Micro and Nano-Tomography of Biological Tissues

Marco Stampanoni and Kevin Mader

Swiss Light Source, Paul Scherrer Institut
Institute for Biomedical Engineering, University and ETH Zürich

ETH-227-0965-00 L

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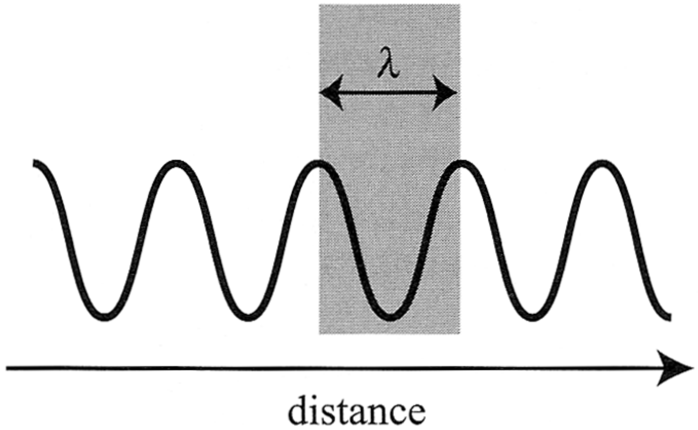
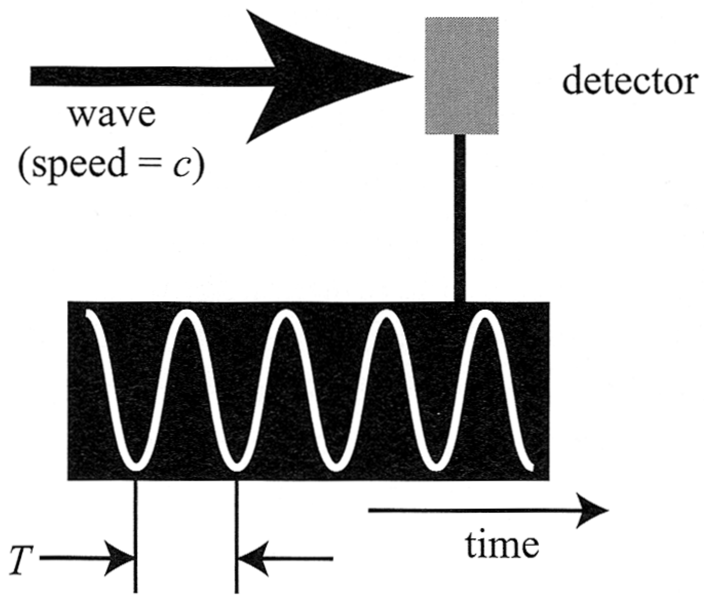
  - Mirrors (a few hints)
  - Focusing devices (a few hints as well)
  - Gratings monochromators
  - Crystal monochromators

Wavelength - Period - Frequency

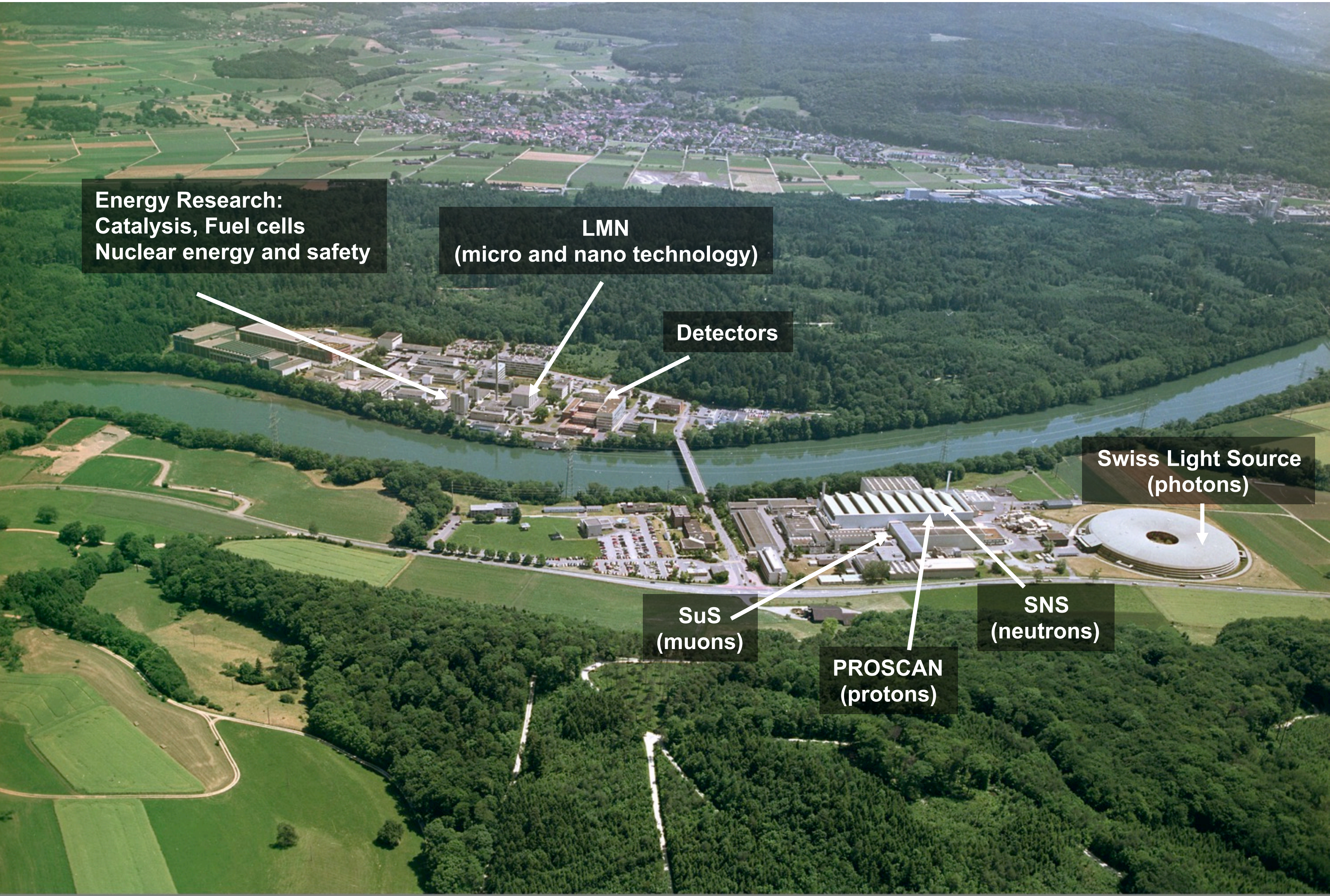




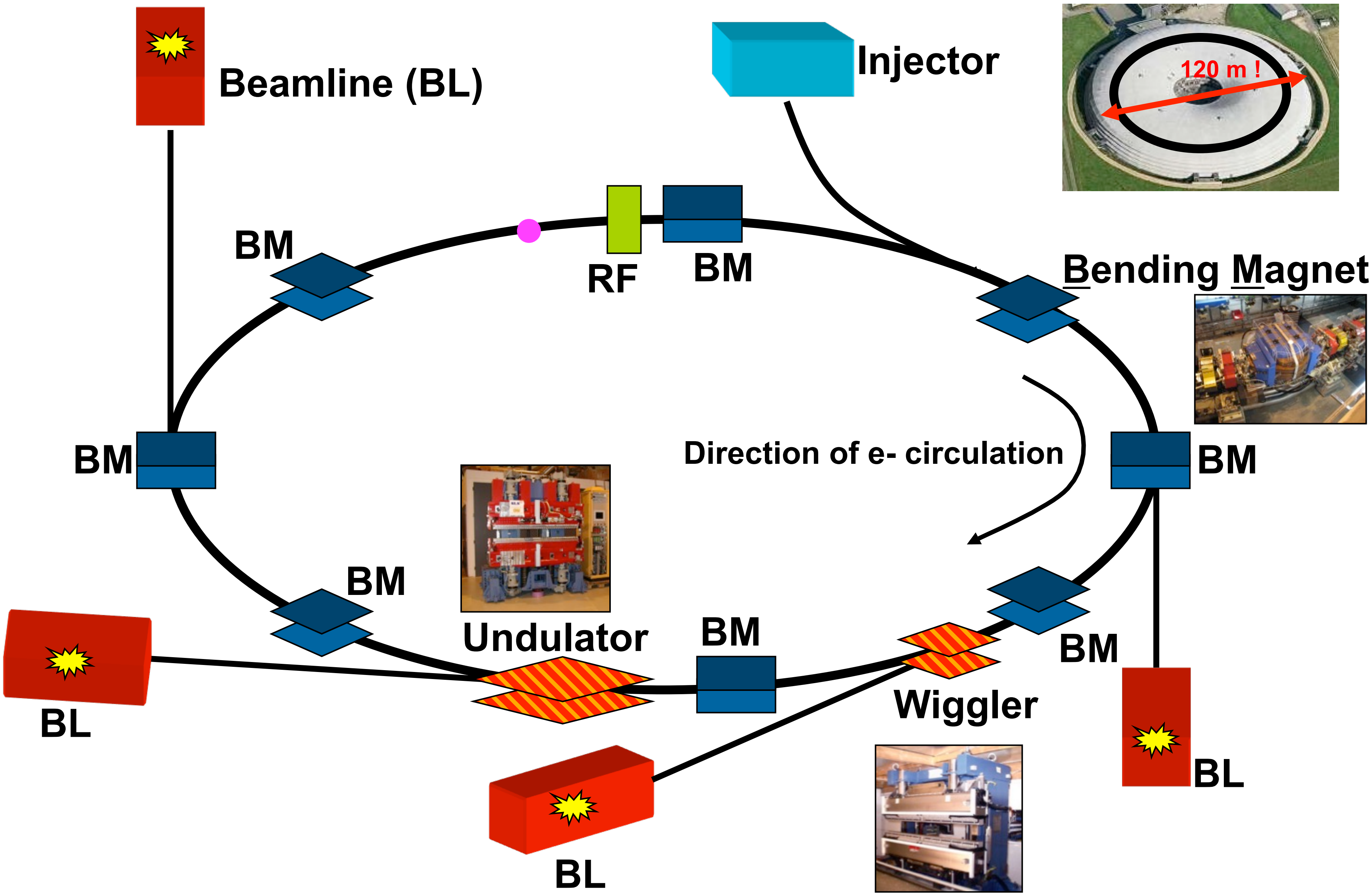
- **Wavelength λ** : distance over which the wave repeats itself
- For a wave propagating at the speed of light, $c \approx 3 \cdot 10^8$ m/s, we define the **period** of the wave to be given by **T= λ/c**
- During a time interval Δt we detect Δt/T oscillations. Thus the number of detected oscillations per unit time is given by ν**=1/T**, called **frequency**.
- So, **wavelength and frequency** are linked by: ν**=c/ λ**
- According to Plank who stated that electro-magnetic radiation behaves itself as a wave and as a large ensemble of particles (photons) it holds: **E =** $\boldsymbol{h \cdot \nu}$, *h* being the Plank constant
- For $h=6.6262*10^{-34}$ J/s and E=100 eV, typical for chemical bounds, we obtain the corresponding wavelength around 10^{-8} m and a frequency of 10^{16} Hz.

→ **You need X-rays / Ultraviolet radiation to explore chemical bonds, molecules and biosystems**

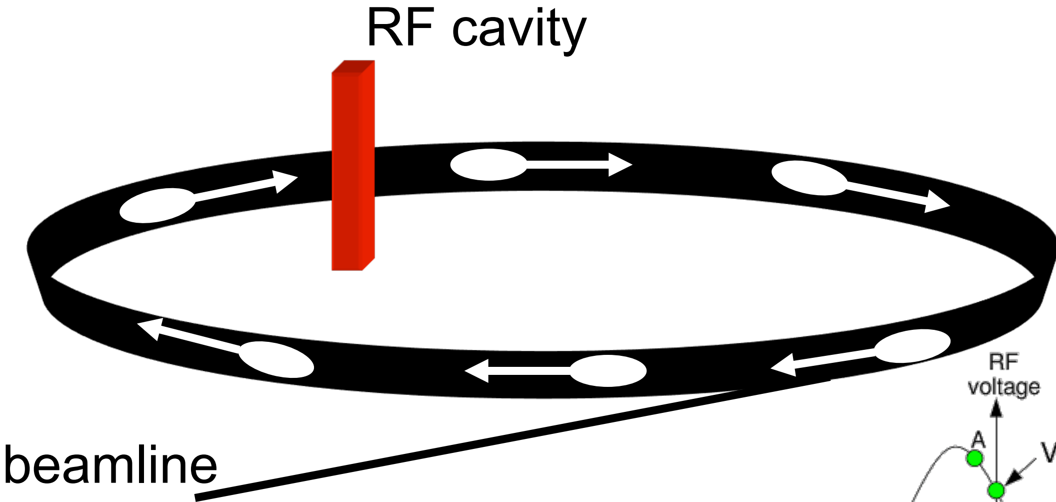
The Paul Scherrer Institut

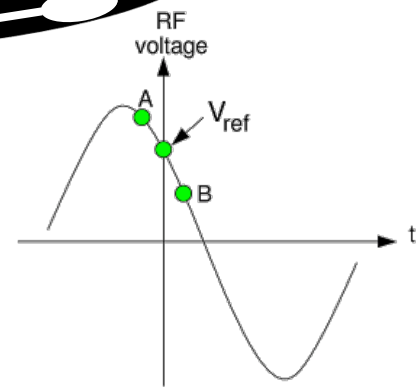


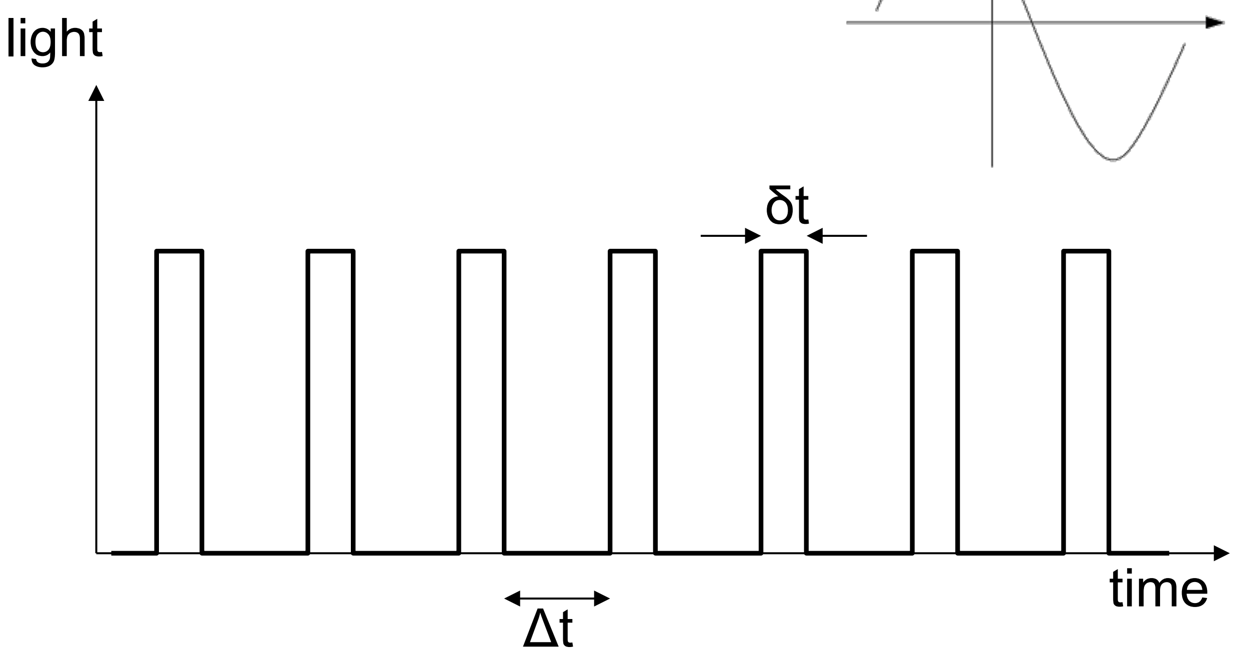
Synchrotron’s working principle



Feeding energy to the flying electrons



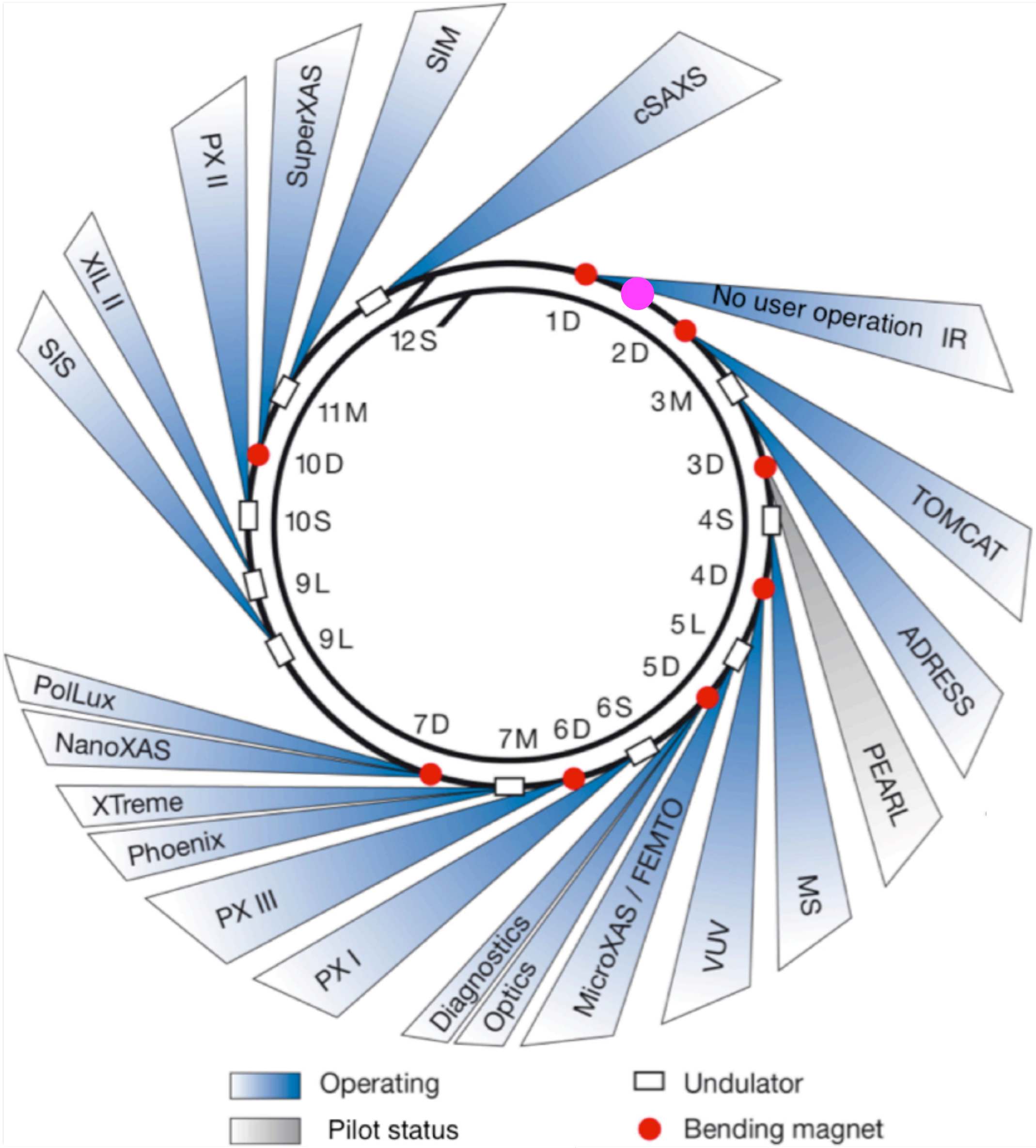




- Electrons circulate around the ring as discrete bunches. When a bunch enters the RF cavity, a pulsed electric field is applied to ‘kick’ the electrons at the end of the bunch, i.e. the electrons that lost energy by emitting sr while turning.

- Synchrotron light is present at the beamline anytime the electrons bunch passes its frontend, thus the light consists of a series of pulses of duration δt, separated by “dead times” of duration Δt .

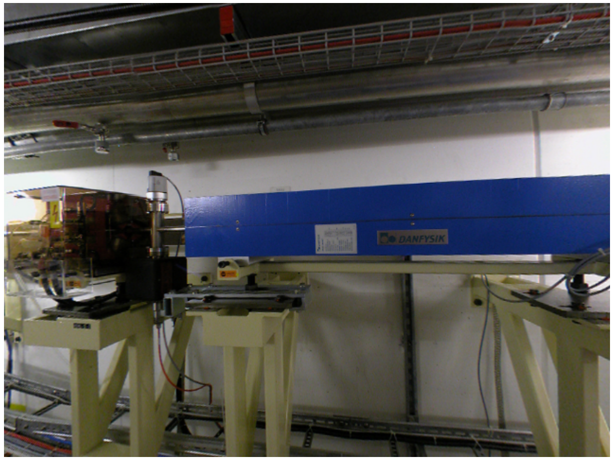
The Swiss Light Source



http://sls.web.psi.ch/view.php/beamlines/index.html

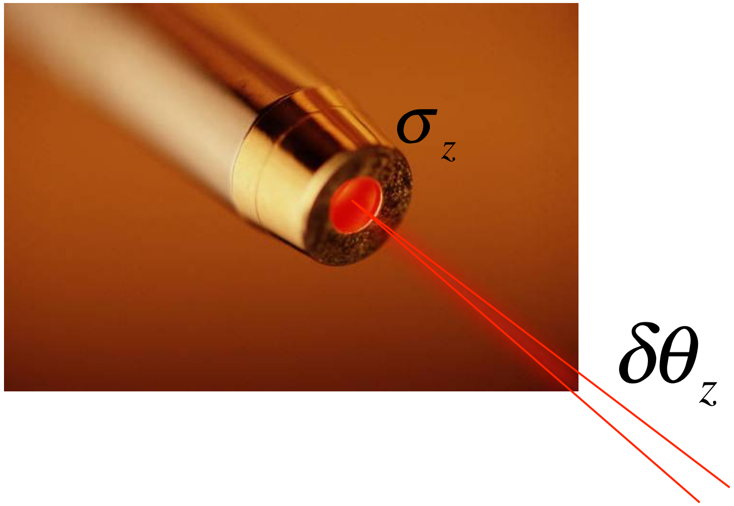
Machine:
- 100 MeV Linac
- 2.4 GeV Booster
- 2.4 GeV Storage Ring
- 400 mA electron current

Beamlines (10. 2015)
- 20 beamlines in operation



Brightness

- A good source is usually **a powerful source**, i.e. a source which emits a large amount of energy (photons) per unit time
- However, high power is not enough!
- A source is much more useful if its power can be concentrated into a small area, i.e. the source **is bright**, it has a **high brilliance**, which means that it emits a lot of light from a small area and also that the emission occurs within a narrow angular cone.



Definition of brightness

- Source brightness is by definition proportional to the **emitted power** *F* and inversely proportional to the **source size** in the horizontal and vertical direction , $\sigma_x \sigma_y$ and to the **angular spreads** in the same directions, $\delta\theta_x \delta\theta_y$:

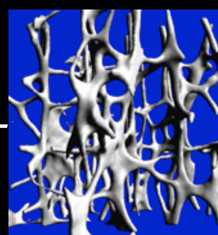
$$\text{brightness} = \text{constant} \times \frac{\mathrm{F}}{\sigma_x \sigma_y \delta\theta_x \delta\theta_y}$$

- Implications:
  - Brightness can be increased by increasing emitted power and/or by decreasing source size and angular spread
  - A conventional X-ray source (X-rays tube) does usually not match the requirements for high brightness: the emitting area is large and the emission occurs in a broad range of directions
  - The bad geometry of conventional X-ray sources cannot be easily corrected by optical devices, being X-ray lens quite inefficient, delicate and rather expensive
  - A good source geometry is therefore much more important for X-rays than for visible light. It took several years before the brightness of X-ray sources could be dramatically improved.
  - Synchrotrons produce bright X-rays thanks to a clever use or Einstein’s relativity theory!

Brightness over the last century…

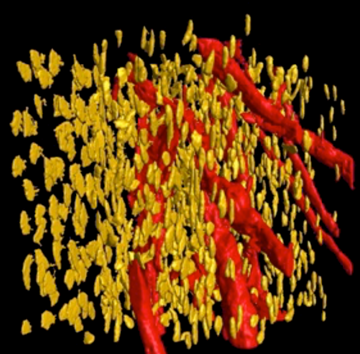
1000-500 µm

Hospital



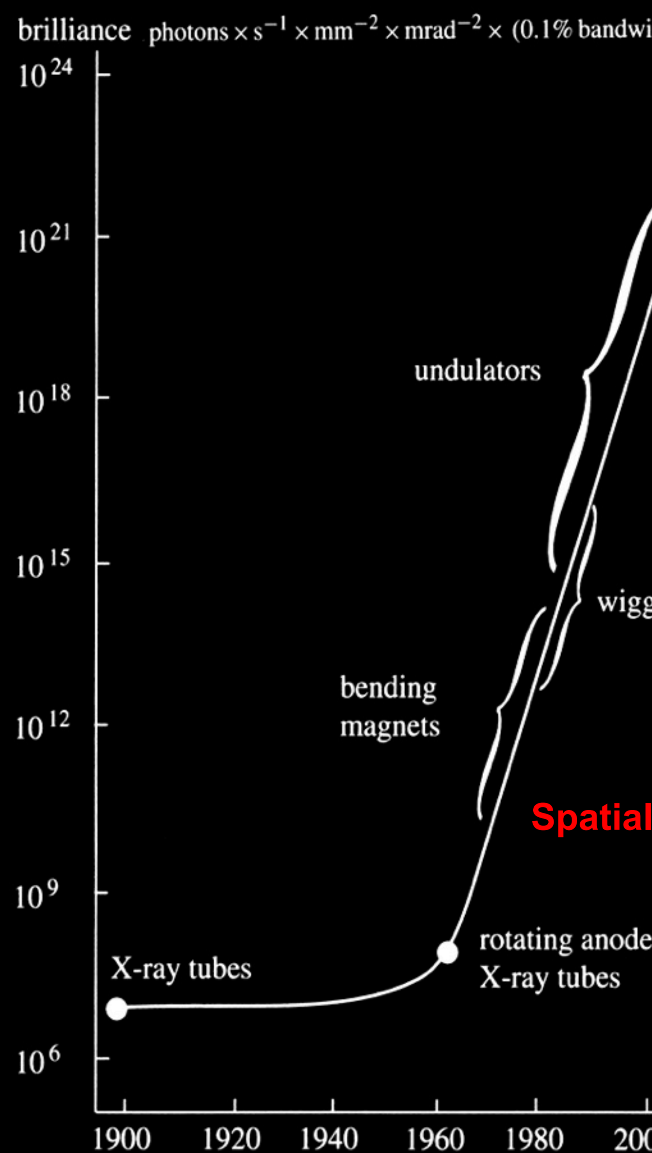
50-10 µm

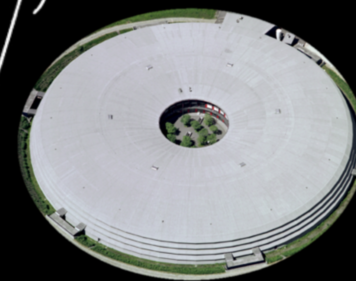
Table Top micro CT



1-0.1 µm

SR - Tomographic Microscopy





Contrast

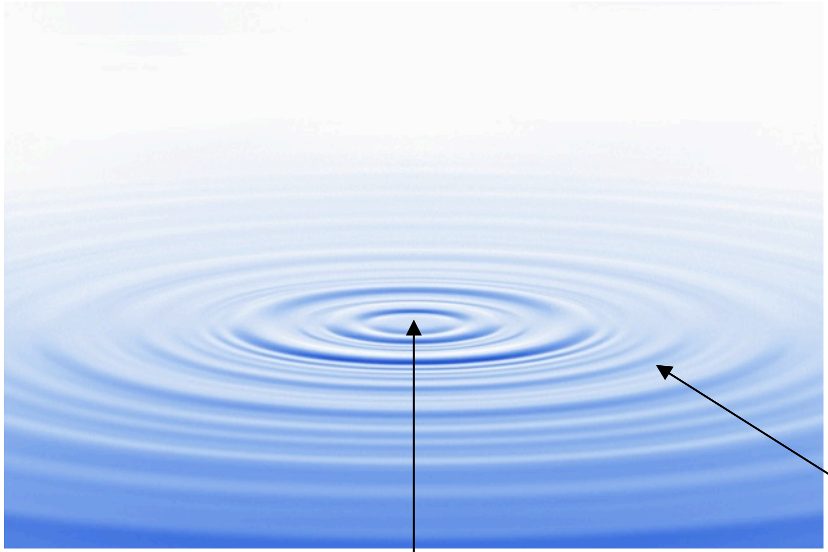
$$\Phi \propto \frac{SNR^2}{\Delta x^4 \cdot \Delta\mu^2}$$

Spatial resolution

Bonse et al.,
Prog. Biophys. Molec. Biol.,
Vol. 65, No.1, pp. 133-169, 1996

Electromagnetic waves

- First : how are electromagnetic waves generated?
- An electromagnetic wave is a perturbation of the electromagnetic field which, after being created at a given site, propagates away from it at the speed of light
- Example: stone hitting water surface or radio antenna !



