ETH
Eidgenössische Technische Hochschule Zürich
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I. Preprocessing: Cleaning up Images

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Topics

1. Motivation
2. Correcting imaging defects
3. Image processing in the spatial domain
4. Image processing in frequency space

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Book Material

John C. Russ, “The Image Processing Handbook”,
Boca Raton, CRC Press
Available online within domain ethz.ch (or proxy.ethz.ch / public VPN)
http://dx.doi.org/10.1201/9780203881095

More detailed Slides:
Quantitative Big Imaging Course
http://kmader.github.io/Quantitative-Big-Imaging-Course/

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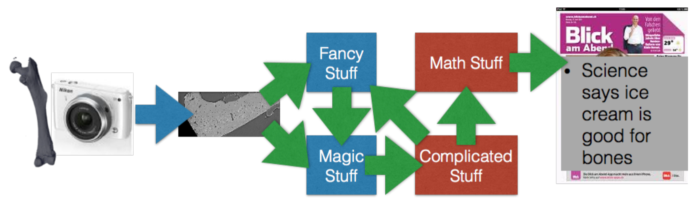
1. Motivation

Why do we make micro- and nano-measurements?

1. Observe smaller scale behavior / structure for understood macrostructures — Crack formation in metals
2. Observe inside opaque structures — Cells inside bones
3. Look for new previously unseen structures / behavior — Looking for jaw bones inside tiny fossils!

1. Motivation



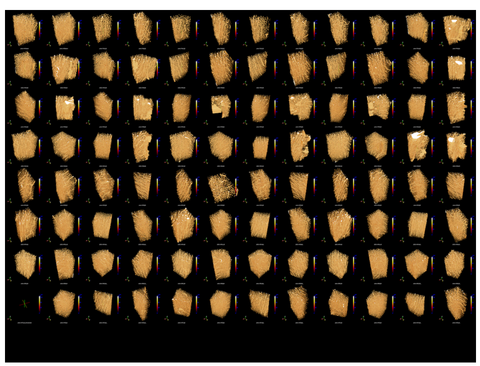
- The utility of automated, quantitative metrics
- To understand the basic steps from collecting an image to extracting useful, scientifically-relevant information from it

1. Motivation: Automated

- Count how many cells are in the bone slice
- Ignore the ones that are 'too big' or shaped 'strangely'
- Are there more on the right side or left side?
- Are the ones on the right or left bigger, top or bottom?

Overwhelmed?

- Do it all over again for 96 more samples, this time with 2000 slices instead of just one!

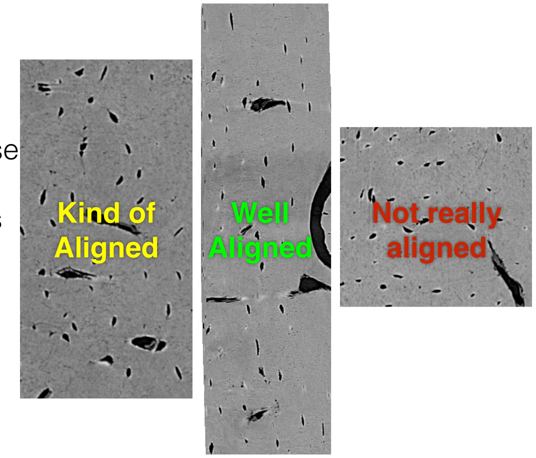


Overwhelmed?

- Now again with 1090 samples!

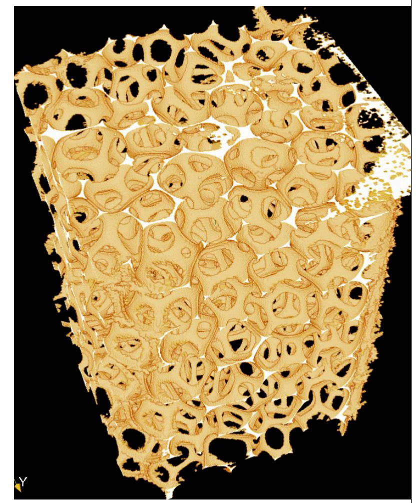
Quantitative?

- How aligned are these cells?
- Are they more or less aligned than these?
- or these?



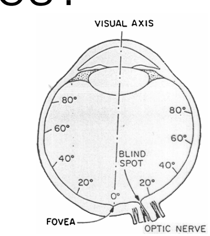
Overwhelmed?

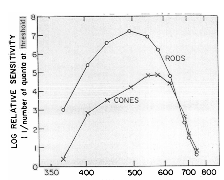
- How many bubbles are here?
- How fast are they moving?
- Do they all move the same speed?
  - Do bigger bubbles move faster?
  - Do bubbles near the edge move slower?
- Are they rearranging?



Why do we need quantitative metrics?

- Human vision is far from perfect
- Good at some tasks
  - Seeing patterns and trends
  - Detecting artifacts and erroneous data
  - Removing noise
- Bad at other things
  - Blind spots
  - Interpreting color
  - Situationally dependent performance
  - Counting many things
  - Biased



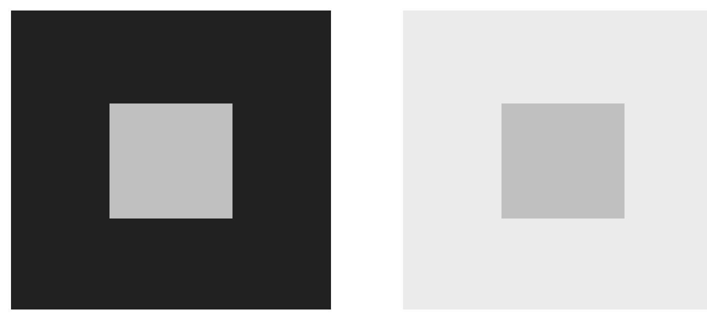


B. Wandell, Foundations of Vision, 1995

Why do we need quantitative metrics?

- **Situationally dependent performance**

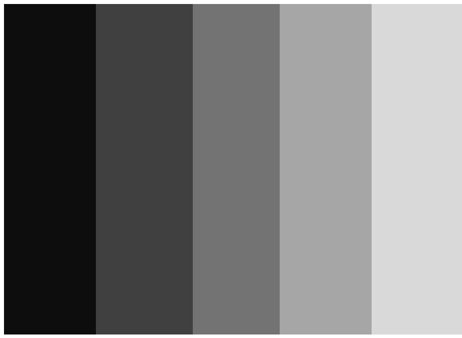
Which center square seems brighter?



Why do we need quantitative metrics?

- **Situationally dependent performance**

Are the bands uniform in brightness?



Why do we need quantitative metrics?

- **Situationally dependent performance**

Yes, but the line spread function of the human eye creates a gradient in the image so the subjective no longer matches the objective

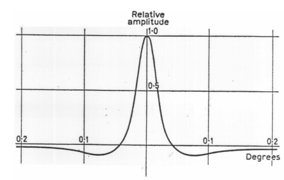


Figure 2.7 Line-spread function of the human eye (after Campbell, Carpenter and Levinson, 1969).

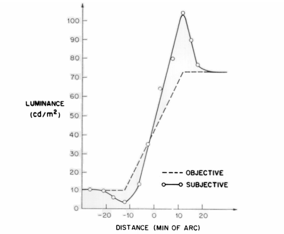


Fig. 4.3.5 Subjective brightness distribution across Mach bands at a graded luminance edge (from Lowry et al. [4.34]).

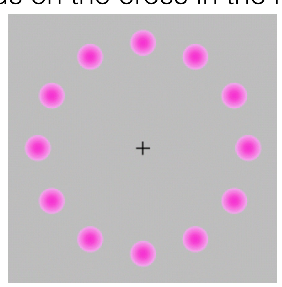
[Pearson 1975]

[Netravali & Haskell 1988]

Why do we need quantitative metrics?

- **Just plain strange**

Focus on the cross in the middle



The green circle only appears when you focus on the middle

What if instead of green ball it's a tumor?

Why do we need quantitative metrics?

- **Biased - Many types: confirmation, contextual, interpretive**

Context

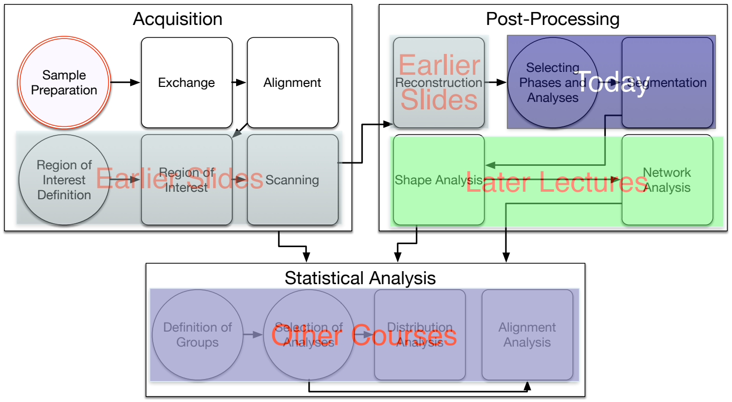
Doctor's who have in the past seen more cases of a given disease are more likely to diagnose it in future cases

Decisions are based on context rather than statistics and presenting the same information to multiple doctor's elicits multiple responses

Egglin, T. K., & Feinstein, A. R. (1996). Context bias. A problem in diagnostic radiology. JAMA : the journal of the American Medical Association, 276(21), 1752–5.

