Representation Learning for Text

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an introduction and applications

P. Meladianos, A. Tixier, M. Vazirgiannis

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“a word is defined by “the company it keeps” (Firth, 1957)

Language model

- Goal: determine $P(s = w_1 \dots w_k)$ in some domain of interest

$$P(s) = \prod_{i=1}^{k} P\left(w_i \mid w_1 \dots w_{i-1}\right)$$

e.g., $P\left(w_1 w_2 w_3\right) = P\left(w_1\right) P\left(w_2 \mid w_1\right) P\left(w_3 \mid w_1 w_2\right)$

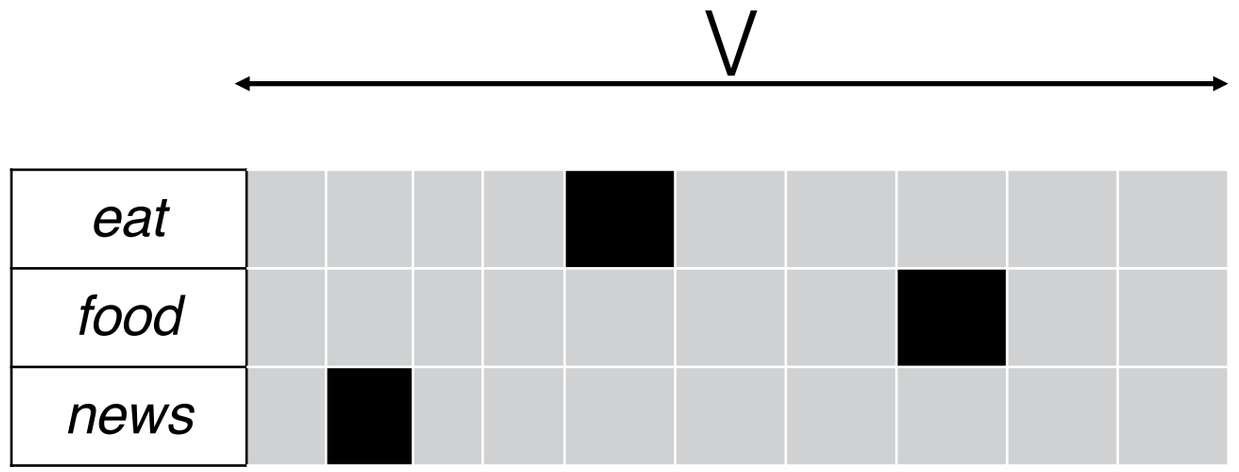
- Traditional n-gram language model assumption:
  "the probability of a word depends only on **context** of $n - 1$ previous words"

$$\Rightarrow \widehat{P}(s) = \prod_{i=1}^{k} P\left(w_i \mid w_{i-n+1} \dots w_{i-1}\right)$$

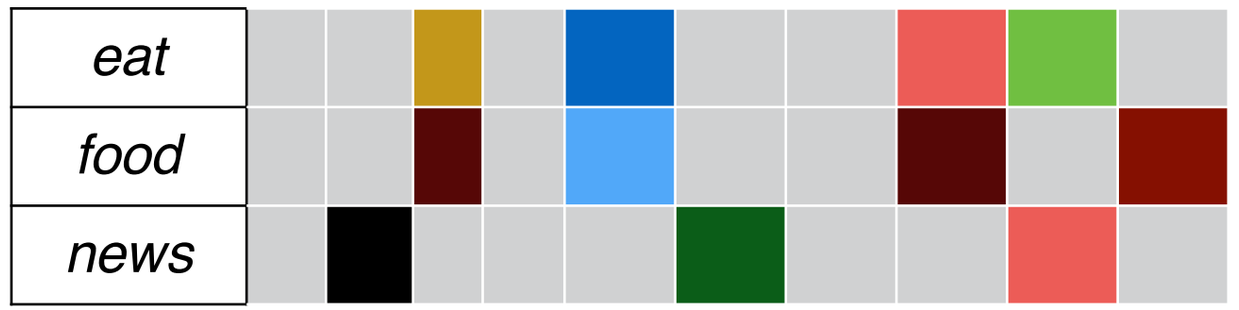
- Typical ML-smoothing learning process (e.g., Katz 1987):
  1. compute $\widehat{P}\left(w_i \mid w_{i-n+1} \dots w_{i-1}\right) = \frac{\#w_{i-n+1} \dots w_{i-1} w_i}{\#w_{i-n+1} \dots w_{i-1}}$ on training corpus
  2. smooth to avoid zero probabilities

Representing Words

- **One-hot vector**
  - high dimensionality
  - sparse vectors
  - dimensions=|V| (10^6<|V|)
  - unable to capture semantic similarity between words



- **Distributional vector**
  - words that occur in similar contexts, tend to have similar meanings
  - each word vector contains the frequencies of all its neighbors
  - dimensions=|V|
  - computational complexity for ML algorithms



Representing Words

- **Word embeddings**
  - store the same contextual information in a low-dimensional vector
  - **densification** (sparse to dense)
  - **compression**
    - dimensionality reduction
    - dimensions=m 100<m<500
  - able to capture semantic similarity between words
  - learned vectors (unsupervised)
  - Learning methods
    - **SVD**
    - **word2vec**
    - **GloVe**

Example

- We should assign similar probabilities (discover similarity) to *Obama speaks to the media in Illinois* and the *President addresses the press in Chicago*

- This does not happen because of the "one-hot" vector space representation

One hot

$$\left.\begin{array}{r} \text{obama} = [0\ 0\ 0\ 0\ \ldots\ 0\ 1\ 0\ 0] \\ \text{president} = [0\ 0\ 0\ 1\ \ldots\ 0\ 0\ 0\ 0] \end{array}\right\} \overrightarrow{\text{obama}}.\overrightarrow{\text{president}} = \vec{0}$$

$$\left.\begin{array}{r} \text{speaks} = [0\ 0\ 1\ 0\ \ldots\ 0\ 0\ 0\ 0] \\ \text{addresses} = [0\ 0\ 0\ 0\ \ldots\ 0\ 0\ 1\ 0] \end{array}\right\} \overrightarrow{\text{speaks}}.\overrightarrow{\text{addresses}} = \vec{0}$$

$$\left.\begin{array}{r} \text{illinois} = [1\ 0\ 0\ 0\ \ldots\ 0\ 0\ 0\ 0] \\ \text{chicago} = [0\ 1\ 0\ 0\ \ldots\ 0\ 0\ 0\ 0] \end{array}\right\} \overrightarrow{\text{illinois}}.\overrightarrow{\text{chicago}} = \vec{0}$$

Word embeddings

$$w_i \in V \xrightarrow{\text{mapping C}} \mathbb{R}^m$$

$$\text{obama} = [0 \ldots \ldots 1 \ldots \ldots 0] \qquad \text{obama} = [0.12 \ldots -0.25]$$

(w_1, obama, $w_{|V|}$ index the one-hot vector; feature_1, feature_m index the feature vector)

$|V|$ — "one-hot" vector

$m \ll |V|$ — feature vector

SVD word embeddings

- Dimensionality reduction on co-occurrence matrix
- Create a $|V|x|V|$ word co-occurrence matrix X
- Apply SVD $X = USV^T$
- Take first k columns of U
- Use the k-dimensional vectors as representations for each word
- Able to capture semantic and syntactic similarity

SVD application - Latent Structure in documents

- Documents are represented based on the Vector Space Model
- Vector space model consists of the keywords contained in a document.
- In many cases baseline keyword based performs poorly – not able to detect synonyms.
- Therefore document clustering is problematic
- Example where of keyword matching with the query: "IDF in computer-based information look-up"

	access	document	retrieval	information	theory	database	indexing	computer
Doc1	x	x	x			x	x	
Doc2				x	x			x
Doc3			x	x				x

Indexing by Latent Semantic Analysis (1990) Scott Deerwester, Susan T. Dumais, George W. Furnas, Thomas K. Landauer, Richard Harshman, Journal of the American Society of Information Science

Latent Semantic Indexing (LSI)-I

- Finding similarity with exact keyword matching is problematic.
- Using SVD we process the initial document-term document.
- Then we choose the k larger singular values. The resulting matrix is of order k and is the most similar to the original one based on the Frobenius norm than any other k-order matrix.

Latent Semantic Indexing (LSI) - II

- The initial matrix is SVD decomposed as: $A=ULV^T$
- Choosing the top-k singular values from L we have:

$$A_k=U_kL_kV_k^T ,$$

- L_k square kxk - top-k singular values of the diagonal in matrix L,
- U_k, mxk matrix - first k columns in U (left singular vectors)
- V_k^T, kxn matrix - first k lines of V^T (right singular vectors)

Typical values for $\kappa \sim 200\text{-}300$ (empirically chosen based on experiments appearing in the bibliography)

LSI capabilities

- - Term to term similarity: $A_k A_k^T = U_k L_k^2 U_k^T$
- Where Ak=UkLkVt
- - Document-document similarity: $A_k^T A_k = V_k L_k^2 V_k^T$
- - Term document similarity (as an element of the transformed – document matrix)
- - Extended query capabilities transforming initial query q to q_n $q_n = q^T U_k L_k^{-1}$
- - Thus q_n can be regarded a line in matrix V_k

LSI – an example

LSI application on a term – document matrix

C1: Human machine Interface for Lab ABC computer application

C2: A survey of user opinion of computer system response time

C3: The EPS user interface management system

C4: System and human system engineering testing of EPS

C5: Relation of user-perceived response time to error measurements

M1: The generation of random, binary unordered trees

M2: The intersection graph of path in trees

M3: Graph minors IV: Widths of trees and well-quasi-ordering

M4: Graph minors: A survey

• The dataset consists of 2 classes, 1st: "human – computer interaction" (c1-c5) 2nd: related to graph (m1-m4). After feature extraction the titles are represented as follows.

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LSI – an example

	C1	C2	C3	C4	C5	M1	M2	M3	M4
human	1	0	0	1	0	0	0	0	0
Interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
User	0	1	1	0	1	0	0	0	0
System	0	1	1	2	0	0	0	0	0
Response	0	1	0	0	1	0	0	0	0
Time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
Survey	0	1	0	0	0	0	0	0	1
Trees	0	0	0	0	0	1	1	1	0
Graph	0	0	0	0	0	0	1	1	1
Minors	0	0	0	0	0	0	0	1	1

LSI – an example

$A = ULV^T$

A=

1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0
0	1	1	0	1	0	0	0	0
0	1	1	2	0	0	0	0	0
0	1	0	0	1	0	0	0	0
0	1	0	0	1	0	0	0	0
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	1
0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1

LSI – an example

$A = ULV^T$

U=

0.22	-0.11	0.29	-0.41	-0.11	-0.34	0.52	-0.06	-0.41
0.20	-0.07	0.14	-0.55	0.28	0.50	-0.07	-0.01	-0.11
0.24	0.04	-0.16	-0.59	-0.11	-0.25	-0.30	0.06	0.49
0.40	0.06	-0.34	0.10	0.33	0.38	0.00	0.00	0.01
0.64	-0.17	0.36	0.33	-0.16	-0.21	-0.17	0.03	0.27
0.27	0.11	-0.43	0.07	0.08	-0.17	0.28	-0.02	-0.05
0.27	0.11	-0.43	0.07	0.08	-0.17	0.28	-0.02	-0.05
0.30	-0.14	0.33	0.19	0.11	0.27	0.03	-0.02	-0.17
0.21	0.27	-0.18	-0.03	-0.54	0.08	-0.47	-0.04	-0.58
0.01	0.49	0.23	0.03	0.59	-0.39	-0.29	0.25	-0.23
0.04	0.62	0.22	0.00	-0.07	0.11	0.16	-0.68	0.23
0.03	0.45	0.14	-0.01	-0.30	0.28	0.34	0.68	0.18

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

LSI – an example

$$A = ULV^T$$

$L =$

3.34	0	0	0	0	0	0	0	0
0	2.54	0	0	0	0	0	0	0
0	0	2.35	0	0	0	0	0	0
0	0	0	1.64	0	0	0	0	0
0	0	0	0	1.50	0	0	0	0
0	0	0	0	0	1.31	0	0	0
0	0	0	0	0	0	0.85	0	0
0	0	0	0	0	0	0	0.56	0
0	0	0	0	0	0	0	0	0.36
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

LSI – an example

$$A = U L V^T$$

$V =$

0.20	-0.06	0.11	-0.95	0.05	-0.08	0.18	-0.01	-0.06
0.61	0.17	-0.50	-0.03	-0.21	-0.26	-0.43	0.05	0.24
0.46	-0.13	0.21	0.04	0.38	0.72	-0.24	0.01	0.02
0.54	-0.23	0.57	0.27	-0.21	-0.37	0.26	-0.02	-0.08
0.28	0.11	-0.51	0.15	0.33	0.03	0.67	-0.06	-0.26
0.00	0.19	0.10	0.02	0.39	-0.30	-0.34	0.45	-0.62
0.01	0.44	0.19	0.02	0.35	-0.21	-0.15	-0.76	0.02
0.02	0.62	0.25	0.01	0.15	0.00	0.25	0.45	0.52
0.08	0.53	0.08	-0.03	-0.60	0.36	0.04	-0.07	-0.45

LSI – an example

Choosing the 2 largest singular values we have

$U_k =$

0.22	-0.11
0.20	-0.07
0.24	0.04
0.40	0.06
0.64	-0.17
0.27	0.11
0.27	0.11
0.30	-0.14
0.21	0.27
0.01	0.49
0.04	0.62
0.03	0.45

$L_k =$

3.34	0
0	2.54

$V_k^T =$

0.20	0.61	0.46	0.54	0.28	0.00	0.02	0.02	0.08
-0.06	0.17	-0.13	-0.23	0.11	0.19	0.44	0.62	0.53

LSI reconstruction (2 singular values)

$A_k =$

	C1	C2	C3	C4	C5	M1	M2	M3	M4
human	0.16	0.40	0.38	0.47	0.18	-0.05	-0.12	-0.16	-0.09
Interface	0.14	0.37	0.33	0.40	0.16	-0.03	-0.07	-0.10	-0.04
Compute r	0.15	0.51	0.36	0.41	0.24	0.02	0.06	0.09	0.12
User	0.26	0.84	0.61	0.70	0.39	0.03	0.08	0.12	0.19
System	0.45	1.23	1.05	1.27	0.56	-0.07	-0.15	-0.21	-0.05
Respons e	0.16	0.58	0.38	0.42	0.28	0.06	0.13	0.19	0.22
Time	0.16	0.58	0.38	0.42	0.28	0.06	0.13	0.19	0.22
EPS	0.22	0.55	0.51	0.63	0.24	-0.07	-0.14	-0.20	-0.11
Survey	0.10	0.53	0.23	0.21	0.27	0.14	0.31	0.44	0.42
Trees	-0.06	0.23	-0.14	-0.27	0.14	0.24	0.55	0.77	0.66
Graph	-0.06	0.34	-0.15	-0.30	0.20	0.31	0.69	0.98	0.85
Minors	-0.04	0.25	-0.10	-0.21	0.15	0.22	0.50	0.71	0.62

LSI Example

- Query: "human computer interaction" retrieves documents: c_1,c_2, c_4 but *not* c_3 and c_5.
- If we submit the same query (based on the transformation shown before) to the transformed matrix we retrieve (using cosine similarity) all c_1-c_5 even if c_3 and c_5 have no common keyword to the query.
- According to the transformation for the queries we have:

Query transformation

	query
human	1
Interface	0
computer	1
User	0
System	0
Response	0
Time	0
EPS	0
Survey	0
Trees	0
Graph	0
Minors	0

q=

1
0
1
0
0
0
0
0
0
0

Query transformation

$q^T =$

1	0	1	0	0	0	0	0	0	0	0	0

$U_k =$

0.22	-0.11
0.20	-0.07
0.24	0.04
0.40	0.06
0.64	-0.17
0.27	0.11
0.27	0.11
0.30	-0.14
0.21	0.27
0.01	0.49
0.04	0.62
0.03	0.45

