ETH zürich University of Zurich[UZH]

# Biomedical Imaging

## Summary

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# Exam

| | |
|---|---|
| Date: | 21.1.2020 (Tuesday) |
| Time: | 09:00-11:00 |
| Room: | ?? |
| Type: | Closed-book |
| Aid: | Own single A4 sheet formulary* (double sided) |
| Structure: | Open text (text, equations, drawings)<br>X-ray/Computed tomography<br>Magnetic Resonance Imaging<br>Nuclear Imaging<br>Ultrasound |

*https://amiv.ethz.ch/en/studydocuments

# Lecture documentation

## Lecture notes, summary slides and exercises

https://moodle-app2.let.ethz.ch

## Book

Introduction to Medical Imaging

Cambridge University Press

Nadine Barrie Smith and Andrew Webb

www.cambridge.org/9780521190657

## Contact

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# Biomedical Imaging

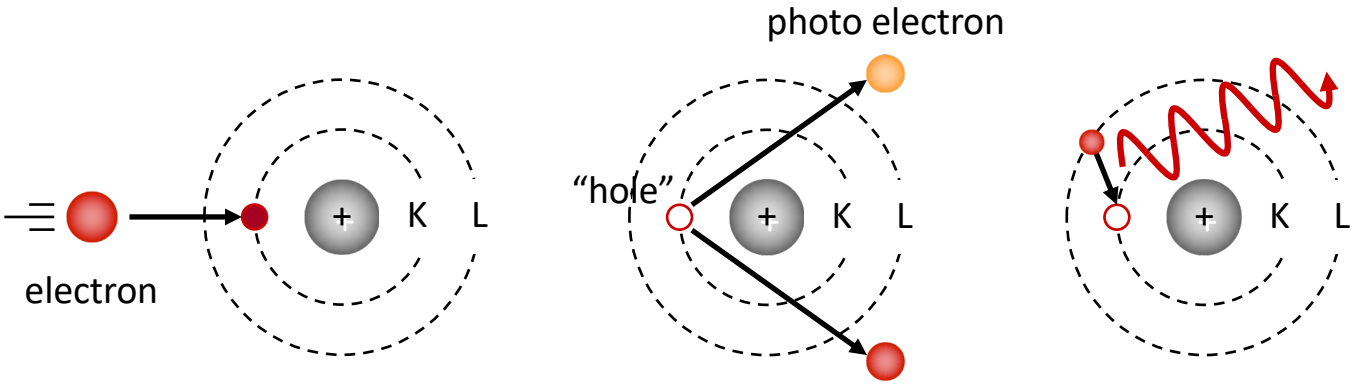
## Imaging Mode: X-ray Imaging

- Probe: X-ray photons
- Wavelength: 10 pm – 10 nm
- Matter interaction: absorption, scatter
- Modalities: Projection imaging
  Digital Subtraction Angiography (DSA)
  Computed tomography

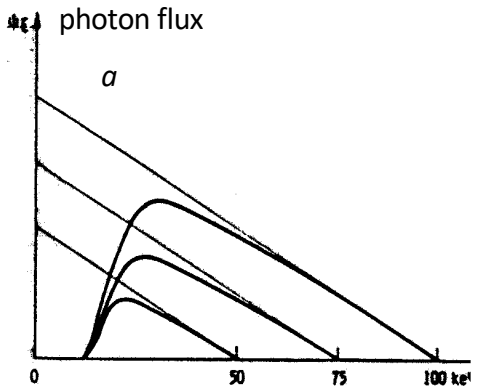
# X-ray/Computed tomography

## X-ray production

Describe mechanisms resulting in characteristic radiation!



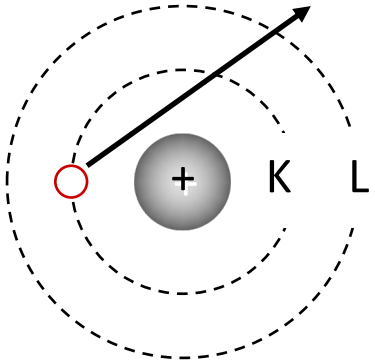

How does photon flux vary if the anode voltage is increased?

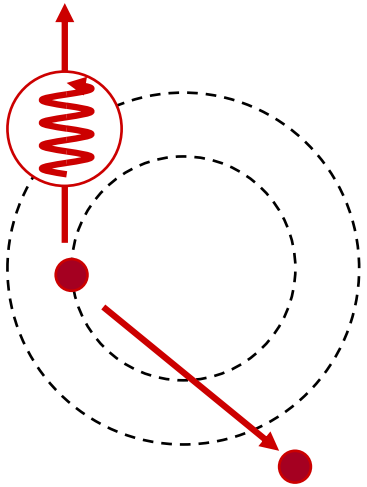


# X-ray/Computed tomography

## X-ray attenuation

Which effects dominate attenuation at diagnostic X-ray energies?



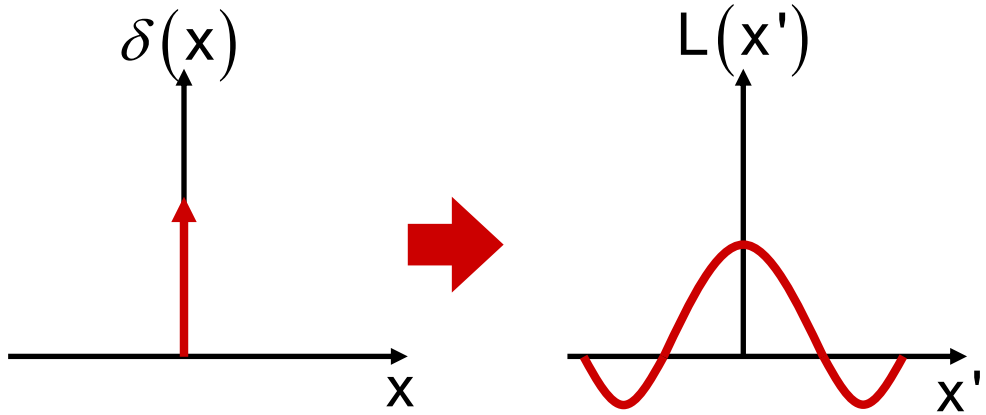
Compute the thickness of a radiation protection sheet!

$$I = \int_0^{E_{max}} I_0(E)\, e^{-\int_{-\infty}^{\infty} \mu(E,x)dx}\, dE$$

# X-ray/Computed tomography

## Spatial resolution and contrast

How are PSF, LSF and MTF defined?



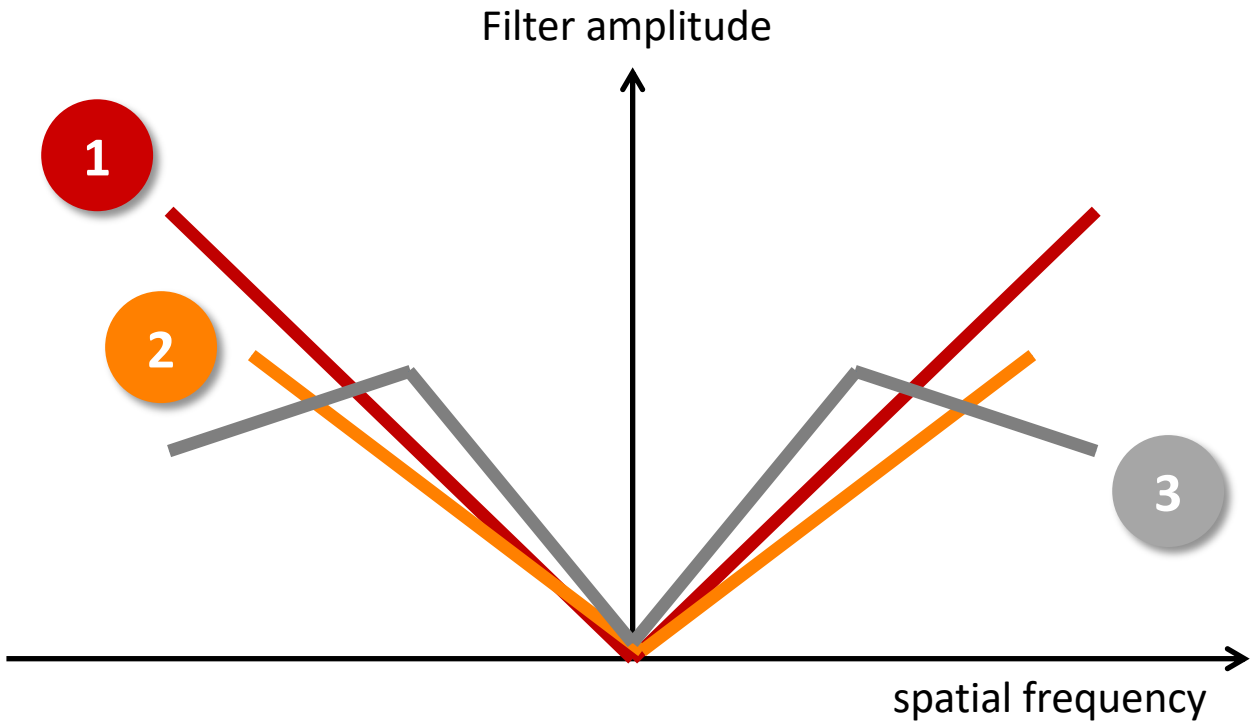

How is contrast defined and what does it depend on?

$$\text{Contrast} \propto (\mu_1 - \mu_0)\text{d}$$

# X-ray/Computed tomography

## Spatial resolution and signal-to-noise ratio

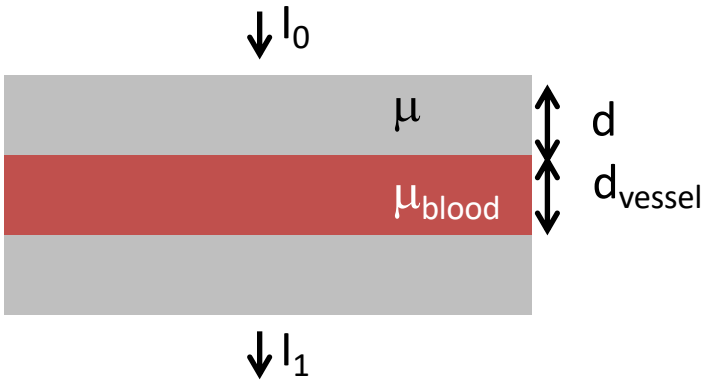
Which of the following filters gives best trade-off between resolution and signal-to-noise ratio?



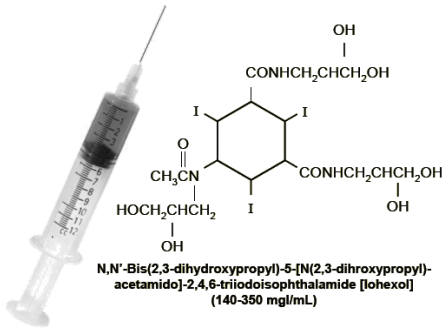

# X-ray/Computed tomography

## Digital Subtraction Angiography

Compute the intensity for following object?



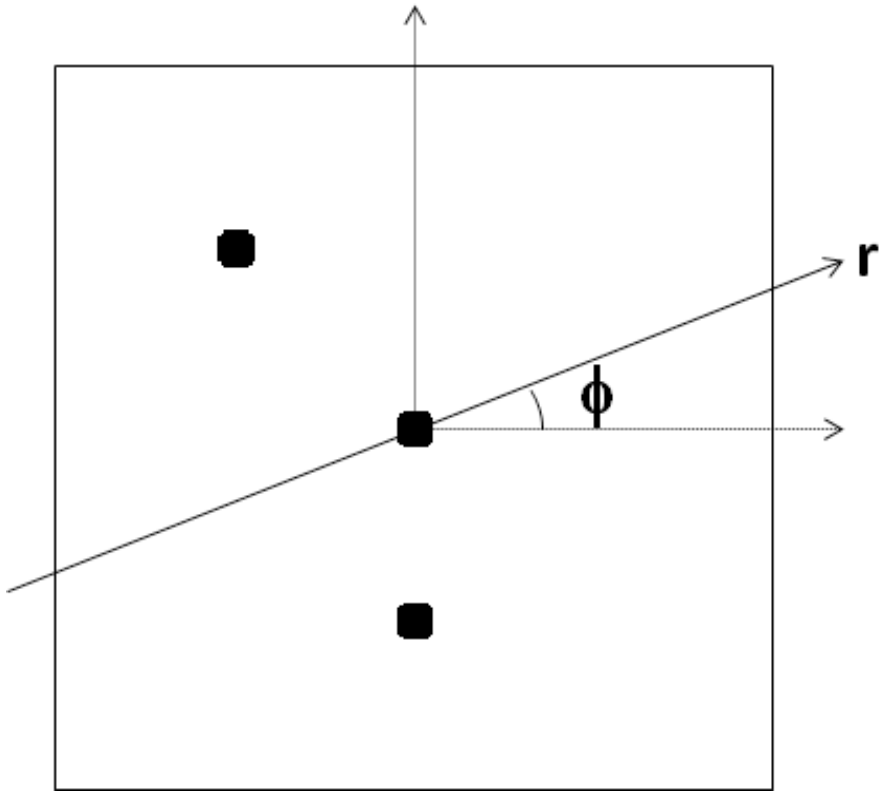

How does intensity change after iodine injection?