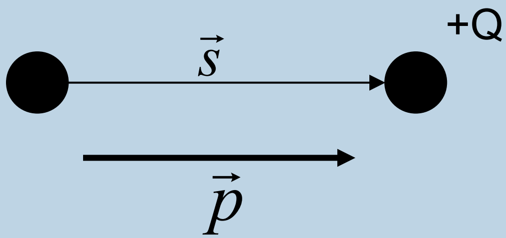
Wave propagation

Local perturbation

Radiation from an accelerated particle

- Electric dipole:

$$\vec{p} = Q\vec{s}$$



- Electric field:

$$\vec{E}(\vec{r}) = -gradU(\vec{r}) \text{ with } U(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\cdot\vec{r}}{r^3}$$

- If an initially quite charge q is accelerated with an acceleration $\vec{a}(t)$ we can write:

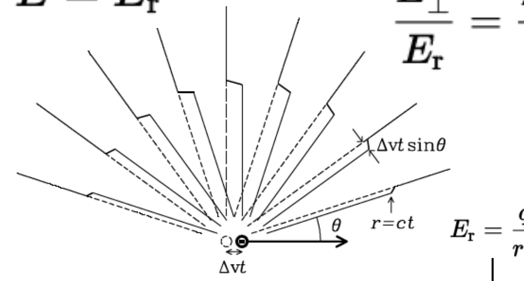
$$\dot{\vec{p}} = \vec{v} \cdot q \quad \text{and} \quad \ddot{\vec{p}} = \vec{a} \cdot q$$

- According to Larmor, the total emitted power of such charge is:

$$P = \frac{q^2}{6\pi\varepsilon_0 c^3}[a]^2 \quad \text{with } [\vec{a}] = \vec{a}(t - \frac{r}{c})$$

The Larmor equation states that any charged particle radiates when accelerated and that the total radiated power is proportional to the square of the acceleration.

$$E = E_r \qquad \frac{E_\perp}{E_r} = \frac{\Delta v\, t \sin\theta}{c\Delta t}$$



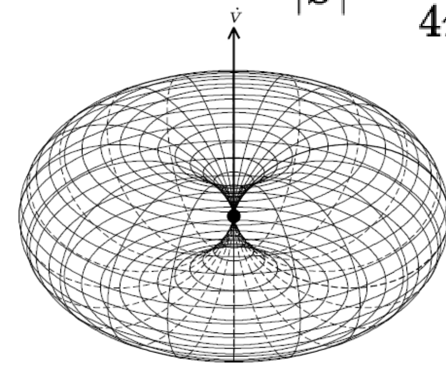
$$E_r = \frac{q}{r^2}$$

$$E_\perp = \frac{q\dot{v}\sin\theta}{rc^2}$$

$$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H}$$

$$|\vec{S}| = \frac{c}{4\pi}E^2$$

$$|\vec{S}| = \frac{1}{4\pi}\frac{q^2\dot{v}^2}{c^3}\frac{\sin^2\theta}{r^2}$$



$$P = \int_{\text{sphere}} |\vec{S}|dA \longrightarrow P = \frac{2}{3}\frac{q^2\dot{v}^2}{c^3}$$

See: Longair, High Energy Physics, 2nd edition, vol. 1, p. 64

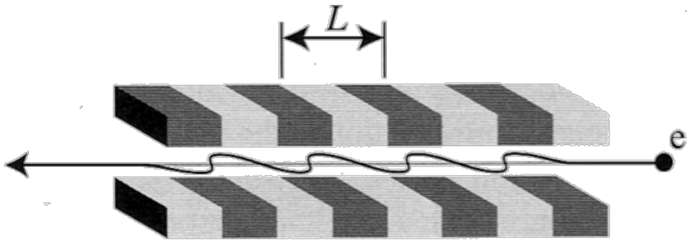
X-rays “antennas” ?

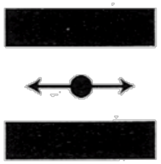
The question is: Can we produce X-rays by exciting a suitable “radio antenna” ?

The answer is: No!! Because the magnitude of the corresponding frequencies and therefore the construction of an electronic circuit oscillating at that frequency is not possible! (radio frq = 10^6, X-ray frq = 10^{16})

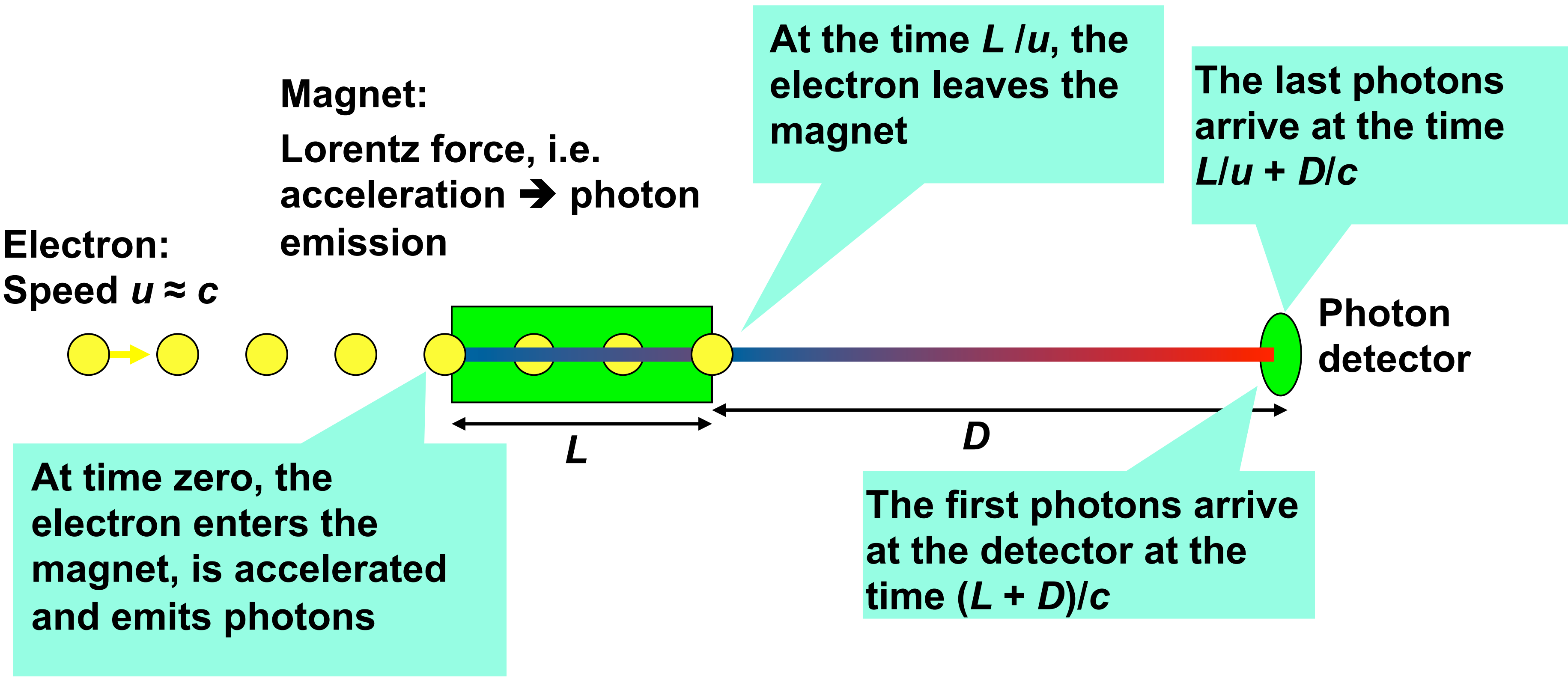
Therefore, follow a different strategy !

Rather than forcing electrons to oscillate in an antenna, let shoot them toward a periodic array of magnets. Each magnet applies a Lorentz force to the moving electrons, slightly deviating their direction. The electrons gently *undulate* around a straight line.





Synchrotron light in a pot



Photon pulse duration: $\Delta t = L/u - L/c = (L/u)\,(1-u/c)$

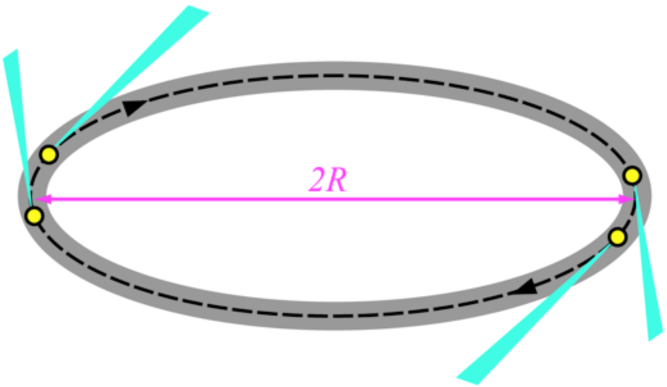
Characteristic frequency: $\nu = 1/\Delta t = u/[L(1-u/c)] = u\,\gamma^2\,(1+u/c)/L$

For $u \approx c$, $(1+u/c) \approx 2$ and $\nu \approx 2c\gamma^2/L$

$\gamma^2 = 1/(1-u^2/c^2)$

For $L = 0.1$ m and $\gamma = 4000$, $\nu \approx 10^{17}\ \mathrm{s}^{-1}$ → **X-rays!**

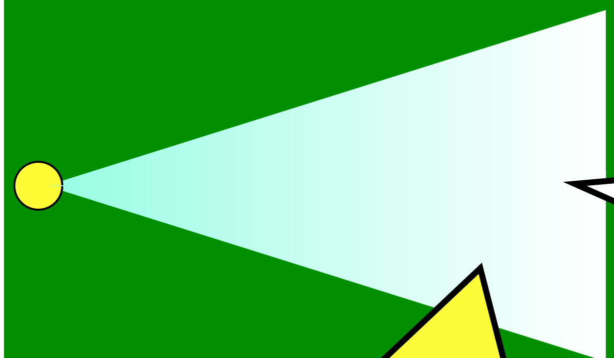
Narrow emission…



Electrons circulating at a speed $u \approx c$ in a storage ring emit photons in a narrow angular cone, like a "flashlight": why?

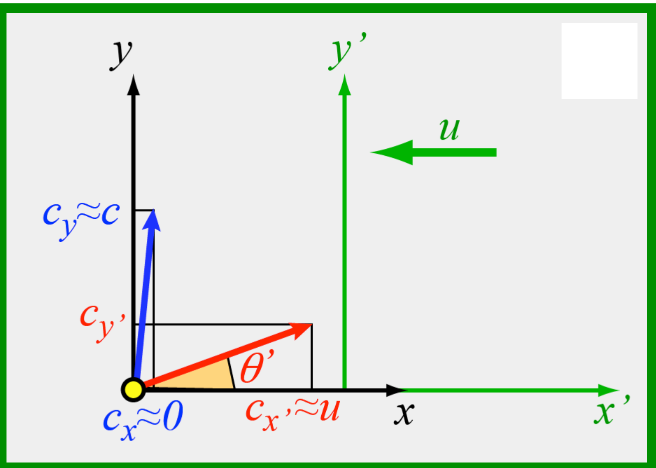
Answer: RELATIVITY

See Exercise



But in the laboratory frame the emission shrinks to a narrow cone

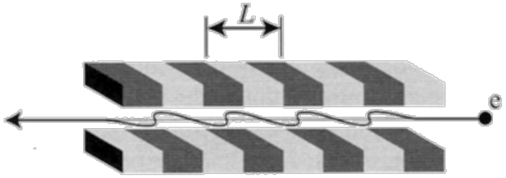
Seen in the electron reference frame, the photon are emitted in a wide angular range



- Take a photon emitted (blue arrow) in a near-transverse direction in the (black) electron frame, with velocity components $c_x \approx 0$, $c_y \approx c$.
- In the (green) laboratory frame the velocity (red arrow) components become $c_x' \approx u$, $c_y' \approx (c^2 - u^2)^{1/2} = c/\gamma$.
- The angle θ' is $\approx c_y'/c = 1/\gamma$ → Very narrow!!!

Undulator radiation

Why does an undulator source work?

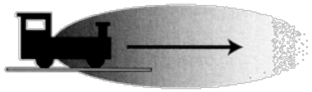


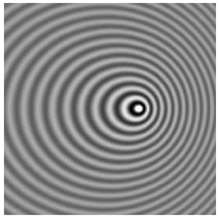
Classical mechanics

- Electrons are sent into the undulator at almost the speed of light. A full undulation is completed in **L/c**. Therefore the emitted frequency ν**=c/ L**, and the emitted wavelength λ=**L**. Not yet enough…

Relativistic effects (see Appendix 2A and 2B)

- Lorentz contraction: The period **L** and the corresponding wavelength λ are automatically shortened by the relativistic effect called "Lorentz contraction". In fact, seen from the point of view of a fast-moving electron, **L** and, therefore, λ shrink.
- Doppler effect: the emitted wavelength is detected in the reference frame of the laboratory, which is different from that of the electron. In the laboratory frame, the wave source moves with the speed close to c. Like the changing pitch of a train whistle as it moves towards us, the Doppler effect further contracts the undulator radiation, producing finally the desired X-ray values.





A source of waves moving to the right. The frequency is higher on the right than on the left.

→ **A synchrotron source emits short-wavelength X-rays by using macroscopic-sized objects and then shortening the emitted wavelength by relativistic effects.**

Undulator emission

- It can be shown that (as a first approximation):

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

wavelength angular spread

$$\lambda_{lab} \approx \frac{L}{2\gamma^2} \quad \text{and} \quad \delta\theta \approx \frac{1}{\gamma}$$

- The γ - parameter corresponds to the ratio between the electron energy (in the laboratory frame) and the electron rest energy.
- For synchrotron sources, the electron energy ranges between 0.25 and 8 GeV and therefore γ between 500 and 16000

- **Implications:**
  - For a typical undulator with period L= 0.05 m, the emitted wavelength would corresponds to radio waves. Considering a $\gamma \approx 2000$ (for an electron energy of 1 GeV) we obtain: $\lambda_{lab} \approx 6 \times 10^{-9}\,\mathrm{m}$, which is definitely in the X-ray range!
  - Again, because of relativistic effects, the resulting angular spread is determined by the λ–factor.

→ **Undulators meet therefore both condition for brightness: a lot of power and emission within a very small angular spread !**

Relativistic background of synchrotron light

Take home formulae:

Gamma factor: $\gamma = \dfrac{1}{\sqrt{1-\dfrac{u^2}{c^2}}}$

Doppler shift: $\lambda_{lab} \approx \dfrac{\lambda_e}{2\gamma}$

Lorentz contraction: $L' = \dfrac{L}{\gamma}$

Emitted wavelength in the lab: $\lambda_{lab} \approx \dfrac{L}{2\gamma^2}$

See Appendix 2B !

Inset A: Relativistic background of synchrotron light

Relativity is the key to understanding how a synchrotron source works. For our discussion, we need a simple introduction to the basic concepts of relativity and ... by synchrotron sources: Lorentz contraction and Doppler shift.

The experimental foundation of relativity is a rather surprising phenomenon: the speed of light, c, does not change when it is measured in two different reference frames, one moving at constant speed u with respect to the other. This fact is surprising because for objects like cars or trains the speed does change with the reference frame.

However surprising, the invariance of c is not a conjecture but a solid fact, supported by many experiments. What are its consequences?

Consider an electron m... synchrotron light. Imagine ... period Δt_e, the pulse travels ...

$$\frac{\Delta x_e}{\Delta t_e} = c\,. \tag{A1}$$

Δx_e and Δt_e are the values measured from the point of view (reference frame) of the moving electron. If we look at the light pulse from the beamline reference frame (the 'laboratory frame'), then the measured position and the measured time change from x_e and t_e to x_L and t_L ...

Before Einstein's relativity, this change was believed to follow the simple rules:

$$x_L = x_e + ut_e \tag{A2}$$

$$t_L = t_e\,. \tag{A3}$$

These rules, however, are in conflict with the experimental fact that c does not change with the reference frame. In fact, according to eqns A2 and A3 the speed of the light pulse in the new frame would be

$$\frac{\Delta x_L}{\Delta t_L} = \frac{\Delta x_e + u\Delta t_e}{\Delta t_e} = \frac{\Delta x_e}{\Delta t_e} + u = c + u\,.$$

Thus, c would not be invariant but change from c to $c + u$, contrary to all experimental evidence. We can eliminate this problem by adopting a modified version of eqns A2 and A3:

$$x_L = \gamma(x_e + ut_e), \tag{A4}$$

$$t_L = \gamma(t_e + \alpha x_e)\,, \tag{A5}$$

where γ and α are two parameters to be determined. How? First of all, we must require the invariance of c. In the laboratory frame:

$$\frac{\Delta x_L}{\Delta t_L} = \frac{\Delta[\gamma(x_e + ut_e)]}{\Delta[\gamma(t_e + \alpha x_e)]} = \frac{\Delta x_e + u\Delta t_e}{\Delta t_e + \alpha\Delta x_e} = \frac{\dfrac{\Delta x_e}{\Delta t_e} + u}{1 + \alpha\dfrac{\Delta x_e}{\Delta t_e}}\,.$$

... $_e/\Delta t_e = c$ (speed of light), then $\Delta x_L/\Delta t_L$ must be also equal to c. Thus:

$$\frac{\dots u}{1+\alpha c} = c\,,$$

and $c + u = c + \alpha c^2$, which gives $\alpha = 1/c^2$.

Equations A4 and A5 can then be written as:

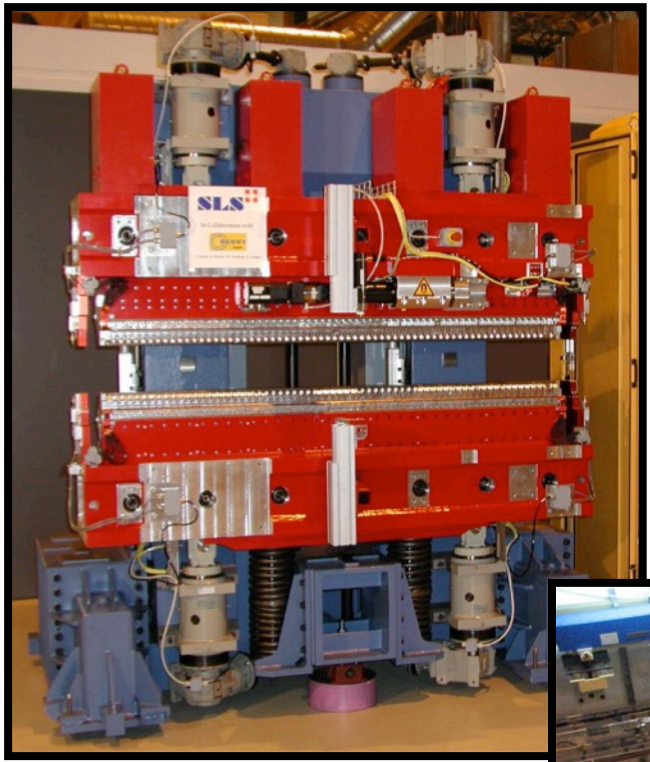
$$x_L = \gamma(x_e + ut_e), \tag{A6}$$

$$t_L = \gamma\left(t_e + \frac{ux_L}{c^2}\right). \tag{A7}$$

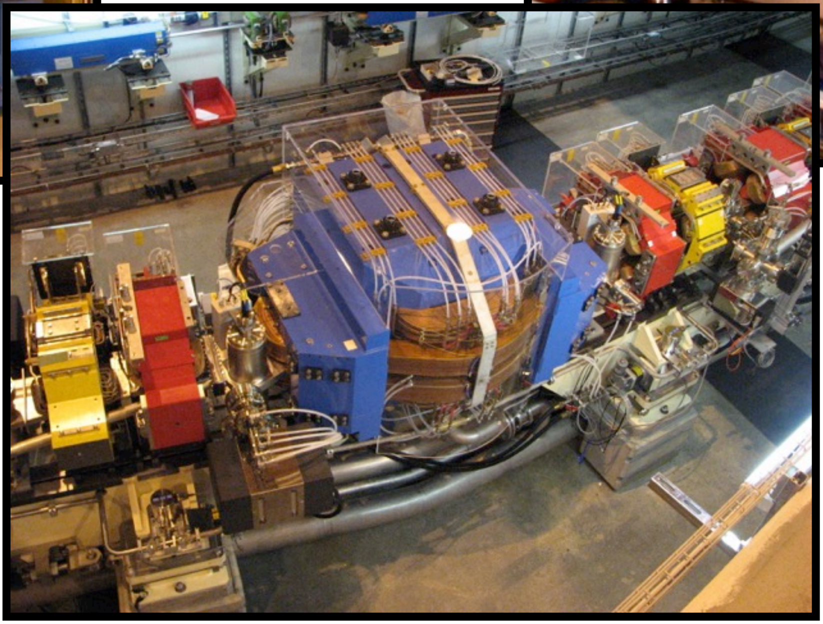
G. Margaritondo, *Elements of Synchrotron Light: For Biology, Chemistry, and Medical Resea...*
Oxford University Press, New York , 2002, ISBN0-19-850931-6

Synchrotron sources (details see Appendix 3)

Wiggler

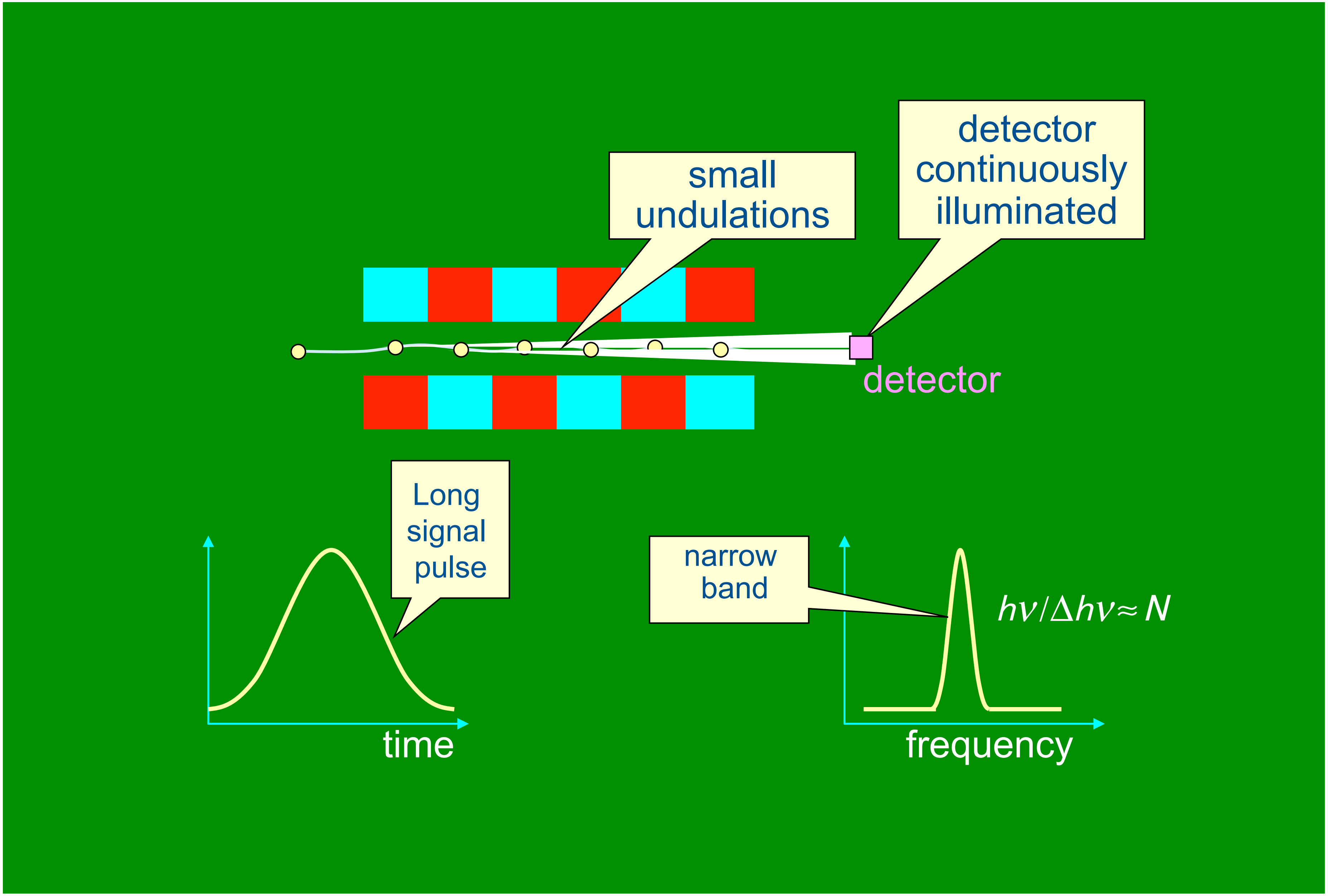


Undulator



Bending Magnet

Synchrotron source: undulators



Changing the wavelength of the undulator radiation

- Consider: $\lambda_{lab} \approx \frac{L}{2\gamma^2}$ Change wavelength by changing γ , i. e. changing the electron energy, but this is rather unrealistic !