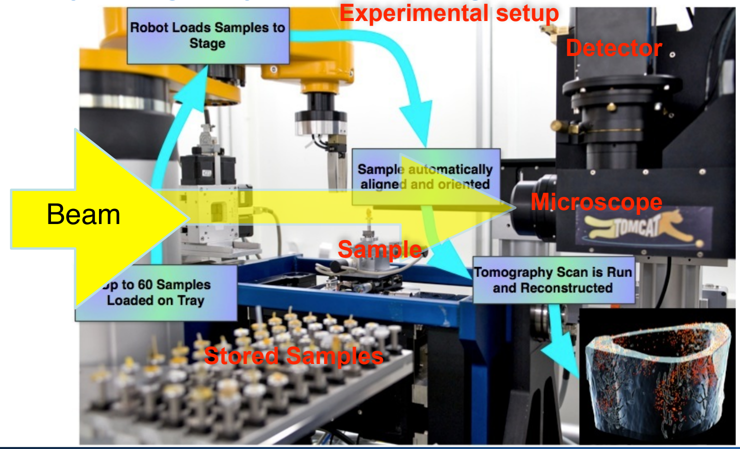
1. Motivation

How do experiments work?



1. Motivation

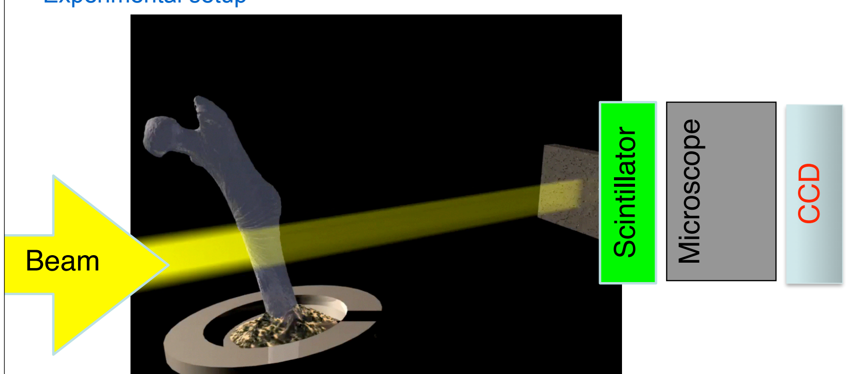
X-ray tomography at the Swiss Light Source



1. Motivation

X-ray tomography at the Swiss Light Source

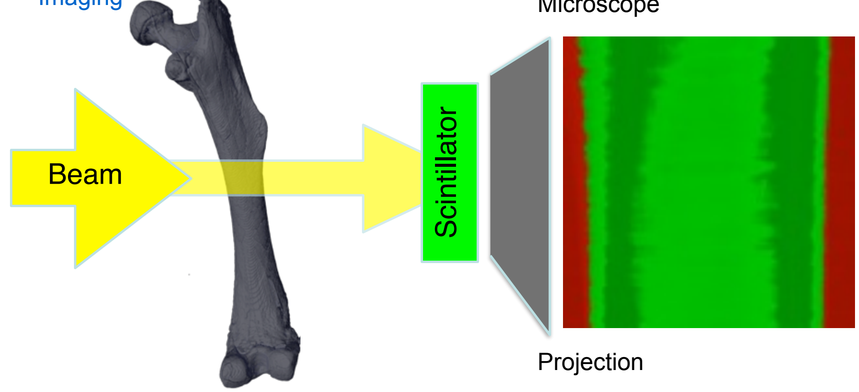
Experimental setup



1. Motivation

X-ray tomography at the Swiss Light Source

Imaging

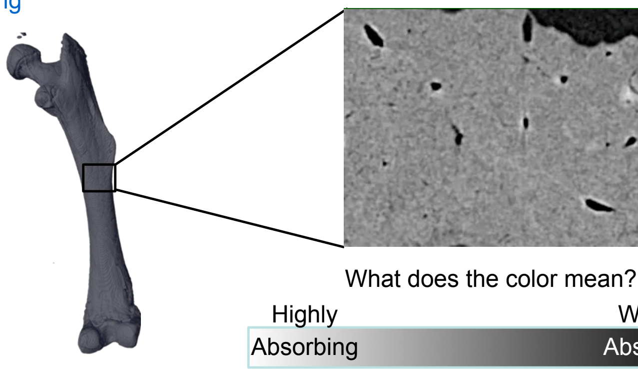


1. Motivation

X-ray tomography at the Swiss Light Source

Imaging

Reconstructed 3D slices

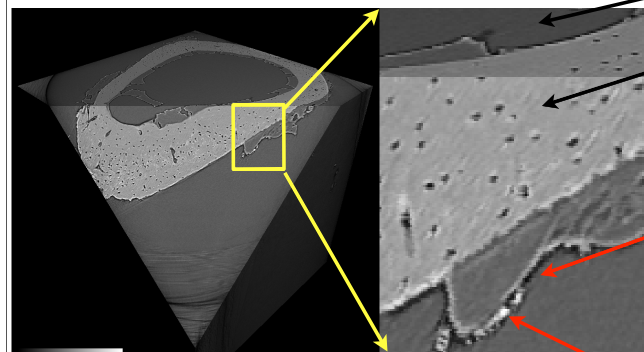


What does the color mean?

Highly Absorbing Regions	Weakly Absorbing Regions

1. Motivation

Interpreting Absorption



Air

Calcified Bone Tissue

Less-than-air absorbing material / Emitting Photons on bone surface!!

Super strong absorbing material (metal?) inside soft tissue!

Highly Absorbing Regions	Weakly Absorbing Regions

2. Correcting imaging defects

Warning : This slide is a wild oversimplification, the reality is MUCH more complicated

Image Defects

Why should the images be defective?

Beam Profile

Free space Propogation $\otimes h_{fs}(x,y,d)$

d

Scintillator

Optical Transfer $*h_{op}(x,y)$

Microscope Optics

Detector

Beam Profile $b(x,y)$ | Sample Absorption $*s_{ab}(x,y)$ | Scintillator Efficiency $*h_{sc}(x,y)$ | Detector $*h_{det}(x,y)+d_{dark}(x,y)$

Image Saved by Camera =

$$([b(x,y)*s_{ab}(x,y)]\otimes h_{fs}(x,y))*h_{sc}(x,y)*h_{op}(x,y)]*h_{det}(x,y)+d_{dark}(x,y)$$

What we want

2. Correcting imaging defects

Image Defects

What can we do?

Image Saved by Camera =

$$[([b(x,y)*s_{ab}(x,y)]\otimes h_{fs}(x,y))*h_{sc}(x,y)*h_{op}(x,y)]*h_{det}(x,y)+d_{dark}(x,y)$$

Beam Profile $b(x,y)$	Can be measured, but changes rapidly and unpredictably at synchrotrons, approach: Flat Field
Free space Propogation $\otimes h_{fs}(x,y,d)$	Cannot be removed without knowing phase, several techniques use approximations Paganin / MBA
Scintillator Efficiency Optical Transfer $*h_{sc}(x,y)$ $*h_{op}(x,y)$	Can be measured, relatively constant over the time scale of hours, approach Flat Field
Detector $*h_{det}(x,y)+d_{dark}(x,y)$	Somewhat predictable, dependent on temperature, radiation protection and other conditions, approach: Flat / Dark Field

2. Correcting imaging defects

Image Defects

Flat Field and Dark Field

d(x,y) — Dark Field: captured with no illumination, shows the natural variatio[n] inside the detector (function of temperature, pixel damage)

f(x,y) — Flat Field: captured with illumination but no sample, captures the beam profile, scintillator transfer, microscope transfer and detector profile

Voila!

Subtract **Dark Field** and Divide by **Flat Field**, **assume** no free space propagation

$$[([\cancel{h(x,y)} * s_{ab}(x,y)] \otimes \cancel{h_{fs}(x,y)}) * \cancel{h_{sc}(x,y)} * \cancel{h_{op}(x,y)}] * \cancel{h_{det}(x,y)} + \cancel{d_{dark}(x,y)}$$

What we want

Eh, not really, the images look much better but not perfect

2. Correcting imaging defects

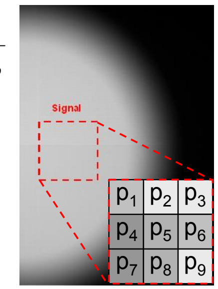
Noise

How can it be measured: Signal-to-noise ratio (SNR)

SNR in engineering: $SNR = \frac{Power_{signal}}{Power_{noise}} = \left(\frac{Amplitude_{signal}}{Amplitude_{noise}}\right)^2$

SNR in image processing: $SNR = \frac{Average(signal)}{Stddev(signal)} = \frac{\bar{p}}{\sigma_p}$

- Signal = pixel intensity = gray value
- $Average(signal) = \bar{p} = \frac{\sum_{i=1}^{n} p_i}{n}$
- $Stddev(signal) = \sigma_p = \sqrt{\frac{\sum_{i=1}^{n}(p_i - \bar{p})^2}{n}}$

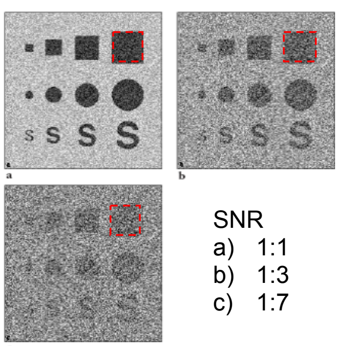


Electro Optical Industries, Inc.

2. Correcting imaging defects

Noise

Signal-to-noise ratio (SNR)



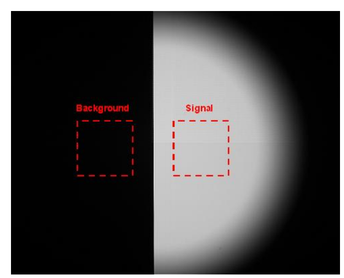
SNR
a) 1:1
b) 1:3
c) 1:7

Image Processing Handbook, J. C. Russ

2. Correcting imaging defects

Noise

Contrast



Electro Optical Industries, Inc.