# X-ray/Computed tomography

## Computed tomography

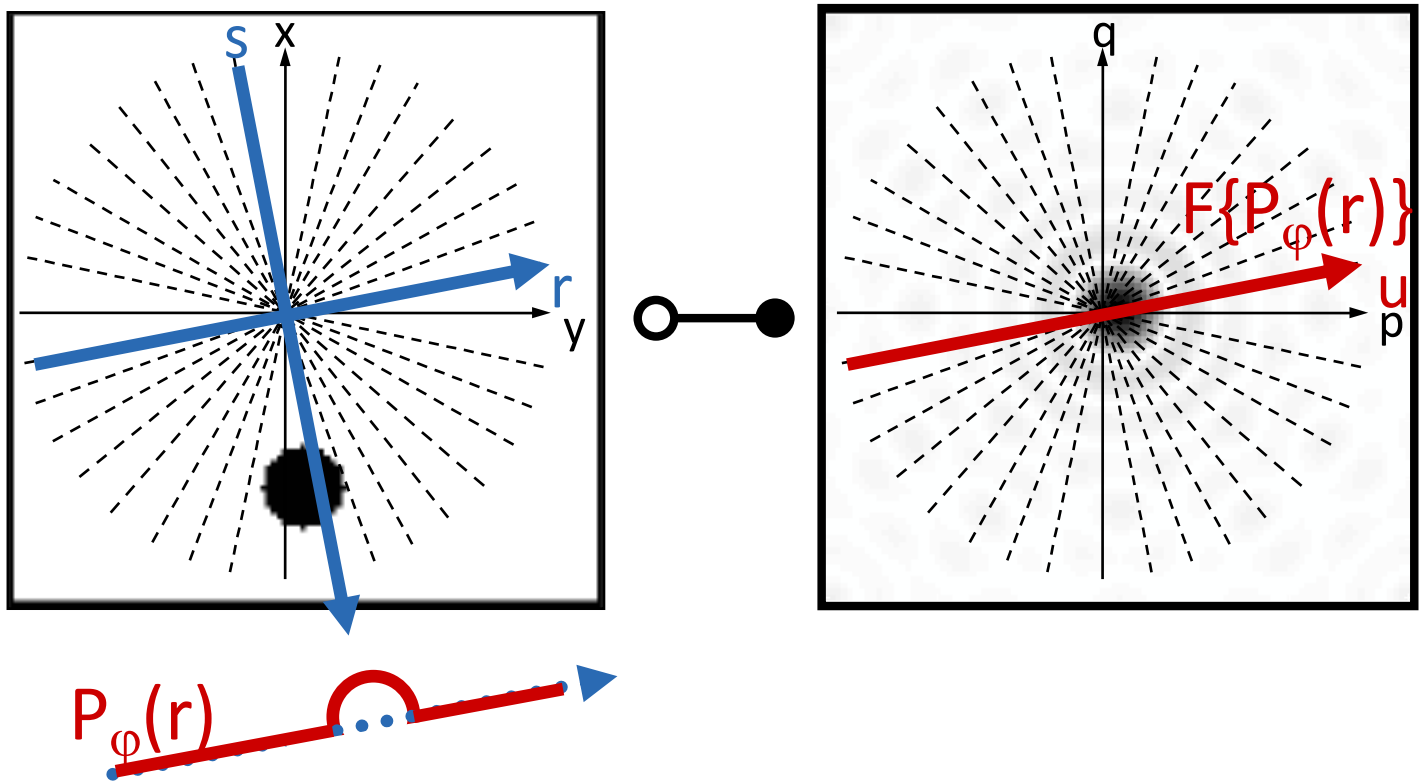
Construct the sinogram of following object!




# X-ray/Computed tomography

## Computed tomography

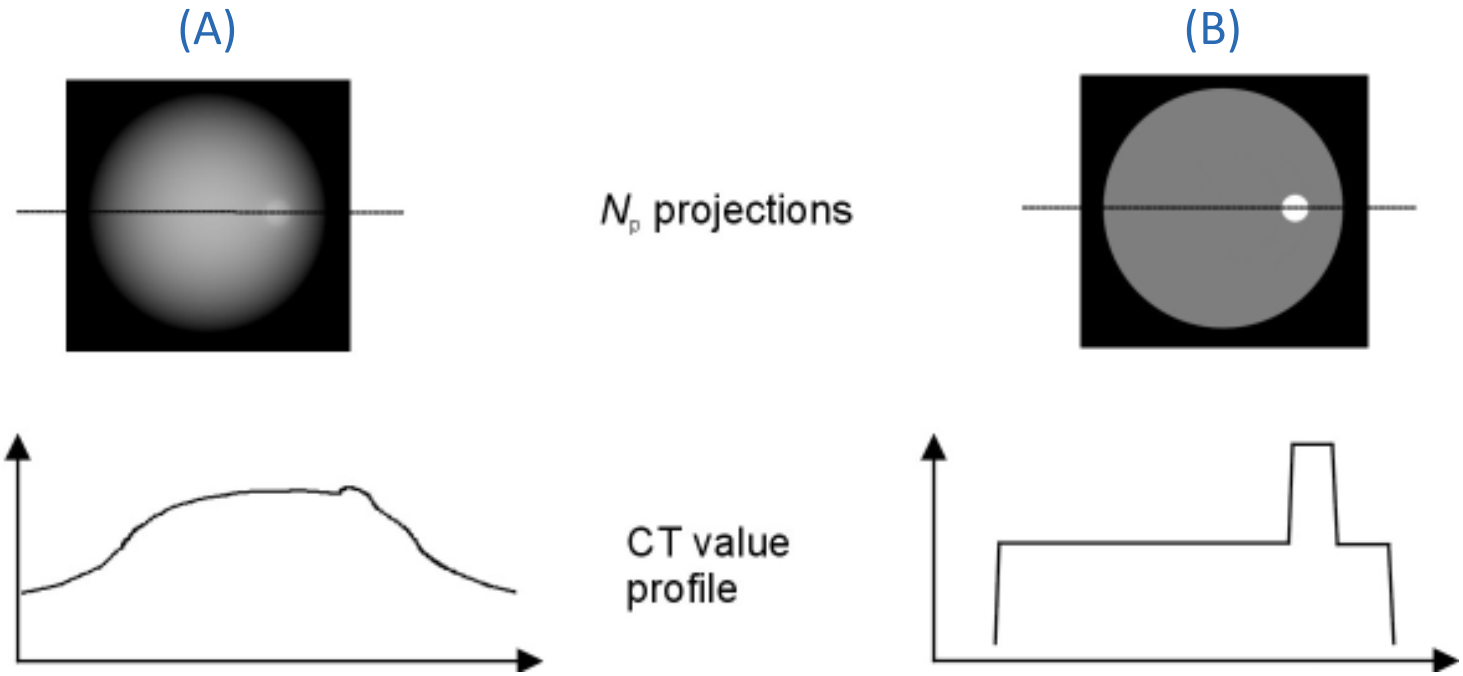
Derive the relation between FT and FBP reconstruction!



# X-ray/Computed tomography

## Computed tomography

What is the difference between the two reconstructions below? Derive the filter function to reconstruct image (B) from (A)!



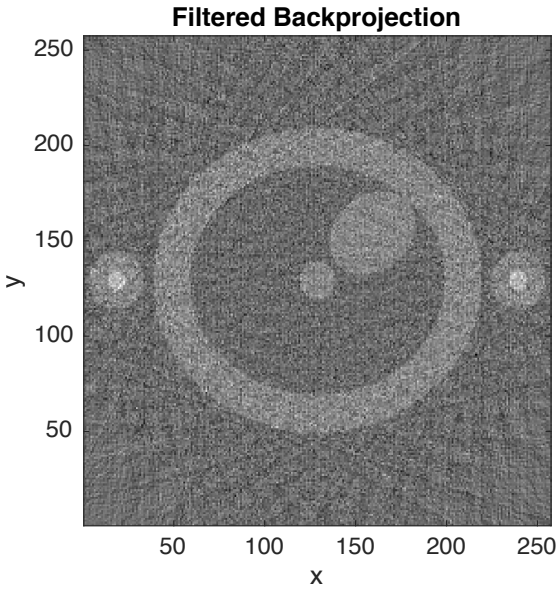
# X-ray/Computed tomography

## Computed tomography

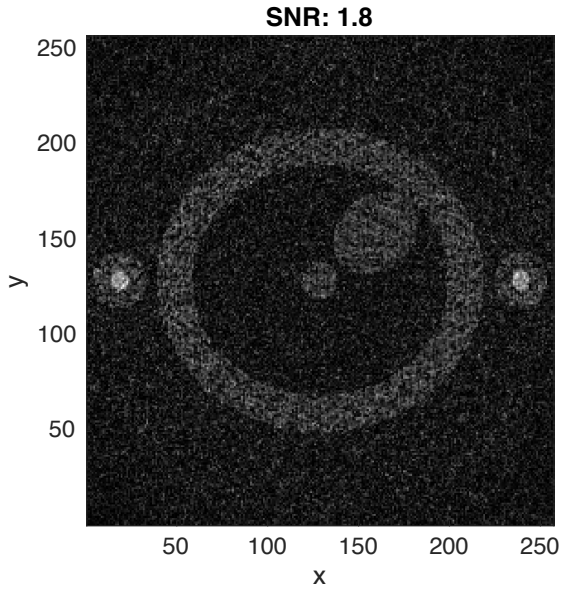
Explain the artefacts in (A) and discuss the type of filtering in (B) and (C).

Derive the relation between signal-to-noise ratio and spatial resolution!

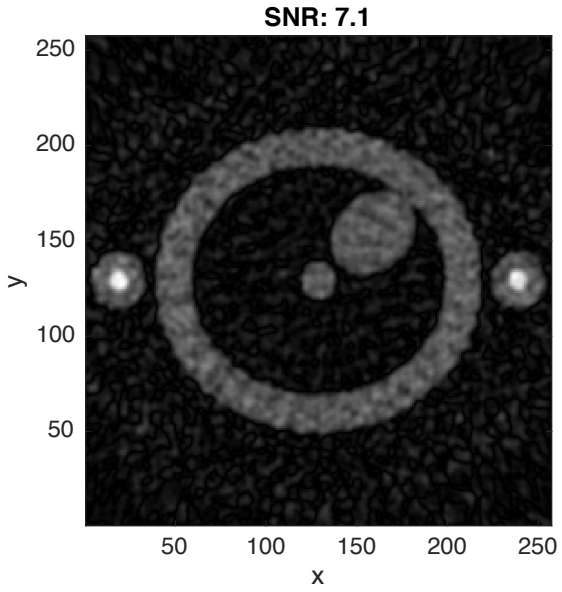
(A)




(B)




(C)




# Biomedical Imaging

Imaging Mode: **Nuclear Imaging (SPECT, PET)**

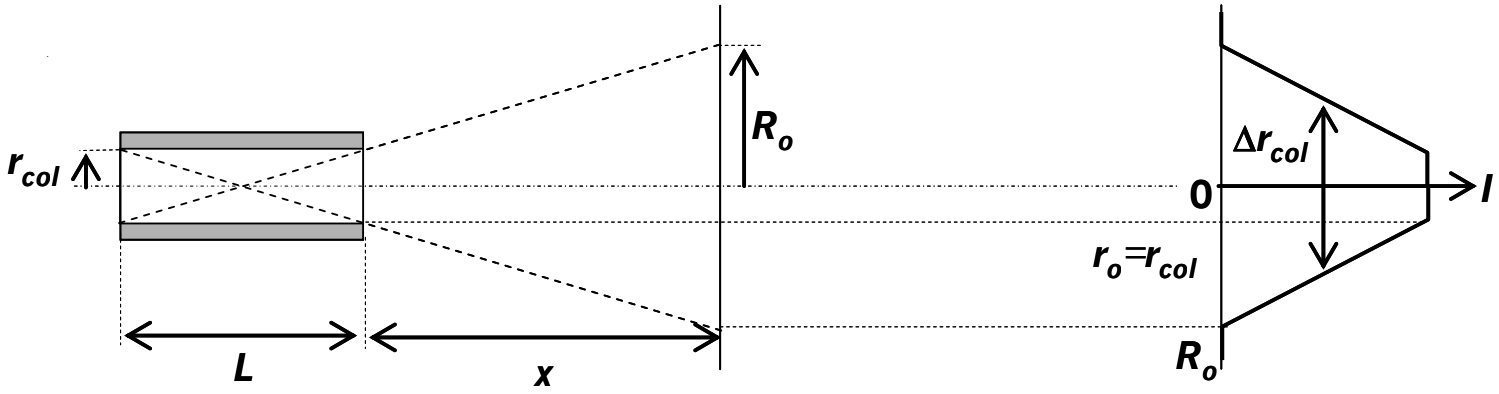
- Probe: $\gamma$-ray photon, positron$\rightarrow$ $\gamma$-ray photons
- Wavelength: 2 pm – 100 pm
- Matter interaction: absorption, scatter, pair production
- Modalities: SPECT (Single Photon Emission Tomography )

  PET (Positron Emission Tomography)

# Nuclear Imaging

## Collimation

Sketch the PSF of a SPECT camera for different source positions!