$L_k =$

0.3	0
0	0.39

$q_n = q^T U_k L_k =$

0.138	-0.0273

Query transformation

Map docs to the 2 dim space $V_k L_k =$

0.20	-0.06
0.61	0.17
0.46	-0.13
0.54	-0.23
0.28	0.11
0.00	0.19
0.01	0.44
0.02	0.62
0.08	0.53

3.34	0
0	2.54

=

0.67	-0.15
2.04	0.43
1.54	-0.33
1.80	-0.58
0.94	0.28
0.00	0.48
0.03	1.12
0.07	1.57
0.27	1.35

$q_n L_k =$

0.138	-0.0273

3.34	0
0	2.54

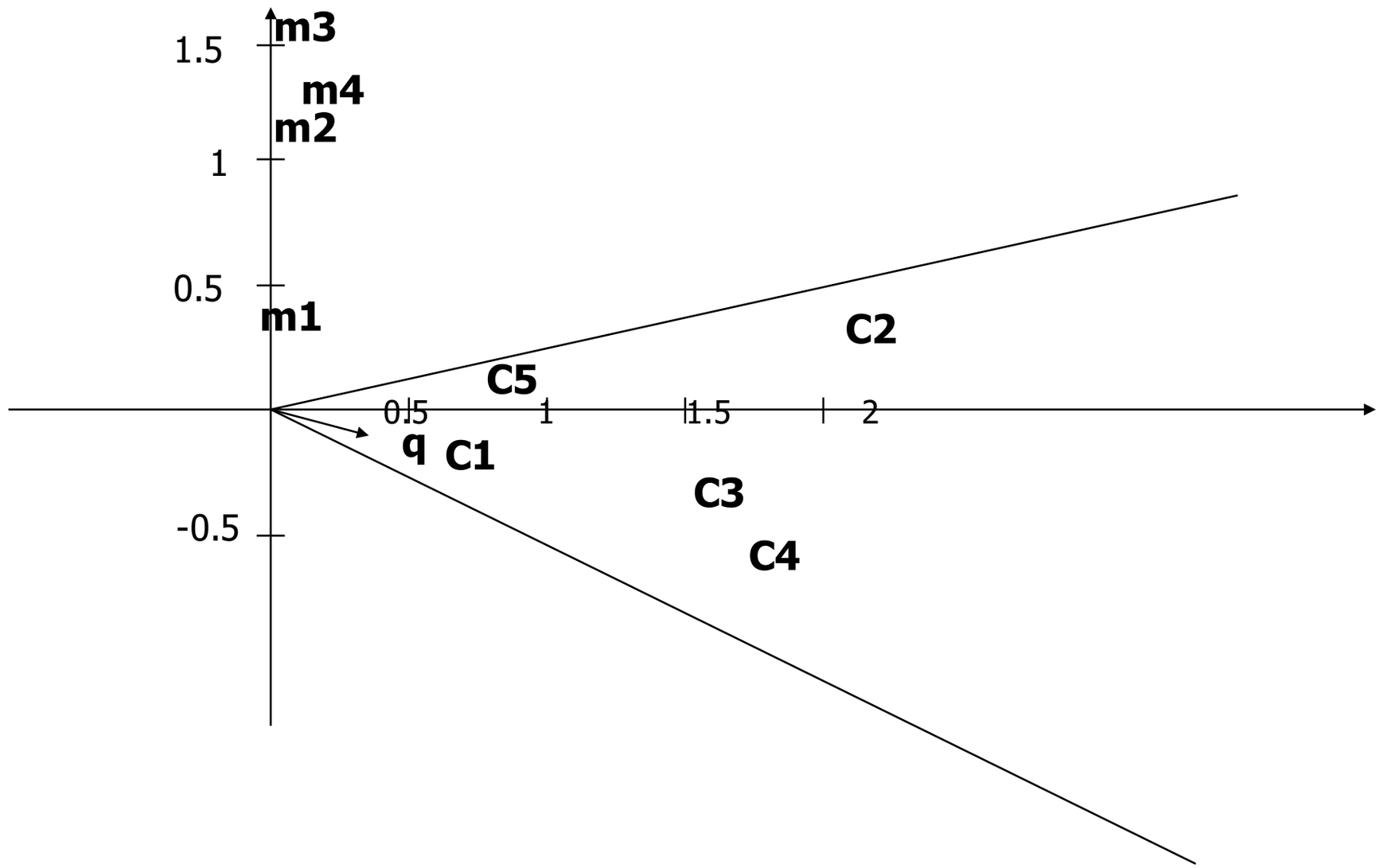
=

0.46	-0.069

Query transformation

- The cosine similarity matrix of query vector to the documents is:

	query
C1	0.99
C2	0.94
C3	0.99
C4	0.99
C5	0.90
M1	-0.14
M2	-0.13
M3	-0.11
M4	0.05



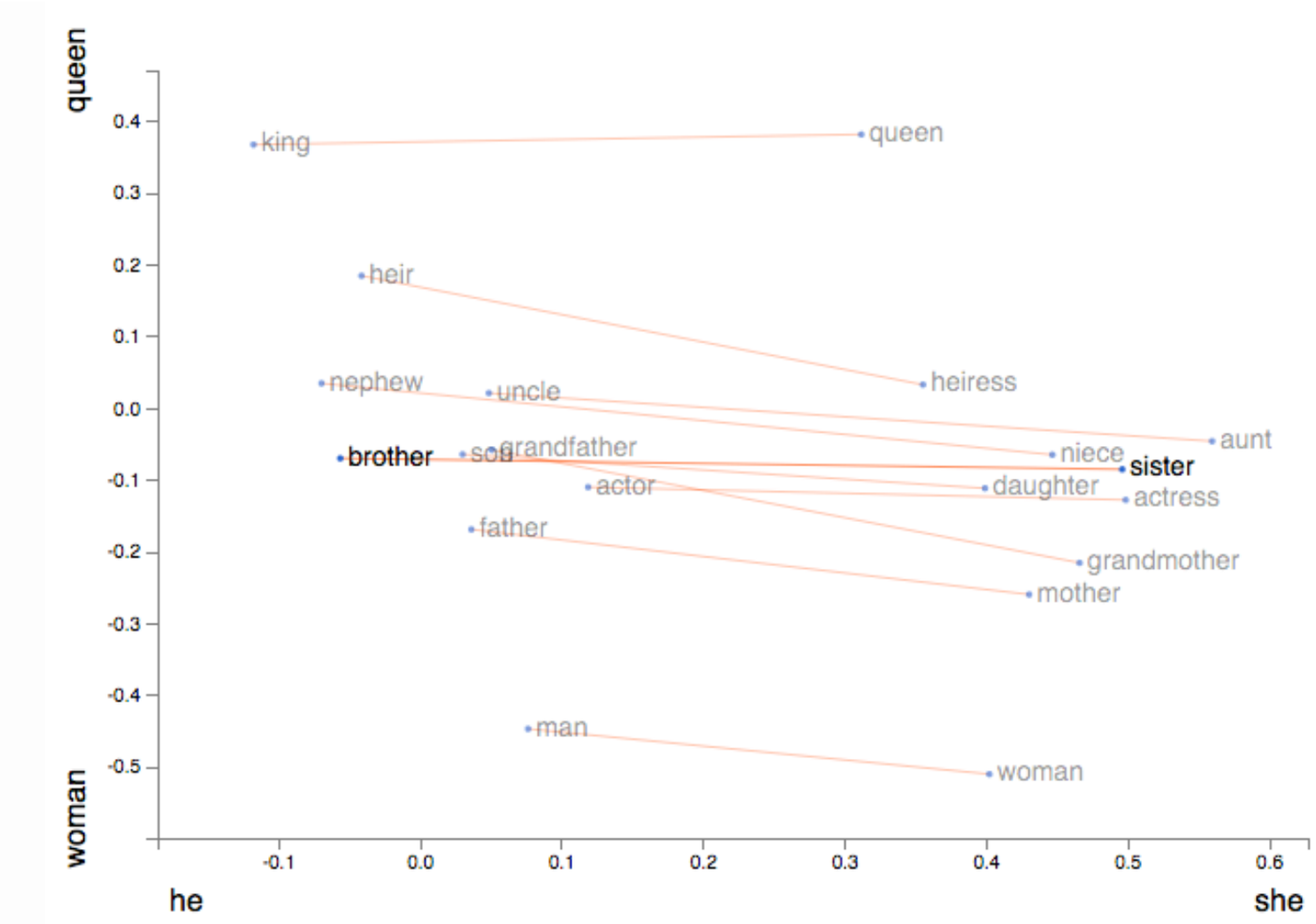
SVD problems

- The dimensions of the matrix change when dictionary changes
- The whole decomposition must be re-calculated when we add a word
- Sensitive to the imbalance of word frequency
- Very high dimensional matrix
- Not suitable for large corpora or dictionaries
- Quadratic cost to perform SVD
- Solution: Directly calculate a low-dimensional representation

Word analogy

- Words with similar meaning end up close to each other
- Words sharing similar contexts may be analogous
  - Synonyms
  - Antonyms
  - Names
  - Colors
  - Places
  - Interchangeable words

- Vector arithmetics to work with analogies
- i.e. **king - man + woman = queen**



https://lamyiowce.github.io/word2viz/

But why?

- what's an analogy?

$$\frac{p(w'|man)}{p(w'|woman)} \approx \frac{p(w'|king)}{p(w'|queen)}$$

Assume PMI is approximated by a low rank approximation of the co-occurrence matrix.

1. $PMI(w',w) \approx v_w v_{w'}$ *inner product*
2. Isotropic: $E_{w'}[(v_{w'} v_u)]^2 = ||v_u||^2$

Then

3. $argmin_w \, E_{w'}[ln \frac{p(w'|w)}{p(w'|queen)} - ln \frac{p(w'|man)}{p(w'|woman)}]^2$

4. $argmin_w \, E_{w'}[\big(PMI(w'|w) - PMI(w'|queen)\big) - (PMI(w'|man) - PMI(w'|woman))]^2$

5. $argmin_w ||(v_w - v_{queen}) - (v_{man} - v_{woman})||^2$
6. $v_w \approx v_{queen} - v_{woman} + v_{man}$ which is an analogy!

- *Arora et al (ACL 2016)* shows that if (2) holds then (1) holds as well
- So we need to construct vectors from co-occurrence that satisfy (2)
- d<<|V| in order to have isotropic vectors

A Latent Variable Model Approach to PMI-based Word Embeddings, *Arora et al (ACL 2016)*

Learning Word Vectors

- Corpus containing a sequence of T training words
- Objective: $f(w_t, \ldots, w_{t-n+1}) = \hat{P}(w_t \mid w_{t-n+1} \ldots w_{t-1})$
- Decomposed in two parts:

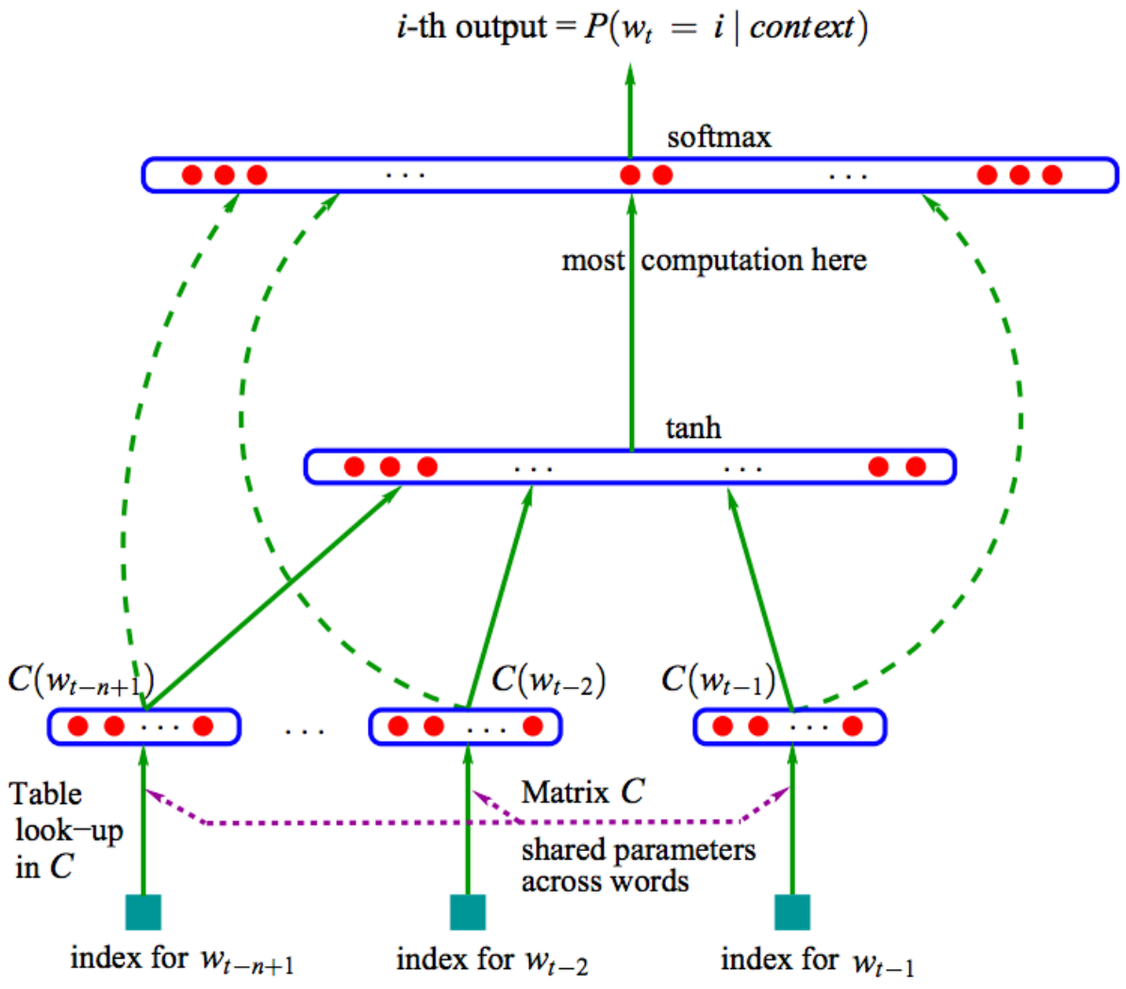
$$w_i \in V \xrightarrow{\text{mapping } C} \mathbb{R}^m$$

  - Mapping **C** (1-hotv => lower dimensions)
  - Mapping any **g** s.t. (estimate prob t+1| t previous)

$$f(w_{t-1}, \cdots, w_{t-n+1}) = g(C(w_{t-1}), \cdots, C(w_{t-n+1}))$$

- C(i) is the i-th word feature vector (Word embedding)

