Biomedical Imaging

Magnetic Resonance Imaging

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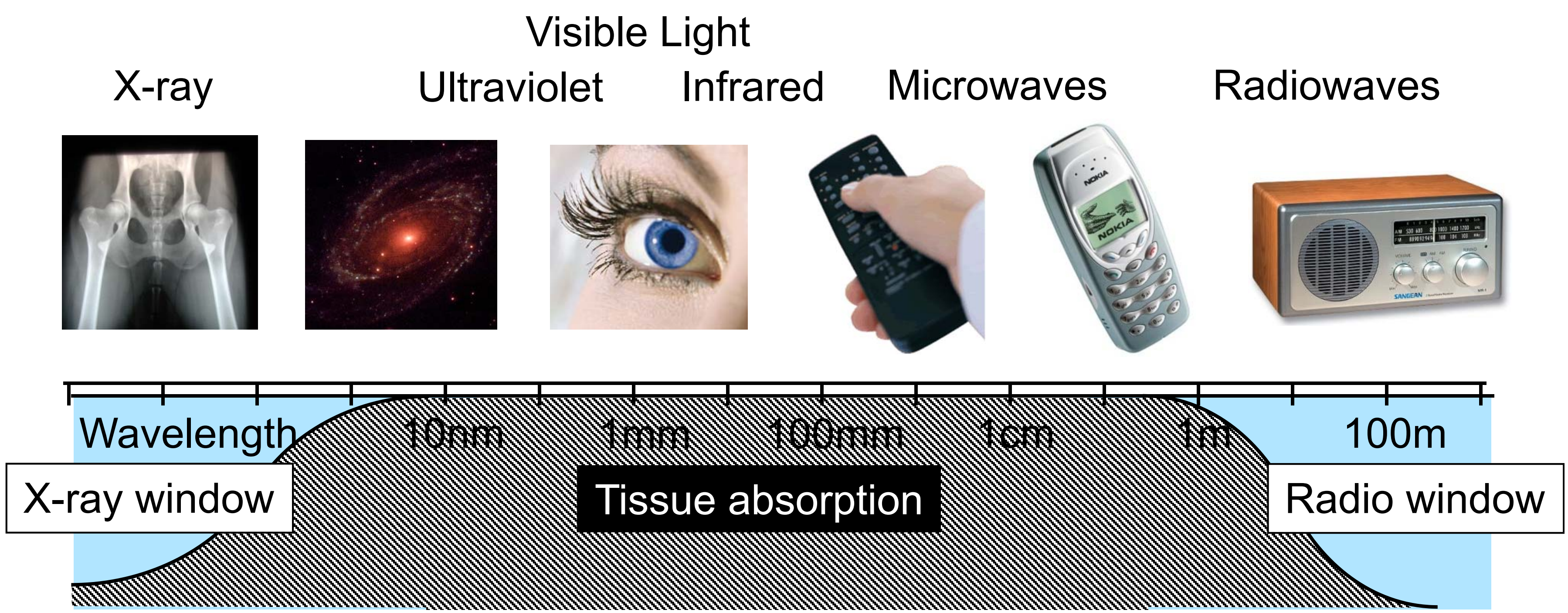
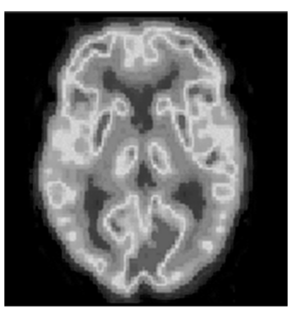
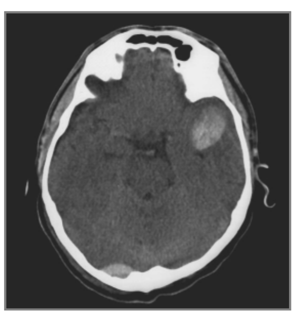
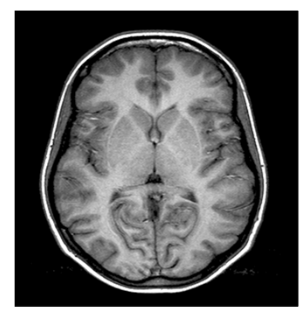
Electromagnetic Waves and Medical Imaging




Nuclear


X-Ray


MRI

Acronyms

- **NMR** **N**uclear **M**agnetic **R**esonance
- **MR** **M**agnetic **R**esonance
- **MRI** **M**agnetic **R**esonance **I**maging
- **MRS** **M**agnetic **R**esonance **S**pectroscopy
- **fMRI** **F**unctional **M**agnetic **R**esonance **I**maging
- **MRT** **M**agnet-**R**esonanz-**T**omographie
- **KST** **K**ern**S**pin-**T**omographie

History

- **1946** **Discovery of NMR by Felix Bloch and Edward Purcell**

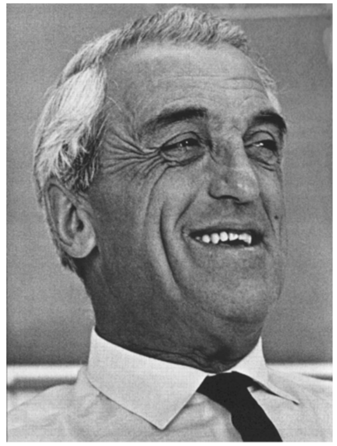
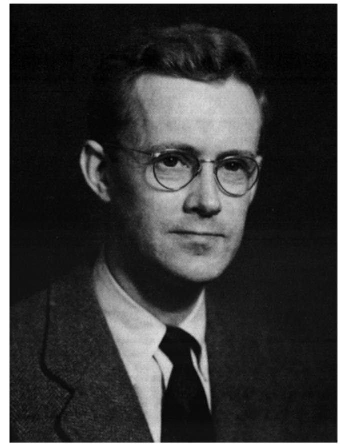
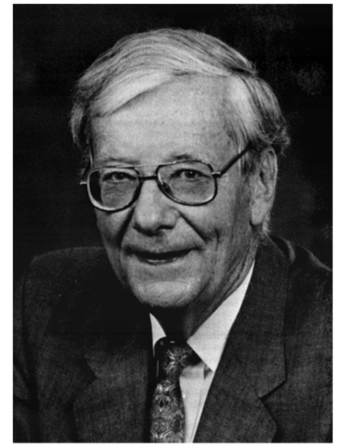
  NMR applications in chemistry und physics

- **1970** **Invention of NMR imaging by Paul Lauterbur**

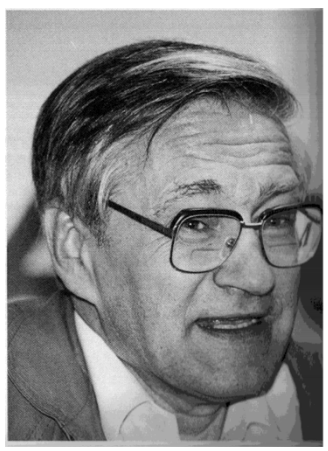
  “zeugmatography”

- **1975** **Invention of Fourier imaging by Richard Ernst (Zurich)**
- **1980** **First commercial medical NMR scanner (“MRI”)**
- **1985** **Clinical and diagnostic MRI starts establishing**
- **Since** **Technology advances, deployment in medicine and research**

Nobel prizes – NMR/MRI

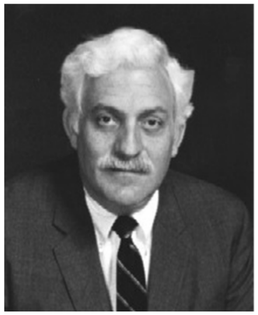





Rabi 1944 | Bloch 1952 | Purcell 1952 | Bloembergen 1981

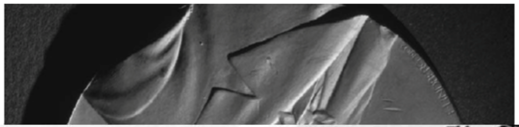

Ramsey 1989 | Ernst 1991 | Wüthrich 2002 | Lauterbur 2003 | Mansfield 2003

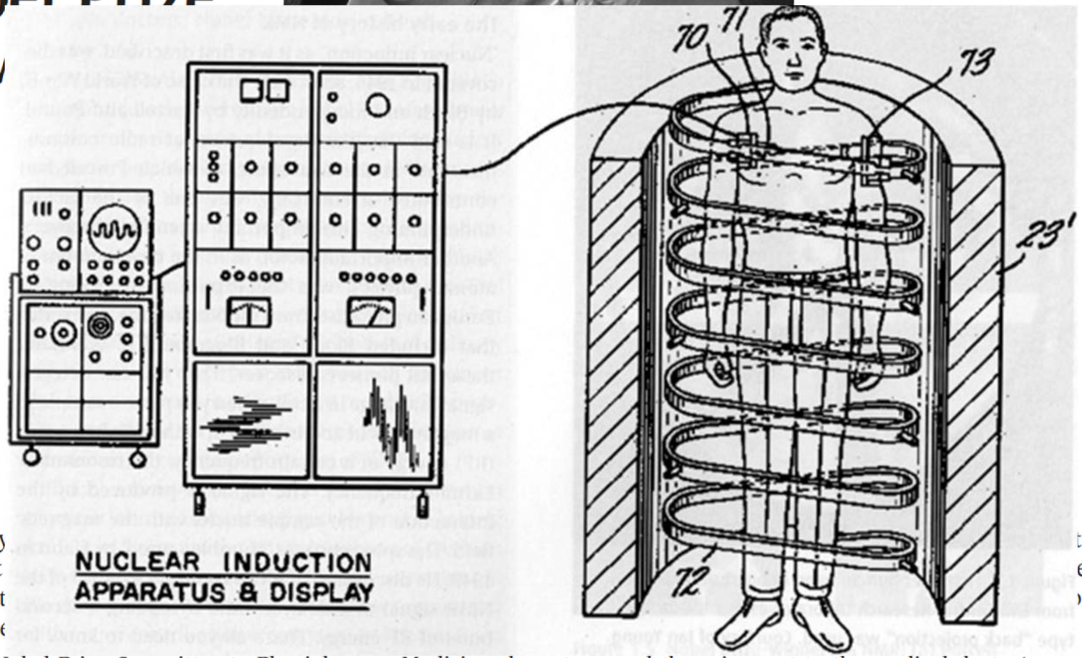
Nobel prize debate



R. Damadian

This Year's Nobel Prize in M





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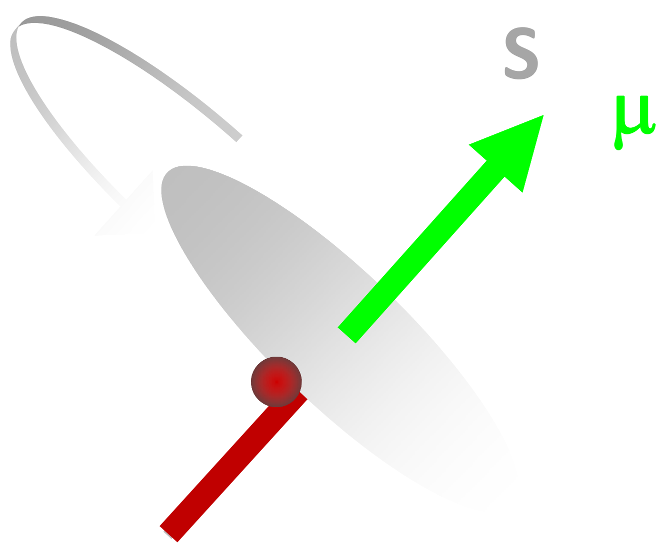
mittee

The Nobel Prize Committee to Physiology or Medicine chose to award the prize, not to the medical doctor/research scientist who made the breakthrough discovery on which all MRI technology is based, but to two scientists who later made technological improvements based on his discovery.

New York Times, Oct. 10, 2003

Magnetic moment

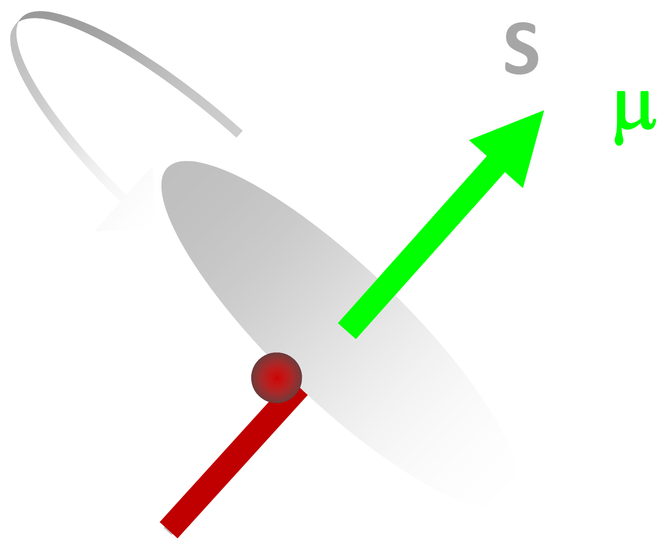


- **Atomic nucleus**
- **Spin $\vec{S}$**
- **Gives rise to magnetic moment $\vec{\mu}$**

$$\boxed{\vec{\mu} = \gamma\,\vec{S}}$$

γ = gyromagnetic ratio

Magnetic moment



- **Quantum mechanics requires spin to occur in quanta:**

$$S_z = \hbar\, m \qquad m = -I, -I+1, \dots, I-1, I$$

$$|\boldsymbol{S}| = \hbar\sqrt{I(I+1)}$$

m = magnetic quantum number, I = spin quantum number

Isotopes

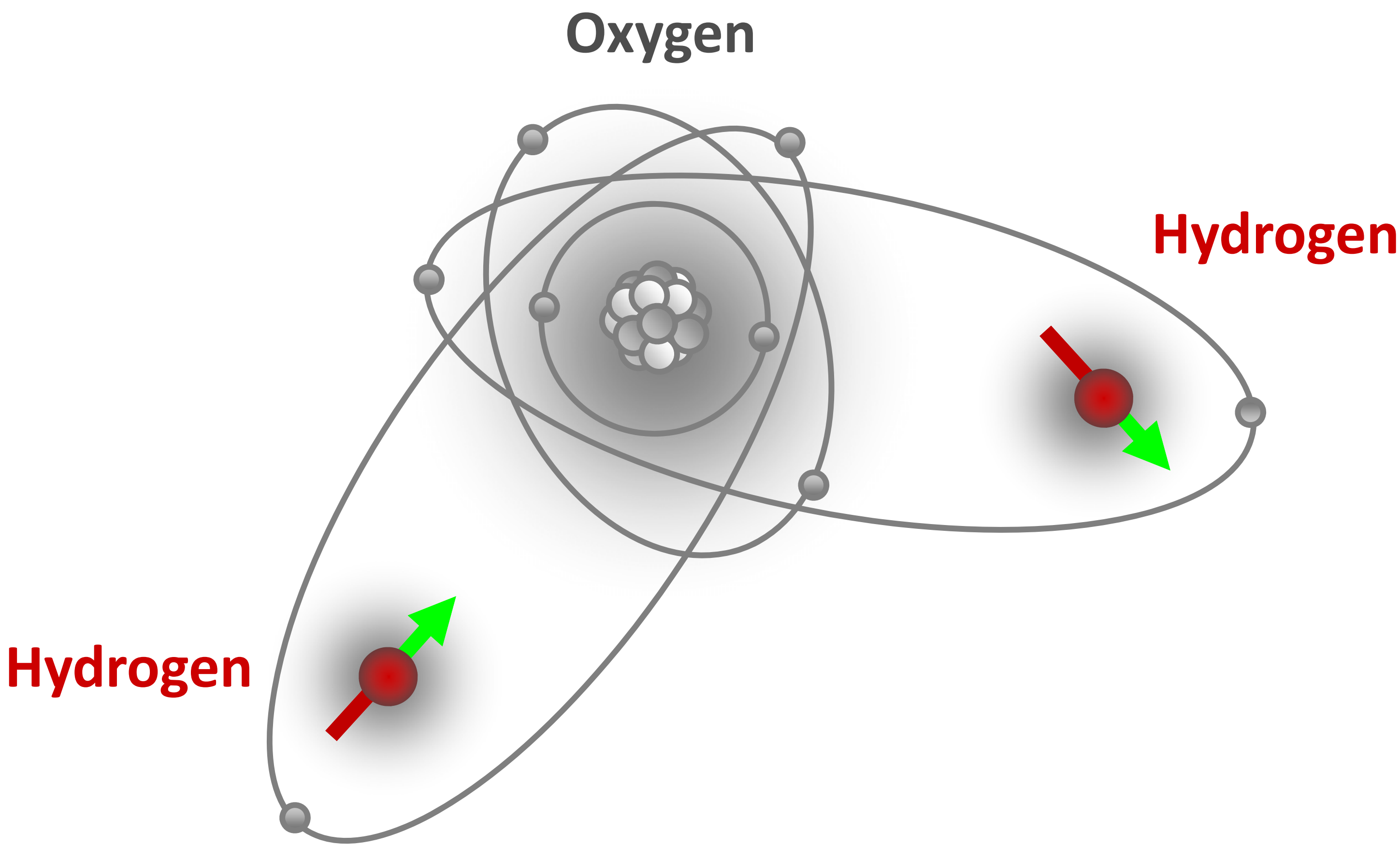
^{1}H Hydrogen	^{13}C Carbogen	^{14}N Nitrogen	^{17}O Oxygen
$I = 1/2$	$I = 1/2$	$I = 1$	$I = 5/2$

$$I = \frac{1}{2} \quad \Rightarrow \quad m = \pm\frac{1}{2} \quad \Rightarrow \quad \mu_z = \gamma\,\hbar\,m = \pm\,\frac{1}{2}\,\gamma\,\hbar$$

Relevant nuclei

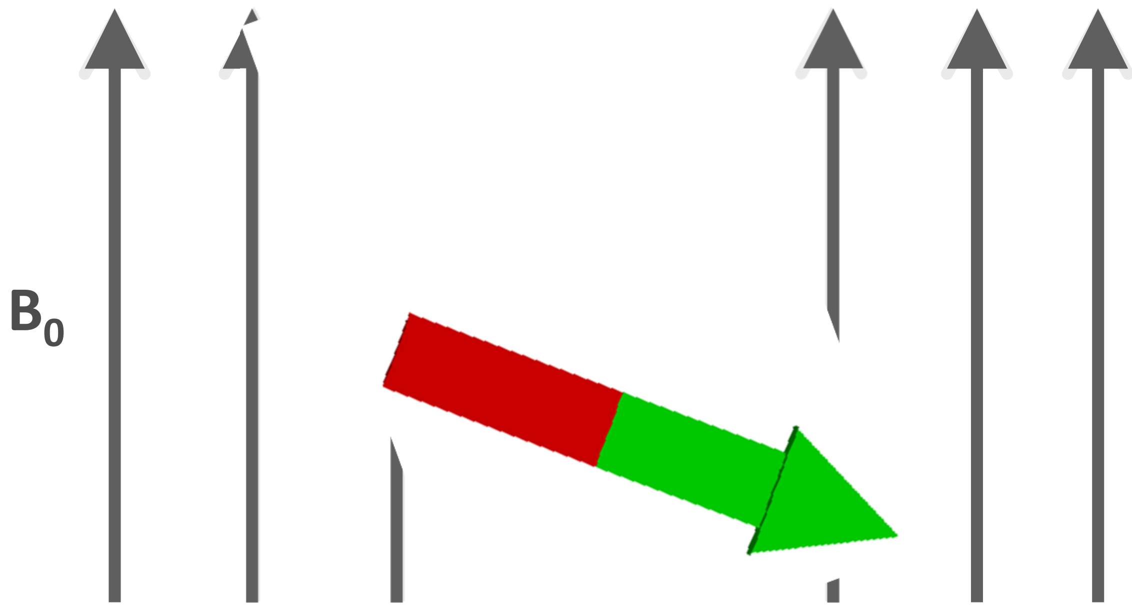
	Spin quantum number	$\gamma/2\pi$ [MHz/T]	Natural abundance [%]	Relative sensitivity
^{1}H	1/2	42.58	99.98	1
^{13}C	1/2	10.71	1.11	1.8×10^{-4}
^{14}N	1	3.08	99.64	1.0×10^{-3}
^{17}O	5/2	5.77	0.04	1.1×10^{-5}
^{19}F	1/2	40.06	100.00	8.3×10^{-1}
^{23}Na	3/2	11.26	100.00	9.3×10^{-2}
^{31}P	1/2	17.24	100.00	6.6×10^{-2}
^{39}K	3/2	1.99	93.08	4.7×10^{-4}
^{43}Ca	7/2	2.87	0.14	9.3×10^{-6}

