DSP Lab, Week 2, Fall 2023

$$y(n) = b_0 x(n) + b_1 x(n-1) - a_1 y(n-1)$$

First-order difference equation

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} \qquad H^f(\omega) = \frac{b_0 + b_1 e^{-j\omega}}{1 + a_1 e^{-j\omega}}$$

Transfer function

Freq. Resp. $\quad H^f(\omega) = \sum_n h_n e^{-jn}$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) - a_1 y(n-1) - a_2 y(n-2)$$

$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{b_0 (z - z_1)(z - z_2)}{(z - p_1)(z - p_2)}$$

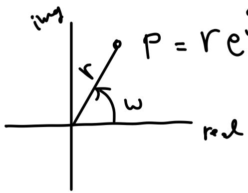
$$H(z) = K + C_1 \frac{z}{z - p_1} + C_2 \frac{z}{z - p_2}$$

$$h(n) = K \delta(n) + C_1 (p_1)^n u(n) + C_2 (p_2)^n u(n)$$

Say a_i & b_i are real, then $p_2 = p_1^*$

& $C_2 = C_1^*$

$$h(n) = K\delta(n) + C p^n u(n) + C^*(p^*)^n u(n)$$



$p = re^{j\omega}$ polar form $C = Re^{j\theta}$

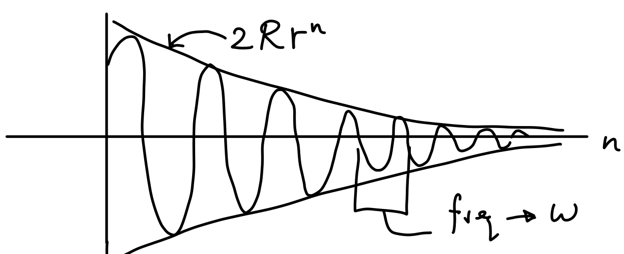
$\cos\theta = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$

$$h(n) = K\delta(n) + Re^{j\theta}(re^{j\omega})^n u(n) + Re^{-j\theta}(re^{-j\omega})^n u(n)$$

$$h(n) = K\delta(n) + R\left[e^{j\theta} r^n e^{jn\omega} + e^{-j\theta} r^n e^{-jn\omega}\right]u(n)$$

$$= K\delta(n) + Rr^n\left[e^{j(\theta+n\omega)} + e^{-j(\theta+n\omega)}\right]u(n)$$

$$h(n) = \underbrace{K\delta(n)} + 2R\underbrace{r^n \cos(\omega n + \theta)\, u(n)}$$



Pole angle determines frequency of impulse response

pole modulus determines decay rate of impulse response

$$(z - p_1)(z - p_2) = (z - re^{j\omega})(z - re^{-j\omega})$$

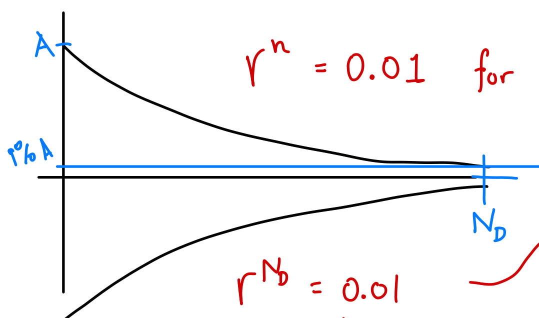
$$= z^2 - zre^{j\omega} - zre^{-j\omega} + r^2 e^{j\omega} e^{-j\omega}$$

$$= z^2 - zr(e^{j\omega} + e^{-j\omega}) + r^2$$

$$= z^2 - 2r\cos(\omega)\, z + r^2$$

$$= z^2 + a_1 z + a_2$$

$$[a_0, a_1, a_2] = [1, -2r\cos(\omega), r^2]$$



$r^n = 0.01$ for what value of n

$$r^{N_D} = 0.01$$

$$N_D = \log_r 0.01 = \frac{\log 0.01}{\log r}$$

$$r = (0.01)^{1/N_D}$$