- Objective function: $J = \frac{1}{T}\sum f(w_t, \ldots, w_{t-n+1})$

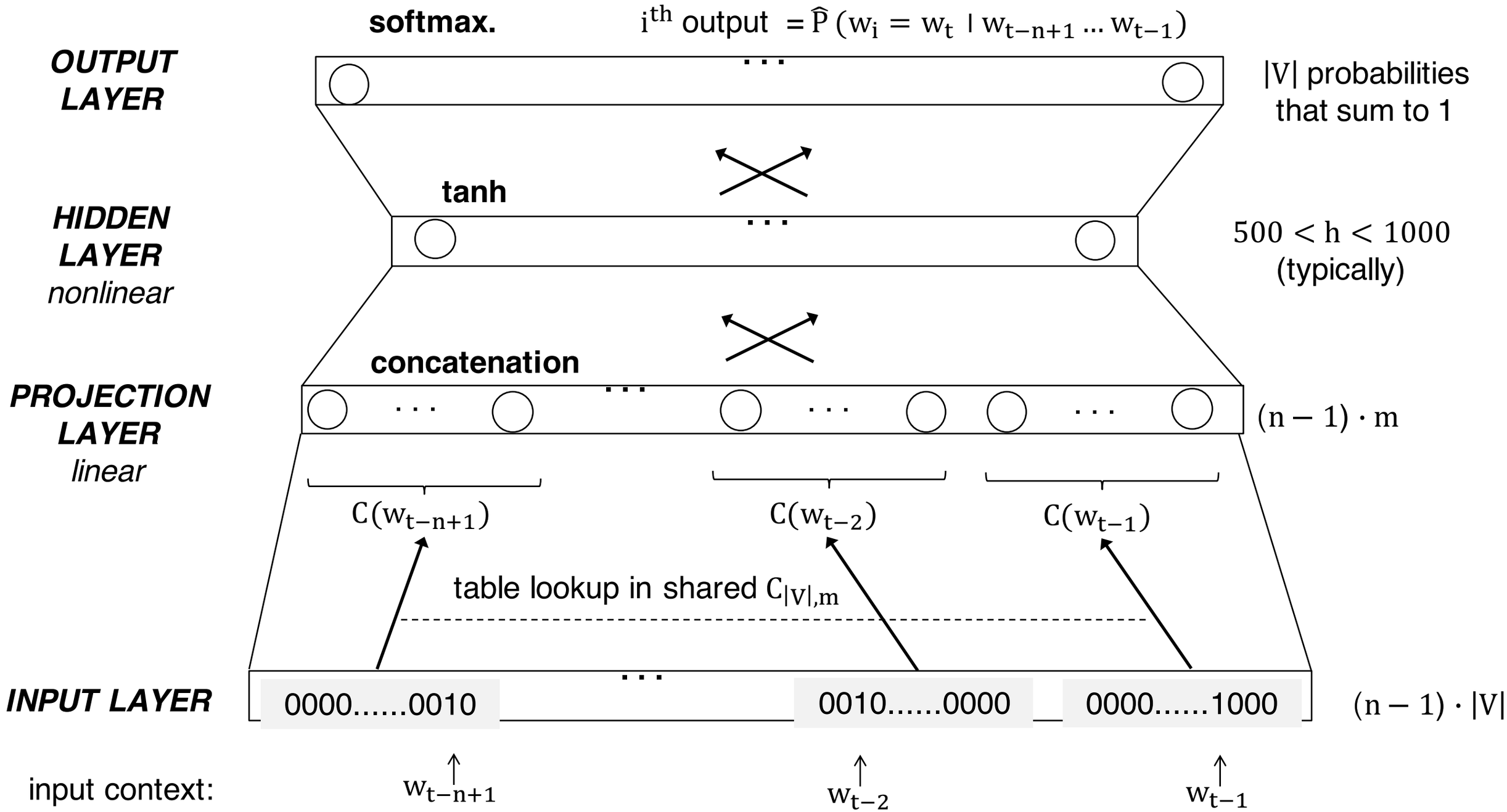


Bengio, Yoshua, et al. "A neural probabilistic language model." *The Journal of Machine Learning Research* 3 (2003): 1137-1155.

Neural Net Language Model

For each training sequence: input = (context, target) pair: $(w_{t-n+1} \ldots w_{t-1}, w_t)$

objective: minimize $E = -\log \hat{P}(w_t \mid w_{t-n+1} \ldots w_{t-1})$



Objective function

- $E = -\log \widehat{P}(w_t \mid w_{t-n+1} \cdots w_{t-1})$
- a probability between 0 and 1.
- On this support, the log is negative => –log term positive.
- makes sense to try to minimize it. ~

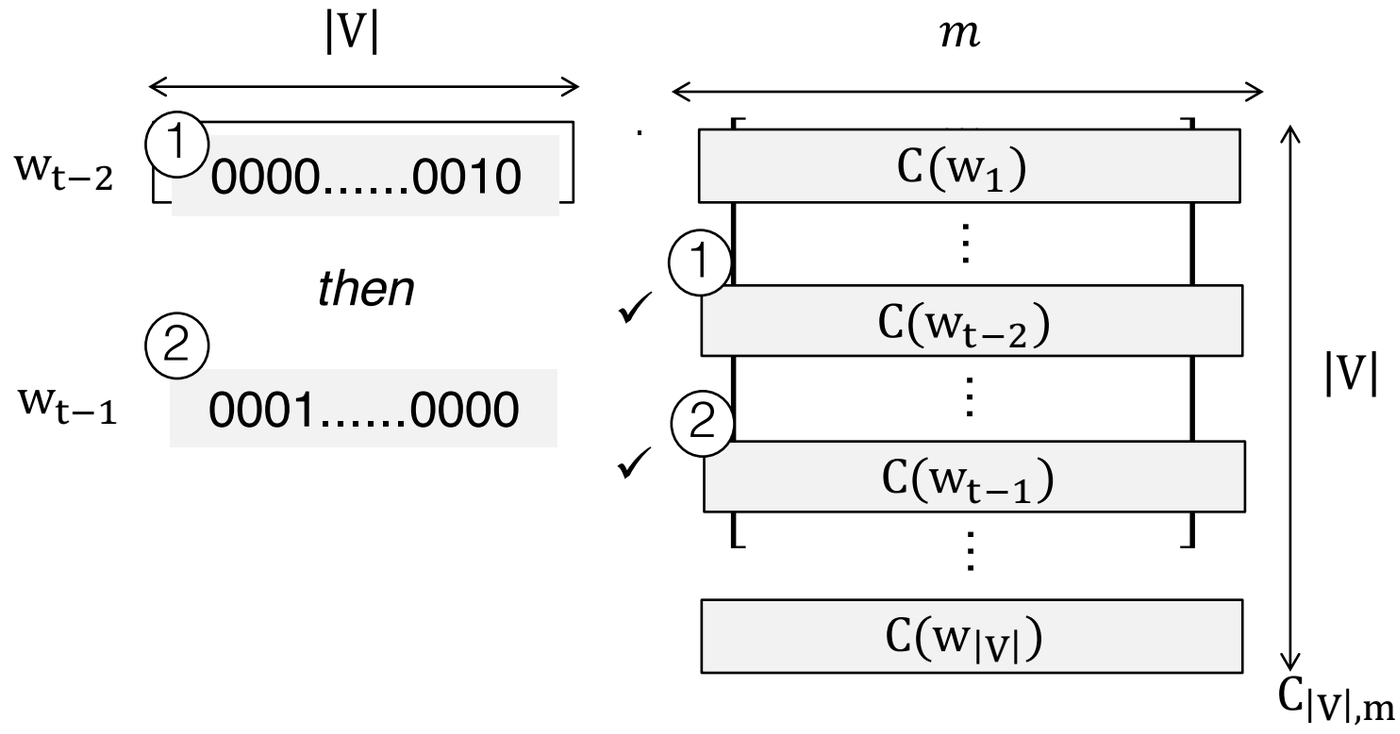
- Probability of word given the context be as high as possible (1 for a perfect prediction).
- case the error is equal to 0 (global minimum).

p	log(p)	-log(p)
0,7	-0,15490196	0,15490196
0,2	-0,698970004	0,698970004

NNLM Projection layer

- Performs a simple table lookup in $C_{|V|,m}$: concatenate the rows of the shared mapping matrix $C_{|V|,m}$ corresponding to the context words

Example for a two-word context $w_{t-2}w_{t-1}$:



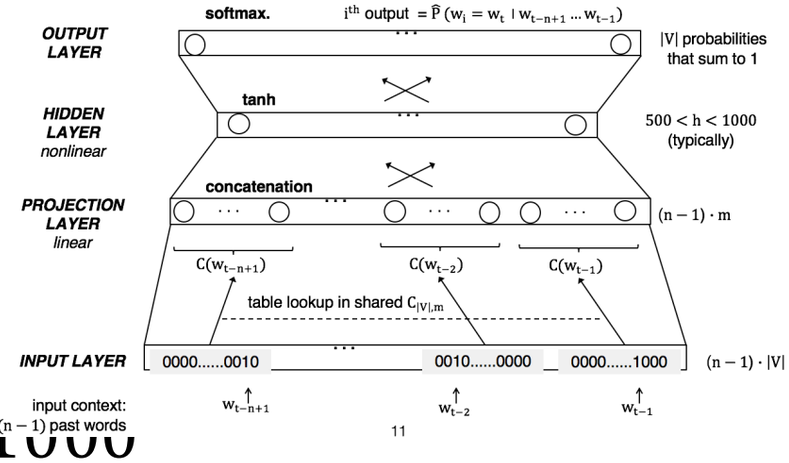
- $C_{|V|,m}$ is **critical**: contains weights tuned at each step. After training, it contains what we're interested in: the **word vectors**

NNLM hidden/output layers and training

- Softmax (log-linear classification model) used to output positive numbers summing to one (a multinomial probability distribution):
- i^{th} unit in the output layer: $\hat{\mathrm{P}}(\mathrm{w_i} = \mathrm{w_t} \mid \mathrm{w_{t-n+1}} \dots \mathrm{w_{t-1}}) = \frac{e^{yw_i}}{\sum_{i'=1}^{|V|} e^{yw'_i}}$

  where:
  - $y = b + U.tanh(d + H.x)$
  - $\tanh$: nonlinear squashing (link) function
  - x : concatenation $\mathrm{C(w)}$ of context weight vectors
  - b : output layer biases ($|\mathrm{V}|$ elements)
  - d : hidden layer biases (h elements), $500 < \mathrm{h} < 1000$
  - U : $|\mathrm{V}| * \mathrm{h}$ matrix storing the *hidden-to-output* weights
  - H : $(\mathrm{h} * (\mathrm{n}-1)\mathrm{m})$ *projection-to-hidden* weights

  $\rightarrow \boldsymbol{\theta} = (\boldsymbol{b}, \boldsymbol{d}, \boldsymbol{U}, \boldsymbol{H}, \boldsymbol{C})$



- Complexity per training sequence: $\mathrm{n} * \mathrm{m} + \mathrm{n} * \mathrm{m} * \mathrm{h} + \mathbf{h} * |\mathbf{V}|$
- **Training** via stochastic gradient descent (learning rate ε):

$$\theta \leftarrow \theta + \varepsilon \cdot \frac{\partial \mathrm{E}}{\partial \theta} = \theta + \varepsilon \cdot \frac{\partial \log \hat{\mathrm{P}}\left(\mathrm{w_t} \mid \mathrm{w_{t-n+1}} \dots \mathrm{w_{t-1}}\right)}{\partial \theta}$$

(weights initialized randomly, updated via backpropagation)

NNLM facts

- tested on Brown (1.2M words, $|V| \cong 16K$) and AP News (14M words, $|V| \cong 150K$ reduced to 18K) corpuses
- Brown: $h = 100$, $n = 5$, $m = 30$
- AP News: $h = 60$, $n = 6$, $m = 100$, **3 week** training using **40 cores**
- 24% and 8% relative improvement (resp.) over traditional smoothed n-gram LMs
- in terms of test *set perplexity*: geometric average

$$1/\hat{P}(w_t \mid w_{t-n+1} \dots w_{t-1})$$

- Due to **complexity**, NNLM can't be applied to large data sets → poor performance on rare words
- Bengio et al. (2003) initially thought their main contribution was a more accurate LM. They let the interpretation and use of the word vectors as **future work**

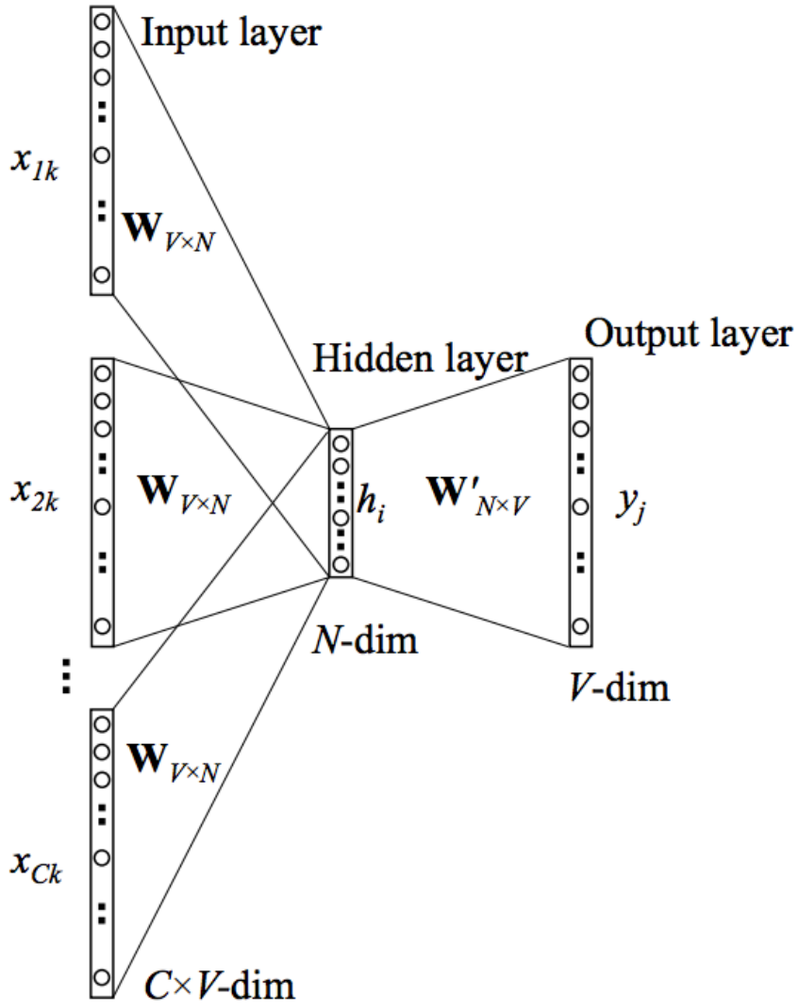
- On the opposite, Mikolov et al. (2013) focus on the **word vectors**

Word2Vec

- Mikolov et al. in 2013
- Word2vec key idea: achieve better performance not by using a more complex model (i.e., with more layers), but by allowing a **simpler (shallower) model** to be trained on **much larger amounts of data**
- no hidden layer (leads to 1000X speedup)
- projection layer is shared (not just the weight matrix) - C
- context: words from both history & future:
- Two algorithms for learning words vectors:
  - **CBOW**: from context predict target
  - **Skip-gram**: from target predict context

CBOW

- Continuous Bag-of-Words
- Continuous representations whose order is of no importance
- Uses the surrounding words to predict the center word
- n-words before and after the target word



