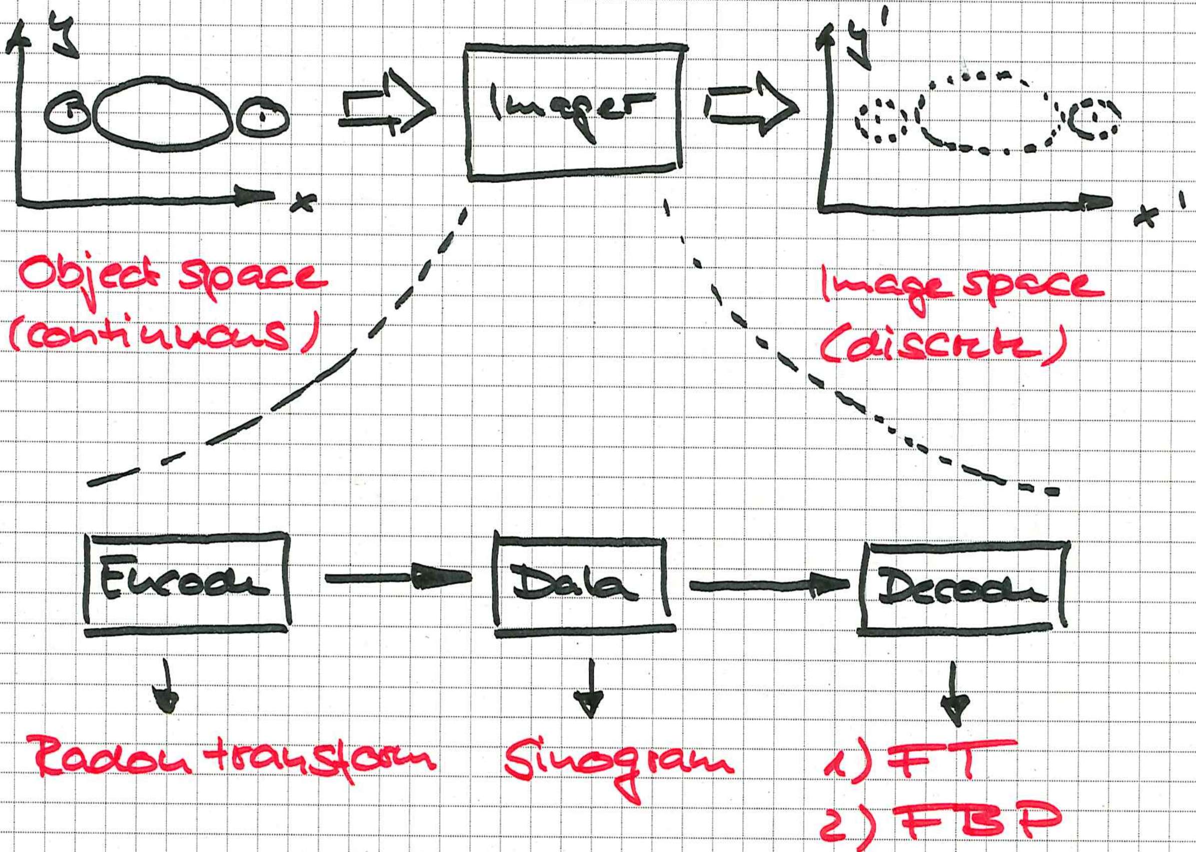
Biomedical Imaging - X-Ray Imaging (III) 1

- CT image reconstruction review



- 1) Fourier transform (FT) recon

a) Acquire projection $P_\varphi(r)$

b) Fourier transform $\rightarrow \mathcal{F}\{P_\varphi(r)\}$

c) Collect $\mathcal{F}\{P_\varphi(r)\}$ for all φ

d) Reconstruct $\rightarrow \mathcal{F}^{-1}\{P_\varphi(p,q)\}$

• 2) Filtered back projection (FBP) recon

a) Acquire projection $P_\varphi(r)$

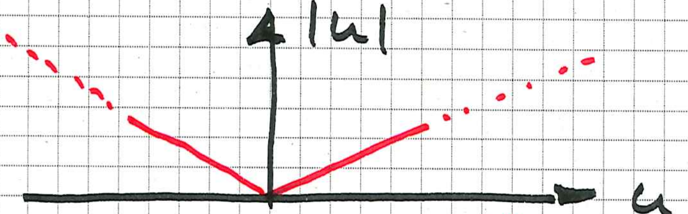
b) Convolve with $\mathcal{F}^{-1}\{|u|\}(r)$

c) Back project for all φ

$$\to \mathcal{F}^{-1}\{\mathcal{F}\{P_\varphi\}\cdot|u|\} \to P_\varphi(r) \underbrace{*}_{\text{Convolution}} \overbrace{\mathcal{F}^{-1}\{|u|\}(r)}^{\text{Filter}}$$