- Again, what is the γ –parameter? It corresponds to the energy of the electron as it moves along a straight trajectory in the forward direction. But the undulator causes small oscillations in the *transverse* direction. $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$

- The creation of a velocity component in the transverse directions requires a decrease in the magnitude of the forward velocity component, therefore affecting the corresponding effective γ -value.

- The magnitude of the transverse oscillations can be controlled by changing the strength B of the periodic magnetic field of the undulator (by changing the “gap” between the poles).

- It holds:

$$\lambda_{lab} \approx \frac{L}{2\gamma^2}\left(1+\frac{K^2}{2}\right) \quad \text{with } K \approx 0.934 \times B\,[\mathrm{T}] \times L\,[\mathrm{cm}]$$

Physics affecting the wavelength

fundamental wavelength

- Off-axis emission (general case): $$\lambda_{lab} \approx \frac{L}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)$$
  - Doppler shift is given by source velocity
  - Velocity components changes with direction
- High harmonic wavelength : $\frac{\lambda_{lab}}{n}$, with $\mathrm{n}=1,2,3,4,\ldots$
  - In analogy to a sound produced by a music instrument you have higher harmonics
- Bandwidth: $$\frac{\Delta\lambda_{lab}}{\lambda_{lab}} = \frac{1}{nN}$$
  - In analogy to diffraction physics: the periodic magnet array acts as a gratings
- Real angular spread: $$\delta\theta \approx \frac{1}{\gamma\sqrt{N}}$$
  - Angular deviation from the undulator axis correspond to wavelength changes. But such changes cannot exceed the “grating” bandwidth without compromising the overall constructive interference that produces the undulator emission

Essential summary: undulators

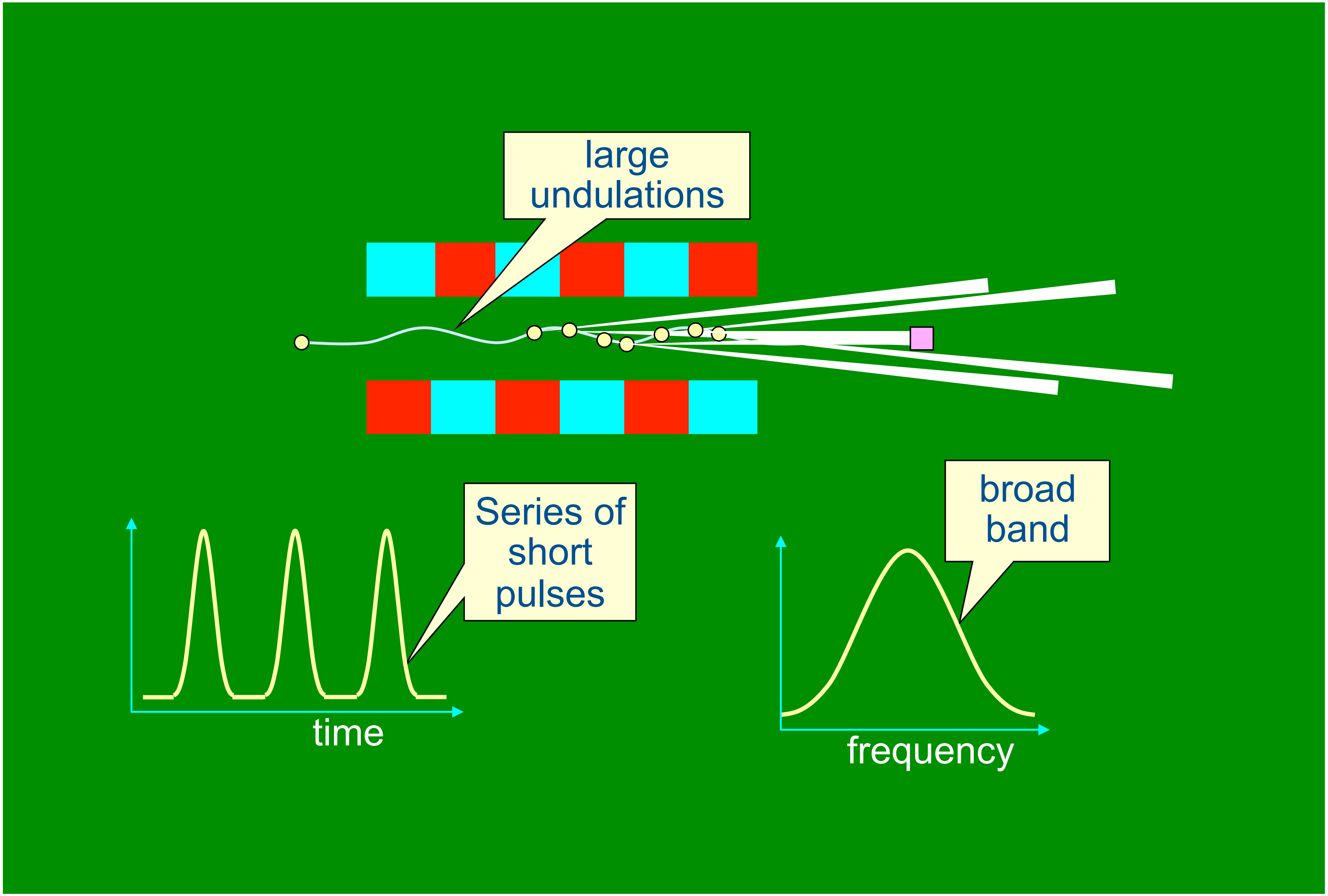
- The **peak emission wavelength** of an undulator is determined by the undulator period, shortened first by the Lorentz contraction and then by the Doppler shift

- The **corresponding shortening factors** must take into account the angular dependence of the Doppler effect and the effect of the magnetic-field-induced electron undulations

- The "**natural bandwidth**" is given by the "diffraction grating" effect of the series of magnets in the undulator

- The **angular spread** is determined by the fact that the corresponding wavelength spread cannot exceed the "natural" bandwidth

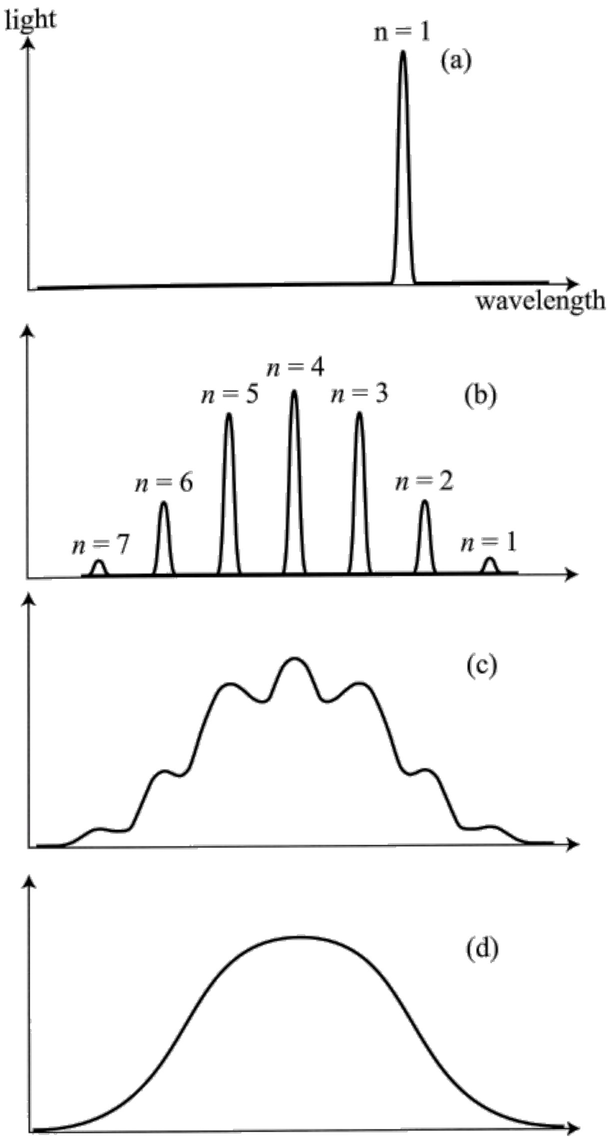
Synchrotron source: wiggler



Wiggler emission (qualitative interpretation)

- A wiggler is similar to an undulator but with a higher magnetic field, i.e. a larger K-parameter.
- Higher B-values cause larger transverse undulations of the electrons.
- We observe that:
  - Fundamental wavelength increases
  - Emitted light intensity shifted from fundamental wavelength to higher harmonics.
  - Broadening of emission bands

→Most of the light is emitted at **shorter wavelengths**, means **higher energy!**



Wiggler emission (qualitative interpretation)

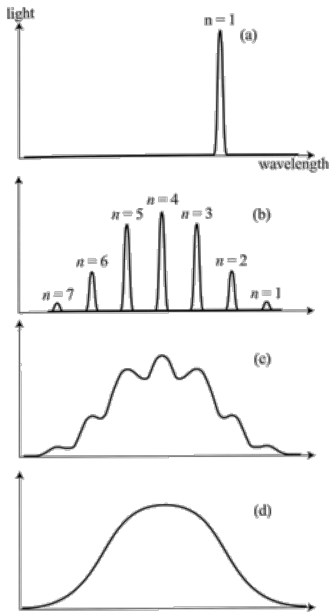
- As B and K increase, the fundamental wavelength (n=1) shifts to higher values, but the weight of higher harmonics increases too.
  → Therefore, the overall emission intensity shifts to smaller wavelengths.

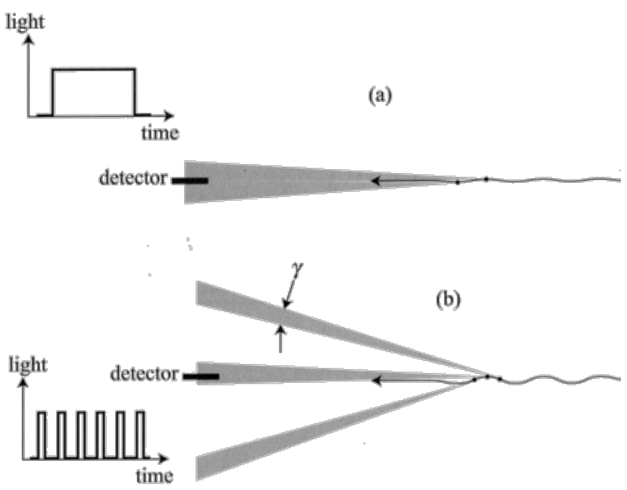
- For larger and larger B- and K-values (and for a small number of periods) the individual bands are broadened

- If B increases, the undulations are larger and the angular deviations make it impossible for the emitted light cone to be continuously detected (series of short pulses)

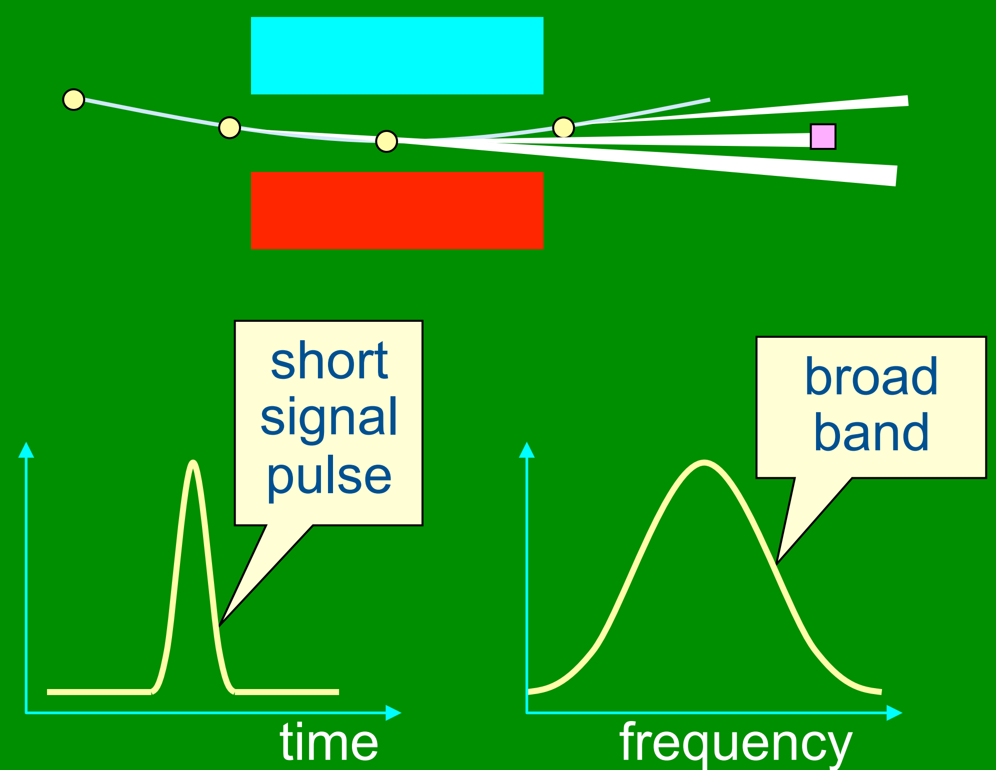
- Since ΔtΔν≈ 1/2π the bandwidth around each emitted wavelength increases.

→ For very high B (and K) the different peaks merge into a broad spectrum



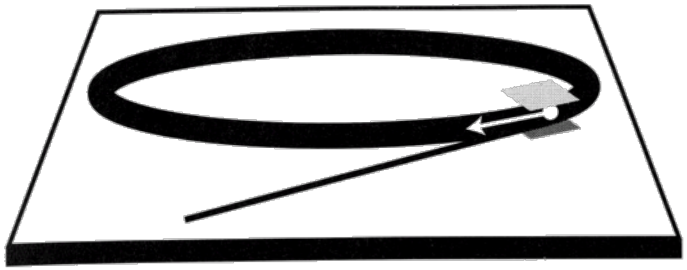


Synchrotron source: bending magnets



Bending magnet radiation

- A circulating electron in the ring looks like an oscillating charge in a radio antenna, with its oscillation frequency and wavelength.



- It can be shown that:

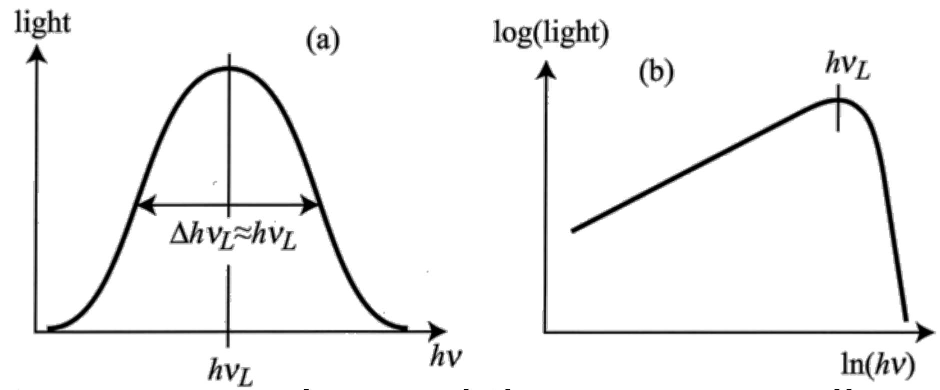
$$\omega_e = \frac{\gamma e B}{m_0} \quad \rightarrow \quad \nu_e = \frac{\omega_e}{2\pi} = \frac{\gamma e B}{2\pi m_0} \quad \rightarrow \quad \lambda_e = \frac{c}{\nu_e} = \frac{2\pi m_0 c}{\gamma e B}$$

$$\text{In the laboratory frame:} \quad \lambda_L = \frac{\lambda_e}{2\gamma} = \frac{\pi m_0 c}{\gamma^2 e B} \quad \rightarrow \quad \lambda_L\,[nm] = \frac{5.3 \times 10^7}{\gamma^2 B[T]}$$

Doppler Effect

See Exercise

Critical energy and wavelength



- The **critical photon energy** $h\nu_c$ and the corresponding **critical wavelength** λ_c are defined by requiring that equal amounts of synchrotron light are emitted above and below $h\nu_c$. It can be shown that:

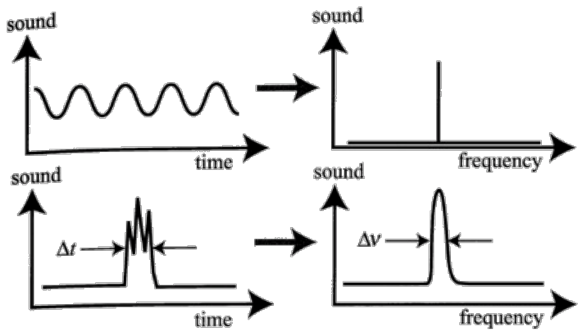
$$h\nu_c = \frac{3}{4} h\nu_L = \frac{3\gamma^2 eBh}{4\pi m_0} \quad \text{and} \quad \lambda_c = \frac{4}{3}\lambda_L = \frac{4\pi m_0 c}{3\gamma^2 eB}$$

In practical units:

$$\boxed{h\nu_c[eV] \approx 1.7\times10^{-4}\gamma^2 B[T] \quad \text{and} \quad \lambda_c[nm] = \frac{7\times10^6}{\gamma^2 B[T]}}$$

Bending magnet radiation

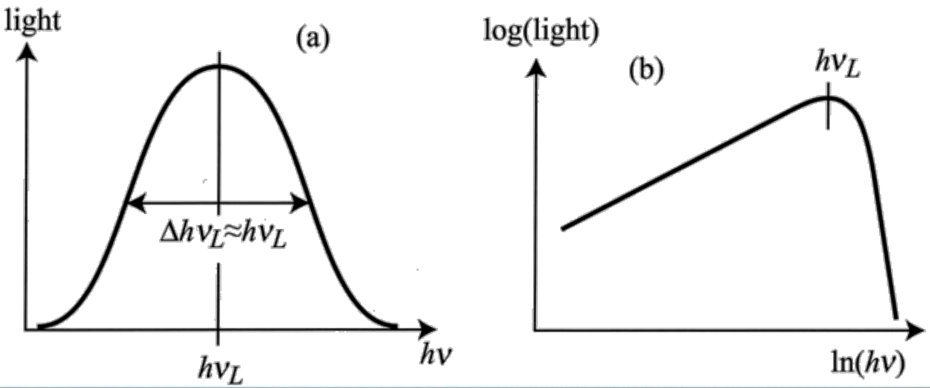
- The wavelength λ_L is not the only emitted wavelength, but the center of an emission band with a large bandwidth $\Delta\lambda_L$. Light can be namely detected only when an electron passes in front of the beamline. The short duration of the single-electron light pulse affects the wavelength bandwidth.



A short pulse corresponds to a large frequency bandwidth and therefore to a large wavelength bandwidth !

- It can be shown that for the bandwidth: $\dfrac{\Delta\lambda_L}{\lambda_L} \approx 1 \rightarrow \dfrac{\Delta h\nu_L}{h\nu_L} \approx 1$

- Bending magnet emission:



Essential summary: bending magnets

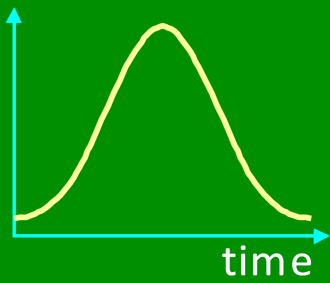
- The **small vertical angular spread** of a bending magnet emission is determined by the electron beam geometry and by the natural angular spread $1/\gamma$.

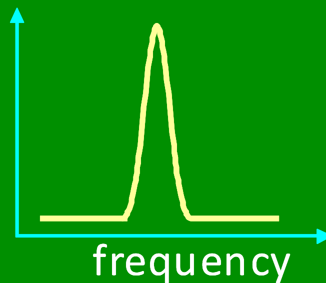
- The **peak emission wavelength** of a bending magnet is determined by the angular speed of the electron motion, which in turn is determined by the bending magnet field strength

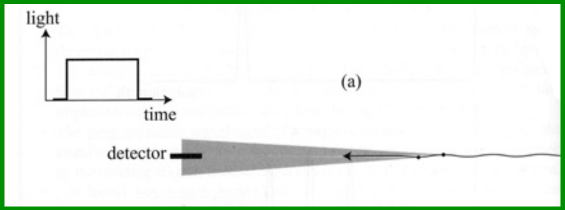
- The **broad wavelength bandwidth** of a bending magnet reflects the short duration in time of the light pulse produced when an electron passes in front of a beamline