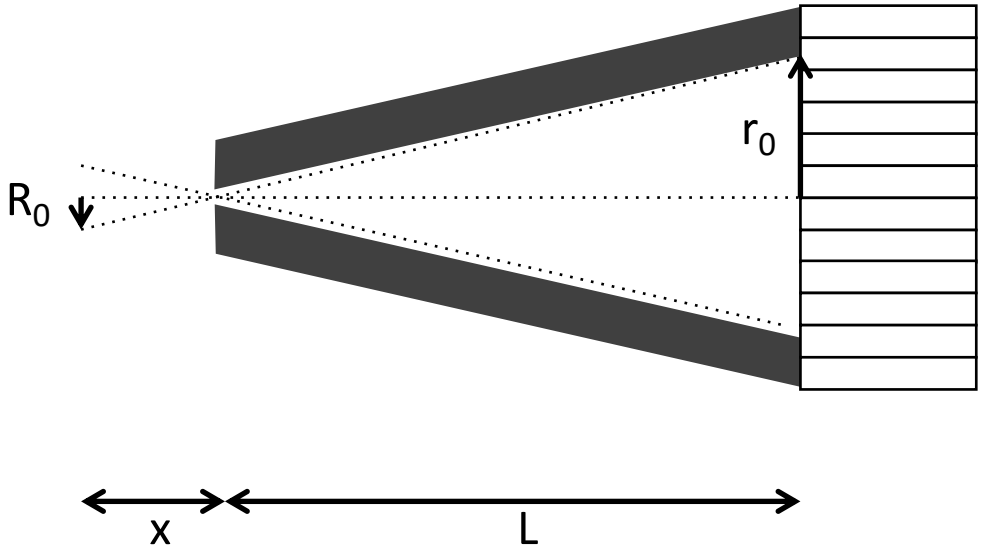



What is the maximum distance from the axis at which photons can still be detected?

# Nuclear Imaging

## Collimation

What is a pinhole and how is spatial amplification defined?



# Nuclear Imaging

## PET detection efficiency

What is detection and geometric efficiency? How is total sensitivity defined?

**Detection efficiency** $\varepsilon = (1 - e^{-\mu \cdot d}) \cdot \Phi$

Fraction of events within selected energy window: $\Phi$
Attenuation by scintillation crystal: $\mu$
Thickness of scintillation crystal: d

For coincidence detection: $\varepsilon^2 = (1 - e^{-\mu \cdot d})^2 \cdot \Phi^2$

**Geometric efficiency**

Axial geometric coverage: $\Omega = 4\pi \cdot \sin(\tan^{-1}(z / D))$

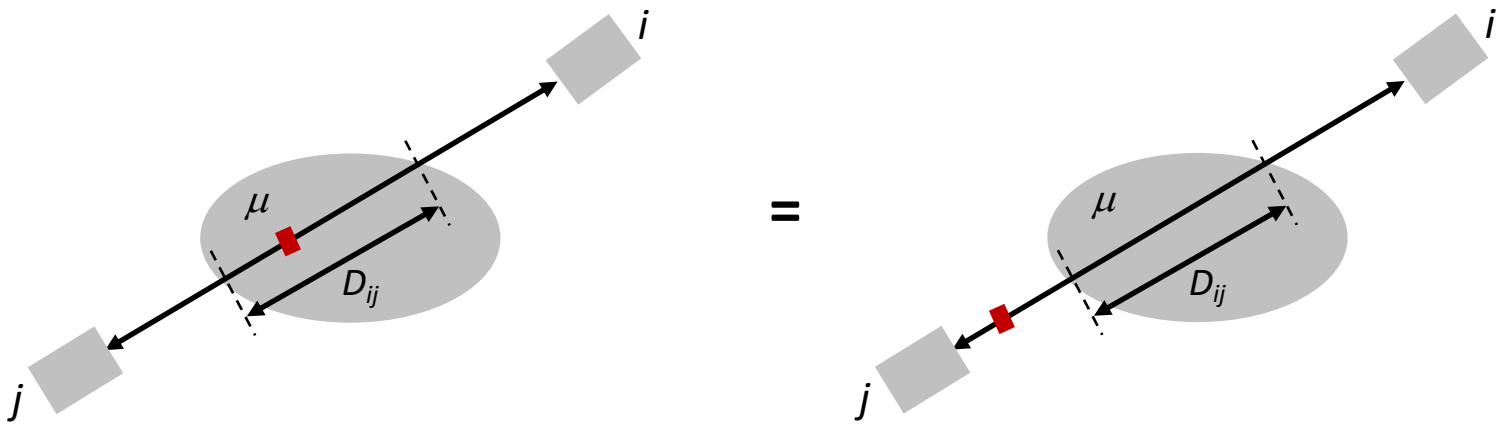
**Total sensitivity**

$$\eta(\%) = 100 \cdot \varepsilon^2 \cdot \phi \cdot \frac{\Omega}{4\pi}$$

# Nuclear Imaging

## Attenuation

How can attenuation correction be performed?

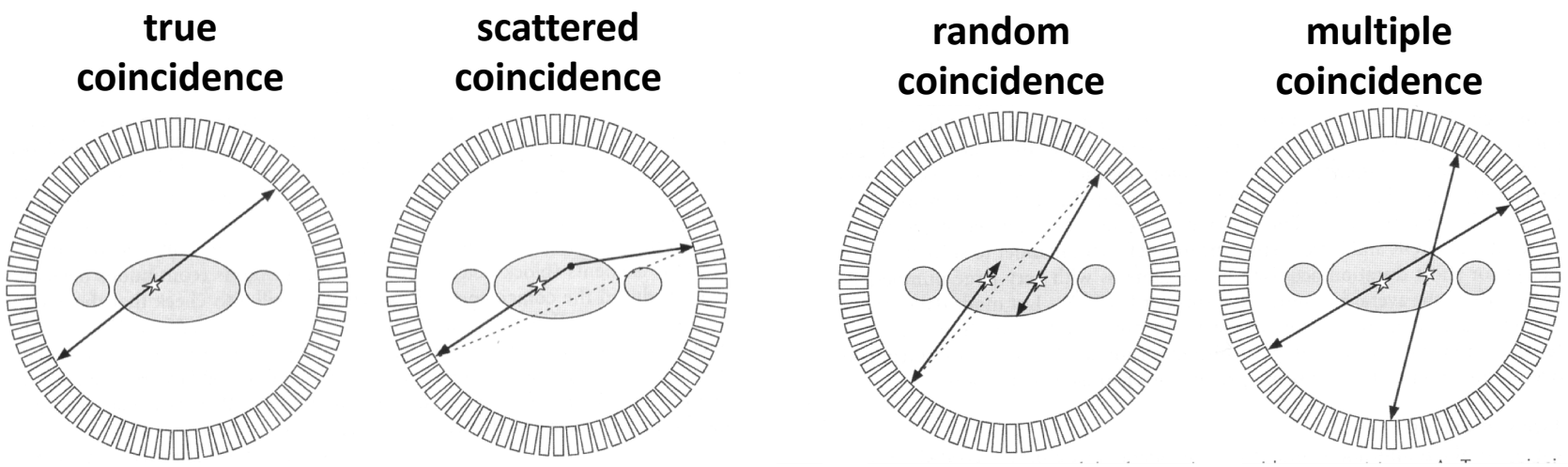



**internal PET source** **external PET source**

# Nuclear Imaging

## Coincidence detection

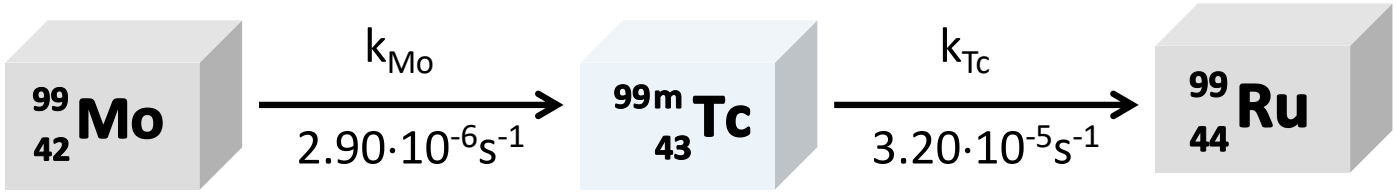
Discuss undesired coincidences in PET and corrections thereof!



# Nuclear Imaging

Tc generator

Describe the processes to create meta-stable Technetium!