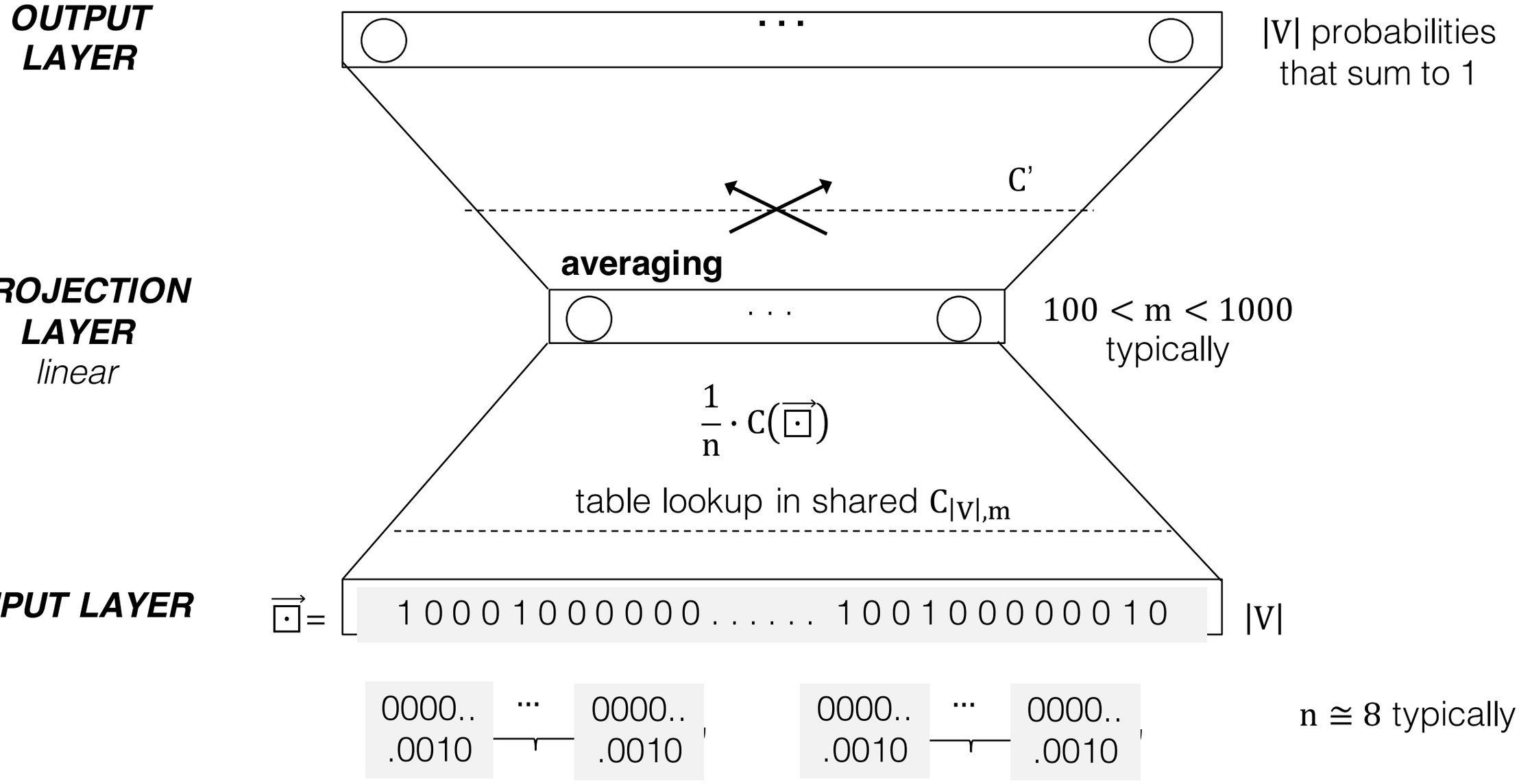
Efficient Estimation of Word Representations in Vector Space- Mikolov et al.

Continuous Bag-of-Words (CBOW)

For each training sequence: input = (context, target) pair: $(w_{t-\frac{n}{2}} \dots w_{t-1} w_{t+1} \dots w_{t+\frac{n}{2}}, w_t)$

objective: minimize $-\log \hat{P}(w_t \mid w_{t-n+1} \dots w_{t-1})$

hierarchical softmax. t^{th} output $= P(w_i = w_t \mid w_{t-n+1} \dots w_{t-1})$

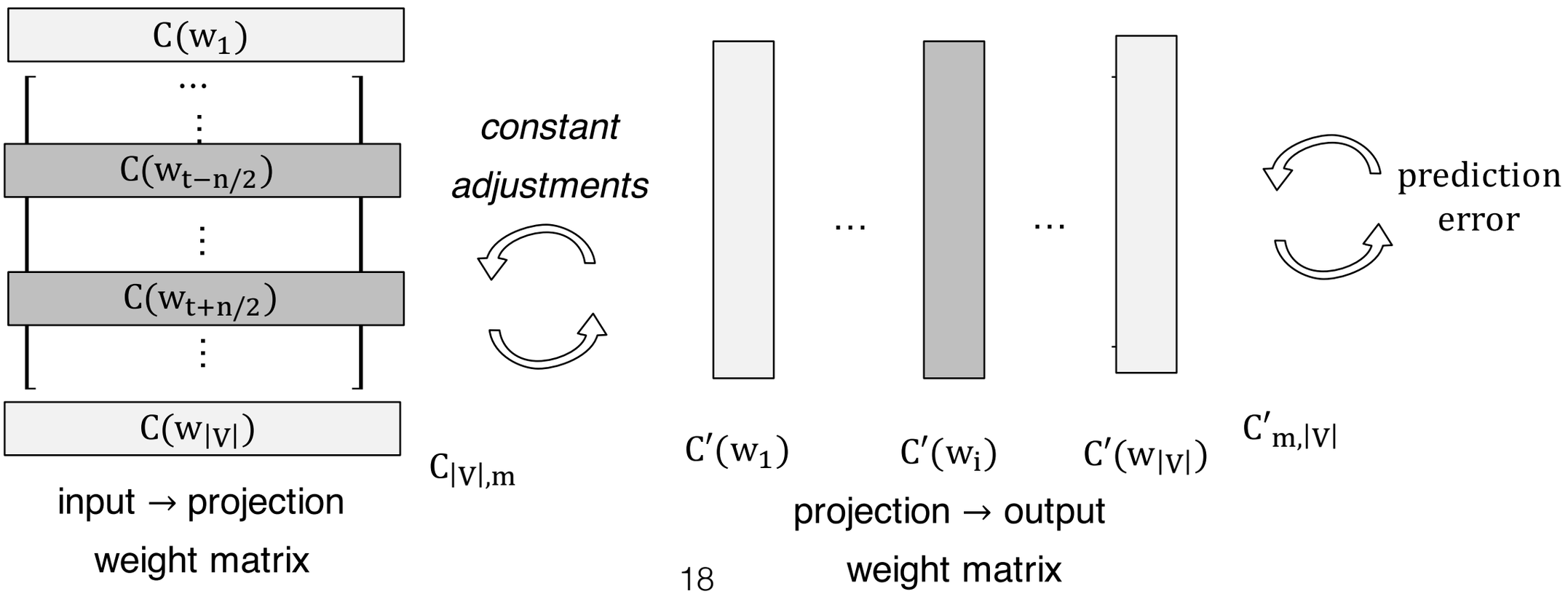


context: $n/2$ history words: $w_{t-\frac{n}{2}} \dots w_{t-1}$ $n/2$ future words: $w_{t+1} + \cdots + w_{t+\frac{n}{2}}$

Weight updating

- For each (context, target=$\mathbf{w_t}$) pair, only the word vectors from matrix $\mathbf{C}$ corresponding to the context words are updated
- Recall that we compute P $(\mathbf{w_i} = \mathbf{w_t} \mid \mathbf{context}) \; \forall \; \mathbf{w_i} \in \mathbf{V}$. We compare this distribution to the true probability distribution (1 for $\mathbf{w_t}$, 0 elsewhere)
- **Back propagation**
- If P $(\mathbf{w_i} = \mathbf{w_t} \mid \mathbf{context})$ is **overestimated** (i.e., > 0, happens in potentially $|\mathbf{V}| - 1$ cases), some portion of $\mathbf{C'(w_i)}$ is **subtracted** from the context word vectors in $\mathbf{C}$, proportionally to the magnitude of the error
- Reversely, if P $(\mathbf{w_i} = \mathbf{w_t} \mid \mathbf{context})$ is **underestimated** (< 1, happens in potentially 1 case), some portion of $\mathbf{C'(w_i)}$ is **added** to the context word vectors in $\mathbf{C}$

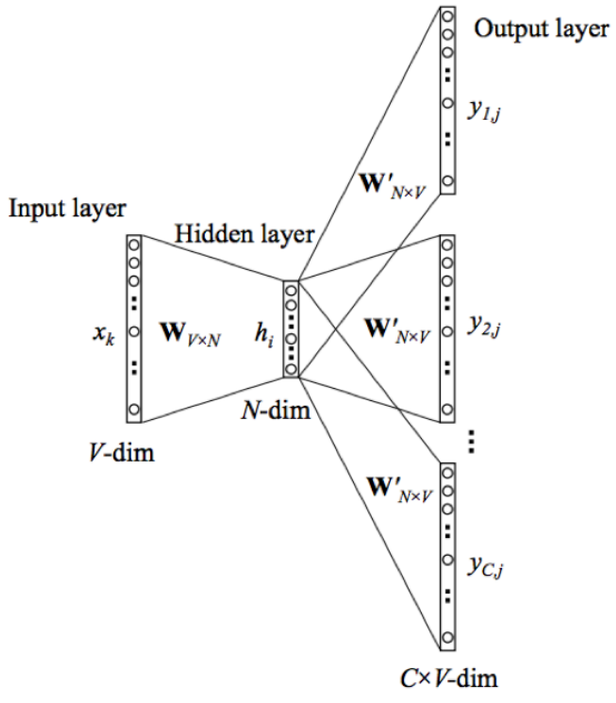
→ at each step the words move away or get closer to each other in the feature space → clustering



Skip-gram

- skip-gram uses the context's center word to predict the surrounding words

- instead of computing the probability of the target word w_t given its previous words, we calculate the probability of the surrounding word w_{t+j} given w_t



- $p(w_{t+j}|w_t) = \frac{\exp(v_{w_t}^T v'_{w_{t+j}})}{\sum_{w\in V}\exp(v_{w_t}^T v'_{w_{t+j}})}$

- $v^T{}_{wt}$ is a column of W_{VxN} and $v'_{w_{t+j}}$ is a column of W'_{NxV}

- Objective function $J = \frac{1}{T}\sum_{t=1}^{T}\sum_{-n\le j\le n}\log p(w_{t+j}|w_t)$

Efficient Estimation of Word Representations in Vector Space- Mikolov et al. 2013

word2vec facts

- Complexity is $\mathbf{n} * \mathbf{m} + \mathbf{m} * \mathbf{log}|\mathbf{V}|$ (Mikolov et al. 2013a)
- **n**:size of the context window (~10) **nxm**: dimensions of the projection layer, **IVI** size of the vocabulary
- On Google news 6B words training corpus, with $|\mathbf{V}| \sim 10^6$:
  - CBOW with $\mathbf{m} = 1000$ took **2 days** to train on **140 cores**
  - Skip-gram with $\mathbf{m} = 1000$ took **2.5 days** on **125 cores**
  - NNLM (Bengio et al. 2003) took **14 days** on **180 cores**, for $\mathbf{m} = 100$ only!
    (note that $\mathbf{m} = 1000$ was not reasonably feasible on such a large training set)
- word2vec training speed ≅ 100K-5M words/s

- Quality of the word vectors:
  - ↗ significantly with **amount of training data** and **dimension of the word vectors** (m), with diminishing relative improvements
  - measured in terms of accuracy on 20K semantic and syntactic association tasks.
    e.g., words in **bold** have to be returned:

Capital-Country	Past tense	Superlative	Male-Female	Opposite
Athens: **Greece**	walking: **walked**	easy: **easiest**	brother: **sister**	ethical: **unethical**

- Best NNLM: 12.3% overall accuracy. Word2vec (with Skip-gram): 53.3%

- References: http://www.scribd.com/doc/285890694/NIPS-DeepLearningWorkshop-NNforText#scribd
  https://code.google.com/p/word2vec/

GloVe

Probability and Ratio	$k = solid$	$k = gas$	$k = water$	$k = fashion$
$P(k\|ice)$	1.9×10^{-4}	6.6×10^{-5}	3.0×10^{-3}	1.7×10^{-5}
$P(k\|steam)$	2.2×10^{-5}	7.8×10^{-4}	2.2×10^{-3}	1.8×10^{-5}
$P(k\|ice)/P(k\|steam)$	8.9	8.5×10^{-2}	1.36	0.96

- Ratio of co-occurrence probabilities best distinguishes relevant words

$$F(w_i, w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}} \quad \Rightarrow \quad w_i^T \tilde{w}_k + b_i + \tilde{b}_k = \log(X_{ik})$$

- *Cast this into a lease square problem:*

- X co-occurrence matrix
- f weighting function,
- b bias terms
- $w_i = word\ vector$
- $\widetilde{w_j} = context\ vector$

$$J = \sum_{i,j=1}^{V} f\left(X_{ij}\right)\left(w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij}\right)^2$$

$$f(x) = \begin{cases} (x/x_{\max})^\alpha & \text{if } x < x_{\max} \\ 1 & \text{otherwise} \end{cases}.$$

model that utilizes
- count data
- bilinear prediction-based methods like word2vec

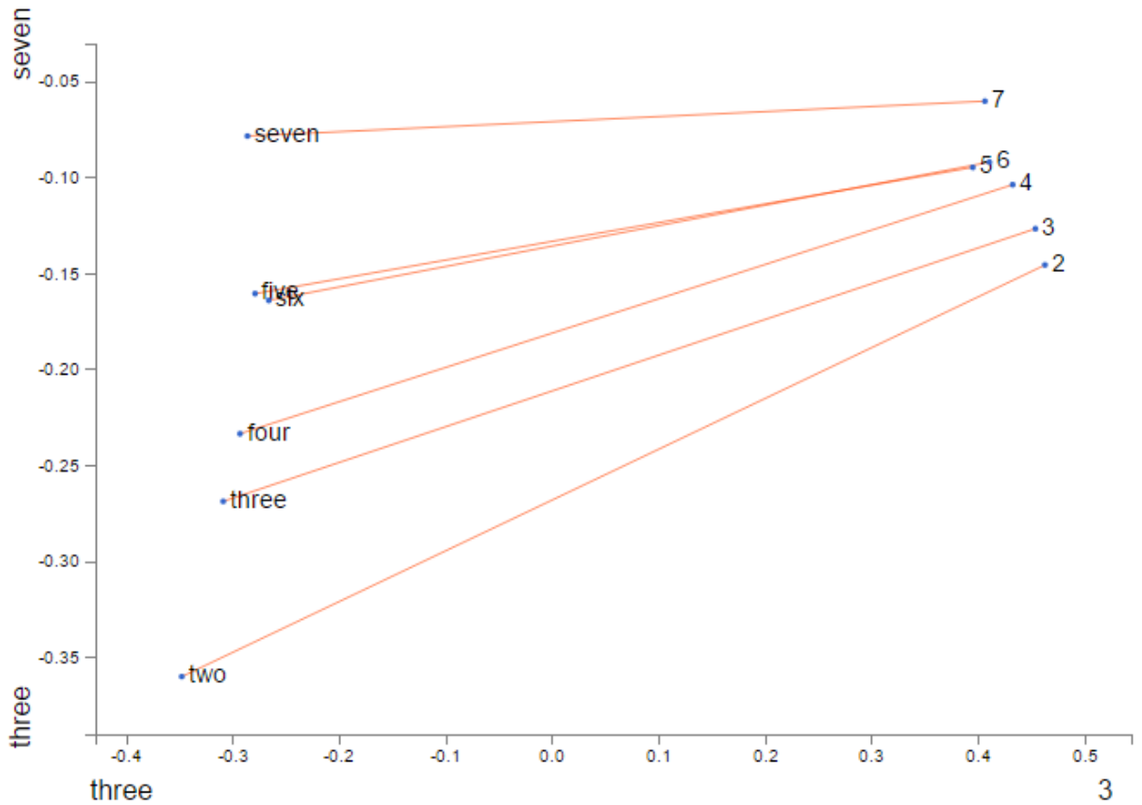
https://nlp.stanford.edu/pubs/glove.pdf

Which is better?