Synchrotron sources

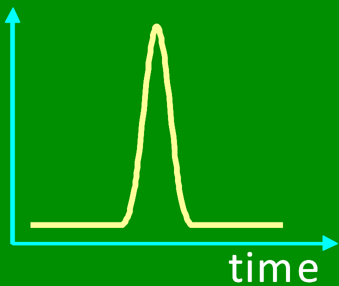
Undulator

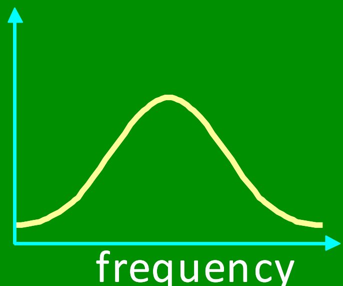


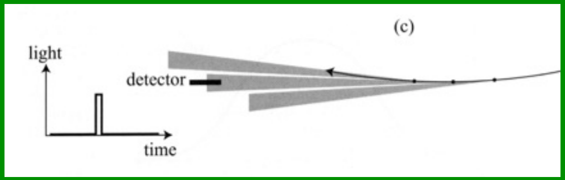




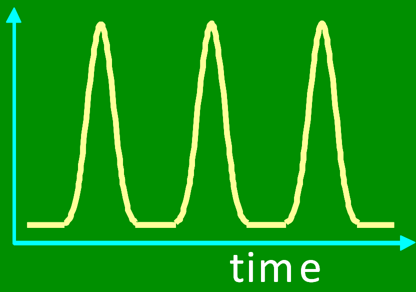
Bending magnet

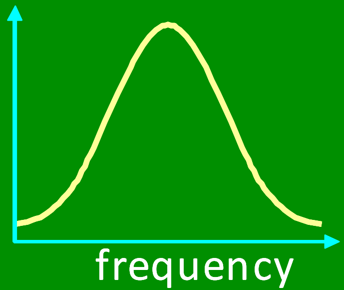


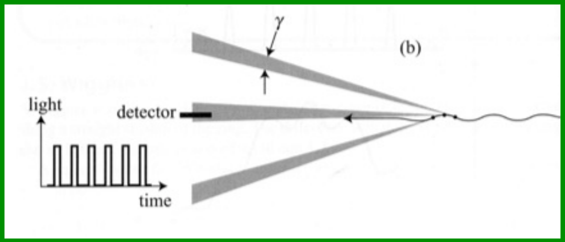




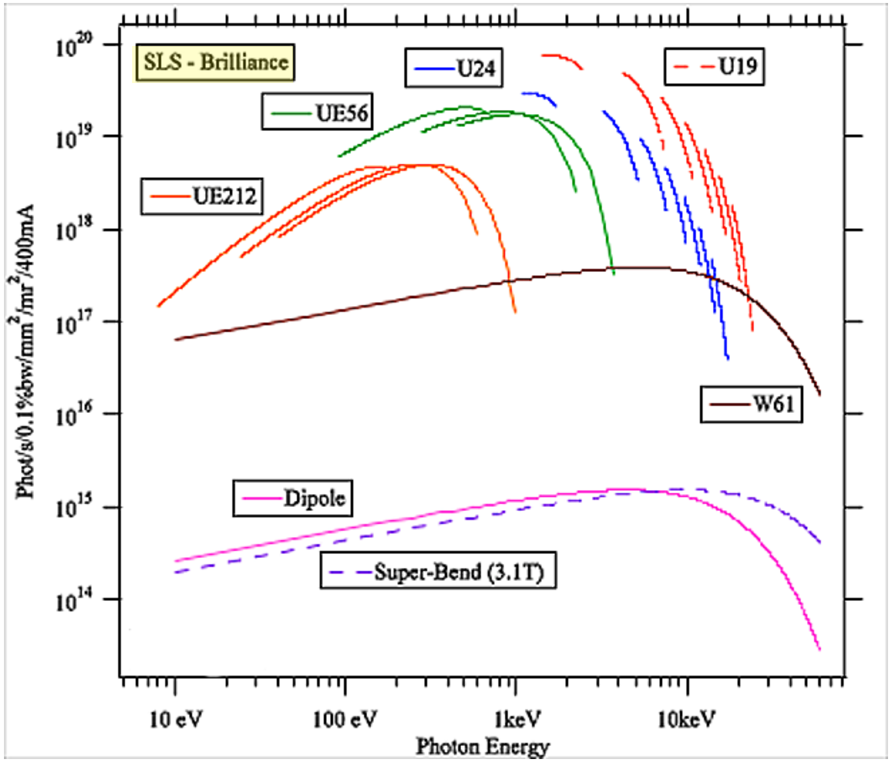
Wiggler

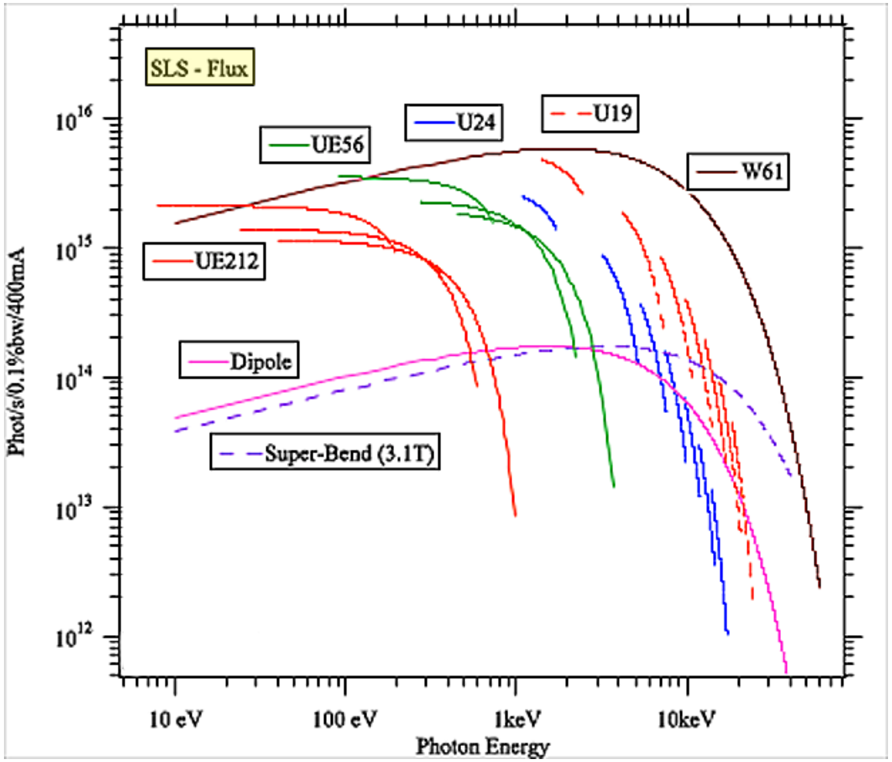






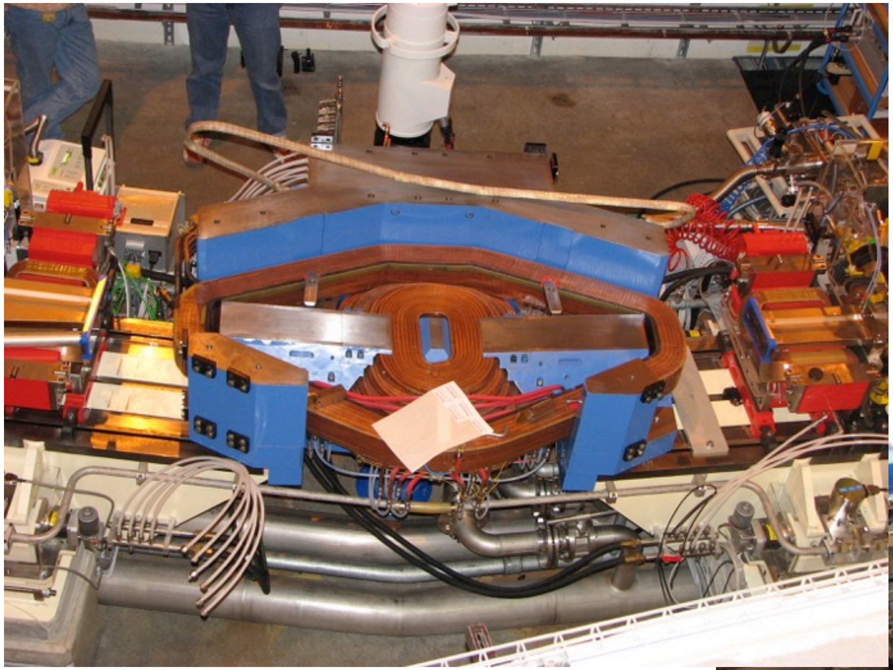
Brilliance and Flux @ SLS

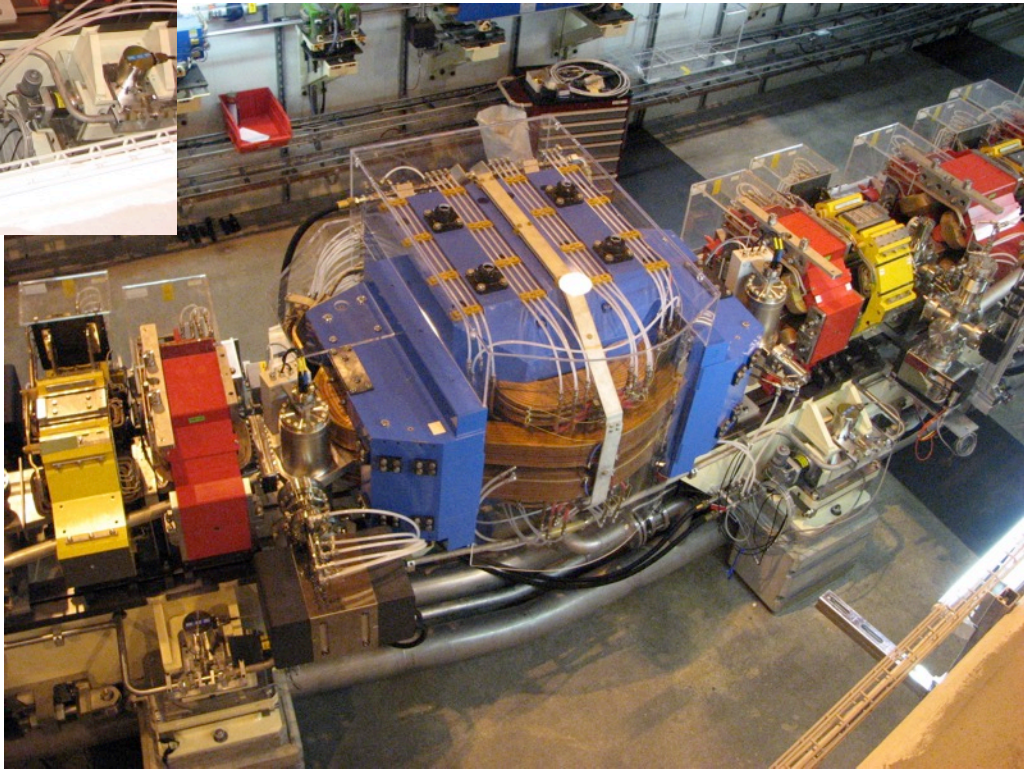




Source: A. Lüdeke, SLS-Machine Team

Installation of a superbend at the SLS

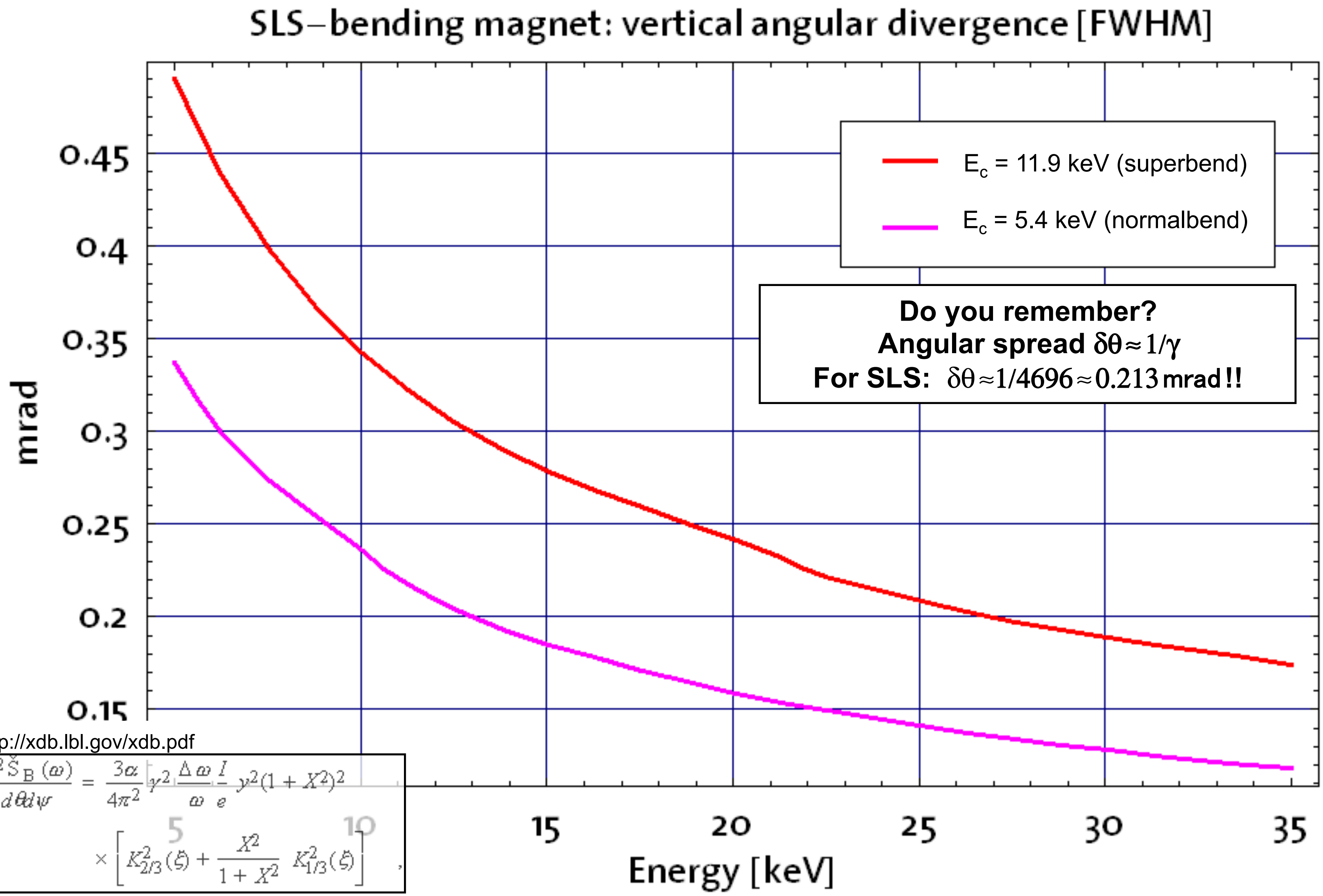




Beam properties

Source characteristics

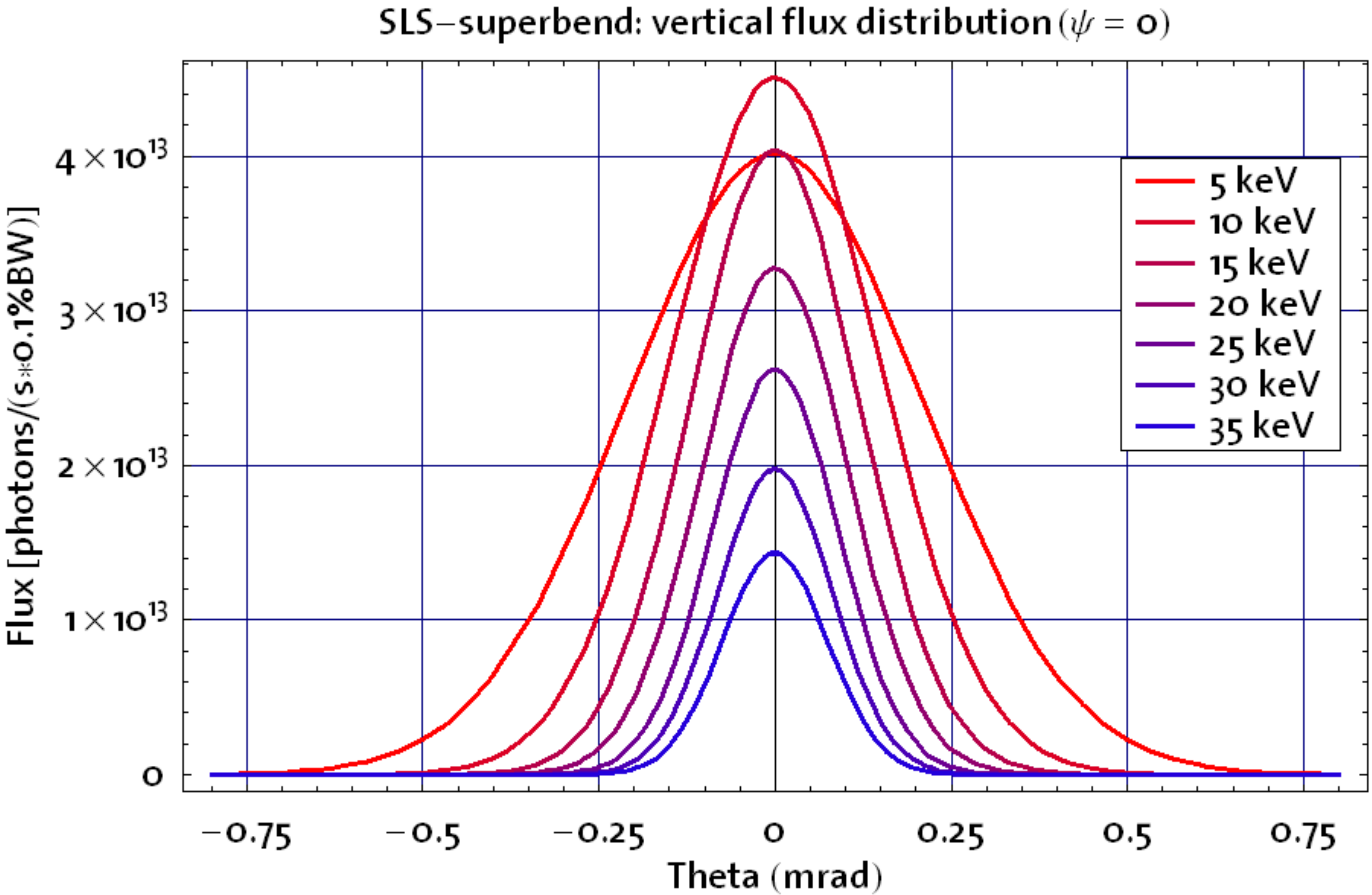
SOURCE: divergence



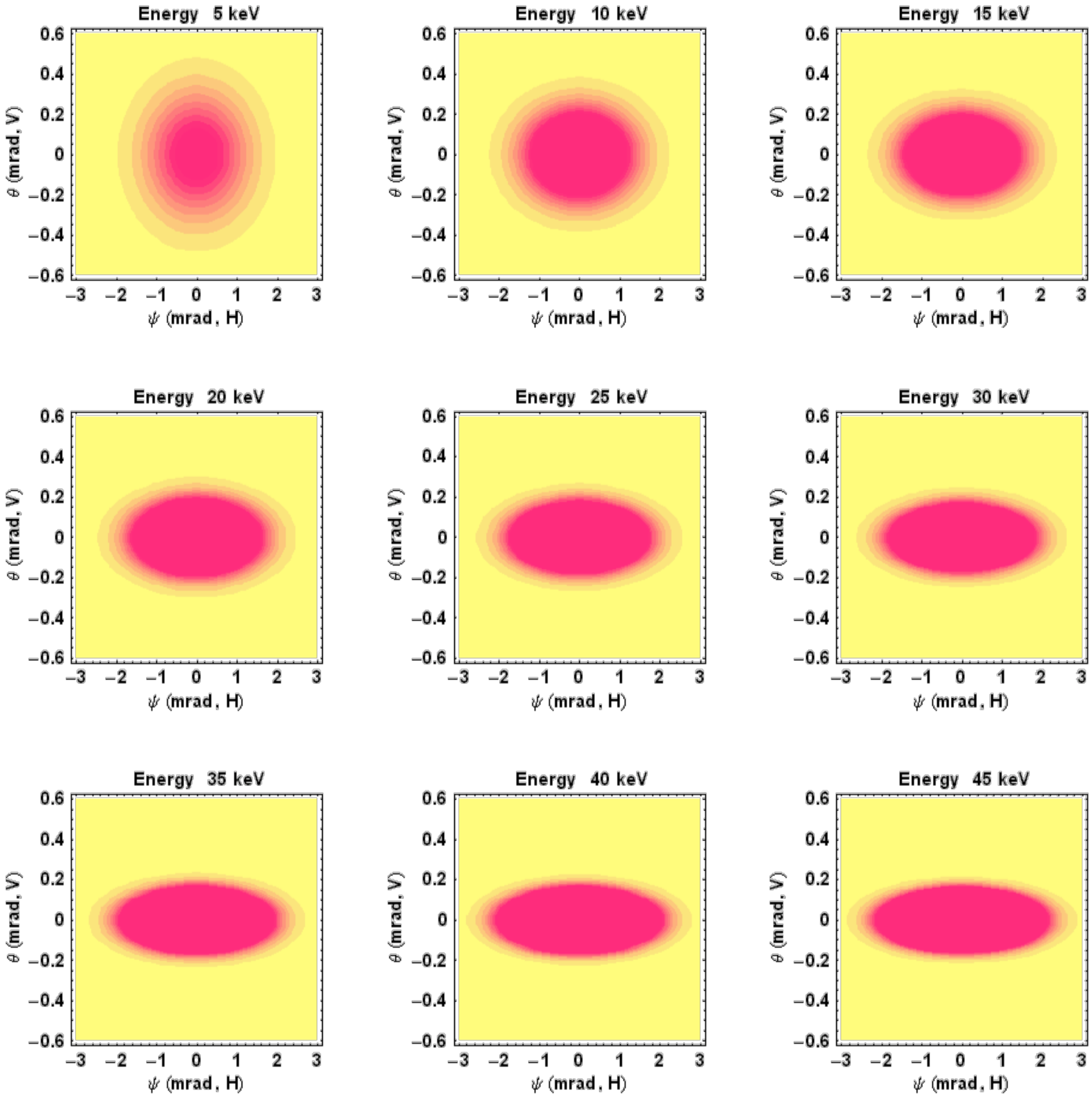
http://xdb.lbl.gov/xdb.pdf

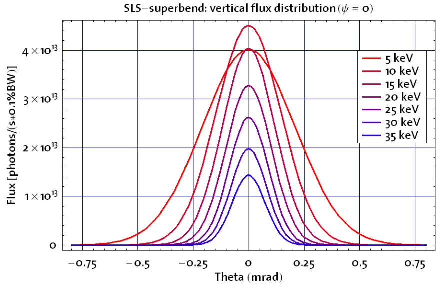
$$\frac{d^2 \tilde{S}_B(\omega)}{d\theta d\psi} = \frac{3\alpha}{4\pi^2}\gamma^2 \frac{\Delta\omega}{\omega}\frac{I}{e} y^2 (1+X^2)^2 \times \left[K_{2/3}^2(\xi) + \frac{X^2}{1+X^2} K_{1/3}^2(\xi) \right],$$

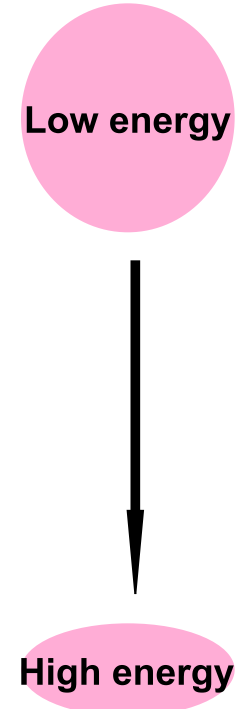
Source energy spectrum



Source energy distribution



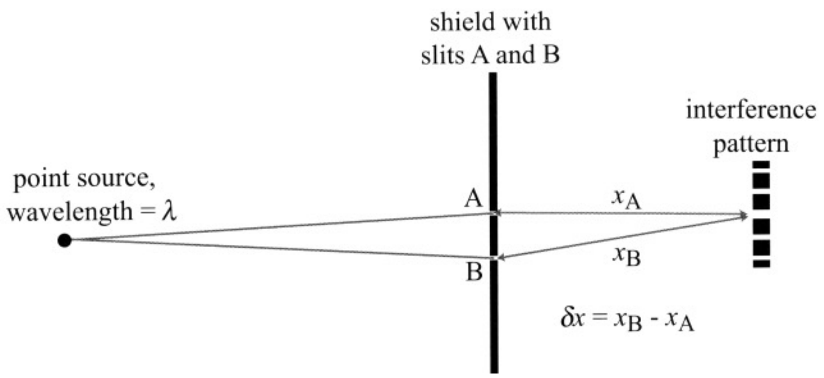




Source coherence

Transverse coherence

- In general, a “coherent wave” is a wave that can produce observable interference and diffraction phenomena.
- To quantify this phenomenon, consider the two-slit experiment:



$$\frac{\delta x}{\lambda} = 1, 2, 3, \ldots \quad \rightarrow \text{intensity maximum}$$

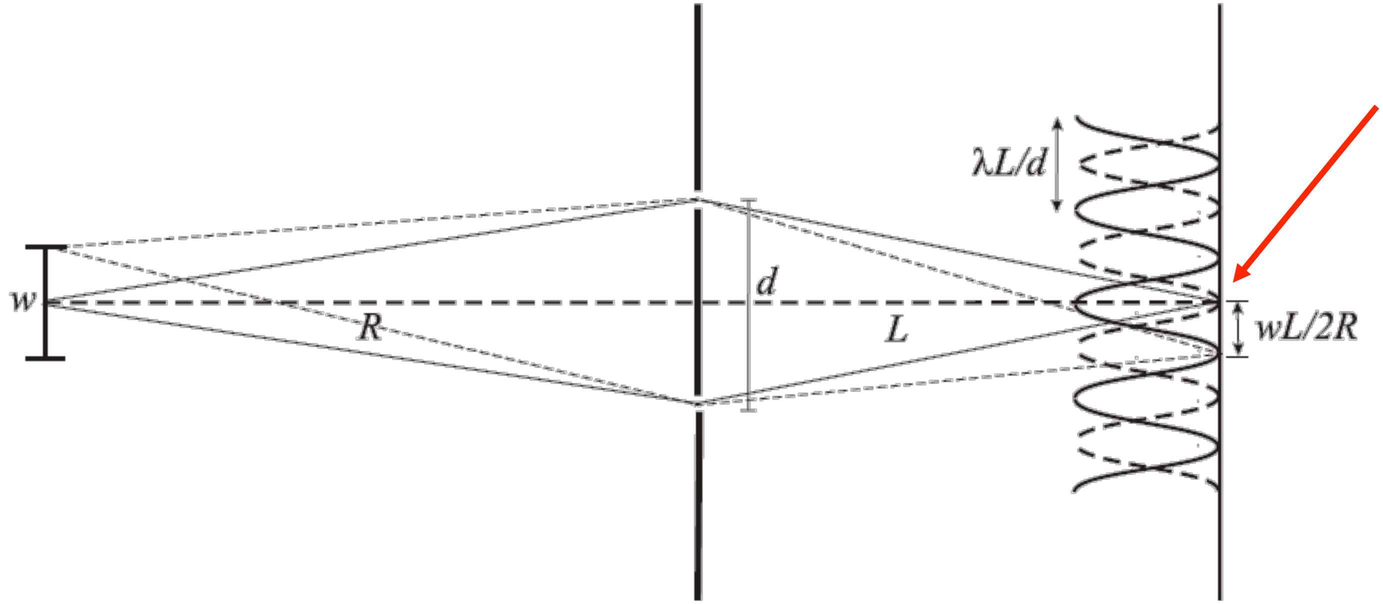
$$\frac{\delta x}{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \quad \rightarrow \text{intensity minimum}$$

- Importance of the “phase factor” x/λ and of the difference between the phase factors of two rays $\delta x/\lambda$.

- Constructive and destructive interference.

- Certainly true if source is a point. If not, transverse coherence length.

Transverse coherence length



- The maxima from a border element of the source coincide with the minima from the central element at a slit separation $d = \lambda R / w$.
- This distance is defined as the transverse coherence length: $\xi_t = \dfrac{\lambda R}{w}$
- A uniform rectangular source with horizontal and vertical widths w_h and w_v, illuminating two pairs of pinholes at right angles at distances d_h and d_v yields the transverse coherence lengths:

$$\xi_h = \lambda R / w_h \qquad \xi_v = \lambda R / w_v$$

Maximum coherence

- For (gaussian) synchrotron radiation sources, the corresponding transverse coherence lengths are given by $\xi_h = \lambda R/(2\pi\sigma_h)$ and $\xi_v = \lambda R/(2\pi\sigma_v)$.

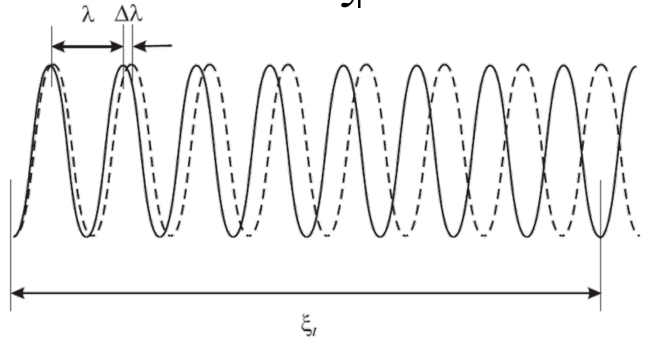
- Third generation synchrotron radiation sources typically have a source size of σ_v 10–50 μm and σ_h 100–500 μm. At a wavelength λ = 0.1 nm and at a typical distance of 40 m from the source, the transverse coherence lengths are in the ranges ξ_v $25-100\mu$m and ξ_h $3-10\mu$m.

- It appears clear that **transverse coherence is more difficult to achieve for short wavelengths** since the coherent power becomes smaller when the wavelength decreases

- Examples:
  - ELETTRA (Italy): fully coherent down to 1000 Å (operative since 1994)
  - SLS (Switzerland): fully coherent down to 100 Å (operative since 2001)
  - SwissFEL (Switzerland) : fully coherent down to 1 Å (operative from 2016)

Longitudinal Coherence

- Let us consider two wavefronts, one at wavelength λ and the other at a slightly different wavelength $\lambda+\Delta\lambda$, which simultaneously depart from a single point. Let us assume that after some distance ξ_l the two wavefronts are in antiphase.

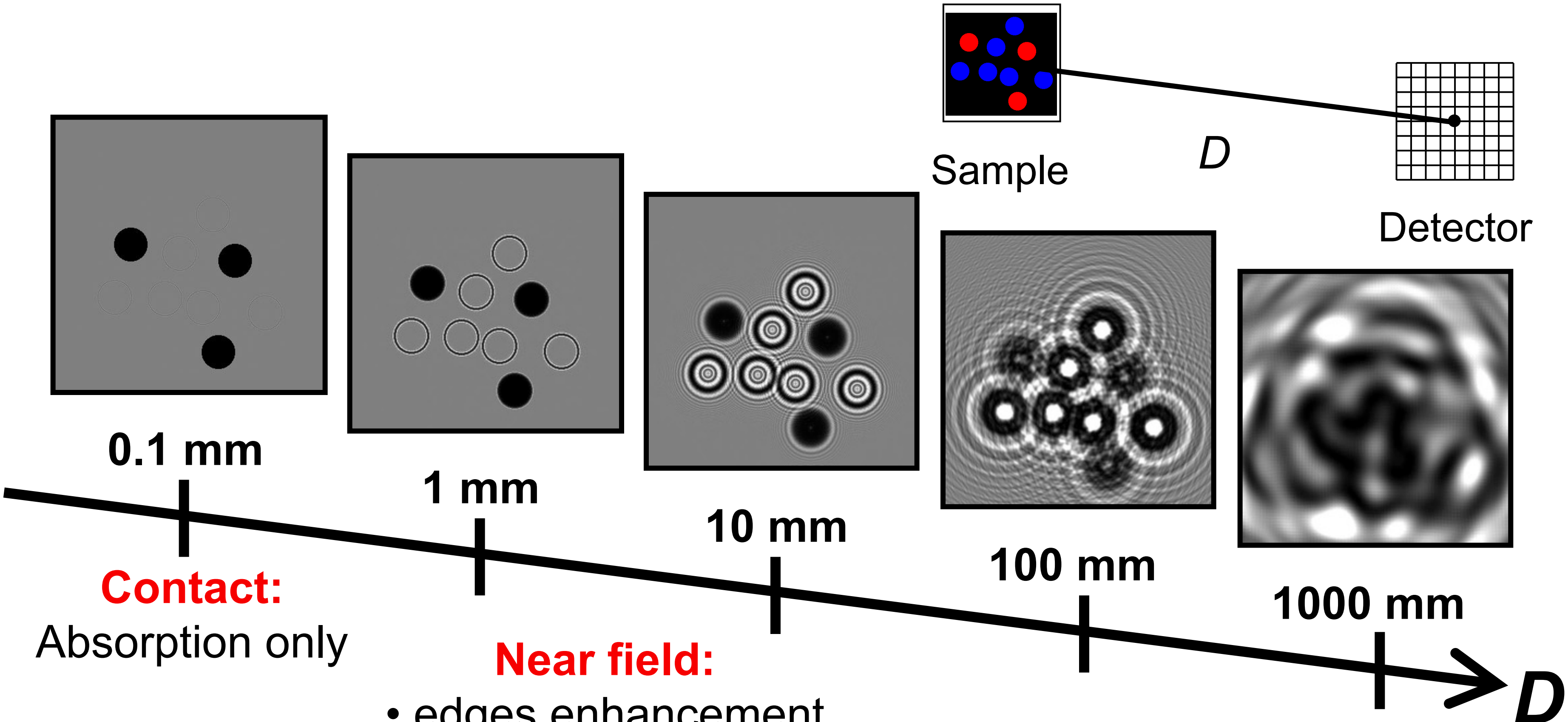


- If the first wave has made N oscillations over that distance, the second wave must have made $N-½$ oscillations. One therefore has $N\lambda=(N-½)(\lambda+\Delta\lambda)$. Solving for N and substituting in $\xi_l=N\lambda$ we find for this distance:

$$\xi_l \simeq \frac{1}{2}\frac{\lambda^2}{\Delta\lambda}$$

- This is the **coherence length** of the wave which in turn defines the **longitudinal coherence** or **temporal coherence**.