Michelson contrast = $\frac{average(signal) - average(background)}{average(signal) + average(background)}$

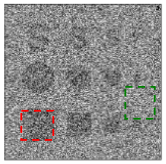
Weber contrast = $\frac{average(signal) - average(background)}{average(background)}$

2. Correcting imaging defects

Noise

Contrast

Signal



Background

$$\text{Michelson contrast} = \frac{\text{average(signal)} - \text{average(background)}}{\text{average(signal)} + \text{average(background)}}$$

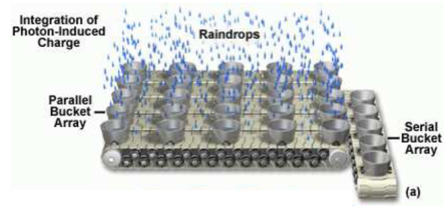
- SNR high <=> contrast high
- SNR low <=> contrast low

2. Correcting imaging defects

Noise

Detector: readout noise of a CCD camera

$$[([b(x,y) * s_{ab}(x,y)] \otimes h_{fs}(x,y)) * h_{sc}(x,y) * h_{op}(x,y)] * h_{det}(x,y) + d_{dark}(x,y)$$



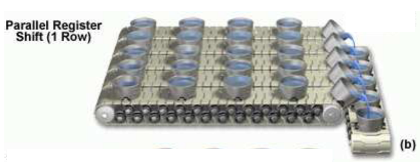
Astronomy with charged coupled devices, R.A. Jansen
http://www.public.asu.edu/~rjansen/ast598/ast598_jansen2006.pdf

Integration

2. Correcting imaging defects

Noise

Detector: readout noise of a CCD camera



Astronomy with charged coupled devices, R.A. Jansen
http://www.public.asu.edu/~rjansen/ast598/ast598_jansen2006.pdf

Parallel registration

2. Correcting imaging defects

Noise

Detector: readout noise of a CCD camera

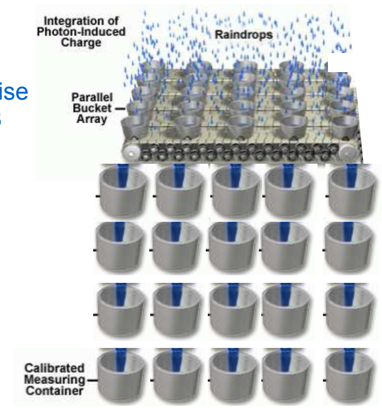
Astronomy with charged coupled devices, R.A. Jansen
http://www.public.asu.edu/~rjansen/ast598/ast598_jansen2006.pdf

Serial registration/readout (amplification and analog-to-digital conversion)

2. Correcting imaging defects

Noise

Detector: readout noise of a **CMOS** camera



Each bucket has it's own measuring container

+ much faster read-out time, cheaper to manufacture

- more wiring, sometimes less dynamic range

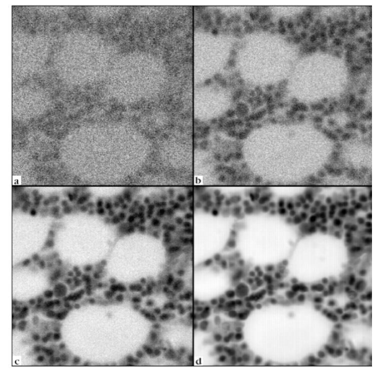
Astronomy with charged coupled devices, R.A. Jansen
http://www.public.asu.edu/~rjansen/ast598/ast598_jansen2006.pdf

Serial registration/readout (amplification and analog-to-digital conversion)

2. Correcting imaging defects

Noise

Noise reduction: frame averaging



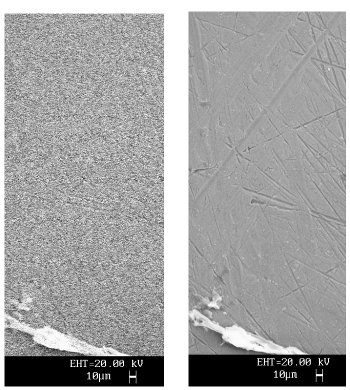
a) 1 frame
b) 4 frames
c) 16 frames
d) 256 frames

Fluorescence light micrograph: bone marrow
Image Processing Handbook, J. C. Russ

2. Correcting imaging defects

Noise

Noise reduction: integration/staring mode



left) 1-second scan
right) 20-second scan

SEM: scratched metal surface
Image Processing Handbook, J. C. Russ

2. Correcting imaging defects

Noise

Frame averaging vs. integration/staring mode

Frame averaging:
- Noise due to incoming photons

Integration/staring mode:
- Noise due to incoming photons
- Noise inherent to the detector (dark current)

=> Cool detector to reduce thermal noise

2. Correcting imaging defects

Neighborhood analysis

Underlying Assumptions

1) If the noise is a random point process it is unlikely to affect neighboring pixels in the same way
2) Many samples are slowly varying and the 'signal' value between neighboring voxels should not change much

The noise can be reduced by taking advantage of this information

Example

c	d	f
b	a	g
k	j	h

$$a = \mu + \delta_a + noise_a$$
$$b = \mu + \delta_b + noise_b$$
$$c = \mu + \delta_c + noise_c$$
$$d = \mu + \delta_d + noise_d$$

The μ can be recovered (if ∂ is small) by averaging all the a-h values

2. Correcting imaging defects

Neighborhood averaging

Definition

Replace each pixel with the average of itself and its neighbors:

$$p_{x,y} = \frac{\sum_{i=-n}^{n} \sum_{j=-n}^{n} w_{i,j} \cdot p_{x+i,y+j}}{\sum_{i=-n}^{n} w_{i,j}}$$

- $w_{i,j}$ weights
- $p_{x,y}$ pixel value

Example

$w_{i,j} =$

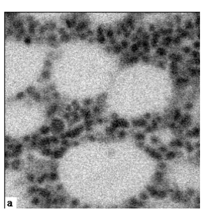
1	1	1
1	1	1
1	1	1

2. Correcting imaging defects

Neighborhood averaging

Noise reduction

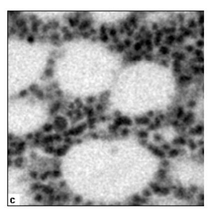
Original



Fluorescence light micrograph: bone marrow
Image Processing Handbook, J. C. Russ

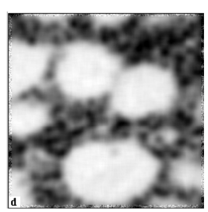
1	1	1
1	1	1
1	1	1

3x3



1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1

11x11



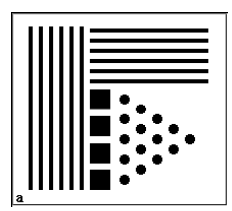
=> Reduction of noise

2. Correcting imaging defects

Neighborhood averaging

Blurring

Original



Kernel

1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1

11x11

Averaged

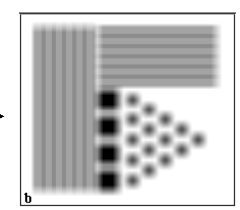


Image Processing Handbook, J. C. Russ

=> Reduction of contrast (blurring)

2. Correcting imaging defects

Neighborhood averaging

Examples

1	1	1
1	1	1
1	1	1

Reduction of blurring

1	2	1
2	4	2
1	2	1

1	4	1
4	12	4
1	4	1

1	2	3	2	1
2	7	11	7	2
3	11	17	11	3
2	7	11	7	2
1	2	3	2	1

Gaussian kernels

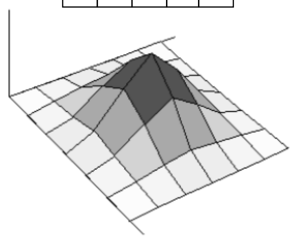


Image Processing Handbook, J. C. Russ

2. Correcting imaging defects

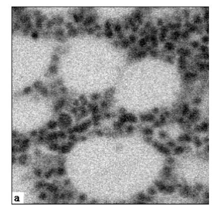
Neighborhood averaging

Noise reduction vs. maximal contrast

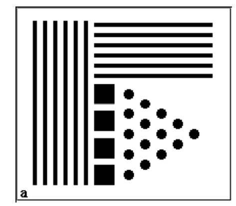
Trade-off

Noise reduction

Maximal contrast (minimal blurring)



=> Gaussian filter optimal



2. Correcting imaging defects

Neighborhood averaging

Noise reduction vs. maximal contrast

Trade-off

Noise reduction

Maximal contrast (minimal blurring)

=> Gaussian filter optimal

The Gaussian filter produces
the least edge blurring for a given amount of noise reduction

2. Correcting imaging defects

Neighborhood ranking

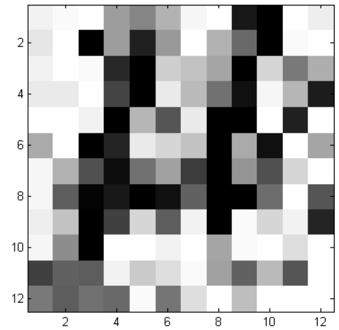
Characterization

- Neighborhood averaging: linear operations
- Neighborhood ranking: non-linear operations

2. Correcting imaging defects

Neighborhood ranking

Median filter

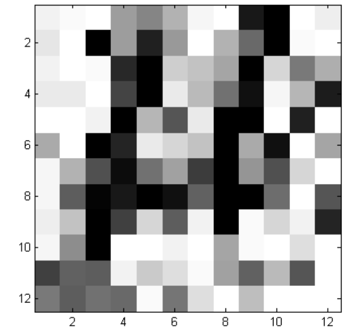


241	249	254	157	134	177	244	255	27	0	255	236
231	255	0	156	34	153	255	176	104	0	249	255
243	255	251	41	0	207	194	167	0	215	121	173
233	233	255	71	0	233	186	112	19	244	183	31
255	255	243	2	182	87	235	0	0	254	33	253
171	255	0	38	234	213	195	0	170	18	255	165
246	176	83	15	115	163	60	0	150	83	212	255
246	95	5	26	0	19	97	0	0	111	255	86
237	194	0	64	212	94	242	0	249	212	241	36
244	140	0	255	253	240	255	164	250	254	222	255
66	98	92	242	202	221	250	162	96	184	87	255
121	92	112	106	251	116	223	255	188	255	255	255