- Open question
- SVD vs word2vec vs GloVe
- All based on co-occurrence
- *Levy, O., Goldberg, Y., & Dagan, I. (2015)*
  - SVD performs best on similarity tasks
  - Word2vec performs best on analogy tasks
  - *No single algorithm consistently outperforms the other methods*
  - *Hyperparameter tuning is important*
  - 3 out of 6 cases, tuning hyperparameters is more beneficial than increasing corpus size
  - word2vec outperforms GloVe on all tasks
  - *CBOW is worse than skip-gram on all tasks*

Applications

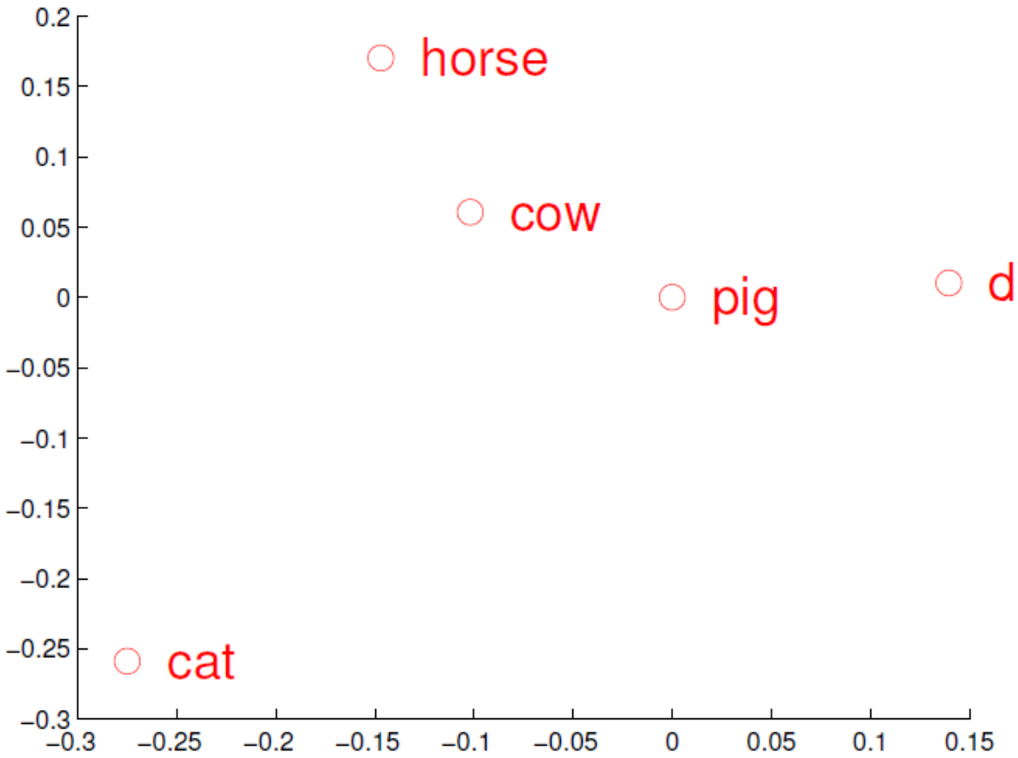
- Word analogies
- Find similar words
  - Semantic similarity
  - Syntactic similarity
- POS tagging
- Similar analogies for different languages
- Document classification

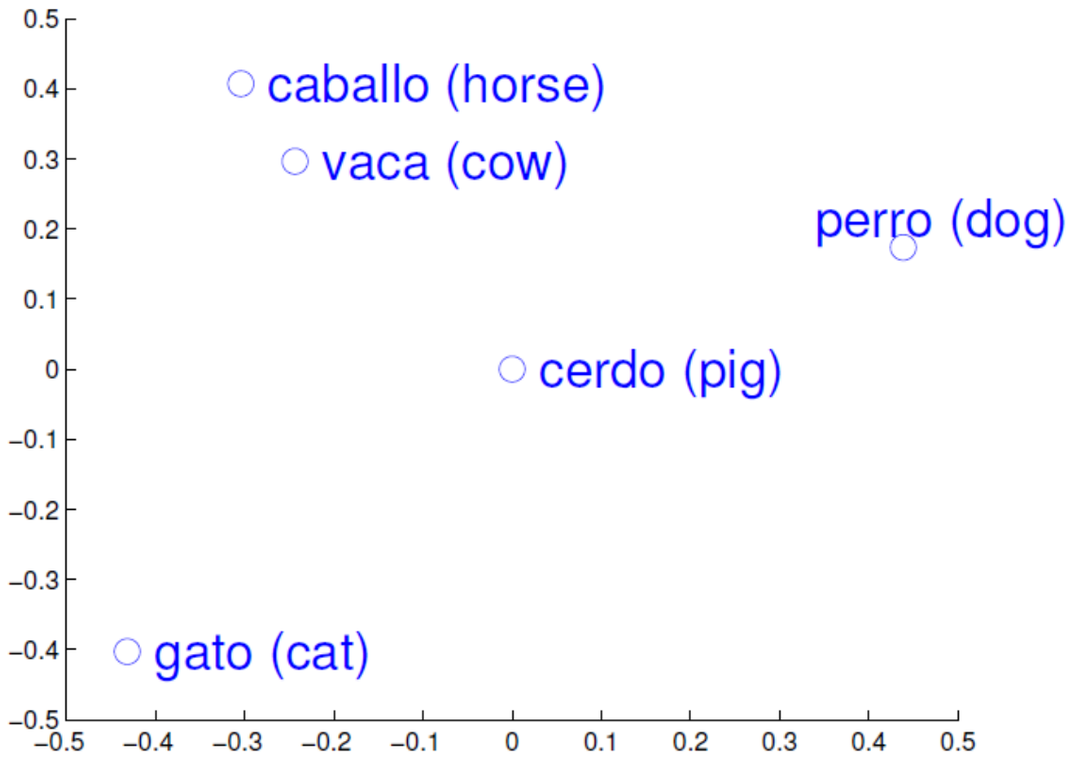


https://lamyiowce.github.io/word2viz/

Applications

- High quality word vectors boost performance of all NLP tasks, including document classification, machine translation, information retrieval…
- Example for English to Spanish machine translation:

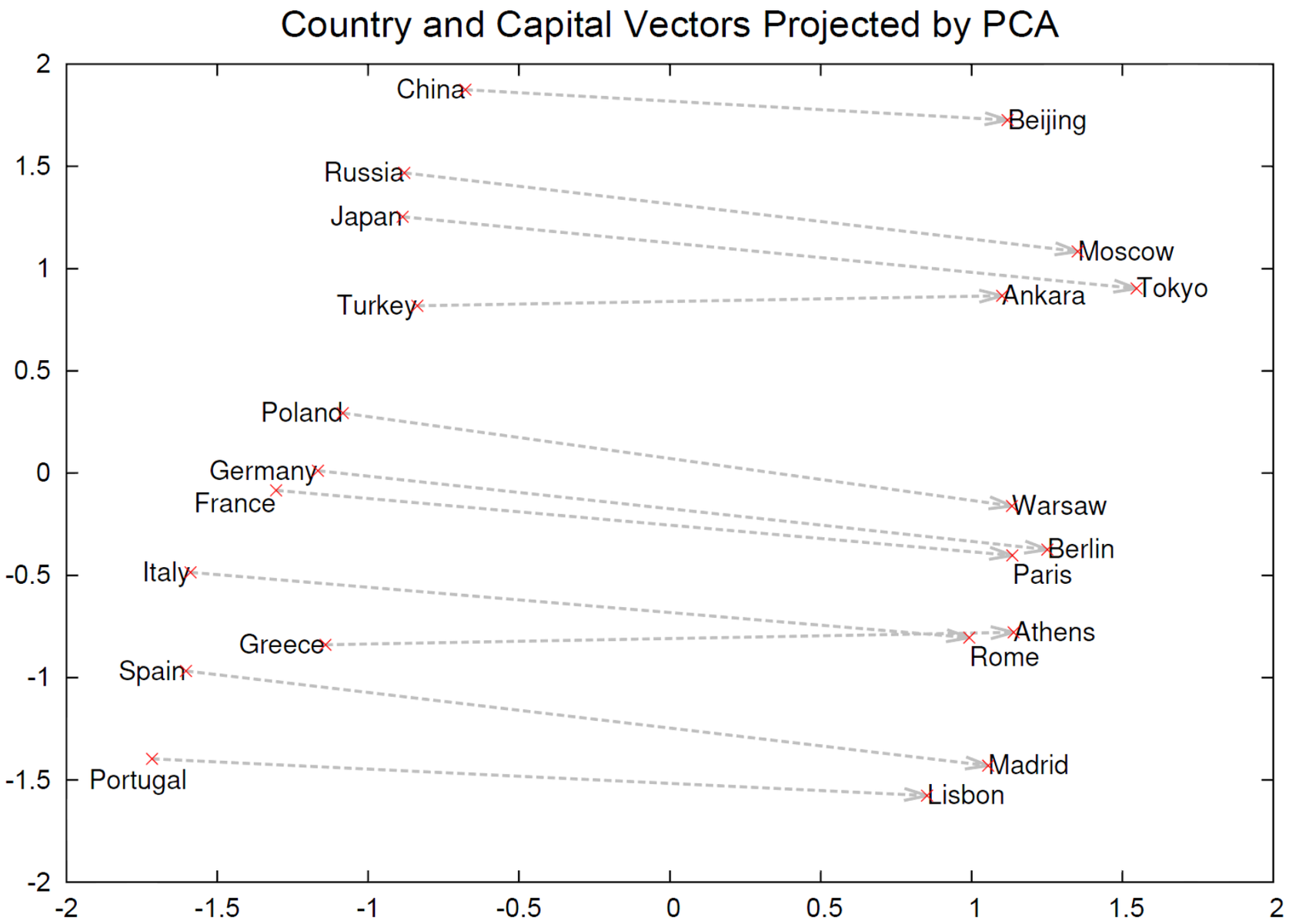




About 90% reported accuracy (Mikolov et al. 2013c)

Mikolov, T., Le, Q. V., & Sutskever, I. (2013). Exploiting similarities among languages for machine translation. *arXiv preprint arXiv:1309.4168.*

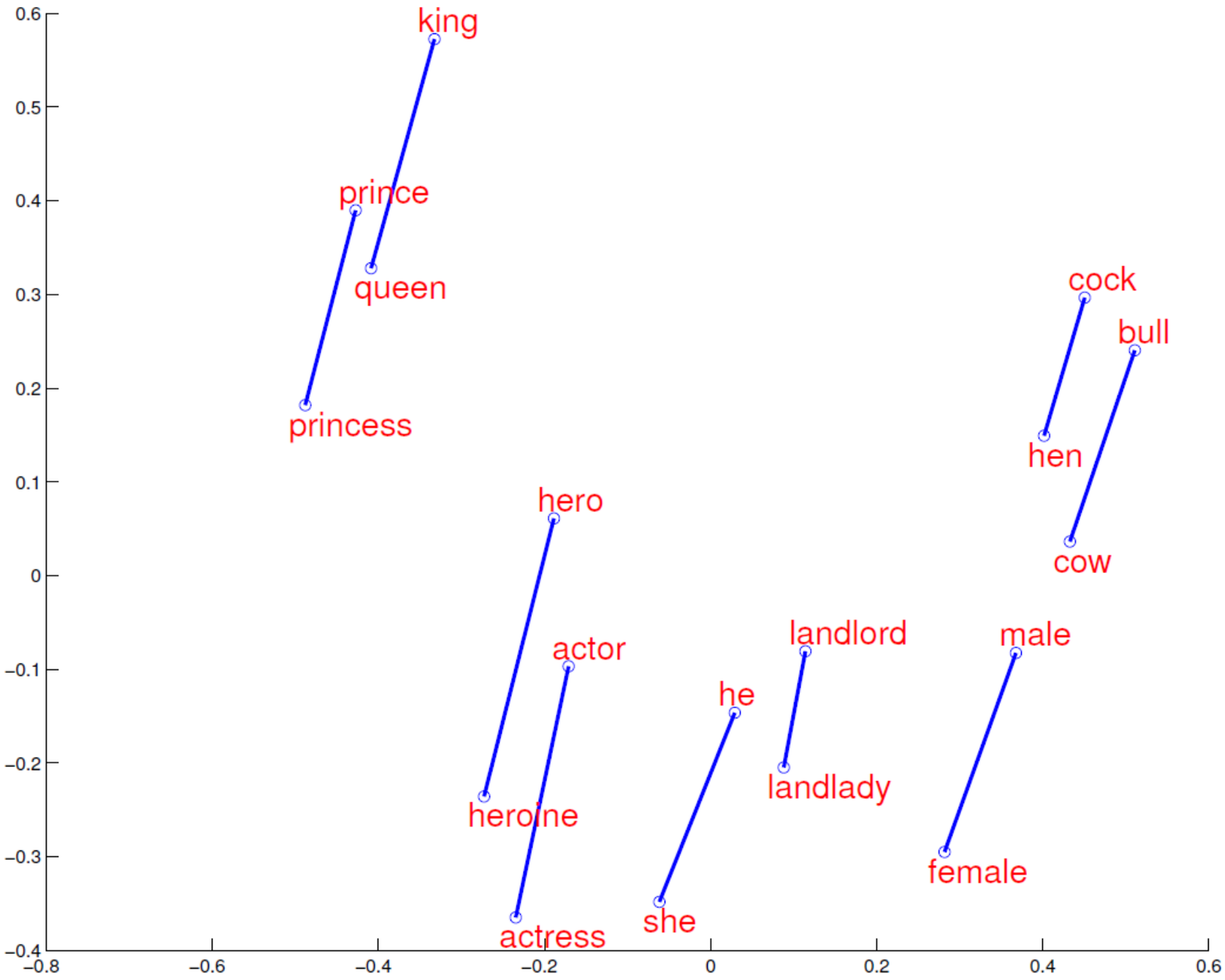
Remarkable properties of word vectors



regularities between words are encoded in the difference vectors
e.g., there is a constant **country-capital** difference vector

Mikolov et al. (2013b)
Distributed representations of words and phrases and their compositionality