Exploiting coherence for imaging



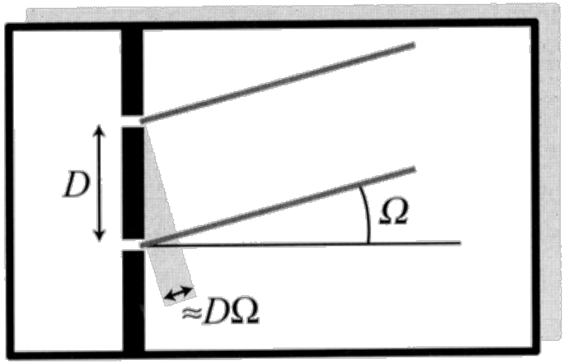
Contact:
Absorption only

Near field:
- edges enhancement
- increases with *D*
- phase objects visible

Towards far field:
- Image loses resemblance with real-space object
- CDI

Kirchoff's propagation integrals beyond…

What else?



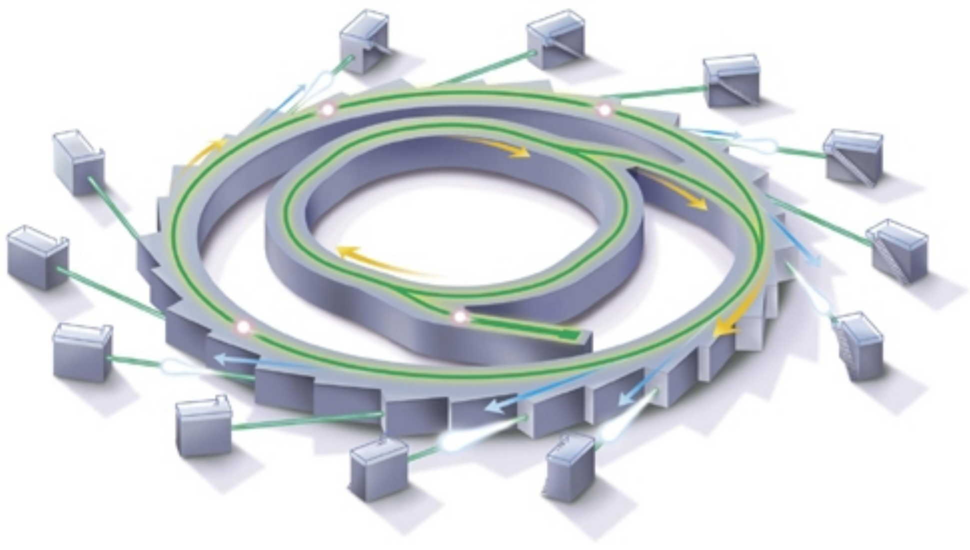
Let's consider the following example:
First dark fringe is observed at $\delta x = \lambda/2$.
Define D to be the distance between the two slits

- It follows that $\delta x \approx D\Omega$ and therefore the first dark fringe corresponds to $\Omega = \lambda/(2D)$
- **This means that by measuring λ and Ω the distance D can be derived!**
- This is the basis of all diffraction-based measurements of distances and position in space
- The value of λ must not be too far from the distance D in order to keep Ω within reasonable values. For X-rays is happens that λ is perfectly suited to study chemical bonds in atoms and molecules.

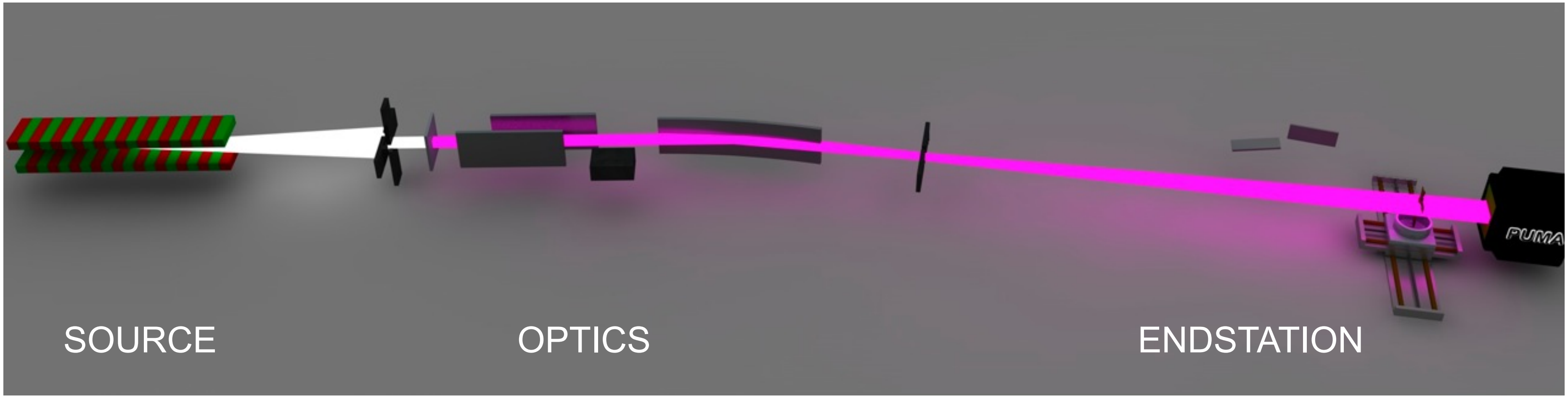
Shaping the X-rays

X-ray optics at beamlines

Synchrotron beamlines

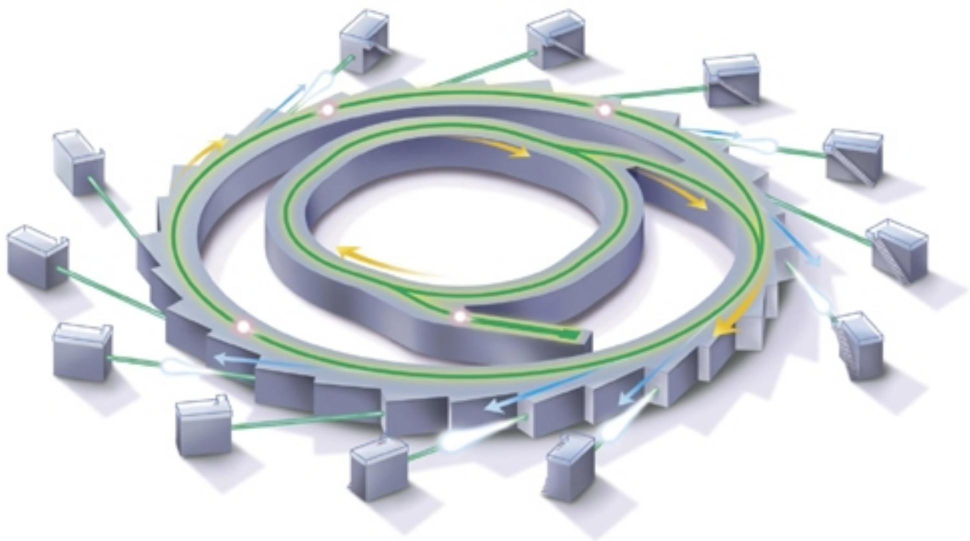


- The storage ring and its many sources of synchrotron light (undulators, wigglers and bending magnets) constitute the core of the synchrotron facility.
- Additional equipment is needed to modify the spectral and geometrical characteristics of the emitted light to meet the requirement of a specific application
- The back-end of the beamline is equipped with experimental stations including different instruments.



http://www.synchrotron-soleil.fr/Recherche/IPANEMA/methodessynchrotron

Synchrotron beamlines



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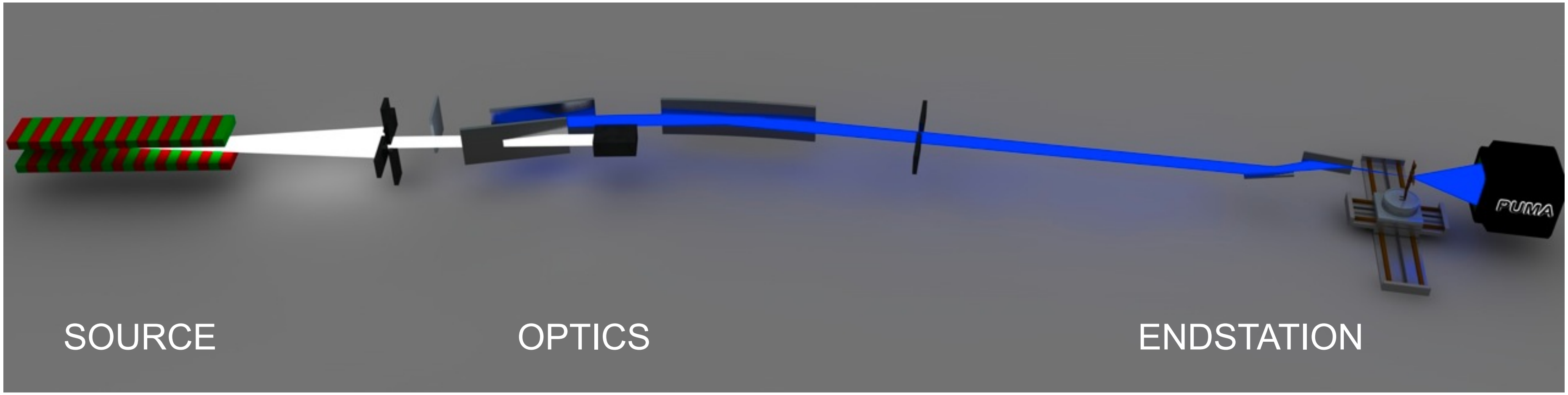
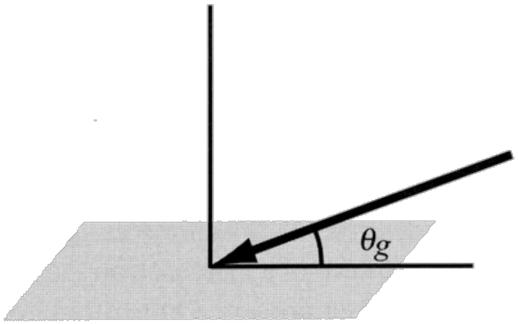
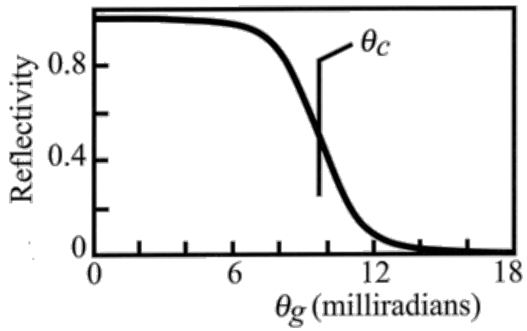
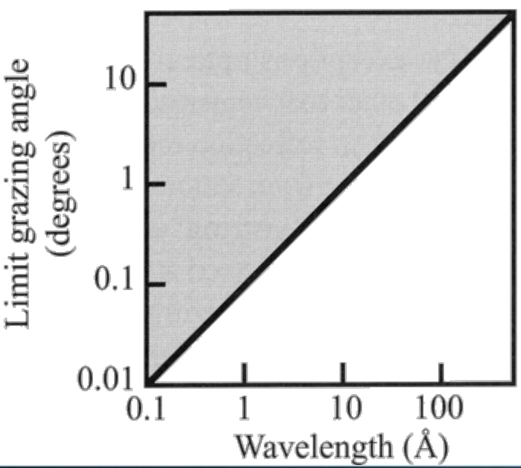


http://www.synchrotron-soleil.fr/Recherche/IPANEMA/methodessynchrotron

Reflection of X-rays







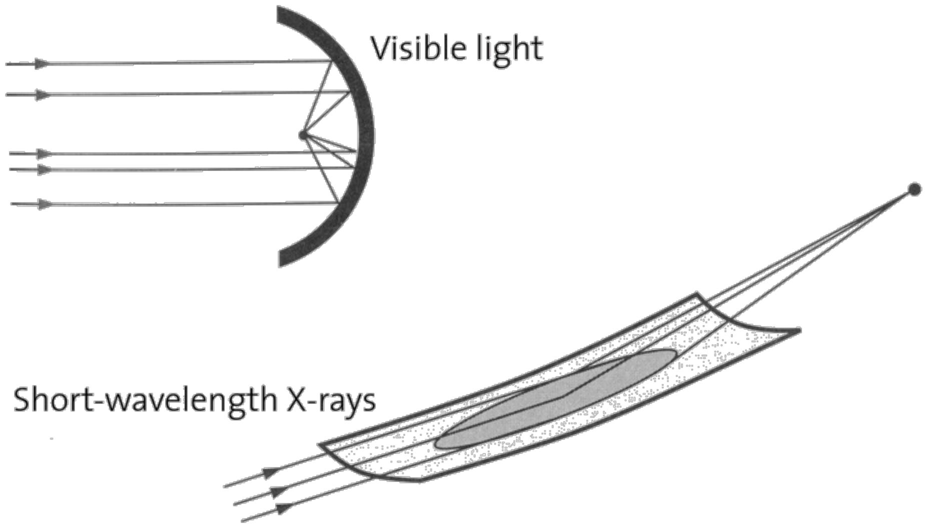
- The reflected intensity sharply decreases when the grazing angle θ_g increases.
- This tendency is enhanced at shorter wavelengths.
- For hard X-rays only beams with $\theta_g \approx 0$ are reflected!

- Practical rule to evaluate the limit of θ_g beyond which the reflection is to week: the grazing angle measured in degree should not exceed one-tenth of the wavelength measured in Å.

Practical implication

- Rule of thumb: $\alpha_{max}[\text{degrees}] = \lambda[\text{Å}]/10$
- Example: 1 Å radiation → $a_c \sim 0.1^o$
  For a beam height = 1 mm → Footprint ≈ 600 mm!!
- This means: X-ray mirrors are big beasts!

Focusing X-rays using reflective optics

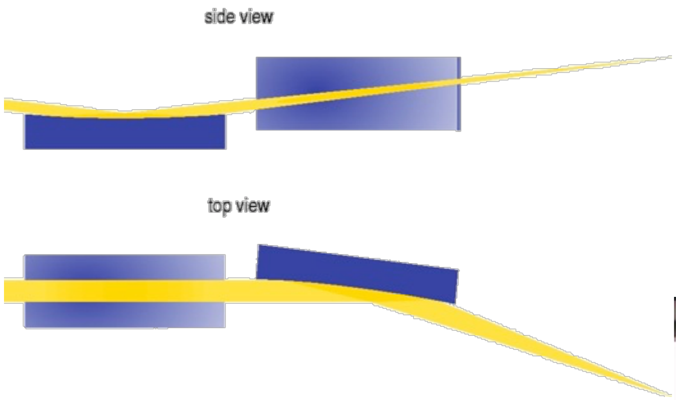


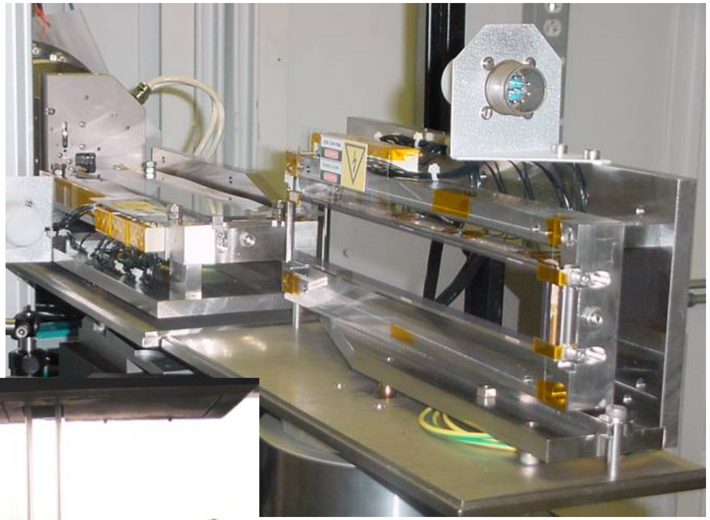
A parabolic mirror focuses a collimated beam of visible light into a point.

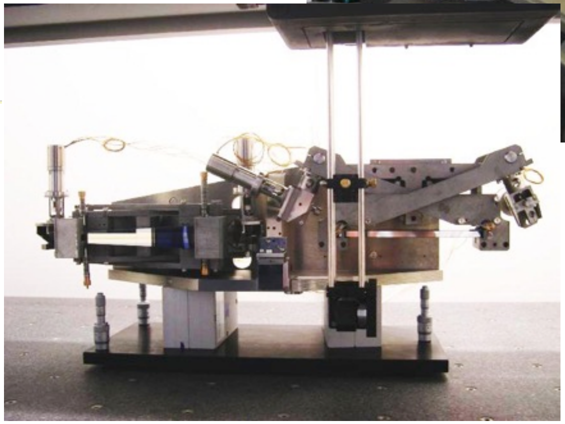
- For shorter wavelength the grazing incidence condition, requires the mirror to be quite long to accommodate the large area that is illuminated by a beam arriving under a small angle.
- The corresponding large surface must be machined or mechanically bent to achieve the required curvature. This is not easy, since the curvature must have high accuracy.

Kirkpatrick-Baez mirrors

- Two cylindrical mirrors in series
  - Flat mirrors + bender actuators
  - **Individual vertical and horizontal focusing**
  - **Achromatic (works for all wavelengths)**







Tuning the wavelength bandwidth

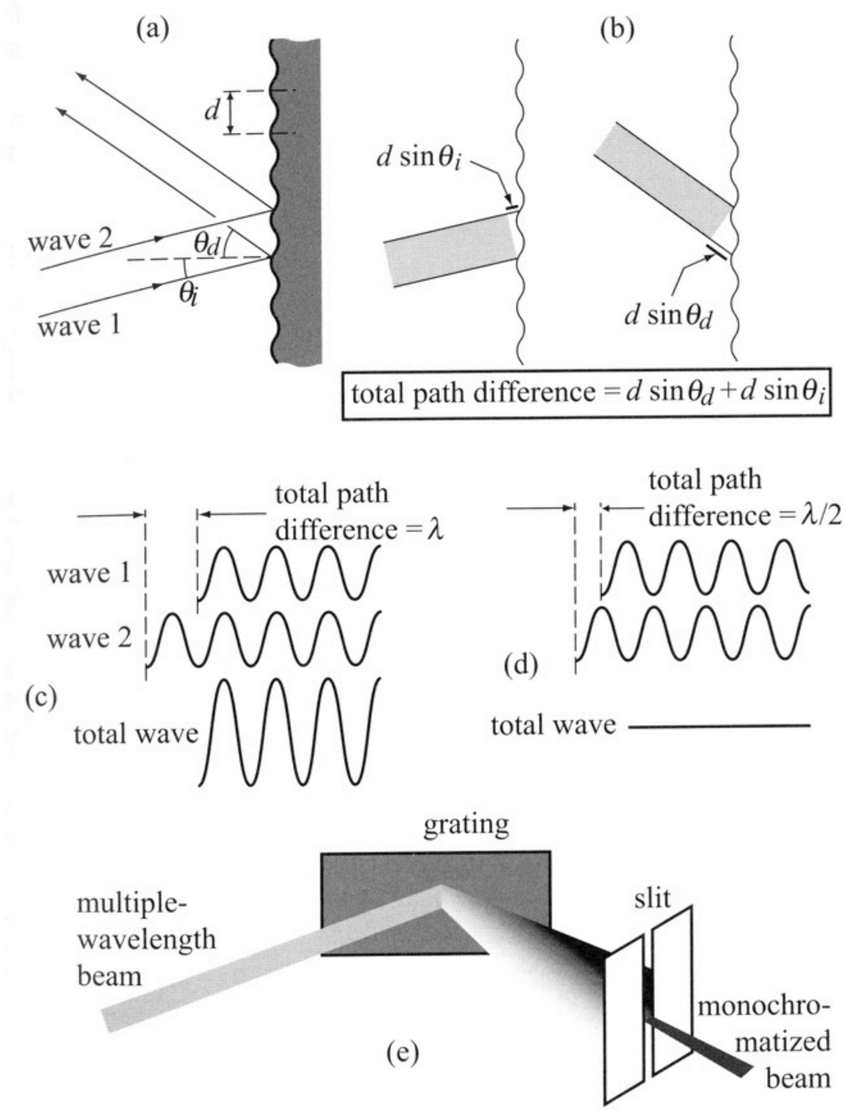
- Most applications of synchrotron light require a very narrow wavelength bandwidth around a given value

- Often it is requested to tune the wavelength during an experiment or to scan it continuously over an extended spectral range

- This filtering process is called “monochromatization” and the corresponding devices are called “monochromators”.

- In general there exist two classes of **monochromators**:
  - Based on **diffraction gratings**, best suited for soft X-rays
  - Based on **crystals**, best suited for shorter wavelengths

Reflection at diffracting grating



(a) The grating is a surface with a periodic array of features (d=period). Two light beams reflected by equivalent points of two adjacent lines travel along different paths

(b) Trigonometry gives the total path difference

(c) If the path difference equals one wavelength (or an integer number of λ) then the superposition of the two waves produces maximum intensity

(d) If the path difference is one-half wavelength (or an odd multiple of λ/2) then the total wave has zero intensity

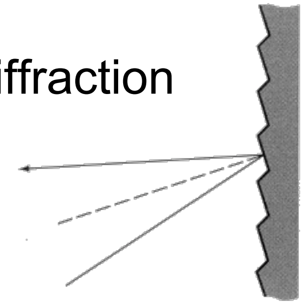
(e) A grating, combined with a slit, produces monochromatization.

Diffraction grating monochromators

- The width Δλ of the monochromatized band of wavelengths is called “absolute resolution”.
- The quality of a monochromator increases when the value of Δλ decreases.
- Usually the figure of merit is the resolving power λ/Δλ. This parameter can be improved by using a large number of lines. It can be shown that:

$$\lambda / \Delta\lambda = mN_L(\sin\theta_i + \sin\theta_d)$$

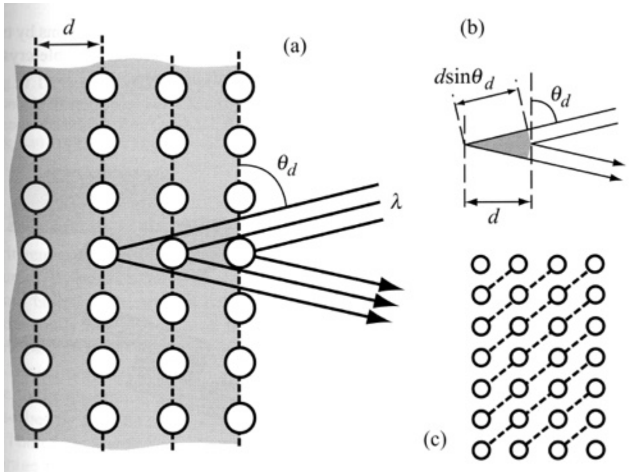
- This means that the filtering effect of each line enhances that of other lines.
- The resolving power improves if the number of illuminated lines increases.
- The preferred line profile is like a “sawtool” because in this case the diffraction condition can be best satisfied and the reflection angle optimized.



Crystal monochromators

- At grazing angles, $\sin\theta_i$ and $\sin\theta_d$ are both $\approx\lambda$ and the grating period d is $\approx\lambda/2$
- For **very short wavelength** the period would become very small and fabricating the grating is technically impossible.
- On the other hand, the **ordered atomic planes** of a crystal automatically constitute the **equivalent of a grating** with a very short period.
- A specific wavelength can be selected by exploiting crystal diffraction.

Bragg Law (diffraction)



- Two beams reflected by adjacent planes travel along different paths and the path difference is 2dsinθ_d.
- If this difference equals one wavelength, then the superposition of the two waves produces maximum intensity. Formally:

$$2d\sin\theta_d = \lambda$$

- The crystal acts as a "natural grating", diffracting with a maximum intensity a given wavelength in the direction given by the Bragg Law.
- Bragg Law applies to each corresponding d-values: the corresponding maximum-intensity direction are called "Bragg reflection".
- A suitable crystal for a monochromator, must have 2d-values that are neither smaller nor too much larger than the wavelength of interest.

