# Micro and Nano-Tomography of Biological Tissues

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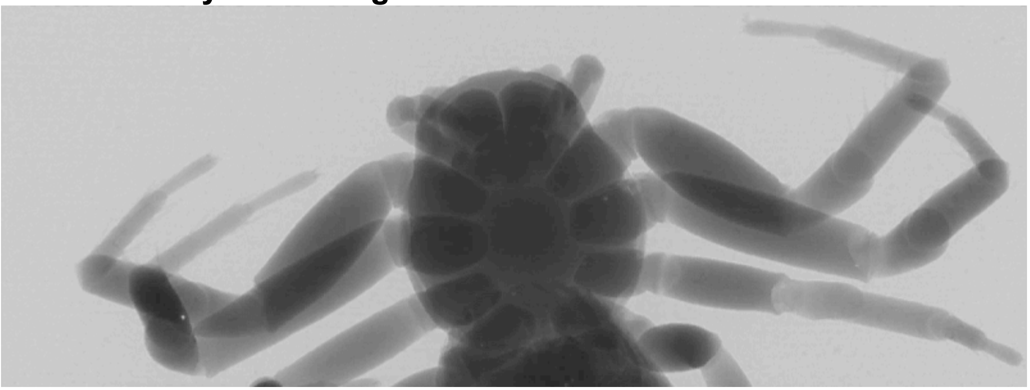
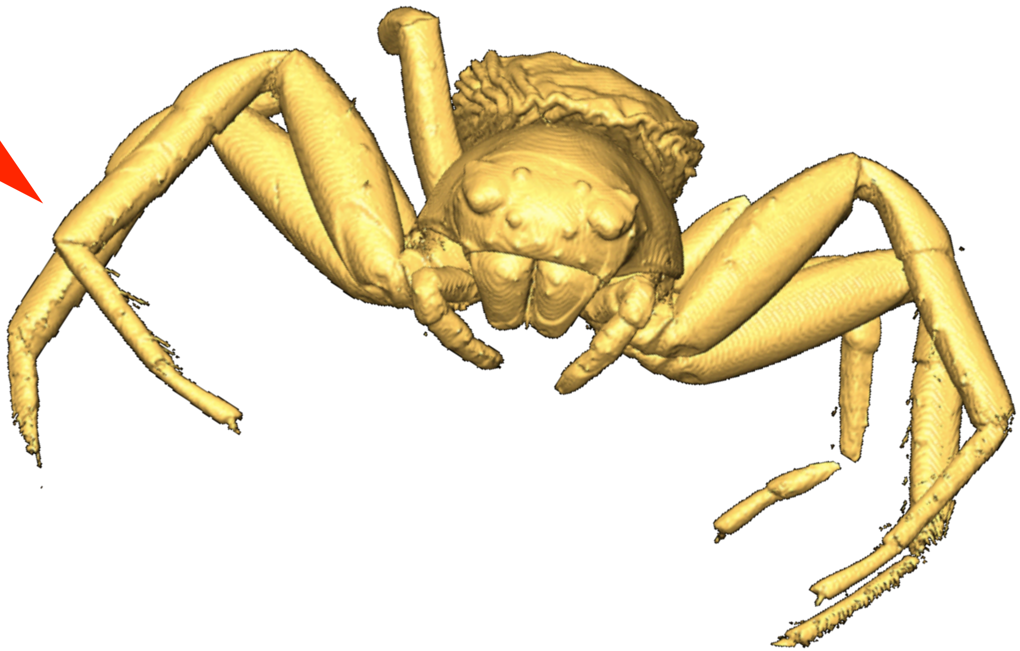
# Aim of this lecture

**2D hard X-ray microimage**



**Contrast mechanism**
**Spatial resolution**

**Reconstruction principles**
**Visualization modalities**



**3D tomographic reconstruction**

# Contents

- **Contrast mechanism**
  - Absorption contrast
  - Factors affecting contrast
- **Spatial resolution**
  - Object function
  - Line/Point spread function
  - Modulation Transfer Function
- **CT-Principles**
  - Why slicing?
  - The CT problem
- **Data acquisition**
  - Beam geometries
  - Computing a CT image
- **Image reconstruction**
  - Backprojection principle
  - Filtered backprojection
- **Visualizing 3D data**
  - SSD, MIP, VR

# X-ray Image Formation

# Contrast

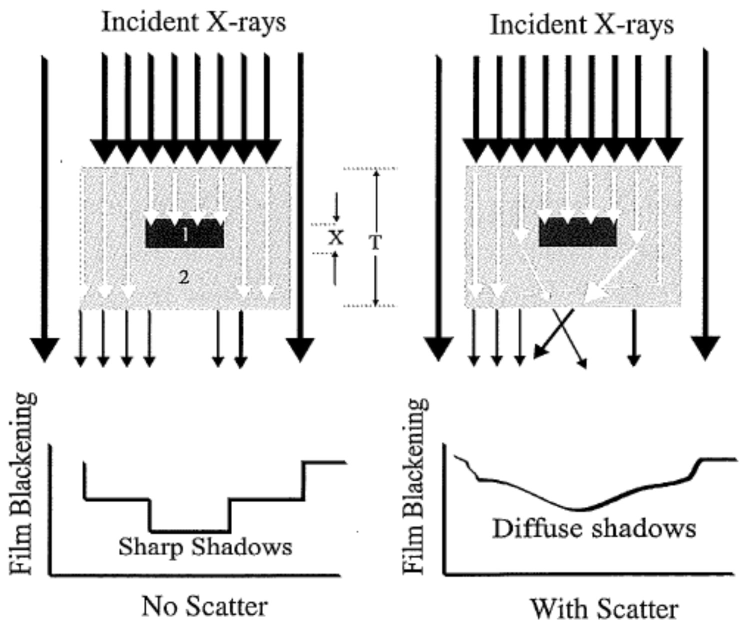
- **Contrast means difference**. In an image, contrast can be in the form of different shades of gray, light intensities, or colors. A detail within a sample will be visible in an image only if it has sufficient physical contrast relative to surroundings.

- The physical contrast of a sample must represent a difference in one or more sample characteristics.
For example:

  - **Classical, absorption-based imaging**: contrast given by difference in absorption properties (density, atomic number and thickness), described by the complex part of the index of refraction.

  - **Phase contrast imaging**: contrast given by difference in electron density within the sample (see later), described by the real part of the index of refraction

# Contrast definition




Definition: $C = \dfrac{I_1 - I_2}{I_2}$

$$I_1 = P_1 + S_1 \qquad I_2 = P_2 + S_2$$

Beer-Lambert →

$$P_1 = Ne^{-\mu_1 x} \times e^{\mu_2 (T-x)}$$
$$P_2 = Ne^{-\mu_2 T}$$

$$C = \frac{N(e^{-(\mu_1-\mu_2)x} e^{-\mu_2 T}) + S_1 - S_2}{Ne^{-\mu_2 T} + S_2}$$

Using $e^{-x} \approx 1 - x$ for small $x$

and assuming $S_1 \cong S_2$ we can write:

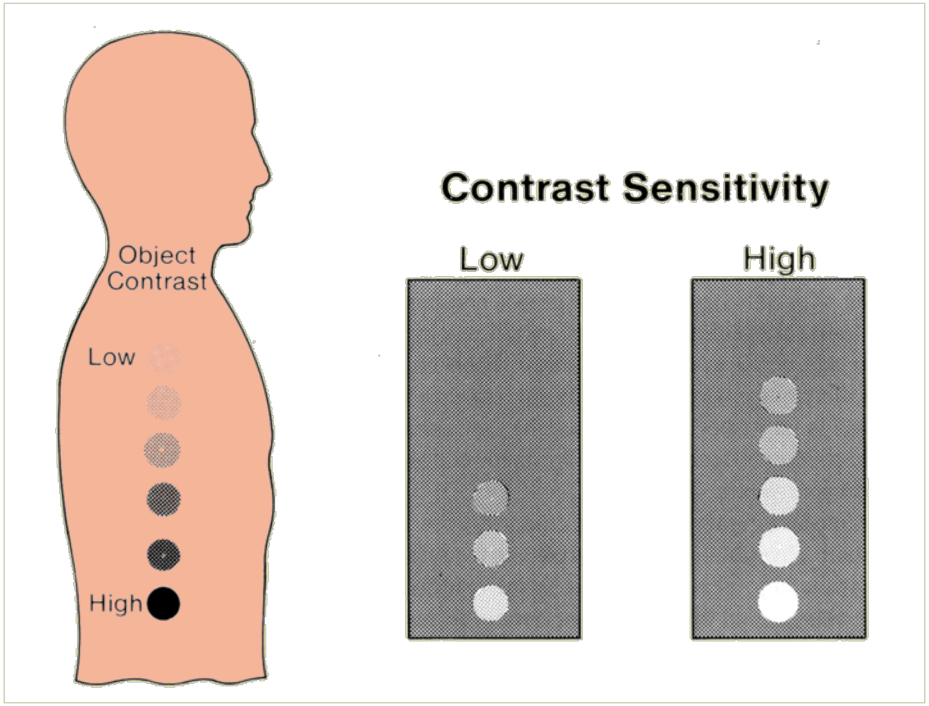
$$C \cong \frac{(\mu_1 - \mu_2)x}{1 + R} \quad \text{where R} \cong \frac{S_2}{Ne^{-\mu_2 T}}$$

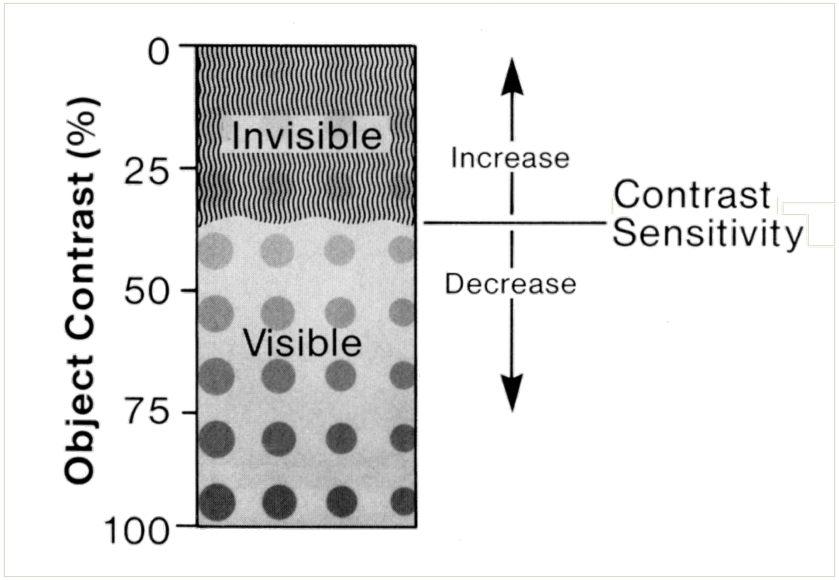
- Contrast maximized by increasing attenuation coefficient difference
- Small scatter contributes to higher contrast.

C. Guy, The Principles of Medical Imaging, 2005

# Contrast sensitivity

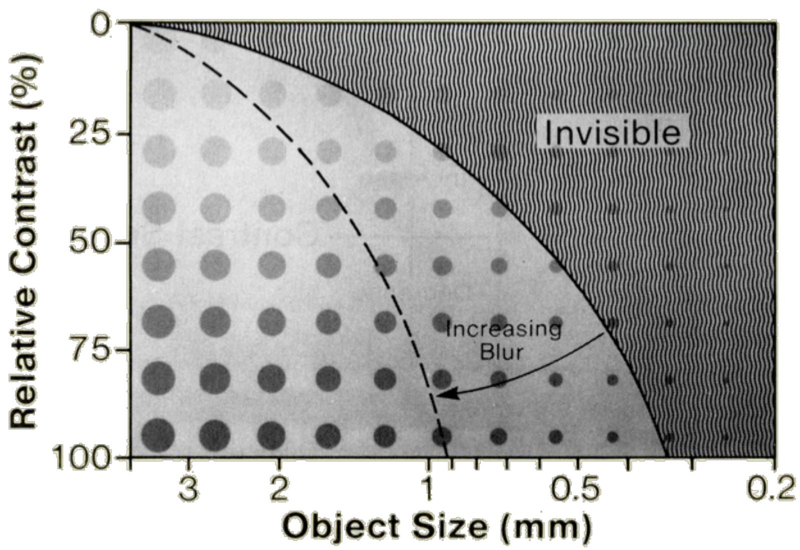
- Establishes the relationship between image contrast and object contrast
- Depends on the imaging method and system
- Is affected by blur, noise and illumination

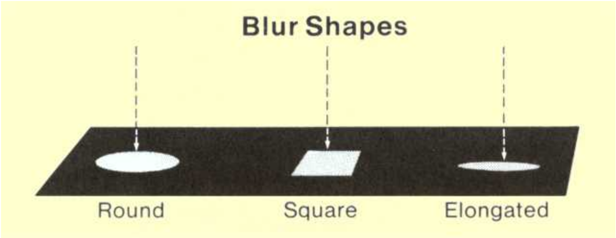


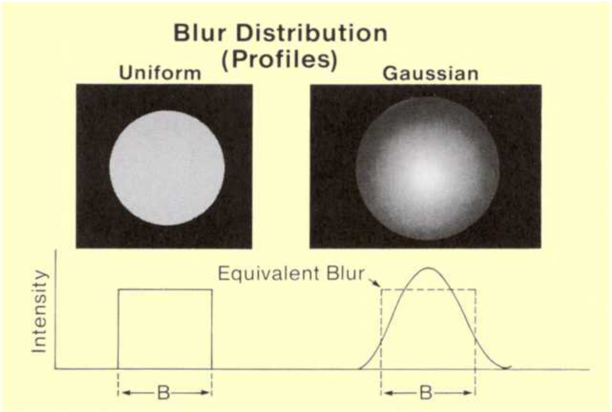



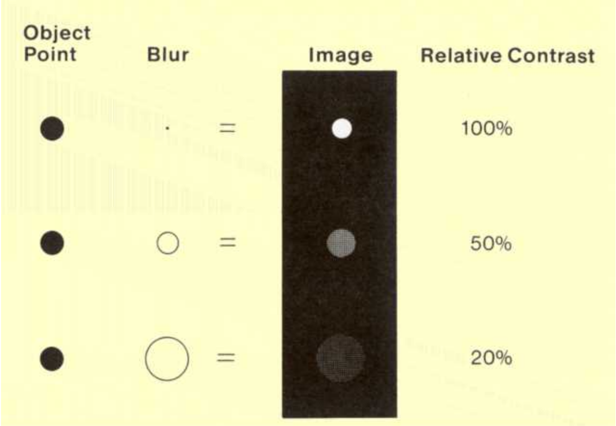
# Blur vs contrast

- Blurring is present in all imaging processes
- In an ideal situation, each small point within an object would be represented by a small, well-defined point within the image. In reality, the "image" of each object point is spread, or blurred, within the image.
- Blur has little effect on the visibility of large objects but it reduces the contrast and visibility of small objects.
- Relationship between spatial resolution and contrast!!