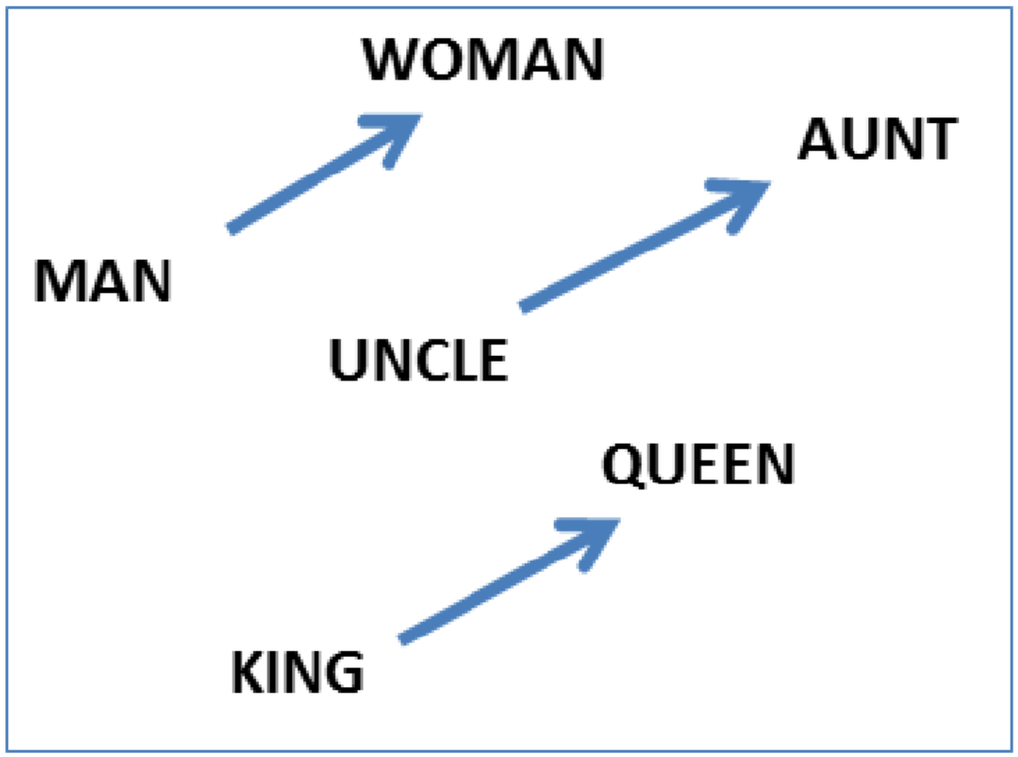
Remarkable properties of word vectors

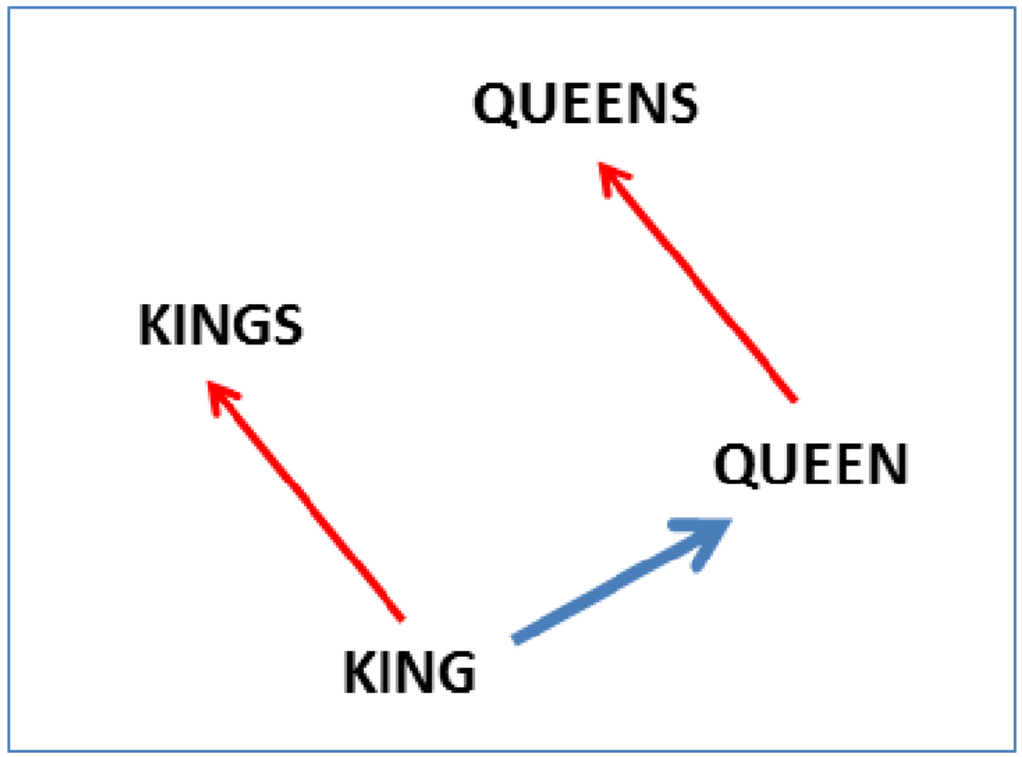


constant **female-male** difference vector

http://www.scribd.com/doc/285890694/NIPS-DeepLearningWorkshop-NNforText#scribd

Remarkable properties of word vectors





constant **male-female** difference vector

constant **singular-plural** difference vector

- Vector operations are supported and make intuitive sense:

$$w_{king} - w_{man} + w_{woman} \cong w_{queen}$$

$$w_{einstein} - w_{scientist} + w_{painter} \cong w_{picasso}$$

$$w_{paris} - w_{france} + w_{italy} \cong w_{rome}$$

$$w_{his} - w_{he} + w_{she} \cong w_{her}$$

$$w_{windows} - w_{microsoft} + w_{google} \cong w_{android}$$

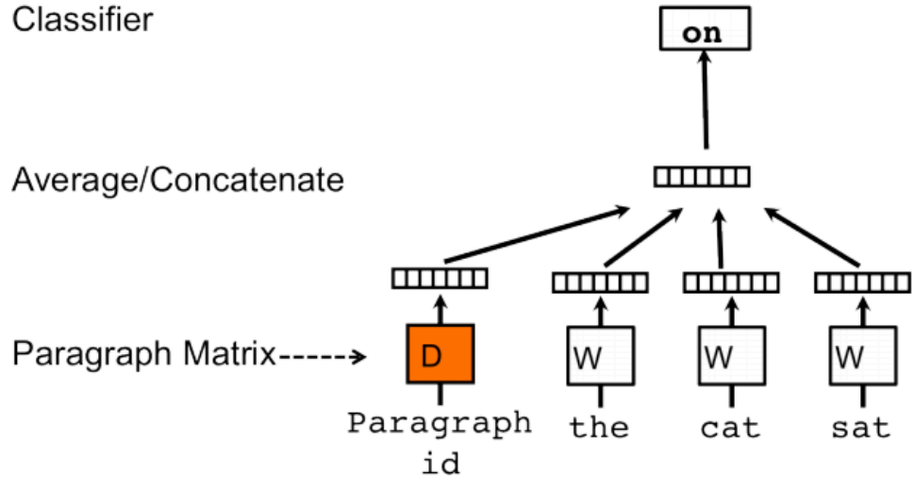
$$w_{cu} - w_{copper} + w_{gold} \cong w_{au}$$

- Online demo (scroll down to end of tutorial)

http://rare-technologies.com/word2vec-tutorial/

Distributed Representations of Sentences and Documents

- **Doc2vec**
- Paragraph or document vectors
- Capable of constructing representations of input sequences of variable length
- Represent each document by a dense vector
- Trained to predict words in the document
- paragraph vector and word vectors are averaged or concatenated to predict the next word in a context
- can be thought of as another word shared across all contexts in document

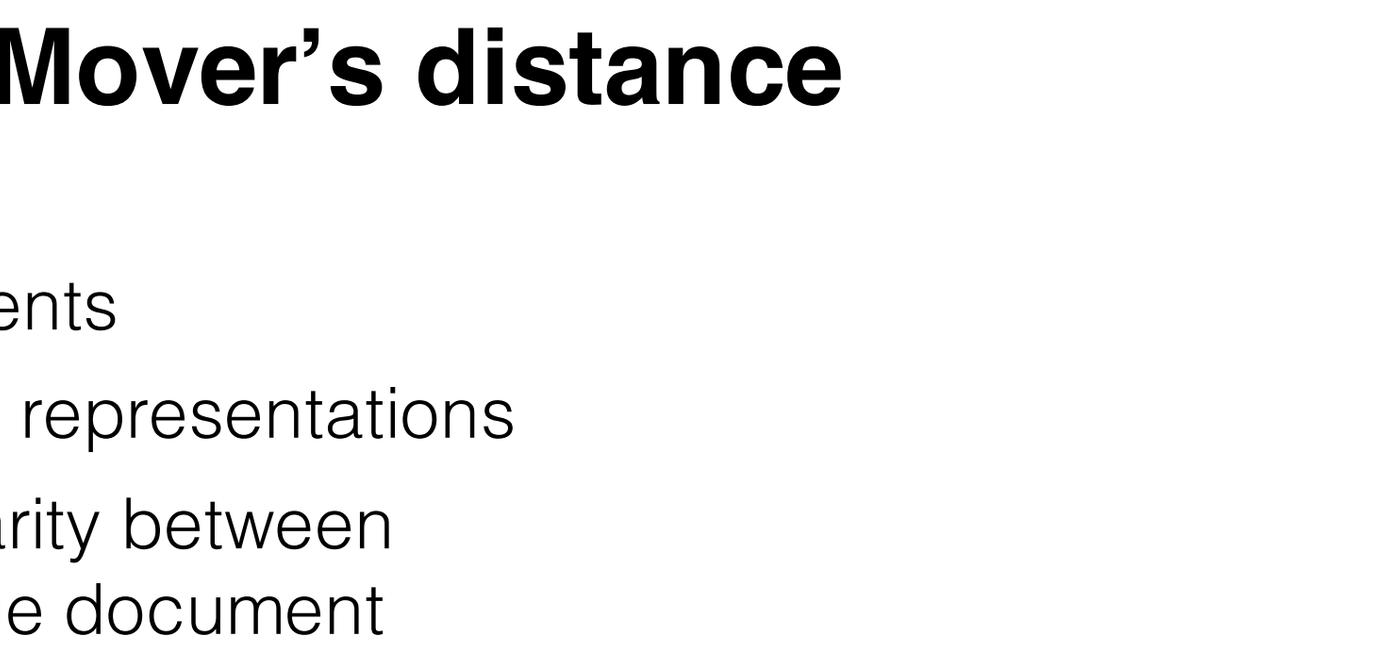


Model	Error rate (Positive/ Negative)	Error rate (Fine-grained)
Naïve Bayes (Socher et al., 2013b)	18.2 %	59.0%
SVMs (Socher et al., 2013b)	20.6%	59.3%
Bigram Naïve Bayes (Socher et al., 2013b)	16.9%	58.1%
Word Vector Averaging (Socher et al., 2013b)	19.9%	67.3%
Recursive Neural Network (Socher et al., 2013b)	17.6%	56.8%
Matrix Vector-RNN (Socher et al., 2013b)	17.1%	55.6%
Recursive Neural Tensor Network (Socher et al., 2013b)	14.6%	54.3%
Paragraph Vector	**12.2%**	**51.3%**

https://cs.stanford.edu/~quocle/**paragraph_vector**.pdf

Word Mover's distance

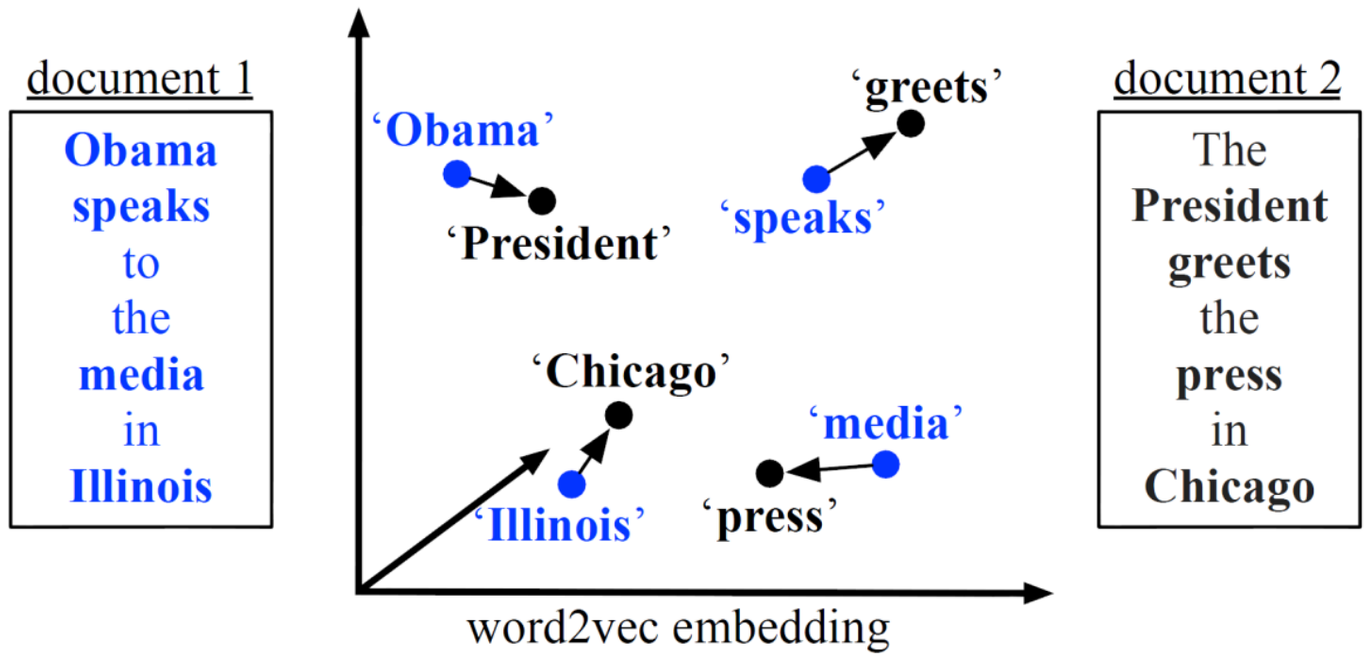
- "Edit" distance of 2 documents
- Based on word embedding representations
- Incorporate semantic similarity between individual word pairs into the document distance metric
- Based on "travel cost" between two words
- Calculates the cost of moving d to d'
- hyper-parameter free
- highly interpretable
- high retrieval accuracy

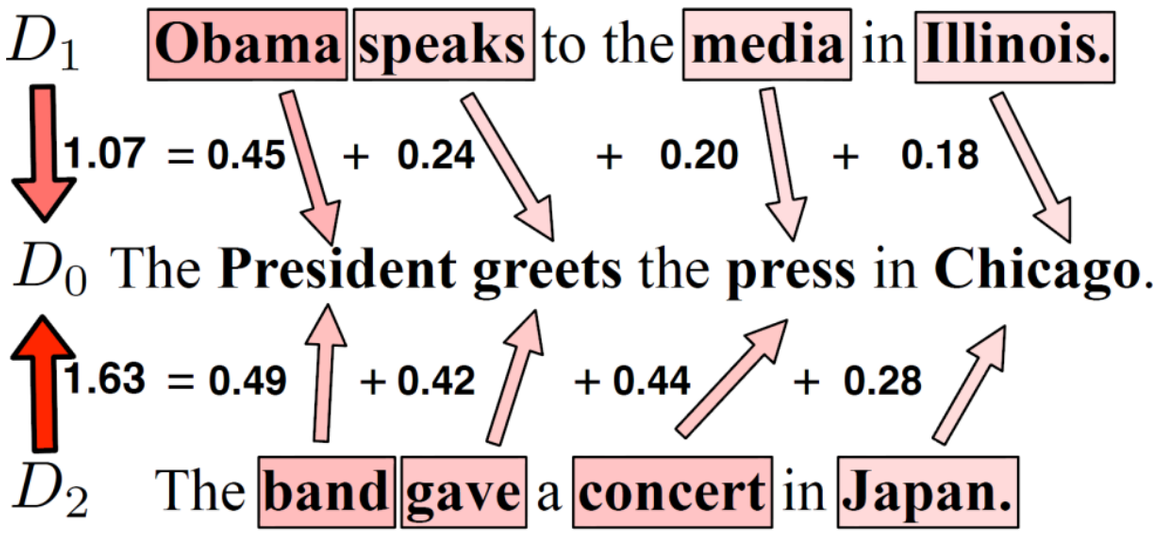


"minimum cumulative distance that all words in document 1 need to travel to exactly match document 2"

Word Mover's distance example

With BOW representation D_1 and D_2 are at equal distance from D_0. Word embeddings allow to capture the fact that D_1 is closer to D_0.





Kusner, M. J., Sun, E. Y., Kolkin, E. N. I., & EDU, W. From Word Embeddings To Document Distances. Proceedings of the 32nd International Conference on Machine Learning, Lille, France, 2015. JMLR: W&CP volume 37.