Water



- **Water abundant in human body** (65% volume fraction)
- **Large gyromagnetic ratio of ^{1}H** (42.6 MHz/T)

Magnetic moment in static magnetic field

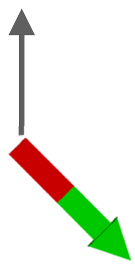


- Choose z aligned with B_0, two values of μ_z
- Transverse component remains
- Energy content:

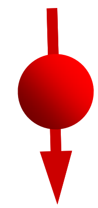
$$E_m = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}} = -\mu_z B_0 = \pm \frac{\hbar}{2} \gamma B_0$$

Convention

high-energy state

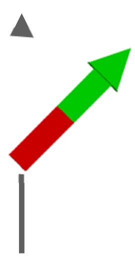


$$E_m = +\frac{1}{2}\gamma\hbar B_0$$

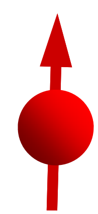


"spin down"

low-energy state

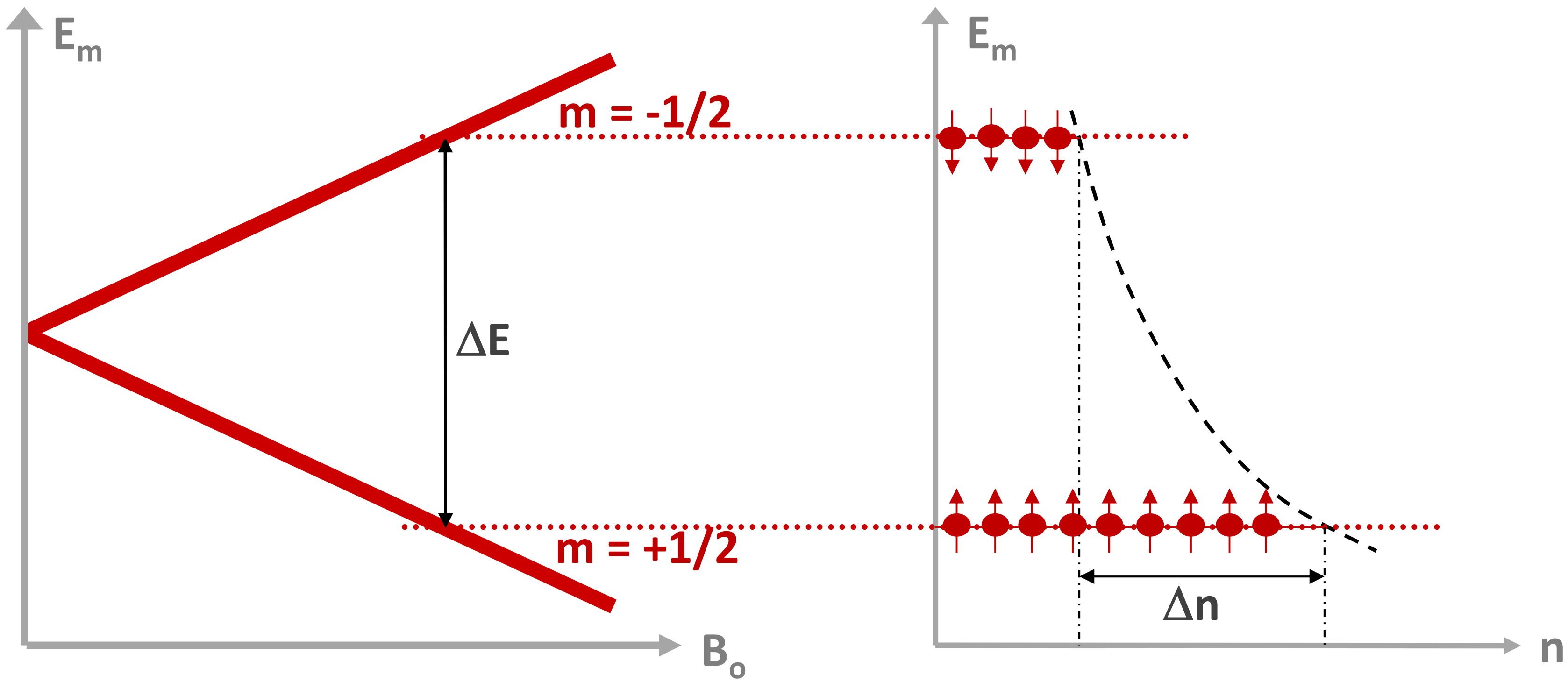


$$E_m = -\frac{1}{2}\gamma\hbar B_0$$



"spin up"

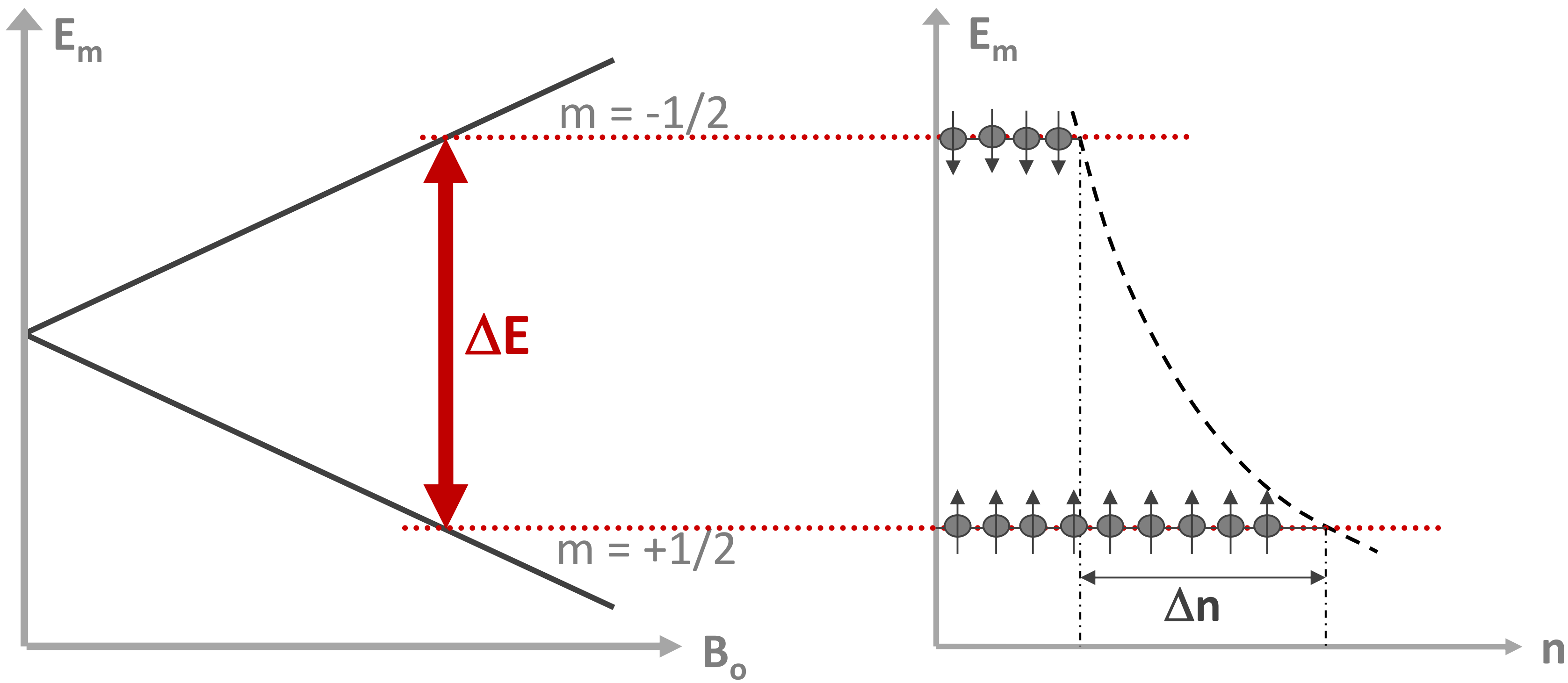
Energy levels and population difference



- **Population numbers** (Boltzmann statistics):

$$\frac{n_{-1/2}}{n_{+1/2}} = \exp\left(-\frac{\Delta E}{k_b T}\right)$$

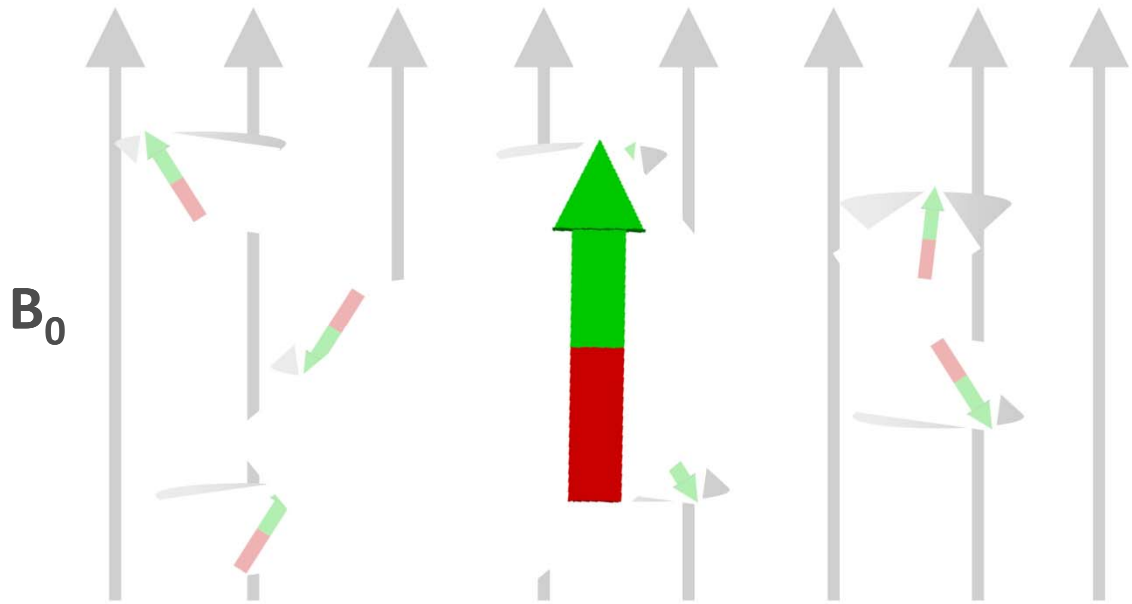
Resonance condition



Match energy gap with photons:

$\boxed{\Delta E = \hbar \omega_L}$ $\boxed{\omega_L = \gamma B_0}$ Larmor frequency

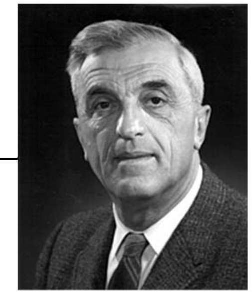
Macroscopic magnetization



- **Macroscopic magnetic moment:**

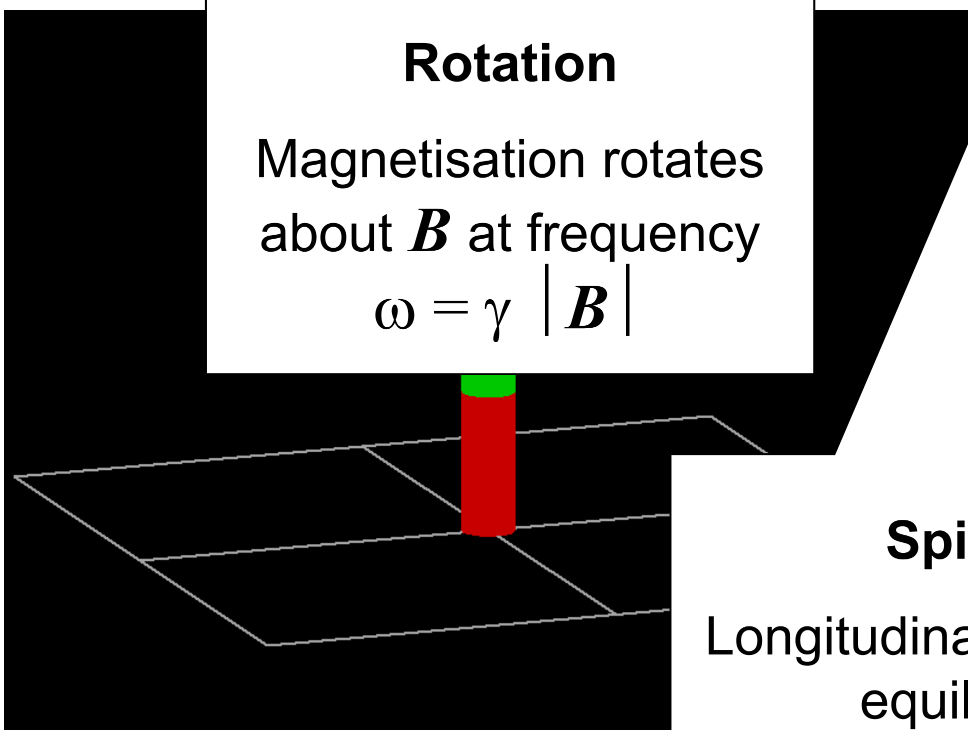
$$M_0 = \sum \mu_z = \Delta n \gamma \frac{\hbar}{2}$$

Magnetization



Governed by Bloch equation(s):

$$\frac{d}{dt}\boldsymbol{M} = \gamma\,\boldsymbol{B} \times \boldsymbol{M} - \begin{pmatrix} M_x / T_2 \\ M_y / T_2 \\ (M_z - M_0)/T_1 \end{pmatrix}$$



Rotation

Magnetisation rotates about $\boldsymbol{B}$ at frequency $\omega = \gamma \left|\boldsymbol{B}\right|$

Spin-spin relaxation

Transverse components (M_x, M_y) decay with rate $1 / T_2$

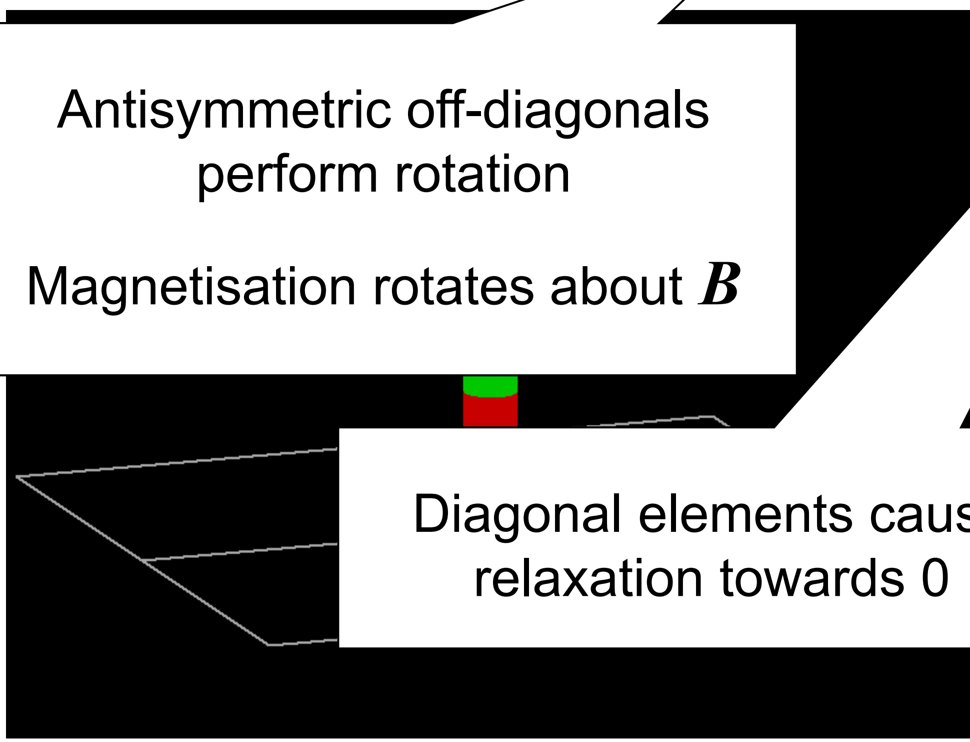
Spin-lattice relaxation

Longitudinal component (M_z) returns to equilibrium with rate $1 / T_1$

Magnetization

Governed by Bloch equation(s):

$$\frac{d}{dt}\boldsymbol{M} = \begin{pmatrix} -1/T_2 & -\gamma B_z & \gamma B_y \\ \gamma B_z & -1/T_2 & -\gamma B_x \\ -\gamma B_y & \gamma B_x & -1/T_1 \end{pmatrix} \boldsymbol{M} + \begin{pmatrix} 0 \\ 0 \\ M_0 / T_1 \end{pmatrix}$$



Antisymmetric off-diagonals perform rotation

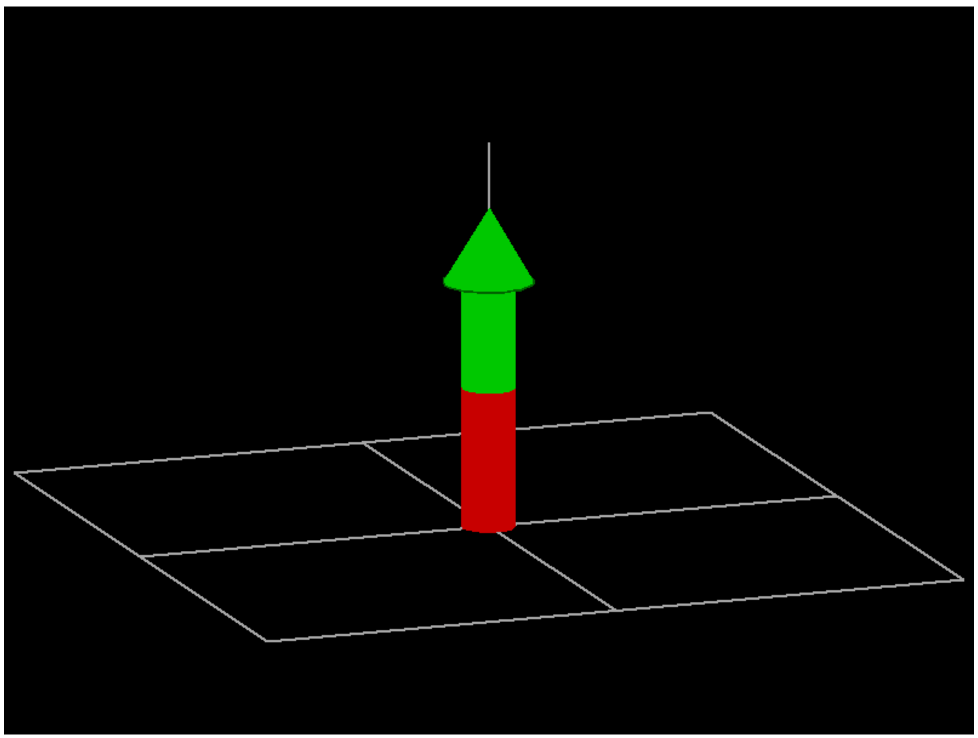
Magnetisation rotates about $\boldsymbol{B}$