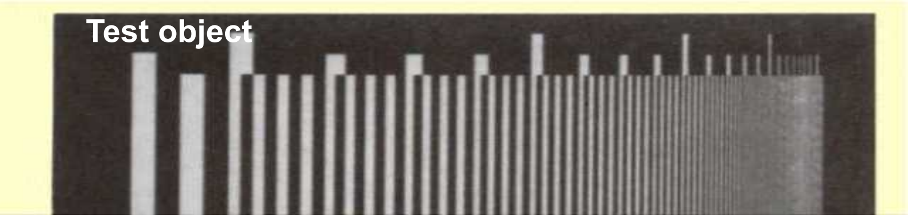



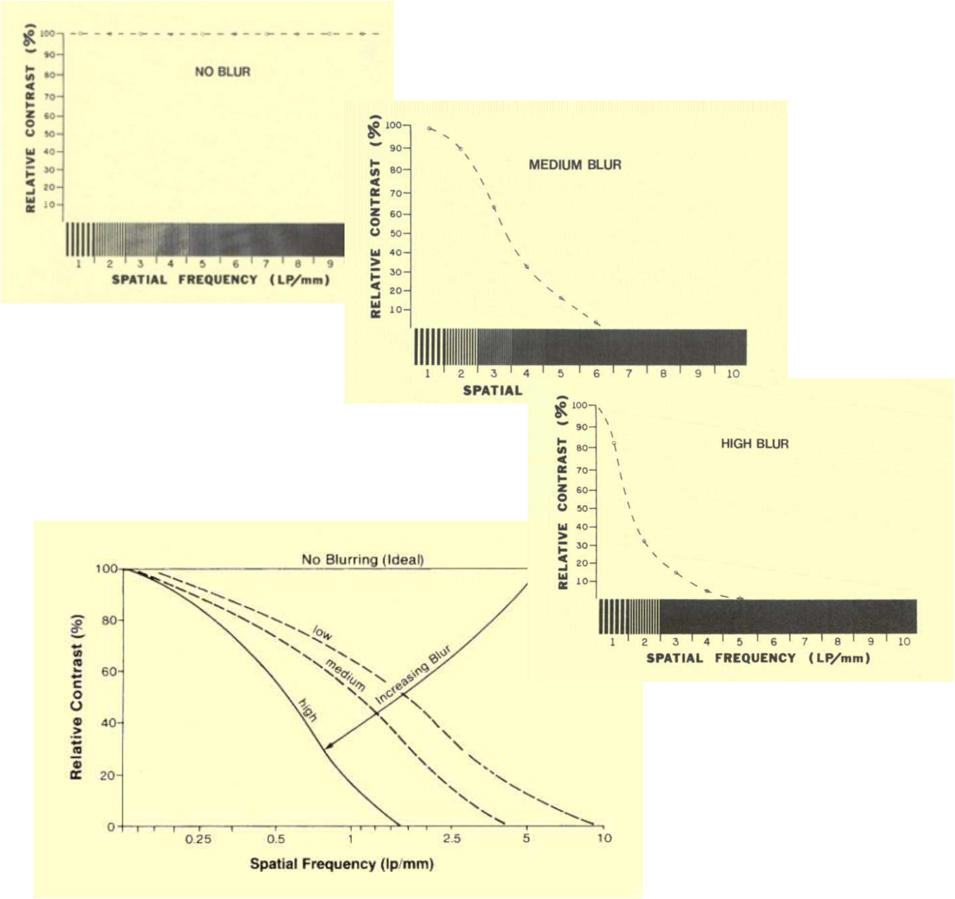



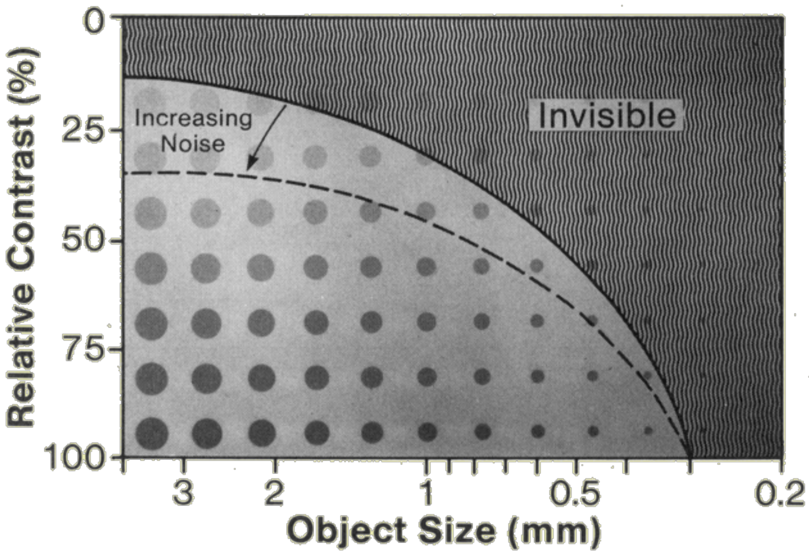
# Contrast Transfer Curves : evaluating system blur

- The resolving ability, or resolution, of an imaging system is relatively easy to measure and is often used to evaluate system blur.
- The common practice is to describe the line width and separation distance in terms of line pairs (lp) per unit distance (millimeters or centimeters). One line pair consists of one lead strip and one adjacent separation space.
- The number of line pairs per millimeter is actually an expression of spatial frequency. As the lines get smaller and closer together, the spatial frequency (line pairs per millimeter) increases.
-  A typical test pattern contains areas with different spatial frequencies.
- An imaging system is evaluated by imaging the test object and observing the highest spatial frequency (or minimum separation) at which the separation of the lines is visible



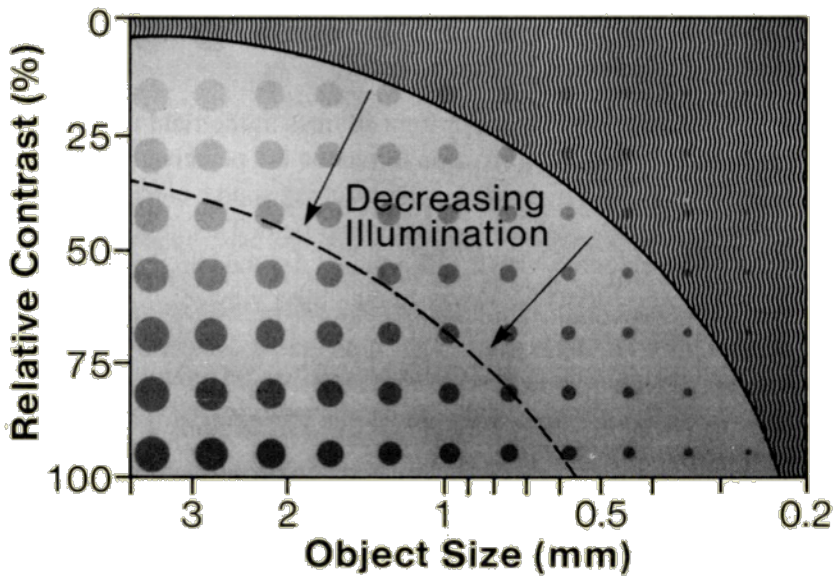



# Affecting contrast




## NOISE

- Image noise gives an image a textured or grainy appearance.
- Increasing image noise usually reduces object visibility.
- Low-contrast objects are more affected by noise effects.




## IMAGING VIEWING CONDITIONS:

- Background conditions and structure
- Object size
- Viewing distance
- Glare

# Spatial resolution - Intro

- In order to image (view) an object, the instrument (microscope, X-ray detector or whatever) has to **probe** the object.

- There is an **interaction** between the object and the instrument. This interaction will necessary affect the image one obtains from it, usually by blurring it.

- We try to describe this process by simple (linear) mathematics while keeping in mind that spatial resolution and contrast are strongly correlated.

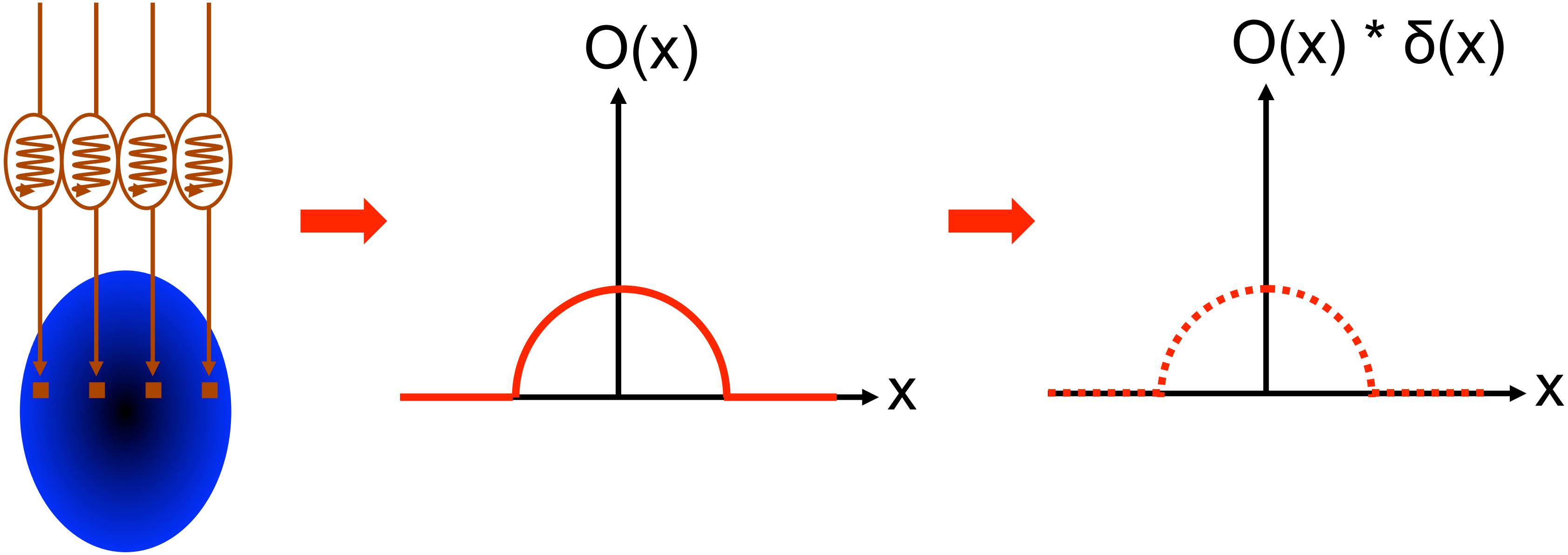
- Spatial resolution can be described in the form of the **Modulation Transfer Function**, i.e. the grey-level contrast in the image space as a function of the frequency (line pairs / mm) in the object space.

# Spatial resolution : line spread function

## Object representation:

$$O(x,y)=\iint O(x',y')\cdot\delta(x-x',y-y')\,dx'\,dy'$$

## In 1 dimension:

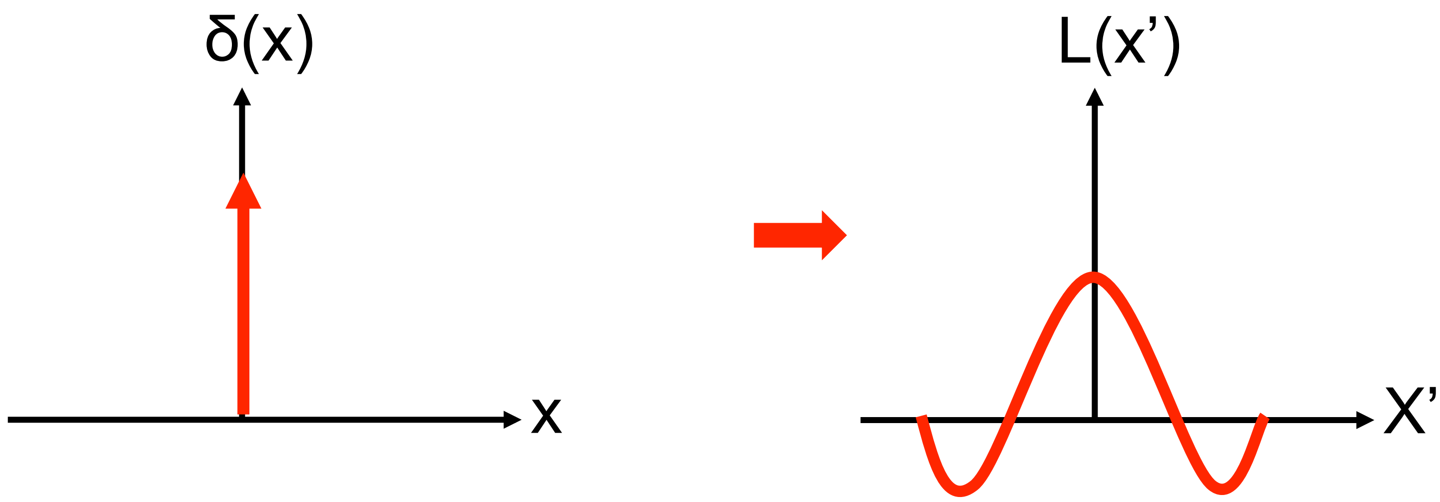


# Spatial resolution : Line Spread Function

- Mapping the object space into image space (1 dimension):
- **Def:** L(x') is the Line Spread Function (LSF) which maps a line at x=0 in the object space onto x' in the image space.
- Then: A line at location X with intensity O(X) yields an image:

$$\boxed{O(X)\,\delta(x-X) \rightarrow O(X)\,L(x'-X)}$$

**object space** **image space**




**"Line Spread Function"**

# Spatial resolution: Modulation Transfer Function

**Let consider a general object O(x) and its Fourier decomposition:**

$$O(x) = \int_{-\infty}^{\infty} \left[ a(k)\sin(kx) + b(k)\cos(kx) \right] \, dk$$

with a(k),b(k) amplitudes , and $k = 2\pi/\lambda$ wavenumber

**Now, lets map this Fourier representation onto the image space:**

$$A_k(x) = a(k)\sin(kx) = a(k)\int_{-\infty}^{\infty} \delta(x - X)\sin(kX)dX \longrightarrow A_{k'}(x') = a(k)\int_{-\infty}^{\infty} L(x' - X)\sin(kX)dX$$

$$B_k(x) = b(k)\cos(kx) = b(k)\int_{-\infty}^{\infty} \delta(x - X)\cos(kX)dX \longrightarrow B_{k'}(x') = b(k)\int_{-\infty}^{\infty} L(x' - X)\cos(kX)dX$$

# Spatial resolution: Modulation Transfer Function

- Exploiting the addition properties for “sin” and “cos” function one obtains:

$$\boxed{\begin{array}{l} A_k(x) = a(k)\sin(kx) \longrightarrow A_{k'}(x') = a(k)\eta(k)\sin(kx' - \phi) \\ B_k(x) = b(k)\cos(kx) \longrightarrow B_{k'}(x') = b(k)\eta(k)\cos(kx' - \phi) \end{array}}$$

- The Fourier components are imaged onto the image space as harmonic functions of the **same frequency *k***, but displaced by **a (spatial) phase Φ** and with and **amplitude *a(k)η(k)*** .This holds for every k!

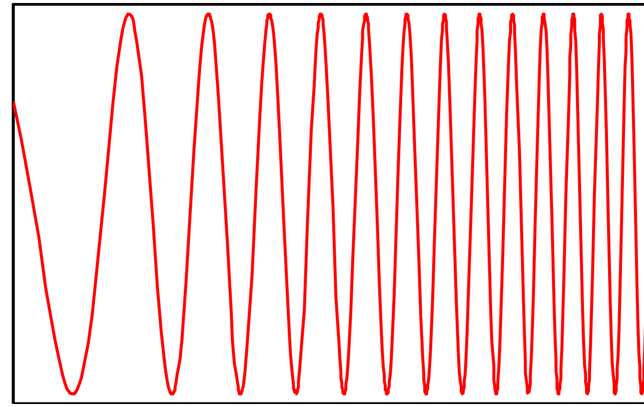
- The function η*(k)* is defined as **MTF: Modulation Transfer Function.**
- **The MTF describes the dependence of the amplitude (i.e the contrast) of the original intensity function in the image space as a function of k. Since this holds for every *k*, we can write:**

$$\boxed{F(image) = MTF \cdot F(original) \quad \text{with } F\text{: Fourier Transformation}}$$

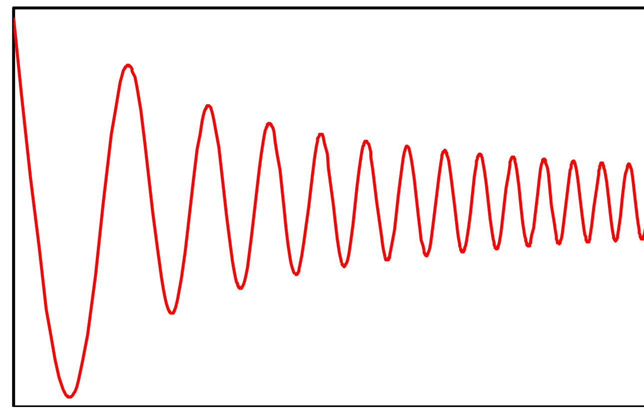
# Modulation Transfer Curves

- The modulation transfer function (MTF) is a graphical description of the resolution characteristics of an imaging system (or its individual components)
- MTF is similar to the CTF: it describes a system's ability to image sine-wave shapes, or spatial frequencies. It is a the response of the imaging system to sinusoids of different spatial frequencies.
- In order to form an unblurred image of the object, the imaging system must be able to produce sufficient contrast for all spatial frequencies contained in the object.
- If some of the frequency components are lost in the imaging process, the image will not be a true representation of the object.
- If the high frequency components are not present in the image, the image will be blurred.
- → Link between resolution and contrast!!

