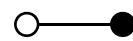


## Fourier Transform

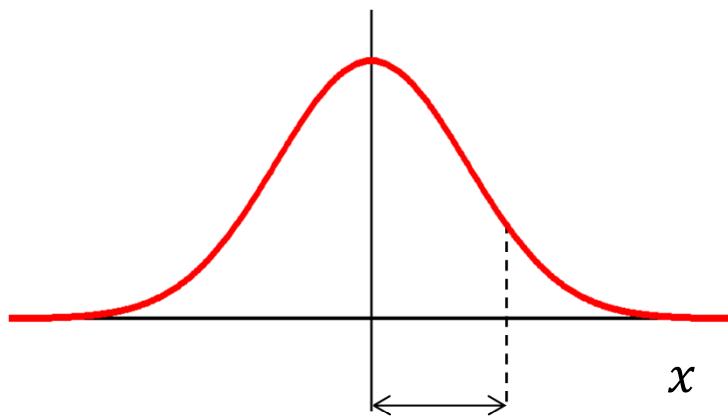
---

$$f(x) = \sqrt{\frac{2\sigma^2}{\pi}} e^{-\frac{\sigma^2 x^2}{2}}$$

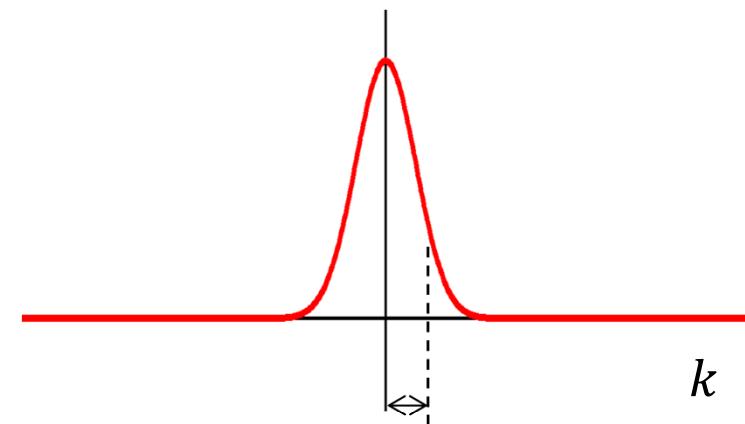


$$F(k) = e^{-\frac{k^2}{2\sigma^2}}$$

Gaussian



Gaussian



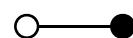
$$\sigma^{-1} = 10 \text{ mm}$$

$$\sigma = 0.1 \text{ mm}^{-1}$$

## Fourier Transform

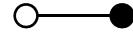
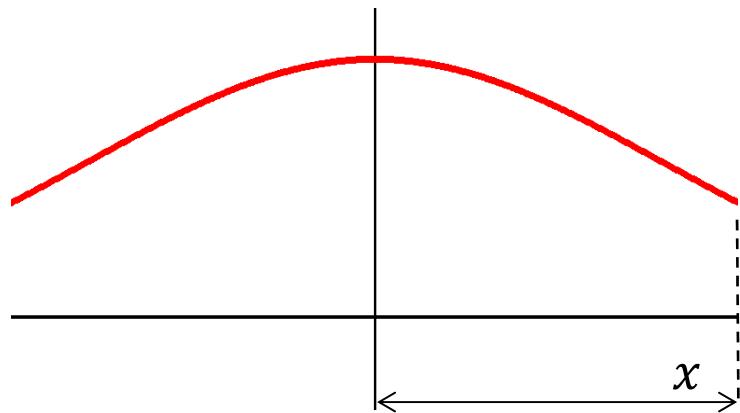
---

$$f(x) = \sqrt{\frac{2\sigma^2}{\pi}} e^{-\frac{\sigma^2 x^2}{2}}$$

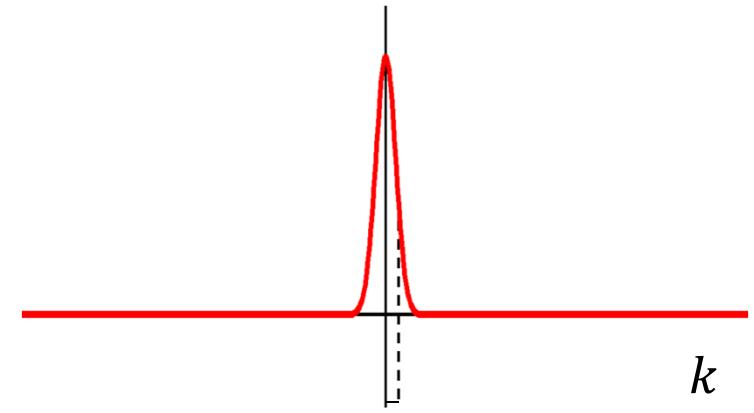


$$F(k) = e^{-\frac{k^2}{2\sigma^2}}$$

Gaussian



Gaussian



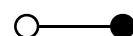
$$\sigma^{-1} = 30 \text{ mm}$$

$$\sigma = 0.033 \text{ mm}^{-1}$$

## Fourier Transform

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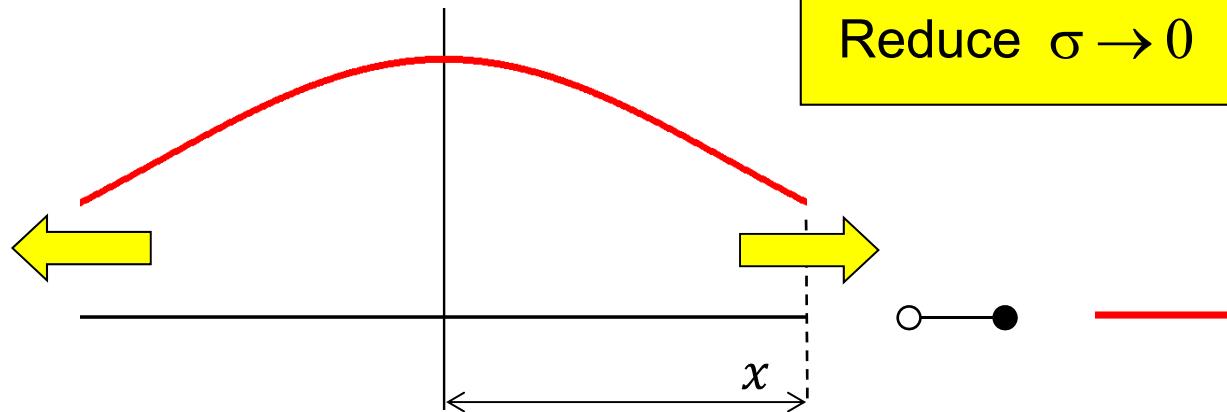
$$f(x) = \sqrt{\frac{2\sigma^2}{\pi}} e^{-\frac{\sigma^2 x^2}{2}}$$



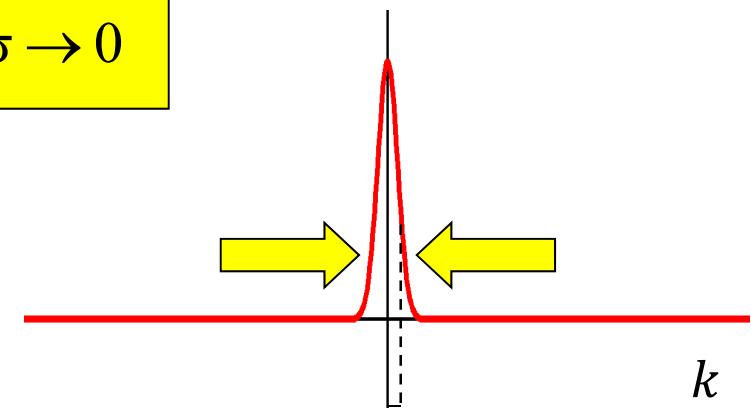
$$F(k) = e^{-\frac{k^2}{2\sigma^2}}$$

Gaussian

Reduce  $\sigma \rightarrow 0$



Gaussian



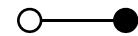
$$\sigma^{-1} = 30 \text{ mm}$$

$$\sigma = 0.033 \text{ mm}^{-1}$$

## Fourier Transform

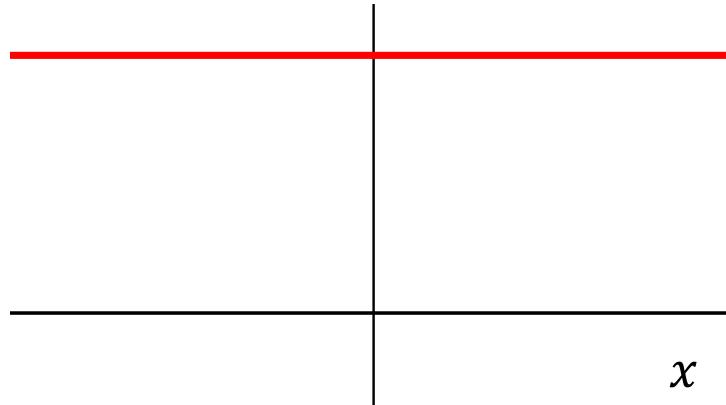
---

$$f(x) = 1$$



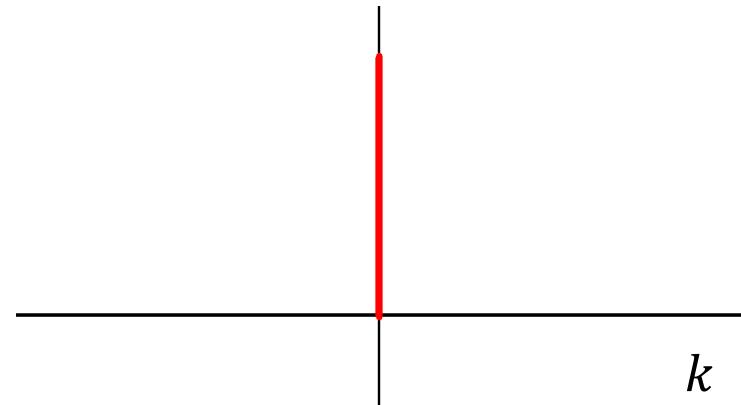
$$F(k) = \delta(k)$$

Constant



Dirac  $\delta$  function  
Unit area:

$$\int_{-\infty}^{+\infty} F(k) dk = 1$$



$$\sigma^{-1} \rightarrow \infty$$

$$\sigma \rightarrow 0$$

# Fourier Transform

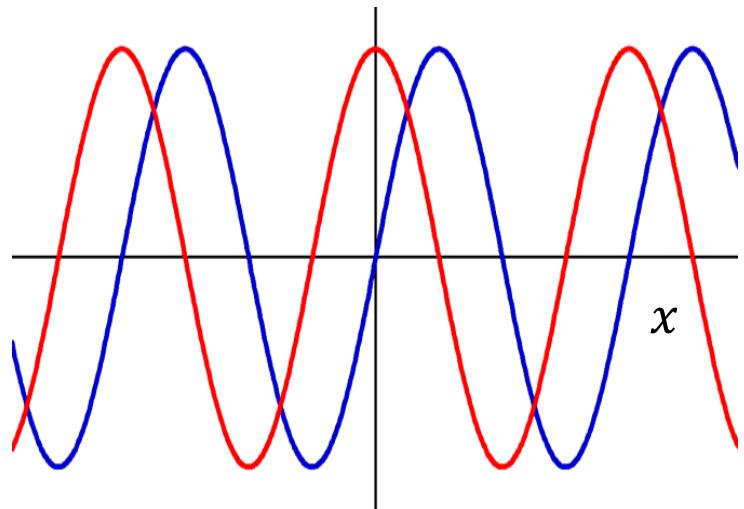
---

$$f(x) = e^{ik_0x}$$

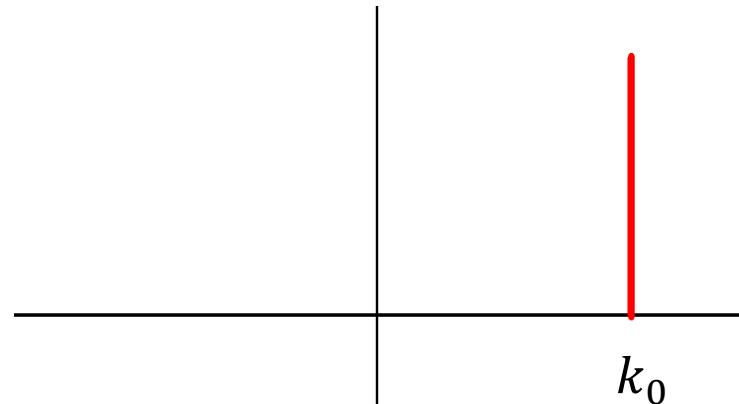


$$F(k) = \delta(k - k_0)$$

Harmonic



Shifted Dirac  $\delta$  function



## Fourier Transform

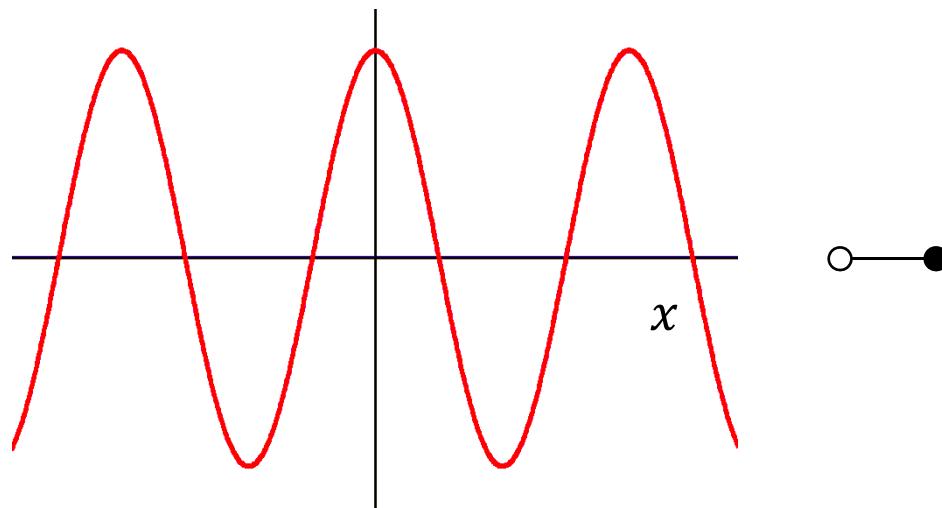
---

$$f(x) = \cos(k_0 x) = \frac{e^{ik_0 x}}{2} + \frac{e^{-ik_0 x}}{2}$$

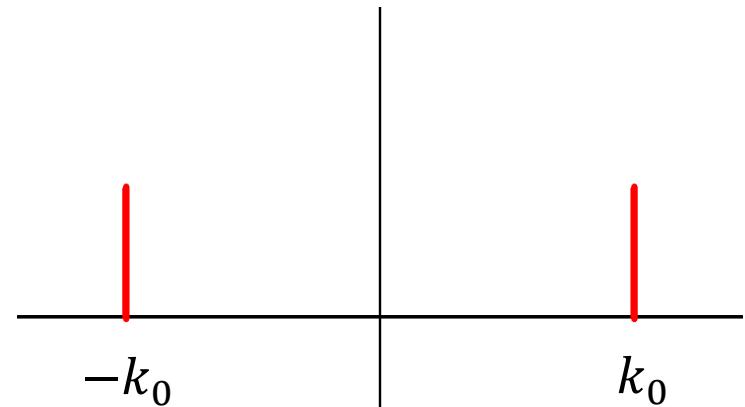


$$F(k) = \frac{\delta(k-k_0)}{2} + \frac{\delta(k+k_0)}{2}$$

Cosine



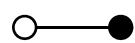
Symmetric pair of  $\delta$  functions



# Fourier Transform

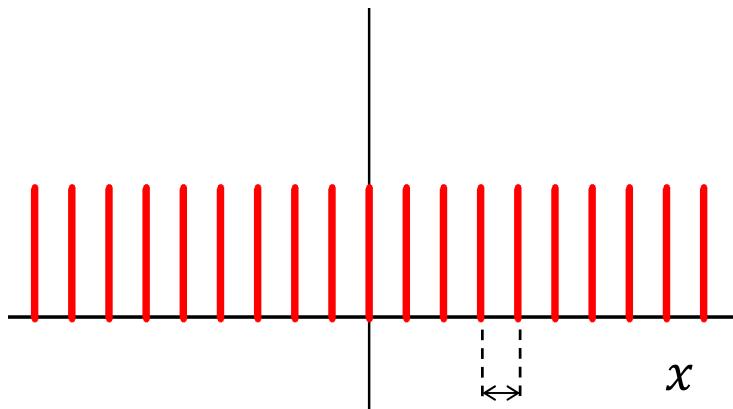
---

$$f(x) = \sum_{n=-\infty}^{\infty} \delta(x - n \Delta x)$$

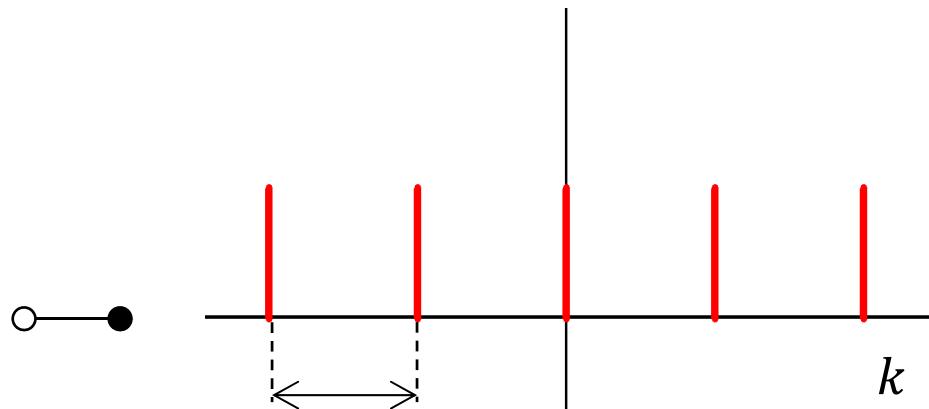


$$F(k) = \sum_{n=-\infty}^{\infty} \delta(k - n \Delta k)$$

Dirac comb (Shah function)



Dirac comb (Shah function)



$$\Delta k = \frac{2\pi}{\Delta x}$$

## Fourier Transform

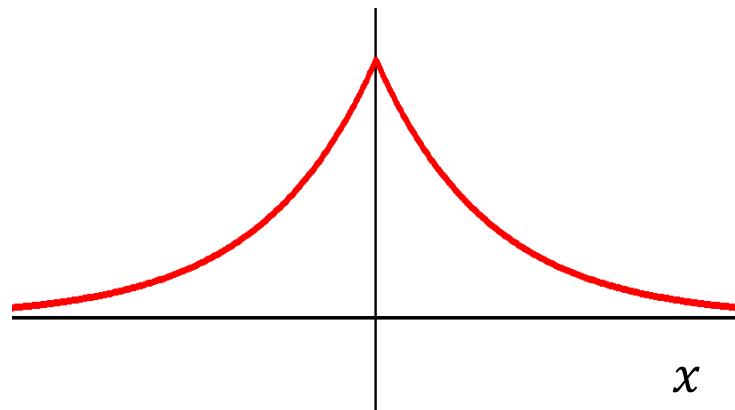
---

$$f(x) = e^{-a|x|}$$

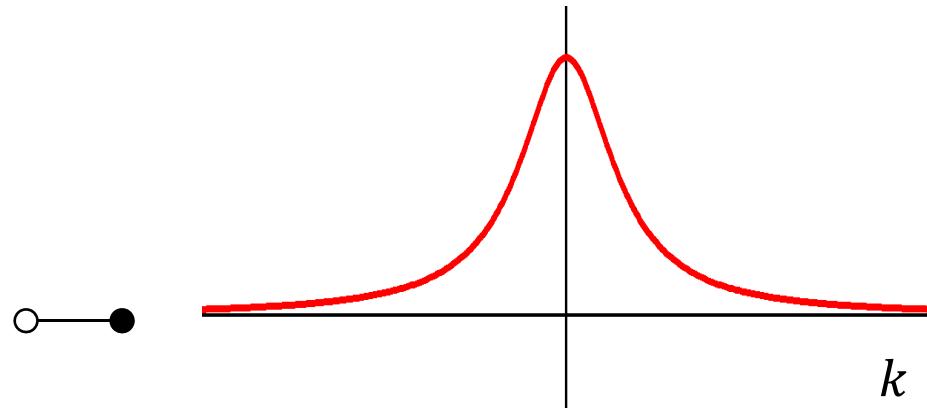


$$F(k) = \frac{2a}{k^2 + a^2}$$

Exponential decay



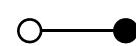
Lorentzian



## Fourier Transform

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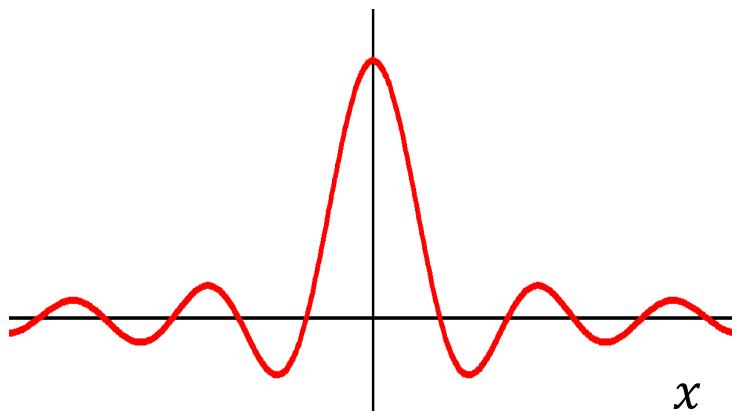
$$f(x) = 2k_c \text{sinc}(k_c t)$$



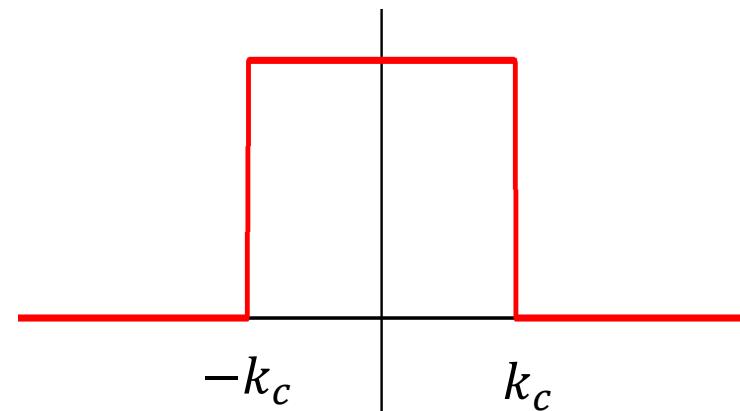
$$F(k) = \begin{cases} 1 & \text{for } |k| \leq k_c \\ 0 & \text{else} \end{cases}$$

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

Sinc

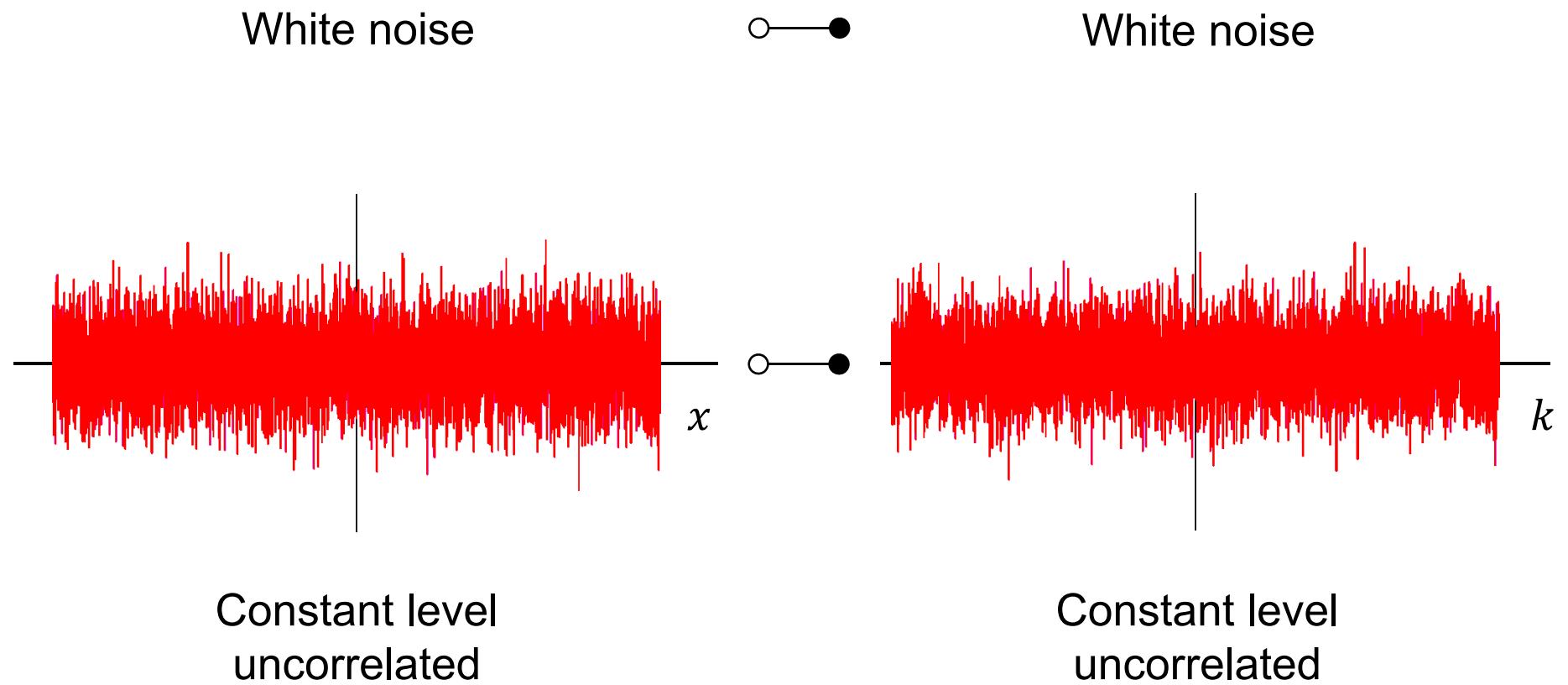


Rectangle



## Fourier Transform

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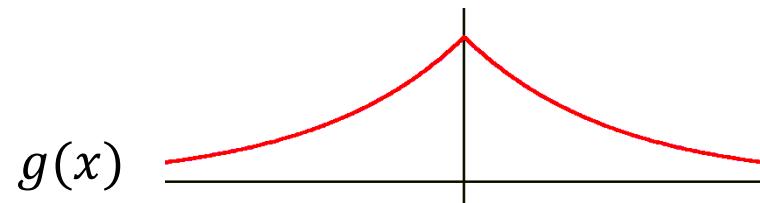