2. Correcting imaging defects

Neighborhood ranking

Median filter



241	249	254	157	134	177	244	255	27	0	255	236
231	255	0	156	34	153	255	176	104	0	249	255
243	255	251	41	0	207	194	167	0	215	121	173
233	233	255		0	233	186	112	19	244	183	31
255	255	243	2	182	87	235	0	0	254	33	253
171	255	0	38	234	213	195	0	170	18	255	165
246	176	83	15	115	163	60	0	150	83	212	255
246	95	5	26	0	19	97	0		111	255	86
237	194	0	64	212	94	242	0	249	212	241	36
244	140	0	255	253	240	255	164	250	254	222	255
66	98	92	242	202	221	250	162	96	184	87	255
121	92	112	106	251	116	223	255	188	255	255	255

= median ( )

2. Correcting imaging defects

Neighborhood ranking

Median filter

[251,41,0,255,71,0,243,2,182] ->
[0,0,2,41,71,182,243,251,255]
Median = 71

[0,170,18,60,0,150,83,212,97,0,0
111,255,242,0,249,212,241,164,
250,254] ->
[0,0,0,0,0,18,60,83,97,
111,150,164,170,212,212,241,
242,249,250,254,255]
Median = 150

241	249	254	157	134	177	244	255	27	0	255	236
231	255	0	156	34	153	255	176	104	0	249	255
243	255	251	41	0	207	194	167	0	215	121	173
233	233	255		0	233	186	112	19	244	183	31
255	255	243	2	182	87	235	0	0	254	33	253
171	255	0	38	234	213	195	0	170	18	255	165
246	176	83	15	115	163	60	0	150	83	212	255
246	95	5	26	0	19	97	0		111	255	86
237	194	0	64	212	94	242	0	249	212	241	36
244	140	0	255	253	240	255	164	250	254	222	255
66	98	92	242	202	221	250	162	96	184	87	255
121	92	112	106	251	116	223	255	188	255	255	255

= median ( )

2. Correcting imaging defects

Neighborhood ranking

Example

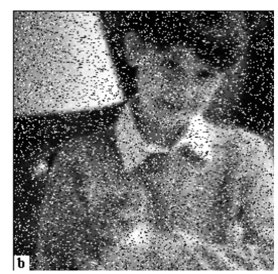


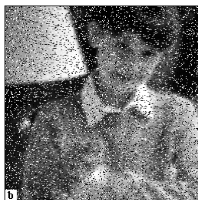
Image Processing Handbook, J. C. Russ

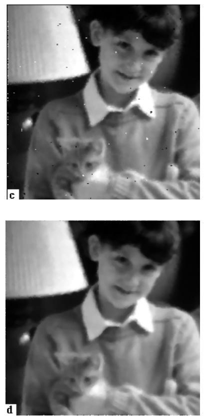
Shot/impulse noise:
e.g. if detector pixels damaged or dead

2. Correcting imaging defects

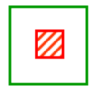
Neighborhood ranking

Median filter





3x3 square



5x5 octagonal

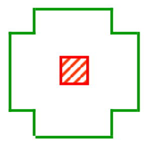


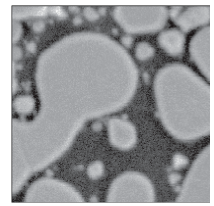
Image Processing Handbook, J. C. Russ

=> Median filter removes small artifacts

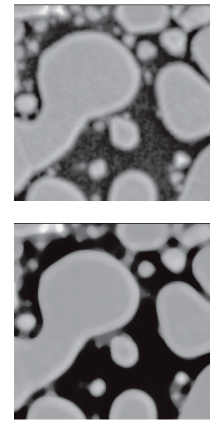
2. Correcting imaging defects

Neighborhood ranking

Gaussian vs. median filtering



SEM: deposited gold particles
Image Processing Handbook, J. C. Russ



Gaussian filtered

Median filtered

=> Median filter preserves edge sharpness

2. Correcting imaging defects

Neighborhood ranking

Median filter: Multiple median filtered: contouring/ posterization

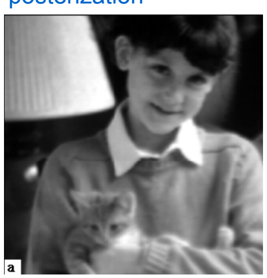


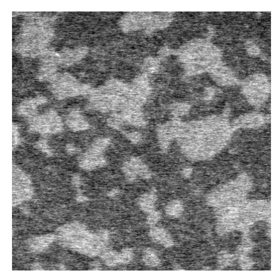
Image Processing Handbook, J. C. Russ

Original

2. Correcting imaging defects

Neighborhood ranking

Median filter



Low-voltage SEM image
Image Processing Handbook, J. C. Russ

O	O	O	O	O	O	O
O	O	O	X	O	O	O
O	O	O	X	O	O	O
O	O	O	X	O	O	O
O	O	O	X	O	O	O
O	O	O	X	O	O	O
O	O	O	O	O	O	O

Support for vertical median filtering

2. Correcting imaging defects

Neighborhood ranking

Median filter

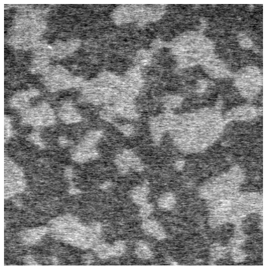
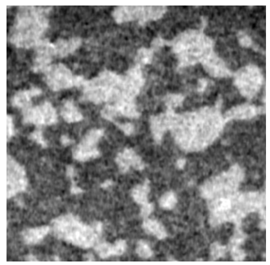


Image Processing Handbook, J. C. Russ

Original



Vertical median filtered

2. Correcting imaging defects

Other neighborhood methods

Top hat filter

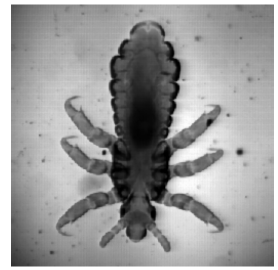


Image Processing Handbook, J. C. Russ

Impurities (e.g. dust particles)

2. Correcting imaging defects

Other neighborhood methods

Top hat filter

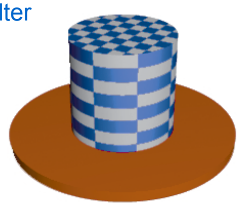
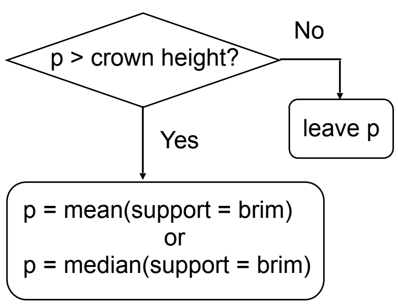


Image Processing Handbook, J. C. Russ

- Crown height: max accepted gray value (compared to brim average)
- Crown diameter: support for inspection
- Brim diameter: support for replacement



2. Correcting imaging defects

Other neighborhood methods

Top hat filter

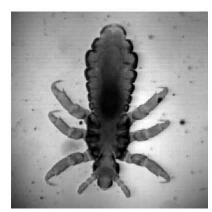
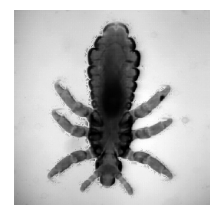
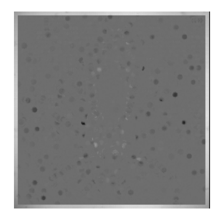


Image Processing Handbook, J. C. Russ

Original



Top hat filtered



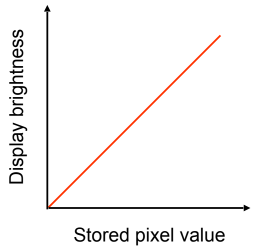
Replaced pixels (dust)

3. Image processing in the spatial domain

Contrast manipulation

Display transfer function

Image Processing Handbook, J. C. Russ



3. Image processing in the spatial domain

Contrast manipulation

Display transfer function

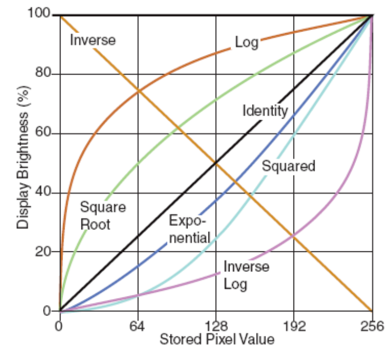


Image Processing Handbook, J. C. Russ

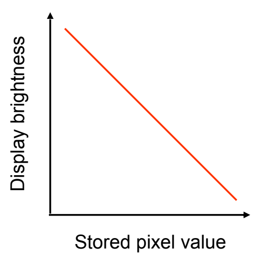
Tables of transfer functions precalculated:
lookup table (LUT)
Also called Gamma
(non-linearities in display)

3. Image processing in the spatial domain

Contrast manipulation

Display transfer function

Image Processing Handbook, J. C. Russ



Original

Reversed contrast

3. Image processing in the spatial domain

Contrast manipulation

Display transfer function

Image Processing Handbook, J. C. Russ

Original

Negative

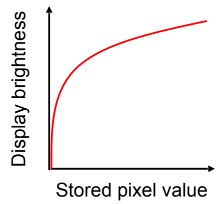
3. Image processing in the spatial domain