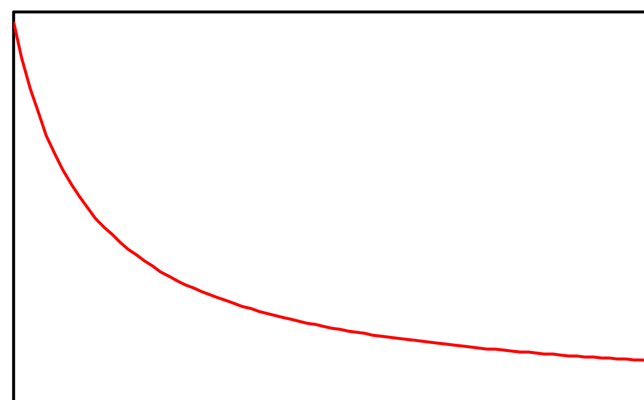
OBJECT



IMAGE



MTF

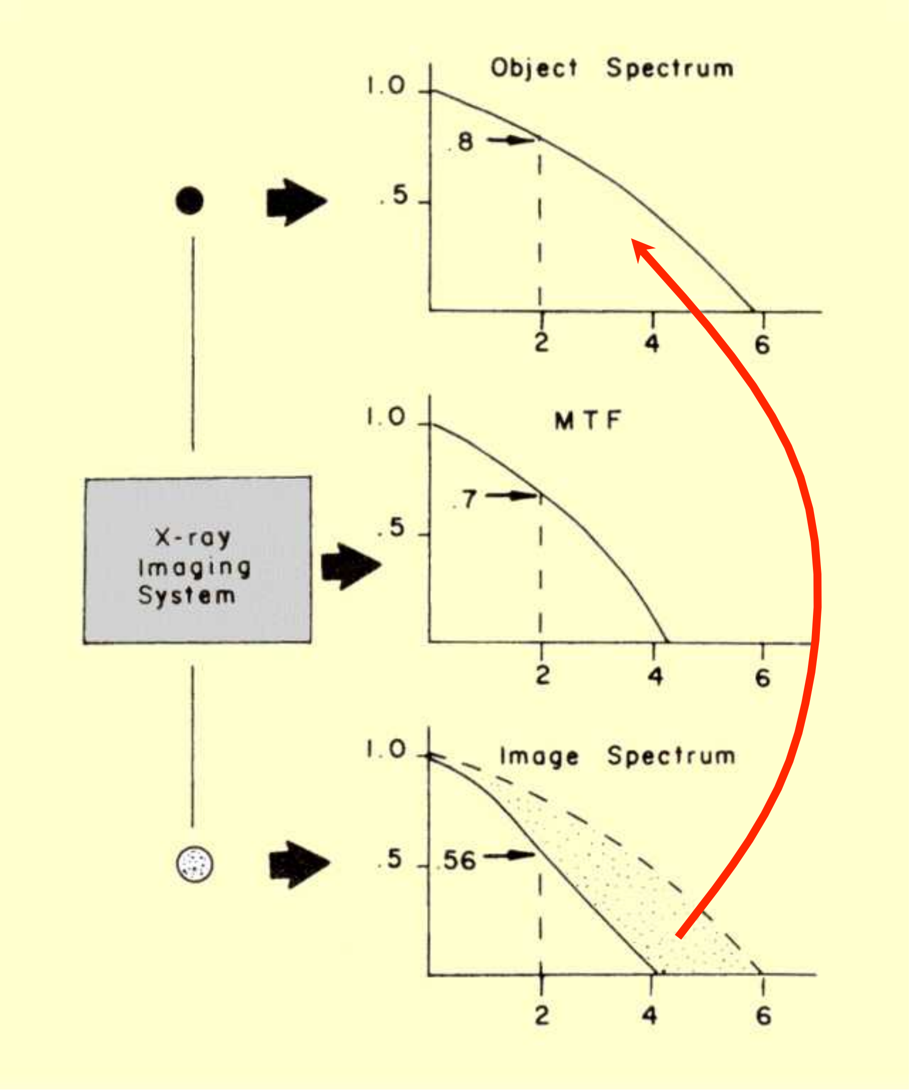


# Modulation Transfer Curve: an example

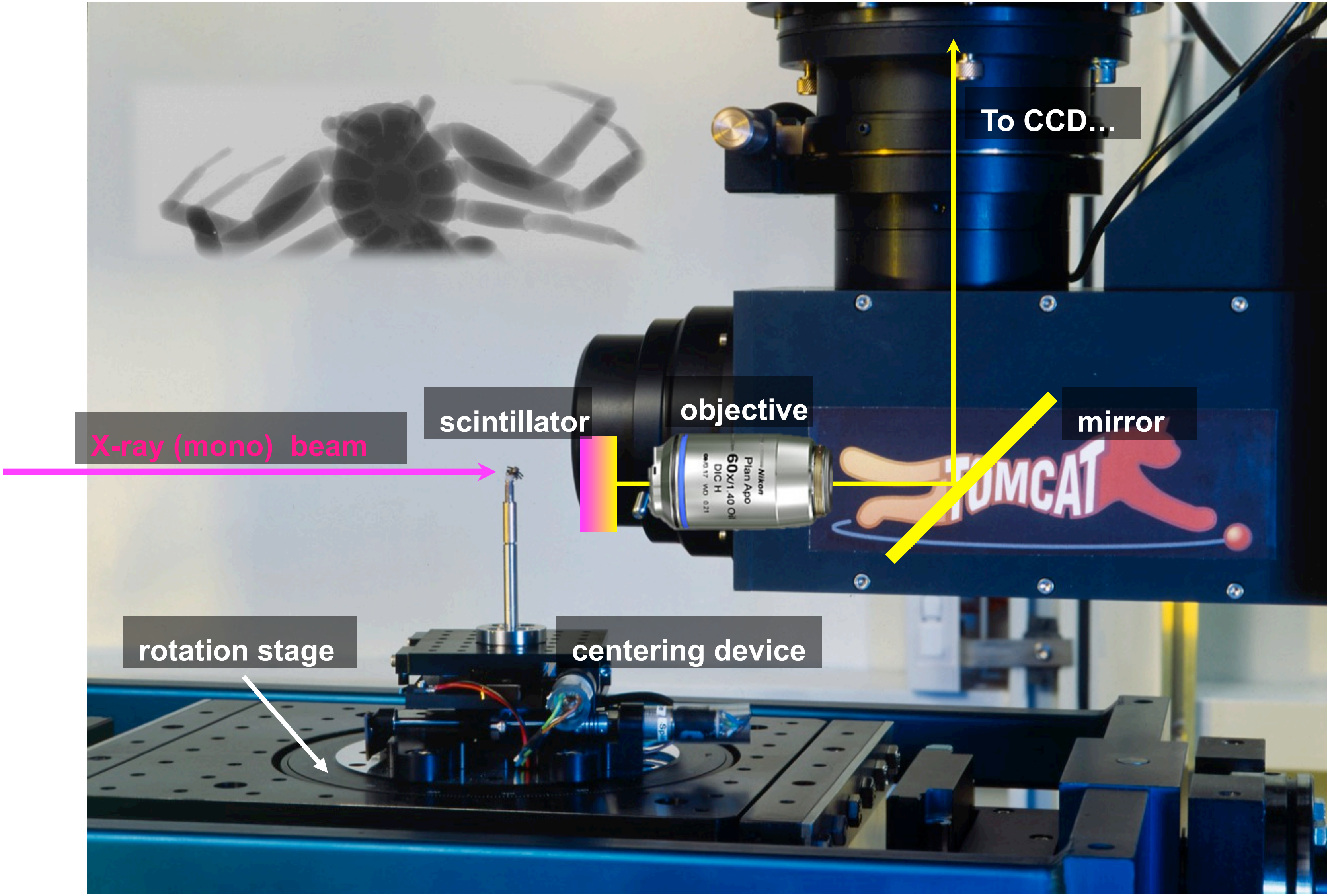
The image content at a specific frequency is found by multiplying the object content by the MTF.

An example

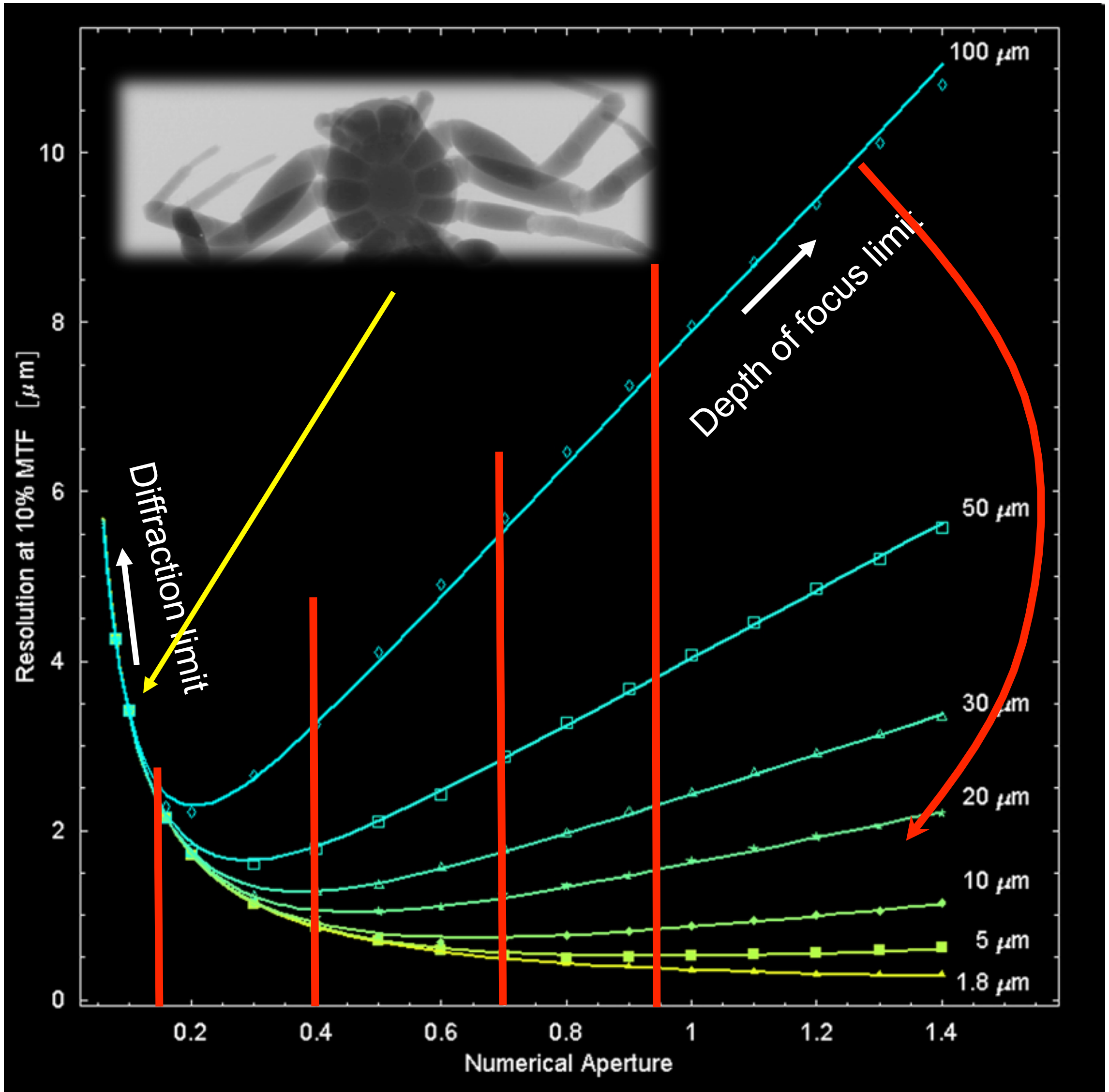
- An object contains 0.8 (80%) at a frequency of 2 cycles per mm.
- The MTF at this frequency is 0.7.
- Multiplying these two quantities shows that the image will contain only 0.56 (56%) at this frequency.
- The shaded area is the portion of the object's spatial frequency spectrum that is lost because of the MTF of the imaging system.
- Any frequency components of the object that are above the resolution limit of the system are completely lost.
- In effect, the MTF of the imaging system can cut out the higher frequency components associated with an object, and the image will be made up only of lower frequency components.
- Since low frequency components are associated with large objects with gradual changes in thickness, as opposed to sharp edges, the image will be blurred.