# Convolution Theorem

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Multiplication

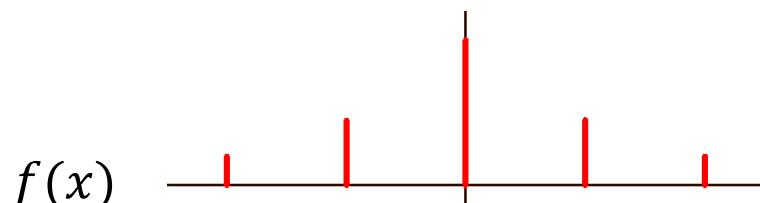
$$f(x) = g(x) \cdot h(x)$$



$\times$

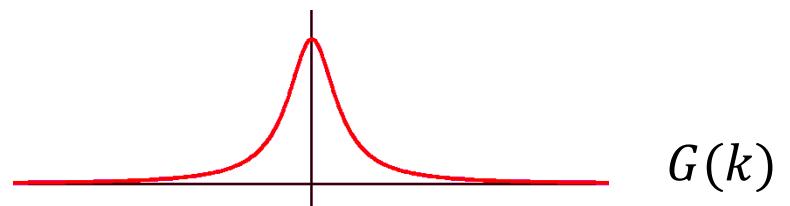
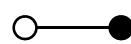


$=$

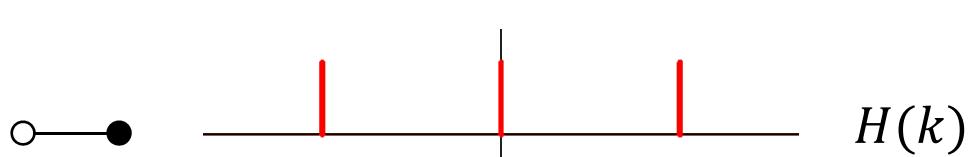


Convolution

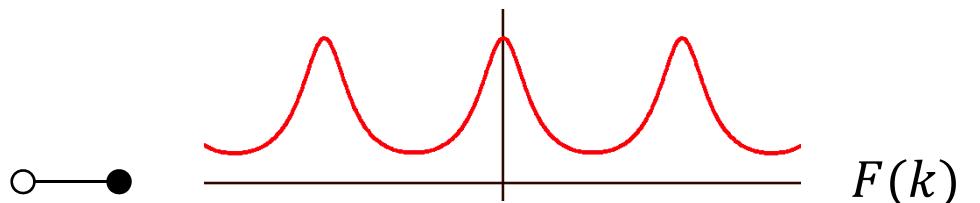
$$F(k) = G(k) * H(k)$$



$*$



$=$

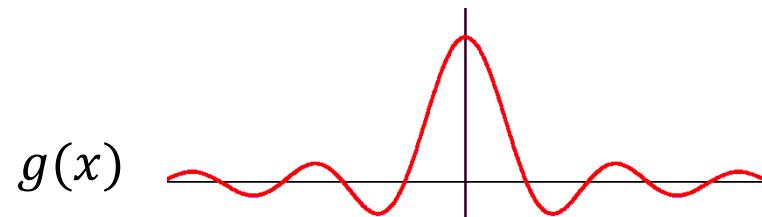


# Convolution Theorem

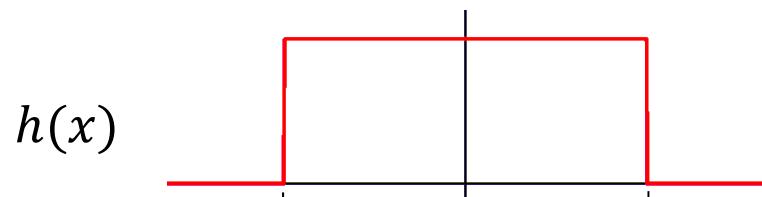
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Multiplication

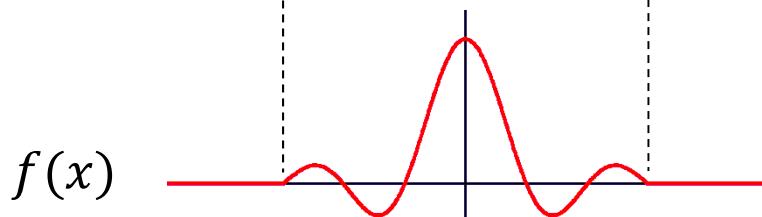
$$f(x) = g(x) \cdot h(x)$$



$\times$

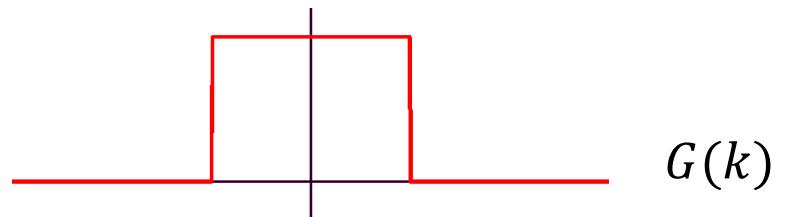
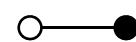


$=$

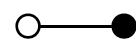
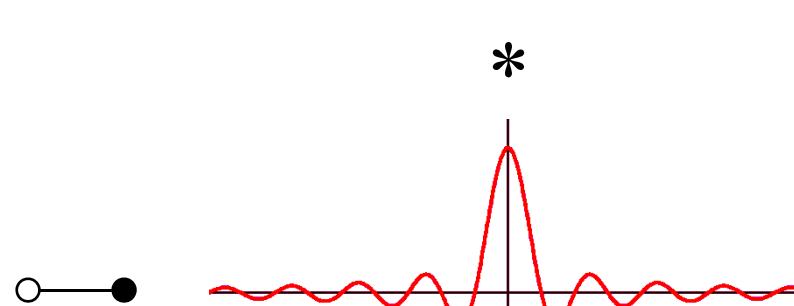