• FBP filter

Ideal:



$\to g_\varepsilon(u) = |u|e^{-\varepsilon|u|}$

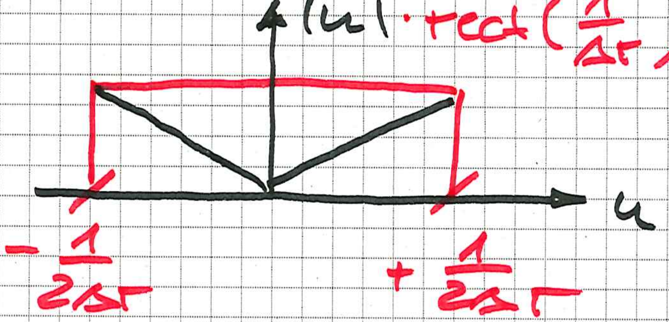
for $\varepsilon \to 0$ $g_\varepsilon(u) = |u|$

$$\mathcal{F}^{-1}\{g_\varepsilon(u)\} \propto \frac{\varepsilon^2 - r^2}{(\varepsilon^2 + r^2)^2}$$

$\to \mathcal{F}^{-1}\{g_\varepsilon(u)\} \propto \frac{1}{r^2}$ for $\varepsilon \to 0$

↳ High-pass

Recall:



Δr ... pixel width

$$\mathcal{F}^{-1}\left\{\mathrm{rect}\left(\frac{1}{\Delta r}\right)\right\} = \int_{-\frac{1}{2\Delta r}}^{+\frac{1}{2\Delta r}} e^{jur}\, du = \frac{1}{jr} e^{jur}\Bigg|_{-\frac{1}{2\Delta r}}^{+\frac{1}{2\Delta r}}$$

$$= \frac{\sin\left(\frac{r}{2\Delta r}\right)}{r} = \underline{\underline{\frac{\mathrm{sinc}\left(\frac{r}{2\Delta r}\right)}{\Delta r}}}$$

$$\mathcal{F}^{-1}\left\{g_\varepsilon(u)\cdot \mathrm{rect}\left(\frac{1}{\Delta r}\right)\right\} \propto \underbrace{\boxed{\frac{1}{r^2} * \frac{\mathrm{sinc}\left(\frac{r}{2\Delta r}\right)}{\Delta r}}}_{\text{Ram-Lak filter}}$$

- **Analytical versus algebraic recon**

Analytical: $P_\varphi(r) = \int \mu(r,s)\, ds$

Algebraic: $P_{\varphi j} = \sum_i \mu_i \cdot w_{ij}$

spatial weights

$$P_{\varphi j} = \sum_i \mu_i \cdot w_{ij}$$

Object is discrete !!