Contrast manipulation

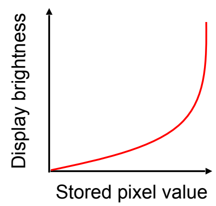
Display transfer function

Image Processing Handbook, J. C. Russ

Original



Log



Inverse Log

3. Image processing in the spatial domain

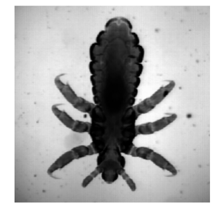
Contrast manipulation

Display transfer function

Image Processing Handbook, J. C. Russ

Original

Log

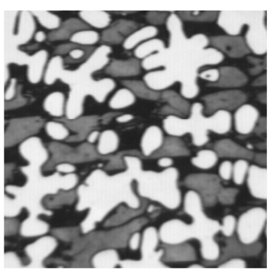


Inverse Log

3. Image processing in the spatial domain

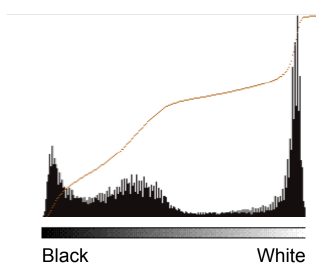
Histogram equalization

Histogram and cumulative histogram



Light micrograph: polished metal with 3 phases
Image Processing Handbook, J. C. Russ

Image

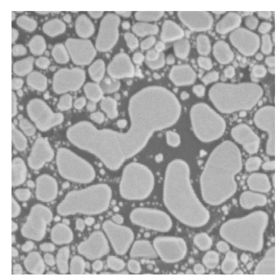


Histogram & cumulative histogram

3. Image processing in the spatial domain

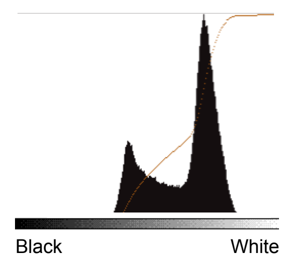
Histogram equalization

Cumulative histogram and transfer function



SEM: deposited gold particles
Image Processing Handbook, J. C. Russ

Original

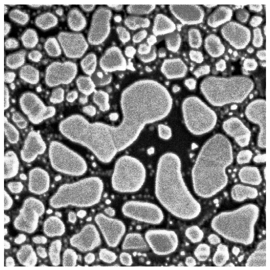


Histogram & cumulative histogram

3. Image processing in the spatial domain

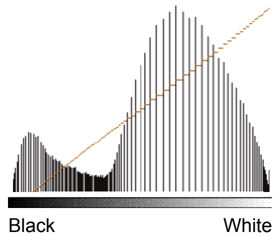
Histogram equalization

Cumulative histogram and transfer function



SEM: deposited gold particles
Image Processing Handbook, J. C. Russ

Equalized

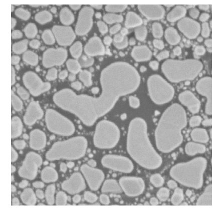


Histogram & cumulative histogram

3. Image processing in the spatial domain

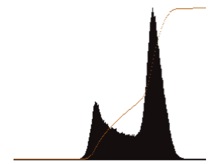
Histogram equalization

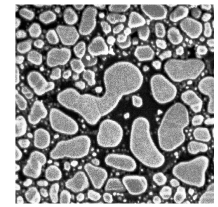
Cumulative histogram and transfer function



Original

Transfer function = Cumulative histogram of original



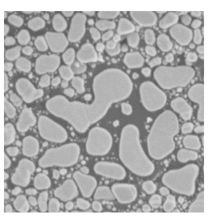


Equalized

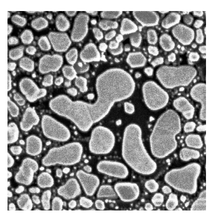
3. Image processing in the spatial domain

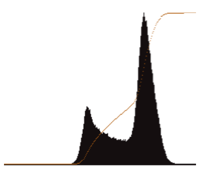
Histogram equalization

Cumulative histogram and transfer function

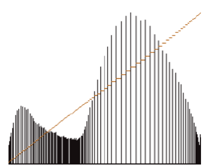


Transfer function = Cumulative histogram of original





=> Cumulative histogram linear



3. Image processing in the spatial domain

Laplacian

Remember: noise smoothing

1	1	1
1	1	1
1	1	1

Reduction of blurring

1	2	1
2	4	2
1	2	1

1	4	1
4	12	4
1	4	1

1	2	3	2	1
2	7	11	7	2
3	11	17	11	3
2	7	11	7	2
1	2	3	2	1

Gaussian kernels

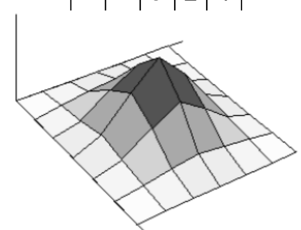


Image Processing Handbook, J. C. Russ

3. Image processing in the spatial domain

Laplacian

Laplacian vs. Gaussian

1	4	1
4	12	4
1	4	1

Gaussian

- Maximal noise reduction at minimal edge blurring
- Only positive kernel entries

-1	-1	-1
-1	8	-1
-1	-1	-1

Laplacian

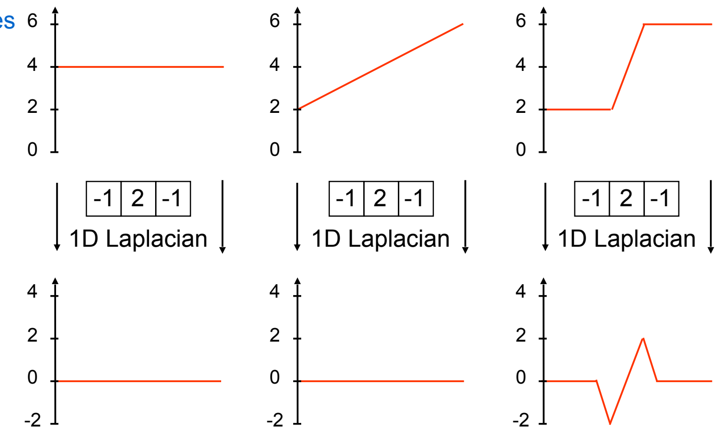
- Also negative kernel entries
- Sum of kernel entries = 0
- Approximation of the 2nd derivative:

$$\nabla^2 p(x,y) = \frac{\partial^2 p(x,y)}{\partial x^2} + \frac{\partial^2 p(x,y)}{\partial y^2}$$

3. Image processing in the spatial domain

Laplacian

Examples

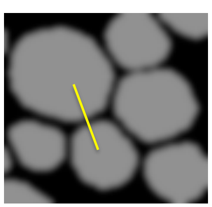


3. Image processing in the spatial domain

Laplacian

Edge enhancement

Original

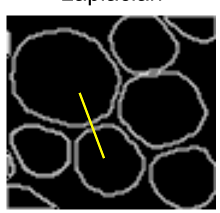


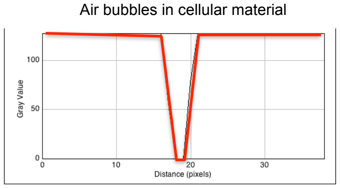
Air bubbles in cellular material

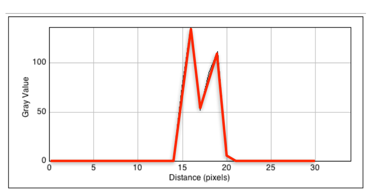
Laplacian

-1	-1	-1
-1	8	-1
-1	-1	-1

Laplacian







3. Image processing in the spatial domain

Laplacian

Physical explanation

- Blurred image: $p(x,y,t)$
- Brightness spreads out across image (diffusion process):

$$J(x,y,t) = -D\cdot\frac{\partial p(x,y,t)}{\partial t} \quad \text{(1) Fick's first law}$$

$$\frac{\partial p(x,y,t)}{\partial t} + \nabla J(x,y,t) = 0 \quad \text{(2) Continuity equation}$$

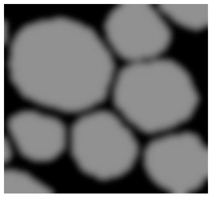
$$\frac{\partial p(x,y,t)}{\partial t} = D\cdot\nabla^2 p(x,y,t) \quad \text{(3) Diffusion equation}$$

- Unblurred image: $q(x,y) = p(x,y,\tau) - \tau\frac{\partial p(x,y,t)}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2 p(x,y,t)}{\partial t^2} - \ldots \approx p(x,y,\tau) - D\tau\cdot\nabla^2 p(x,y,\tau)$

3. Image processing in the spatial domain

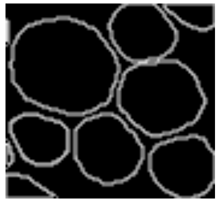
Laplacian

Sharpening operator



Air bubbles in cellular material

$$q(x,y) \approx \overbrace{p(x,y,\tau)}^{\text{Original}} - D\tau \cdot \underbrace{\nabla^2 p(x,y,\tau)}_{\text{Laplacian}}$$



Laplacian

-1	-1	-1
-1	8	-1
-1	-1	-1

Laplacian + Original

-1	-1	-1
-1	9	-1
-1	-1	-1

3. Image processing in the spatial domain

Derivatives

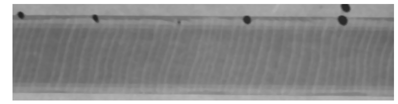
Edge enhancement

- Derivative:

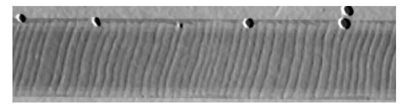
1	0	-1
1	0	-1
1	0	-1

- Smoothed derivative:

1	0	-1
2	0	-2
1	0	-1



Original



Smoothed derivative

Light micrograph: tree rings in a drill core
Image Processing Handbook, J. C. Russ

=> Sensitive to orientation

3. Image processing in the spatial domain

Finding edges

Kirsch operator

k_1:

1	0	-1
2	0	-2
1	0	-1

k_2:

2	1	0
1	0	-1
0	-1	-2

k_3:

1	2	1
0	0	0
-1	-2	-1

k_4:

0	1	2
-1	0	1
-2	-1	0

k_5:

-1	0	1
-2	0	2
-1	0	1

k_6:

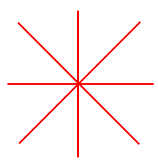
-2	-1	0
-1	0	1
0	1	2

k_7:

-1	-2	-1
0	0	0
1	2	1

k_8:

0	-1	-2
1	0	-1
2	1	0

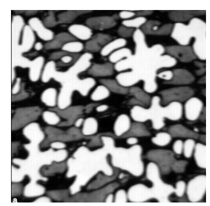


Idea: Take max(p,k_n) for every pixel => insensitive to edge orientation

3. Image processing in the spatial domain

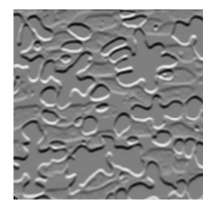
Finding edges

Kirsch operator

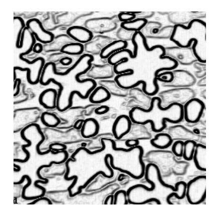


Light micrograph: polished metal with 3 phases
Image Processing Handbook, J. C. Russ

Original



Vertical derivative



Kirsch image

3. Image processing in the spatial domain

Finding edges

Sobel operator

- Magnitude:

$$\sqrt{\left(\frac{\partial p(x,y)}{\partial x}\right)^2 + \left(\frac{\partial p(x,y)}{\partial y}\right)^2}$$

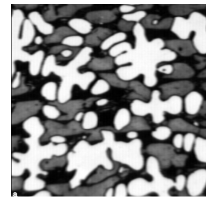
- Direction:

$$\arctan\left(\frac{\partial p(x,y)}{\partial y} \Big/ \frac{\partial p(x,y)}{\partial x}\right)$$

3. Image processing in the spatial domain

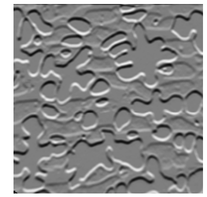
Finding edges

Sobel operator

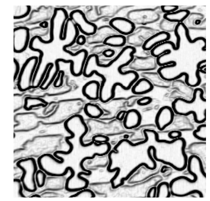


Light micrograph: polished metal with 3 phases
Image Processing Handbook, J. C. Russ

Original



Vertical derivative

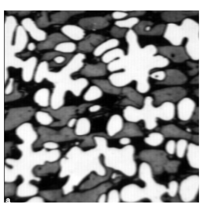


Sobel magnitude

3. Image processing in the spatial domain

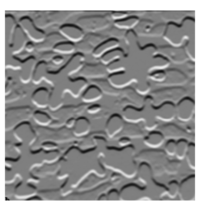
Finding edges

Sobel operator

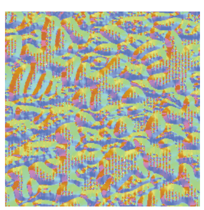


Light micrograph: polished metal with 3 phases
Image Processing Handbook, J. C. Russ

Original



Vertical derivative

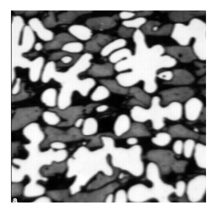


Sobel direction

3. Image processing in the spatial domain

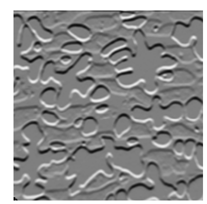
Finding edges

Sobel operator

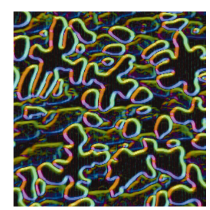


Light micrograph: polished metal with 3 phases
Image Processing Handbook, J. C. Russ

Original



Vertical derivative



Sobel orientation & magnitude

3. Image processing in the spatial domain

Finding edges

Frei and Chen operator

Edge:

$$fc_1 = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad fc_2 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Ripple:

$$fc_3 = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \quad fc_4 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

Line:

$$fc_5 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \quad fc_6 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

Point:

$$fc_7 = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} \quad fc_8 = \begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$

No structure:

$$fc_9 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

3. Image processing in the spatial domain

Finding edges

Frei and Chen operator

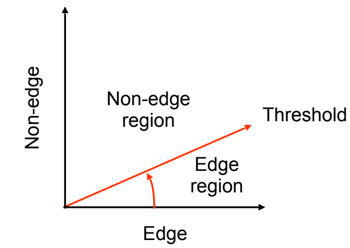
- Edge/Non-wedge weights:

$$E = \sum_{i=1}^{2} (p, fc_i)^2 \qquad NE = \sum_{i=3}^{9} (p, fc_i)^2$$

- Projection:

$$Edgeness = \arctan\left(\frac{NE}{E}\right)$$

- Advantage:
  Independent of brightness magnitude



3. Image processing in the spatial domain

Finding edges

Frei and Chen operator

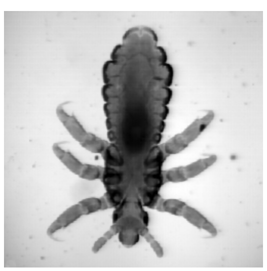
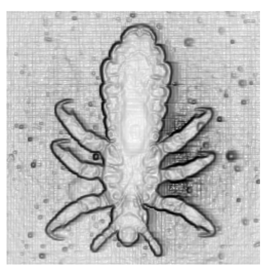


Image Processing Handbook, J. C. Russ

Original



Frei and Chen image (edgeness)

3. Image processing in the spatial domain

Finding edges

Variance operator

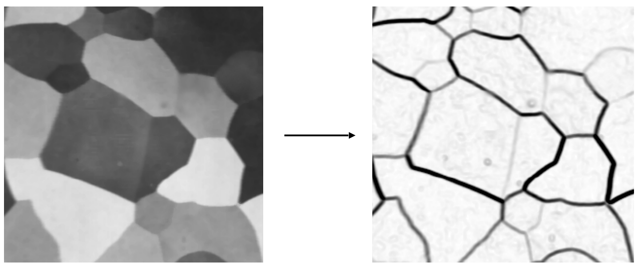
- Edge enhancement operator
- Statistical method
- Local neighborhood operation
- Kernel without weights (only support)
- Magnitude:

$$\sum_{x,y}\left(p(x,y) - \frac{1}{m \cdot n}\sum_{x,y} p(x,y)\right)^2 = \sum_{x,y}(p(x,y) - \mu)^2$$

3. Image processing in the spatial domain

Finding edges

Variance operator



Light micrograph: polished and etched aluminum
Image Processing Handbook, J. C. Russ

Original Variance image

4. Image processing in frequency space

The Fourier transform

Fourier theorem

- Fourier theorem:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{-2\pi iux} dx$$

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{2\pi iux} du$$

- Euler's formula:

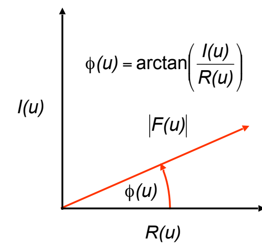
$$e^{2\pi iux} = cos(2\pi ux) + i \cdot sin(2\pi ux)$$

- Polar coordinates:

$$F(u) = R(u) + i \cdot I(u) = |F(u)| \cdot e^{i\phi(u)}$$

- Power spectrum/spectral density:

$$|F(u)|^2 = R(u)^2 + I(u)^2$$

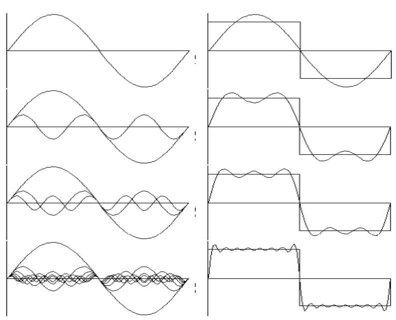


4. Image processing in frequency space

The Fourier transform

Example

Frequency terms Summed terms & step function



1 term

2 terms

3 terms

7 terms

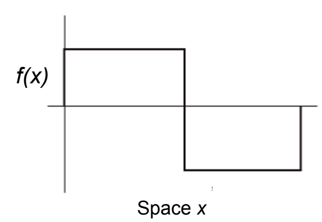
Image Processing Handbook, J. C. Russ

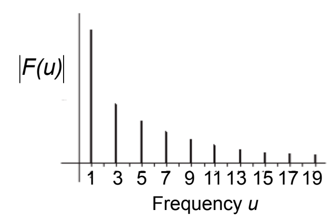
4. Image processing in frequency space

The Fourier transform

Duality of the Fourier transform

Spatial domain Frequency domain





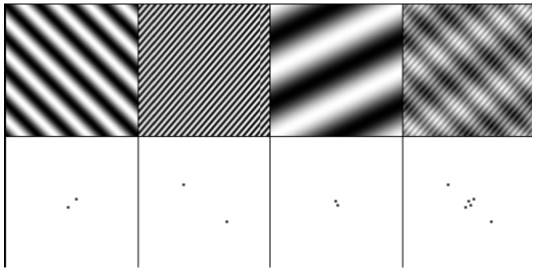
Fourier transform

Inverse Fourier transform

4. Image processing in frequency space

Power spectrum

Example



Spatial domain

Power spectrum

Image Processing Handbook, J. C. Russ

Sinusoid + Sinusoid + Sinusoid = Sum

4. Image processing in frequency space

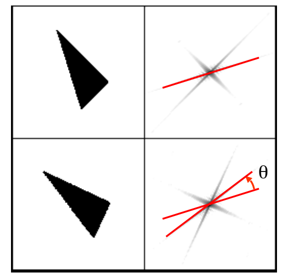
Power spectrum

Example

Spatial domain

Power spectrum

Rotation θ



Rotation θ of an image
<=>
Rotation θ of Fourier transform

Image Processing Handbook, J. C. Russ

4. Image processing in frequency space

Power spectrum

Example

Power spectrum

Spatial domain

Line profiles (spatial domain)