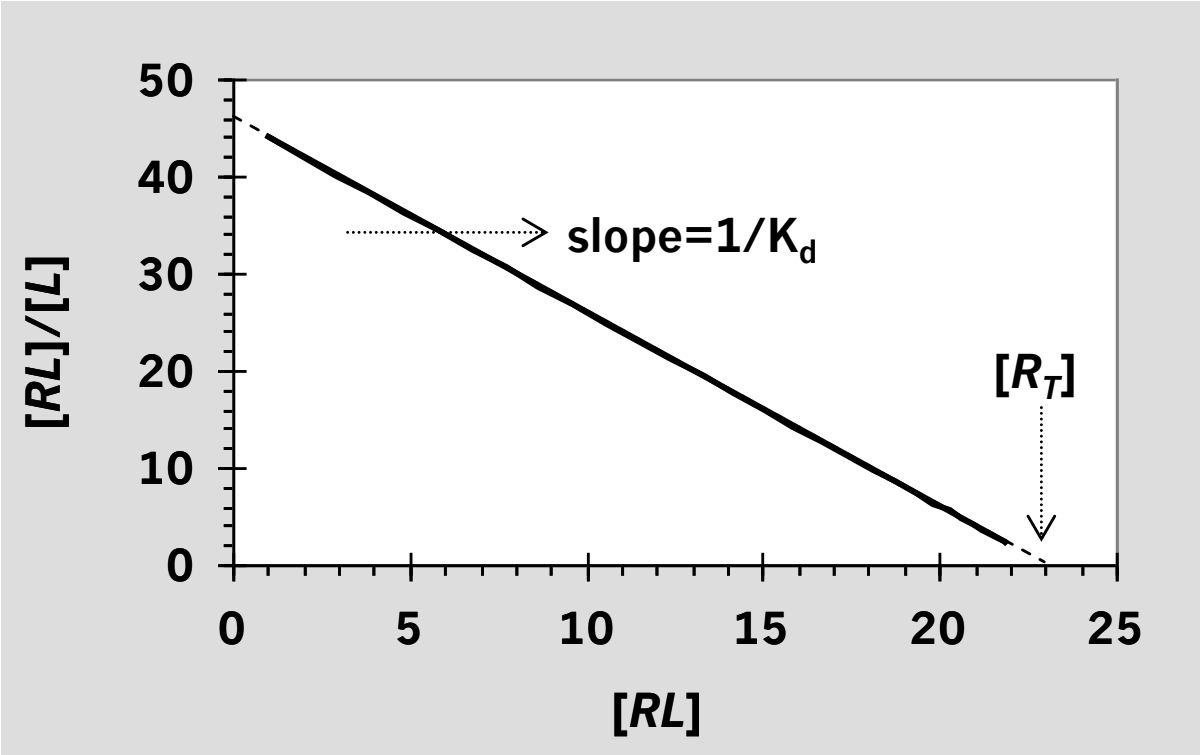

$$\frac{d}{dt}[^{99}Mo] = -k_{Mo} \cdot [^{99}Mo]$$

$$\frac{d}{dt}[^{99m}Tc] = +k_{Mo} \cdot [^{99}Mo] - k_{Tc} \cdot [^{99m}Tc]$$

# Nuclear Imaging

## Quantitative PET

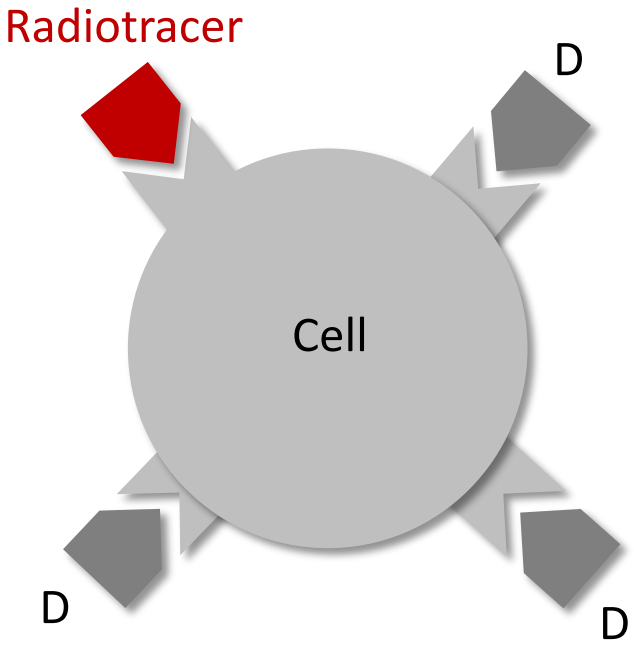
### What is a Scatchard plot?



# Nuclear imaging

## Quantitative PET

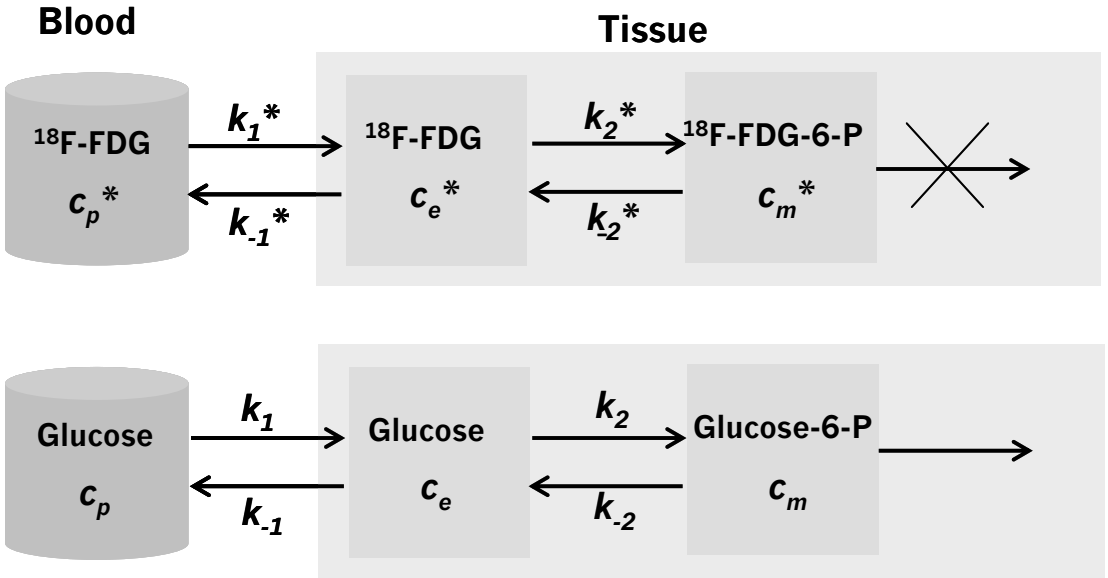
In a competition experiment using a cold drug D the maximum binding capacity is reduced to 25% of its value in the absence of the drug. What is the receptor occupancy (in %) of the drug D.




# Nuclear Imaging

## Kinetic modeling

Sketch compartment models for FDG-PET! What is a lumped constant?




$$v_{GluMR} \approx \frac{k_1 \cdot k_2}{k_{-1} + k_2} \cdot c_p(t) \approx \frac{1}{LC} \cdot \frac{k_1^* \cdot k_2^*}{k_{-1}^* + k_2^*} \cdot c_p(t)$$

# Biomedical Imaging

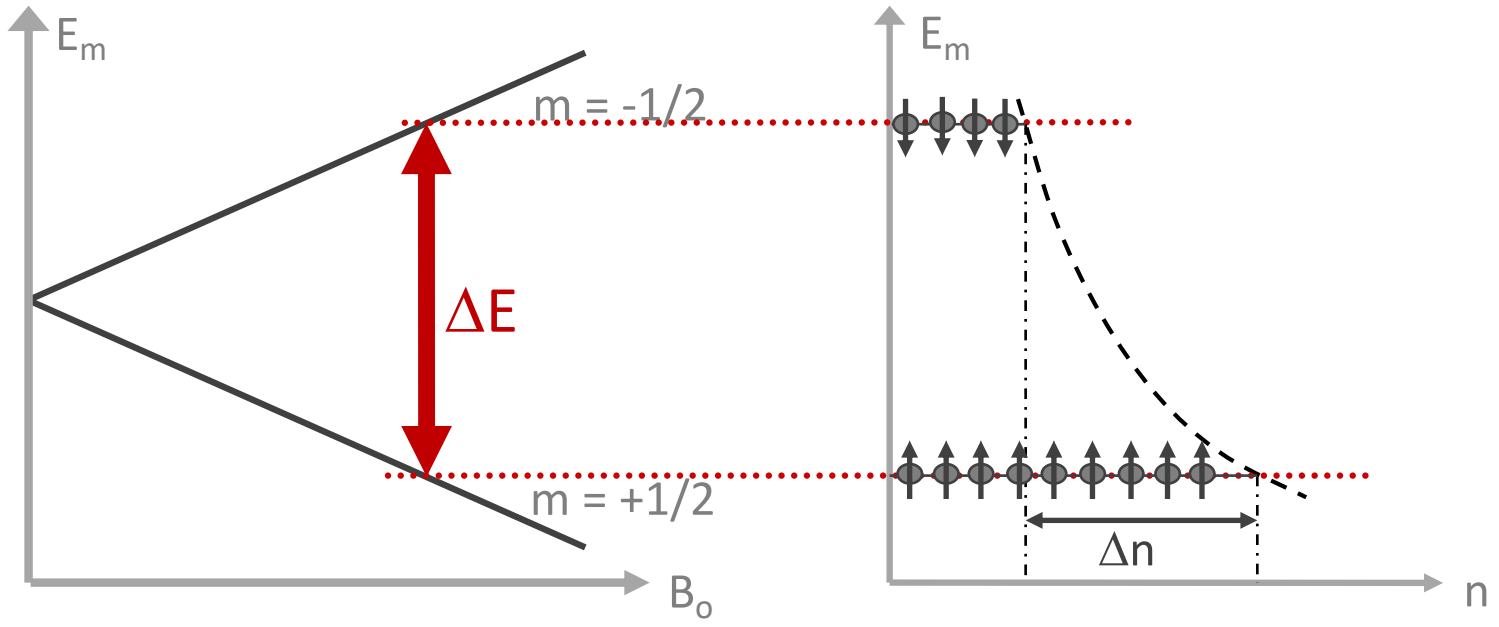
Imaging Mode: **Magnetic Resonance (MR)**

- Probe: radiofrequency waves
- Wavelength: 5 – 150 cm
- Matter interaction: nuclear spin transitions
- Modalities: Magnetic Resonance Imaging (MRI)
  Magnetic Resonance Spectroscopy (MRS)
  Functional MRI (fMRI)

# Magnetic Resonance Imaging

## Nuclear magnetism

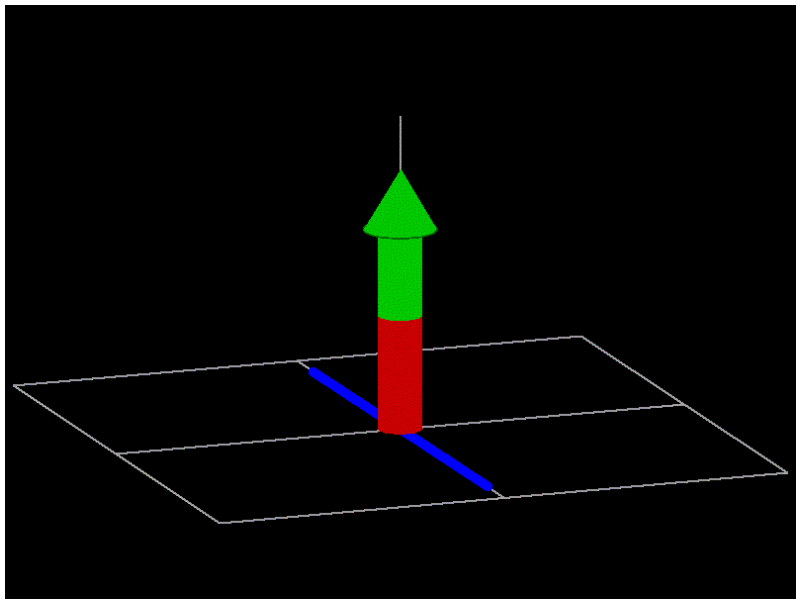
Compare MR sensitivity of different spin-1/2 nuclei ($^{1}H$, $^{19}F$, $^{13}C$)!



# Magnetic Resonance Imaging

## Excitation

Compute RF pulse duration for 90 deg on-resonant excitation!



Fast excitation: $T_1,\ T_2 = \infty$

$$\frac{d}{dt}\boldsymbol{M} = \begin{pmatrix} 0 & -\gamma B_0 & \gamma B_y \\ \gamma B_0 & 0 & -\gamma B_x \\ -\gamma B_y & \gamma B_x & 0 \end{pmatrix} \boldsymbol{M}$$