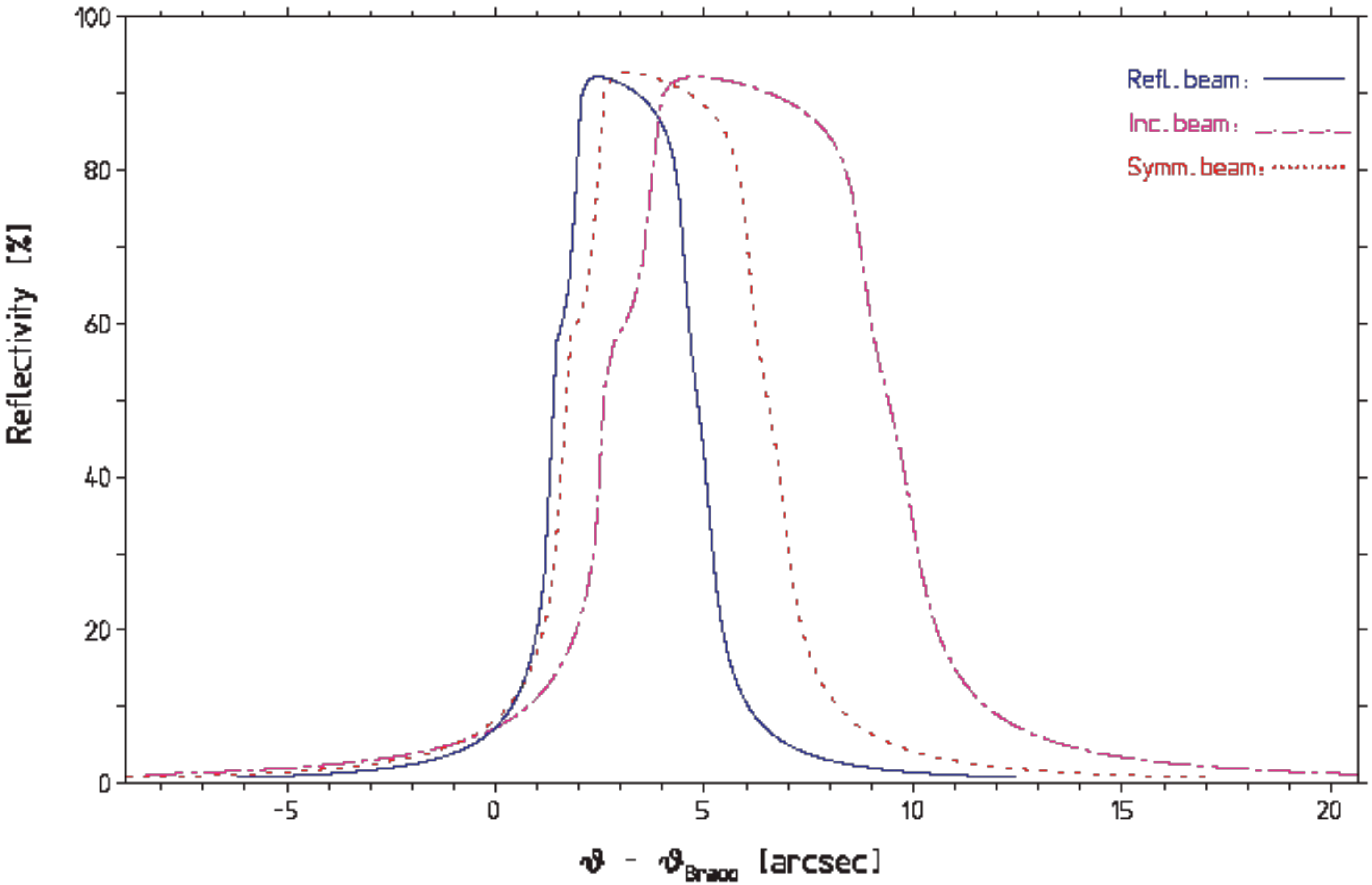
Implications of Bragg Law

- For a high resolving power there is the need of:
  - **Good crystal quality**: a poor-quality crystal (formed by grains in different orientation) produces a "spread" of each Bragg reflection angle θ_d, corresponding to a spread in the wavelength and therefore to a limited $\Delta\lambda$.
  - **Collimated beam**: the angular spread of an insufficiently collimated beam negatively affects the wavelength resolution.
  - **Thermal stability**: the lattice space can change as a function of the temperature and therefore affect the reflectivity properties.
- The overall quality of a crystal monochromator is characterized by the so-called **rocking curve**, which plots the Bragg reflected intensity as a function of the θ_d-angle.

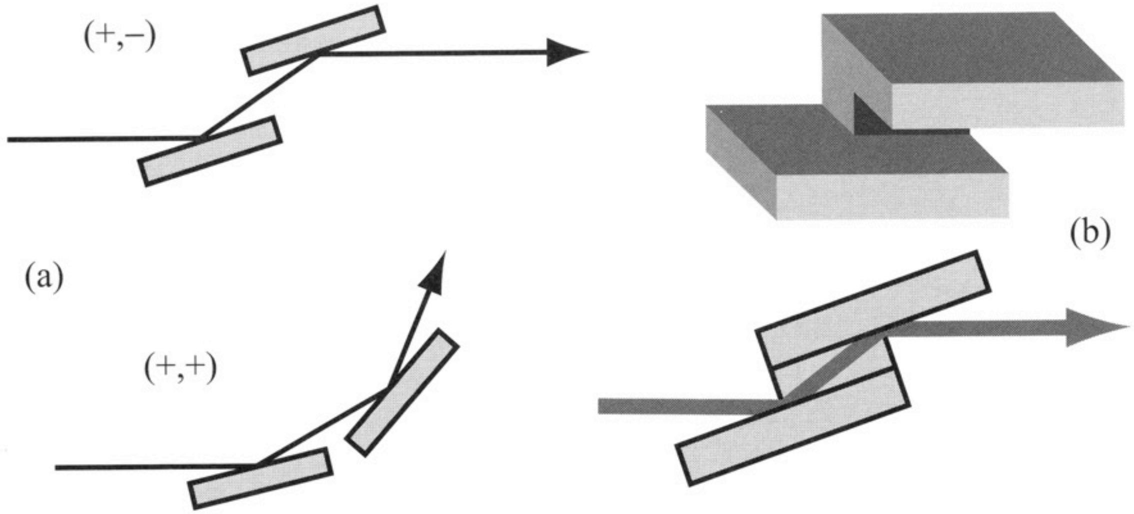
Rocking curve from Si-crystal

SI(2 2 0) E= 8047.8eV d= 1.92014A ϑ_{Bragg}=23.651° α= 8.00° T= 5.00mm unpol
Mag= 1.9 L(abs)= 69.72μm ΔE/Δϑ= 0.8909E-01 eV/arcs
χ in μrad: 0(15.154, 0.352) σ:h(9.323, 0.344)-h(9.323, 0.344) π:h(9.323, 0.233)-h(9.323, 0.233)

σ&π	Rpk %	RI μrad	R-power	FW arcs	μrad	ITh arcs	μrad	Δϑ arcs	μrad	ΔTh arcs	μrad	σEx.L μm	πEx.L μm
Refl.	92.18	18.18	25317.	3.57	17.30	3.75	18.20	3.17	15.36	3.22	15.61	6.21	9.17
Incid.	92.17	35.31	12863.	7.02	34.05	7.30	35.41	6.15	29.83	6.26	30.37	6.21	9.17
Symm.	92.74	25.48	18153.	4.98	24.13	5.24	25.39	4.18	20.26	4.25	20.62	6.62	9.77



Crystal monochromator designs

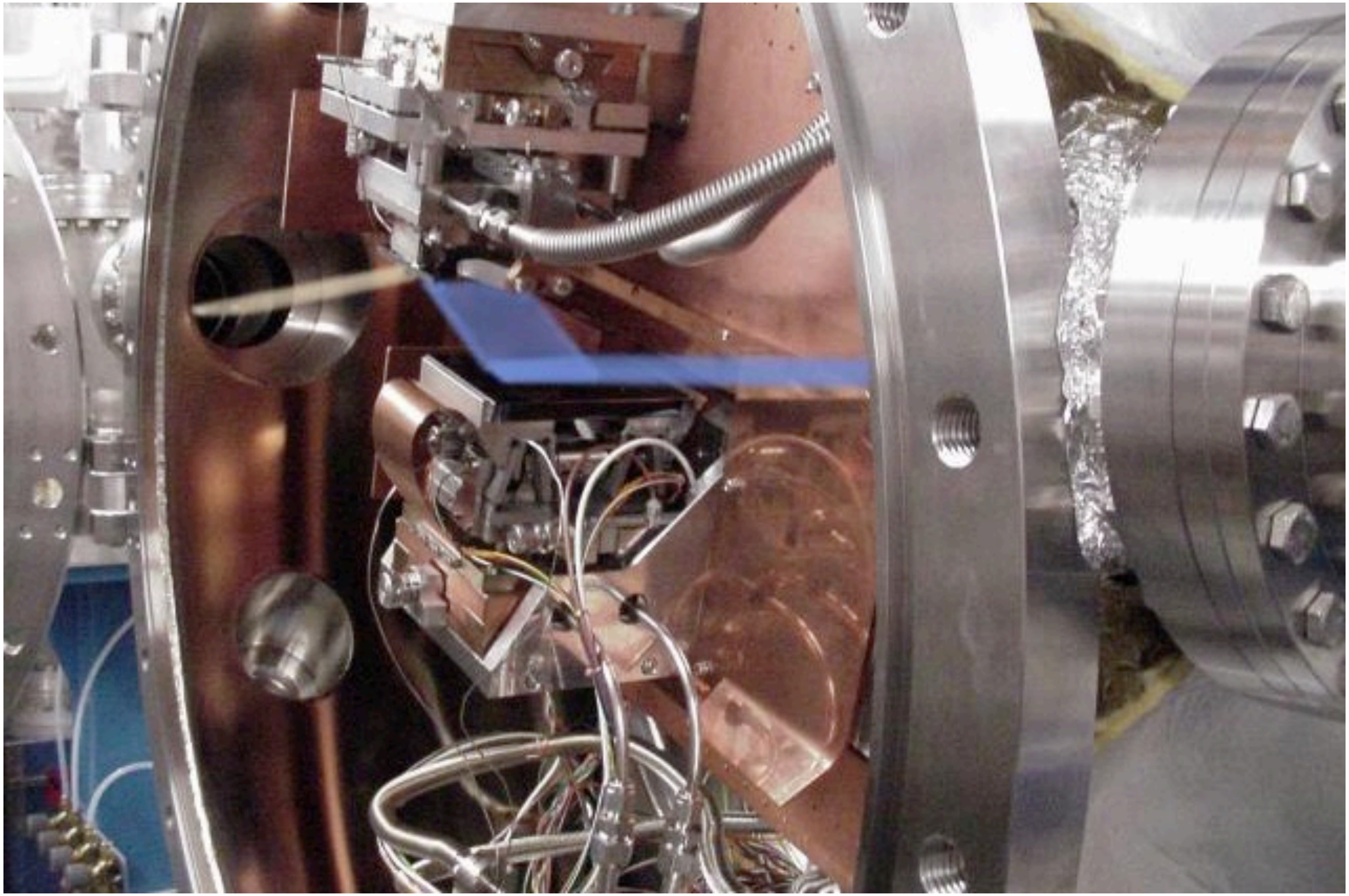


- Double crystals monochromator:
  - High monochromaticity
  - Fixed exit

- Example: (a) (+,-) and (+,+) monochromator
  (b) channel cut monochromator

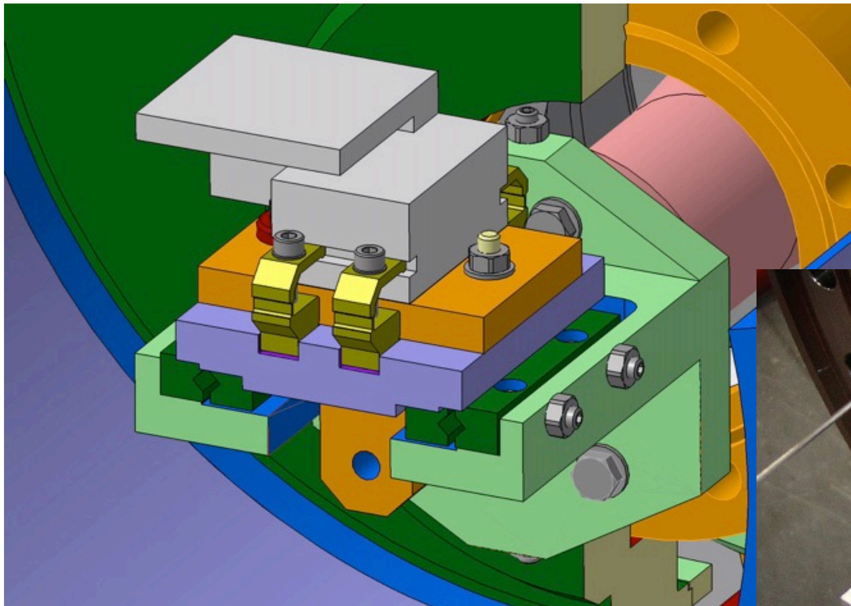
Example of monochromators I

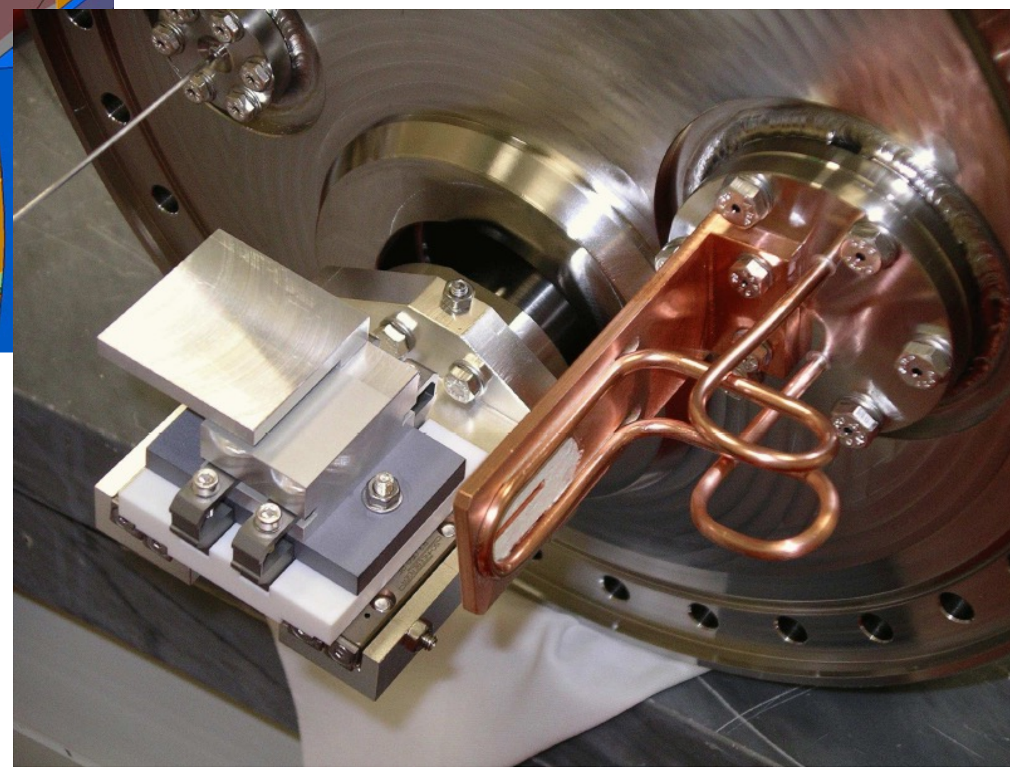
- Double-crystal, fixed exit, Si111 monochromator at 4S (Materials Science)-SLS



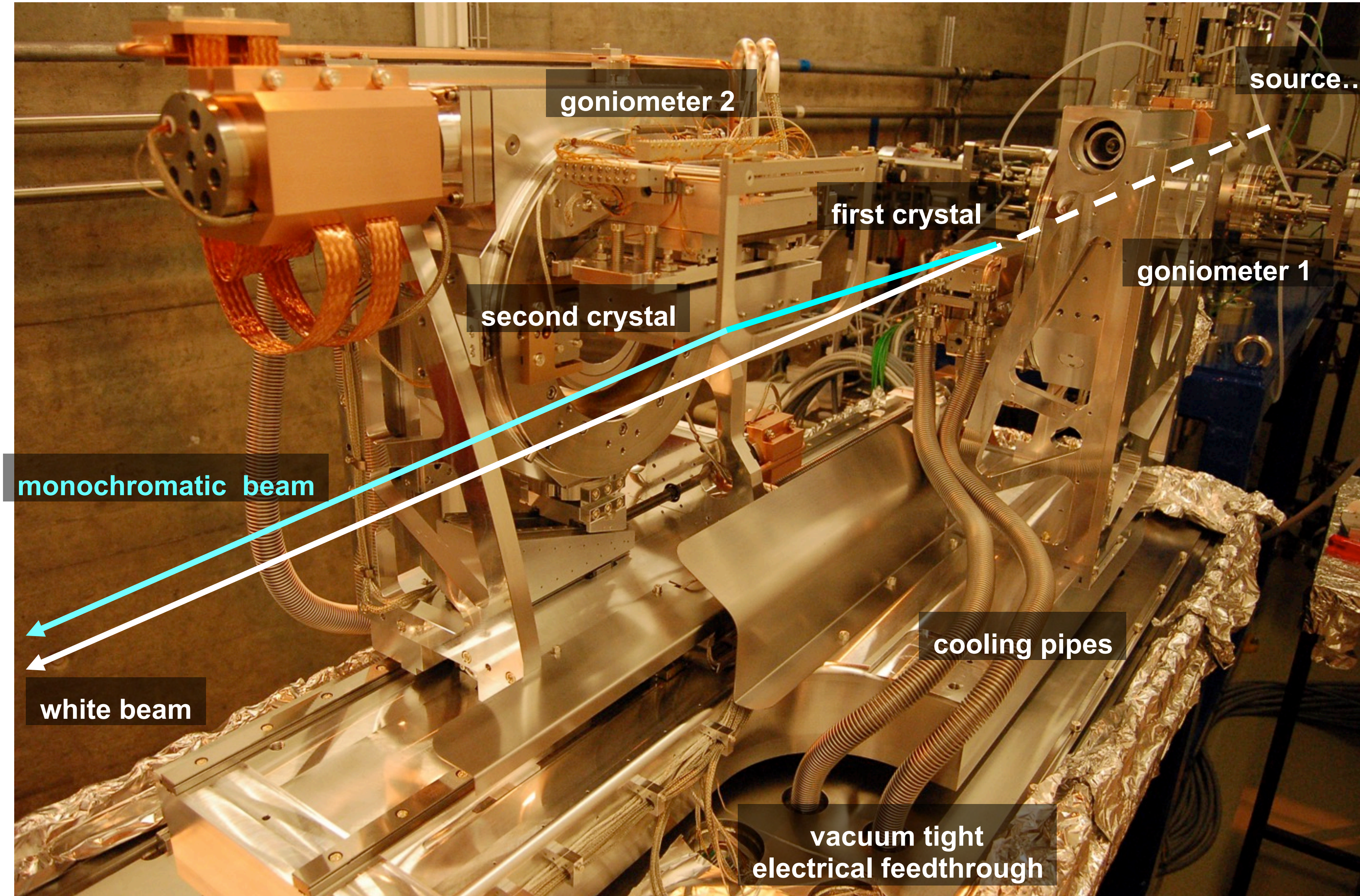
Example of monochromators II

- Channel cut monochromator at Optics Beamline – SLS (courtesy U. Flechsig)

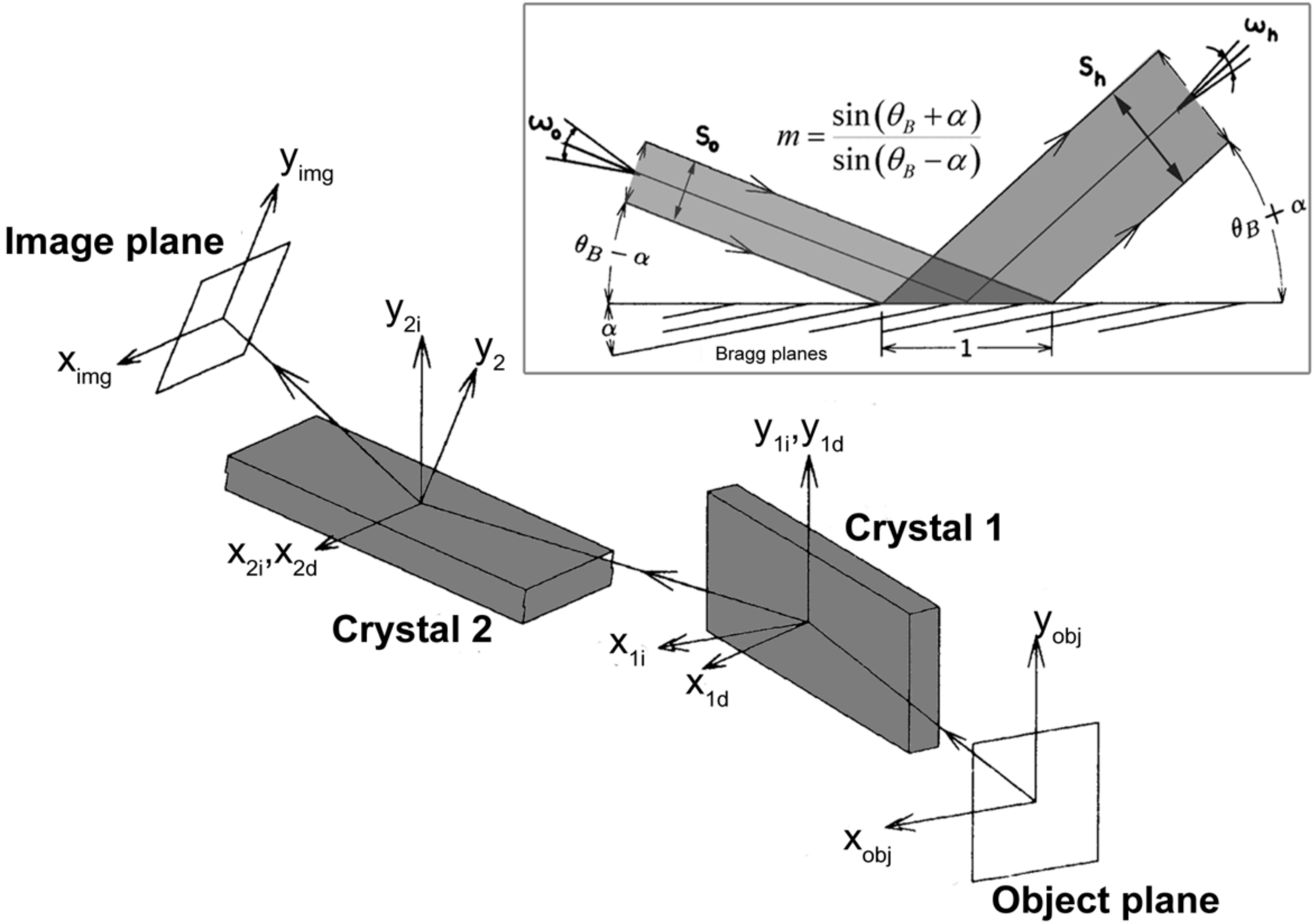




Example of monochromators III (Double-crystal, fixed-exit multilayer monchromator, TOMCAT)

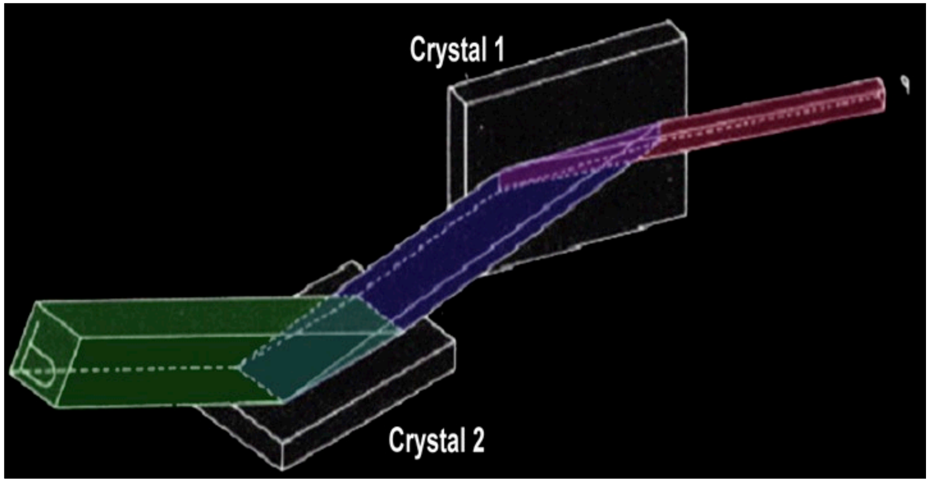


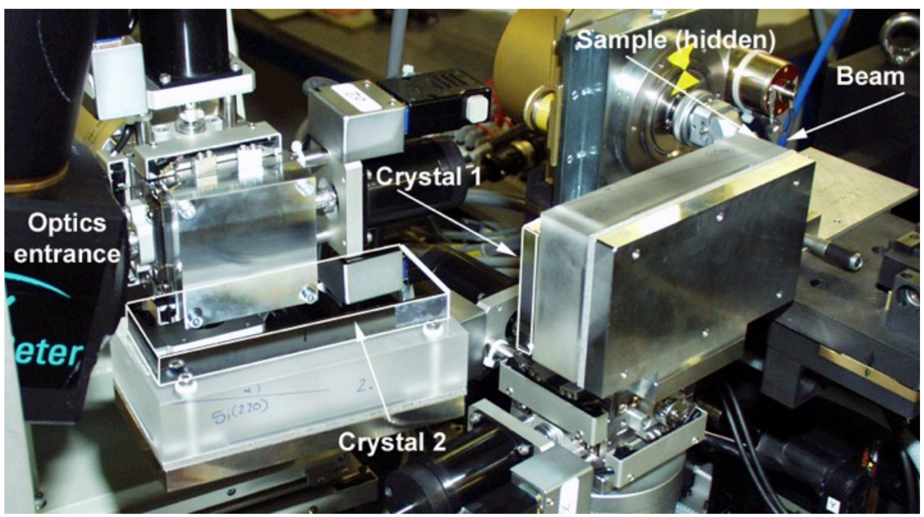
X-ray beam expansion with asymmetrical Bragg diffraction

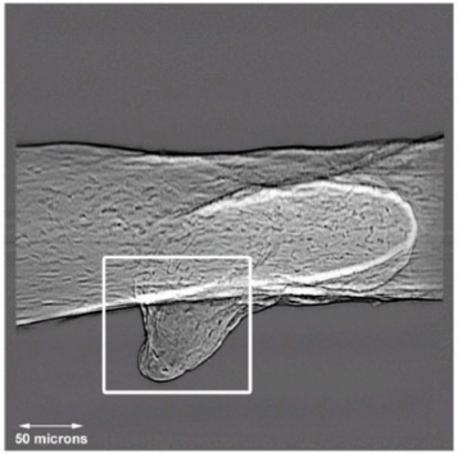


M. Stampanoni et al., J. Appl. Phys. 92, 7630 (2002)

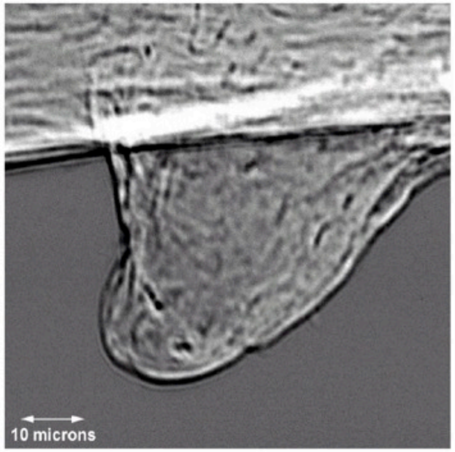
Magnification with asymmetric Bragg diffraction



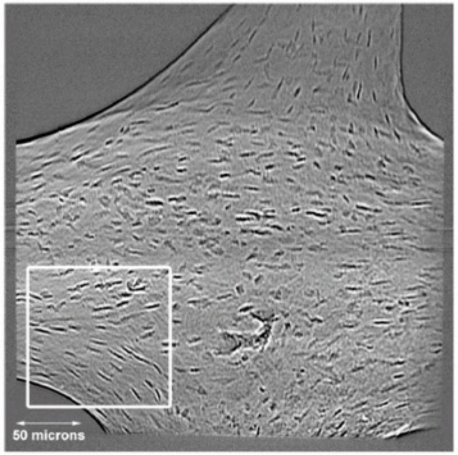




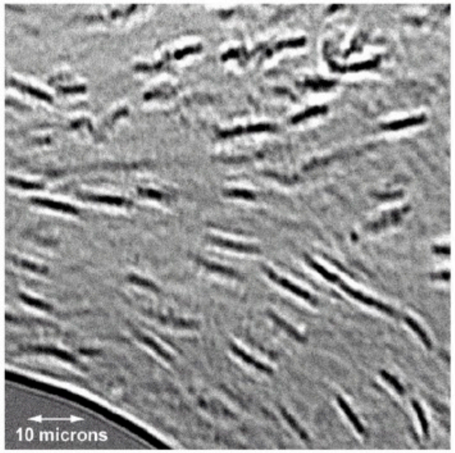
(b) Human trabecula imaged at 50x50.