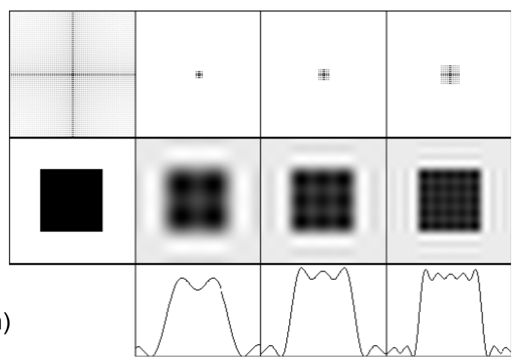


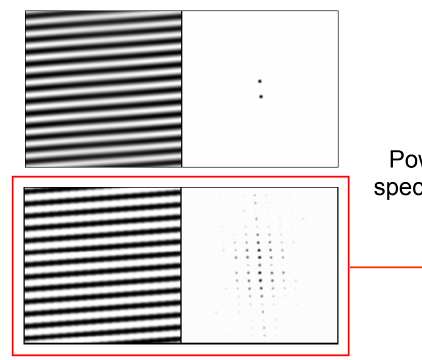
Image Processing Handbook, J. C. Russ

4. Image processing in frequency space

Power spectrum

Aliasing

Spatial domain



Power spectrum

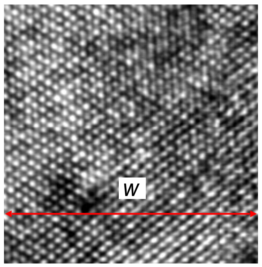
Aliasing

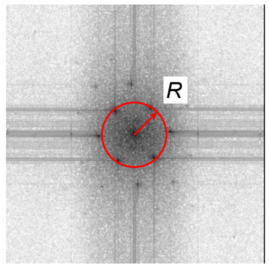
Image Processing Handbook, J. C. Russ

4. Image processing in frequency space

Spacing and orientation

Spacing





TEM: atomic lattice in silicon
Image Processing Handbook, J. C. Russ

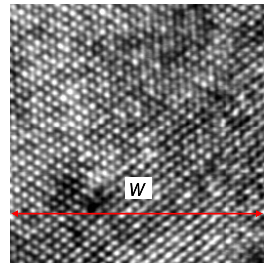
Spatial domain

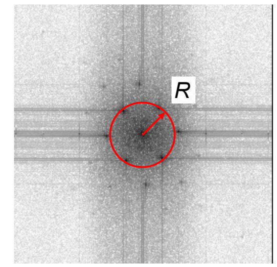
Power spectrum
<=>
Diffraction pattern

4. Image processing in frequency space

Spacing and orientation

Spacing





TEM: atomic lattice in silicon
Image Processing Handbook, J. C. Russ

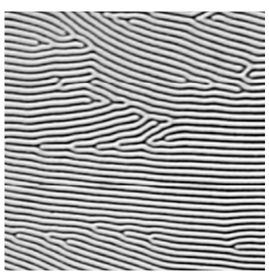
Spatial domain

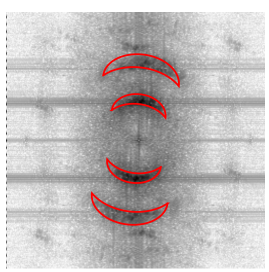
=> Atomic spacing $\propto \frac{W}{R}$

4. Image processing in frequency space

Spacing and orientation

Orientation





Polarized light micrograph: magnetic domains
Image Processing Handbook, J. C. Russ

=> Preferred orientation

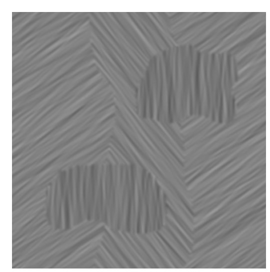
Spatial domain

Power spectrum

4. Image processing in frequency space

Spacing and orientation

Orientation



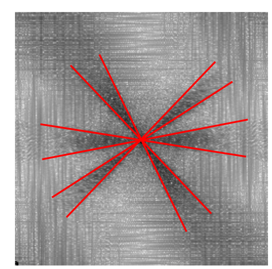


Image Processing Handbook, J. C. Russ

=> Preferred orientations

Spatial domain

Power spectrum

4. Image processing in frequency space

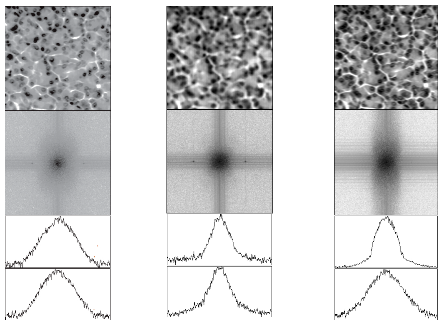
Spacing and orientation

Focus and astigmatism

Spatial domain

Power spectra

Line profiles (power spectrum)



In-focus | Out-of-focus | Astigmatism

Image Processing Handbook, J. C. Russ

4. Image processing in frequency space

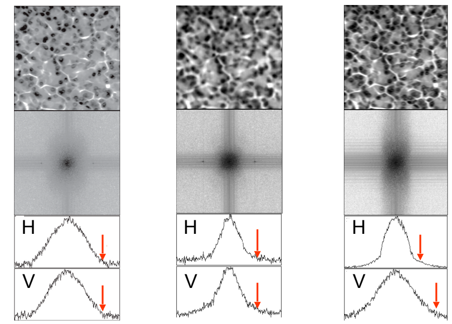
Spacing and orientation

Resolution

Spatial domain

Power spectra

Line profiles (power spectrum)



Optimal resolution | Lower resolution | Astigmatism

Image Processing Handbook, J. C. Russ

4. Image processing in frequency space

Spacing and orientation

Resolution

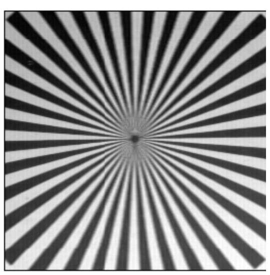
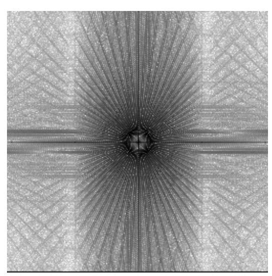


Image Processing Handbook, J. C. Russ

Spatial domain



Power spectrum

4. Image processing in frequency space

Spacing and orientation

Resolution

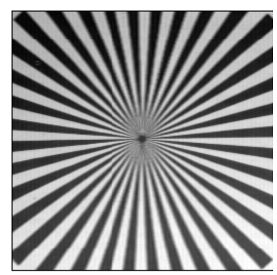
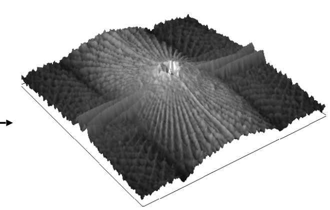


Image Processing Handbook, J. C. Russ

Spatial domain



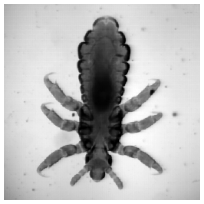
=> Resolution asymmetrical

Power spectrum

4. Image processing in frequency space

Filtering

Low- and high-pass filter



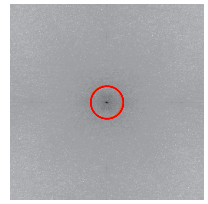
[Image Processing Handbook](), J. C. Russ