WMD

$d_i = \frac{c_i}{\sum_{j=1}^{n} c_j}$ Normalized frequency of word ***i***

$c(i,j) = \|\mathbf{x}_i - \mathbf{x}_j\|_2$ *word embeddings distance among* ***i,j***

- *Assume documents* ***d, d'.***
- *Assume each word* ***i*** *from* ***d*** *can be transformed into any word* ***j*** *in* ***d'***
- ***Tij ≥ 0*** denotes how much of word ***i*** in ***d*** travels to word ***j*** in ***d' .***
- To transform ***d*** entirely into ***d' :*** entire outgoing flow from word ***i*** equals d_i : .
- Transportation problem:

$$\min_{\mathbf{T} \geq 0} \sum_{i,j=1}^{n} \mathbf{T}_{ij} c(i,j) \qquad \sum_j \mathbf{T}_{ij} = d_i.$$

$$\text{subject to: } \sum_{j=1}^{n} \mathbf{T}_{ij} = d_i \quad \forall i \in \{1,\dots,n\} \qquad \sum_i \mathbf{T}_{ij} = d'_j$$

$$\sum_{i=1}^{n} \mathbf{T}_{ij} = d'_j \quad \forall j \in \{1,\dots,n\}.$$

- *Learn parameters* $\boldsymbol{T_{ij}}$ *then the distance is:* $\sum_{i,j=1}^{n} \mathbf{T}_{ij} c(i,j)$

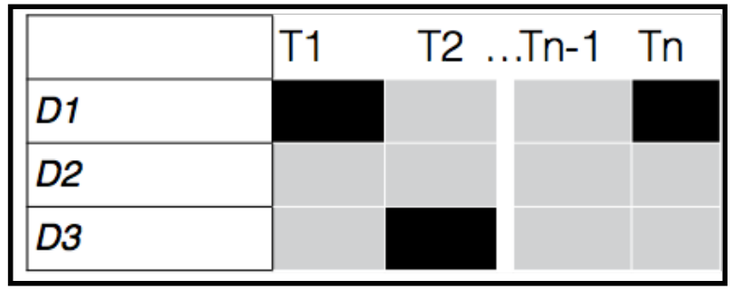
Gaussian Document Representation from Word Embeddings

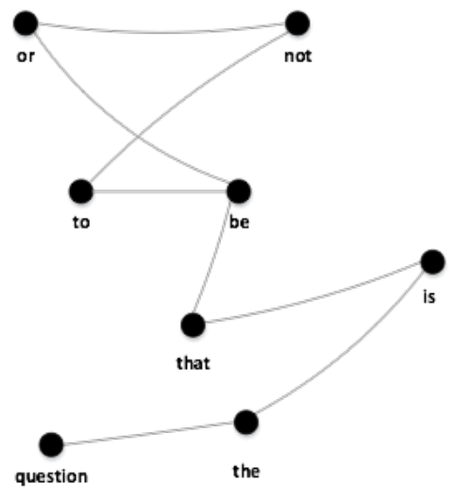
DASCIM

EACL 2017

Document representation

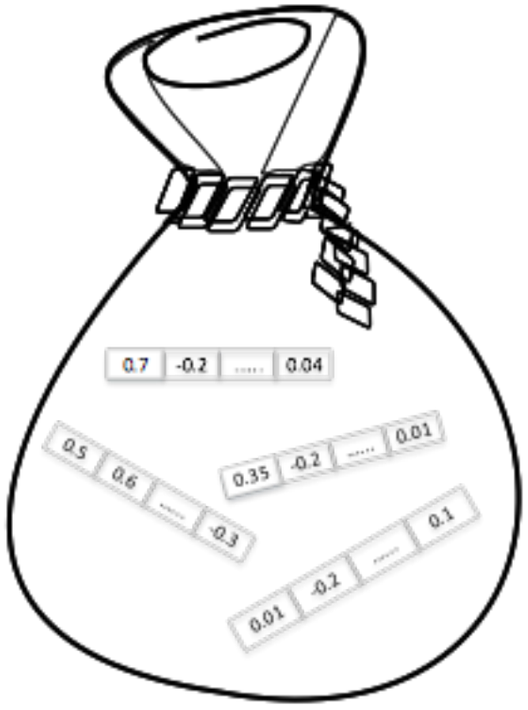
- Bag-of-Words
  - Document-term matrix
  - high dimensionality
  - sparse vectors
  - dimensions=|V| |V|>10^6
  - unable to capture semantic similarity
- Graph-of-Words
  - captures co-occurrence
  - graph algorithms and techniques
  - graph kernels





Bag-of-Vectors

- Distributed representations of words
  - store contextual information in a low-dimensional vector
  - dimensionality reduction
  - dimensions=m 100<m<500
  - able to capture semantic similarity between words
  - many learning methods (word2vec, GloVe, SVD)
- Document is represented by a bag-of-vectors
- Goal: meaningful document representations and distance metrics based on representations of their words



Related work

- Centroid of vectors *[Lebret and Collobert, 2015]*
- Paragraph Vector *[Mikolov, 2014]*
  - vector representations for paragraphs by inserting an additional memory vector in the input layer.
- Word Mover's Distance *[Kusner, 2015]*
  - cumulative edit distance between two documents
- CNN for document classification *[Kim, 2014]*
  - Use the high quality embeddings as input for Convolutional Neural Network

Gaussian Document Representation from Word Embeddings

- Assumption that words w present in a document are i.i.d. samples drawn from a multivariate Gaussian distribution
- Document is represented as a multivariate Gaussian distribution
  - mean vector $$\boldsymbol{\mu} = \frac{1}{|d|} \sum_{w \in d} \mathbf{w}$$
  - covariance matrix $$\boldsymbol{\Sigma} = \frac{1}{|d|} \sum_{w \in d} (\mathbf{w} - \boldsymbol{\mu})(\mathbf{w} - \boldsymbol{\mu})^T$$
- Words contained in the vocabulary, but not contained in the embeddings model are randomly initialized

Document Similarity

- centroid similarity $$sim(\boldsymbol{\mu}_1, \boldsymbol{\mu}_2) = \frac{\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2}{\|\boldsymbol{\mu}_1\| \|\boldsymbol{\mu}_2\|}$$
- covariance similarity $$sim(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2) = \frac{\sum \boldsymbol{\Sigma}_1 \circ \boldsymbol{\Sigma}_2}{\|\boldsymbol{\Sigma}_1\|_F \times \|\boldsymbol{\Sigma}_2\|_F}$$
- $(\cdot \circ \cdot)$ is the Hadamard or element-wise product

- similarity between two documents

$$sim(d_1, d_2) = \alpha\big(sim(\boldsymbol{\mu}_1, \boldsymbol{\mu}_2)\big) + (1 - \alpha)\big(sim(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2)\big) \quad \alpha \in [0, 1]$$

- valid kernel function

Why Gaussian?

- Each document follows a distribution described by its topic
- Word embeddings capture lexico-semantic regularities
- Words with similar syntactic and semantic properties are found to be close to each other in the embedding space
- Semantically related words are localized in space
- Gaussian distributions capture a notion of centrality in space
- Gaussian parameterization justified
  - analytic convenience
  - Euclidean distances between embeddings correlate with semantic similarity

Experiments

Datasets

Dataset	# training examples	# test examples	# classes	vocabulary size	*word2vec* size
Reuters	5,485	2,189	8	23,585	15,587
Amazon	8,000	CV	4	39,133	30,526
TREC	5,452	500	6	9,513	9,048
Snippets	10,060	2,280	8	29,276	17,067
BBCSport	348	389	5	14,340	13,390
Polarity	10,662	CV	2	18,777	16,416
Subjectivity	10,000	CV	2	21,335	17,896
Twitter	3,115	CV	3	6,266	4,460

Baselines

- BOW-SVM
- NBSVM
- Centroid-SVM
- WMD-KNN
- CNN

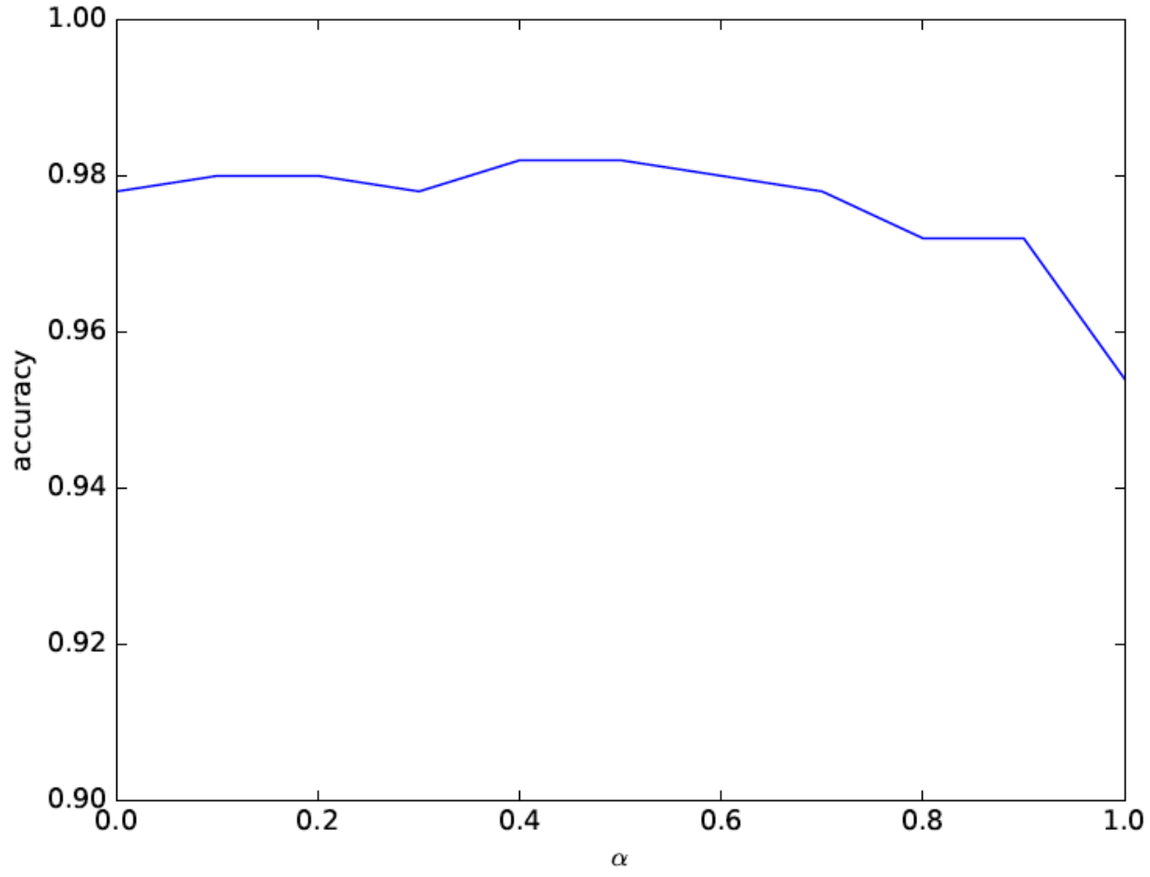
Results

Dataset / Method	Reuters		Amazon		TREC		Snippets	
	Accuracy	**F1-score**	**Accuracy**	**F1-score**	**Accuracy**	**F1-score**	**Accuracy**	**F1-score**
BOW (binary)	0.9571	0.8860	0.9126	0.9127	0.9660	0.9692	0.6171	0.5953
Centroid	0.9676	0.9171	0.9311	0.9312	0.9540	0.9586	0.8123	0.8170
WMD	0.9502	0.8204	0.9200	0.9201	0.9240	0.9336	0.7417	0.7388
NBSVM	**0.9712**	0.9155	0.9486	0.9486	0.9780	0.9805	0.6474	0.6357
CNN	0.9707	0.9297	0.9448	0.9449	0.9800	0.9800	**0.8478**	**0.8466**
Gaussian	**0.9712**	**0.9388**	**0.9498**	**0.9497**	**0.9820**	**0.9841**	0.8224	0.8244

Dataset / Method	BBCSport		Polarity		Subjectivity		Twitter	
	Accuracy	**F1-score**	**Accuracy**	**F1-score**	**Accuracy**	**F1-score**	**Accuracy**	**F1-score**
BOW (binary)	0.9640	0.9690	0.7615	0.7614	0.9004	0.9004	0.7467	0.6205
Centroid	0.9923	0.9915	0.7783	0.7782	0.9100	0.9100	0.7361	0.5727
WMD	0.9871	0.9866	0.6642	0.6639	0.8604	0.8603	0.7031	0.4436
NBSVM	0.9871	0.9892	**0.8698**	**0.8698**	**0.9369**	**0.9368**	**0.7852**	0.6191
CNN	0.9486	0.9461	0.8037	0.8031	0.9315	0.9314	0.7549	0.6137
Gaussian	**0.9974**	**0.9974**	0.8021	0.8020	0.9310	0.9310	0.7534	**0.6443**

Results

- Parameter a sensitivity
- TREC dataset
- Centroid performance drops significantly
- Highest accuracy a=0.5

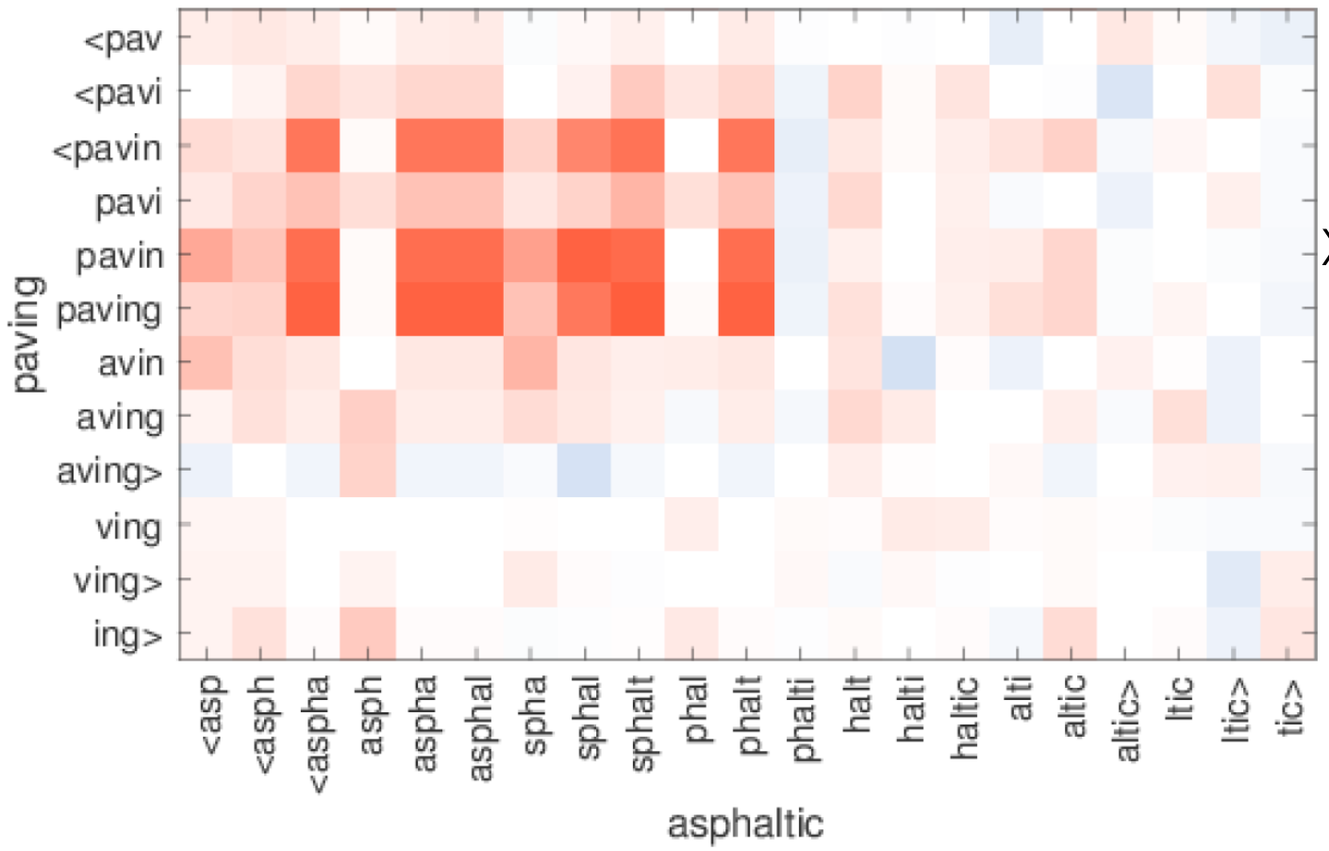


Conclusion

- Model each document as a Gaussian distribution based on the embeddings of its words
- Similarity between two documents based on the similarity of their distributions
- Empirical evaluation demonstrates the effectiveness of the approach across a range of data
- Performance gain is attributed to the high quality of the embeddings and the ability to effectively utilize them

Enriching Word Vectors with Subword Information

FAIR 2017, 860 cites



X'11-MVA'12

Piotr Bojanowski

Facebook AI Research
Verified email at fb.com - Homepage

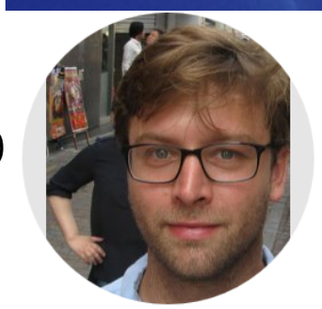
Computer Vision Machine Learning

X'9-MVA'10

Edouard Grave

Research Scientist

X'8-MVA'9



Armand Joulin

Research scientist at Facebook
Verified email at fb.com - Homepage

Artificial Intelligence Machine Learning

Tomas Mikolov

Research scientist, Facebook
Verified email at fb.com

Artificial Intelligence Machine Learning

Enriching Word Vectors with Subword Information

Simple problem: word2vec/glove etc. ignore the internal structure of words

E.g., knowledge about *luck* is not used when learning a representation for *unlucky or luckily*

=> parameters are not shared => difficult to learn good vectors for rare words, and impossible for out-of-vocabulary words

Simple solution: learn vectors for *character n-grams*. Compose word vectors from their n-gram vectors.