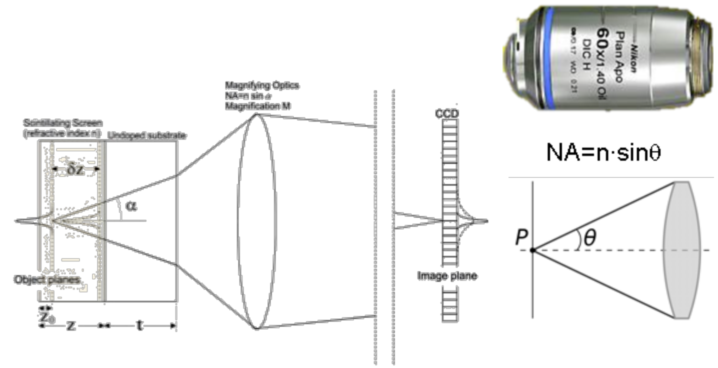


# State-of-the-art 2D microimaging (1-50 um)



# Implication of optical coupling




Depth of focus $\approx \delta z\, NA$

Diffraction $\approx \lambda / NA$

Spherical aberration $\approx t\, NA^3$

$$R = \sqrt{(p/NA)^2 + (qzNA)^2}$$

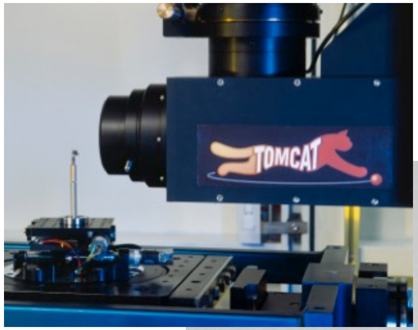
Take home message:

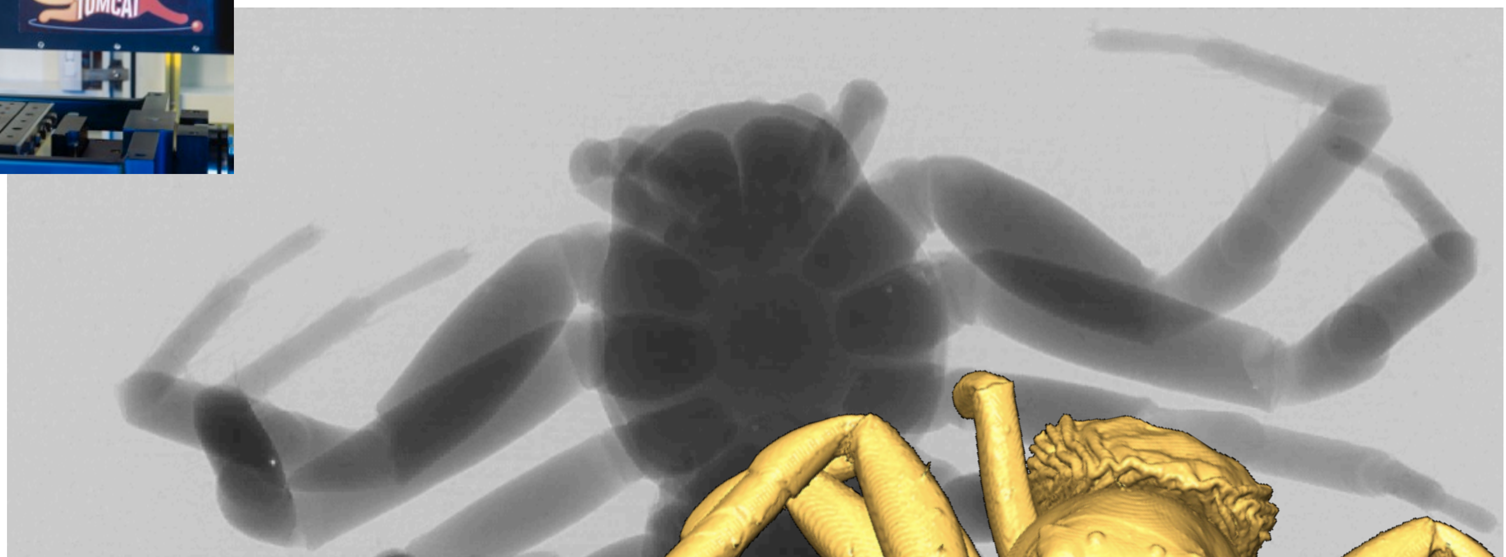
- Geom. Pixel Size ≠ True Resolution
- Thin scintillator → better resolution
- DOF limited at high NA
- Diffraction limit at low NA

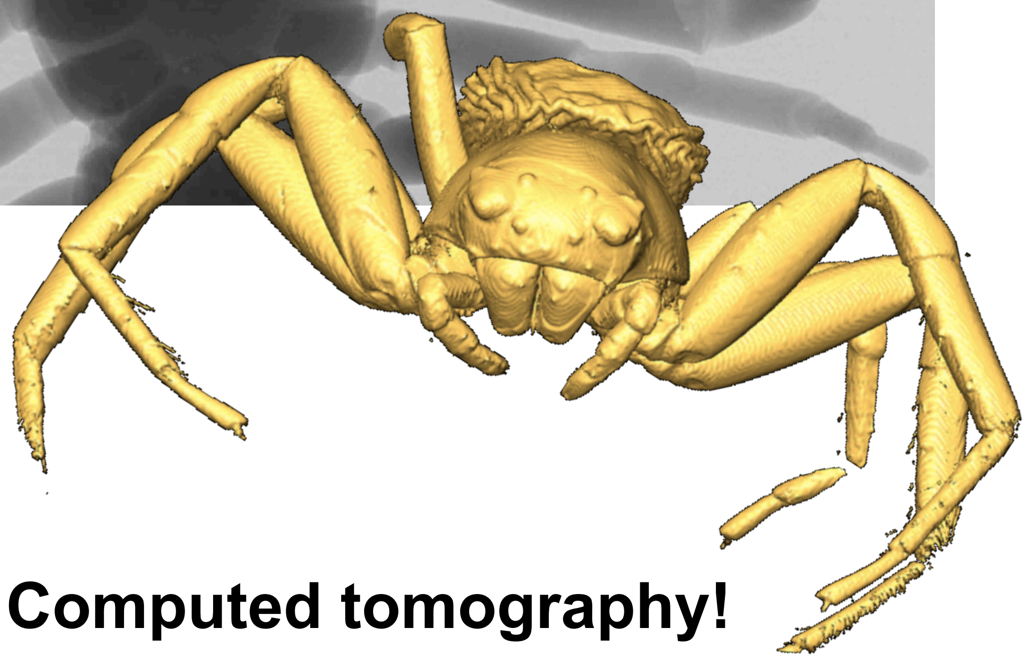
A. Koch *et al.*, J. Opt. Soc. Am. A 15 (7) (1998) 1940 and M. Stampanoni *et al.*, NIM A 491 (2002) 291–301

# Computed Tomography

# 2D→ 3D, how?



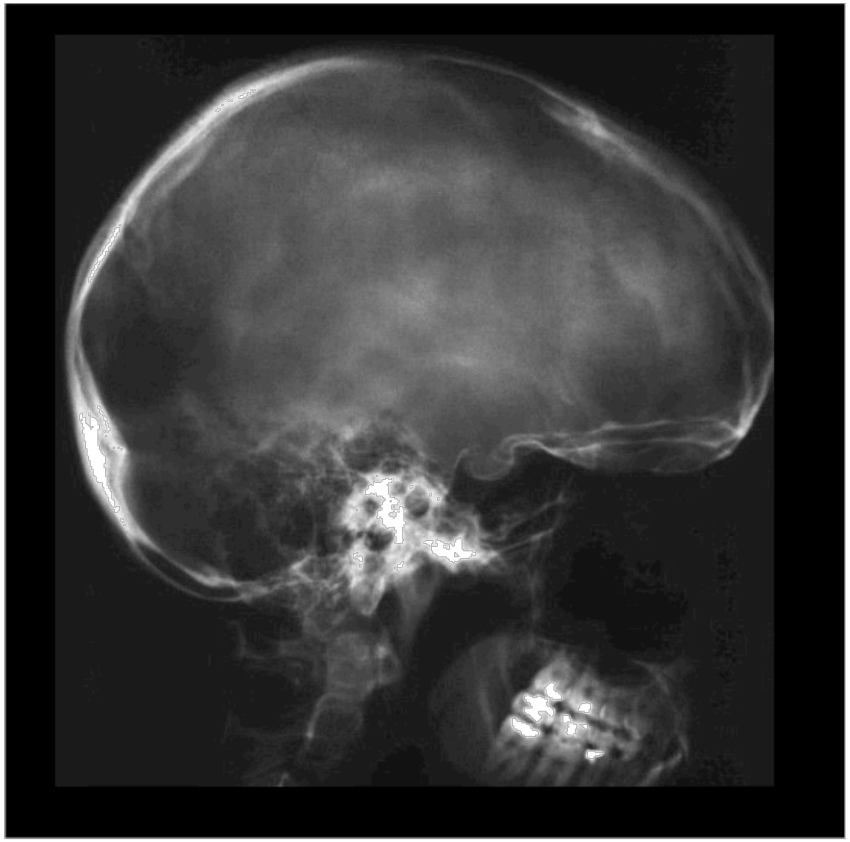





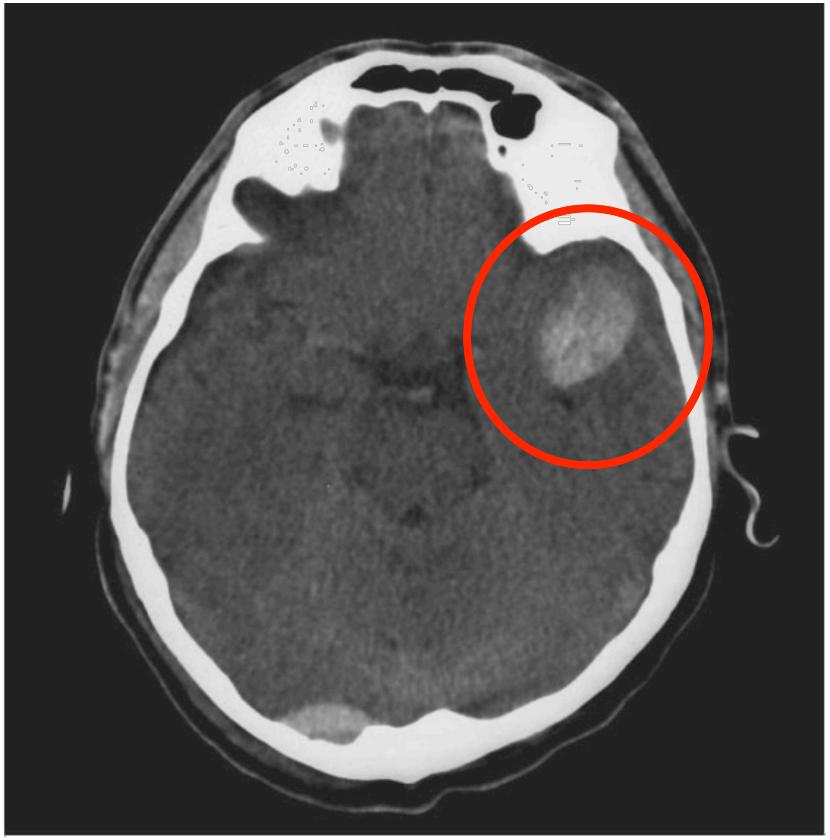
**→ Computed tomography!**

# Slicing Imaging – Why ?

- **Tomography** means imaging by sections or slices.
  (From the Greek word *tomos*, meaning "a section" or "a cutting )



Radiographic projection



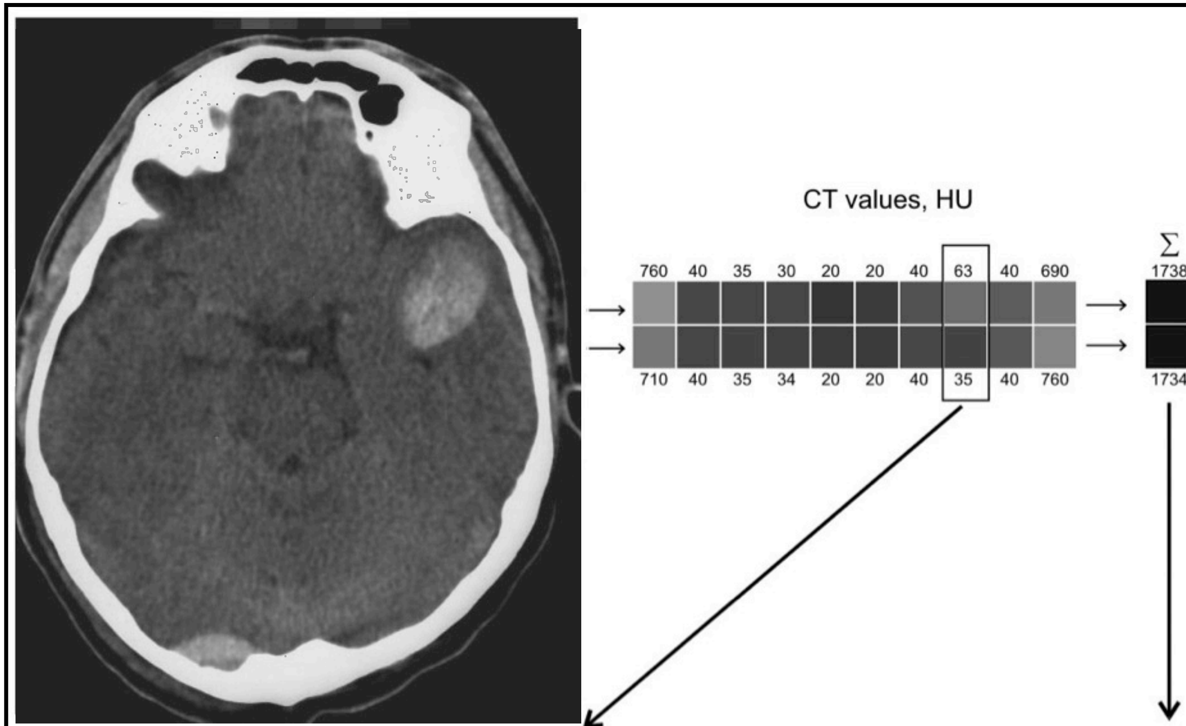
Tomographic slice

# Tomographic vs Radiographic Imaging

- **Radiographic imaging**:
  - 3D structures collapsed
  - Not quantitative
- **Tomographic imaging:**
  - 3D structure solved
  - Computed image reconstruction

**Note:**

**Radiographic imaging and CT share the same principles of X-ray generation, interaction and detection.**




**CT image:**

High local contrast of soft tissue structures

$$\text{contrast} = \Delta CT = I_1 - I_2 =$$
$$= 63 - 35 = 28\ \text{HU}$$
$$\approx 50\%$$

**X-ray image:**

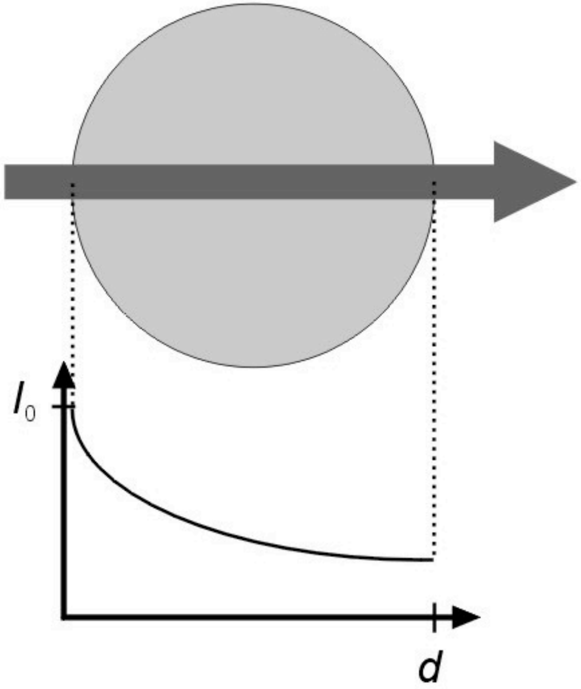
Low contrast of soft tissue structures due to superposition of bones

$$\text{contrast} = \frac{I_1 - I_2}{(I_1 + I_2)/2} =$$
$$= \frac{1738 - 1734}{(1738 + 1734)/2} \cdot 100\% = 0{,}23\%$$

**HIGH CONTRAST !!**

# The (absorption) CT problem

- Homogeneous object, monochromatic radiation.
- Beer-Lambert's law:



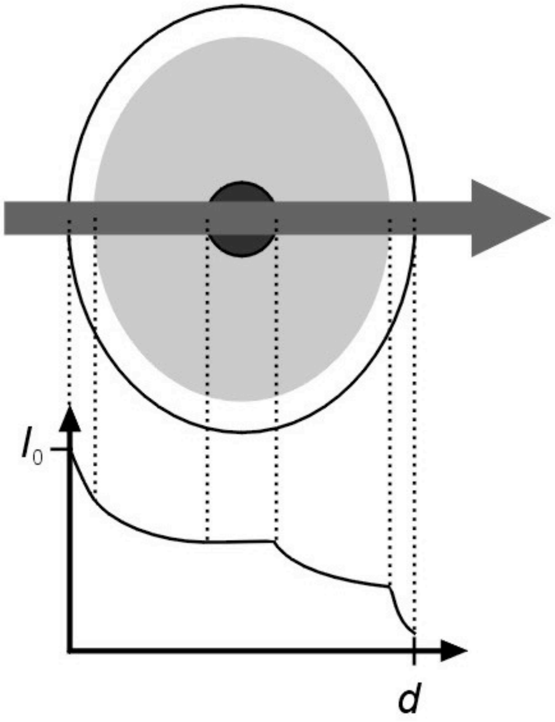

$$I = I_0 \cdot e^{-\mu \cdot d}$$

$$P = \ln \frac{I_0}{I} = \mu \cdot d$$

$$\boxed{\mu = \frac{1}{d} \cdot \ln \frac{I_0}{I}}$$

# The (absorption) CT problem

- Inhomogeneous object, monochromatic radiation.
- Beer-Lambert's law:




$$I = I_0 \cdot e^{-\mu_1 \cdot d_1 - \mu_2 \cdot d_2 - \mu_3 \cdot d_3 - \dots} =$$