→ $\vec{p} = W\vec{\mu}$

projections

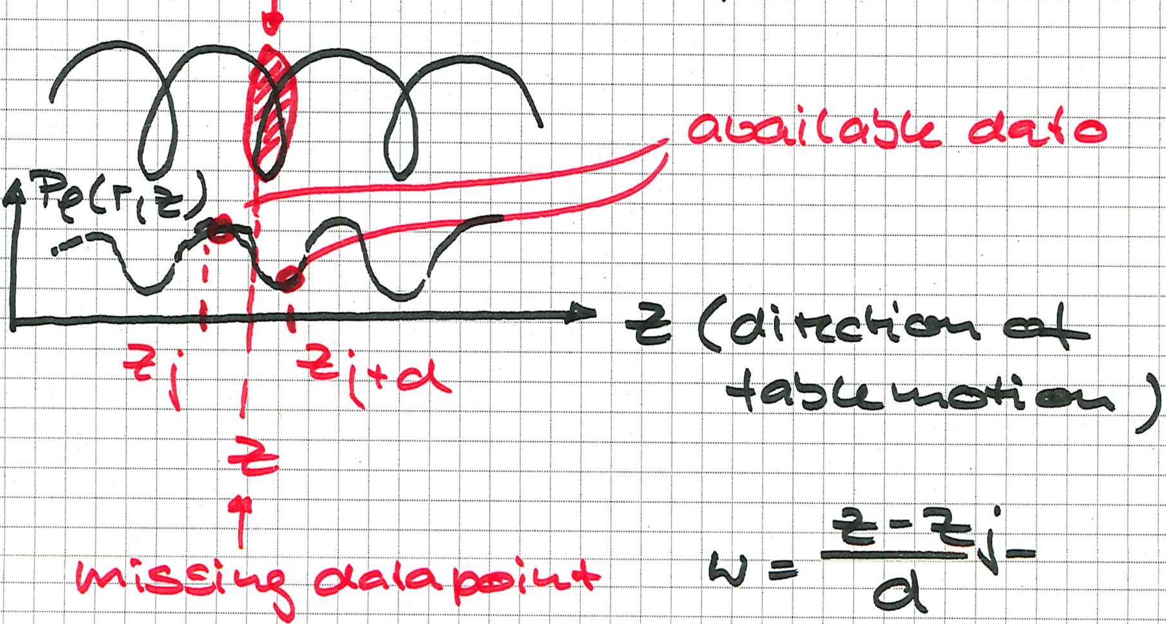
spatial weights

discrete object

→ solve using $\vec{\mu} = (W^T W)^{-1} W^T \vec{p}$

- Spiral CT

→ reconstruct by interpolating to 2D slices and apply 2D FBP

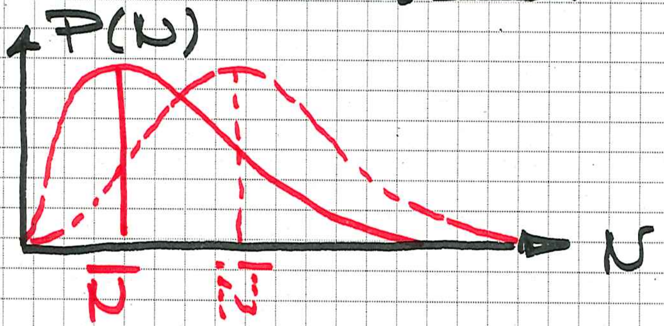


$$w = \frac{z - z_j}{d}$$

→ $P_\varphi(r,z) = (1-w)P_\varphi(r,z_j) + wP_\varphi(r,z_j+d)$

- Image noise
  → Quantum or shot noise
  → Poisson distribution:

$$\boxed{P_\lambda(i) = \frac{i^\lambda e^{-\lambda}}{i!}}$$



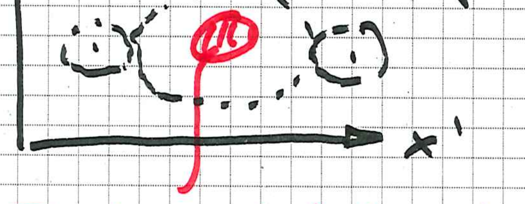
Average $E\{P\} = \lambda = \bar{N}$

Variance $V\{P\} = \lambda = \bar{N}$

- Signal-to-noise ratio (SNR)

$$\boxed{SNR = \frac{E\{P\}}{\sqrt{V\{P\}}} = \frac{\bar{N}}{\sqrt{\bar{N}}} = \sqrt{\bar{N}}}$$

- Image SNR



$$SNR_{Image} = \frac{\bar{I}}{SD(\bar{I})}$$

Standard deviation

- **Radiation dose**

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$$\text{Dose} \propto \underbrace{I_a}_{\text{Anode current}} \cdot \underbrace{t}_{\text{Scan duration}}$$

$$\rightarrow \quad SNR_{image} \propto \sqrt{\underbrace{Q}_{I_a \cdot t} \cdot \underbrace{\Delta z}_{\text{Detector thickness}}}$$

- **Radiation risk**

  - **Relative risk (RR)**

e.g. 1 Mio people

200k cancer (w/o CT/XR) — 220k cancer (with CT/XR)

$\rightarrow$ $\underline{RR = 1.1}$

  - **Excess relative risk (ERR)**

$$ERR = RR - 1 \longrightarrow ERR \approx \frac{0.9}{Sv}$$

$$\boxed{\text{CT (10mSv)}: \; ERR_{CT} \sim 0.09}$$