Diagonal elements cause relaxation towards 0

Absolute term causes build-up of equilibrium

Magnetization

$$\frac{d}{dt}\boldsymbol{M} = \begin{pmatrix} -1/T_2 & -\gamma B_0 & 0 \\ \gamma B_0 & -1/T_2 & 0 \\ 0 & 0 & -1/T_1 \end{pmatrix} \boldsymbol{M} + \begin{pmatrix} 0 \\ 0 \\ M_0/T_1 \end{pmatrix}$$

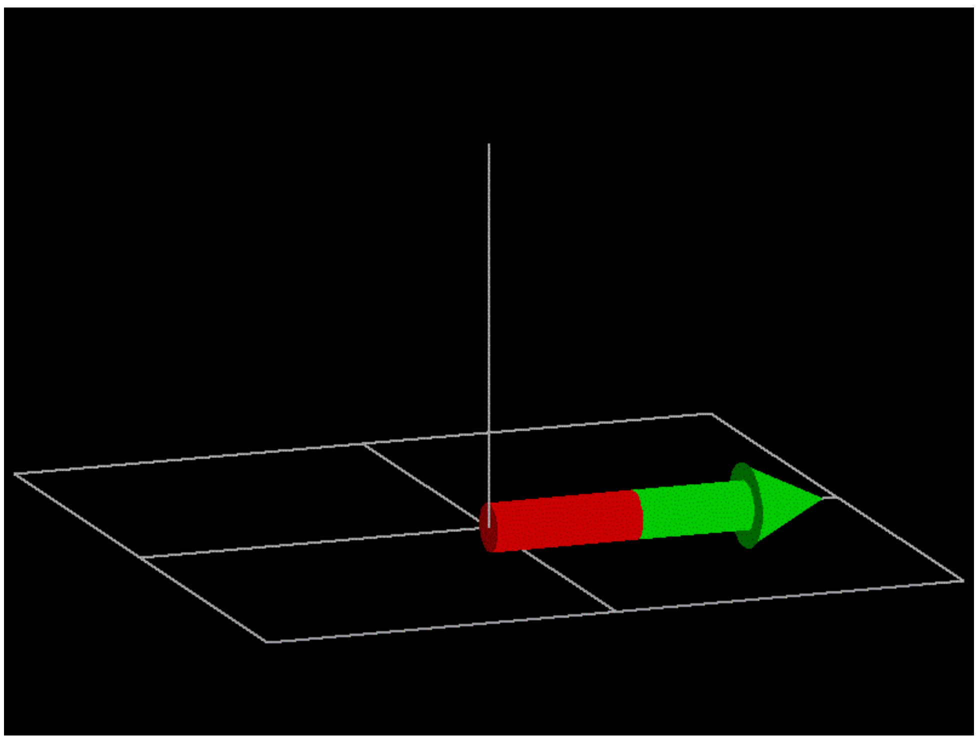


Equilibrium:

$$\boldsymbol{M}(0) = \begin{pmatrix} 0 \\ 0 \\ M_0 \end{pmatrix}$$

Magnetization

$$\frac{d}{dt}\boldsymbol{M} = \begin{pmatrix} 0 & -\gamma B_0 & 0 \\ \gamma B_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \boldsymbol{M}$$



Free precession:

$$B_x = B_y = 0$$

$$\omega = \omega_L = \gamma B_0$$

$$T_1,\ T_2 = \infty$$

RF transmission

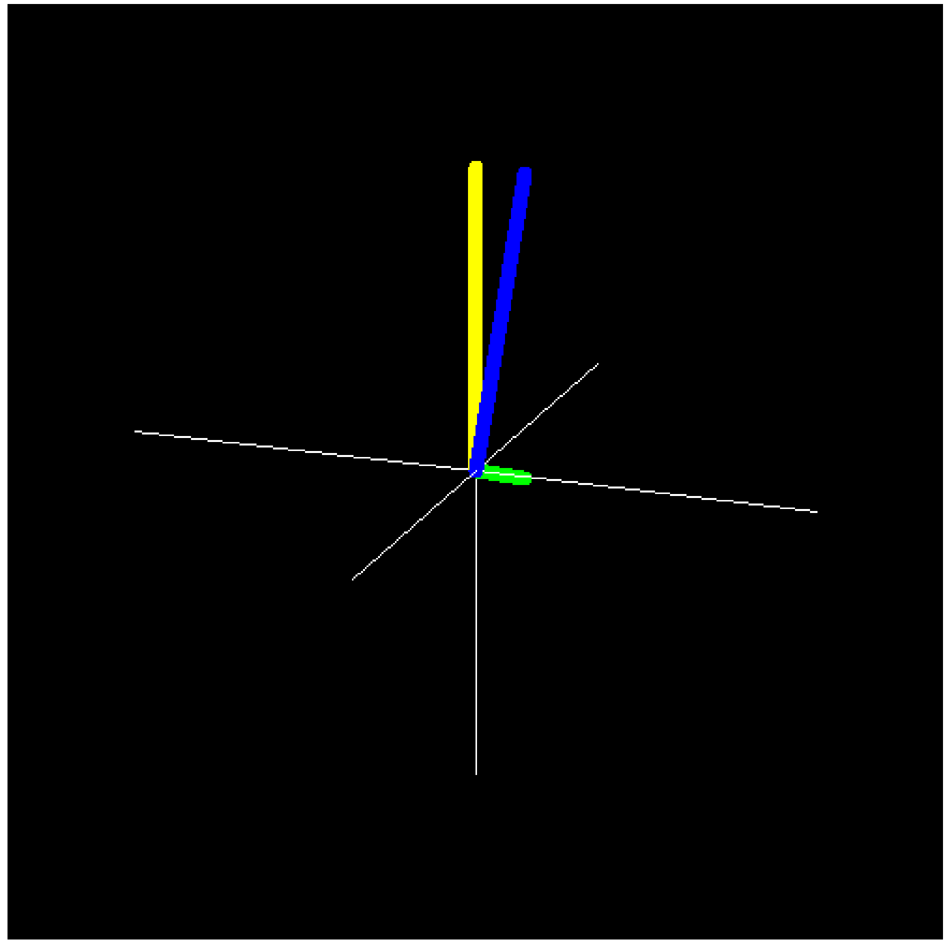
Frequency ω_{RF}

Constant amplitude B_1:

$$\begin{cases} B_x = B_1 \cos(\omega_{RF} t) \\ B_y = B_1 \sin(\omega_{RF} t) \\ \text{circularly polarized} \end{cases}$$

$$B_z = B_0$$

Net field



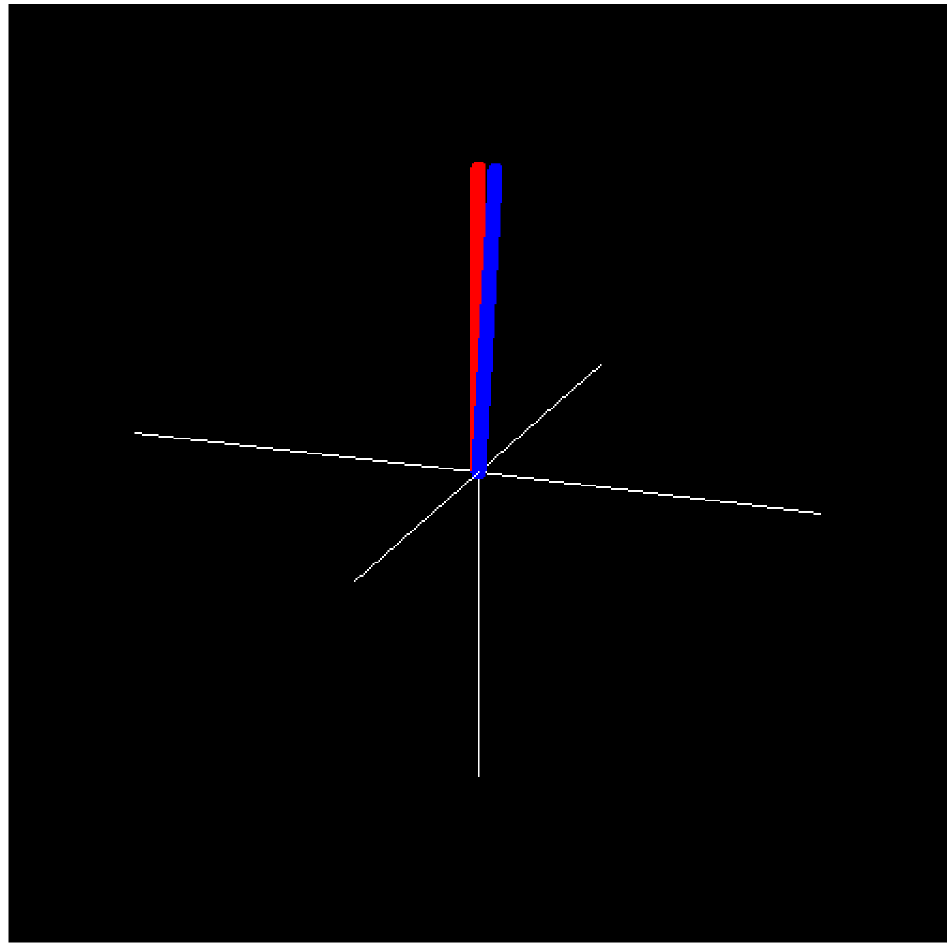
RF transmission

Frequency ω_{RF}

Constant amplitude B_1:

Net field

Magnetization



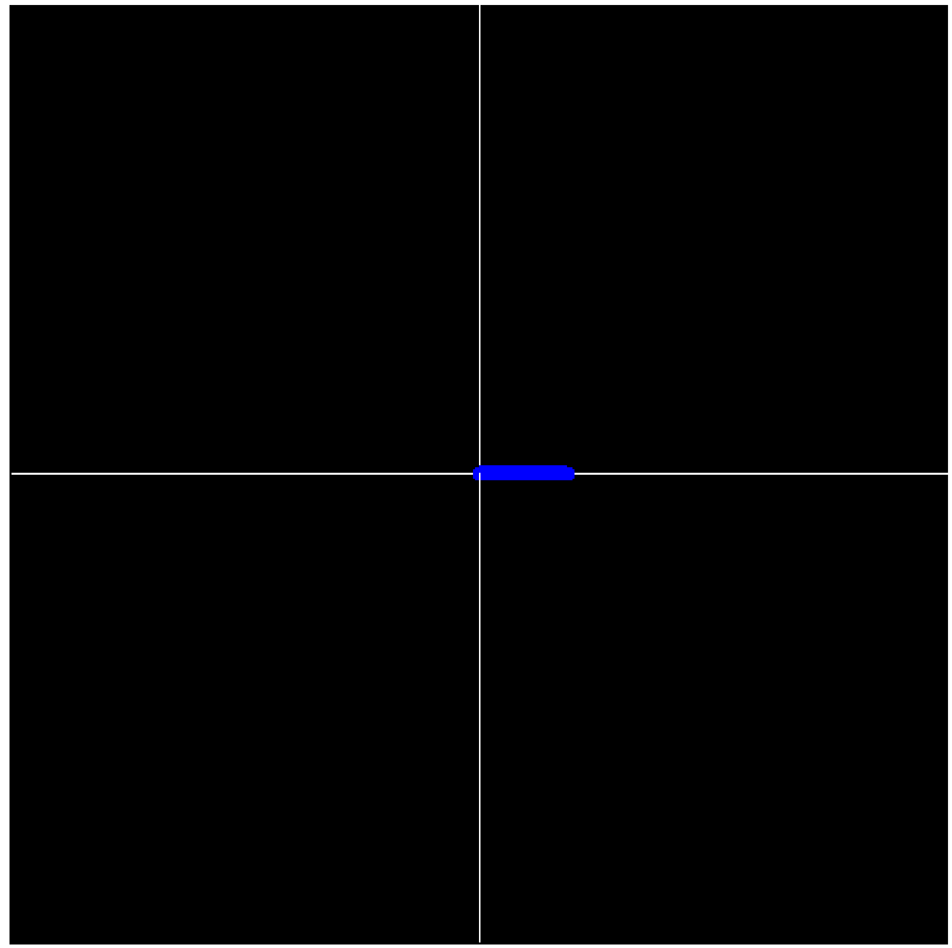
RF transmission

Frequency ω_{RF}

Constant amplitude B_1:

Net field

Magnetization



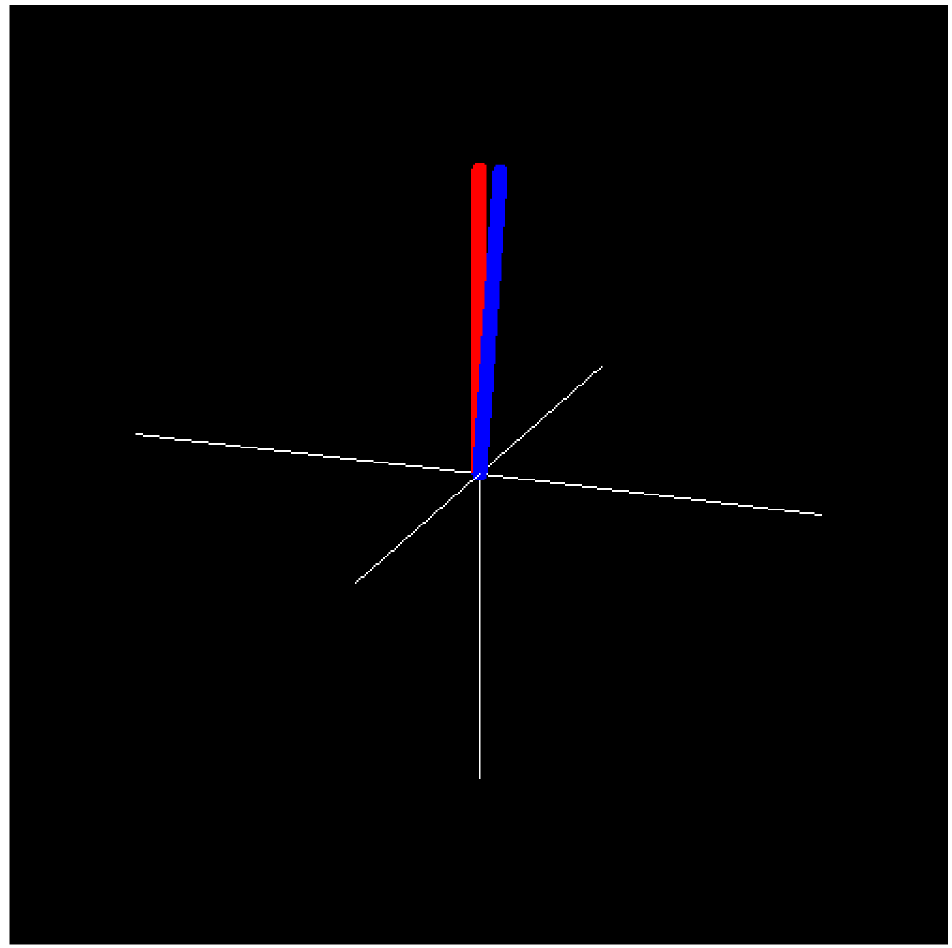
RF transmission

Increase frequency to $\omega_{RF} = \gamma B_0$

Constant amplitude B_1:

Net field

Magnetization



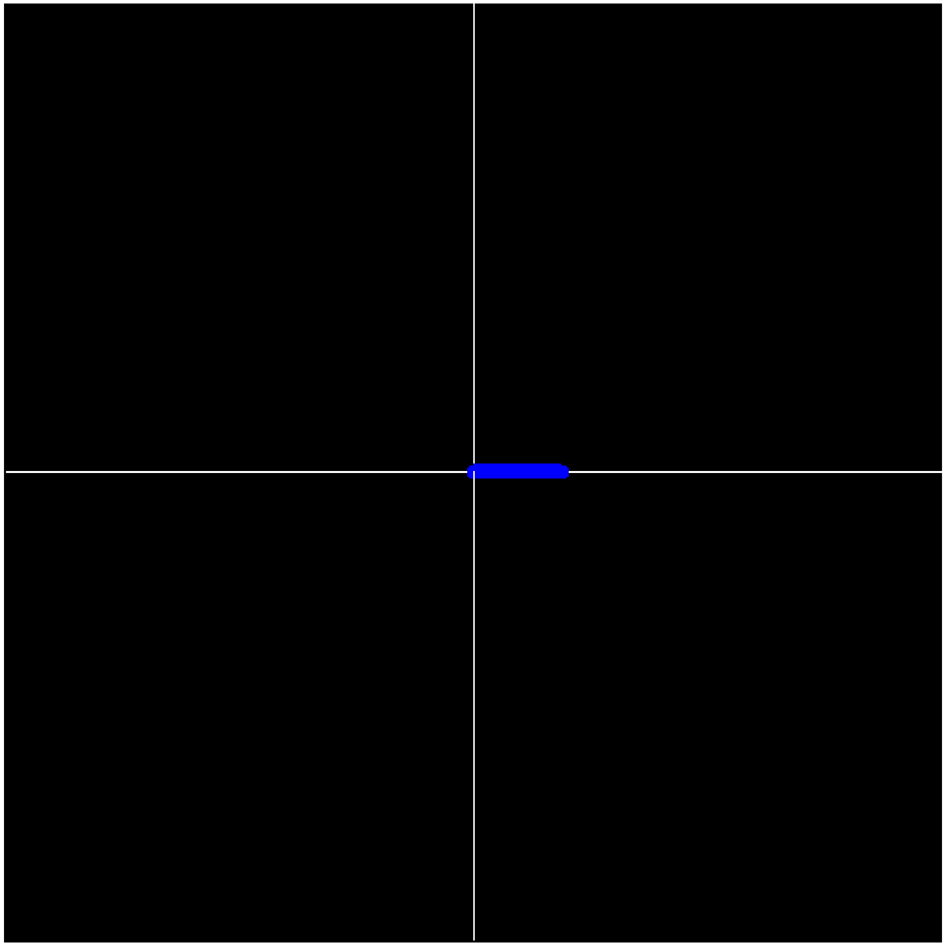
RF transmission

Frequency $\omega_{RF} = \gamma B_0$

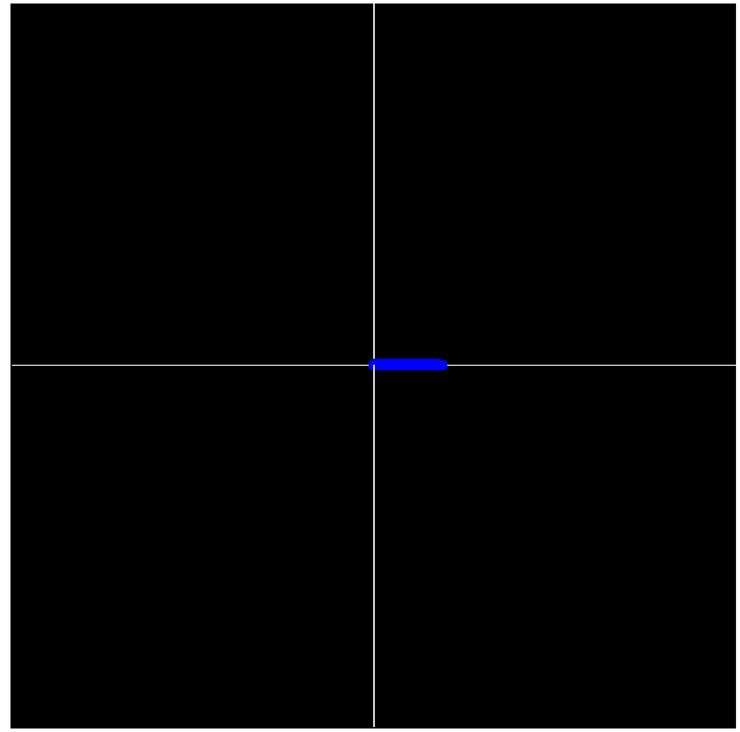
Constant amplitude B_1:

Net field

Magnetization



RF transmission



Frequency $\omega_{RF} = \gamma B_0$

Constant amplitude B_1:

$\begin{pmatrix} B_x \\ B_y \end{pmatrix}$ stays orthogonal to $\begin{pmatrix} M_x \\ M_y \end{pmatrix}$

⇨ Maximal rate of change of M_z

"on resonance"

$$\frac{d}{dt}\boldsymbol{M} = \begin{pmatrix} 0 & \gamma B_z & -\gamma B_y \\ -\gamma B_z & 0 & \gamma B_x \\ \gamma B_y & -\gamma B_x & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

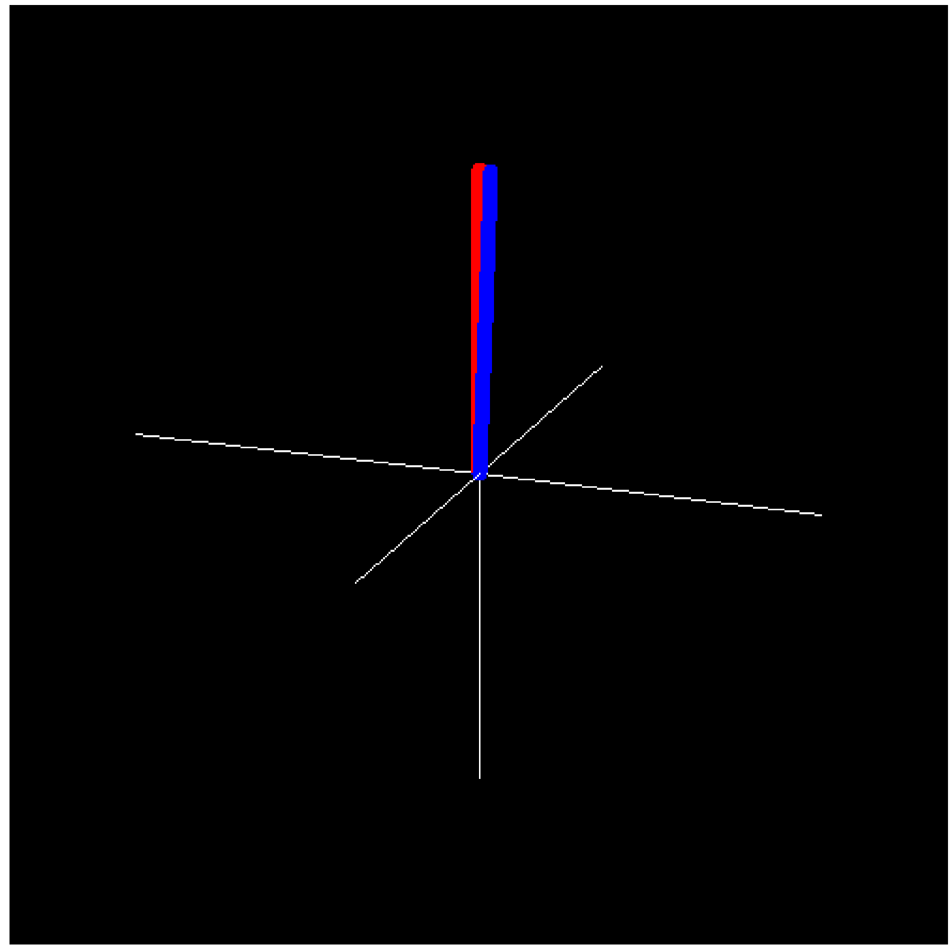
RF transmission

Decrease frequency to $\omega_{RF} \ll \gamma B_0$

Constant amplitude B_1:

Net field

Magnetization

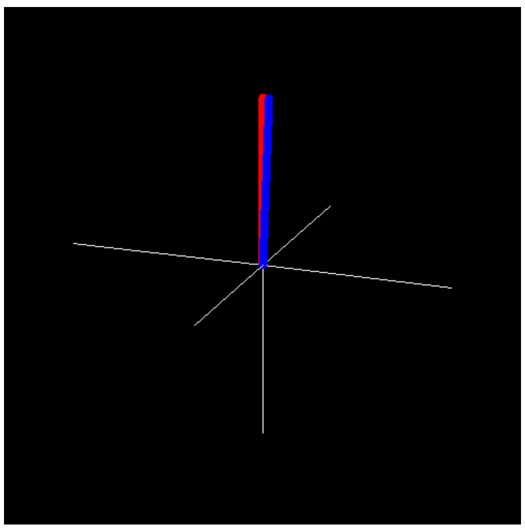


RF transmission

Constant amplitude B_1

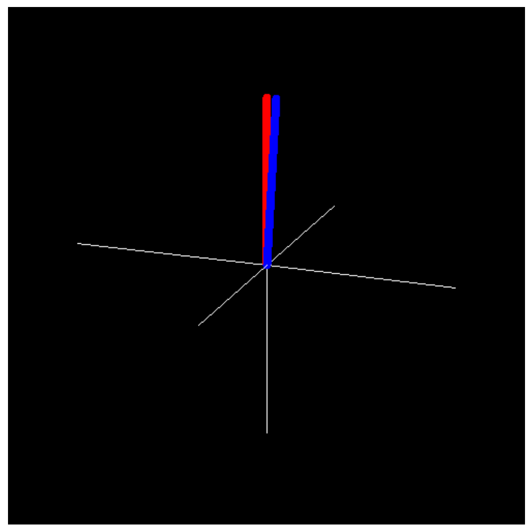
$\omega_{RF} \ll \gamma B_0$

off resonance



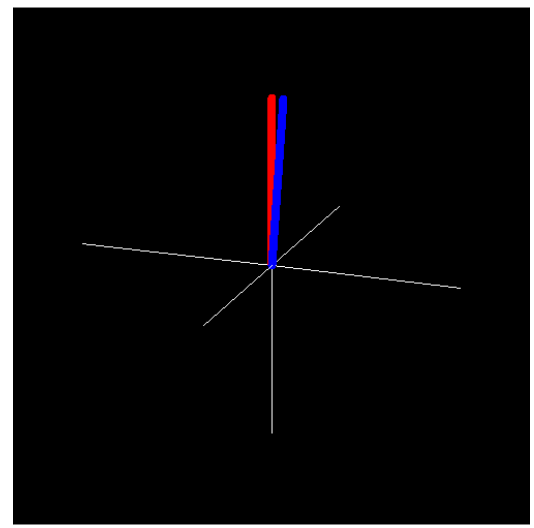
$\omega_{RF} < \gamma B_0$

near resonance



$\omega_{RF} = \gamma B_0$

on resonance



 Net field

 Magnetization

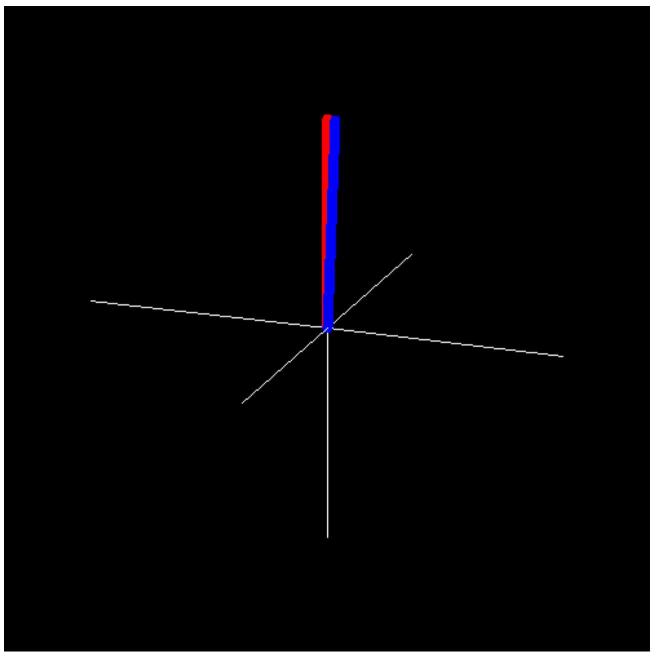
Rotating frames of reference

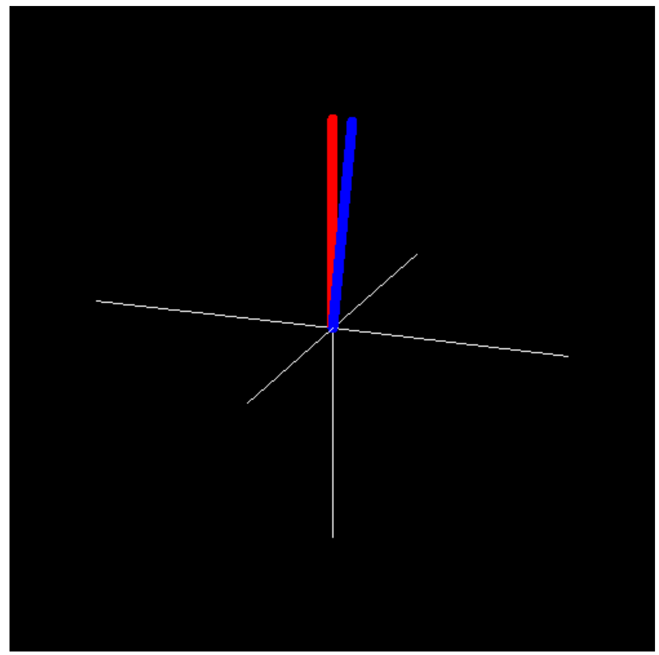
Off resonance: $\omega_{RF} \ll \gamma B_0$

RF frame: $\omega_{Frame} = \omega_{RF}$

$B_z = B_0 - \omega_{RF}/\gamma$

Lab frame: $\omega_{Frame} = 0$





Effective field

Magnetization

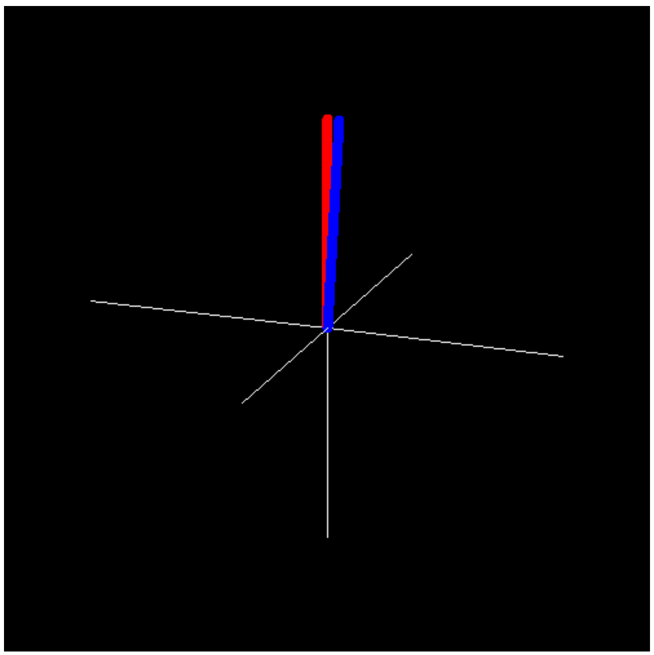
Rotating frames of reference

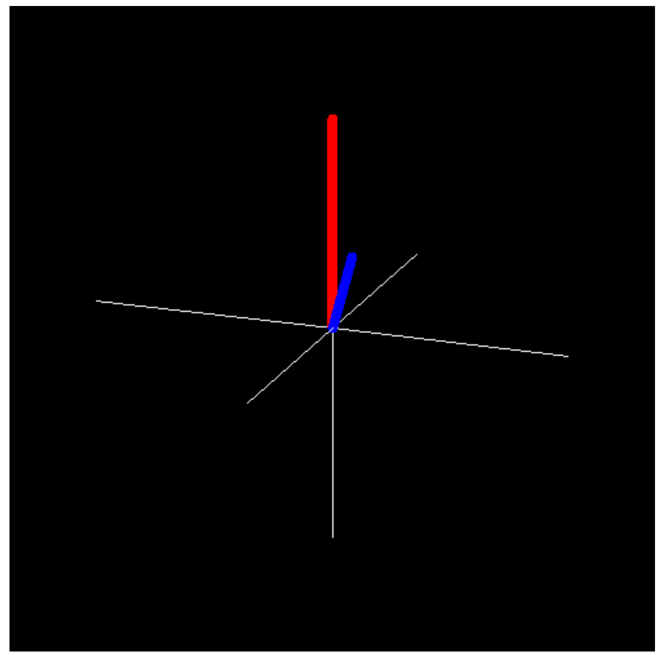
Near resonance: $\omega_{RF} < \gamma B_0$

RF frame: $\omega_{Frame} = \omega_{RF}$

$B_z = B_0 - \omega_{RF}/\gamma$

Lab frame: $\omega_{Frame} = 0$





Effective field

Magnetization

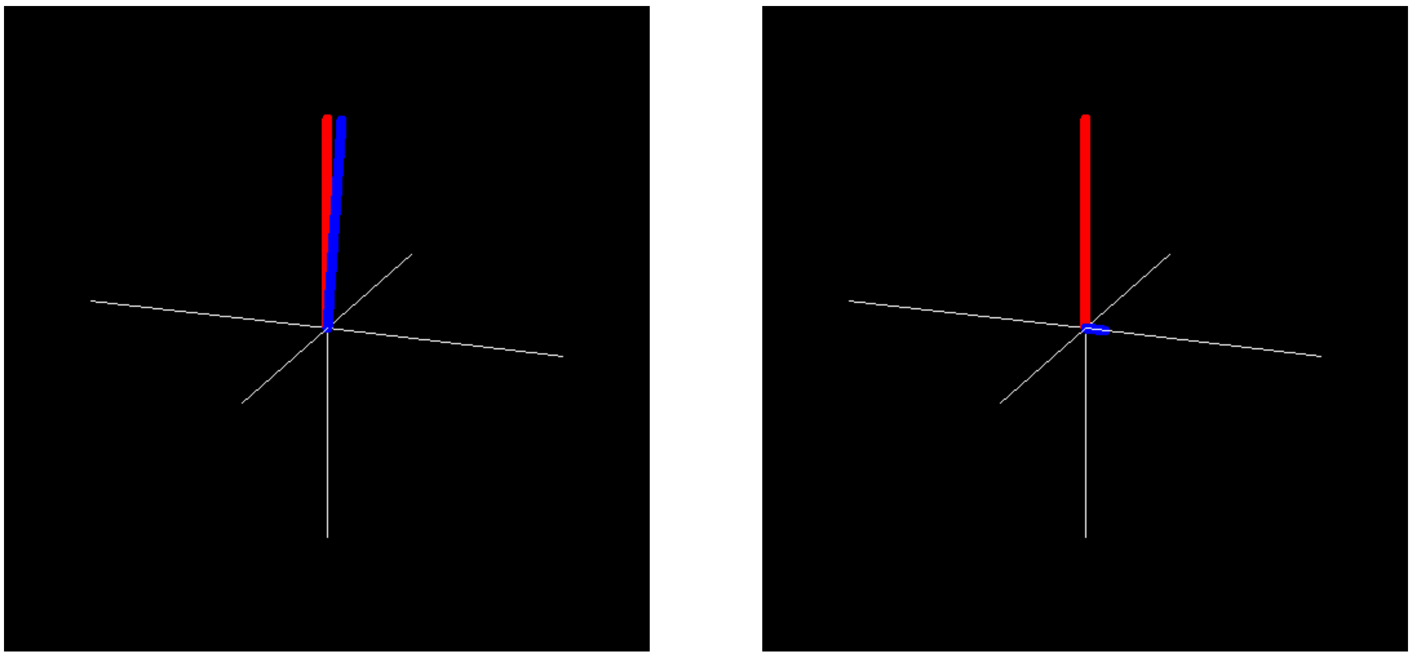
Rotating frames of reference

On resonance: $\omega_{RF} = \gamma B_0$

RF frame: $\omega_{Frame} = \omega_{RF}$

$B_z = B_0 - \omega_{RF}/\gamma$

Lab frame: $\omega_{Frame} = 0$

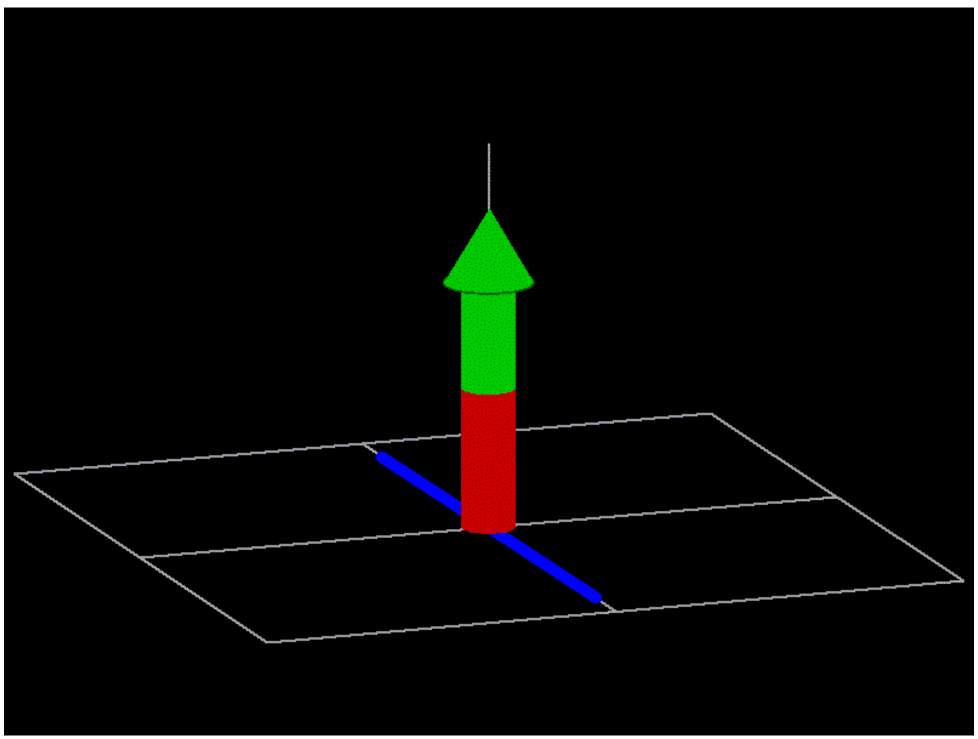


Effective field

Magnetization

Magnetization

$$\frac{d}{dt}\boldsymbol{M} = \begin{pmatrix} 0 & -\gamma B_0 & \gamma B_y \\ \gamma B_0 & 0 & -\gamma B_x \\ -\gamma B_y & \gamma B_x & 0 \end{pmatrix} \boldsymbol{M}$$