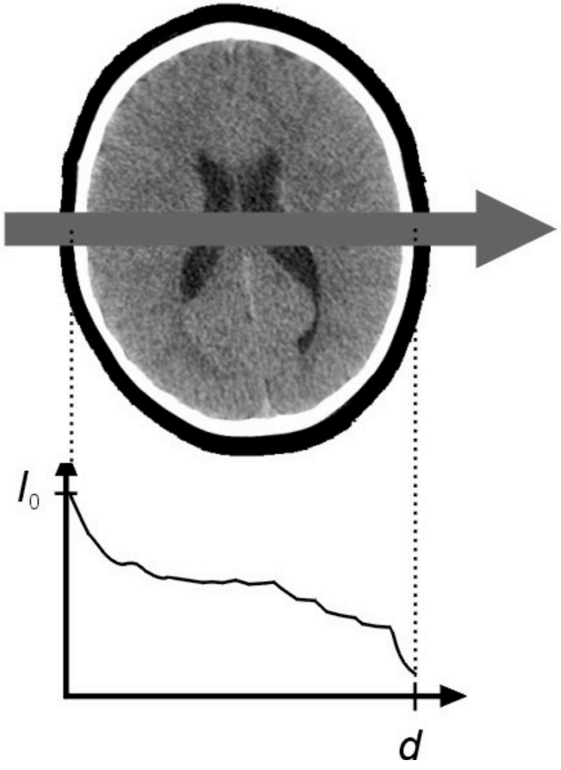
$$= I_0 \cdot e^{-\left[\sum_{i=1}^{n} \mu_i d_i\right]} = I_0 \cdot e^{-\int_0^d \mu \, ds}$$

$$P = \ln \frac{I_0}{I} = \sum \mu_i d_i$$

$$\mu_i = ?$$

# The (absorption) CT problem

- Inhomogeneous object, polychromatic radiation.
- Beer-Lambert's law:




$$I = \int_0^{E_{max}} I_0(E) \cdot e^{-\int_0^d \mu(E)\,ds} \, dE$$

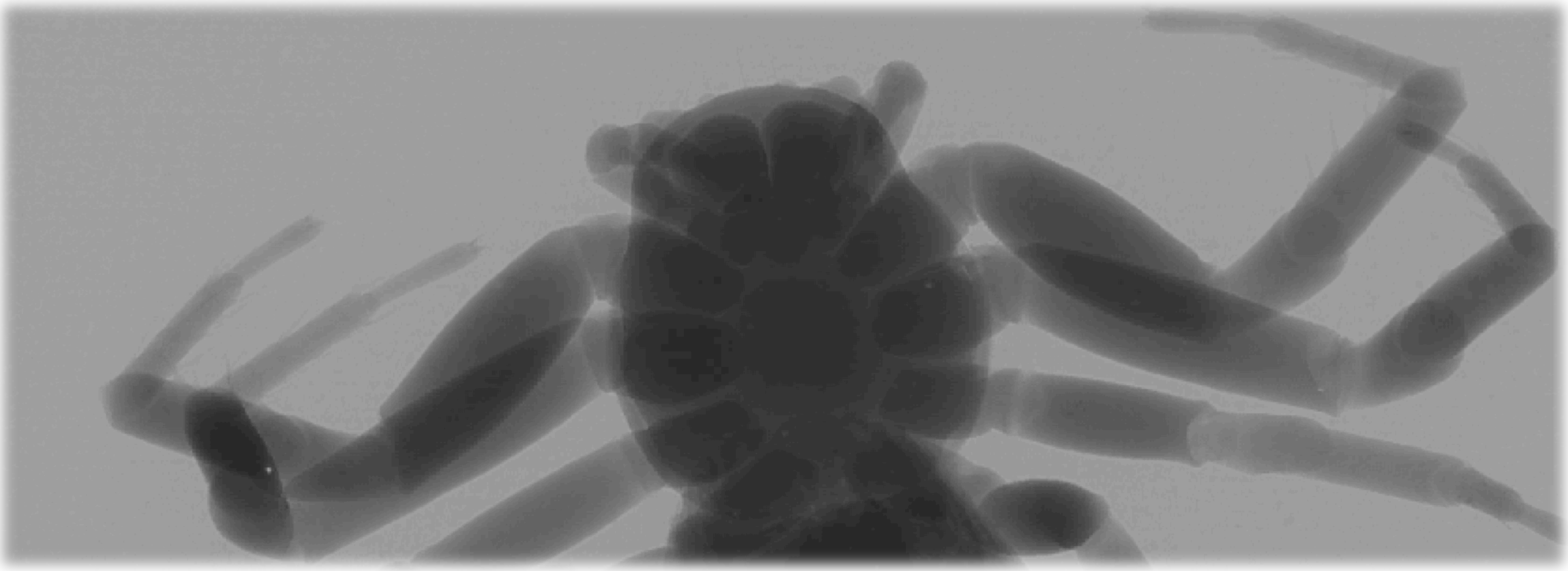
$$P = \ln \frac{I_0}{I}$$

$$\mu(x, y) = ?$$

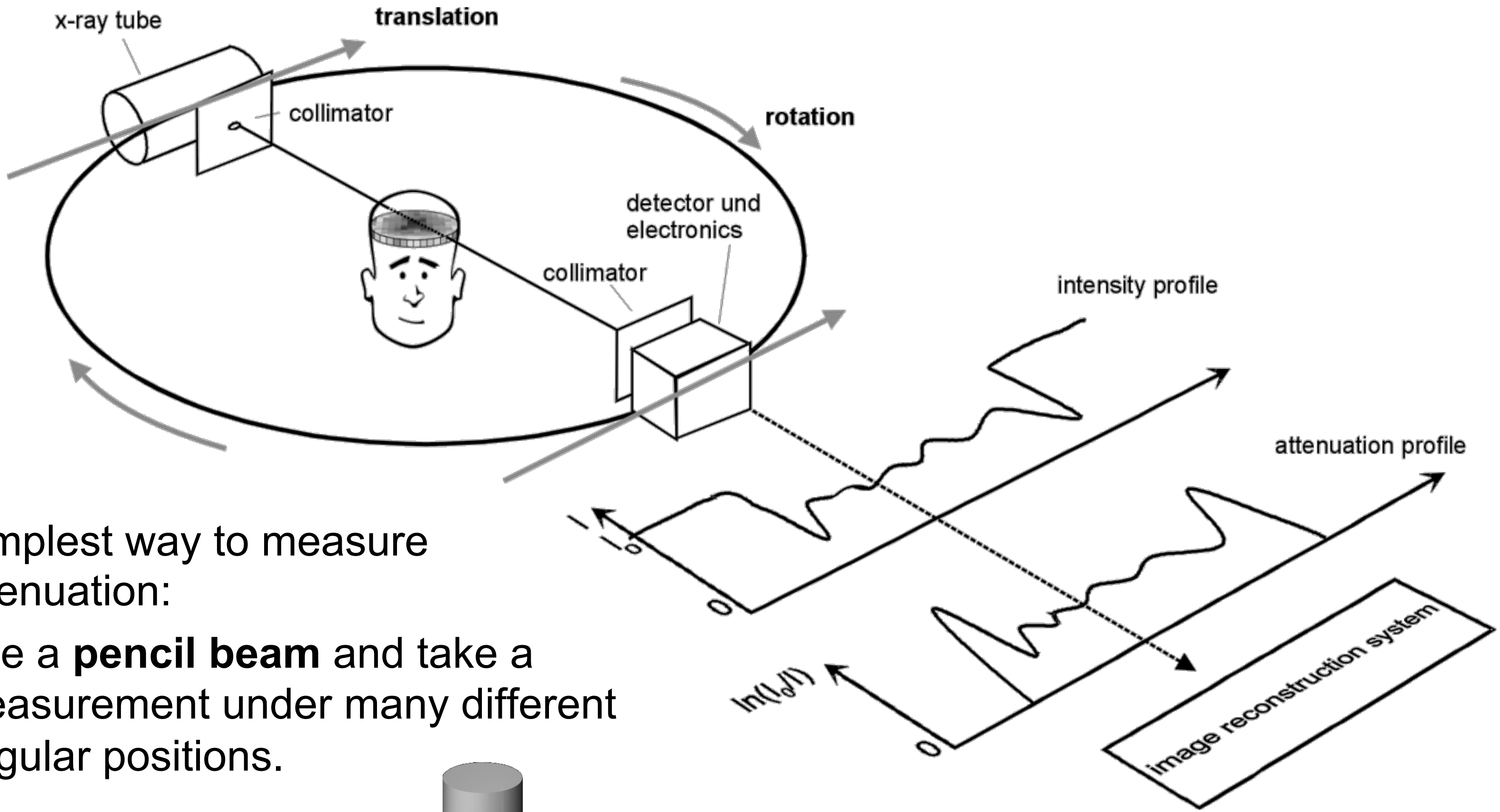
# Basics of CT

Primary Goal: Reconstruct an object, considered as a 2D distribution of some kind of *function*.

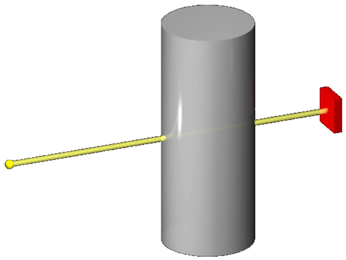
For us now: The *function* represents the linear attenuation coefficient of the object considered.



# CT Principle

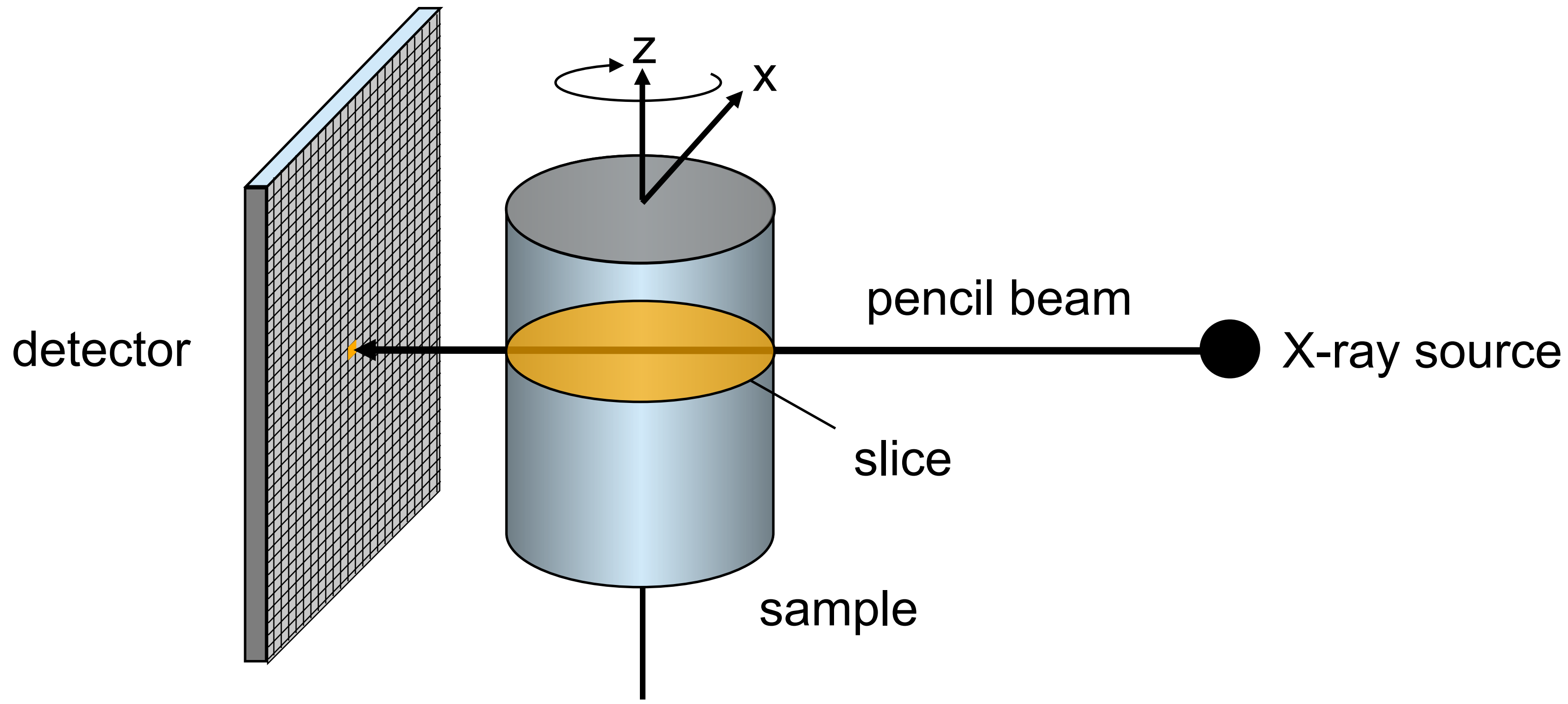



- Simplest way to measure attenuation:
- Use a **pencil beam** and take a measurement under many different angular positions.
- Schematic:



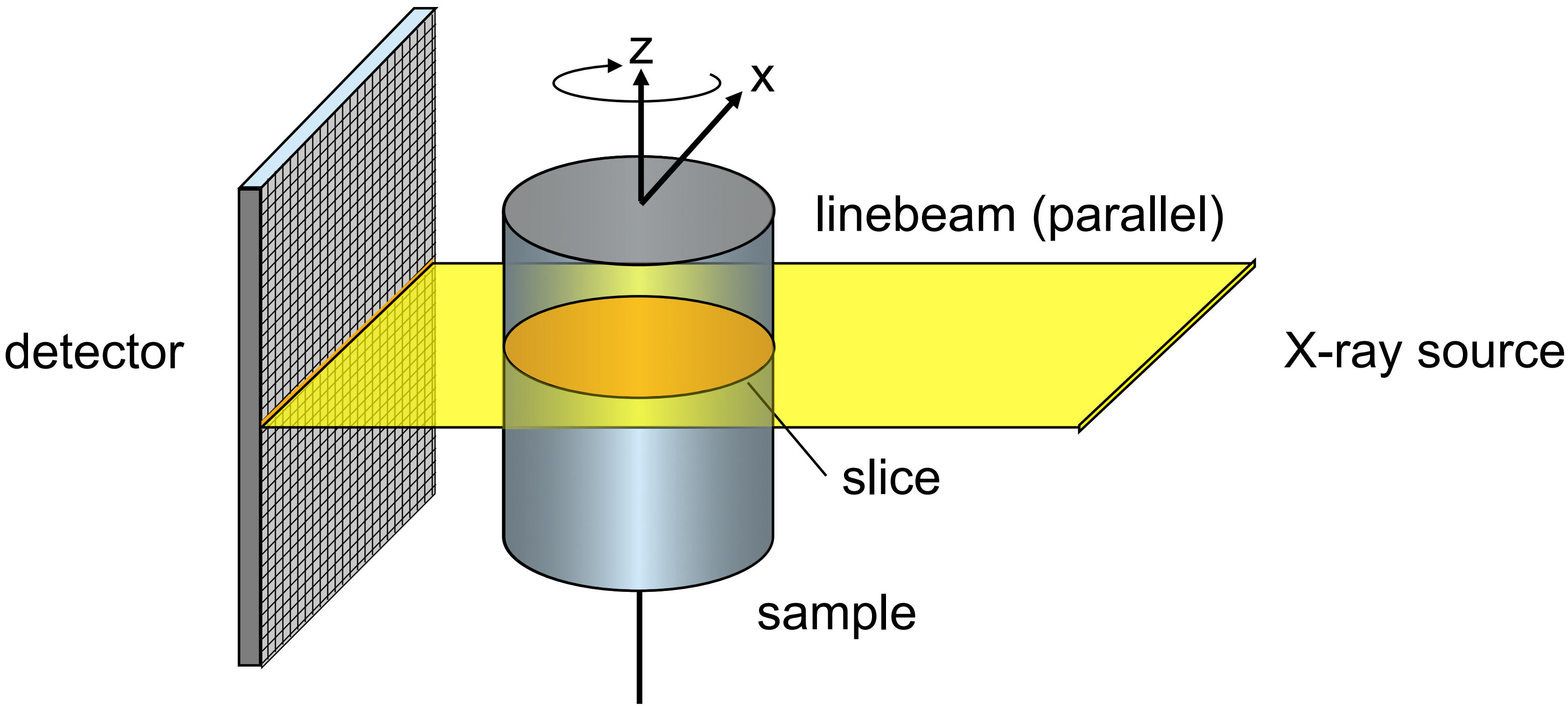
# Beam geometry: pencil beam

- Simplest beam geometry
- Integrating one detector pixel at a time



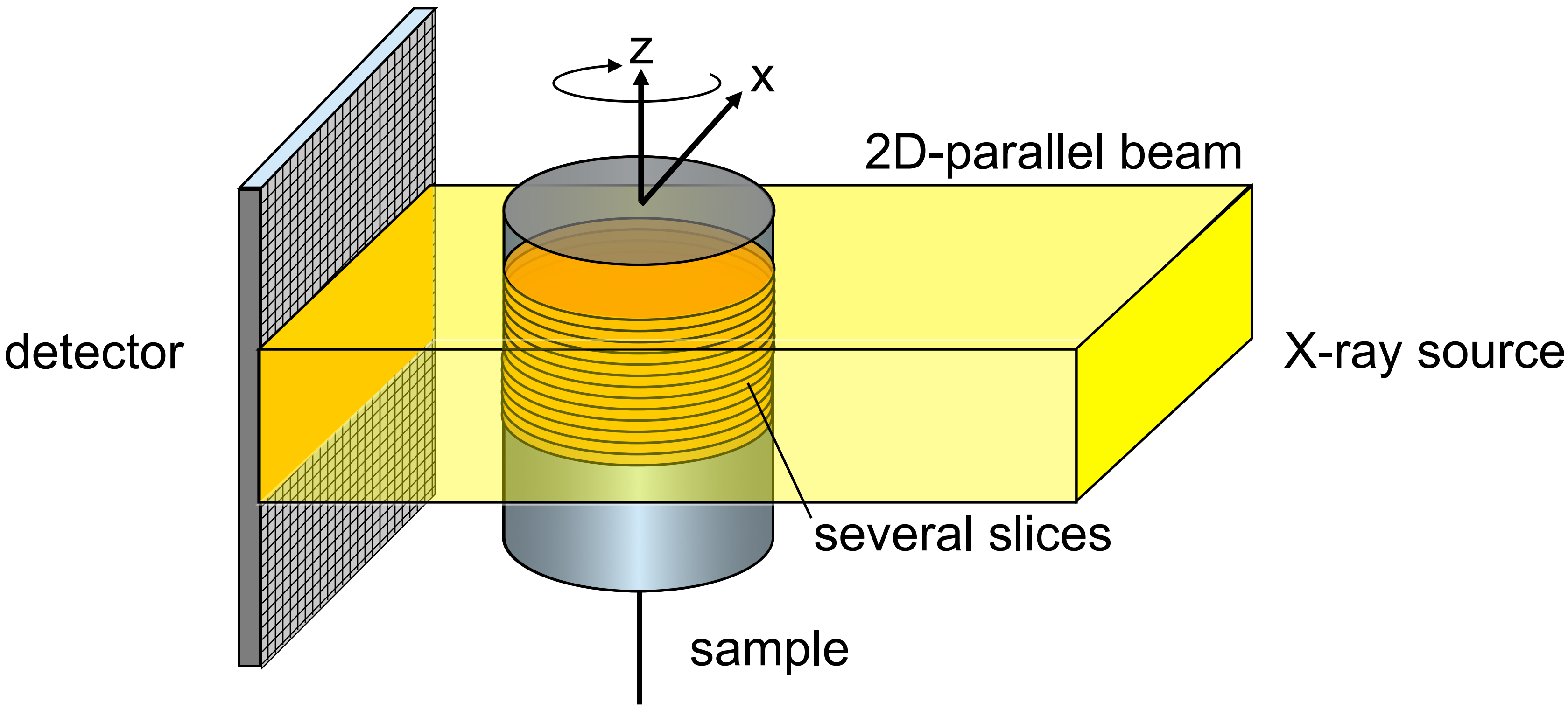
# Beam geometry : linebeam

- Integrating a row of detector pixels at a time (-> faster scans)



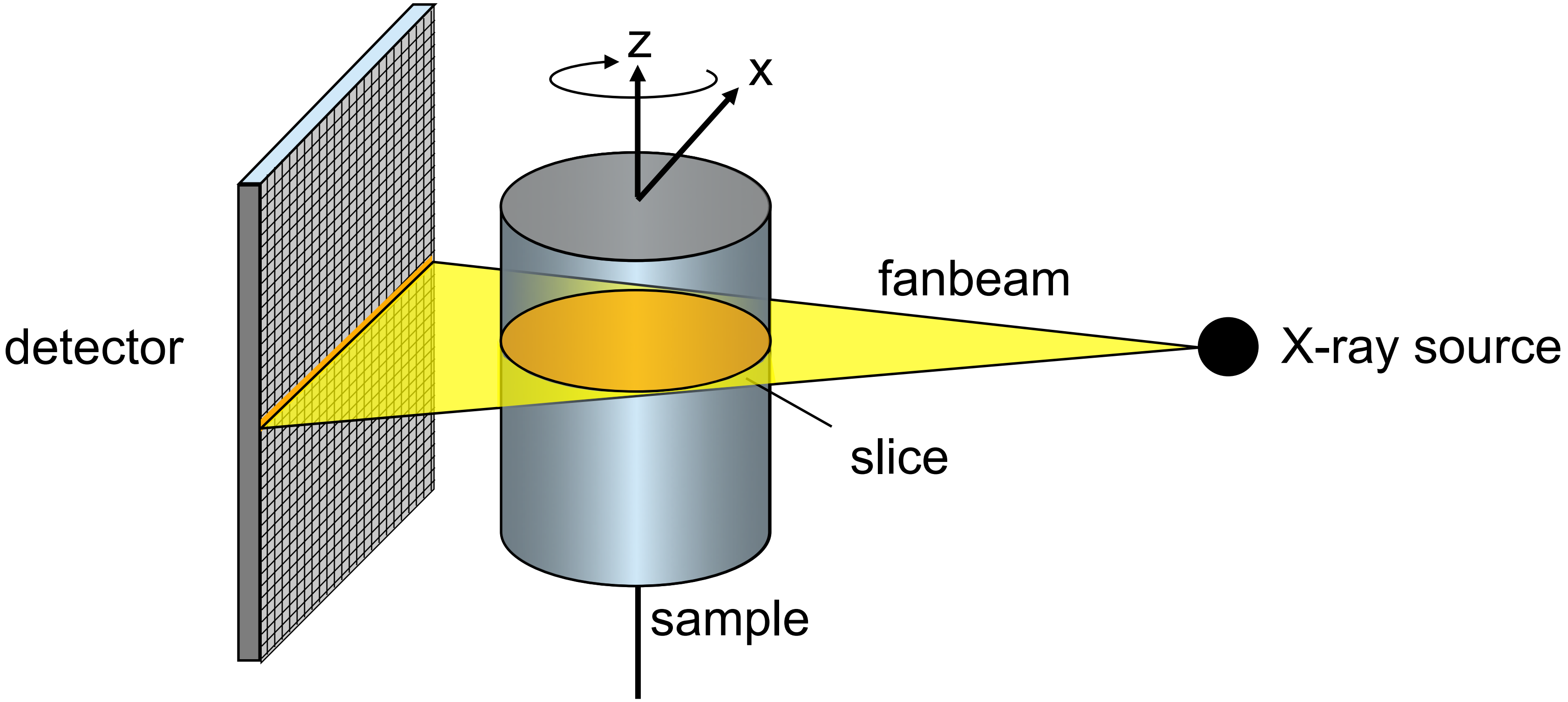
# Beam geometry: 2D-Parallel Beam

- Integrating a 2D array of pixels at a time, very fast
- Simple reconstruction



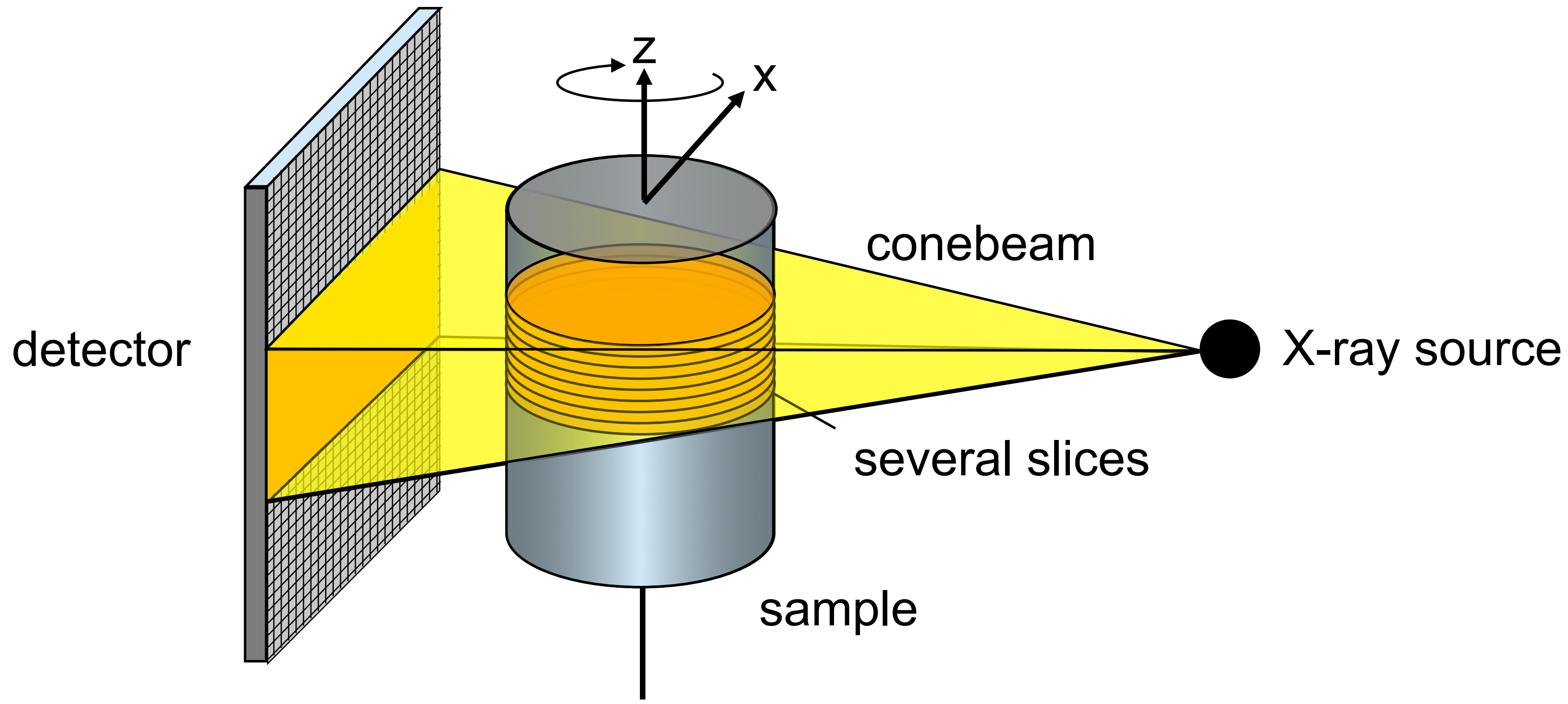
# Beam geometry: fanbeam

- Integrating a row of pixels at a time
- Magnification effect by fan geometry



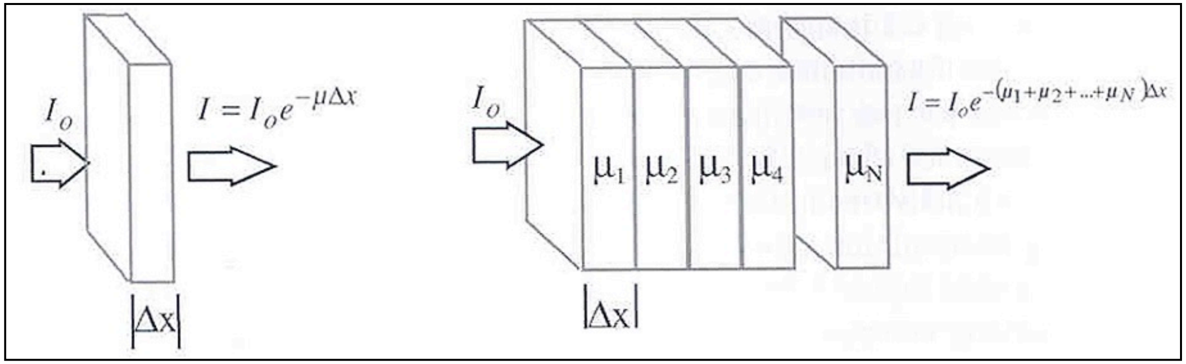
# Beam geometry: conebeam

- Integrating a 2D array of pixels at a time
- Magnification effect by cone geometry
- Reconstruction is non-trivial, since beam travels through different slices



# How to compute a CT image?

- Arbitrary Object:




- It holds : $$I = I_0 e^{-\mu_1 \Delta x} e^{-\mu_2 \Delta x} e^{-\mu_3 \Delta x} .. e^{-\mu_n \Delta x} = I_0 e^{-\sum_{n=1}^{N} \mu_n \Delta x}$$

- Define the *projection measurement*: $$\boxed{p = -\ln\left(\frac{I}{I_0}\right) = \sum_{n=1}^{N} \mu_n \Delta x = \int_L \mu(x) dx}$$

- The question is: Given the measured line integrals of the object, how do we estimate or calculate its attenuation distribution?