(c) Detail showing Fresnel patterns.



(d) Same trabecula, rotated by 90°.



(e) Detail showing osteocytes.

M. Stampanoni *et al.*, Applied Physics Letters, Vol. 82 (17), 2003

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Appendix 1: Storage ring parameters: an overview

- **Beam current I [mA]:** defines the intensity of electron current circulating in the ring. Typical values are 10-500 mA.
- **Beam energy E [GeV]:** defines the energy of the electrons circulating into the ring. It can be shown that: $\gamma \approx 1957$ E [GeV]
- **Beam size σ_X , σ_Y [microns]:** beam widths corresponding to x and y, expressed in terms of standard deviation of the number of electrons along each coordinate
- **Beam emittance ε_X , ε_Y [nm · rad]:** describes the extent occupied by the electrons in space and momentum phase space as it travels. A low emittance particle beam is a beam where the particles are confined to a small distance and have nearly the same momentum. Keeping the emittance small means that the likelihood of particle interactions will be greater resulting in higher brilliance. Typical values 0.1-10 nm·rad
- **Beam life time τ_b :** describes the slow decay electron current circulating in the ring. Is usually in the order of several hours!
- **Number and length of bunches N_b:** the length is usually expressed in time length σ_t or space length $\sigma_L = c\, \sigma_t$. Rings can be operated in different modes, corresponding to different values of N_b. We speak about filling patterns.

Appendix 1: Storage ring parameters: an overview

- **Radius of curvature ρ [m]:** of the electron trajectory determined by the action of the dipole bending magnets. It can be shown that $\rho \approx E/B$.

- **Pressure p [mbar]** of the ring vacuum chamber. Typical values are in the 10^{-10}-10^{-11} range.

- **Energy lost per turn δE_p [keV]:** expresses the loss of electron energy due to emission of synchrotron radiation. This energy is restored by the radiofrequency cavity. Ranges between 0.01 and 1000 keV.

- **Frequency ω_{rf}** of the radiofrequency cavity. The electrons entering the cavity must find the right electric field to recover the energy lost by emitting synchrotron radiation. Thus ω_{rf} is related to the time necessary for an electron to travel around the ring , T_0 by: $\omega_{rf} = h_A (2\pi/T_0)$, where h_A is an integer number, called harmonic number. Since the electrons speed apporoaches c, the time of flight is 10^{-7}-10^{-6} s, therefore ω_{rf} is in the MHz range

Appendix 2A: Doppler Shift

The relationship between observed frequency f' and emitted frequency f is given by:

$$f' = \left(\frac{v}{v + v_{s,r}}\right) f$$

where

v is the velocity of waves in the medium
$v_{s,r}$ is the radial component of the velocity of the source (the object emitting the wave) along a line from the source to the observer

Because we are using an inertial reference system, the velocity of an object moving towards the observer is considered as negative, so the observed frequency is higher than its emitted frequency (this is because the source's velocity is in the denominator). Conversely, the velocity of an object moving away from the observer is considered as positive, so the observed frequency is lower than its emitted frequency. When the object is at the same position as the observer, the observed frequency is briefly equal to its emitted frequency.

For all paths of an approaching object, the observed frequency that is first heard is higher than the object's emitted frequency. Thereafter there is a monotonic decrease in the observed frequency as it gets closer to the observer, through equality when it is level with the observer, and a continued monotonic decrease as it recedes from the observer. When the observer is very close to the path of the object, the transition from high to low frequency is very abrupt. When the observer is far from the path of the object, the transition from high to low frequency is gradual.

In the limit where the speed of the wave is much greater than the relative speed of the source and observer (this is often the case with electromagnetic waves, e.g. light), the relationship between observed frequency f' and emitted frequency f is given by:

Change in frequency

$$\Delta f = \frac{fv}{c} = \frac{v}{\lambda}$$

Observed frequency

$$f' = f + \frac{fv}{c}$$

where

f is the transmitted frequency
v is the velocity of the transmitter relative to the receiver in meters per second: positive when moving towards one another, negative when moving away
c is the speed of wave (3×10^8 m/s for electromagnetic waves travelling in a vacuum)
λ is the wavelength of the transmitted wave subject to change.

As mentioned previously, these two equations are only accurate to a first order approximation. However, they work reasonably well in the case considered by Doppler: when the speed between the source and receiver is slow relative to the speed of the waves involved and the distance between the source and receiver is large relative to the wavelength of the waves. If either of these two approximations are violated, the formulae are no longer accurate.

Source: wikipedia

Appendix 2B: Relativistic background of synchrotron light

Take home formulae:

Gamma factor: $\gamma = \dfrac{1}{\sqrt{1-\dfrac{u^2}{c^2}}}$

Doppler shift: $\lambda_{lab} \approx \dfrac{\lambda_e}{2\gamma}$

Lorentz contraction: $L' = \dfrac{L}{\gamma}$

Emitted wavelength in the lab: $\lambda_{lab} \approx \dfrac{L}{2\gamma^2}$

Inset A: Relativistic background of synchrotron light

Relativity is the key to understanding how a synchrotron source works. For our discussion, we need a simple introduction to the basic concepts of relativity and an equally simple discussion of the two relativistic phenomena used by synchrotron sources: Lorentz contraction and Doppler shift.

The experimental foundation of relativity is a rather surprising phenomenon: the speed of light, c, does not change when it is measured in two different reference frames, one moving at constant speed u with respect to the other. This fact is surprising because for objects like cars or trains the speed does change with the reference frame.

However surprising, the invariance of c is not a conjecture but a solid fact, supported by many experiments. What are its consequences?

Consider an electron moving at speed u along the x-axis, and emitting synchrotron light. Imagine an emitted light pulse along the x-axis; during a time period Δt_e, the pulse travels along a distance Δx_e such that

$$\frac{\Delta x_e}{\Delta t_e} = c\,. \qquad \text{(A1)}$$

Δx_e and Δt_e are the values measured from the point of view (reference frame) of the moving electron. If we look at the light pulse from the beamline reference frame (the 'laboratory frame'), then the measured position and the measured

Before Einstein's relativity, this change was believed to follow the simple rules:

$$x_L = x_e + ut_e \qquad \text{(A2)}$$

$$t_L = t_e\,. \qquad \text{(A3)}$$

These rules, however, are in conflict with the experimental fact that c does not change with the reference frame. In fact, according to eqns A2 and A3 the speed of the light pulse in the new frame would be

$$\frac{\Delta x_L}{\Delta t_L} = \frac{\Delta x_e + u\Delta t_e}{\Delta t_e} = \frac{\Delta x_e}{\Delta t_e} + u = c + u\,.$$

Thus, c would not be invariant but change from c to $c + u$, contrary to all experimental evidence. We can eliminate this problem by adopting a modified version of eqns A2 and A3:

$$x_L = \gamma(x_e + ut_e), \qquad \text{(A4)}$$

$$t_L = \gamma(t_e + \alpha x_e)\,, \qquad \text{(A5)}$$

where γ and α are two parameters to be determined. How? First of all, we must require the invariance of c. In the laboratory frame:

$$\frac{\Delta x_L}{\Delta t_L} = \frac{\Delta[\gamma(x_e + ut_e)]}{\Delta[\gamma(t_e + \alpha x_e)]} = \frac{\Delta x_e + u\Delta t_e}{\Delta t_e + \alpha\Delta x_e} = \frac{\dfrac{\Delta x_e}{\Delta t_e} + u}{1 + \alpha\dfrac{\Delta x_e}{\Delta t_e}}\,.$$

If $\Delta x_e/\Delta t_e = c$ (speed of light), then $\Delta x_L/\Delta t_L$ must be also equal to c. Thus:

$$\frac{c+u}{1+\alpha c} = c\,,$$

and $c + u = c + \alpha c^2$, which gives $\alpha = 1/c^2$.

Equations A4 and A5 can then be written as:

$$x_L = \gamma(x_e + ut_e), \qquad \text{(A6)}$$

$$\left(\ldots \frac{ux_L}{c^2} \right). \qquad \text{(A7)}$$

G. Margaritondo, *Elements of Synchrotron Light: For Biology, Chemistry, and Medical Research*, Oxford University Press, New York , 2002, ISBN0-19-850931-6

Appendix 2B: Relativistic background of synchrotron light

Note a remarkable implication of eqn A7: a time interval Δt_e measured from the point of view of the electron (at the position $x_e = 0$) changes to:

$$\Delta t_L = \gamma \Delta t_e \qquad \text{(A8)}$$

when measured in the laboratory frame. Thus, since $\gamma > 1$ (see later), the time interval *appears expanded*.

We must now evaluate the parameter γ in eqns A6 and A7. Imagine (see Fig. A-1) a light pulse that, instead of traveling along the x-axis, follows a direction y perpendicular to x. From the point of view of the electron, after a time Δt_e the light pulse reaches the distance:

$$y_e = c\,\Delta t_e\,. \qquad \text{(A9)}$$

Seen from the laboratory (see again Fig. A-1), the pulse travels in an oblique direction. After a time $\Delta t_L = \gamma \Delta t_e$, it reaches the distance y_L along the y-direction and the distance $x_L = u\Delta t_L$ along the x-direction. Thus, the total distance traveled $c\,\Delta t_L$ is equal to $\sqrt{y_L{}^2 + (u\Delta t_L)^2}$, so $c^2\Delta t_L{}^2 = y_L{}^2 + u^2\Delta t_L{}^2$, and

$$y_L = \sqrt{c^2 - u^2}\;\Delta t_L\,;$$

considering eqn A8, we obtain:

$$y_L = \sqrt{c^2 - u^2}\;\gamma\Delta t_e\,. \qquad \text{(A10)}$$

On the other hand, the y-coordinate, being perpendicular to the motion, cannot change when we change the reference frame:

$$y_e = y_L\,, \qquad \text{(A11)}$$

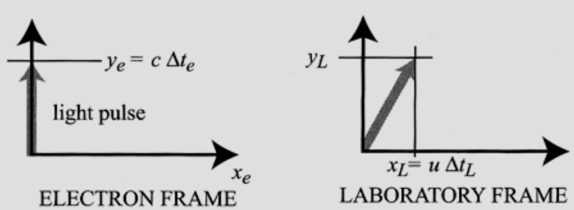


Fig. A-1

which, using the results of eqns A9 and A10 for y_e and y_L gives

$$c = \sqrt{c^2 - u^2}\;\gamma\,,$$

or:

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}\,. \qquad \text{(A12)}$$

The Doppler shift

We now possess all the ingredients to calculate the wavelength and frequency changes of synchrotron light when we change our point of view from the electron frame to the laboratory frame. Imagine (Fig. A-2) a moving source emitting a series of ultrashort pulses at a time distance T_e from each other. The frequency in the source reference frame is $\nu_e = 1/T_e$, and the corresponding wavelength is $\lambda_e = c/\nu_e = cT_e$.

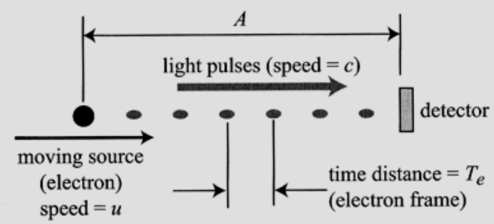


Fig. A-2

We can evaluate the frequency in the laboratory frame from the measured time distance between two successive pulses. Assume that the first pulse is emitted at the time $t_e = t_L = 0$ s and detected by a detector at distance A: the first-pulse detection occurs A/c s after the emission. The second pulse is emitted after the time period T_e, which expands (eqn A8) to γT_e in the laboratory frame. In the meantime, the source motion has shortened the distance A to $A - u(\gamma T_e)$. Thus, the second pulse is detected at the time:

$$\gamma T_e + \frac{A - u\gamma T}{c}\,.$$

The interval between the two detection times is:

G. Margaritondo, *Elements of Synchrotron Light: For Biology, Chemistry, and Medical Research*, Oxford University Press, New York , 2002, ISBN0-19-850931-6

Appendix 2B: Relativistic background of synchrotron light

$$T_L = \gamma T_e + \frac{A - u\gamma T_e}{c} - \frac{A}{c} = \gamma T_e\left(1-\frac{u}{c}\right).$$

Thus, the frequency in the laboratory frame is

$$\nu_L = \frac{1}{T_L} = \frac{T_e}{\left(1-\frac{u}{c}\right)} = \frac{\nu_e}{\gamma\left(1-\frac{u}{c}\right)}. \qquad \text{(A13)}$$

Equation A13 shows the Doppler shift in frequency between the source (electron) frame and the laboratory frame. The corresponding Doppler shift between the wavelengths $\lambda_e = c/\nu_e$ and $\lambda_L = c/\nu_L$ is

$$\lambda_L = \lambda_e \gamma\left(1-\frac{u}{c}\right). \qquad \text{(A14)}$$

In a synchrotron, the electron speed u is $\approx c$, and therefore $u/c \approx 1$. Thus* we have $\gamma(1-u/c) \approx 1/(2\gamma)$, and eqn A14 becomes:

$$\lambda_L \approx \frac{\lambda_e}{2\gamma}, \qquad \text{(A15)}$$

which is the approximate Doppler shift that will be used to derive eqn 1.4.

The Lorentz contraction
Why does the undulator period L appear contracted from the point of view of the moving electron? To answer, consider a modern way to measure L: interferometry. One could for example (Fig. A-3) use mirrors to mark the distance L, and adjust λ_L to obtain constructive interference between the initial wave and the wave that has traveled back and forth along L.

The condition for constructive interference at the semi-transparent mirror is:

$$\frac{L}{\lambda_L} + \frac{L}{\lambda_L} = n,$$

(*) $$\gamma\left(1-\frac{u}{c}\right) = \frac{1-\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{\sqrt{1-\frac{u}{c}}}{\sqrt{1+\frac{u}{c}}} = \frac{\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}} = \frac{1}{\gamma\left(1+\frac{u}{c}\right)} \approx \frac{1}{2\gamma}.$$

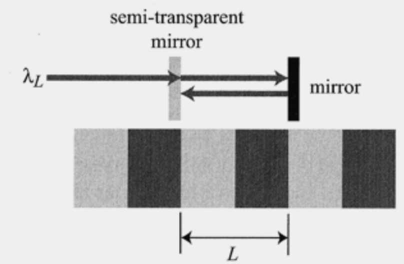


Fig. A-3 Interferometric measurement of the distance L in the laboratory frame.

where n = an integer number; thus:

$$n = \frac{2L}{\lambda_L}. \qquad \text{(A16)}$$

Imagine now repeating the interferometric measurement in the electron frame, calling L' the result. When the wave travels back from the mirror, its wavelength is Doppler shifted (eqn A14) to $\lambda_L\,\gamma(1+u/c)$. During the path to the mirror, the Doppler shift is given by the corresponding expression with reversed speed direction, $\lambda_L\,\gamma(1-u/c)$.

Thus, the constructive-interference condition for the entire wave path becomes:

$$\frac{L'}{\gamma\left(1-\frac{u}{c}\right)\lambda_L} + \frac{L'}{\gamma\left(1-\frac{u}{c}\right)\lambda_L} = n = \frac{2L}{\lambda_L}.$$

After a few steps, this equation gives

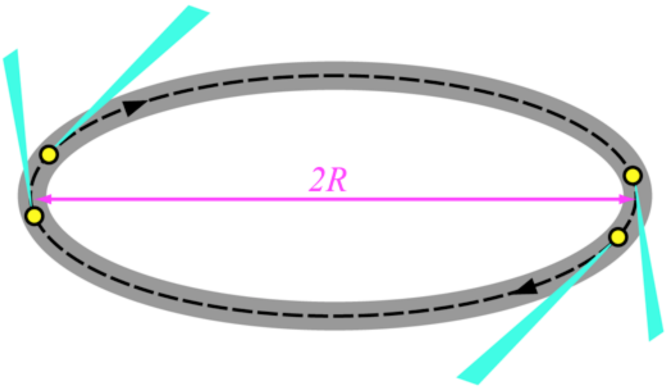
$$L' = \frac{L}{\gamma} \qquad \text{(A17)}$$

confirming the Lorentz contraction from L to L/γ that will be used to derive eqn 1.4.

The electron energy
Einstein's best known law says that the electron energy is m_0c^2, where the electron (rest) mass m_0 is $\approx 9 \times 10^{-31}$ kg. The quantitative value is $m_0c^2 \approx 9 \times 10^{-31} \times (3 \times 10^8)^2 \approx 8 \times 10^{-14}$ J. If we measure the energy in electronvolts [1 electronvolt (eV) = 1.6×10^{-19} J], then m_0c^2 is approximately equal to 0.5×10^6 eV, or 0.5 MeV [1 MeV equals one million electronvolts].

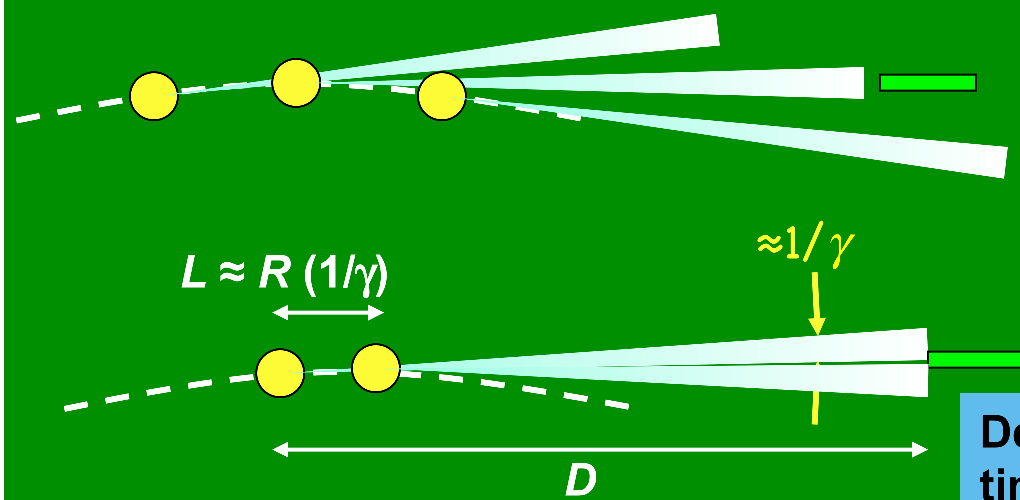
G. Margaritondo, *Elements of Synchrotron Light: For Biology, Chemistry, and Medical Research*, Oxford University Press, New York , 2002, ISBN0-19-850931-6

Appendix 2B: X-ray emission…another look



Seen from the side of the ring, each electron looks like an oscillating charge in an antenna, emitting photons with a frequency 2πR/c , but this lies in the radio wave range.

What shifts the emission to the x-ray range?
→ RELATIVITY AGAIN!



A torchlight-electron illuminates a small-area detector once per turn around the ring for a short time Δt

Photons start to be detected at the time D/c

Detection ends at the time $L/u + (D - L)/c$

Photon pulse duration:

$$\Delta t = L/u + (D - L)/c - D/c = L/u - L/c = (L/u)\,(1-u/c) = (L/u)\gamma^2/(1+u/c)$$

For $u \approx c$, $(1+u/c) \approx 2$ and $\Delta t \approx L/(2c\gamma^2) \approx R/(2c\gamma^3)$.

Characteristic frequency $\nu = 1/\Delta t \approx 2c\gamma^3/R$ → **Again, x-rays!**

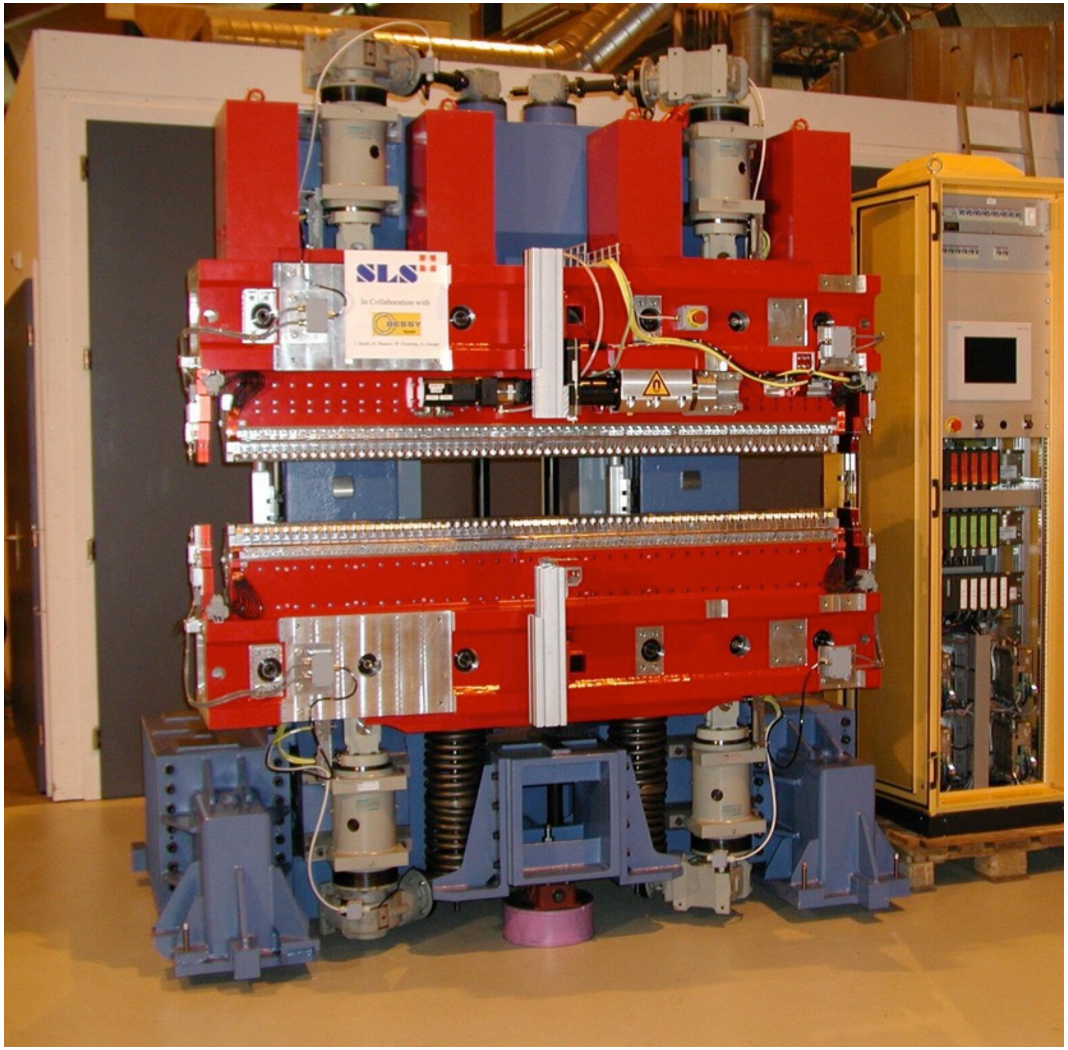
Appendix 3: Example of synchrotron sources I

Elliptical twin undulator UE56 at SIM (Surface Interface Microscopy), SLS

- The source of the SIM beamline is a pair of pure permanent magnet helical undulators.
- It produces up to 10^{15} photons/sec in a range of 95 - 2000 eV.
- This photon energy range covers most relevant core levels from the Si-2p states (99eV) up to the rare earth d-states (~1500 - 2000eV).

Technical specs:

Period length:	56 mm
Number of periods:	2 x 32
Gap:	18 - 150 mm
Polarization:	Linear
	Circular (R or L)
K value:	2.2/4.4
Operating on harm.:	1 - 7
Maximum field:	0.83 T / 0.66 T (H / V)
Energy range:	95 - 2000 eV



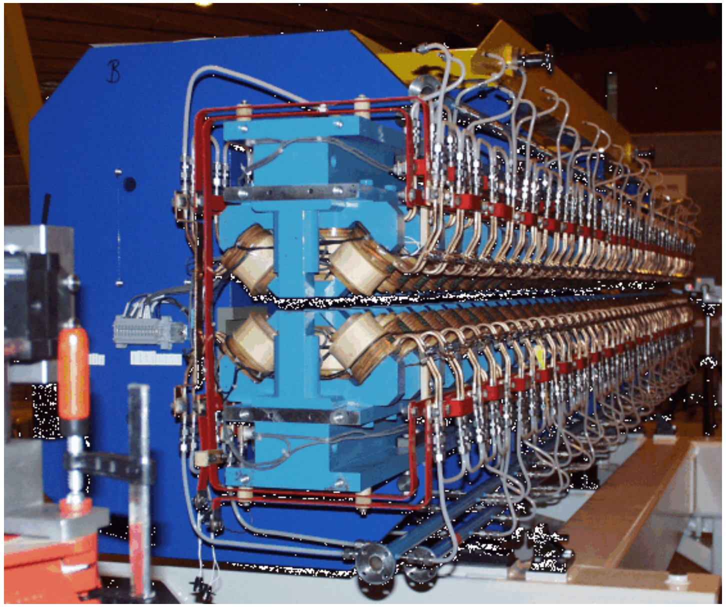
Appendix 3: Example of synchrotron sources II

EM crossed field undulator at SIS (Surface Interface Spectroscopy), SLS

- The light is produced in two electromagnetic, elliptically undulators.

- The undulators can be operated independently or together appearing as a single device

- The emitted light can be linearly polarized (horizontal and vertical) and circular (left or right handcircular polarization) or a mixture of both called elliptical.