Fast excitation:

$$T_1,\ T_2 = \infty$$

$$B_x = B_1\, cos\left(\omega_L t\right)$$

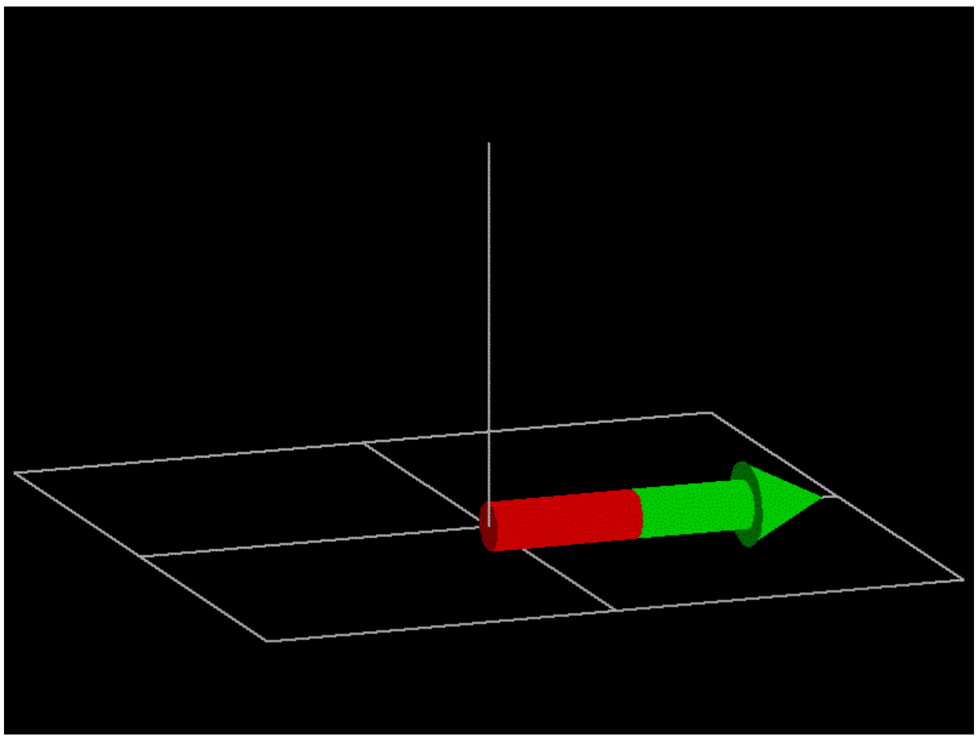
$$B_y = B_1\, sin\left(\omega_L t\right)$$

$$\omega_L = \omega_{Larmor} = \gamma B_0$$

Duration such that
flip angle = 90°

Magnetization

$$\frac{d}{dt}\boldsymbol{M} = \begin{pmatrix} -1/T_2 & -\gamma B_0 & 0 \\ \gamma B_0 & -1/T_2 & 0 \\ 0 & 0 & -1/T_1 \end{pmatrix}\boldsymbol{M} + \begin{pmatrix} 0 \\ 0 \\ M_0/T_1 \end{pmatrix}$$



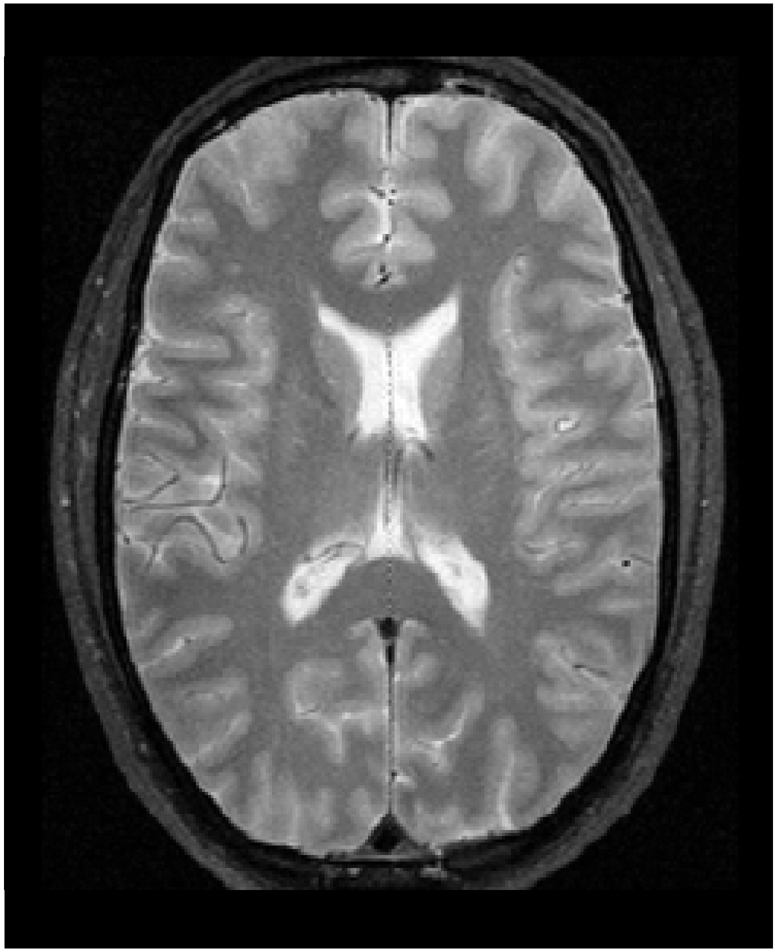
Relaxation:

$$\boldsymbol{M}(0) = \begin{pmatrix} M_{xy} \\ 0 \\ 0 \end{pmatrix}$$

Relaxation times

	T1 @ 1.5T [ms]	T1 @ 0.5T [ms]	T2 [ms]
White matter	790	539	92
Gray matter	920	656	100
CSF	>2500	>2500	>2000
Skeletal muscle	870	600	47
Kidney	650	450	60
Liver	490	320	40
Fat	260	215	85

Relaxation times and contrast



How detect precessing magnetisation?

Standard in MRI and NMR:

Faraday induction

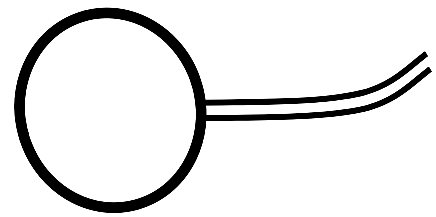
Precessing magnetisation generates oscillating magnetic field $\boldsymbol{B}(\boldsymbol{r},t)$

Faraday‘s law: $$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial}{\partial t}\boldsymbol{B}(\boldsymbol{r},t)$$

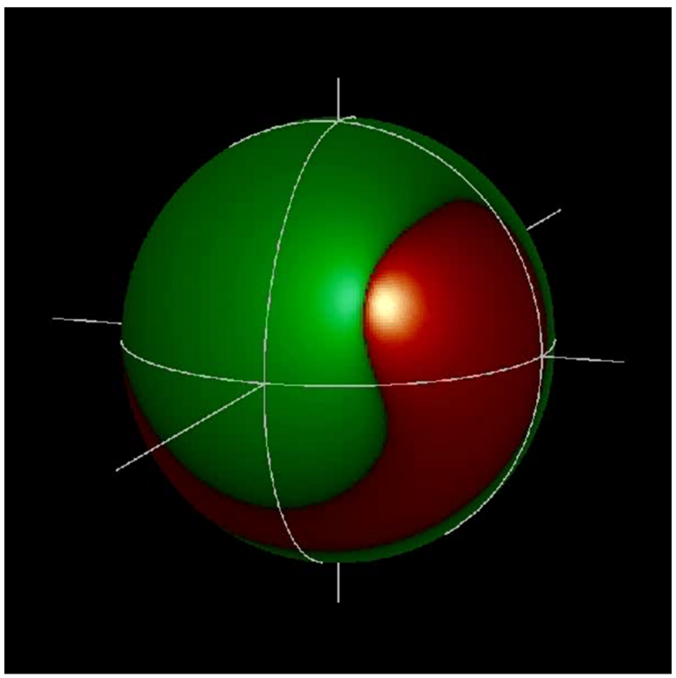
Closed curve: $$\oint_C \boldsymbol{E}(\boldsymbol{r},t)\, d\boldsymbol{l} = -\frac{d}{dt}\int_S \boldsymbol{B}(\boldsymbol{r},t)\, d\boldsymbol{A}$$

MR Detection

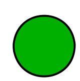
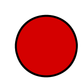
Perform integration of E field with conductive loop



$$U_{ind}(t) = \oint_C \boldsymbol{E}(\boldsymbol{r},t)\, d\boldsymbol{l} = -\frac{d}{dt}\int_S \boldsymbol{B}(\boldsymbol{r},t)\, d\boldsymbol{A}$$

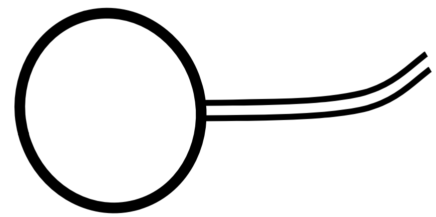


Magnetic flux through surface:

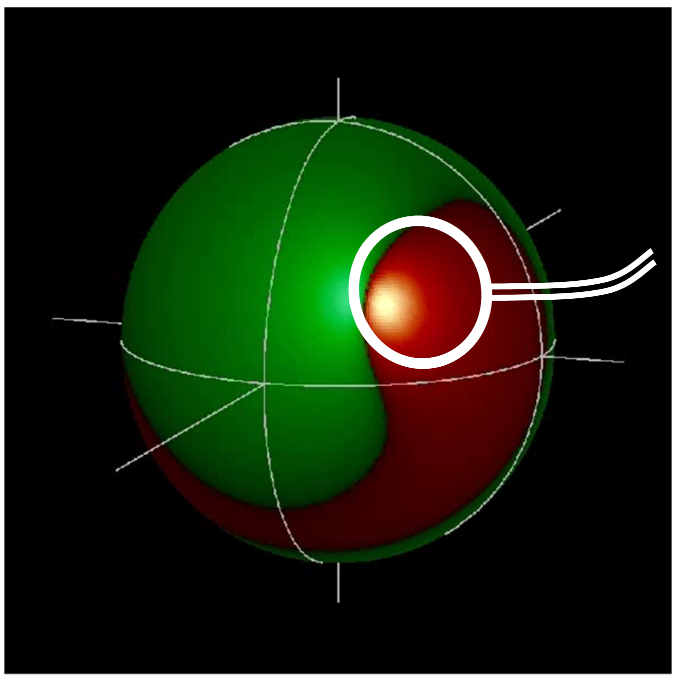
 Positive  Negative

MR Detection

Perform integration of E field with conductive loop



$$U_{ind}(t) = \oint_C \boldsymbol{E}(\boldsymbol{r},t)\, d\boldsymbol{l} = -\frac{d}{dt}\int_S \boldsymbol{B}(\boldsymbol{r},t)\, d\boldsymbol{A}$$

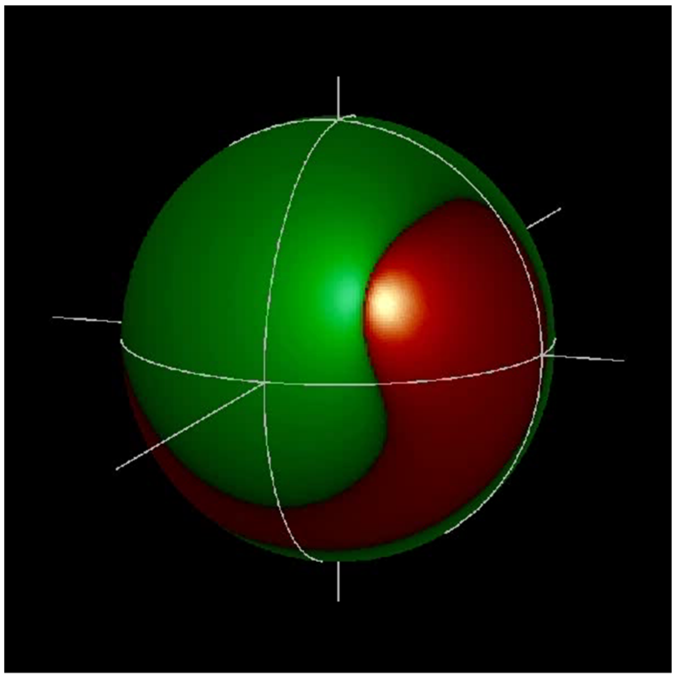


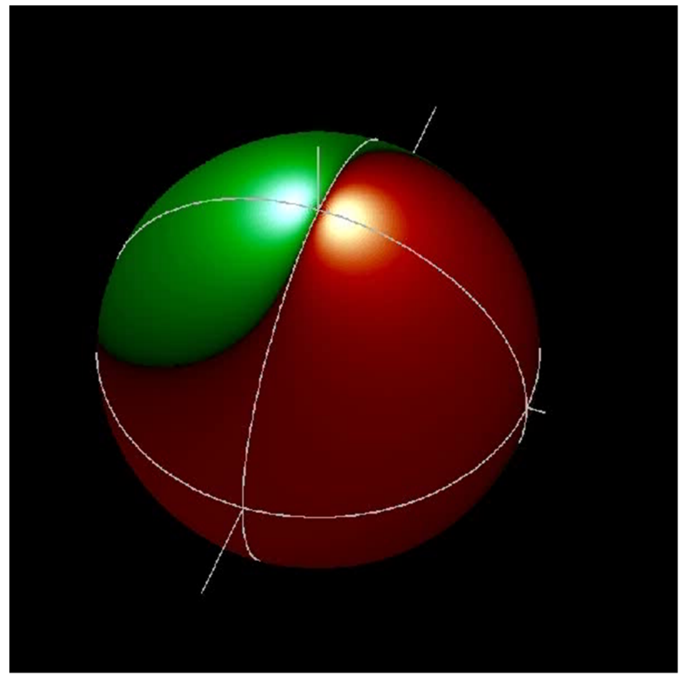
Magnetic flux through surface:

Positive Negative

MR Detection

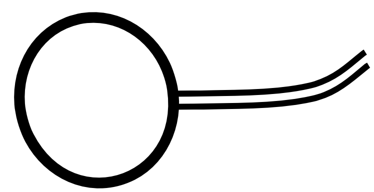
How much voltage does a precessing magnetic moment induce?





Induced voltage depends on

- position of the moment
- position, geometry of the detector coil

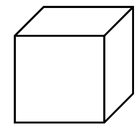


→ Coil sensitivity $s(\mathbf{r})$

Coil Sensitivity

How much voltage does a precessing magnetic moment induce?

Consider a small volume V at position $\boldsymbol{r}_0$



Magnetisation at this position: $\boldsymbol{M}(\boldsymbol{r}_0, t) = \hat{\boldsymbol{M}}(\boldsymbol{r}_0) e^{i\omega t}$

Magnetic moment: $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{M}}(\boldsymbol{r}_0) V$

generates magnetic field $\boldsymbol{B}(\boldsymbol{r}, t) = \hat{\boldsymbol{B}}(\boldsymbol{r}) e^{i\omega t}$

whose flux oscillation induces $U_{ind}(t) = -\frac{d}{dt} \int_{coil} \boldsymbol{B}(\boldsymbol{r}, t)\, d\boldsymbol{A}$

$$\hat{U}_{ind} = i\omega \int_{coil} \hat{\boldsymbol{B}}(\boldsymbol{r})\, d\boldsymbol{A}$$

Solve Maxwell equations for source position $\boldsymbol{r}_0$!

Reciprocity

Principle of reciprocity says roughly

« electromagnetic interaction between two points in space is the same in both directions »

For MRI: „Lorentz reciprocity“, requiring that all materials involved

- are linear, i.e.

$$\boldsymbol{M} = \chi_m \boldsymbol{H}, \quad \boldsymbol{P} = \chi_e \varepsilon_0 \boldsymbol{E}, \quad \boldsymbol{j} = \sigma \boldsymbol{E}$$

- with symmetric tensors χ_m, χ_e, σ

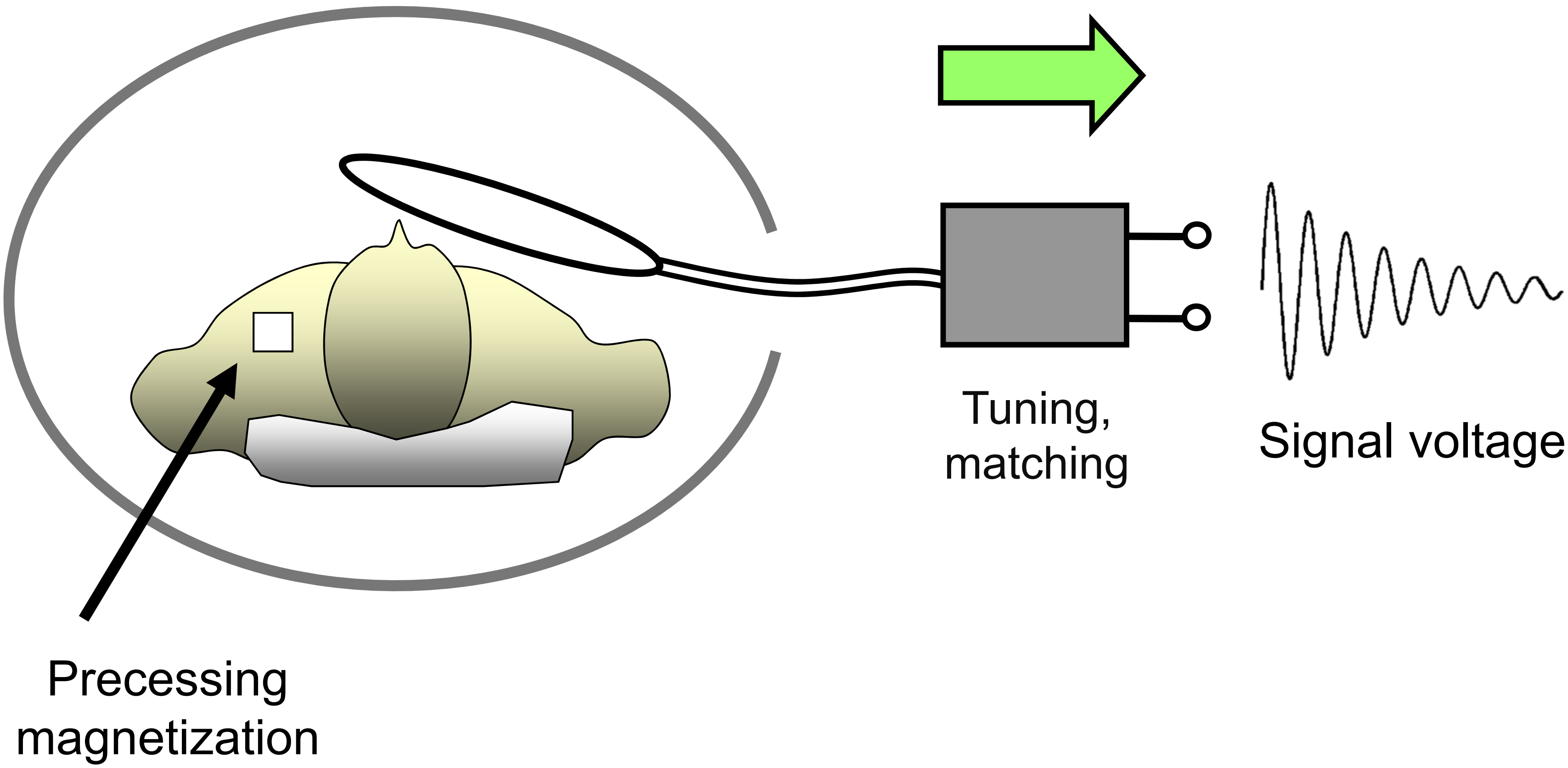
Violations: e.g., semiconductors, gyrotropism