Convolution

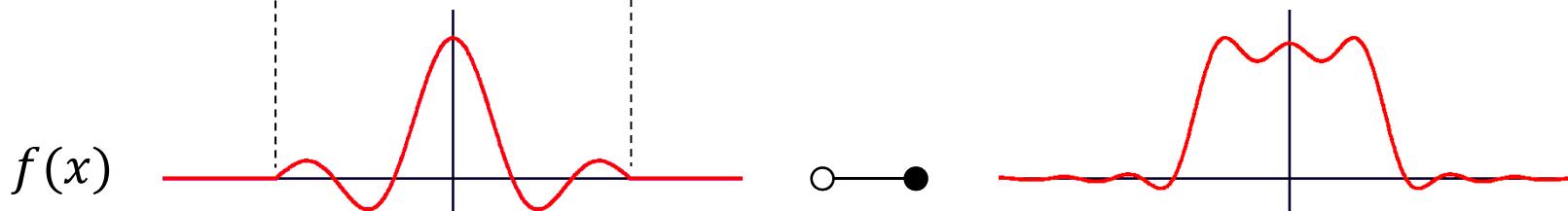
$$F(k) = G(k) * H(k)$$



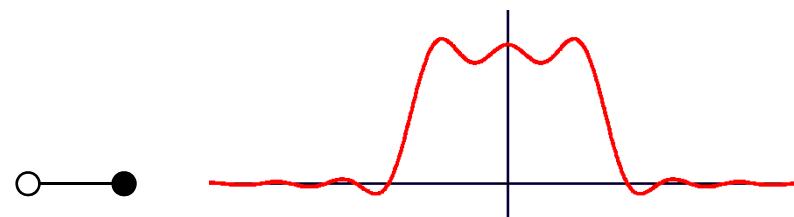
$*$



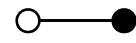
$$H(k)$$



$=$



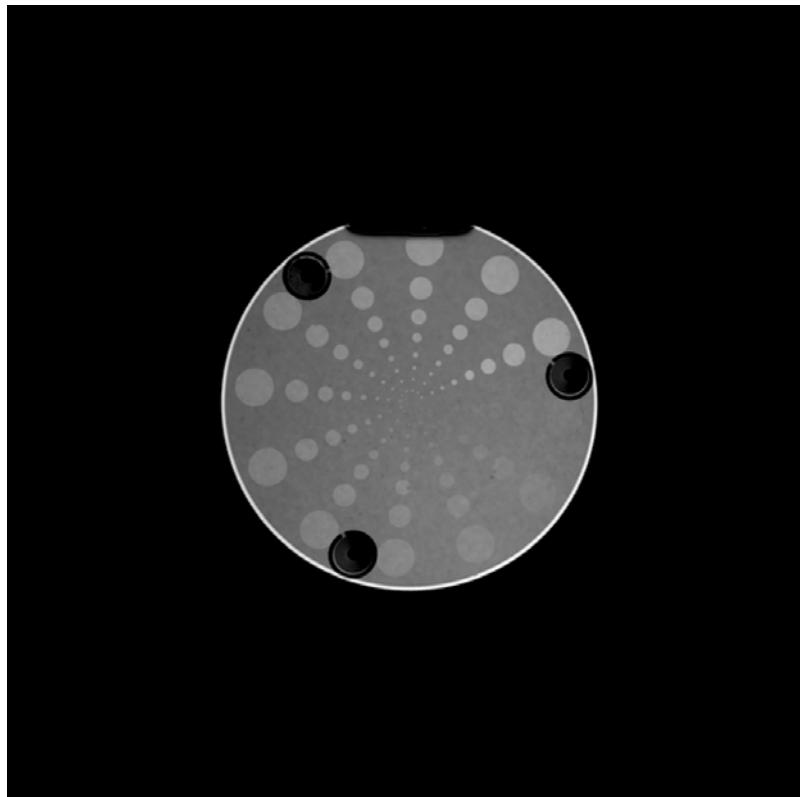
$$F(k)$$



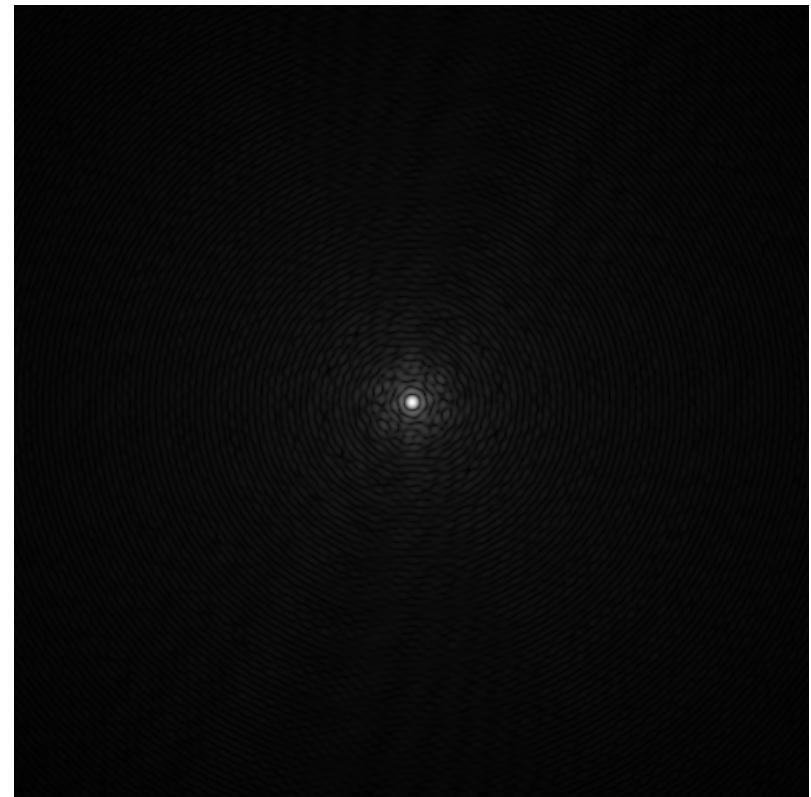
## Sampling k-space

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Image domain



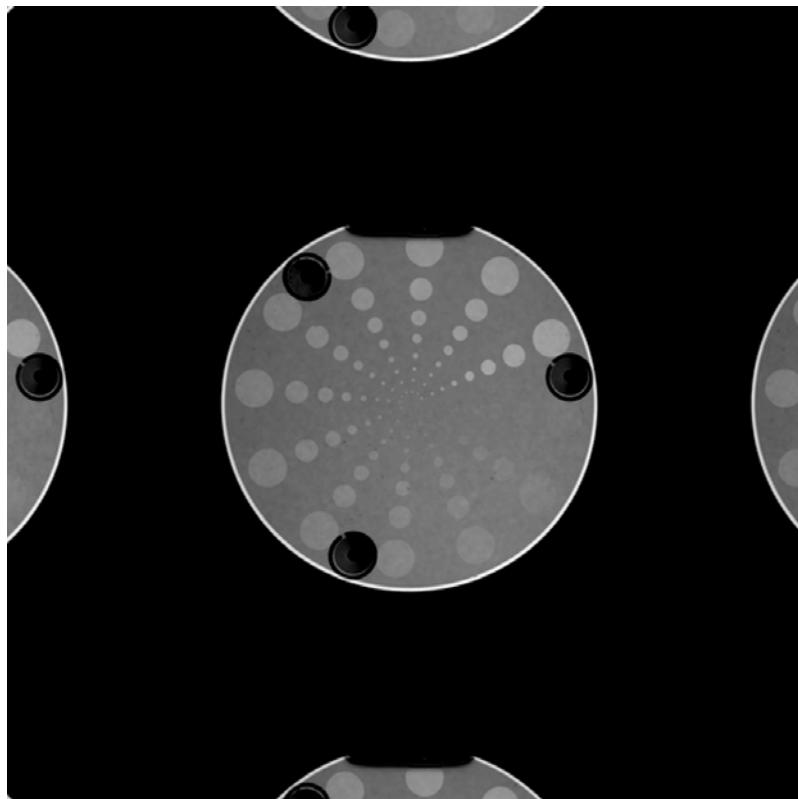
k-Space



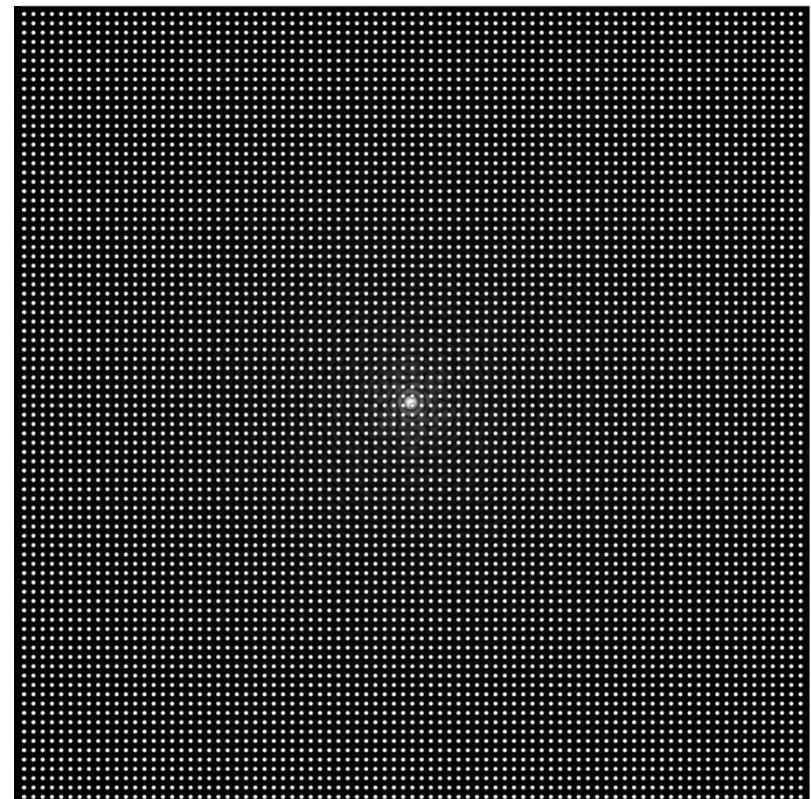
## Sampling k-space

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Image domain



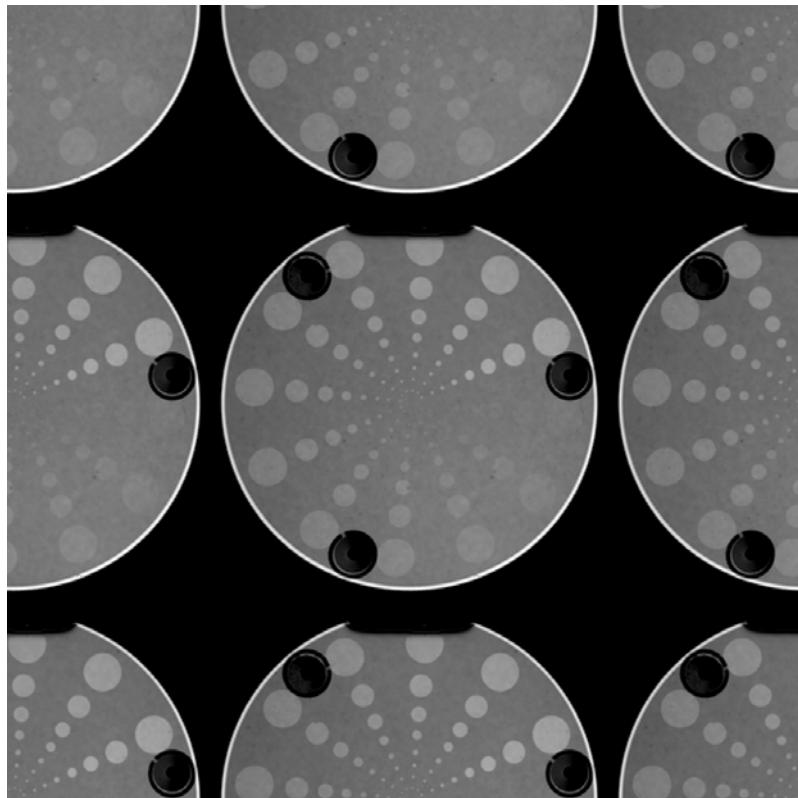
k-Space



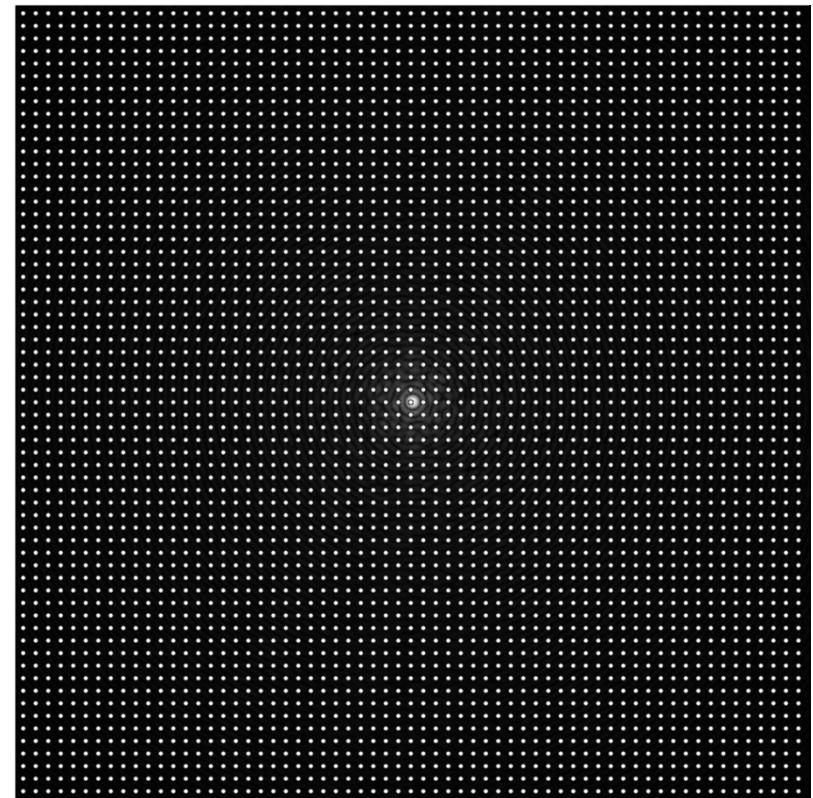
## Sampling k-space

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Image domain



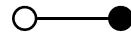
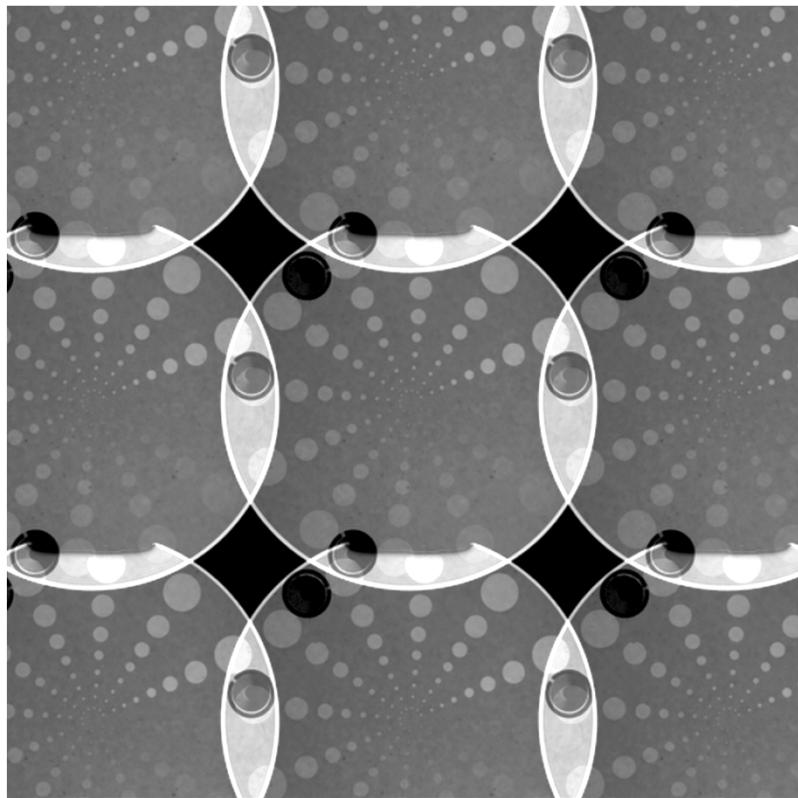
k-Space



