *dr = djikstras(g, source);
    printf("\n\nAll single source (%d) shortest paths and their
costs:\n\n", source);
    print_all_shortest_paths(dr);
}
```

prims.c:

```
#include <stdio.h>
#include <stdlib.h>
#include "utilities.h"

struct prims_result
{
    int cost;
    p_graph mst;
};

struct prims_result *prims(p_graph g)
{
    p_graph mst = malloc(sizeof(struct graph));
    mst->V = g->V;
    mst->E = 0;
    mst->directed = 0;
    mst->edges = malloc(sizeof(p_linked_list) * g->V);
    for (int i = 0; i < mst->V; i++)
        mst->edges[i] = create_linked_list();
    p_priority_queue pq = create_priority_queue(g->E, 0);
    int added[g->V];
    int cost = 0;
    for (int i = 0; i < g->V; i++)
        added[i] = 0;
    int curr_vtx = 0;
    added[curr_vtx] = 1;
    int vtx_counter = 1;
    do
    {
        for (p_edge e = g->edges[curr_vtx]->head; e != NULL; e = e-
>next)
        {
            if (!added[e->dest])
                enqueue(pq, e);
        }
        p_edge min_edge = dequeue(pq);
        while (added[min_edge->source] && added[min_edge->dest])
            min_edge = dequeue(pq);
        add_edge_to_undirected_graph(mst, min_edge->source,
min_edge->dest, min_edge->weight);
        printf("added edge (%d,%d)\n", min_edge->source, min_edge-
>dest);
        mst->E++;
        cost += min_edge->weight;
        added[min_edge->dest] = 1;
        vtx_counter++;
        curr_vtx = min_edge->dest;
```

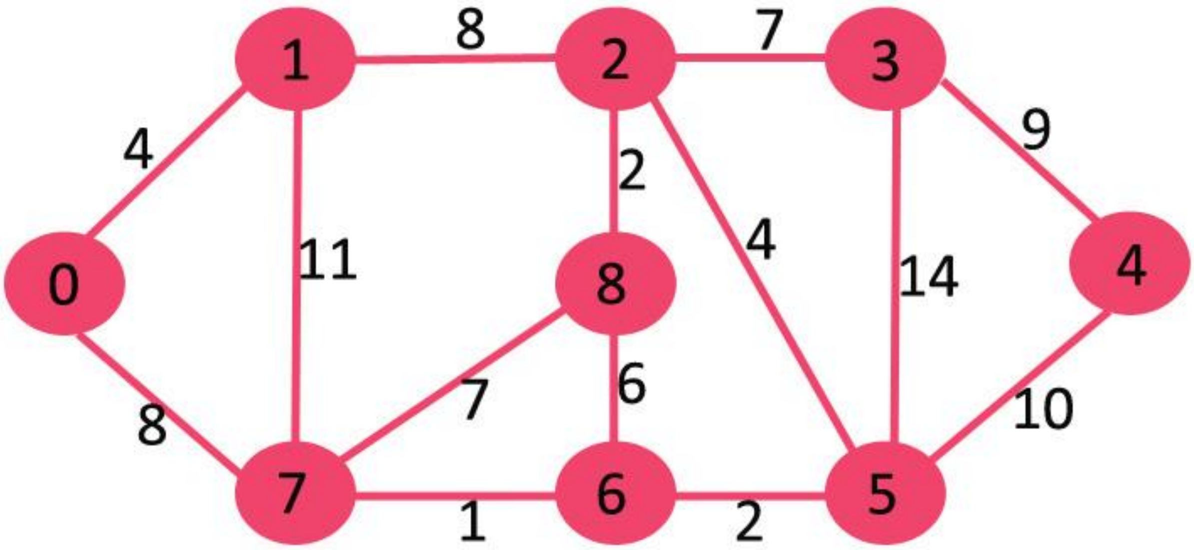
```c
    } while (vtx_counter < g->V);
    struct prims_result *pr = malloc(sizeof(struct prims_result));
    pr->cost = cost;
    pr->mst = mst;
    return pr;
}

int main()
{
    freopen("input_2.txt", "r", stdin);
    p_graph g = take_undirected_graph_input();
    printf("\n\n");
    struct prims_result *pr = prims(g);
    printf("\nCost: %d\n", pr->cost);
    printf("Tree (graph):\n");
    display_graph(pr->mst);
}
```

OUTPUT:

Djikstra’s algorithm:

Input graph:



Output:

```
tilities.c -o prims   (base) PS C:\Users\arifa\Desktop\sem4 work\daa lab\exp6> .\prims.exe            Enter number of vertices: 9
Enter number of edges: 14
For edge 1:
Enter source vertex: 0
Enter destination vertex: 1
Enter weight: 4
For edge 2:
Enter source vertex: 0
Enter destination vertex: 7
Enter weight: 8
For edge 3:
Enter source vertex: 1
Enter destination vertex: 7
Enter weight: 11
For edge 4:
Enter source vertex: 1
Enter destination vertex: 2
Enter weight: 8
For edge 5:
Enter source vertex: 7
Enter destination vertex: 6
Enter weight: 1
For edge 6:
Enter source vertex: 7
Enter destination vertex: 8
Enter weight: 7
For edge 7:
Enter source vertex: 2
Enter destination vertex: 8
Enter weight: 2
For edge 8:
Enter source vertex: 8
Enter destination vertex: 6
Enter weight: 6
```

```
For edge 9:
Enter source vertex: 2
Enter destination vertex: 3
Enter weight: 7
For edge 10:
Enter source vertex: 2
Enter destination vertex: 5
Enter weight: 4
For edge 11:
Enter source vertex: 6
Enter destination vertex: 5
Enter weight: 2
For edge 12:
Enter source vertex: 3
Enter destination vertex: 5
Enter weight: 14
For edge 13:
Enter source vertex: 3
Enter destination vertex: 4
Enter weight: 9
For edge 14:
Enter source vertex: 5
Enter destination vertex: 4
Enter weight: 10
```

```
All single source (0) shortest paths and their costs:

To 0 : 0  :  0
To 1 : 0 -> 1  :  4
To 2 : 0 -> 1 -> 2  :  12
To 3 : 0 -> 1 -> 2 -> 3  :  19
To 4 : 0 -> 7 -> 6 -> 5 -> 4  :  21
To 5 : 0 -> 7 -> 6 -> 5  :  11
To 6 : 0 -> 7 -> 6  :  9
To 7 : 0 -> 7  :  8
To 8 : 0 -> 1 -> 2 -> 8  :  14
(base) PS C:\Users\arifa\Desktop\sem4 work\daa lab\exp6>
```

Output: *0 4 12 19 21 11 9 8 14*

Explanation: *The distance from 0 to 1 = 4.*

The minimum distance from 0 to 2 = 12. 0->1->2

The minimum distance from 0 to 3 = 19. 0->1->2->3

The minimum distance from 0 to 4 = 21. 0->7->6->5->4

The minimum distance from 0 to 5 = 11. 0->7->6->5

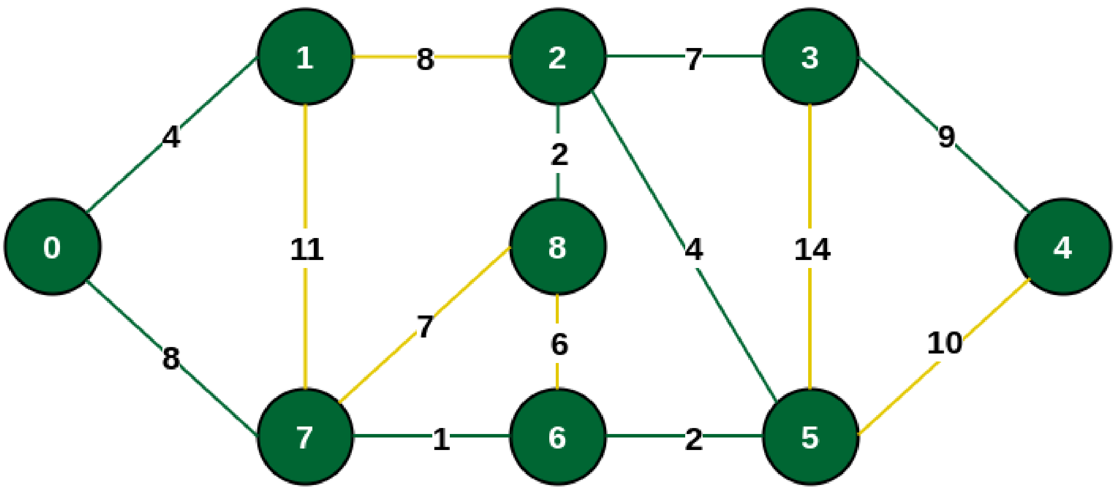
The minimum distance from 0 to 6 = 9. 0->7->6

The minimum distance from 0 to 7 = 8. 0->7

The minimum distance from 0 to 8 = 14. 0->1->2->8

Prim’s algorithm:

Input graph:



Output:

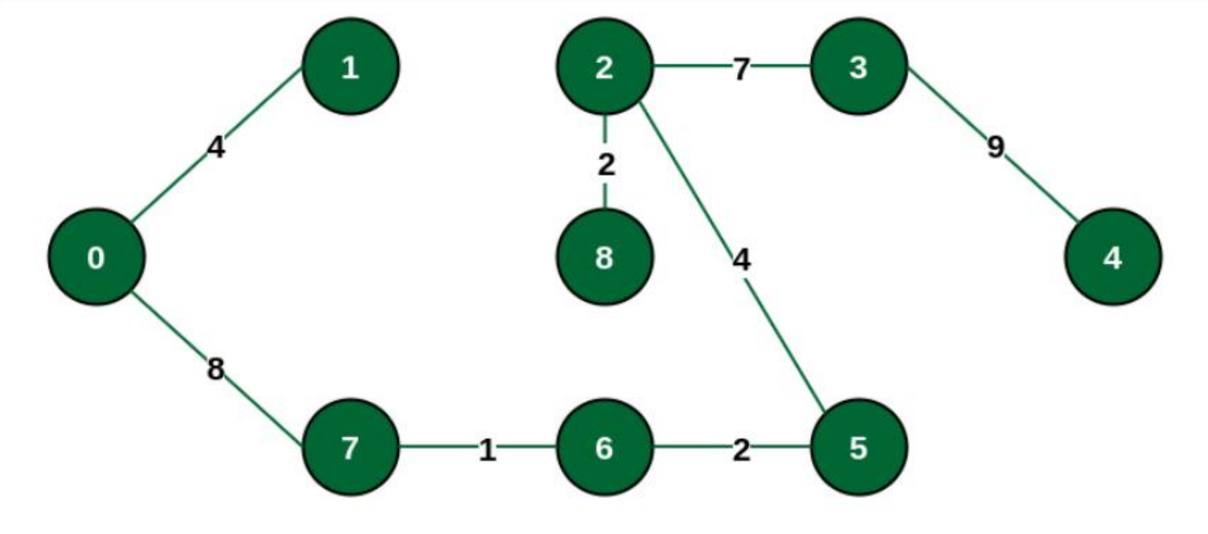
```
tilities.c -o prims   (base) PS C:\Users\arifa\Desktop\sem4 work\daa lab\exp6> .\prims.exe          Enter number of vertices: 9
Enter number of edges: 14
For edge 1:
Enter source vertex: 0
Enter destination vertex: 1
Enter weight: 4
For edge 2:
Enter source vertex: 0
Enter destination vertex: 7
Enter weight: 8
For edge 3:
Enter source vertex: 1
Enter destination vertex: 7
Enter weight: 11
For edge 4:
Enter source vertex: 1
Enter destination vertex: 2
Enter weight: 8
For edge 5:
Enter source vertex: 7
Enter destination vertex: 6
Enter weight: 1
For edge 6:
Enter source vertex: 7
Enter destination vertex: 8
Enter weight: 7
For edge 7:
Enter source vertex: 2
Enter destination vertex: 8
Enter weight: 2
For edge 8:
Enter source vertex: 8
Enter destination vertex: 6
Enter weight: 6
```