Lorentz reciprocity holds for

- biological tissues
- MR detectors

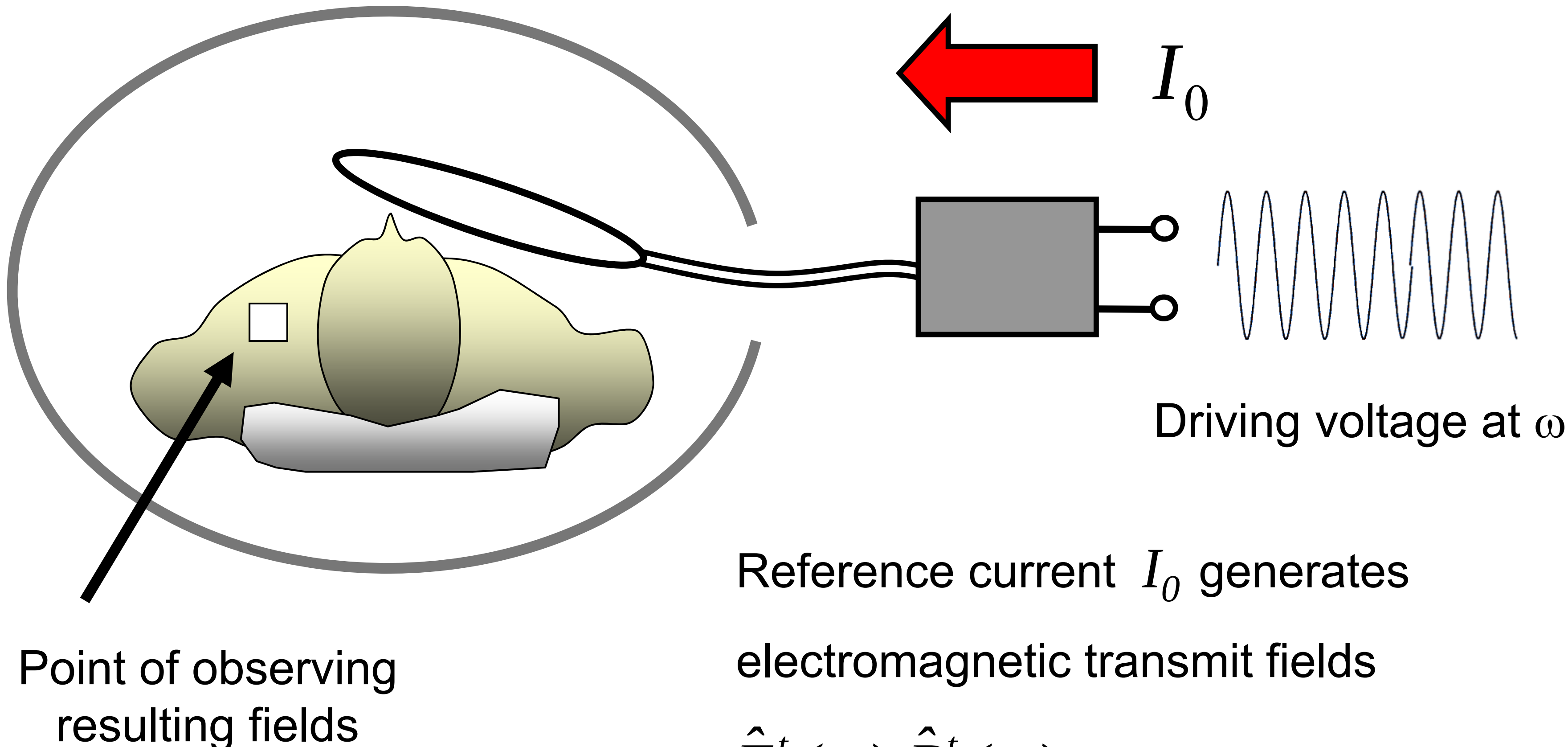
Reciprocity

Receive mode



Reciprocity

<u>Transmit mode</u>

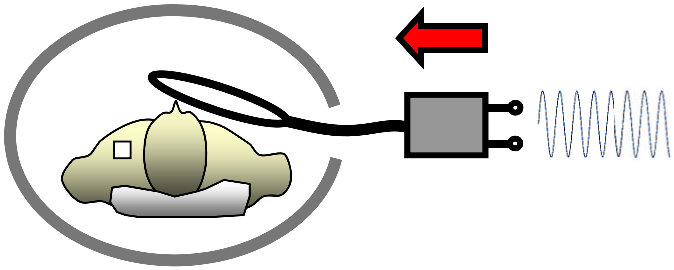


Reference current I_0 generates electromagnetic transmit fields

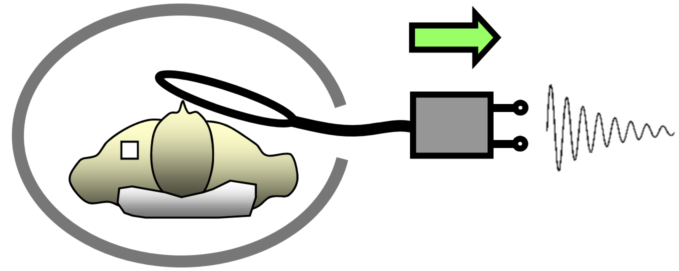
$$\hat{\boldsymbol{E}}^t(\boldsymbol{r}), \hat{\boldsymbol{B}}^t(\boldsymbol{r})$$

Reciprocity

Transmit mode



Receive mode



Magnetic transmit field

$$\hat{\boldsymbol{B}}^{t}(\boldsymbol{r}_0)$$

when driven with I_0

Signal voltage

$$U_{ind} = i\,\omega\, I_0^{-1}\,\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{B}}^{t}(\boldsymbol{r}_0)$$

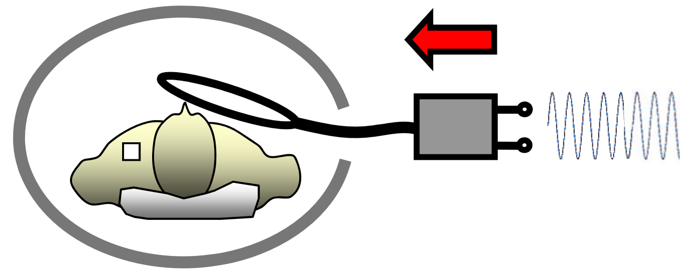
$$\hat{\boldsymbol{\mu}} = V\left(M_{xy}, -iM_{xy}, 0\right)^{T}$$

$$U_{ind} = M_{xy}V\, i\,\omega\, I_0^{-1}\left(\hat{B}_x^{t}(\boldsymbol{r}_0) - i\hat{B}_y^{t}(\boldsymbol{r}_0)\right)$$

Coil sensitivity $s(\boldsymbol{r}) = i\,\omega\, \underbrace{I_0^{-1}\left(\hat{B}_x^{t}(\boldsymbol{r}) - i\hat{B}_y^{t}(\boldsymbol{r})\right)}_{\text{called } B_1^{(-)}}$

Coil sensitivity

Still need to solve Maxwell equations, but only once



$$\nabla \times \boldsymbol{E} = -i\omega \boldsymbol{B} \qquad \nabla \cdot \boldsymbol{D} = \rho \qquad \boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + i\omega \boldsymbol{D} \qquad \nabla \cdot \boldsymbol{B} = 0 \qquad \boldsymbol{B} = \mu \boldsymbol{H}$$

Resulting $\boldsymbol{B}$, $\boldsymbol{E}$ are

- linearly polarized for low frequencies, small coils/samples, low σ
- elliptically polarized otherwise

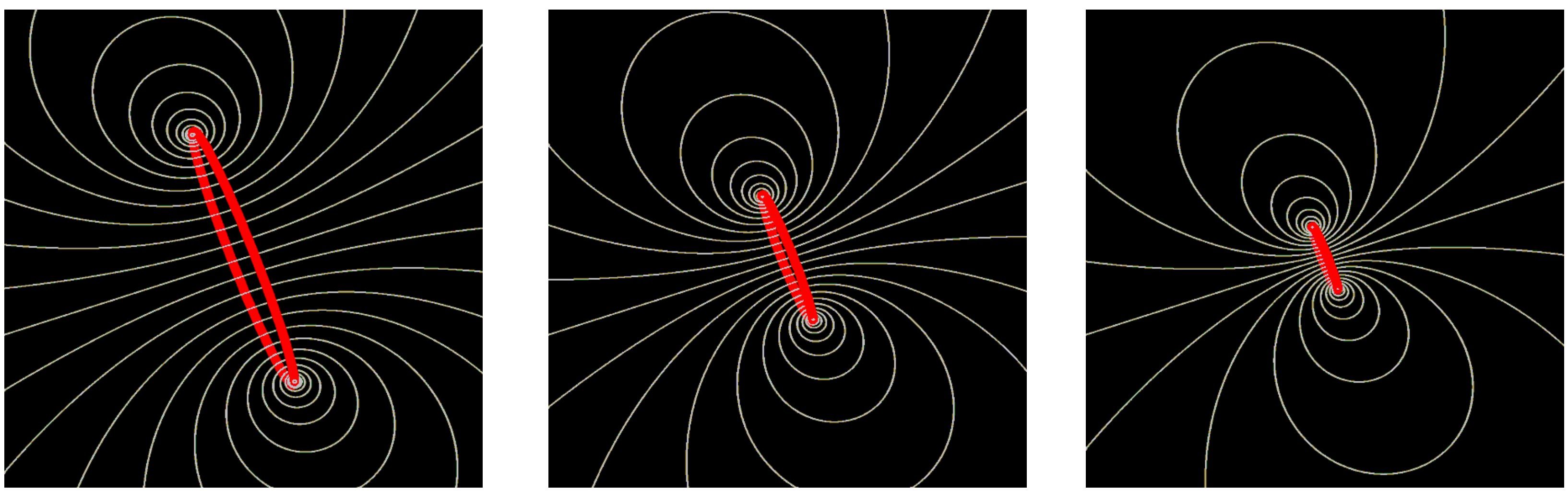
Coil sensitivity

Low-frequency (= quasi-stationary) approximation:

$$\nabla \times \widehat{\boldsymbol{H}} = \widehat{\boldsymbol{j}}_{coil} \qquad \nabla \cdot \widehat{\boldsymbol{B}} = 0$$ Magnetostatics

Solution: $$\widehat{\boldsymbol{B}}(\boldsymbol{r}) = \frac{\mu \hat{I}}{4\pi} \oint_{coil} \frac{d\boldsymbol{l} \times (\boldsymbol{r} - \boldsymbol{r}_l)}{|\boldsymbol{r} - \boldsymbol{r}_l|^3}$$ Biot-Savart law

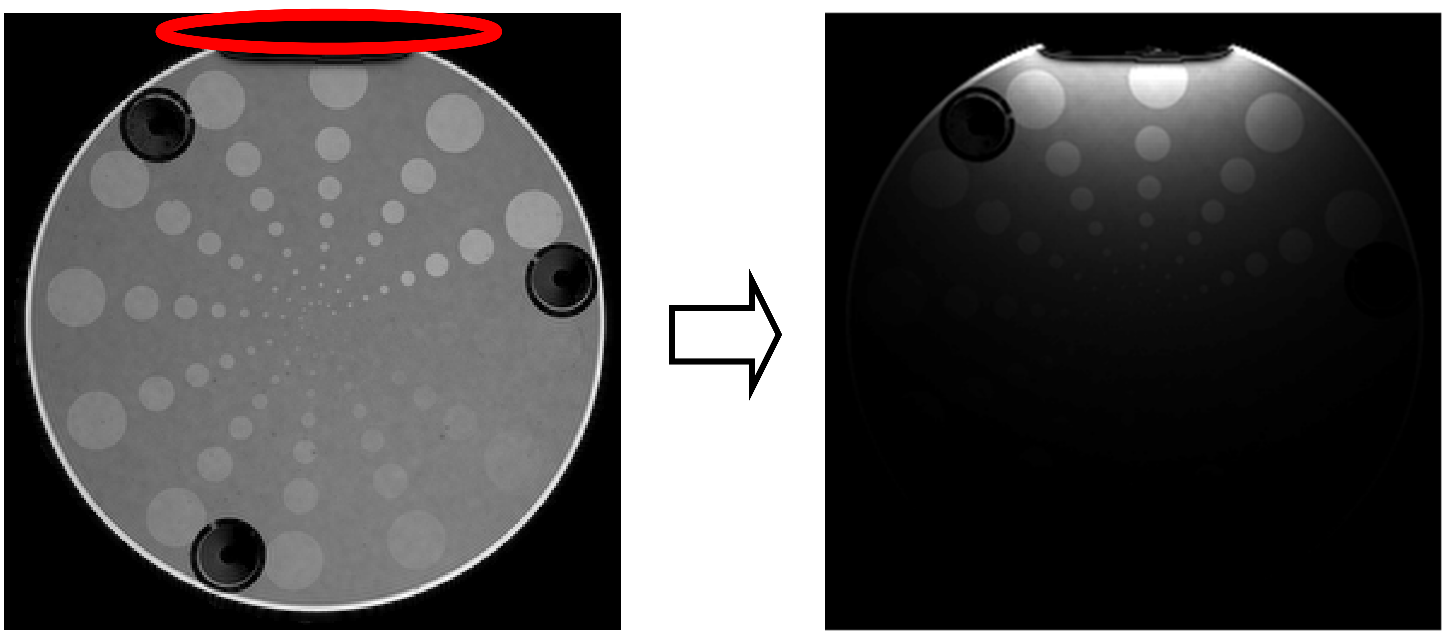
$\boldsymbol{B}$ of circular loop coils in the transverse plane:



∅ = 15 cm ∅ = 10 cm ∅ = 5 cm

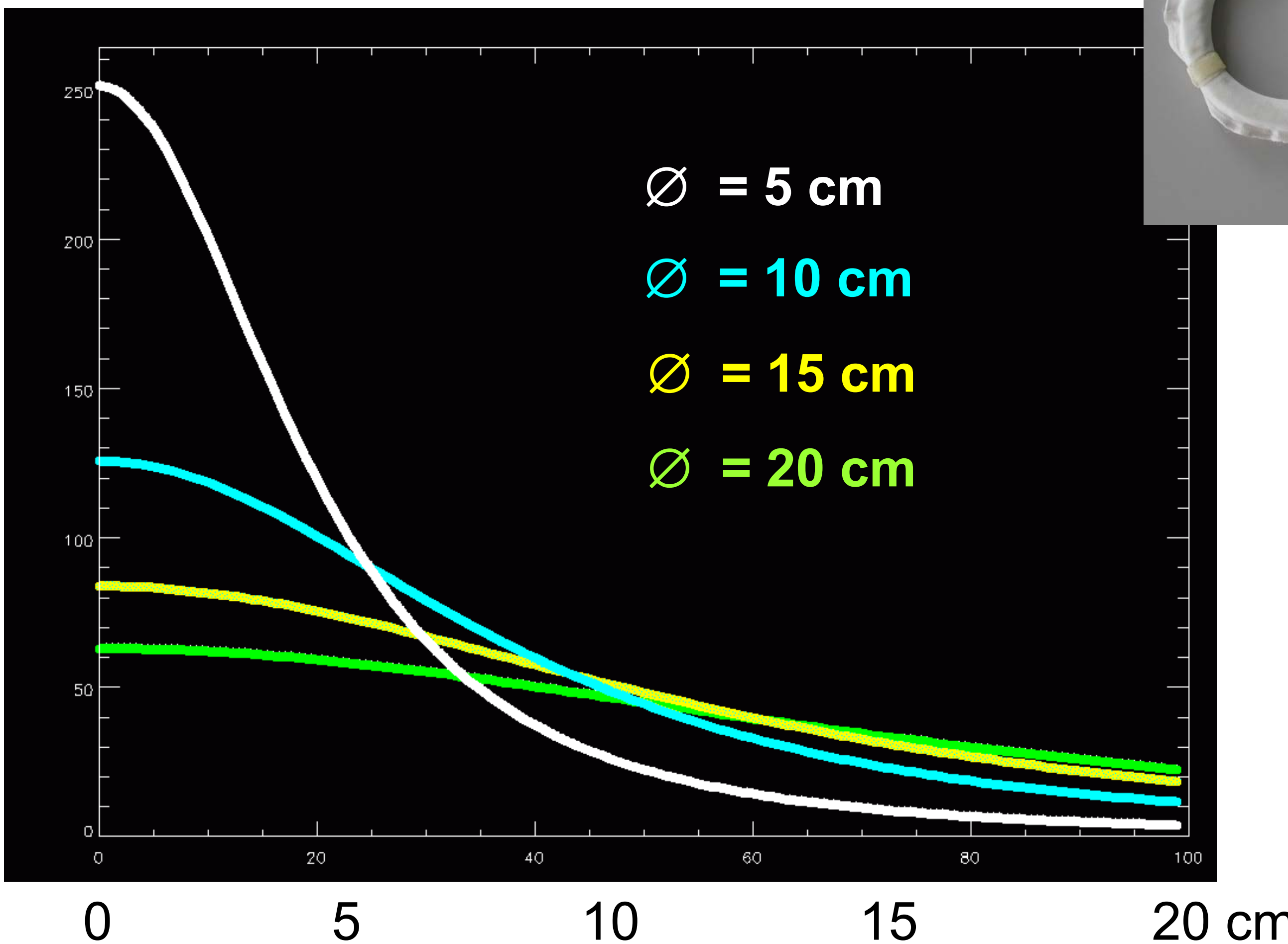
Coil sensitivity

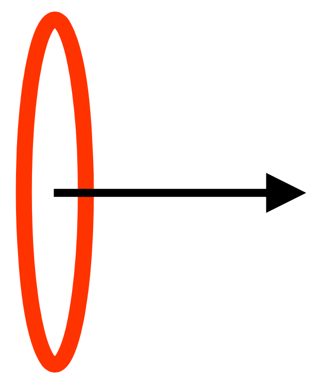
Surface coil
63 MHz (^1H at 1.5 T)