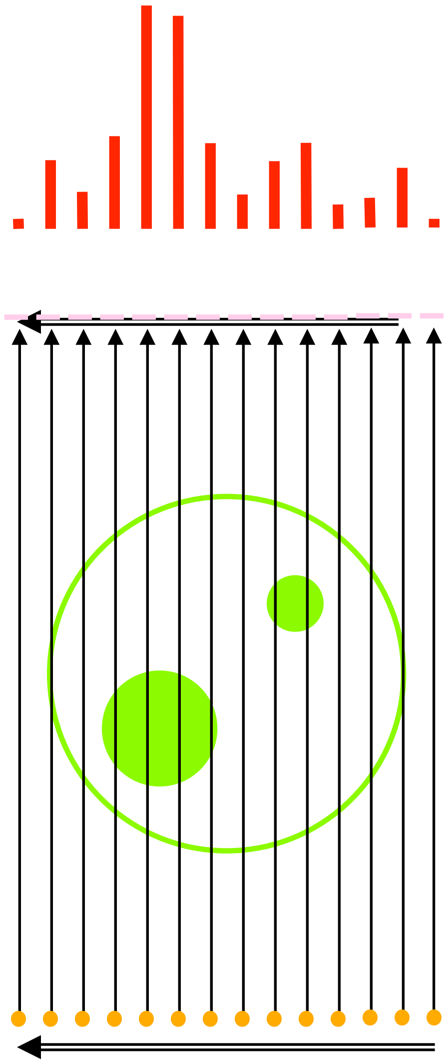
# Basics CT

- Discussion of the projection:

$$\boxed{p = -\ln\left(\frac{I}{I_0}\right) = \sum_{n=1}^{N} \mu_n \Delta x = \int_L \mu(x)dx}$$

- Basic assumptions:
  - **Monoenergetic** nature of the incoming X-ray beam
    If not satisfied: beam-hardening artifact (cupping, shading, streaking)
  - **Parallel beam**, **no scattering**
    If scattered radiation considered: low-frequency bias to the attenuation measurement
  - **Ideal detector**
    Real detector has dark current, read-out noise, hysteresis
  - **Others source of problems**: sample motion, under-sampling, partial volume, projection truncation, etc.…

# Data acquisition (pencil beam case)

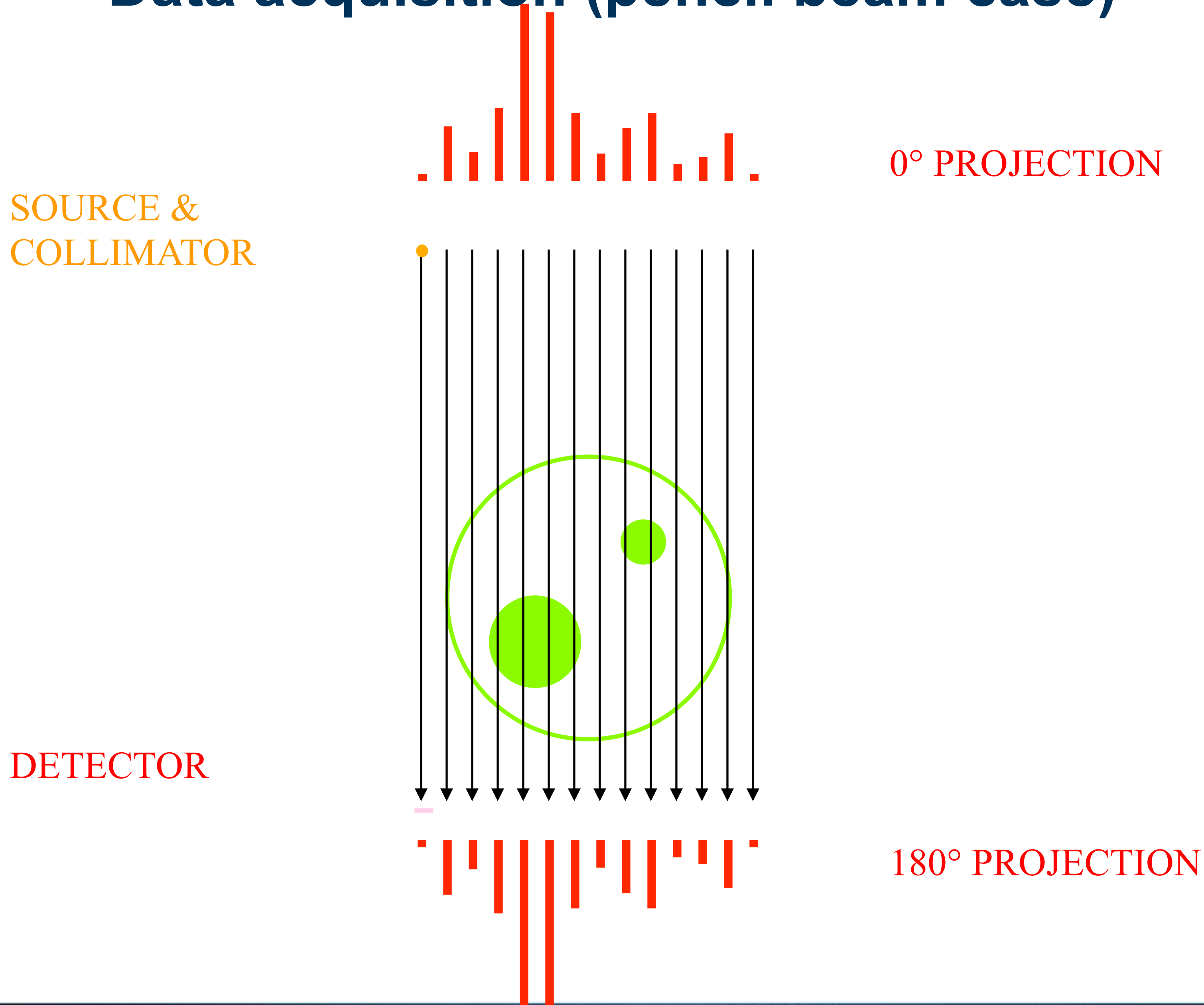


DETECTOR

$$\int \mu(z)dz = -\ln\left(\frac{I}{I_0}\right)$$

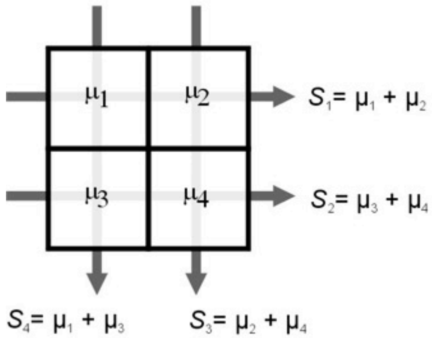
SOURCE

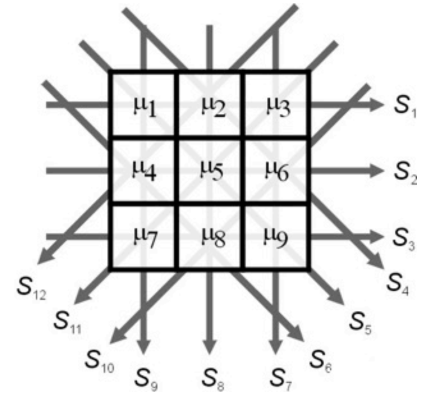
# Data acquisition (pencil beam case)



# Computation of CT images (early times)

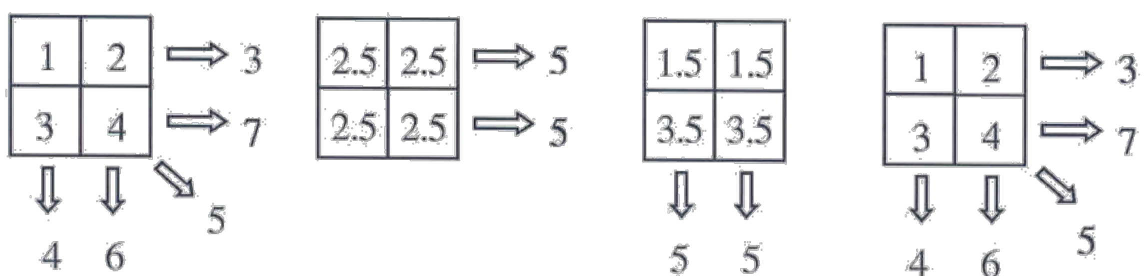
- **Direct inversion** (solving N equations with N unknowns)


  - Houndsfield in 1967 solved 28000 equations simultaneously!
  - Need more than $N^2$ measurements to overcome linear independence

- **Iterative reconstruction methods**



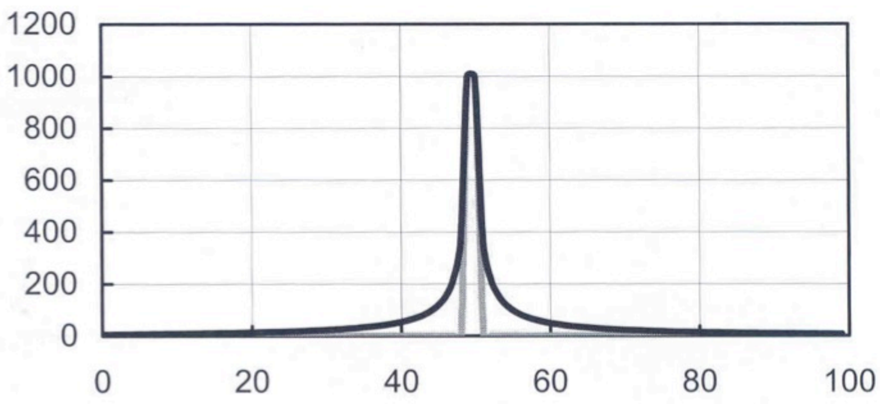

  - Iterative reconstruction methods (image estimation, pseudo-projection, comparison with original projection, error minimization) → conversion speed

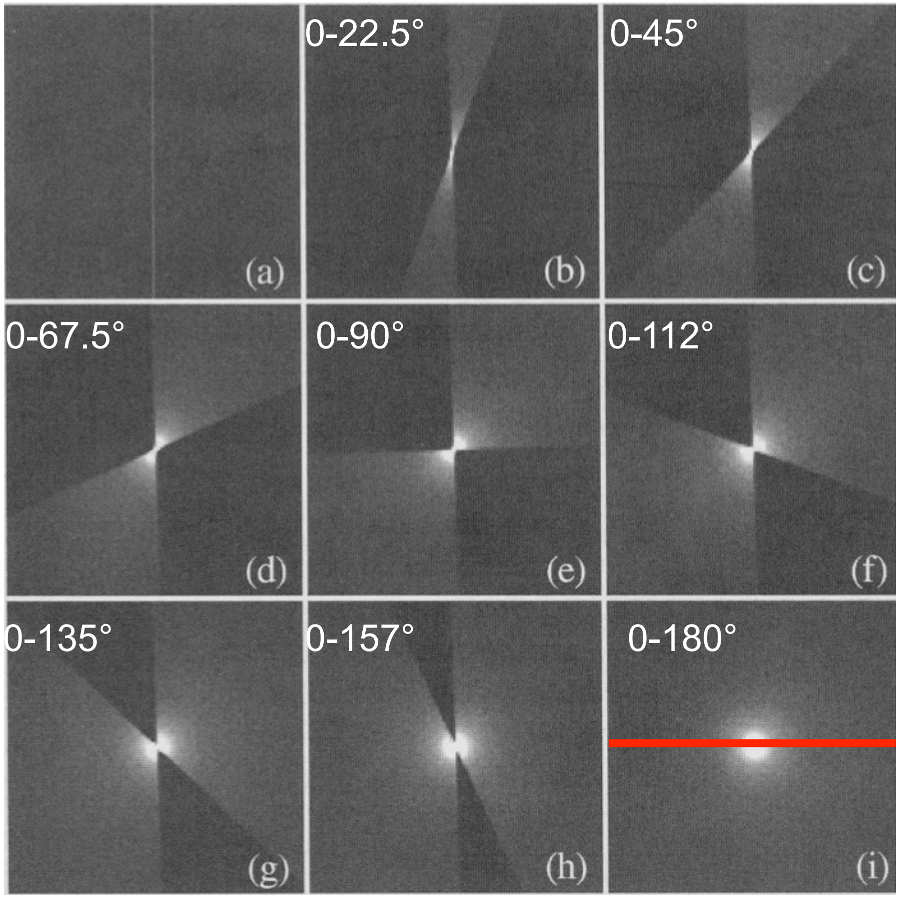
J. Hsieh, Computed Tomography: Principles, Design, Artifacts and Recent Advances, SPIE Press Monograph, ISBN 0-8194-4425-1

# Reconstruction using backprojection

- If **no a priori** information is known, the intensity of the object is assumed to be uniform along the beam path.
- The projection intensity is evenly distributed among all pixels along the ray path

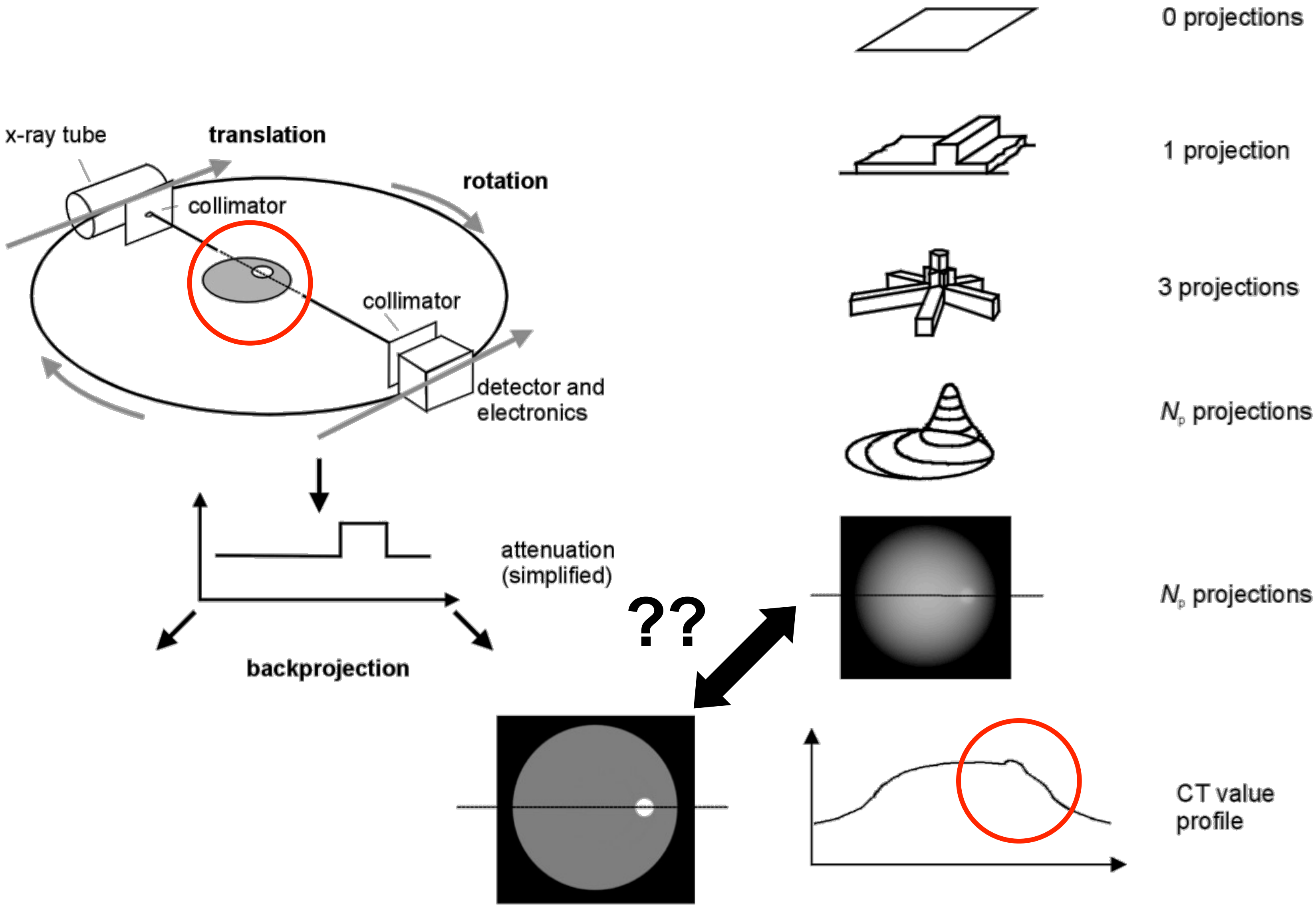
→ **Concept of backprojection!**




Backprojection of a point …

# Intuitive reconstruction using backprojection



# Radon transform

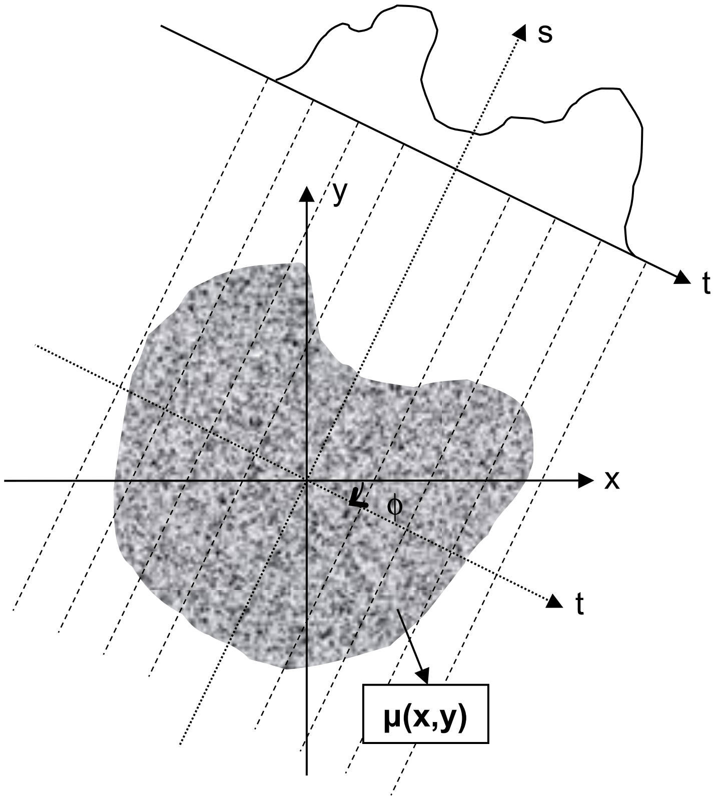
- The Radon transform in 2D is the integral of a function over straight lines and therefore represents the projections data as obtained in a tomographic scan

