ETH zürich

Biomedical Imaging - Nuclear imaging (I) (1)

Rem: X-Rays $10\,pm \le \lambda \le 10\,nm$

γ-Rays $\lambda < 10\,pm$

Photon energy: $E_{ph} = h \cdot \nu = h \cdot \frac{c}{\lambda}$

Radiotracer - instable isotopes

e.g. 18F-FDG

(fluorodeoxyglucose)

- sugar analog,
- uptake in energy-hungry cells → brain example

$H_2{}^{15}O$

(radioactive water)

- freely diffusible,
- metabolically inert → cardiac example

Radioactive nuclei

$^{A}_{Z}X$ (A - mass, Z - number)

$A < 50$ if $\#n = \#p$

$A \geq 50$ if $\#n > \#p$ then stable

otherwise unstable

- **Historical notes**

1896 Bequerel
- Uranium

1898 Curie
- Radium

1911 Rutherford
- p, n, e

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- **α - Decay.** $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$

Half Value Layer (HVL) ~1mm

- **β⁻ - Decay.** $^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e + \bar{\nu}$
(Negation) antineutrino

HVL ~ 4mm

- **β⁺ - Decay.** $^{12}_{7}N \rightarrow ^{12}_{6}C + ^{0}_{+1}e + \nu$
(Positron) neutrino

HVL ~ 4mm

Biological half-life time

- $$N(t_{1/2\,eff}) = \frac{N_0}{2} = N_0 e^{-(\lambda + \lambda_{Bio}) t_{1/2\,eff}}$$

$$\ln(2) = \left(\frac{\ln(2)}{t_{1/2}} + \frac{\ln(2)}{t_{1/2\,Bio}}\right) \cdot t_{1/2\,eff}$$

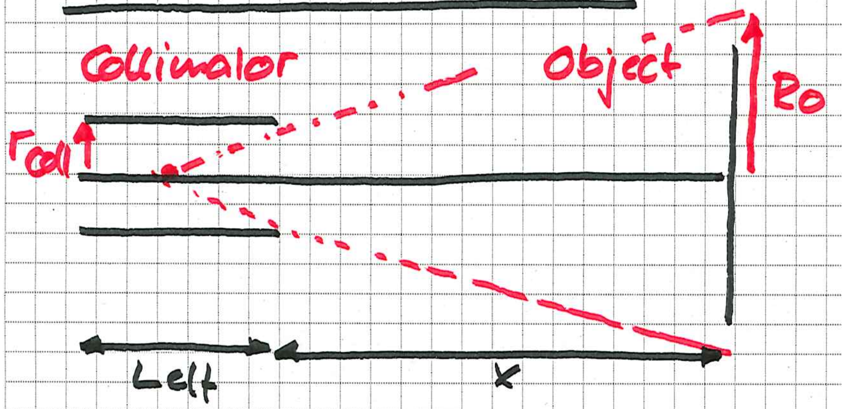
$$\Rightarrow \boxed{t_{1/2\,eff} = \frac{t_{1/2\,Bio} \cdot t_{1/2}}{t_{1/2} + t_{1/2\,Bio}}}$$

Radioisotopes

$$^{99m}Tc \rightarrow ^{99}Tc + \gamma \qquad E_\gamma = 140\,keV$$

metastable = γ emission + internal conversion

Collimation and PSF



$$\frac{R_O}{x + \frac{L_{eff}}{2}} = \frac{r_{col}}{\frac{L_{eff}}{2}}$$

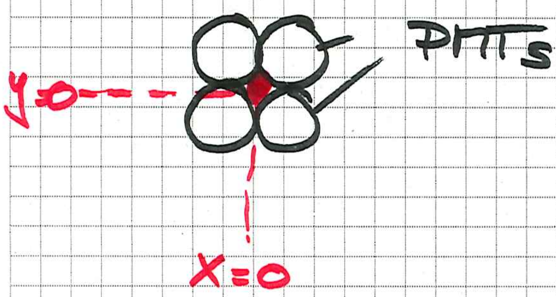
PSF Full-width-at-half-maximum (FWHM)

$$PSF(x) = \Delta r_{col}(x) = \left(\frac{R_O - r_{col}}{2} + r_{col}\right) \cdot 2$$

$$= R_O + r_{col} = \frac{2 r_{col}}{L_{eff}}\left(x + \frac{L_{eff}}{2}\right) + r_{col}$$

$$= \frac{2 r_{col}}{L_{eff}} x + 2 r_{col} \boxed{= \frac{2 r_{col}}{L_{eff}}(x + L_{eff})}$$