Coil sensitivity

Relative sensitivity

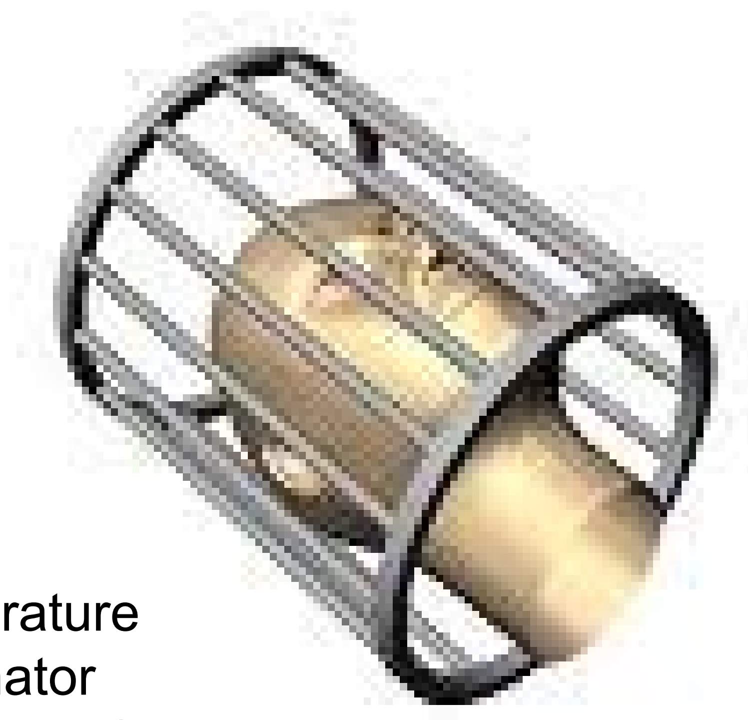




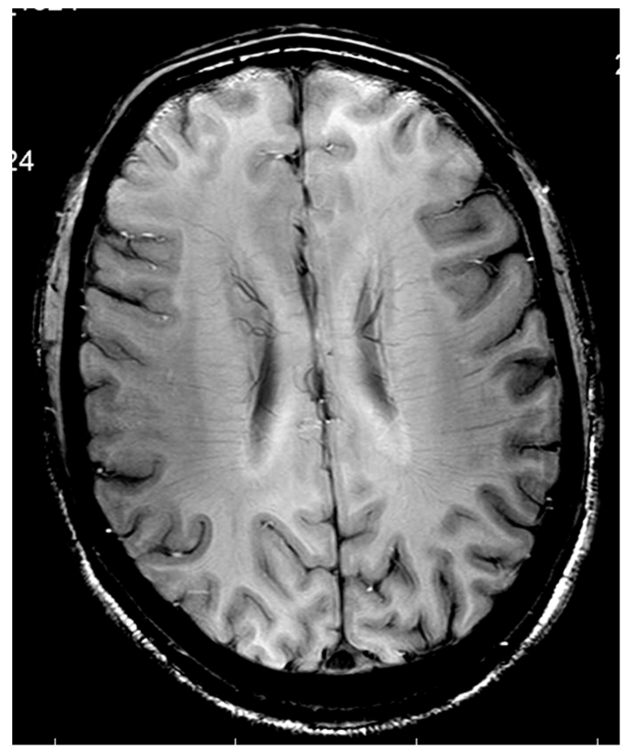
Coil sensitivity

Surface loop coil is the simplest MR detector

Large variety of more advanced designs exist



Quadrature
resonator
('birdcage')



Coil sensitivity

Low-frequency (= quasi-stationary) approximation:

Solution: $$\hat{\boldsymbol{B}}(\boldsymbol{r}) = \frac{\mu \hat{I}}{4\pi} \oint_{coil} \frac{d\boldsymbol{l} \times (\boldsymbol{r} - \boldsymbol{r}_l)}{|\boldsymbol{r} - \boldsymbol{r}_l|^3}$$ Biot-Savart law

Holds well for biological samples with size « wavelength

Approximate RF wavelengths in human tissue:

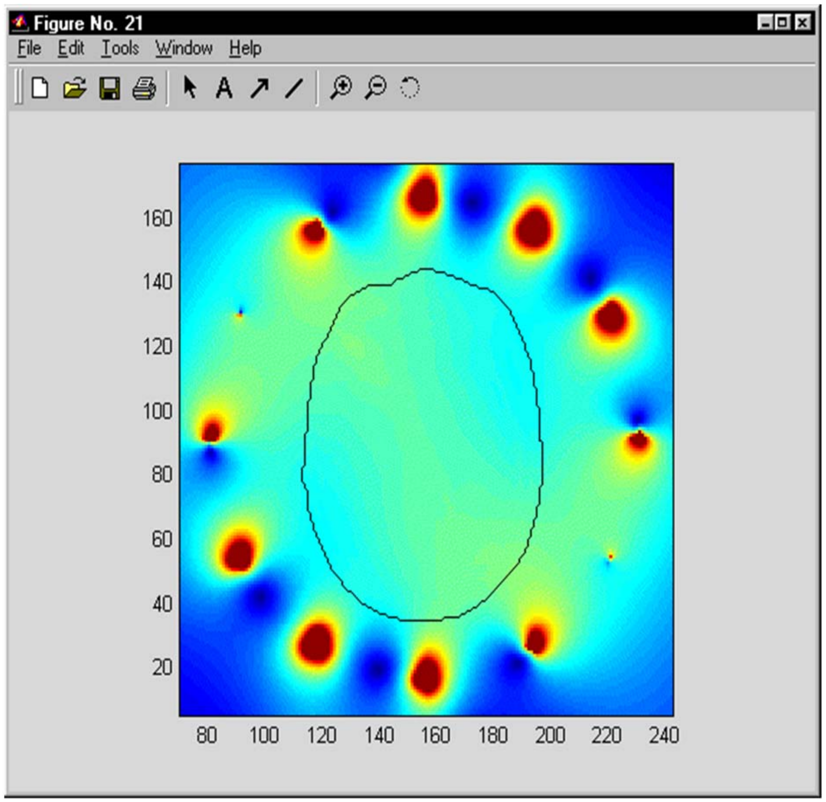
B_0	$\omega_L/2\pi$ (^{1}H)	Wavelength
1.5 T	64 MHz	50 cm
3.0 T	128 MHz	27 cm
7.0 T	298 MHz	13 cm

Full-wave treatment of Maxwell equations increasingly necessary

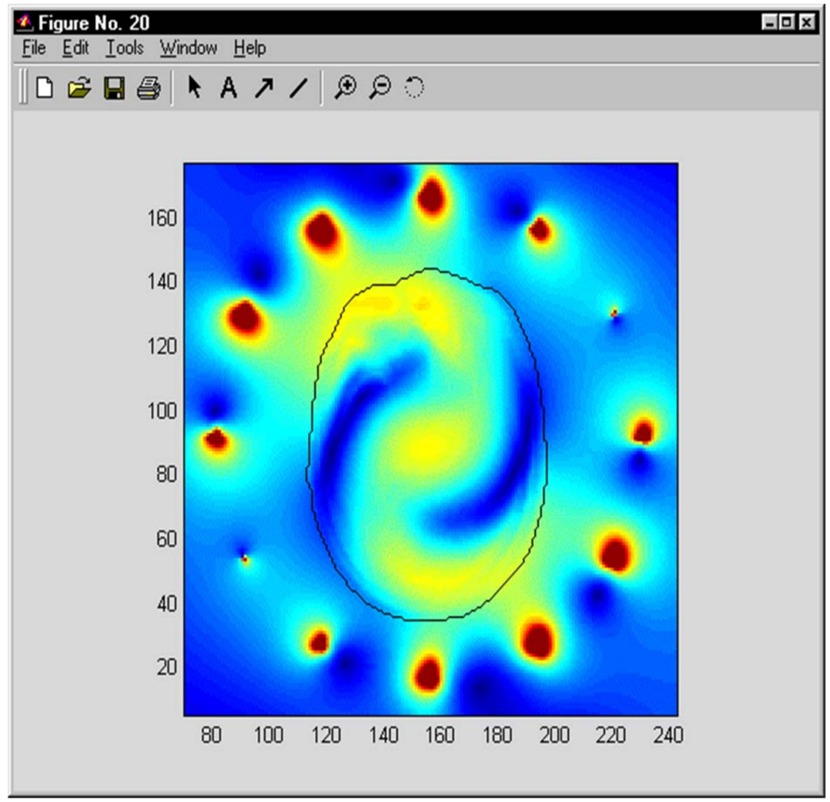
Coil sensitivity

FDTD (Finite Difference Time Domain) calculation of magnetic transmit field

Human Head, 64 MHz



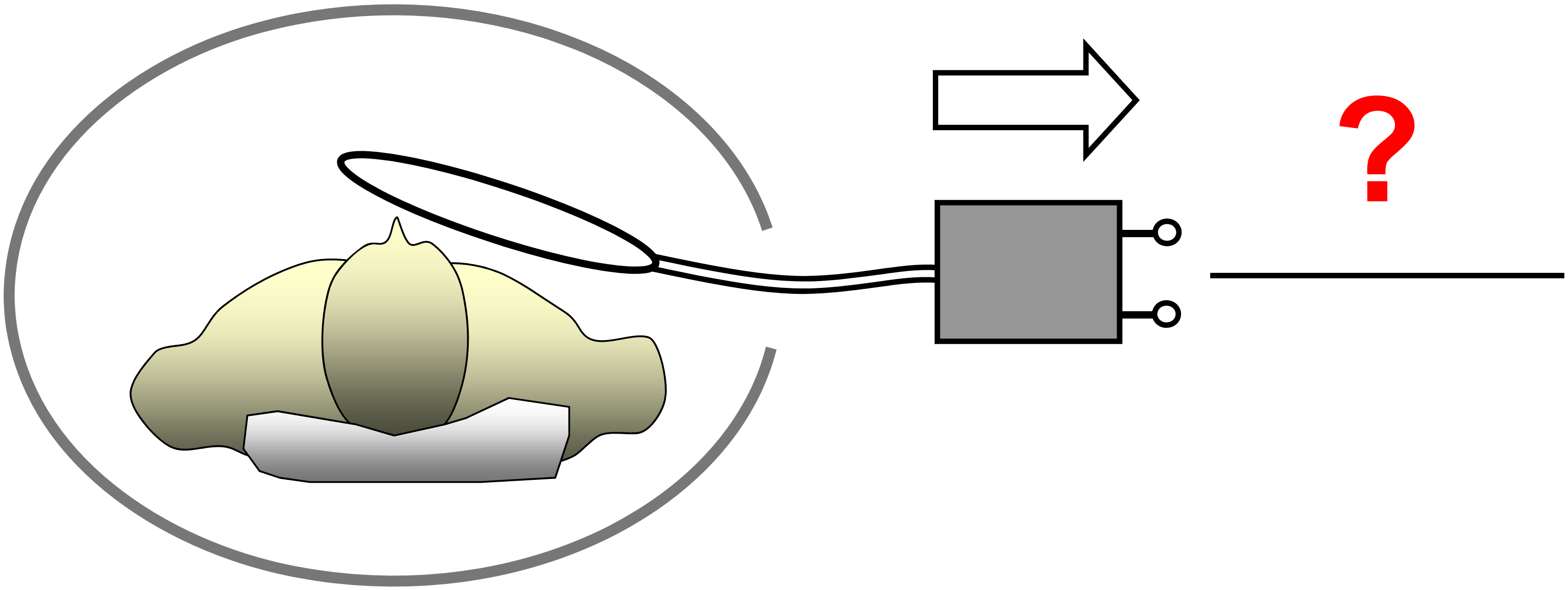
Human Head, 300 MHz



Courtesy M.B. Smith, Q. Yang, C. Collins, Pennsylvania State University

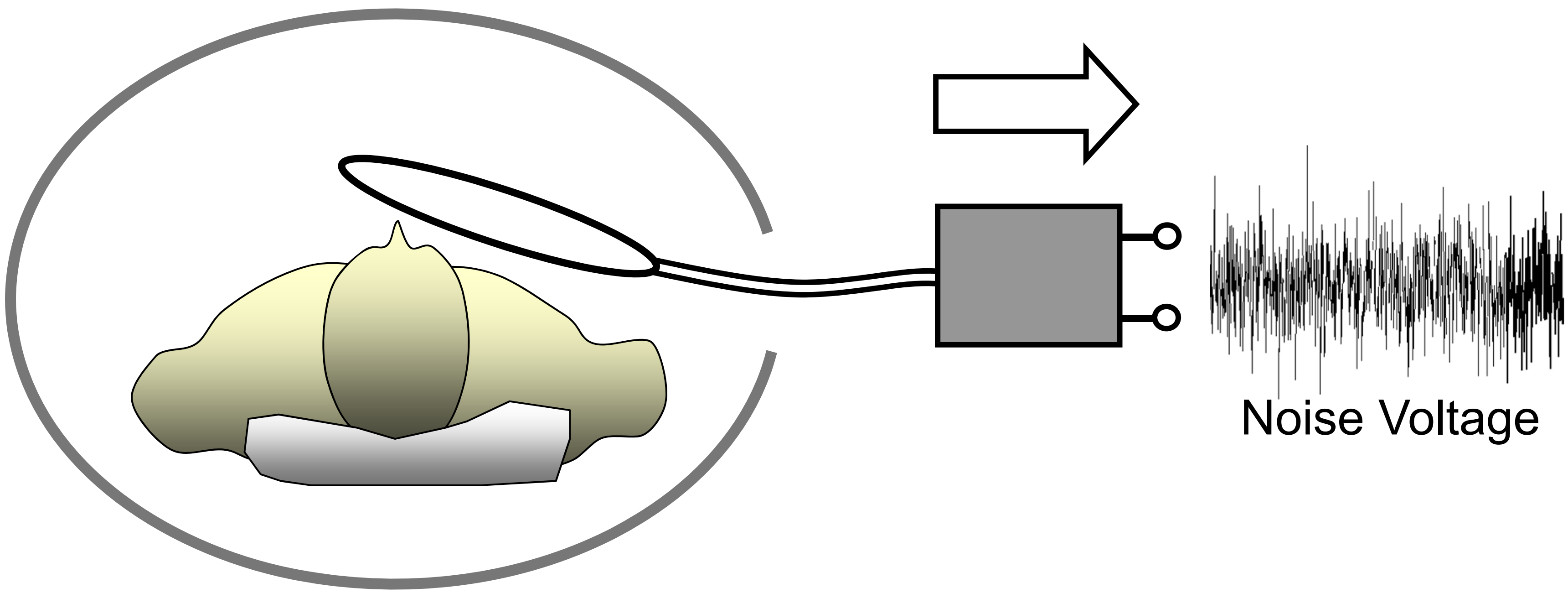
Noise

What if there is no signal ($M_{xy} = 0$ everywhere) ?

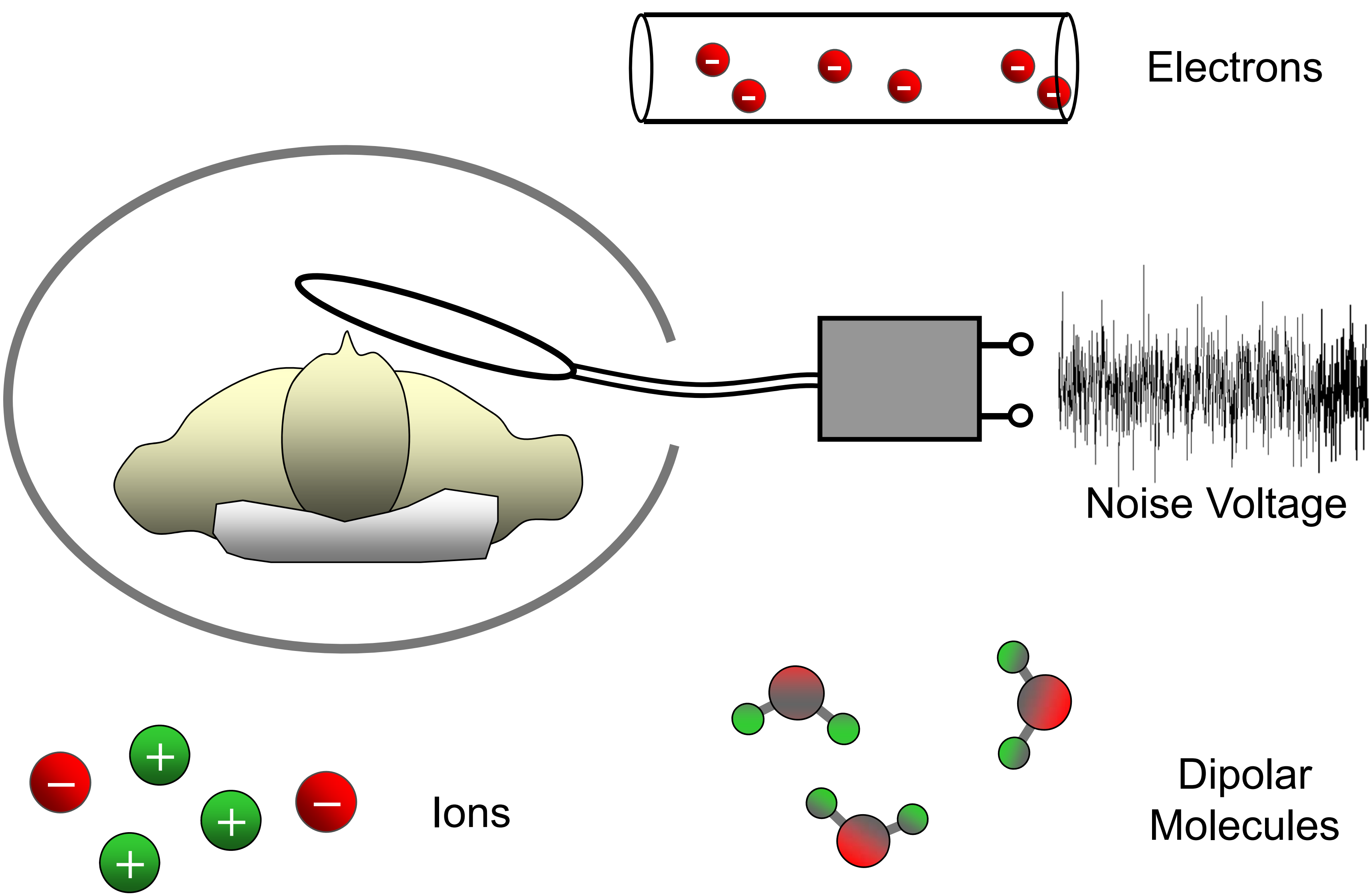


Noise

What if there is no signal ($M_{xy} = 0$ everywhere) ?