Technical specs:

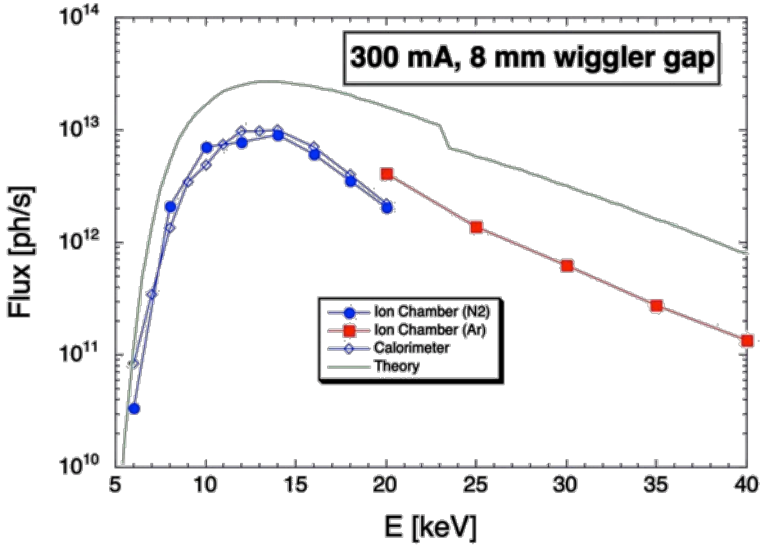
Length:	2 x 4.4 m
Period length:	212 mm
Number of poles:	2 x 21
Fundamental energy @ max. K:	10 eV
Flux @ 20 eV:	2 x 10^{15} ph/s/0.1%BW
Brightness @ 20 eV:	3 x 10^{17} ph/s/mm^2/mrad2/0.1%BW
Photon source size @ 10 eV:	256 x 227 μm^2
Photon source div. @ 10 eV:	71 x 65 μrad^2

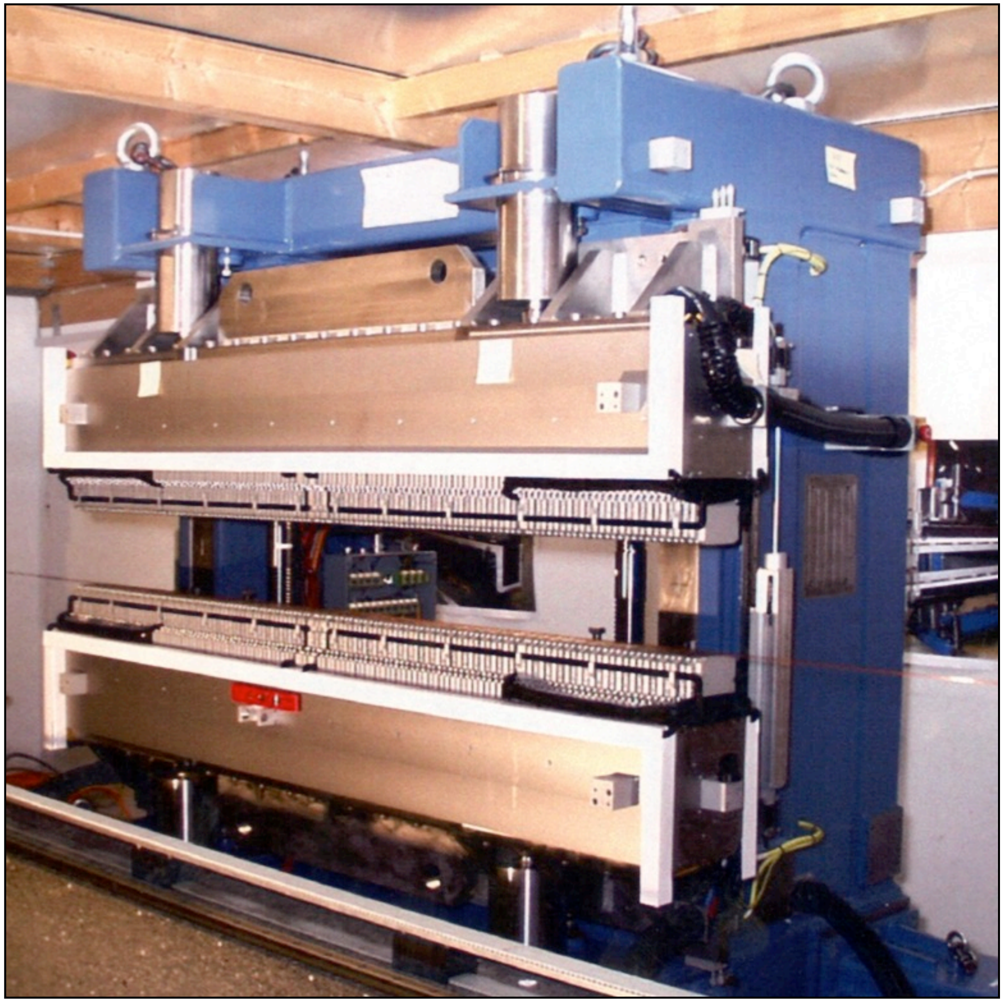
Appendix 3: Example of synchrotron sources III

Hybrid minigap wiggler at 4S (Materials Science) - SLS

Technical specs:

Length:	2.0 m
Period length:	61 mm
Minimum magnetic gap:	7.9 mm
Number of periods:	65
Critical energy:	7.5 keV
Emitted power:	8.8 kW

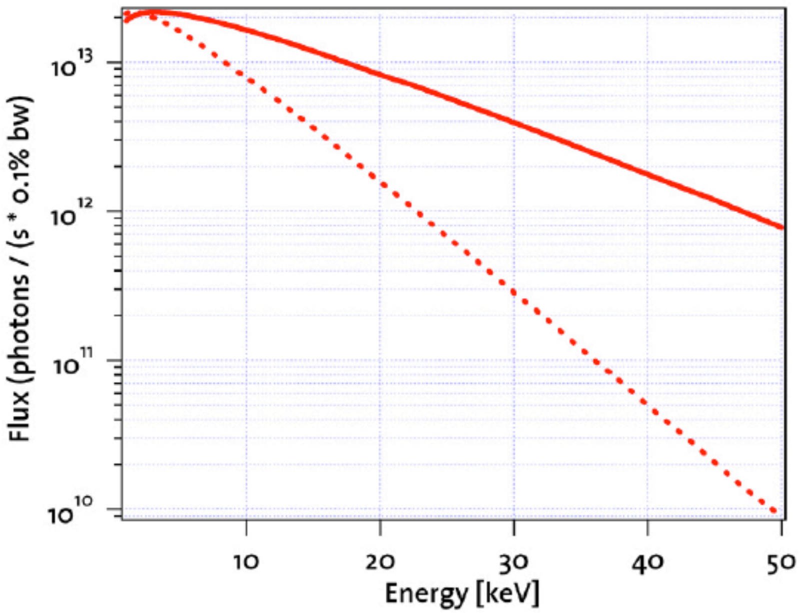


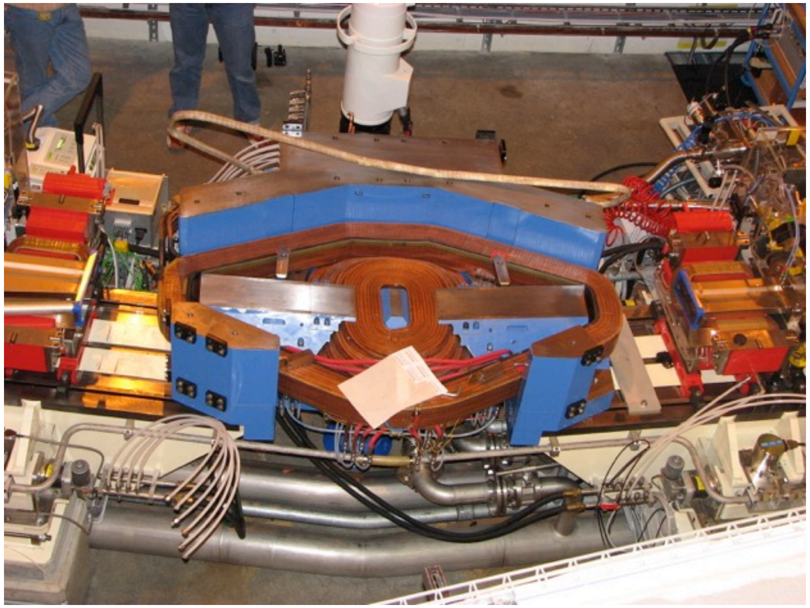


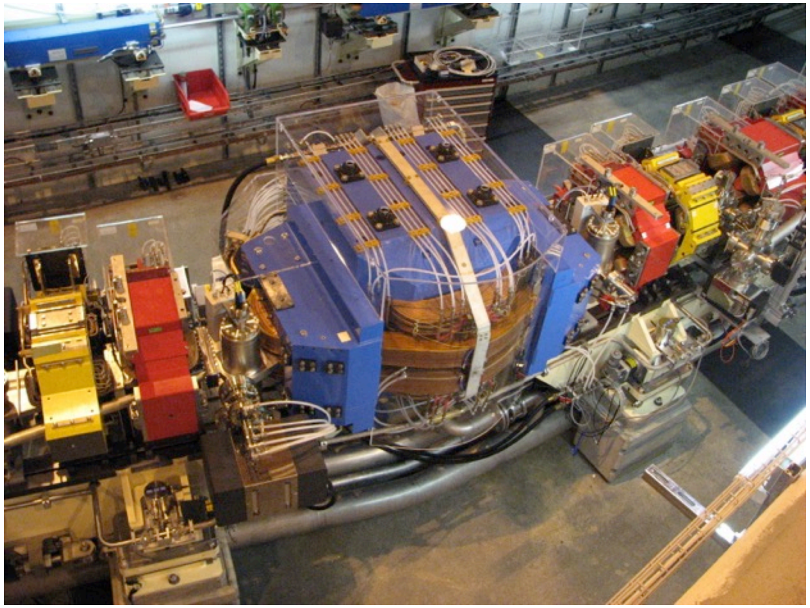
Appendix 3: Example of synchrotron sources IV

Superbend at TOMCAT - SLS

Machine Parameters	
Ring energy	2.4 GeV
Ring current	400 mA
Source parameters	
Magnetic field	2.9 T
Critical energy	11.1 keV / 1.12 Å
Electron source size (σ_x, σ_y)	46 μm, 16 μm
Electron source divergence (σ'_x, σ'_y)	109 μrad,16 μrad
Photon source size (Σ_x, Σ_y)	53 μm, 16 μm
Photon source divergence (Σ'_x, Σ'_y)	2 mrad, 0.6 mrad





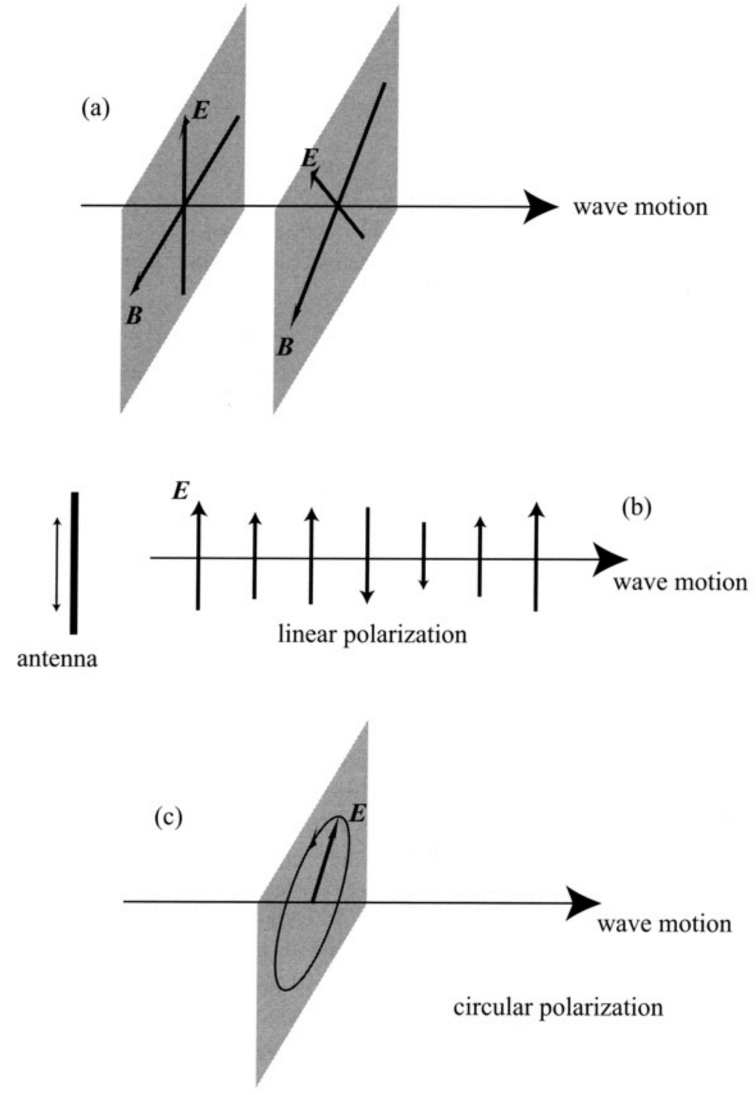


Appendix 4: Polarization

- An electromagnetic wave is a propagating perturbation of the electromagnetic field (**electric and magnetic vectors**) i.e. with strength and direction

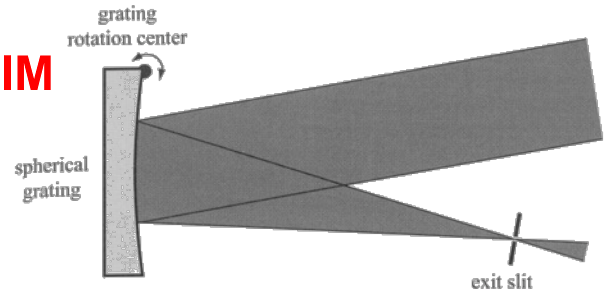
- Electric and magnetic vectors are perpendicular to the propagation direction (transverse wave), and perpendicular to each other

- In addition, the direction in the transverse plan can be arbitrary and we can therefore observe linear, circular or elliptical polarized synchrotron light.

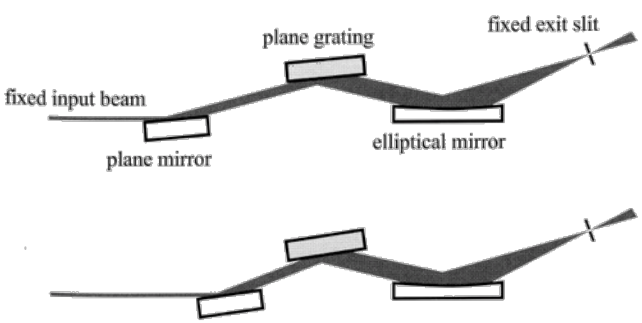


Appendix 5: Gratings monochromators, some designs

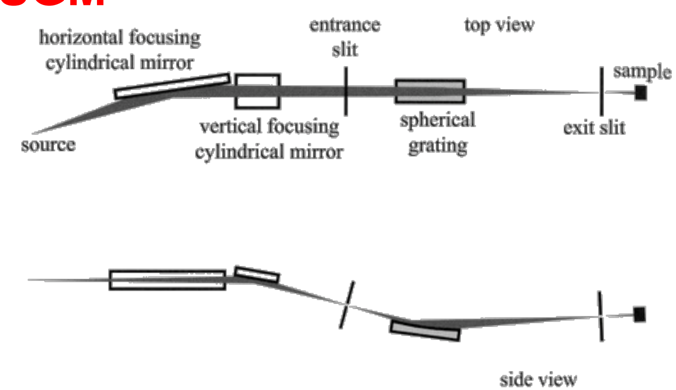
NIM



PGM

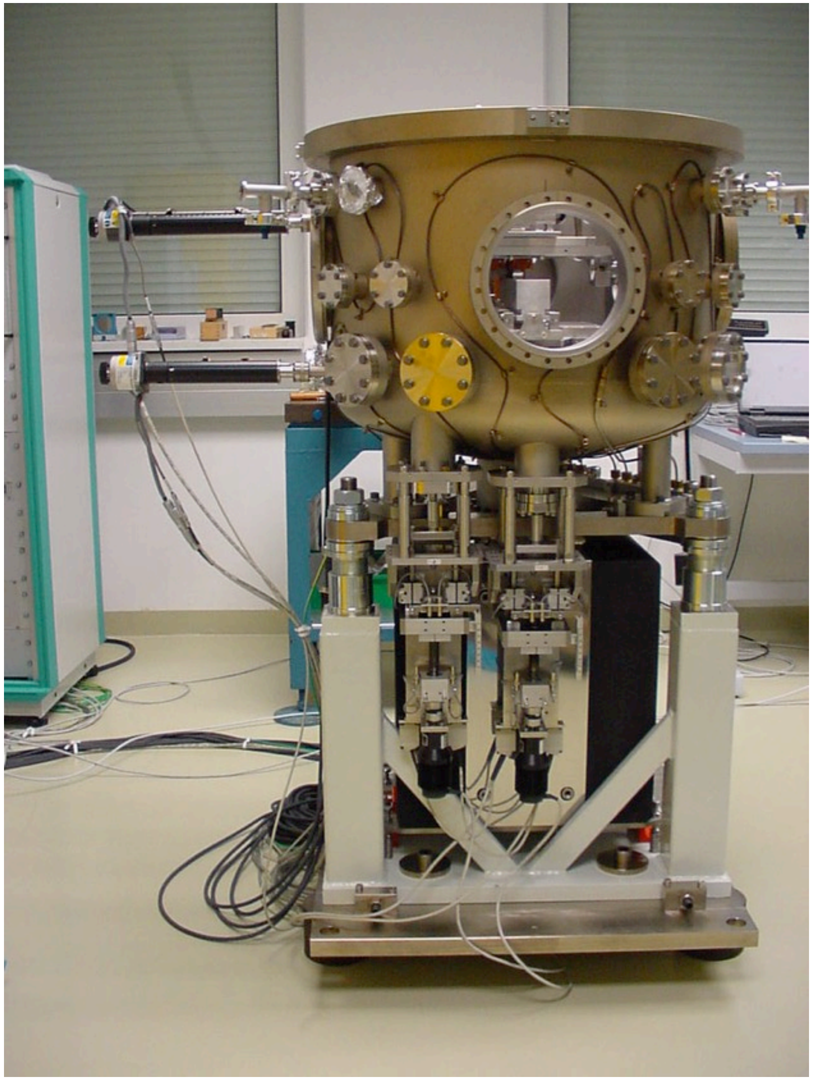


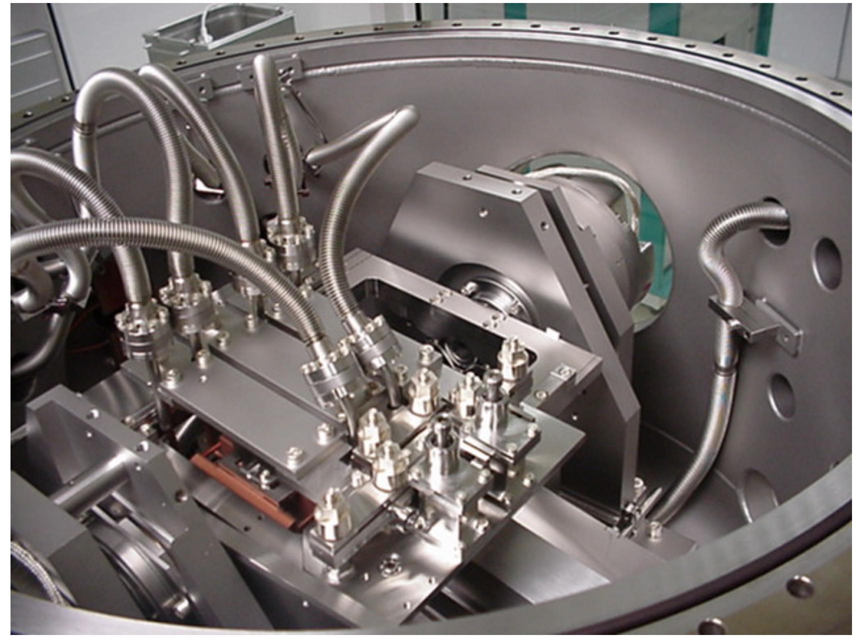
SGM

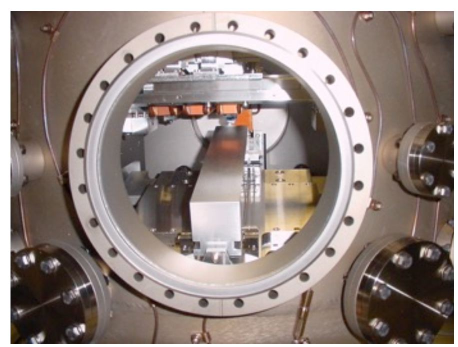


- **Normal-incidence monochromators (NIM):** usable only on a limited range of long wavelength (excellent resolving power)
- **Plane-grating monochromator (PGM):** a rotating plane mirror at the entrance, a rotating plane grating and the focusing elliptical exit mirror provide a high-performance design
- **Spherical-grating monochromator (SGM)**
- **Toroidal grating monochromators (TGM):** the toroidal surface of the grating guaranteees good focusing both parallel and perpendicular to the plane of incidence (widely used in synchrotrons)
- **Variable-line-spacing grating mono**: the focusing action is obtained by a variation of the perodicity of the grating lines

Appendix 5: NIM-PGM Monochromator at SIS-SLS







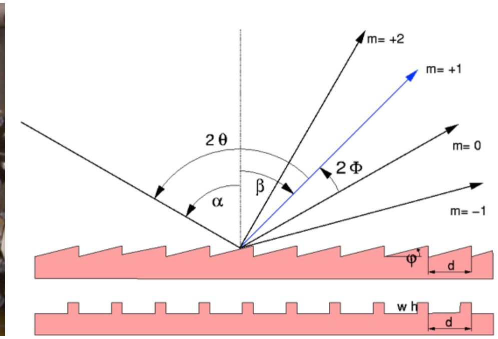
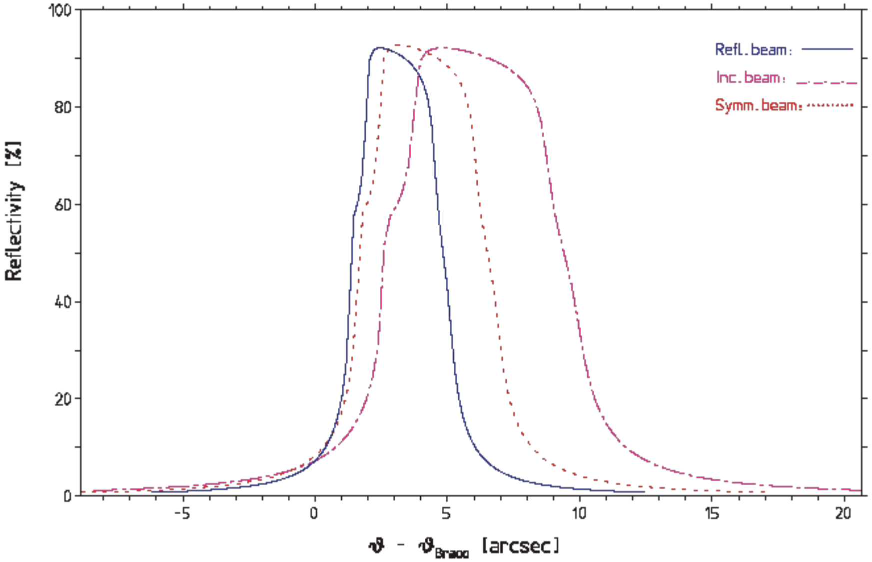


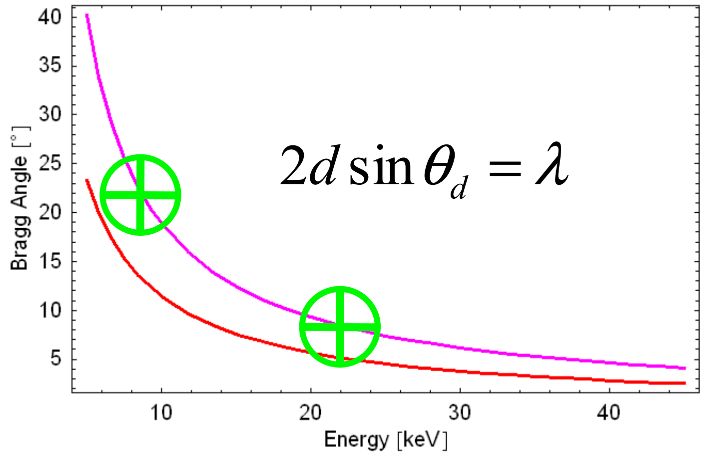
Image courtesy U. Flechsig, SLS-Optics

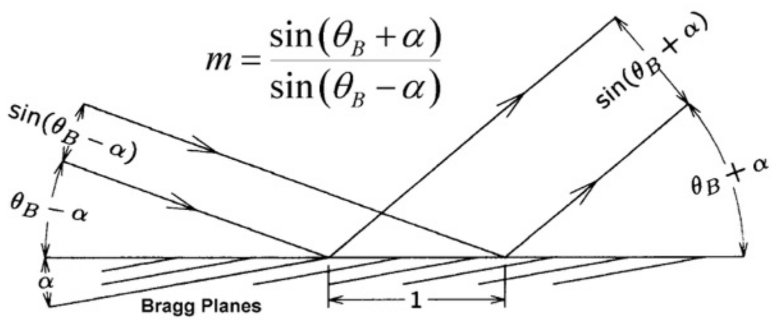
Appendix 6: Asymmetric Bragg diffraction

Si(2 2 0) E= 8047.8eV d= 1.92014A ϑ_{Bragg}=23.651° α= 8.00° T= 5.00mm unpol
Mag= 1.9 L(abs)= 69.72μm ΔE/Δϑ= 0.8909E-01 eV/arcs
χ in μrad:0(15.154, 0.352) σ:h(9.323, 0.344)-h(9.323, 0.344) π:h(9.323, 0.233)-h(9.323, 0.233)

σ&π	Rpk %	RI μrad	R-power	FW arcs	μrad	I'Th arcs	μrad	Δϑ arcs	μrad	ΔTh arcs	μrad	σEx.L μm	πEx.L μm
Refl.	92.18	18.18	25317.	3.57	17.30	3.75	18.20	3.17	15.36	3.22	15.61	6.21	9.17
Incid.	92.17	35.31	12863.	7.02	34.05	7.30	35.41	6.15	29.83	6.26	30.37	6.21	9.17
Symm.	92.74	25.48	18153.	4.98	24.13	5.24	25.39	4.18	20.26	4.25	20.62	6.62	9.77







Si(2 2 0) E= 22100.0eV d= 1.92014A ϑ_{Bragg}= 8.400° α= 8.00° T= 5.00mm unpol
Mag= 40.4 L(abs)= 1386.72μm ΔE/Δϑ= 0.7256 eV/arcs
χ in μrad:0(1.981, 0.006) σ:h(1.209, 0.006)-h(1.209, 0.006) π:h(1.209, 0.006)-h(1.209, 0.006

σ&π	Rpk %	Ri μrad	R-power	FW arcs	μrad	I'Th arcs	μrad	Δϑ arcs	μrad	ΔTh arcs	μrad	σEx.L μm	πEx.L μm
Refl.	96.66	1.63	106434.	0.29	1.39	0.27	1.32	0.72	3.50	0.72	3.51	2.06	2.15
Incid.	96.66	65.61	2644.	11.52	55.85	10.97	53.17	29.24	141.76	29.28	141.96	2.06	2.15
Symm.	99.59	10.64	16782.	1.82	8.80	1.72	8.36	1.41	6.82	1.41	6.85	6.78	7.08