Spatial domain

4. Image processing in frequency space

Filtering

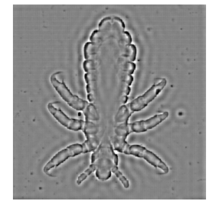
Low- and high-pass filter



[Image Processing Handbook](), J. C. Russ

Power spectrum

Low-pass filtered
(only low frequencies)
<=>
Gaussian smoothing

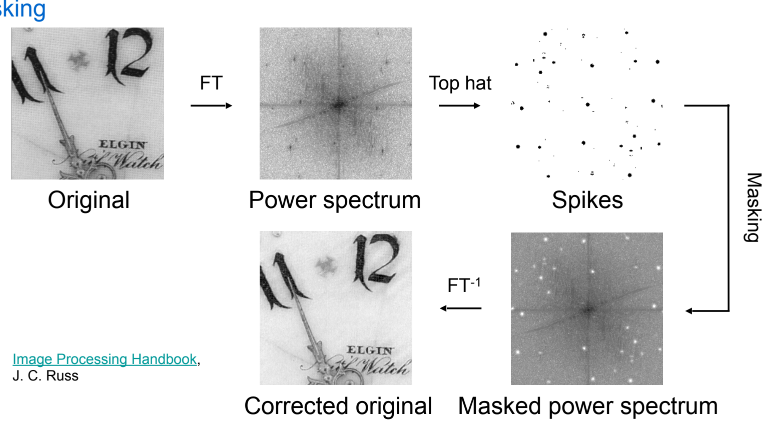


High-pass filtered
(only high frequencies)
<=>
Laplacian sharpening

4. Image processing in frequency space

Filtering

Masking

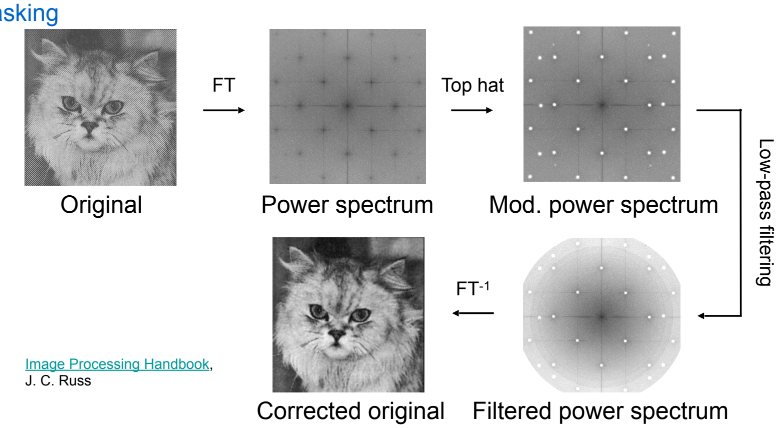


[Image Processing Handbook](),
J. C. Russ

4. Image processing in frequency space

Filtering

Masking

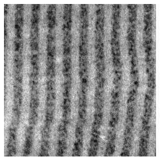


[Image Processing Handbook](),
J. C. Russ

4. Image processing in frequency space

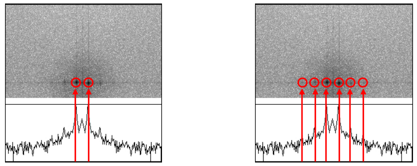
Filtering

Masking

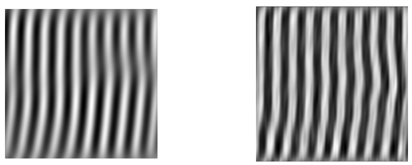


Original

FT →



1st ↓ Power spectra ↓ 1st - 3rd



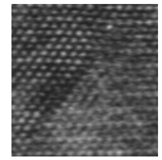
Corrected originals

Light micrograph: stained muscle
Image Processing Handbook,
J. C. Russ

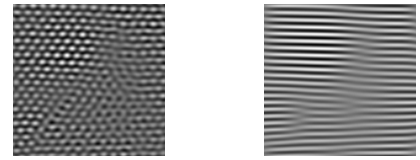
4. Image processing in frequency space

Filtering

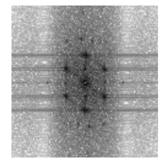
Masking



Original



FT^{-1} ↑ Masked originals ↑ FT^{-1}



Power spectrum

Masking →

Masked power spectra

TEM image of atomic lattice
Image Processing Handbook,
J. C. Russ

4. Image processing in frequency space

Convolution

Kernels

1	1	1
1	1	1
1	1	1

↓ Reduction of blurring

1	2	1
2	4	2
1	2	1

Gaussian kernels

1	4	1
4	12	4
1	4	1

1	2	3	2	1
2	7	11	7	2
3	11	17	11	3
2	7	11	7	2
1	2	3	2	1

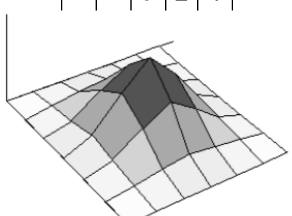


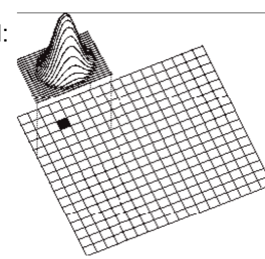
Image Processing Handbook, J. C. Russ

4. Image processing in frequency space

Convolution

Filtering

Gaussian kernel:
$k(x,y)$



Pixel array
(spatial domain)
$p(x,y)$

Filtering/convolution:
$k(x,y) * p(x,y)$

Image Processing Handbook, J. C. Russ

=> Computationally expensive for big images
=> Better to perform Fourier transform first

4. Image processing in frequency space

Convolution

Convolution theorem

$FT(p*k) = FT(p) \cdot FT(k)$ The Fourier transform of a convolution is the product of the Fourier transforms.

=> Convolution implementation (e.g. for filtering):

$$\boxed{p*k = FT^{-1}\left[FT(p) \cdot FT(k)\right]}$$

Attention: only applicable for linear filters
(e.g. not for histogram modifications or rank filtering, without any frequency-domain equivalent)

4. Image processing in frequency space

Convolution

Example

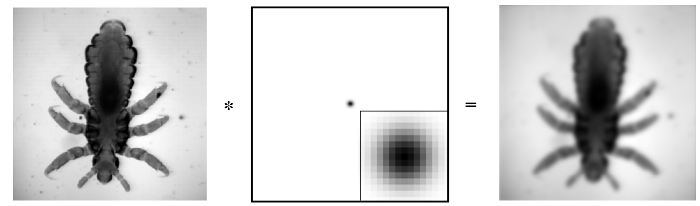


Image Processing Handbook, J. C. Russ

Original Kernel/Filter Convoluted/Filtered

4. Image processing in frequency space

Deconvolution

Point spread function

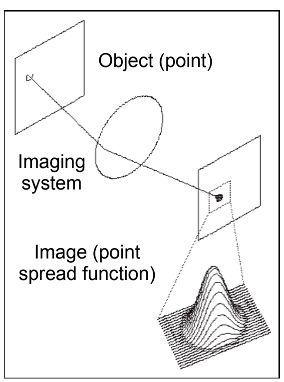


Image Processing Handbook, J. C. Russ

4. Image processing in frequency space

Deconvolution

Convolution theorem

$FT(p*k) = FT(p) \cdot FT(k)$ The Fourier transform of a convolution is the product of the Fourier transforms.

=> Convolution implementation (e.g. for filtering):

$$\boxed{p*k = FT^{-1}\left[FT(p) \cdot FT(k)\right]}$$

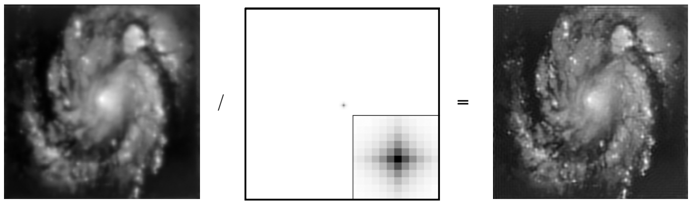
=> Deconvolution implementation:

$$\boxed{p/k = FT^{-1}\left[\frac{FT(p)}{FT(k)}\right]}$$

4. Image processing in frequency space

Deconvolution

Example



Hubble telescope image
Image Processing Handbook, J. C. Russ

Original (out-of-focus) | Kernel/Point spread function | Deconvoluted (in-focus)

4. Image processing in frequency space

Deconvolution

Wiener deconvolution

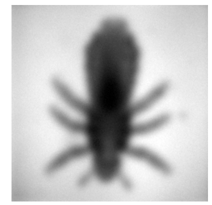
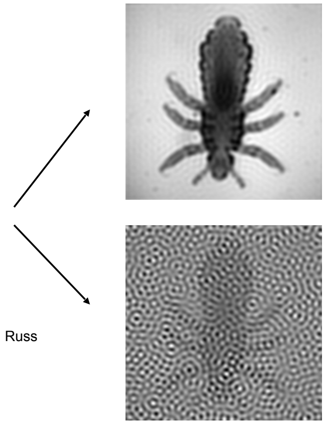


Image Processing Handbook, J. C. Russ

Original (out-of-focus)



Deconvoluted (without noise)

Deconvoluted (with noise)

4. Image processing in frequency space

Deconvolution

Wiener deconvolution

$FT(p * k) = FT(p) \cdot FT(k)$ The Fourier transform of a convolution is the product of the Fourier transforms.

=> Convolution implementation (e.g. for filtering):

$$\boxed{p * k = FT^{-1}\left[FT(p) \cdot FT(k)\right]}$$

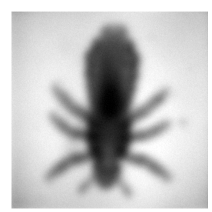
=> Wiener filtering implementation:

$$\boxed{p / k \approx FT^{-1}\left[\frac{FT(p)}{FT(k)} \cdot \frac{\left|FT(k)\right|^2}{\left|FT(k)\right|^2 + const}\right]}$$

4. Image processing in frequency space

Deconvolution

Wiener filtering



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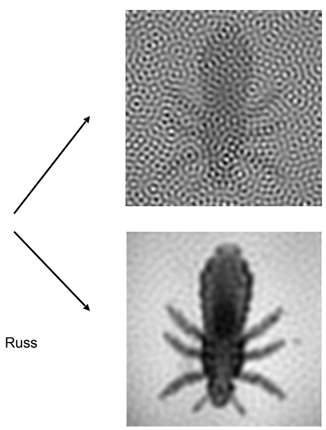
Original (out-of-focus)

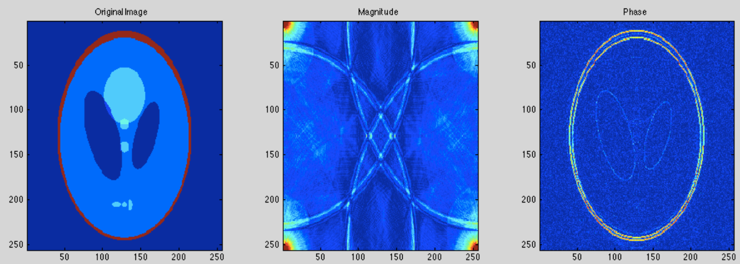


Deconvolved (with noise)

Wiener filtered (with noise)

4. Image processing in frequency space

Inverse FFT / FFT Example

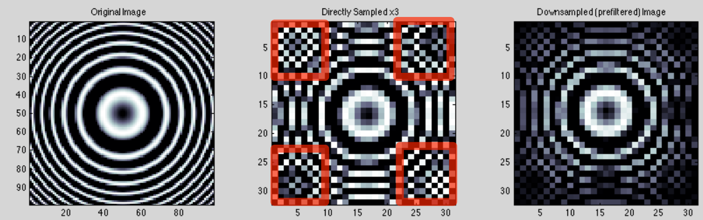


Original | Inverse FFT of FFT with only magnitude | Inverse FFT of FFT with only phase (rescaled)

Phase is much more important for object recognition than amplitude

4. Image processing in frequency space

Aliasing / Moire Pattern Example



Original Image | Directly Downsampled x3 | Prefiltered Downsampling x3

Aliasing shows up when the image is directly downsampled without pre-filtering