$$B_x = B_1 \cos(\omega_L t)$$

$$B_y = B_1 \sin(\omega_L t)$$

$$\omega_L = \omega_{Larmor} = \gamma B_0$$

# Magnetic Resonance Imaging

## Detection

### What are the sources of signal and noise?

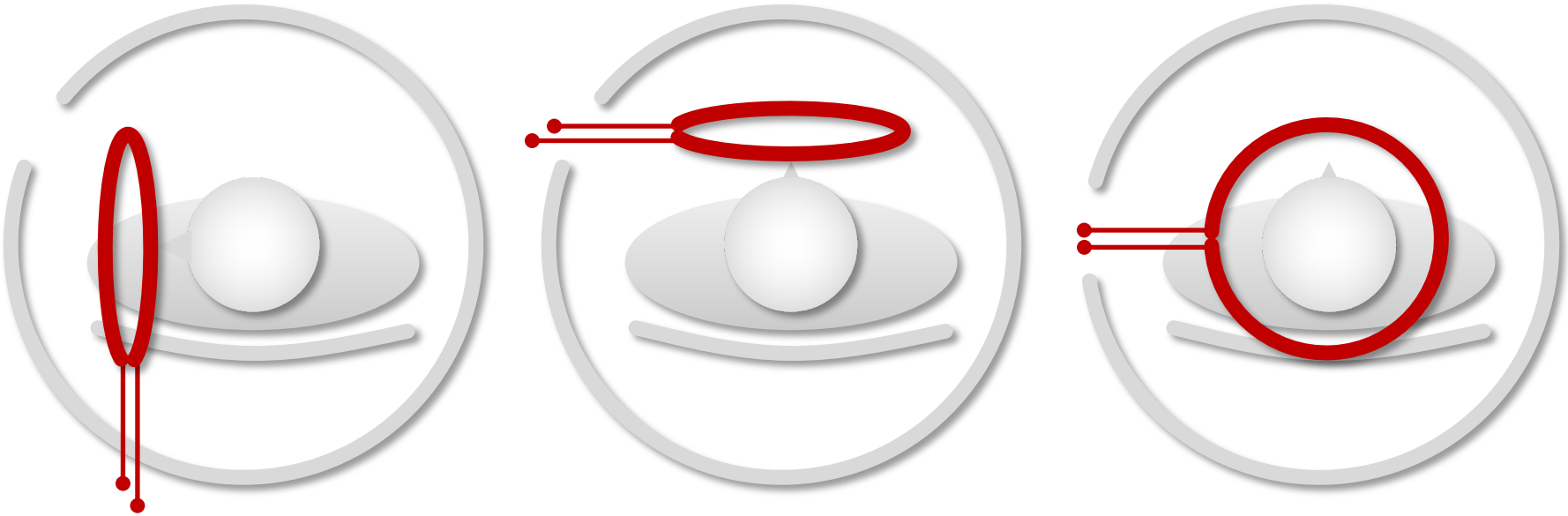
| | Spatial sensitivity to MR signal | Noise variance |
|---|---|---|
| Formal expression | $s(\boldsymbol{r}) = \frac{B_x(\boldsymbol{r}) - iB_y(\boldsymbol{r})}{I_0}$ | $\sigma^2_{noise} = 4k_B \int\int T(\boldsymbol{r})\,\sigma(\omega,\boldsymbol{r}) \frac{\lvert \boldsymbol{E}(\omega,\boldsymbol{r})\rvert^2}{I_0^2} \frac{d\omega}{2\pi} dV$ |
| Sources | spins = magnetic dipoles | electric charges, electric dipoles |
| Relevant transmit field component | magnetic, $B$ „signal sensitivity“ | electric, $E$ „noise sensitivity“ |

# Magnetic Resonance Imaging

## Excitation and detection

Which coil configuration allows excitation and detection of MR signal?

(A) (B) (C)



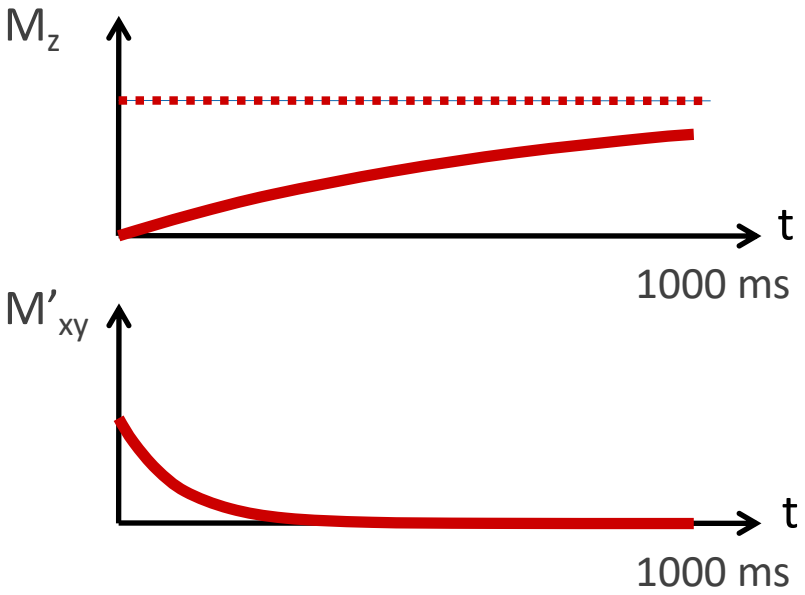
$\otimes B_0$

(direction of $B_0$ is through-plane)

# Magnetic Resonance Imaging

## Relaxation

Plot time evolution of longitudinal and transverse magnetization for tissue with (a) short and (b) long $T_1/T_2$! How would you maximize image contrast for a partial saturation sequence?




"Tissue property"

$M_z = M_0(1-\exp(-t/T_1))$

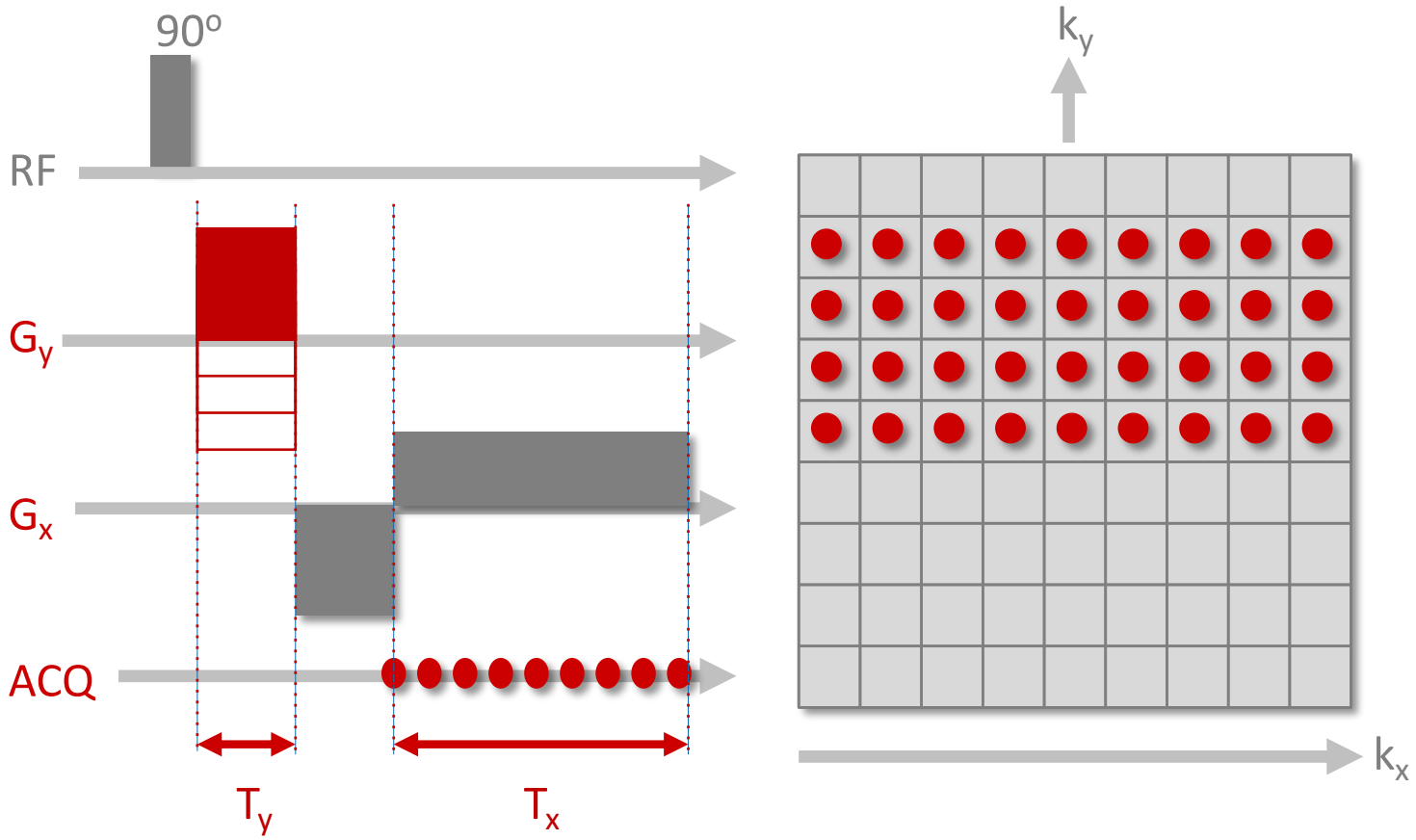
"Tissue property"

$M_{xy} = M_0(\exp(-t/T_2^*))$

# Magnetic Resonance Imaging

## Image formation

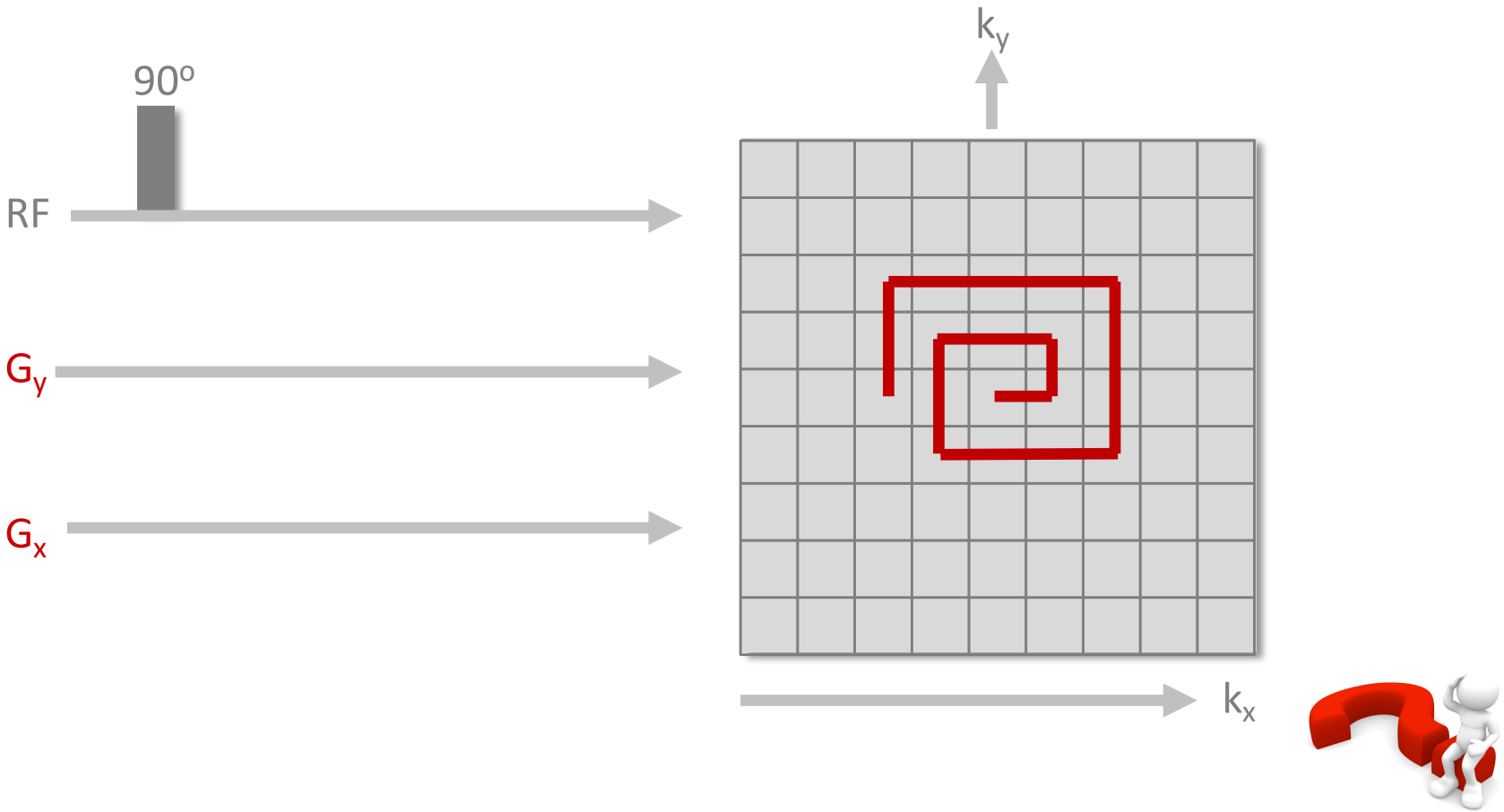
Describe a gradient-echo Fourier encoding experiment!



# Magnetic Resonance Imaging

## Image formation

Prescribe gradient waveforms to generate the follow k-space trajectory! What do k-sampling density and extent depend on?