Positioning network

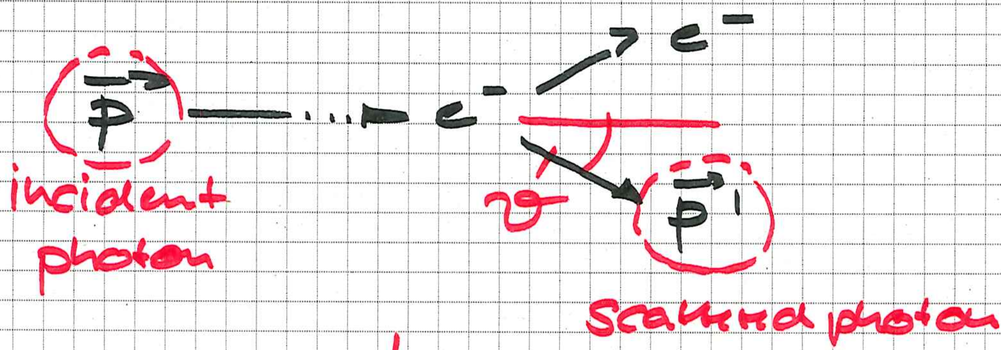


$$x = \frac{1-1}{2} = 0$$

$$y = \frac{1-1}{2} = 0$$

→ 4 PMTs determine location of event
- improved sensitivity + accuracy

Scatter detection



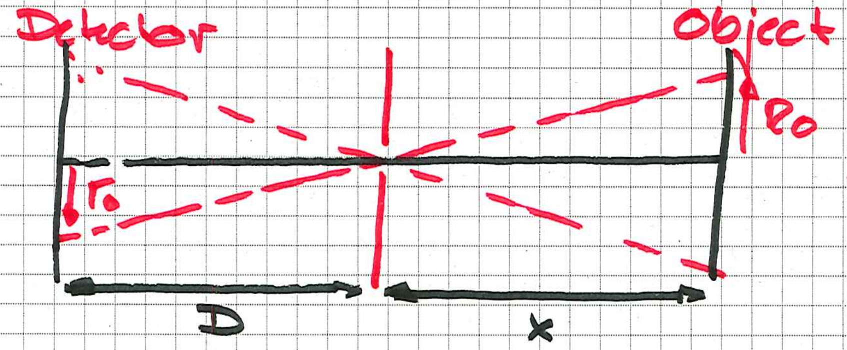
$$\lambda - \lambda' = \frac{h}{m_e c}(1 - \cos\vartheta)$$

$$\rightarrow \boxed{E - E' = h \cdot c \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right)}$$

→ Scattered photon have larger wavelength λ'

⚠ filter out

Pinhole collimation



$$\frac{R_0}{x} = \frac{r_0}{D}$$

Positron emitters

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$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}X + \underbrace{e^+}_{\text{positron}} + \nu$$

e.g. 18F

$${}^{18}_{8}O + {}^{1}_{1}p \rightarrow \underbrace{{}^{18}_{9}F}_{\text{Half-life} \sim 110\,\text{min}} + {}^{1}_{0}n$$

$$\underbrace{e^+ + e^-}_{\text{annihilation}} \longrightarrow \underbrace{\gamma + \gamma}_{2\ \gamma\text{-photons (511 keV)}}$$

Coincidence detection

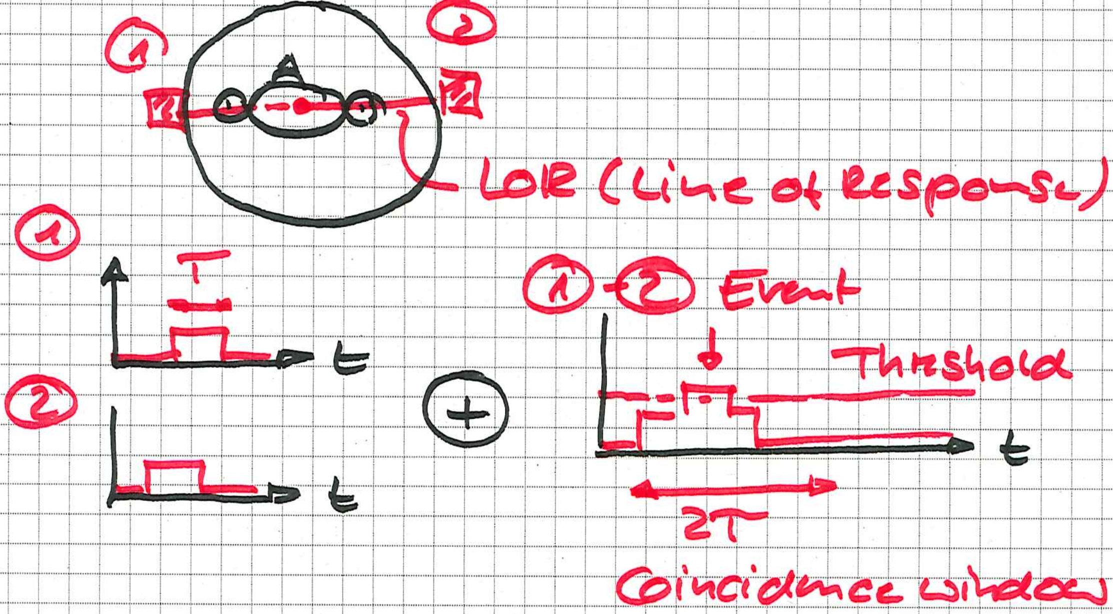

Image reconstruction

- **Activity** $S_\varphi = \int act(s)\,ds$

$\longrightarrow$ CT recon approach

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Point-spread function

(6)

- Activity in iso-centre $\boxed{FWHM = \frac{w}{2}}$

  w... detector width

- Activity off centre $\boxed{FWHM = w \frac{r+x}{2r}}$

  r... radius detector ring

  x... off centre

Attenuation correction

- Activity + attenuation

$$S_\varphi = \int act(s)\,ds \cdot \underbrace{e^{-\int \mu(s)\,ds}}_{\text{Attenuation due to Compton scatter}}$$

- Attenuation correction
  - measure $\int \mu(s)\,ds$ using e.g. CT
  - convert $\mu_{CT} \rightarrow \mu_{511keV}$
  - multiply $S_\varphi \cdot \underbrace{e^{\int \mu_{511keV}(s)\,ds}}_{\text{Attenuation correction}}$