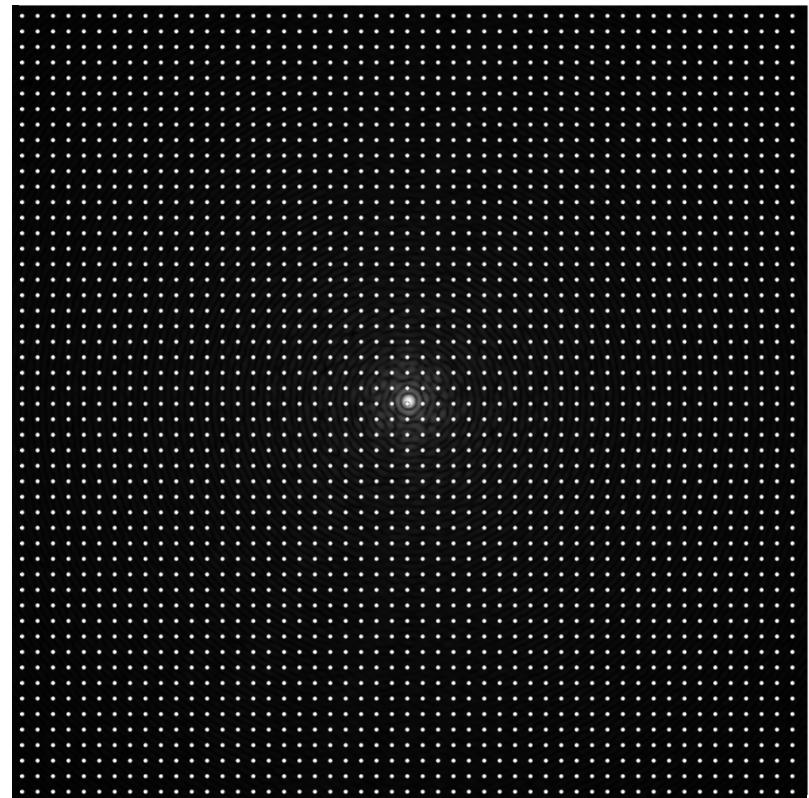
## Sampling k-space

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Image domain



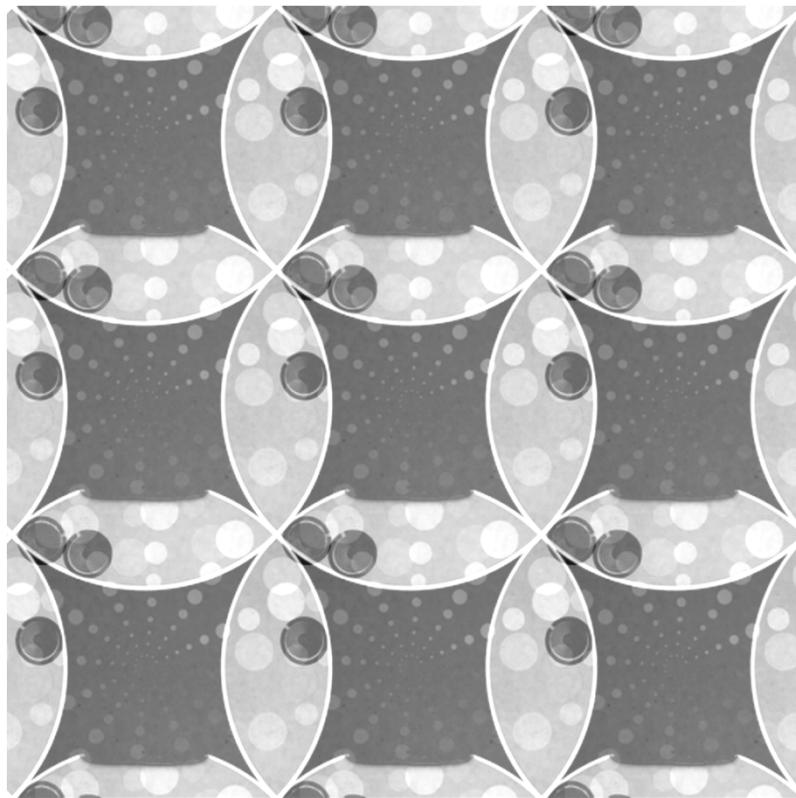
k-Space



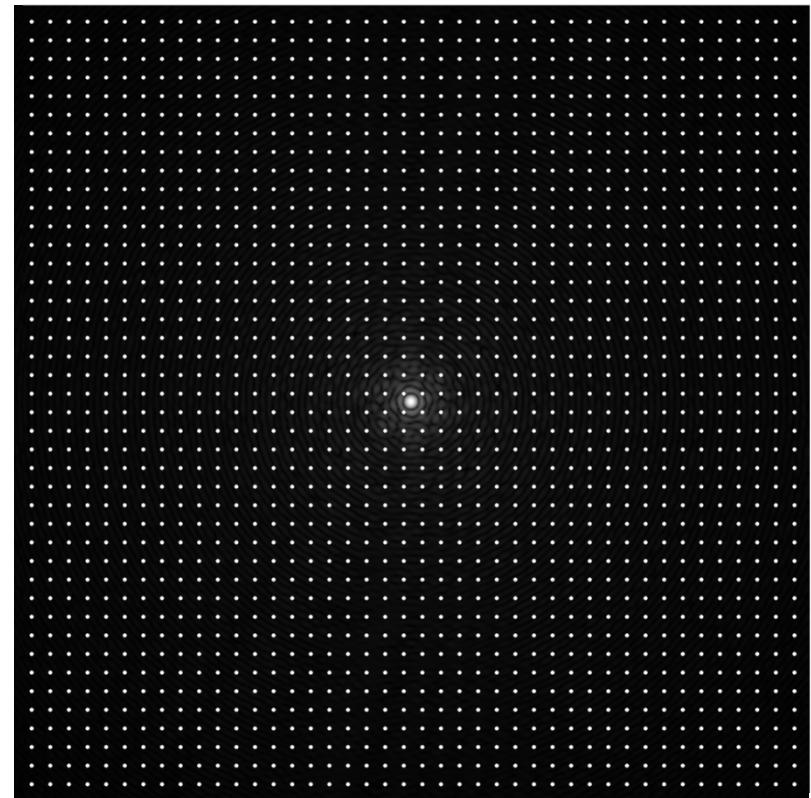
## Sampling k-space

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Image domain



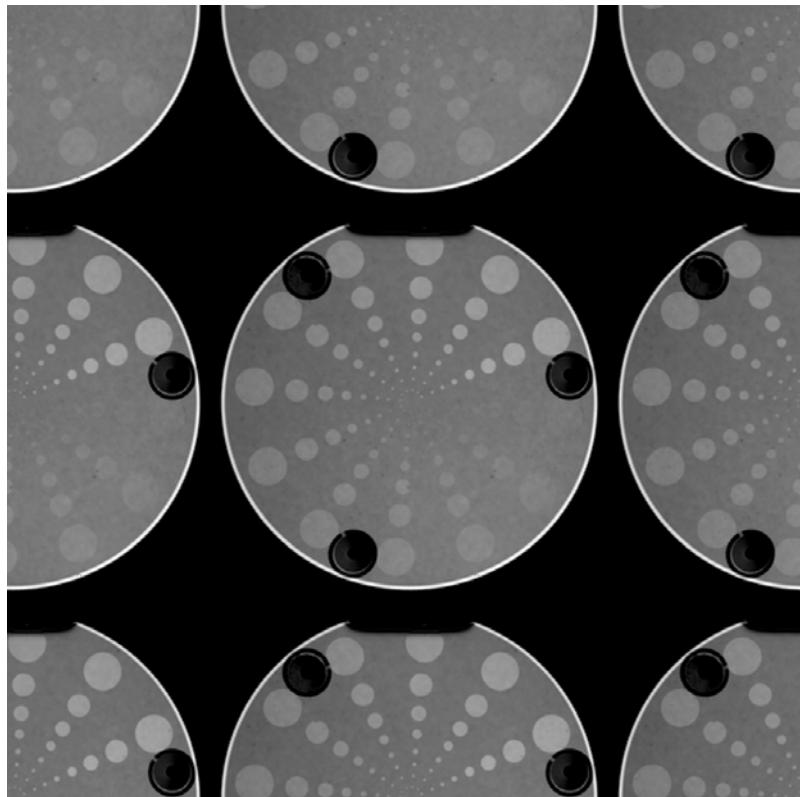
k-Space



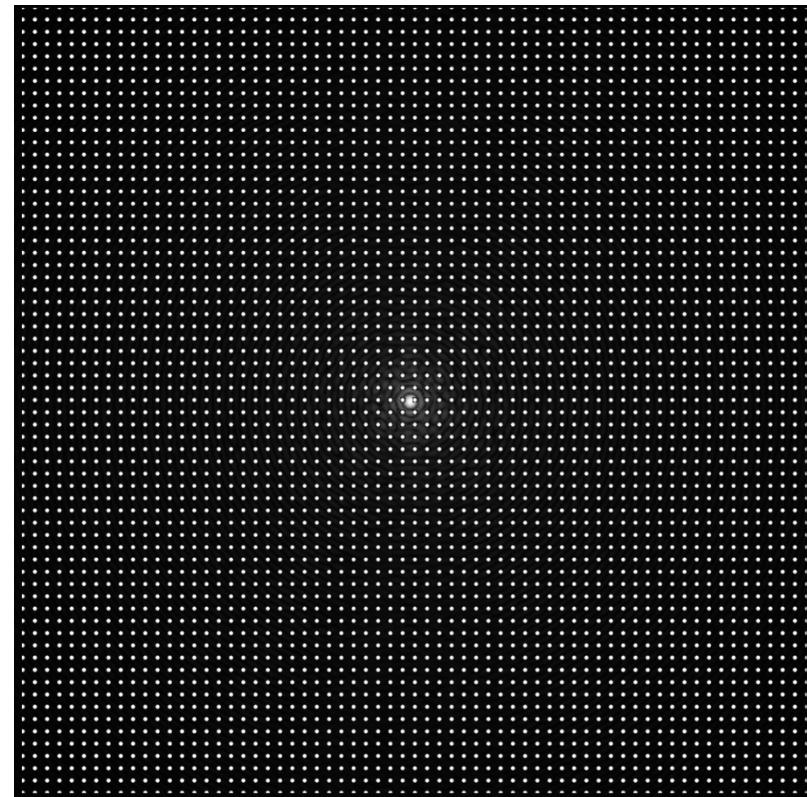
## Sampling k-space

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Image domain



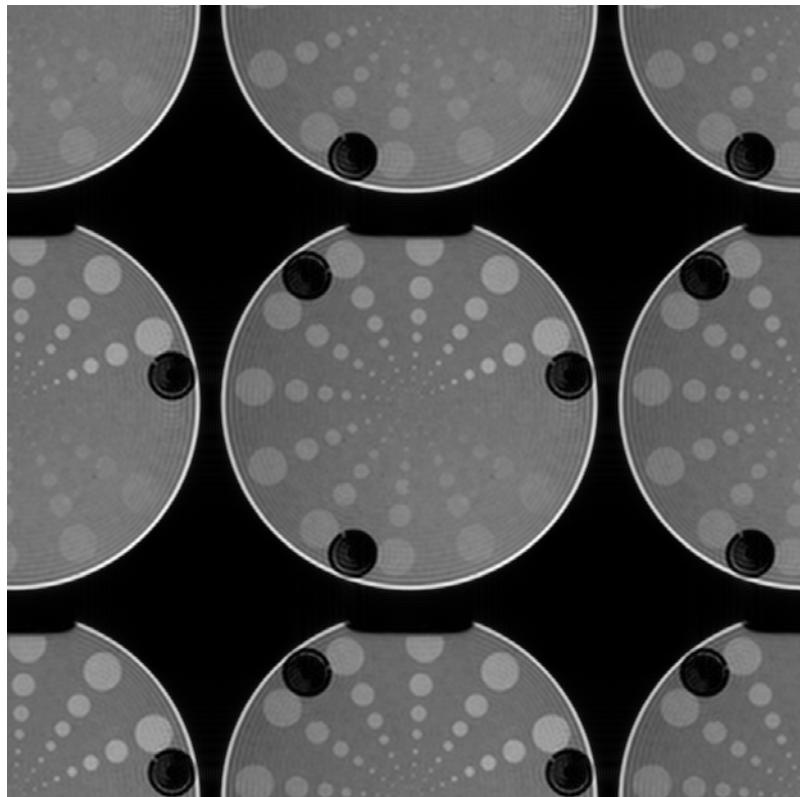
k-Space



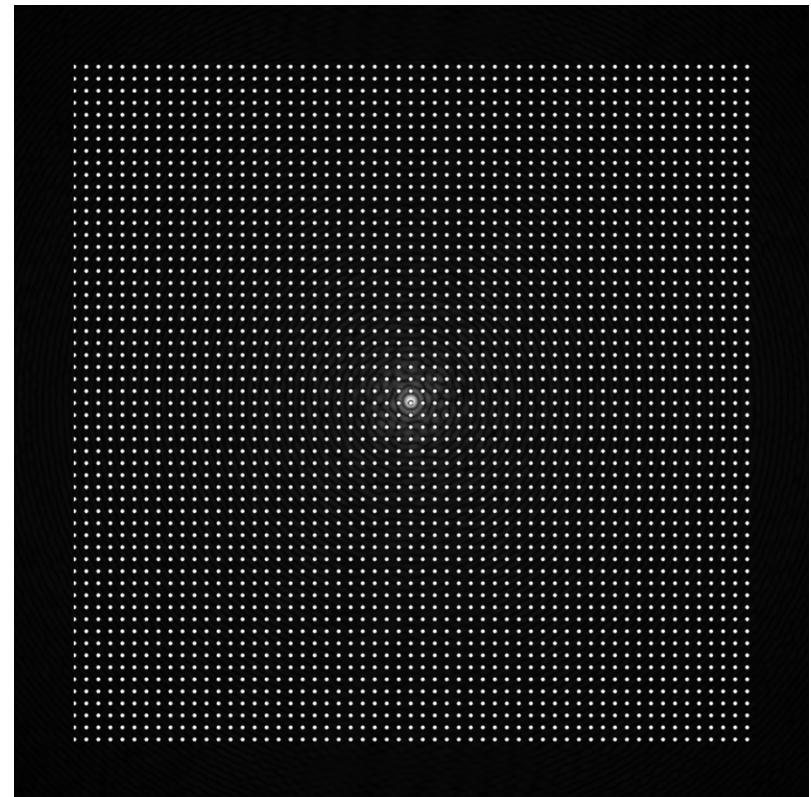
## Sampling k-space

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Image domain



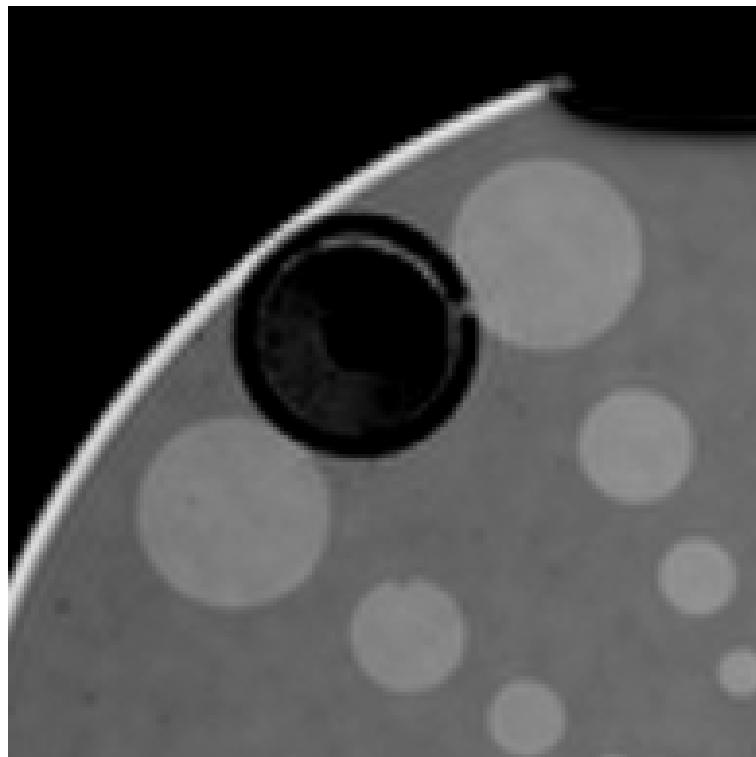
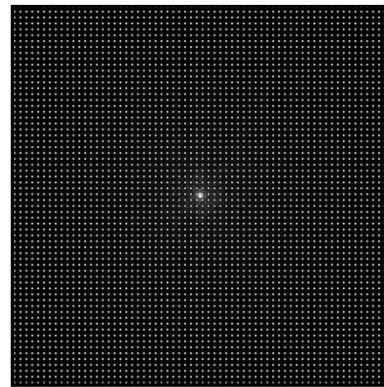
k-Space