# Magnetic Resonance Imaging

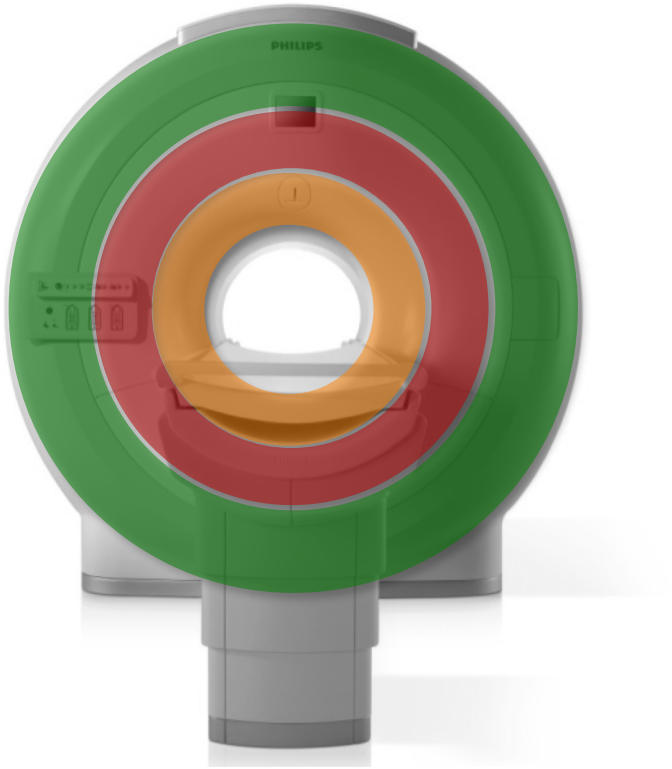
## Hardware

Describe the key components of an MR system! Compare RF excitation and RF detection regarding magnetic flux fields involved!

Magnet, Cryogenics

Gradients, Shims

RF Body Coil

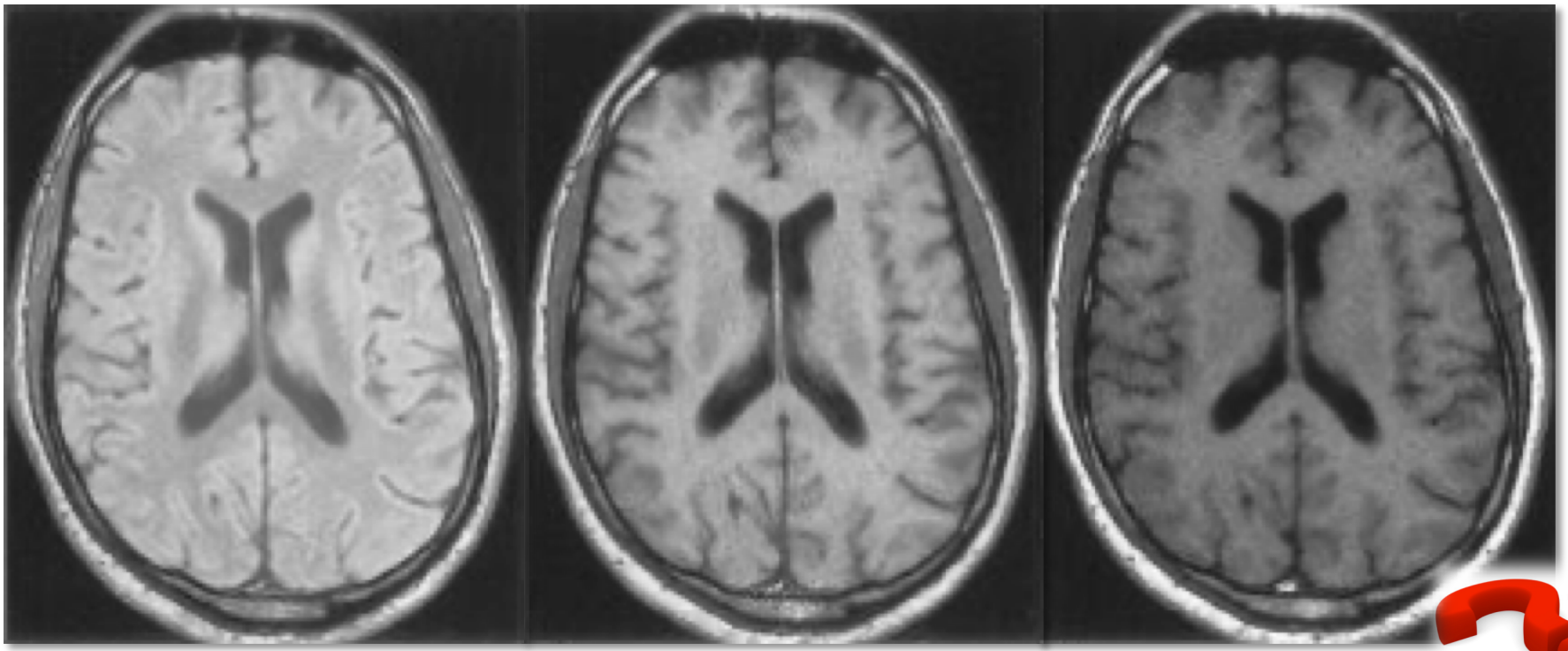


# Magnetic Resonance Imaging

## Image contrast

Which one of the images below was acquired with shortest, intermediate and longest repetition time? What type of sequence was used?

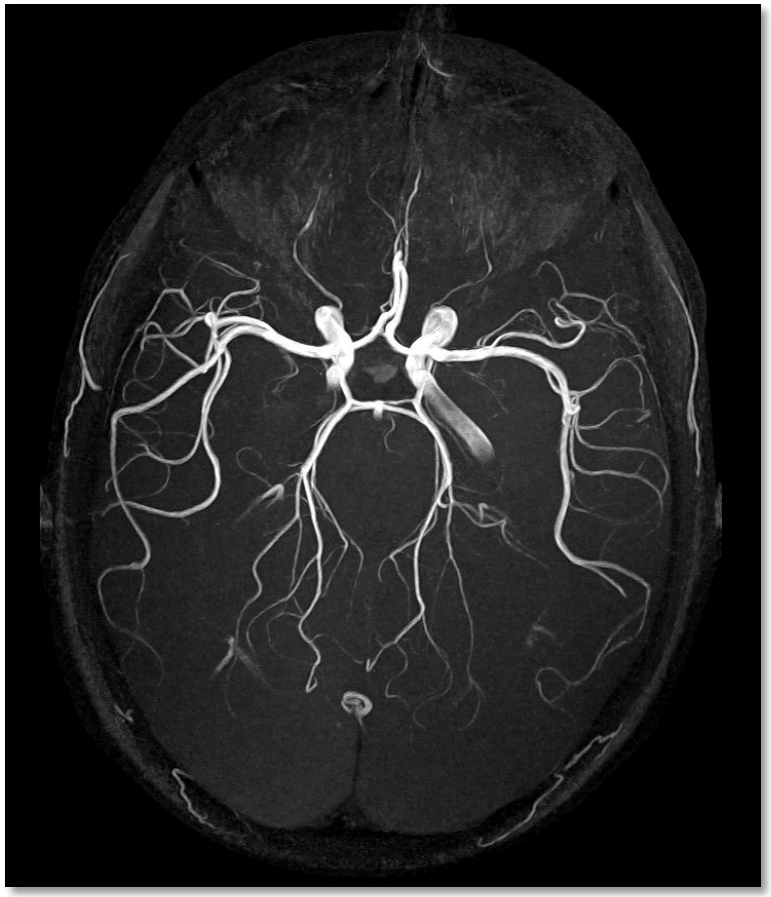
(A) (B) (C)



# Magnetic Resonance Imaging

## Image contrast

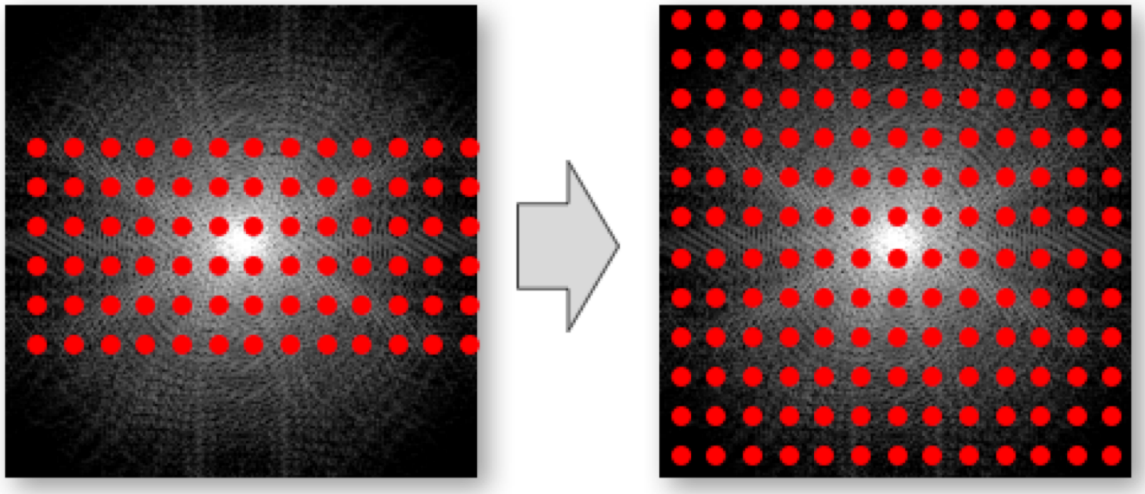
Explain why vessels in the image below appear bright and tissue dark?



# Magnetic Resonance Imaging

## Image signal-to-noise ratio

How does signal-to-noise ratio (SNR) per voxel change if the number of sampling points is doubled as shown in the figure?



# Biomedical Imaging

## Imaging Mode: Ultrasound

- Probe: acoustic waves
- Wavelength: 0.03 – 1.0 mm
- Matter interaction: reflection, absorption, scattering
- Modalities: Echography
  Doppler Imaging
  Thermal Therapy

# Ultrasound

## Sound waves

### How are sound wave attenuation and reflection defined?

Sum of absorption (conversion to heat) and scattering: $p(z) = p_0 e^{-\alpha z}$

Attenuation measure $\alpha_0$ [dB/cm]: $\alpha_0 = 20 \log\left[\frac{p_0}{p(z)}\right] \frac{1}{z} = 8.68\, \alpha\left[\mathrm{dB/cm}\right]$

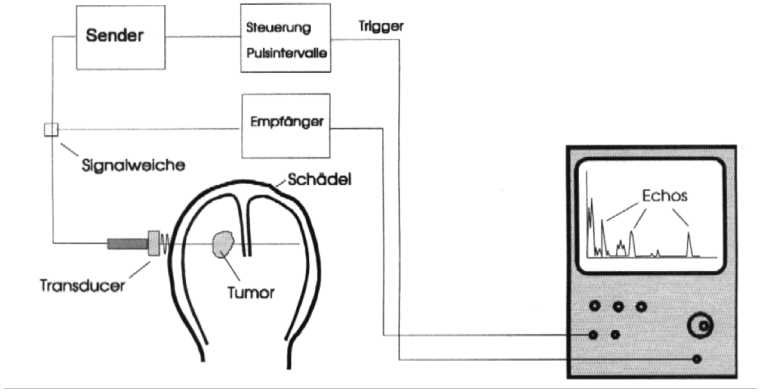
Pressure reflection coefficient: $r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$

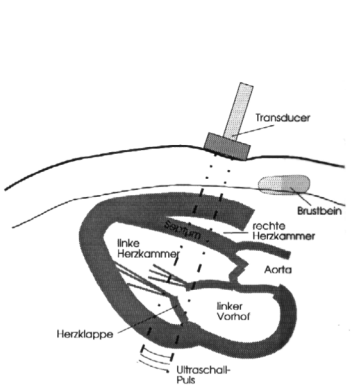
Pressure transmission coefficient: $t = \frac{2Z_2}{Z_2 + Z_1}$

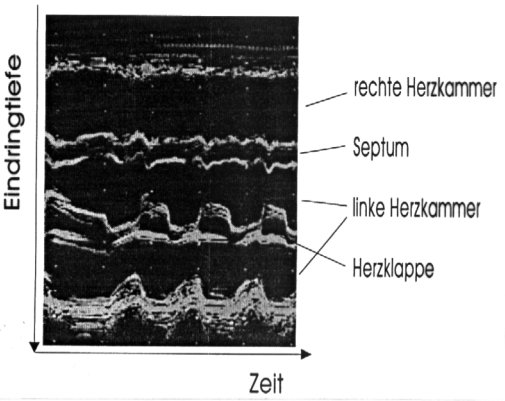
# Ultrasound

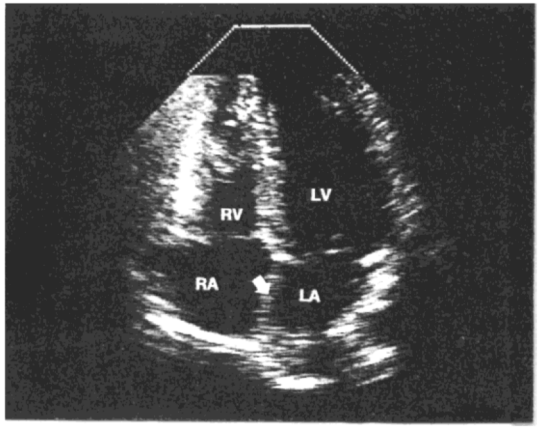
## Scanning modes

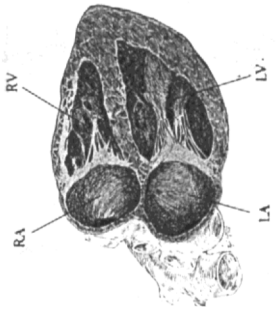
Describe A-, M-, and B-mode scanning!