```
For edge 9:
Enter source vertex: 2
Enter destination vertex: 3
Enter weight: 7
For edge 10:
Enter source vertex: 2
Enter destination vertex: 5
Enter weight: 4
For edge 11:
Enter source vertex: 6
Enter destination vertex: 5
Enter weight: 2
For edge 12:
Enter source vertex: 3
Enter destination vertex: 5
Enter weight: 14
For edge 13:
Enter source vertex: 3
Enter destination vertex: 4
Enter weight: 9
For edge 14:
Enter source vertex: 5
Enter destination vertex: 4
Enter weight: 10
```

```
added edge (0,1)
added edge (0,7)
added edge (7,6)
added edge (6,5)
added edge (5,2)
added edge (2,8)
added edge (2,3)
added edge (3,4)

Cost: 37
Tree (graph):
0 : (1,4) , (7,8)
1 : (0,4)
2 : (5,4) , (8,2) , (3,7)
3 : (2,7) , (4,9)
4 : (3,9)
5 : (6,2) , (2,4)
6 : (7,1) , (5,2)
7 : (0,8) , (6,1)
8 : (2,2)
(base) PS C:\Users\arifa\Desktop\sem4 work\daa lab\exp6>
```

*The final structure of the MST is as follows and the weight of the edges of the MST is (4 + 8 + 1 + 2 + 4 + 2 + 7 + 9) = **37**.*



CONCLUSION:

Greedy approach can be used on graphs to find shortest path from a vertex to all other vertices as well as to construct a minimum spanning tree of a graph.