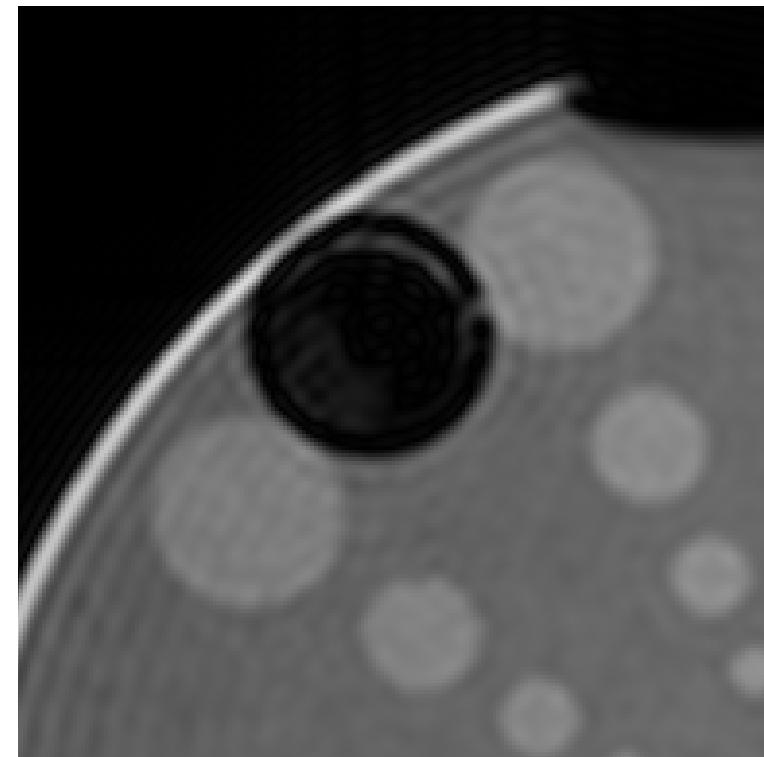
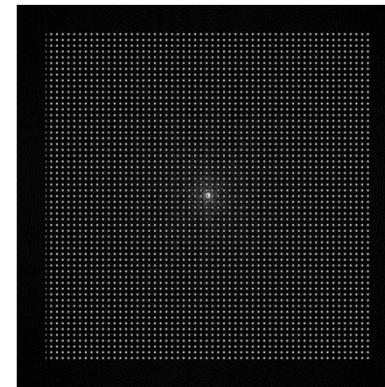
## Sampling k-space