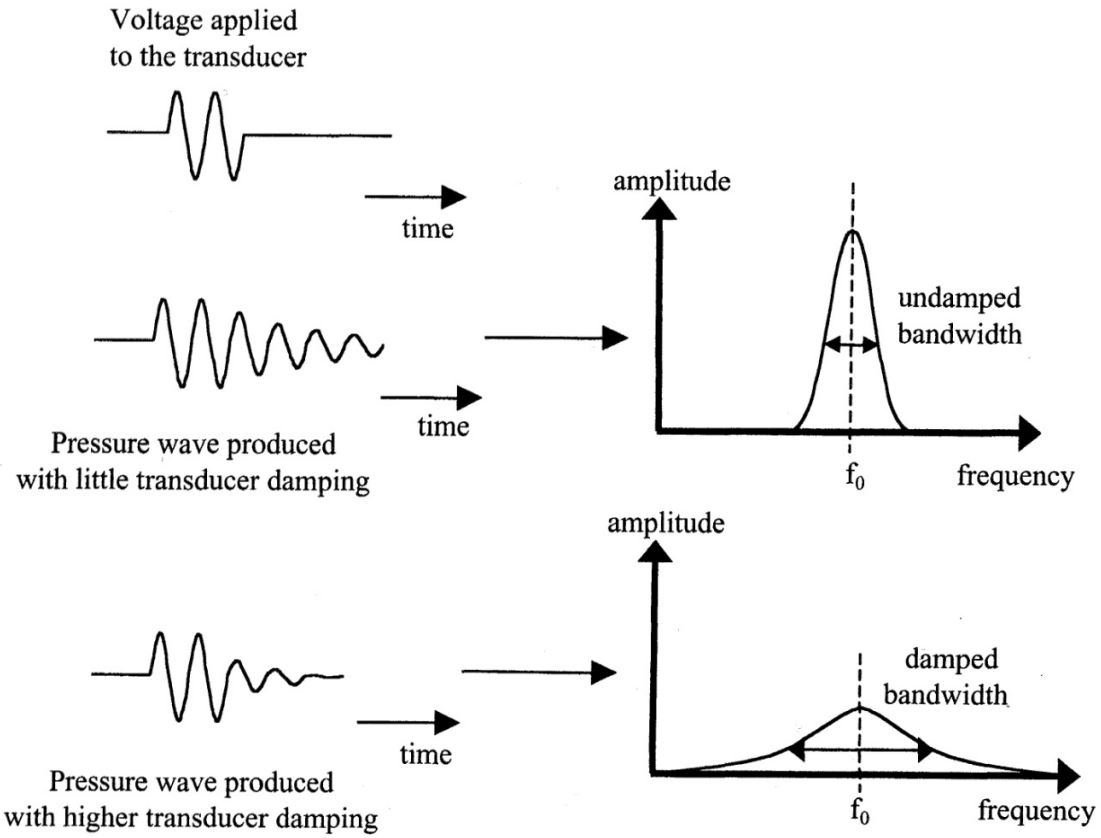



# Ultrasound

## Transducer

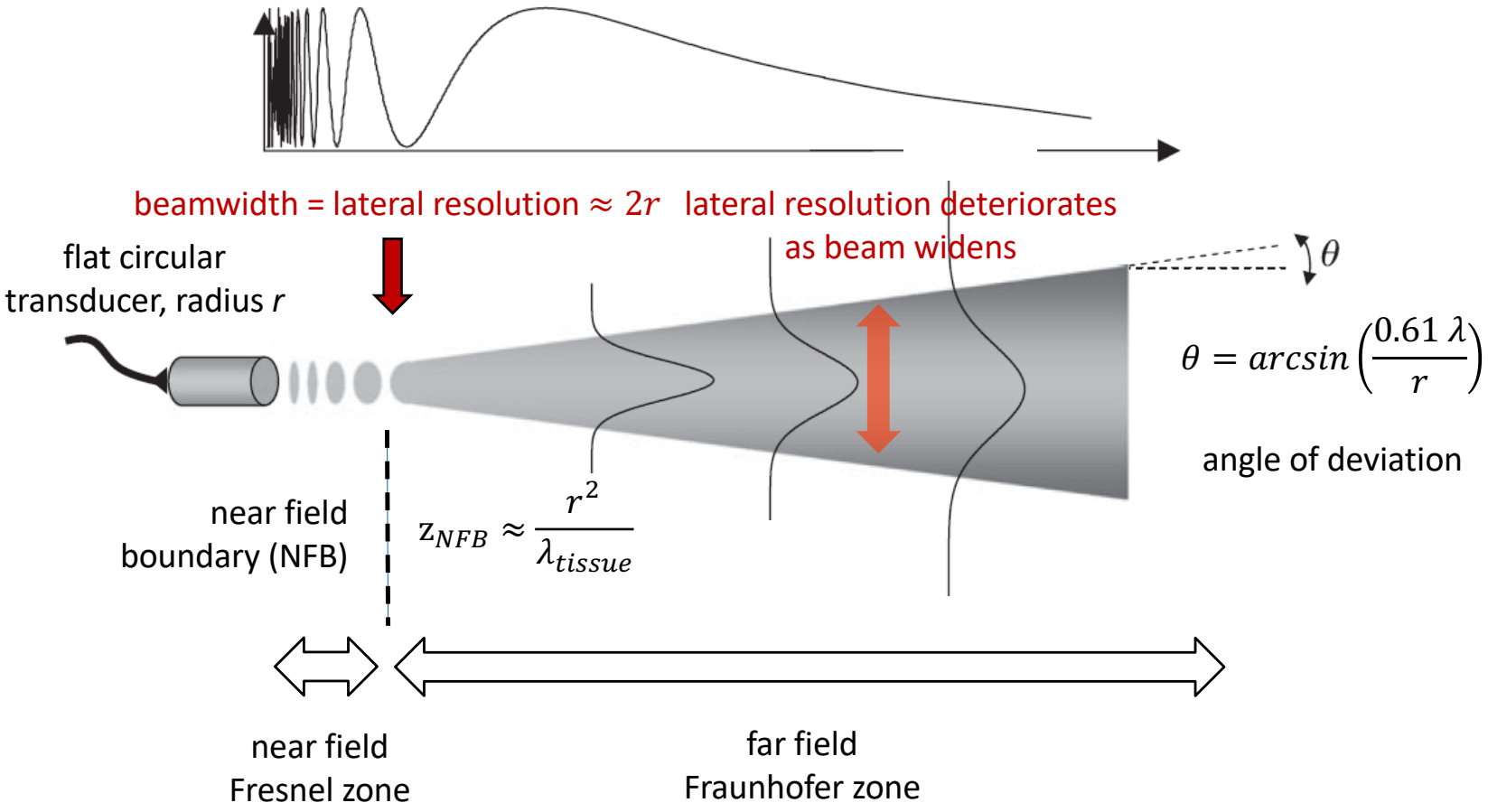
Discuss impact of transducer bandwidth on axial resolution!



# Ultrasound

## Beam geometry

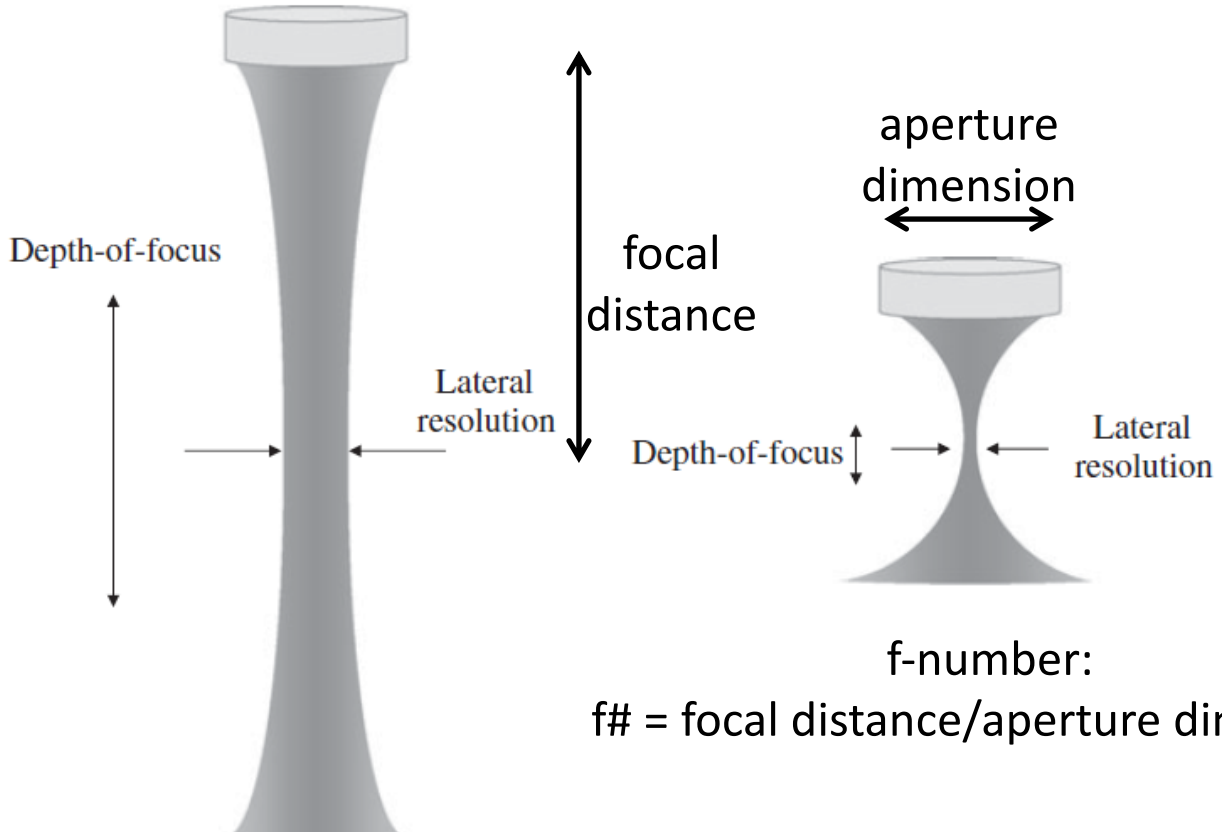
Describe the ultrasound beam from a flat transducer. Which are its characteristics?