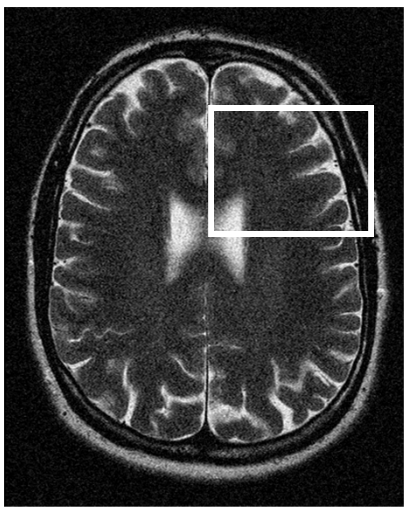


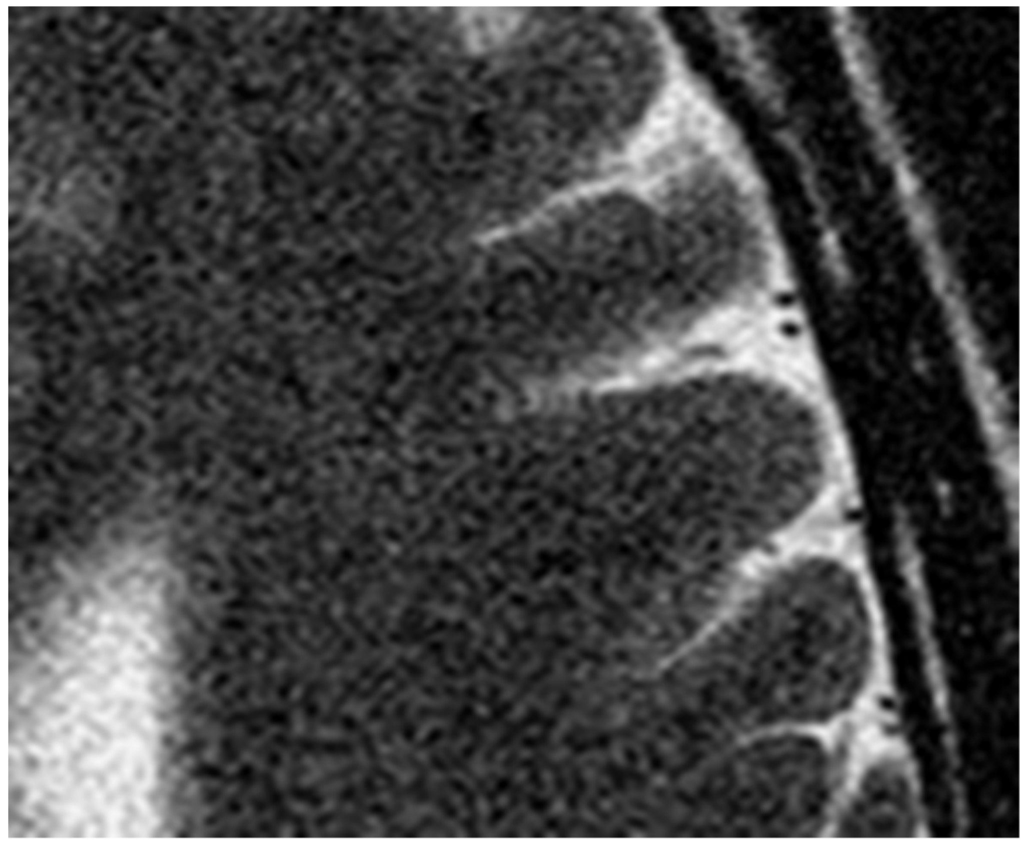
Noise



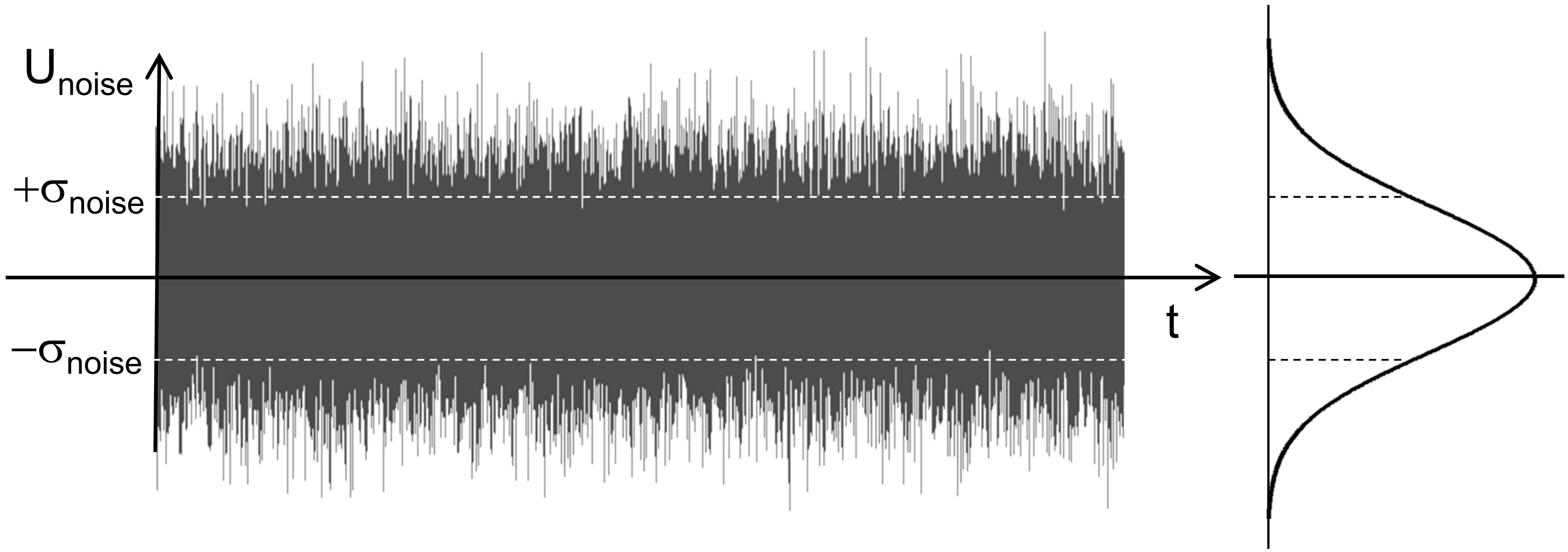
Thermal noise

- Perturbs images, spectra
- Reduces information content
- Limits usefulness of MR data
- A fundamental challenge in MR





Thermal noise



Gaussian noise statistics:

$$\overline{U_{noise}(t)} = 0$$ zero mean

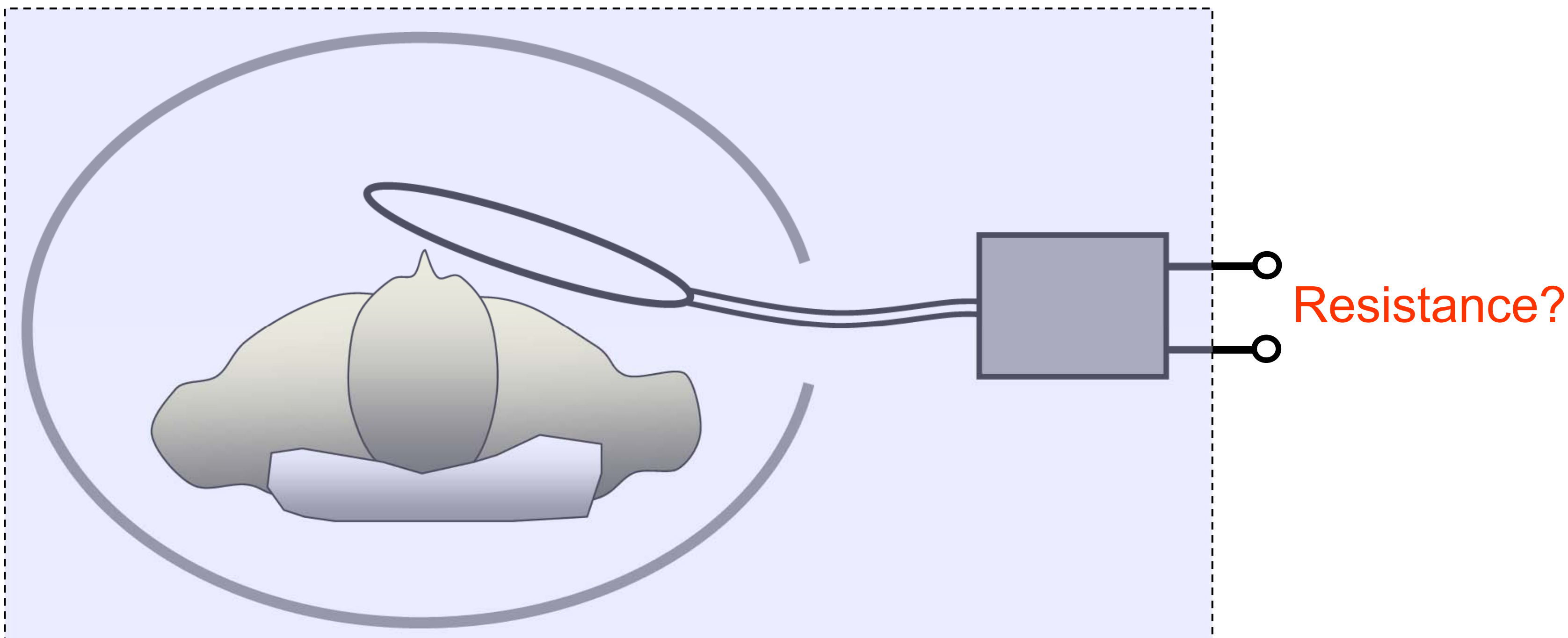
$$\overline{U^2_{noise}(t)} = \sigma^2_{noise}$$ variance = key quantity

Thermal noise

Voltage noise of a resistor (Johnson, Nyquist):

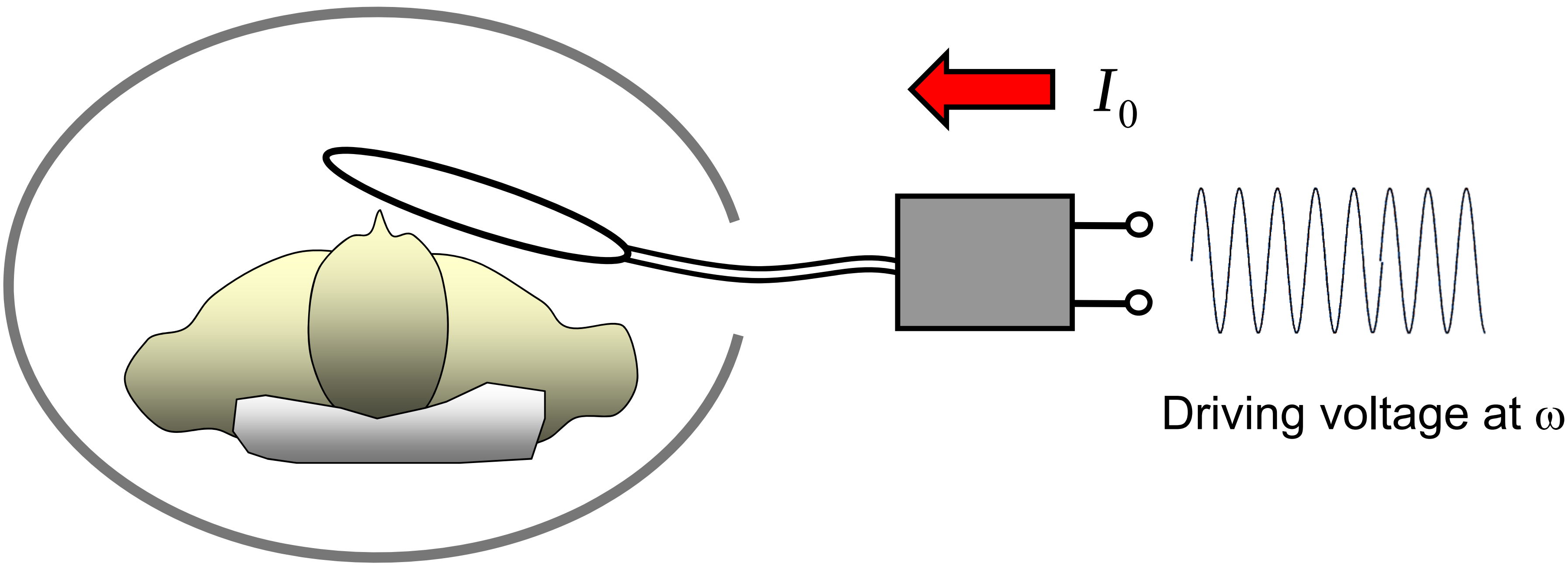
$$\sigma^2_{noise} = 4\, k_B T\ R\ BW$$

k_B	Boltzmann constant
T	temperature
R	resistance
BW	bandwidth



Thermal noise

Study power dissipation when transmitting with current I_0 $\quad (P = R I_0^2)$



Current I_0 generates fields $\quad \boldsymbol{E}(\boldsymbol{r},t) = \hat{\boldsymbol{E}}(\boldsymbol{r})e^{i\omega t}, \ \boldsymbol{B}(\boldsymbol{r},t) = \hat{\boldsymbol{B}}(\boldsymbol{r})e^{i\omega t}$

Dissipated power: $\quad P = \int \boldsymbol{j}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r})\, dV = \int \sigma(\boldsymbol{r}) \left|\boldsymbol{E}(\boldsymbol{r})\right|^2 dV$

Thermal noise

Dissipated power: $$P = \int \boldsymbol{j}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r})\, dV = \int \sigma(\boldsymbol{r}) \left|\boldsymbol{E}(\boldsymbol{r})\right|^2 dV$$

Corresponds to equivalent resistance: $$R = \frac{P}{I_0^2} = \int \sigma(\boldsymbol{r}) \frac{\left|\boldsymbol{E}(\boldsymbol{r})\right|^2}{I_0^2}\, dV$$

Using Johnson/Nyquist: $$\sigma^2_{noise} = 4\, k_B T\; BW\, R$$

Yields noise variance: $$\sigma^2_{noise} = 4\, k_B T\, BW \int \sigma(\boldsymbol{r}) \frac{\left|\boldsymbol{E}(\boldsymbol{r})\right|^2}{I_0^2}\, dV$$

Thermal noise

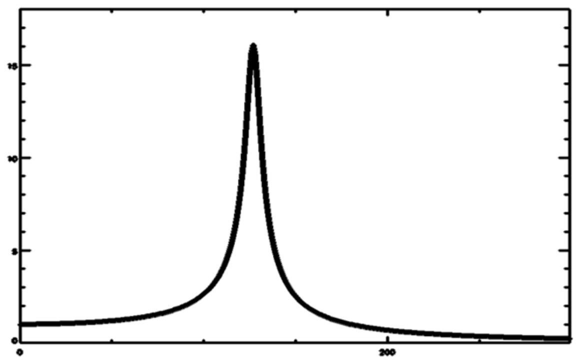
Beware: $\sigma^2_{noise} = 4\, k_B T\ BW\, R$ uses two mild assumptions:

- Resistance R is constant across bandwidth

Receiver coil is a tuned circuit

$\boldsymbol{E}(\boldsymbol{r})$ frequency-dependent

$\sigma(\boldsymbol{r})$ frequency-dependent



- Temperature is constant across the setup

Body: 310 K
Coil: 293 K (or lower: 77 K for nitrogen cooling)
Environment: 293 K or …

Thermal noise

Generalize analysis to accommodate $T(\boldsymbol{r}), \sigma(\omega,\boldsymbol{r}), \boldsymbol{E}(\omega,\boldsymbol{r})$

Integral form:

$$P = \int \sigma(\boldsymbol{r}) \left|\boldsymbol{E}(r)\right|^2 dV$$

$$R = \frac{P}{I_0^2}$$

$$\sigma_{noise}^2 = 4\, k_B T\ BW\, R$$

Differential form:

$$\Longrightarrow \quad dP(\omega,\boldsymbol{r}) = \sigma(\omega,\boldsymbol{r}) \left|\boldsymbol{E}(\omega,\boldsymbol{r})\right|^2 \frac{d\omega}{2\pi} dV$$

$$\Longrightarrow \quad dR(\omega,\boldsymbol{r}) = \frac{dP(\omega,\boldsymbol{r})}{I_0^2}$$

$$\Longrightarrow \quad d(\sigma_{noise}^2) = 4\, k_B T(\boldsymbol{r})\, dR(\boldsymbol{r},\omega)$$

$$\boxed{\sigma_{noise}^2 = 4k_B \int\!\!\int T(\boldsymbol{r})\, \sigma(\omega,\boldsymbol{r}) \frac{\left|\boldsymbol{E}(\omega,\boldsymbol{r})\right|^2}{I_0^2} \frac{d\omega}{2\pi} dV}$$

Thermal noise

To recover „Johnson/Nyquist“ form:

Break up spatial integration into parts of the setup.

Effective resistances: $$R_{sample}^{eff} = \frac{1}{2\pi\, BW} \int\limits_{sample} \int\limits_{\omega_1}^{\omega_2} \sigma(\omega, \boldsymbol{r}) \frac{\left|\boldsymbol{E}(\omega, \boldsymbol{r})\right|^2}{I_0^2}\, d\omega\, dV$$

$$\boxed{\sigma_{noise}^2 = 4\, k_B BW \left(R_{sample}^{eff} T_{sample} + R_{coil}^{eff} T_{coil} + R_{env}^{eff} T_{env} \right)}$$

Usually dominant

Significant for low frequency, small coils

Usually negligible

Thermal noise

$$\sigma^2_{noise} = 4\, k_B\, BW(\ T_{Sample}\ R^{eff}_{sample}\ +\ T_{coil}\ R^{eff}_{coil}\ +\ T_{env}\ R^{eff}_{env}\ )$$

1. Reduce bandwidth BW

    - comes at the expense of scan time, as we will see

2. Reduce temperature T

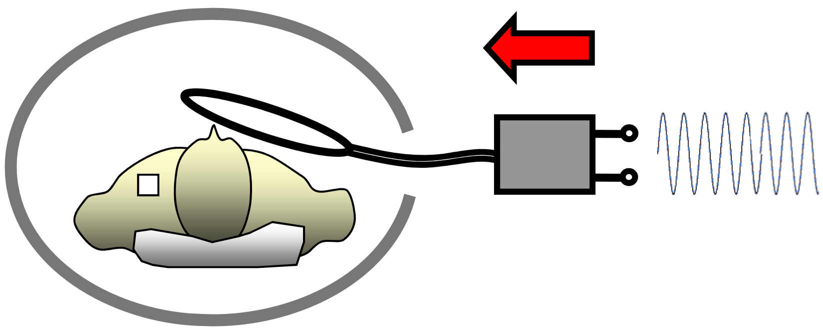
    - practical only for coils and circuitry

3. Reduce resistance R

    - copper, silver for coil conductor
    - cool coils, use superconductor

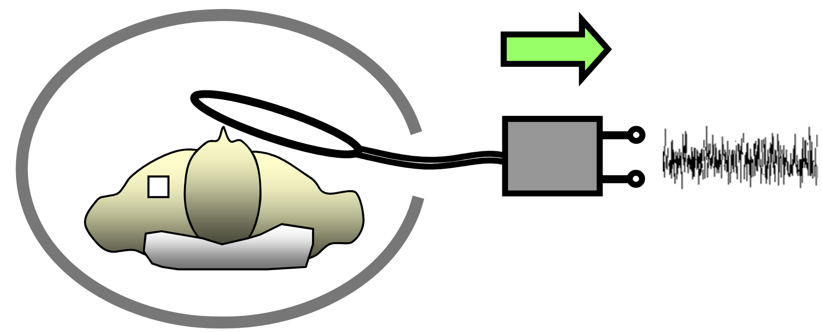
Thermal noise

Transmit mode



Voltage at terminals generates thermal energy

Receive mode



Thermal agitation causes voltage at terminals

Noise variance:

$$d\left(\sigma^2_{noise}\right) = 4k_B T(\boldsymbol{r})\ \sigma(\omega,\boldsymbol{r})\ \frac{\left|\boldsymbol{E}(\omega,\boldsymbol{r})\right|^2}{I_0^2}\frac{d\omega}{2\pi}\,dV$$

Thermal energy available for conversion

Density + mobility of charges

Coil‘s noise sensitivity

Signal detection

Key characteristics of a Faraday MRI detector:

	Spatial sensitivity to MR signal	Noise variance
Formal expression	$s(\boldsymbol{r}) = \dfrac{B_x(\boldsymbol{r}) - iB_y(\boldsymbol{r})}{I_0}$	$\sigma^2_{noise} = 4k_B \int\int T(\boldsymbol{r})\,\sigma(\omega,\boldsymbol{r}) \dfrac{\lvert \boldsymbol{E}(\omega,\boldsymbol{r})\rvert^2}{I_0^2} \dfrac{d\omega}{2\pi}\, dV$
Sources	spins = magnetic dipoles	electric charges, electric dipoles
Relevant transmit field component	magnetic, $\boldsymbol{B}$ „signal sensitivity“	electric, $\boldsymbol{E}$ „noise sensitivity“

MR sensitivity and noise statistics govern

- detector design
- image reconstruction