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Infinite extent



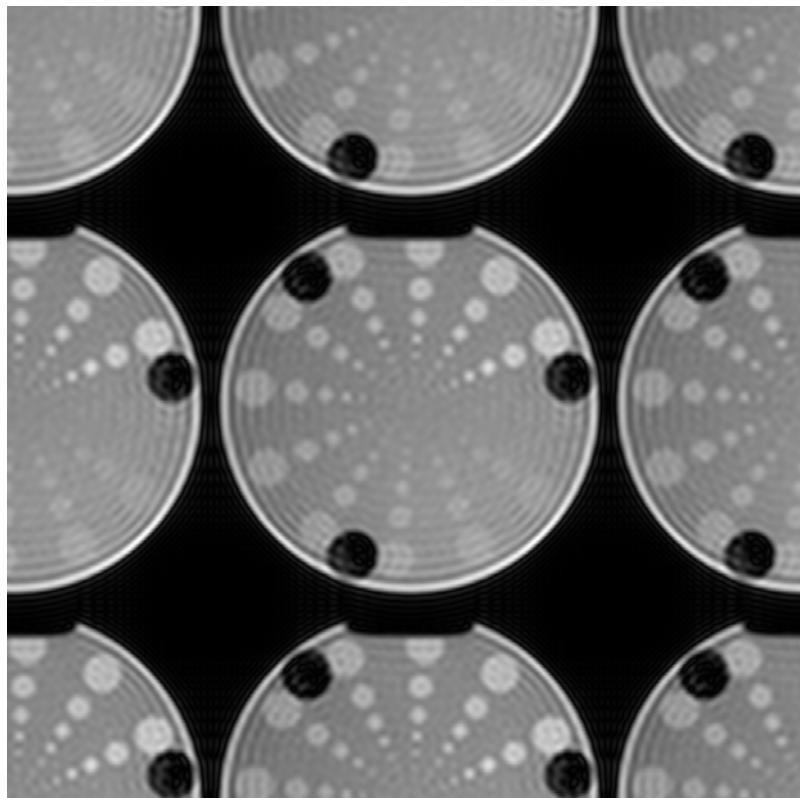
Finite extent



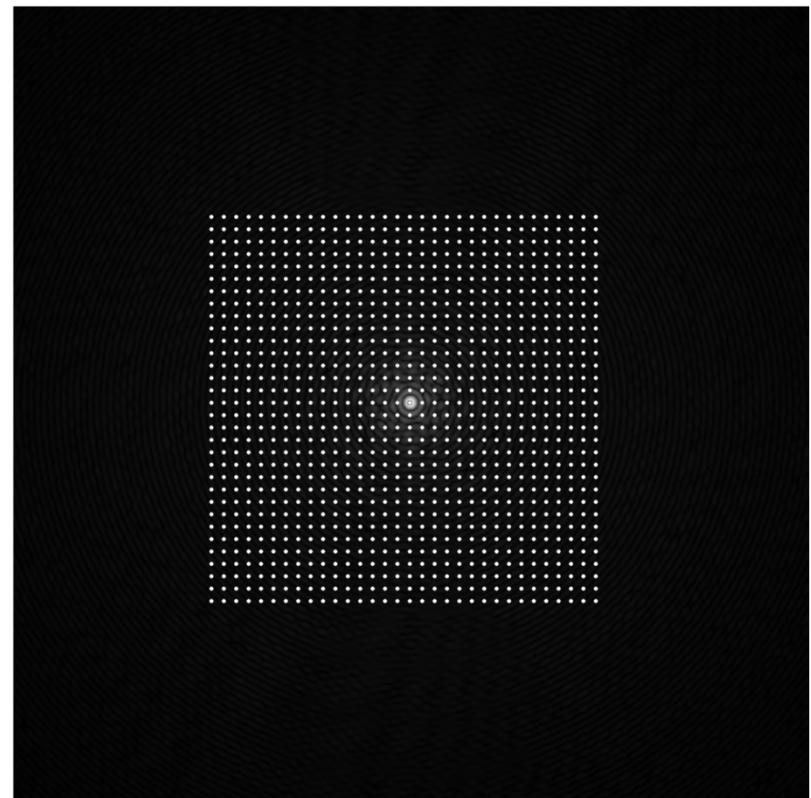
## Sampling k-space

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Image domain



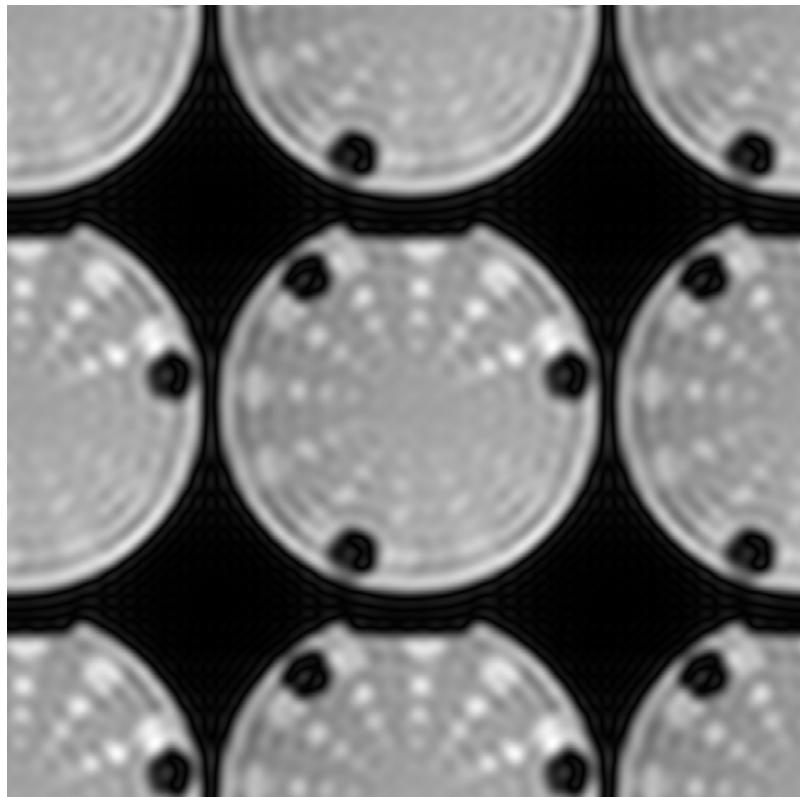
k-Space



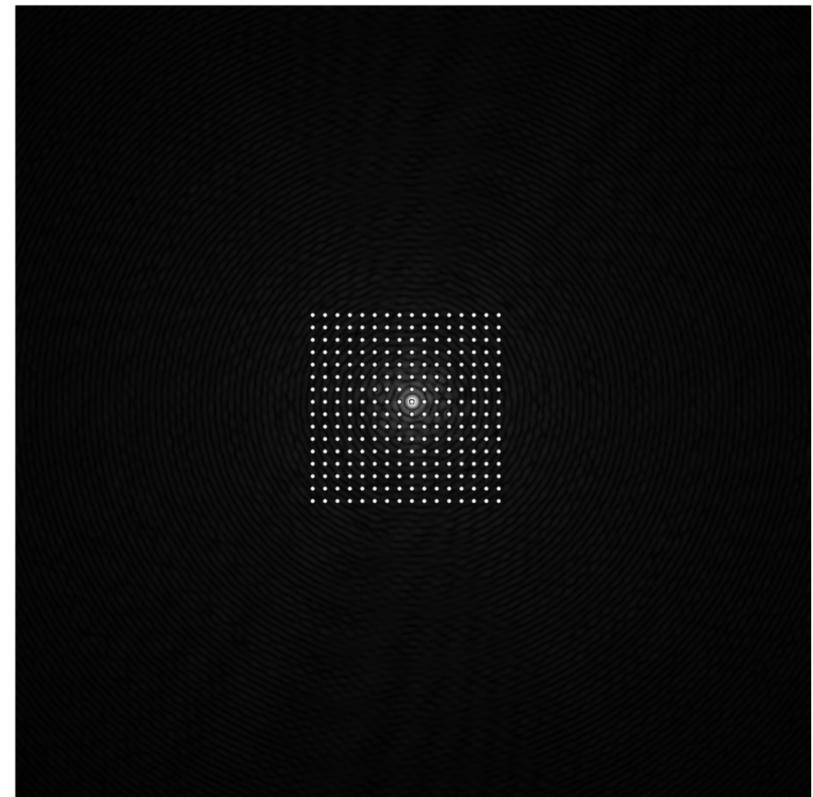
## Sampling k-space

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Image domain



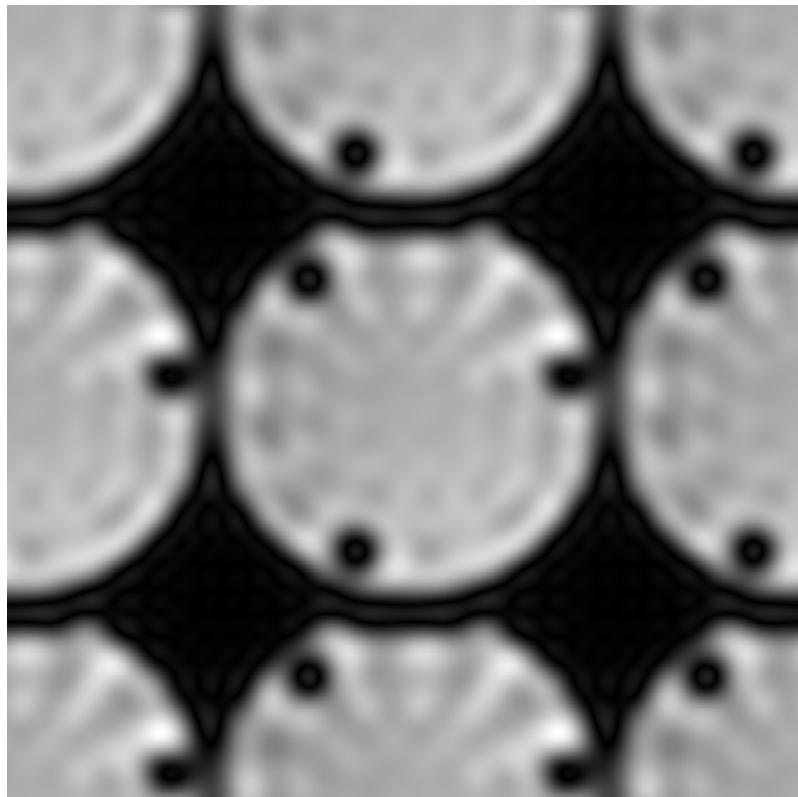
k-Space



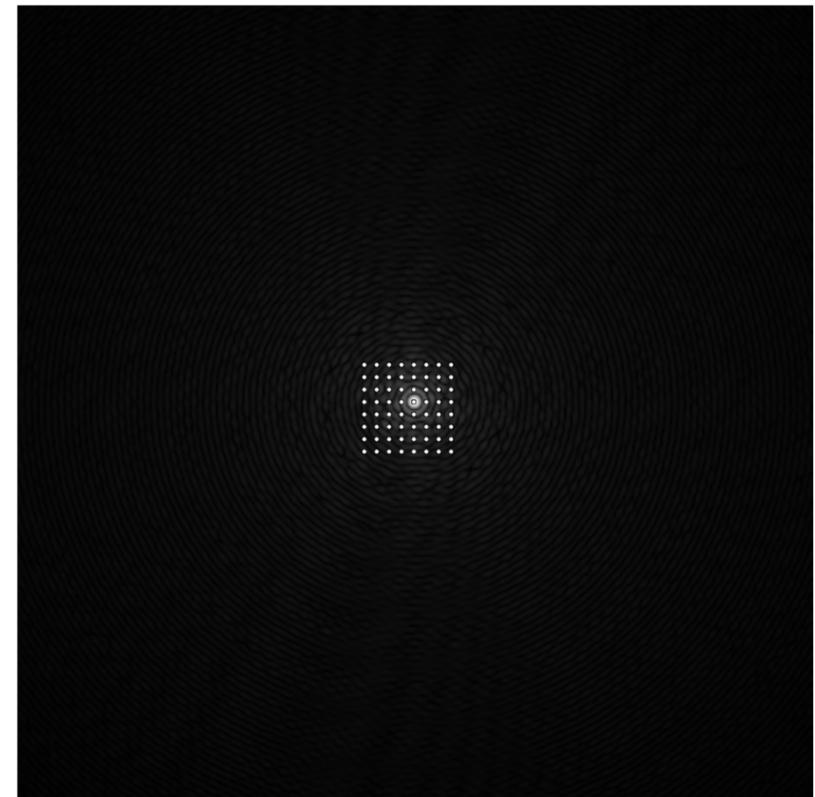
## Sampling k-space

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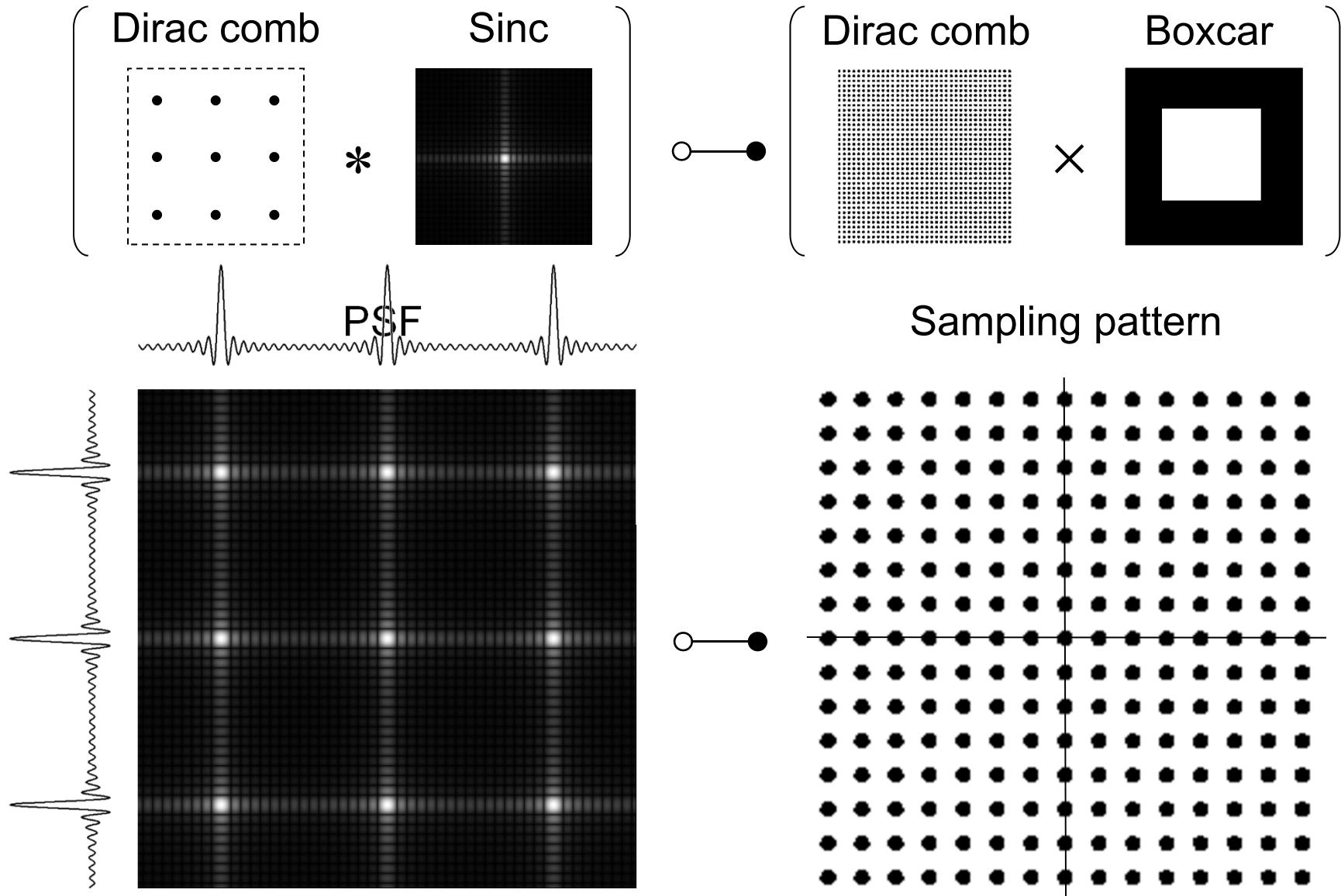
Image domain



k-Space

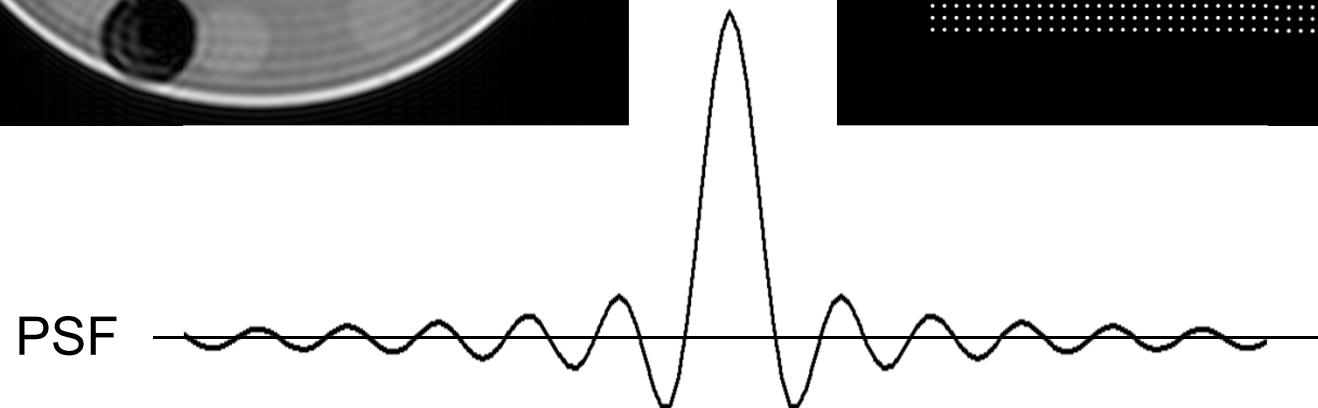
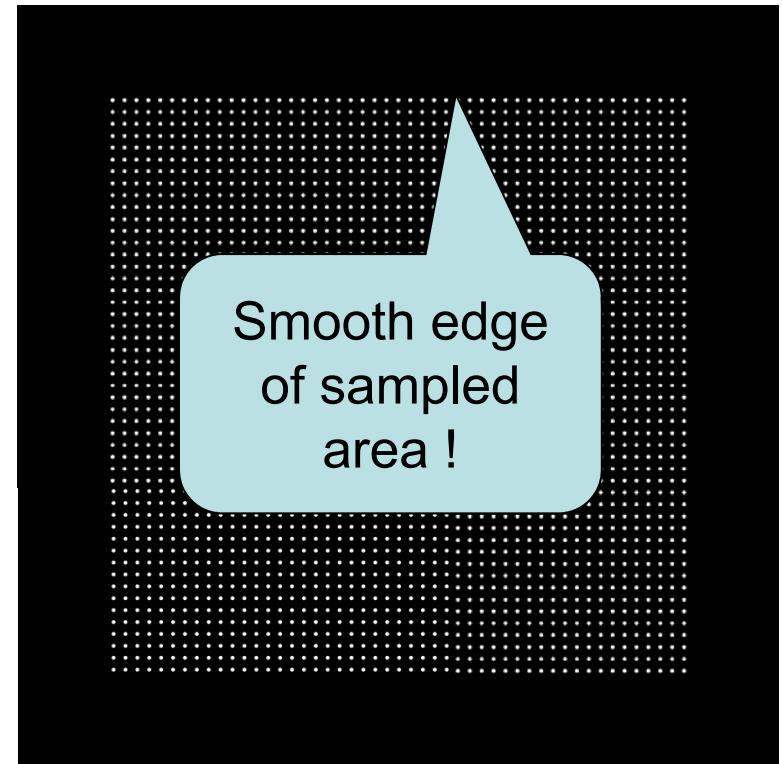
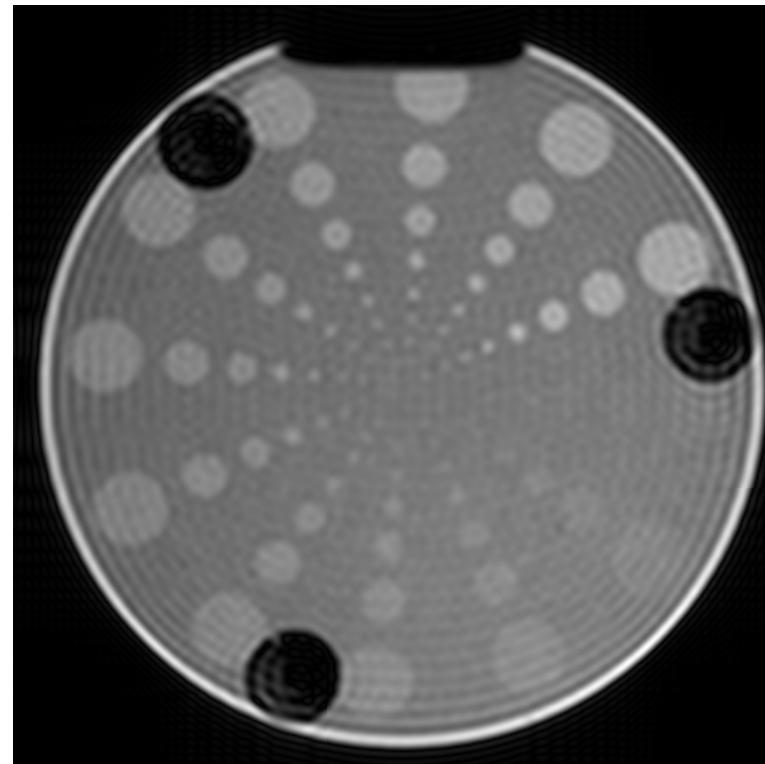


## PSF of Cartesian sampling



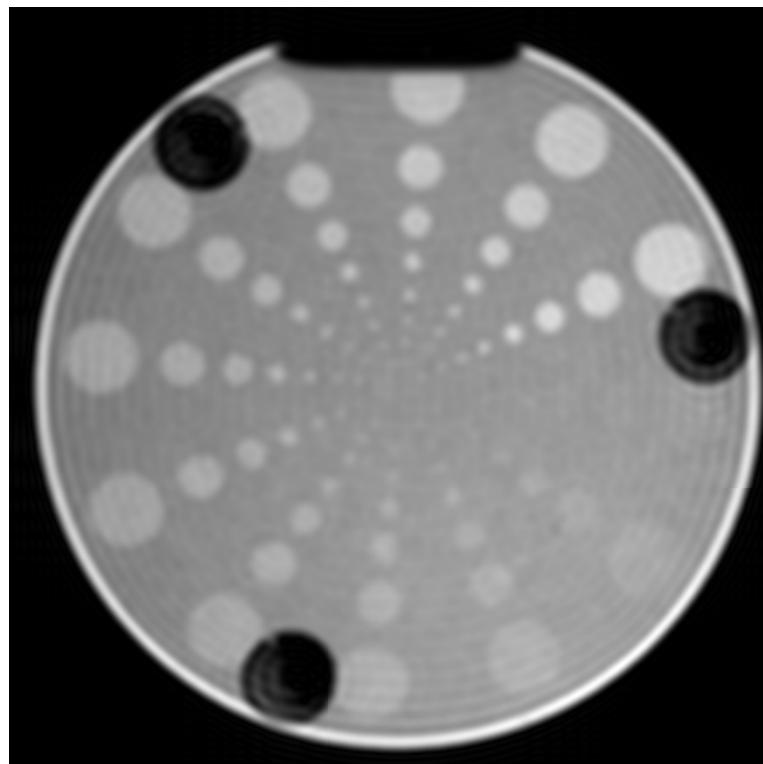
## Gibbs Ringing

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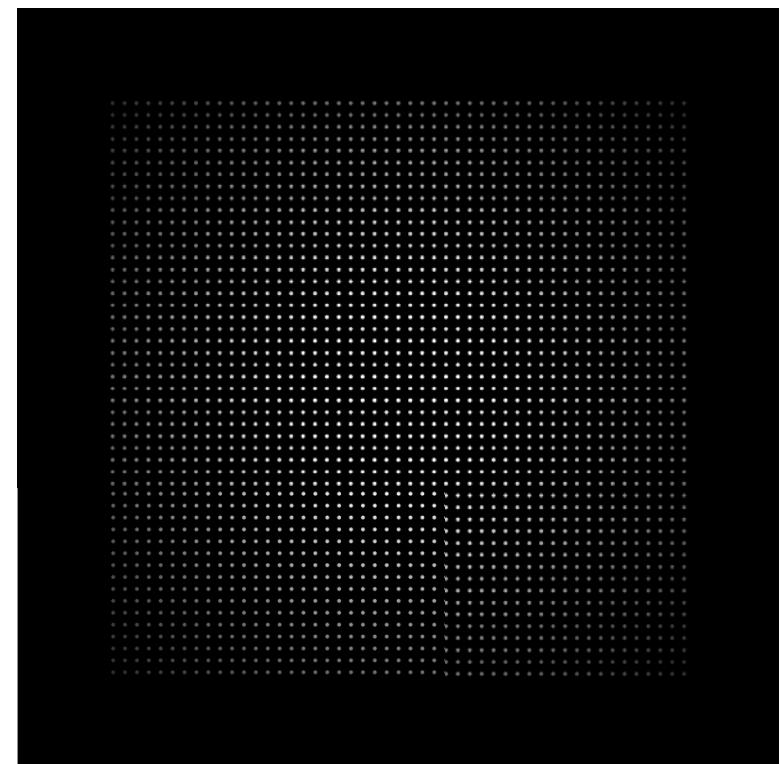