Enriching Word Vectors with Subword Information

Quick recap: in skip-gram (Mikolov et al. 2013), the objective is to maximize:

$$\sum_{t=1}^{T} \sum_{c \in \mathcal{C}_t} \log p(w_c \mid w_t)$$

$w_1, \ldots, w_T$ is the training corpus

C_t is the set of indices of the words around w_t

In English: the objective is to *predict well the context of a word given this word*

$p(w_c \mid w_t)$ parameterized by word vectors through a scoring function **s**

$$s(w_t, w_c) = \mathbf{u}_{w_t}^{\top} \mathbf{v}_{w_c}$$

u and **v** taken from the input and output embedding matrices, resp.

Enriching Word Vectors with Subword Information

Proposed approach:
Each word is represented as a bag of **character n-grams**. E.g., for the word *where* and n=3:
<wh, whe, her, ere, re>

The < and > characters are added at the beginning and end of the word to keep prefix/suffix information.

New scoring function: $$s(w, c) = \sum_{g \in \mathcal{G}_w} \mathbf{z}_g^\top \mathbf{v}_c$$

G_w set of character n-grams in word w. z_g vector of the g^{th} n-gram
V_c vector of the context word c

=> w is represented as the sum of its n-gram vectors

Enriching Word Vectors with Subword Information

Quantitative results: word similarity and word analogy tasks in 7 languages

- similarity: better than original skipgram and CBOW on 6/7 datasets
- analogy: - improves on original skipgram and CBOW for syntactic tasks
  - no improvement for semantic tasks

Qualitative results:

query	tiling	tech-rich	english-born	micromanaging	eateries	dendritic
`sisg`	tile flooring	tech-dominated tech-heavy	british-born polish-born	micromanage micromanaged	restaurants eaterie	dendrite dendrites
`sg`	bookcases built-ins	technology-heavy .ixic	most-capped ex-scotland	defang internalise	restaurants delis	epithelial p53

Nearest neighbors of rare words using subword (sisg) and original (sg) skipgram

Enriching Word Vectors with Subword Information

	anarchy	chy	<anar	narchy
	monarchy	monarc	chy	<monar
	kindness	ness>	ness	kind
	politeness	polite	ness>	eness>
EN	unlucky	<un	cky>	nlucky
	lifetime	life	<life	time
	starfish	fish	fish>	star
	submarine	marine	sub	marin
	transform	trans	<trans	form
	finirais	ais>	nir	fini
FR	finissent	ent>	finiss	<finis
	finissions	ions>	finiss	sions>

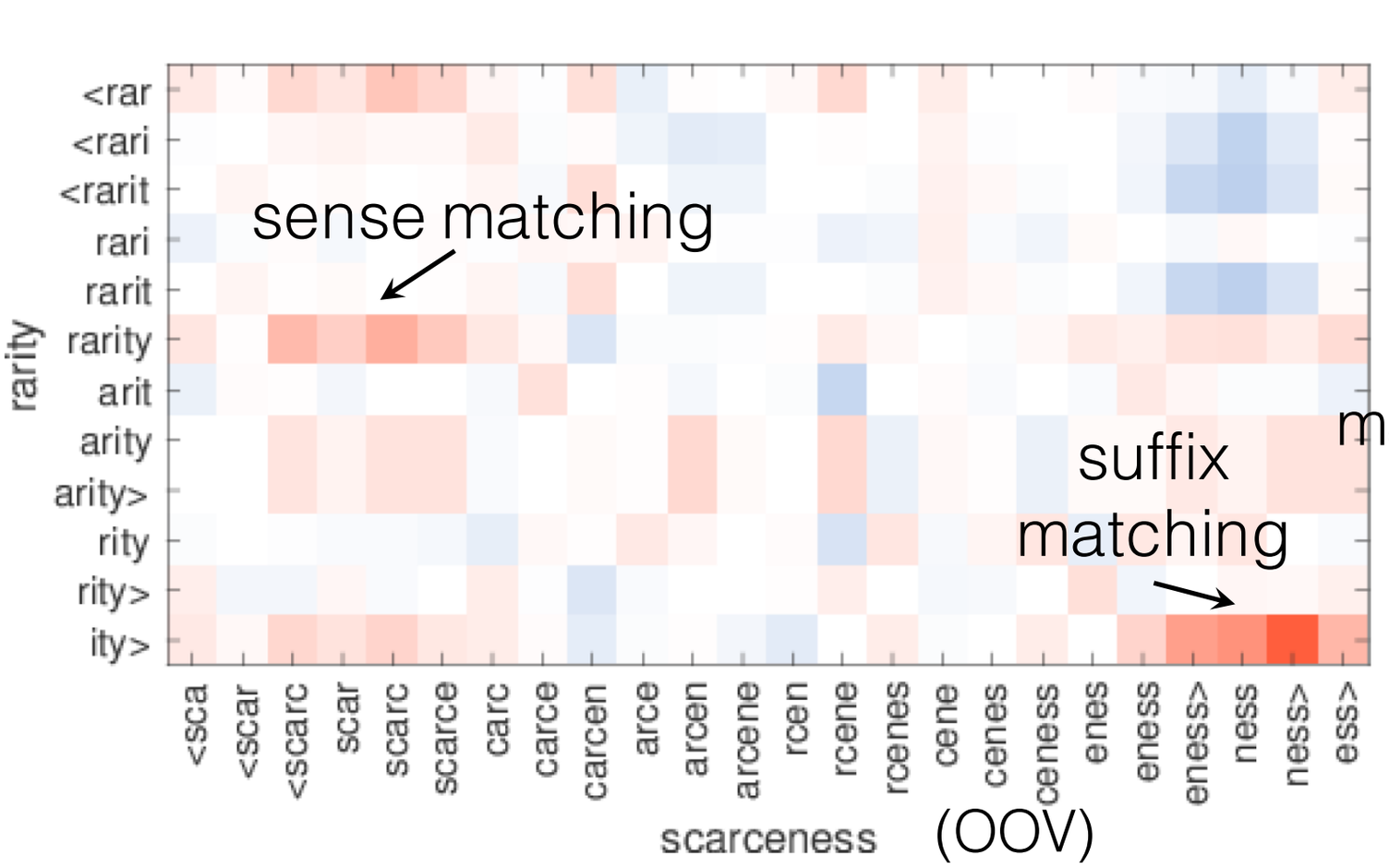
Observations:

the most important n-grams tend to make sense and match:

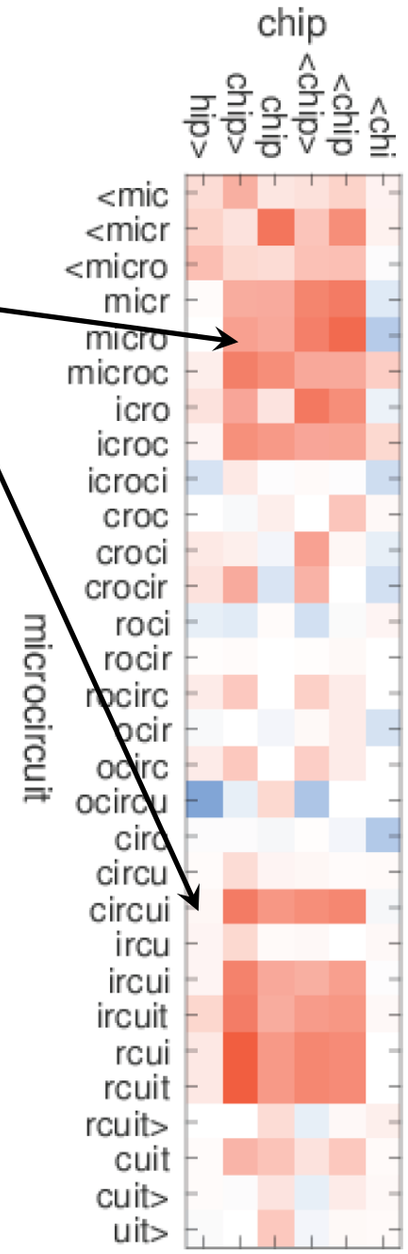
- prefixes & suffixes
- morphemes
- verb inflections

Most important character n-grams for selected words

Enriching Word Vectors with Subword Information



The areas corresponding to *micro* and *circuit* match *chip* the most (OOV)



Similarity between n-grams of in and out of vocabulary words

Observation: matches between n-grams are meaningful.

=> high quality vectors can be constructed for the OOV words (by summing the vectors of the n-grams)

Deep Contextualized Word Representations

Best paper NAACL 2018, 1450 cites

a.k.a. “Embeddings from Language Models” (**ELMo**)

Problem: traditional word vectors map each word to a single context-independent vector
But some words have more than one sense! (polysemy)
e.g., bank, get, wood, play, python…

Solution: use RNN language models to take information about the entire sentence/paragraph into account when computing the vector of a word

Deep Contextualized Word Representations

The proposed approach is **semi-supervised**:

- 1) a deep bidirectional RNN language model is pretrained on a large dataset
- 2) the vector of each word in a given input sentence is computed as a weighted sum of the RNN hidden representations (weights are learned)

1) unsupervised, 2) supervised performed on a task-specific dataset.

$$\mathbf{ELMo}_k^{task} = \gamma^{task} \sum_{j=0}^{L} s_j^{task} \mathbf{h}_{k,j}^{LM}$$

$h_{k,j}^{LM}$ is the k[th] hidden representation of the j[th] layer of the bi-RNN LM

$s^{task}$is the softmax weight vector, γ^{task} is a scaling parameter (for optimization)

The authors use J=2 in all experiments

Deep Contextualized Word Representations

Use of ELMo in practice, on a task-specific dataset:

- 1) use the pretrained language model in prediction mode and store $h_{k,j}^{LM}$ for each word (each k) and each layer (each j)
- 2) concatenate $ELMo_k^{task}$ with the corresponding input vector* of whatever supervised model is used to solve the task (e.g., RNN, CNN, feed-forward…)
- 3) update s^{task} and γ^{task} with the other parameters of the supervised model during training

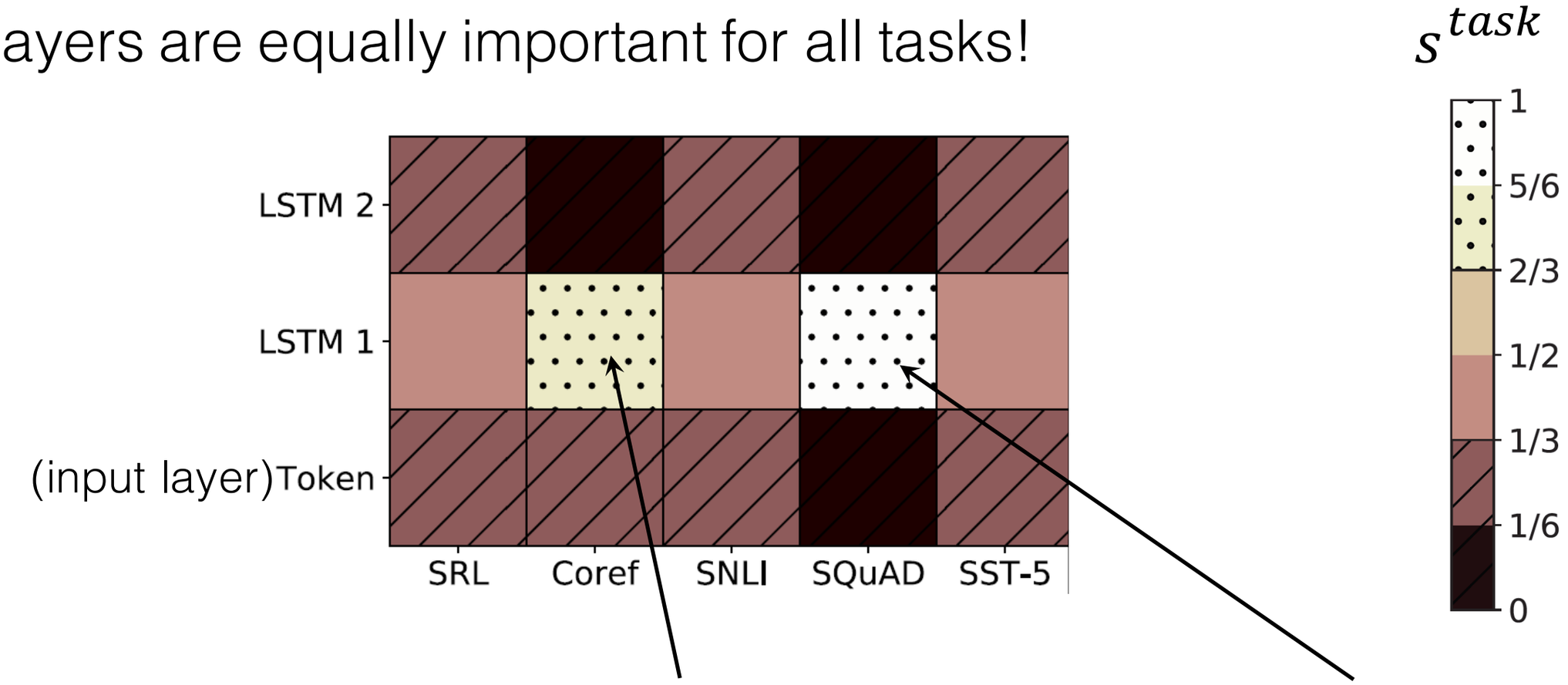
Results:

ELMo improves over the baselines on 8 tasks, ranging from question answering to co-reference resolution, sentiment analysis, POS-tagging, and disambiguation

*given by word2vec, glove, etc.

Deep Contextualized Word Representations

Not all layers are equally important for all tasks!



The 1st layer clearly dominates for coreference resolution and question answering

For the other tasks, the weights are more evenly distributed among layers

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