# Ultrasound

## Resolution

How are lateral and axial resolution controlled?




f-number:
f# = focal distance/aperture dimension

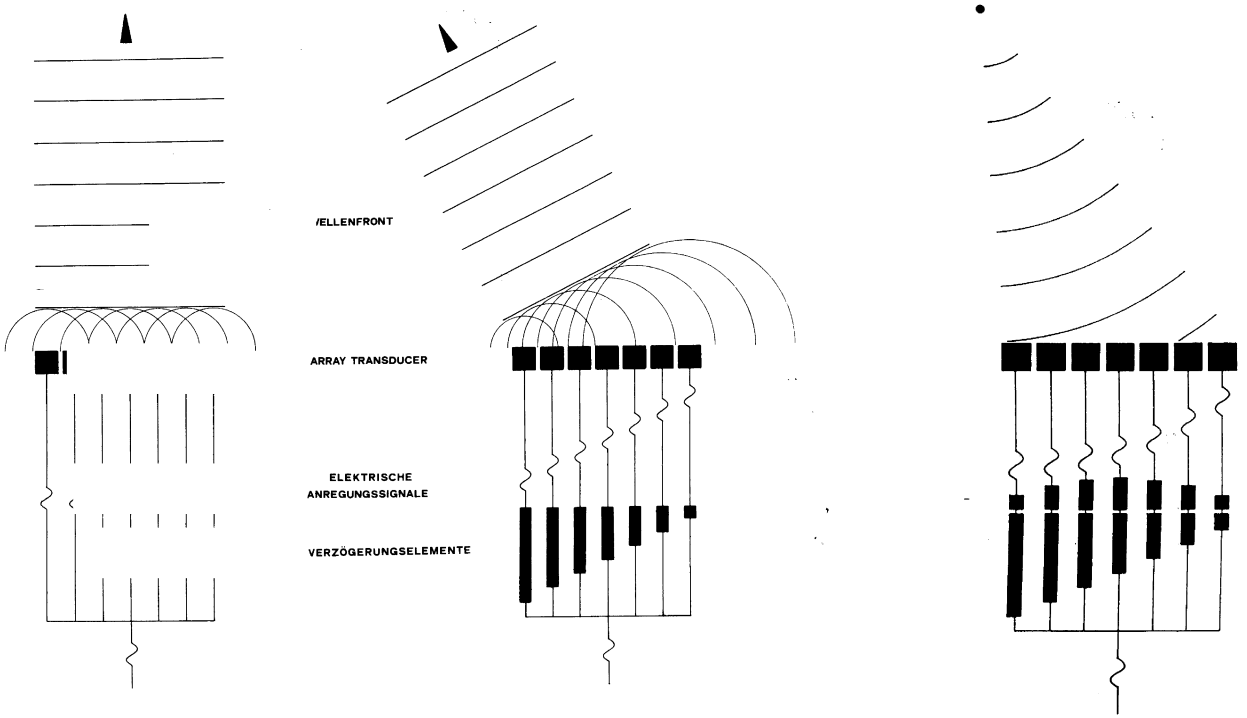
Strong focusing: good lateral resolution but short focal depth

Weak focusing: medium lateral resolution, longer focal depth

# Ultrasound

## Resolution

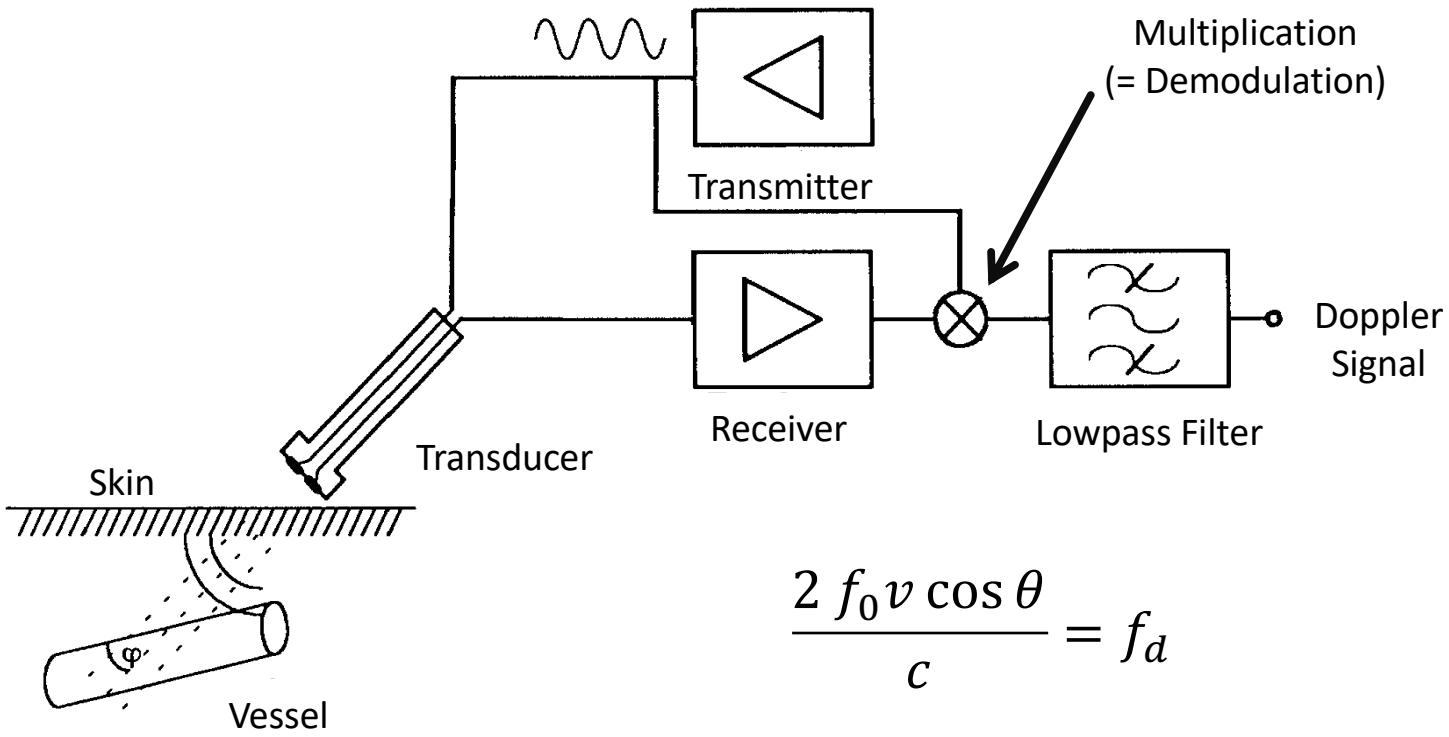
Explain how steering and focusing work with a phased array?



# Ultrasound

## Doppler

Describe a basic (continuous-wave) ultrasound Doppler setup. How does the Doppler shift relate to blood velocity?

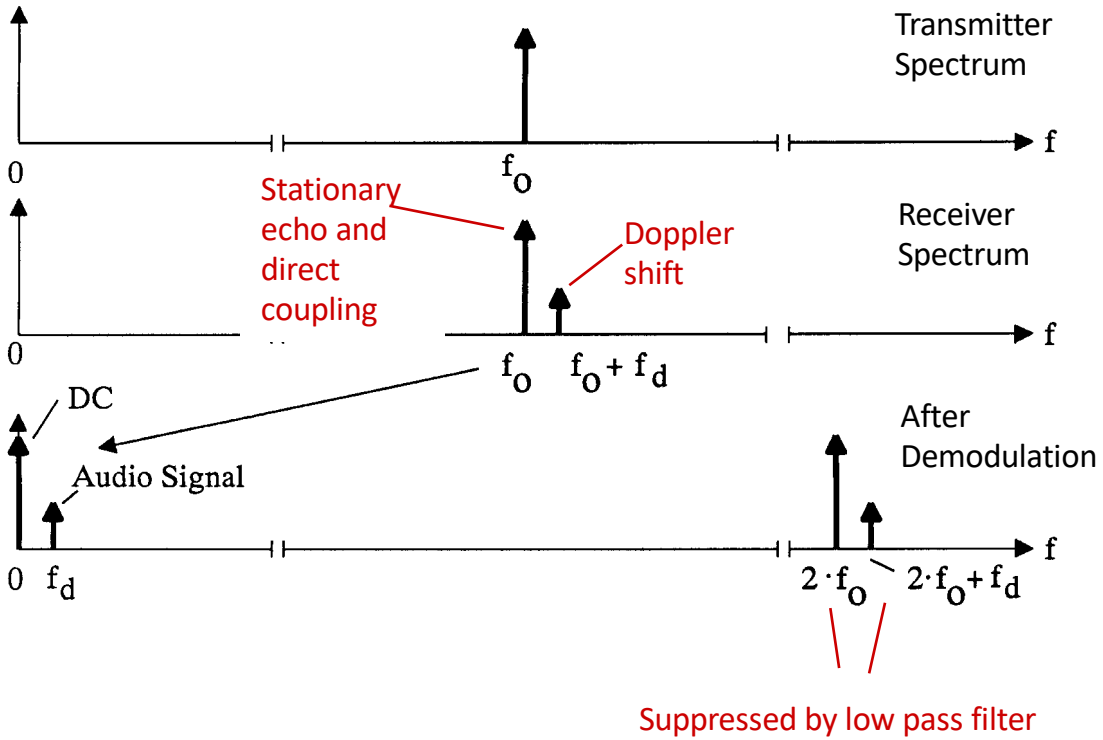



$$\frac{2\, f_0 v \cos\theta}{c} = f_d$$

# Ultrasound

## Doppler

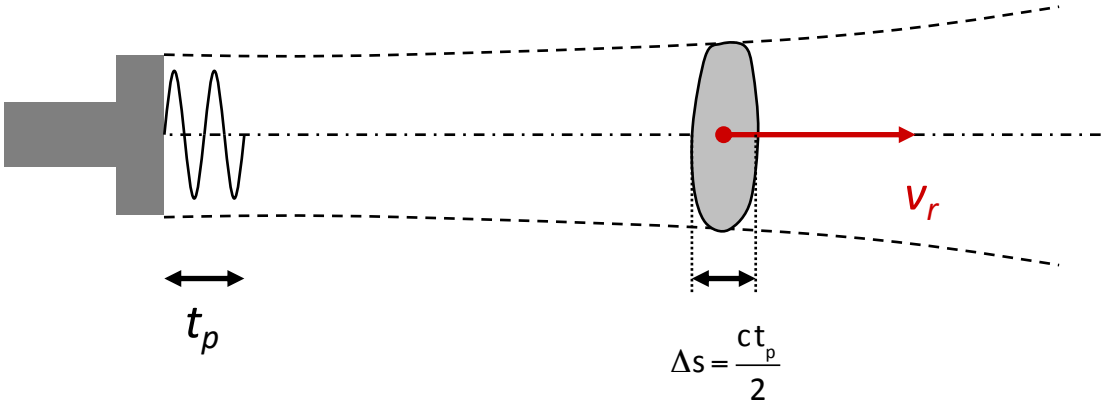
## Analyze a Doppler spectrum!



# Ultrasound

## Doppler

### How are axial resolution and transit time defined?




Pulse bandwidth: $B_p = \frac{1}{t_p}$

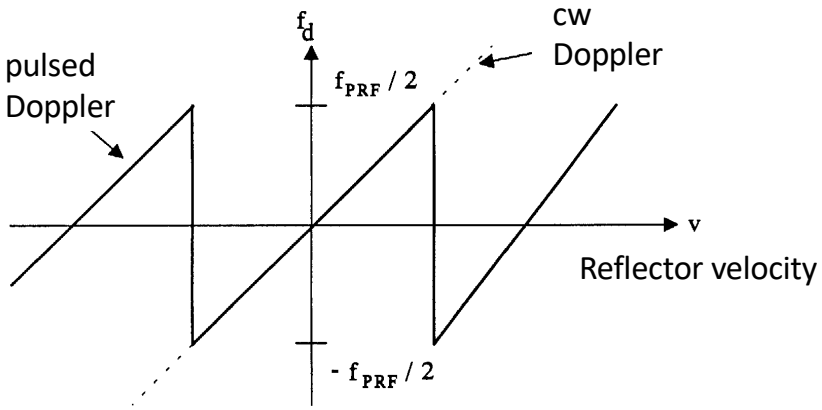
Observation time of particle (transit time): $\Delta t = \frac{\Delta s}{v_r} = \frac{c t_p}{2 v_r}$

Doppler bandwidth due to limited observation time: $B_D \cong \frac{1}{\Delta t} = \frac{2 v_r}{c t_p} = \frac{2 v_r}{c} B_p$

# Ultrasound

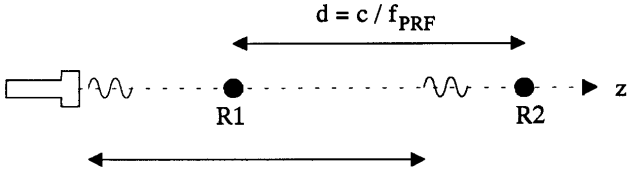
## Doppler

### What is the maximum detectable velocity?




maximum detectable velocity $v_{max}$:

$$v_{max} = +/- \frac{f_{prf}\, c}{4 f_0 \cos\phi}$$




maximum unambiguous measuring distance $d_{max}$

$$d_{max} = \frac{c}{2 f_{prf}}$$

# Merry Christmas