## Ringing Filter

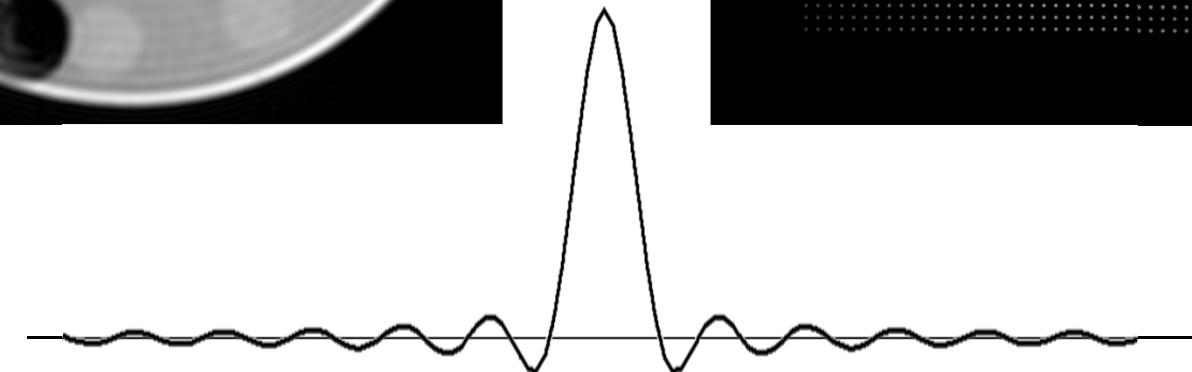
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## Gaussian Filter

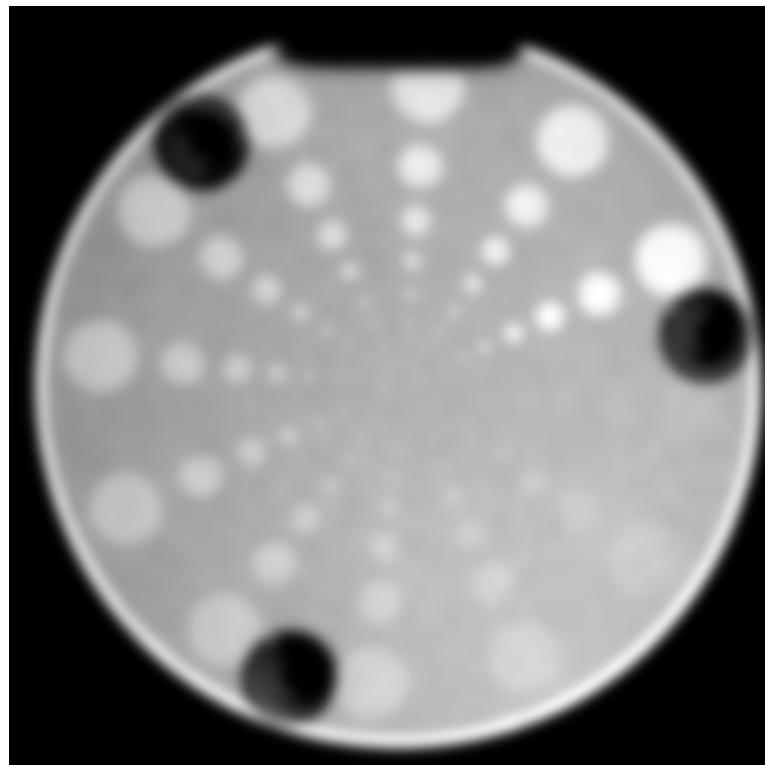


PSF

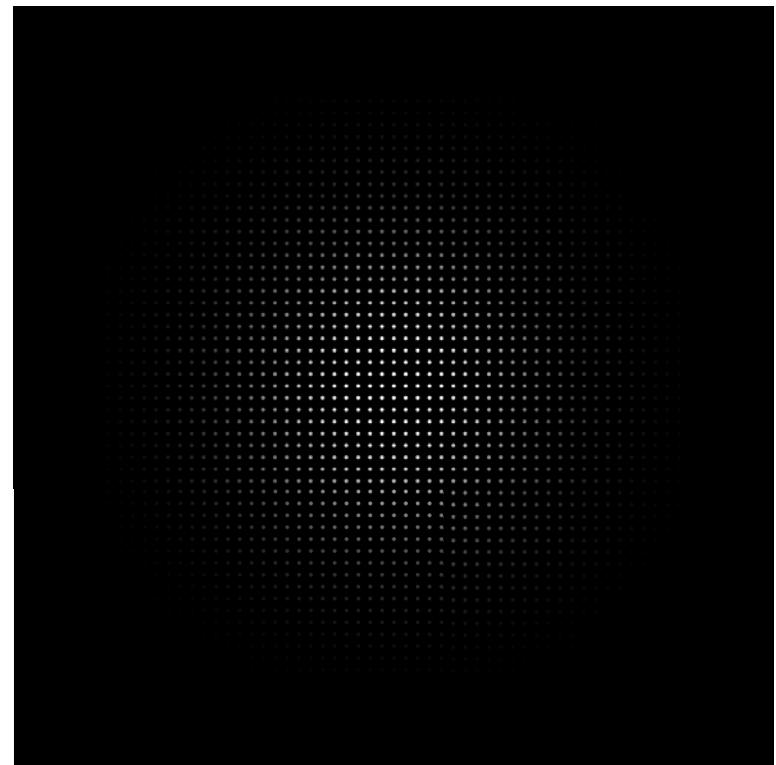


## Ringing Filter

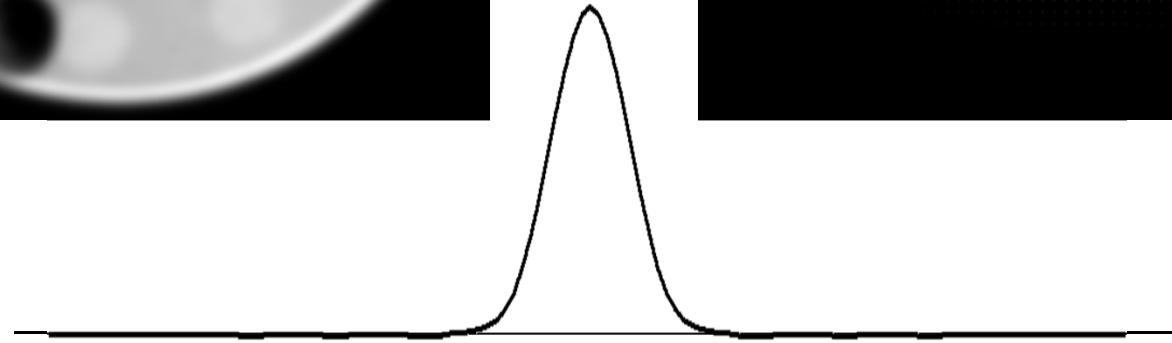
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## Gaussian Filter

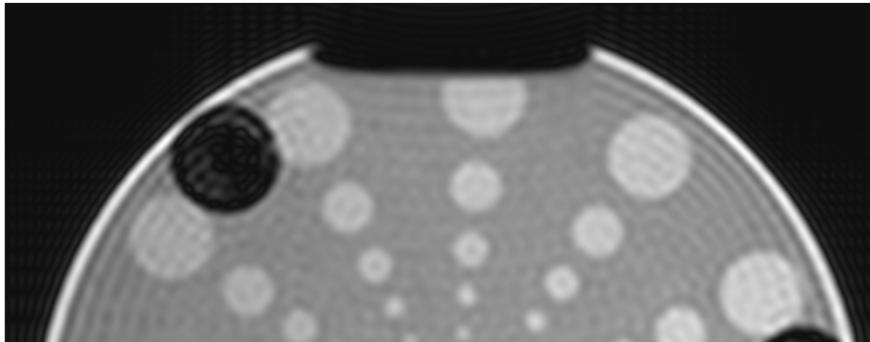


PSF

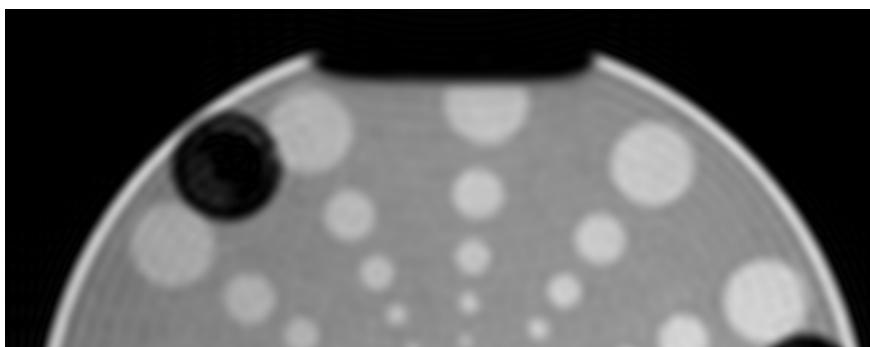
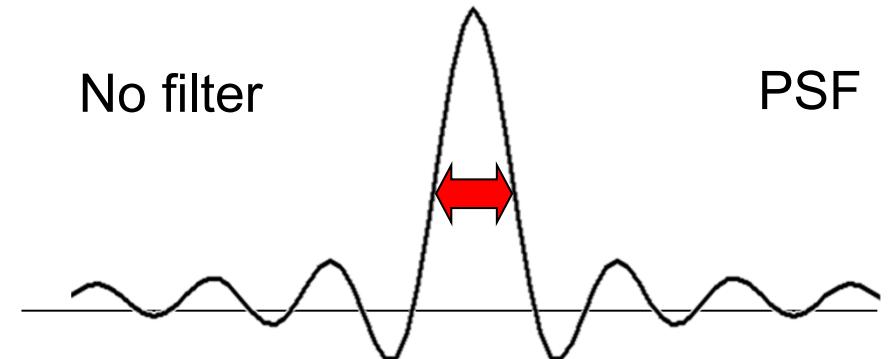


## Ringing Filter

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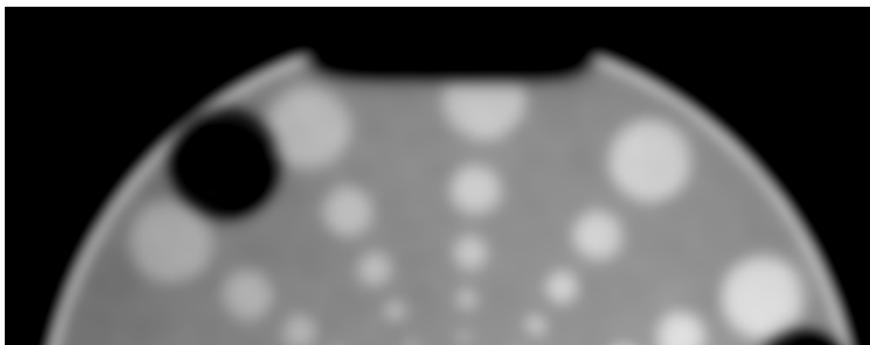
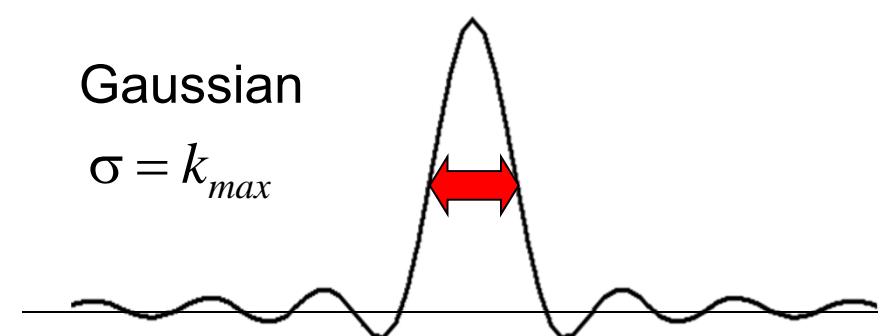


No filter



Gaussian

$$\sigma = k_{max}$$



Gaussian

$$\sigma = \frac{k_{max}}{2}$$