$$R(t,\phi) = \int_l \mu(\mathrm{x},\mathrm{y})\,dl$$

$$R(t,\phi) = P_\phi(t) := -\ln\left(\frac{I_\phi(t)}{I_0}\right)$$

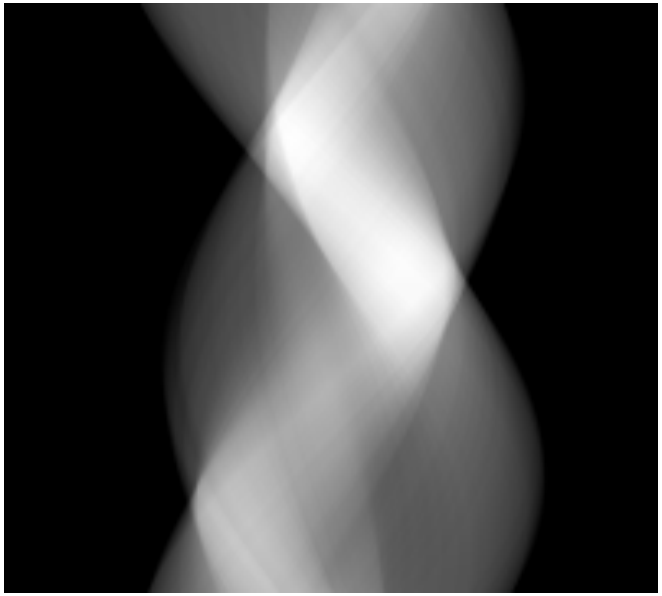
$$R(t,\phi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu(\mathrm{x},\mathrm{y})\,\delta(x\cos\phi + y\sin\phi - t)\,dx\,dy$$

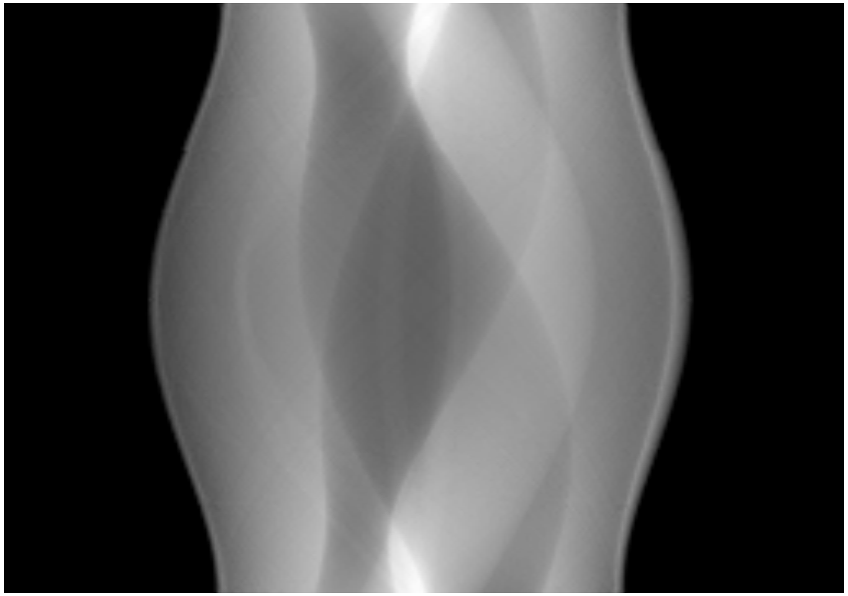
Wanted !



# Radon transform

- The Radon transform is often called a **sinogram** because the Radon transform of a Dirac delta function is a distribution supported on the graph of a sine wave

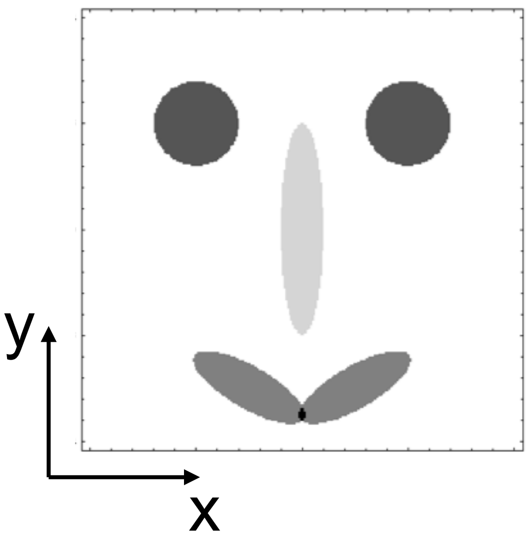


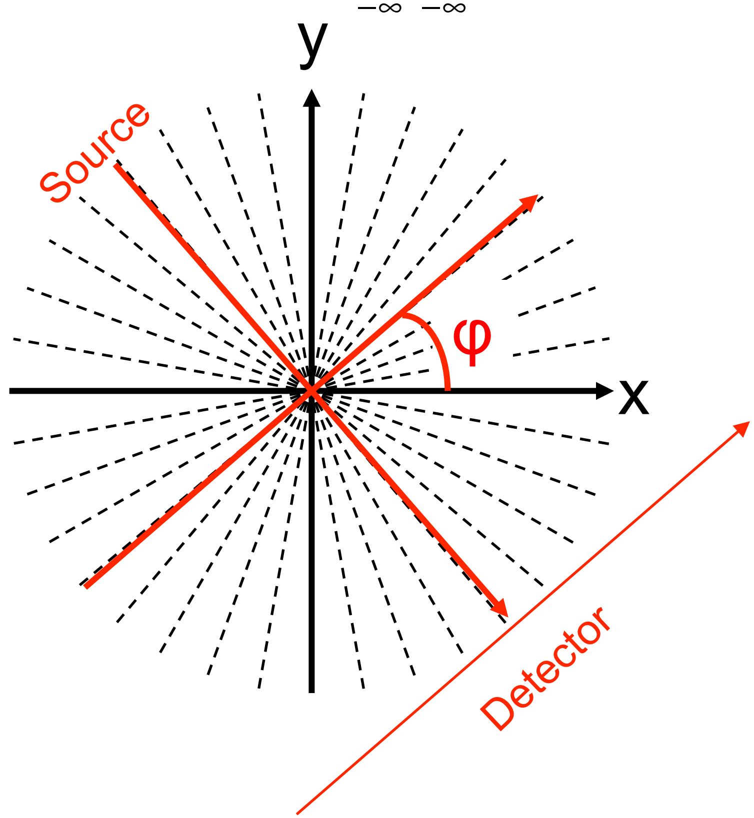


# Radon transform, again, what is this?

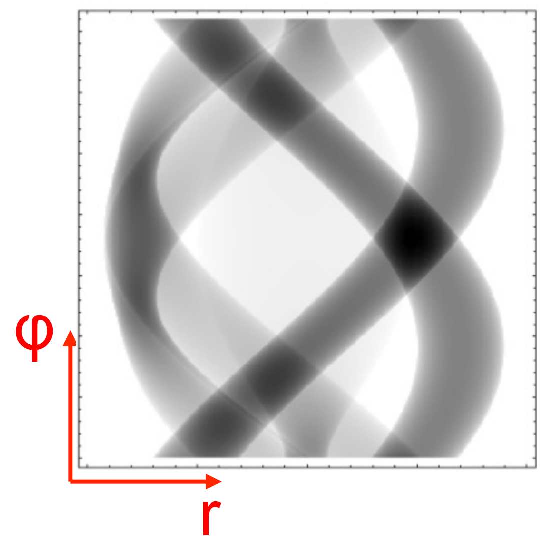
Radon transform:

$$-\ln\left(\frac{I_\varphi(r)}{I_0}\right) := P_\varphi(r) = -R_\mu(r,\varphi) = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mu(\mathrm{x},\mathrm{y})\,\delta(x\cos\varphi + y\sin\varphi - r)\,dx\,dy$$




"Sinogram"



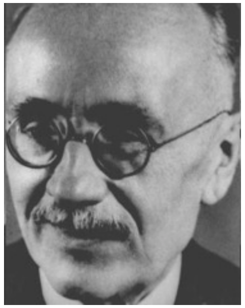

Radon transform = "image encoding principle" in CT

# Radon transform

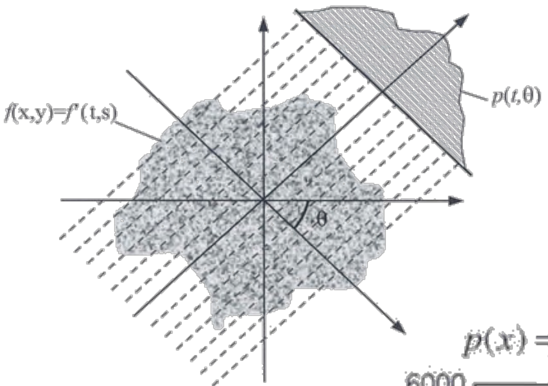
- The Radon transform was introduced by Radon, who also provided a formula for the inverse transform
- The inverse of the Radon transform can be used to reconstruct the original density from the projection data, and thus it forms the mathematical underpinning for tomographic reconstruction
- Radon found the solution to the tomographic problem already in 1917
  - but assumes an **infinite** number of projections and **continuous** projection functions
  - while we only have a **finite** number of projections and a **finite** number of detector points.

$$R(t,\phi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu(\mathrm{x},\mathrm{y})\,\delta(x\cos\phi + y\sin\phi - t)\,dx\,dy$$

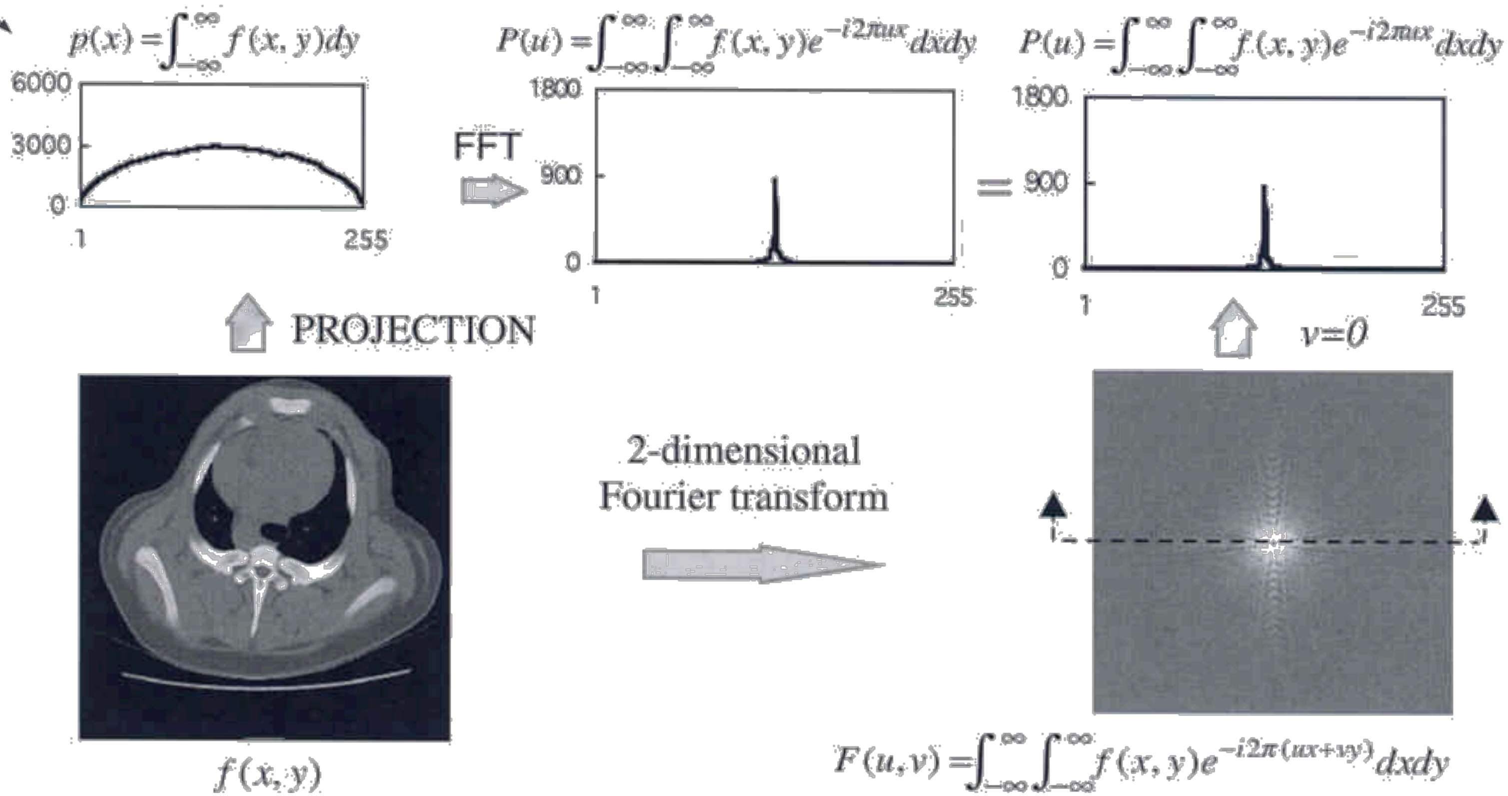
Wanted !



# Fourier Slice Theorem




The Fourier transform of a parallel projection of an object f(x,y) obtained at an angle θ equals a line of the 2D Fourier transform of f(x,y) taken at the same angle.



# Filtered backprojection (mathematical background)

$$-\ln\left(\frac{I_\phi(r)}{I_0}\right) := P_\phi(r) = R(t,\phi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu(\mathrm{x,y})\,\delta(x\cos\phi + y\sin\phi - t)\,dx\,dy$$

**Image function:** $$\mu(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{M}(u,v)\,e^{j2\pi(ux+vy)}\,du\,dv$$

Coordinate transform: $$\mu(x,y) = \int_0^{2\pi} d\theta \int_0^{\infty} \tilde{M}(\omega\cos\theta,\omega\sin\theta)\,e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\left|g\right|du\,dv$$

Cartesian to Polar

with $\begin{cases} u = \omega\cos\theta \\ v = \omega\sin\theta \end{cases}$ and $g = \begin{pmatrix} \frac{\partial u}{\partial \omega} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial \omega} & \frac{\partial v}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\omega\sin\theta \\ \sin\theta & \omega\cos\theta \end{pmatrix}$

Fourier Slice Theorem: $$\mu(x,y) = \int_0^{2\pi} d\theta \int_0^{\infty} \tilde{P}(\omega,\theta)\,e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\,\omega\,d\omega$$

Symmetry properties: $$\tilde{P}(\omega,\theta+\pi) = \tilde{P}(-\omega,\theta)$$

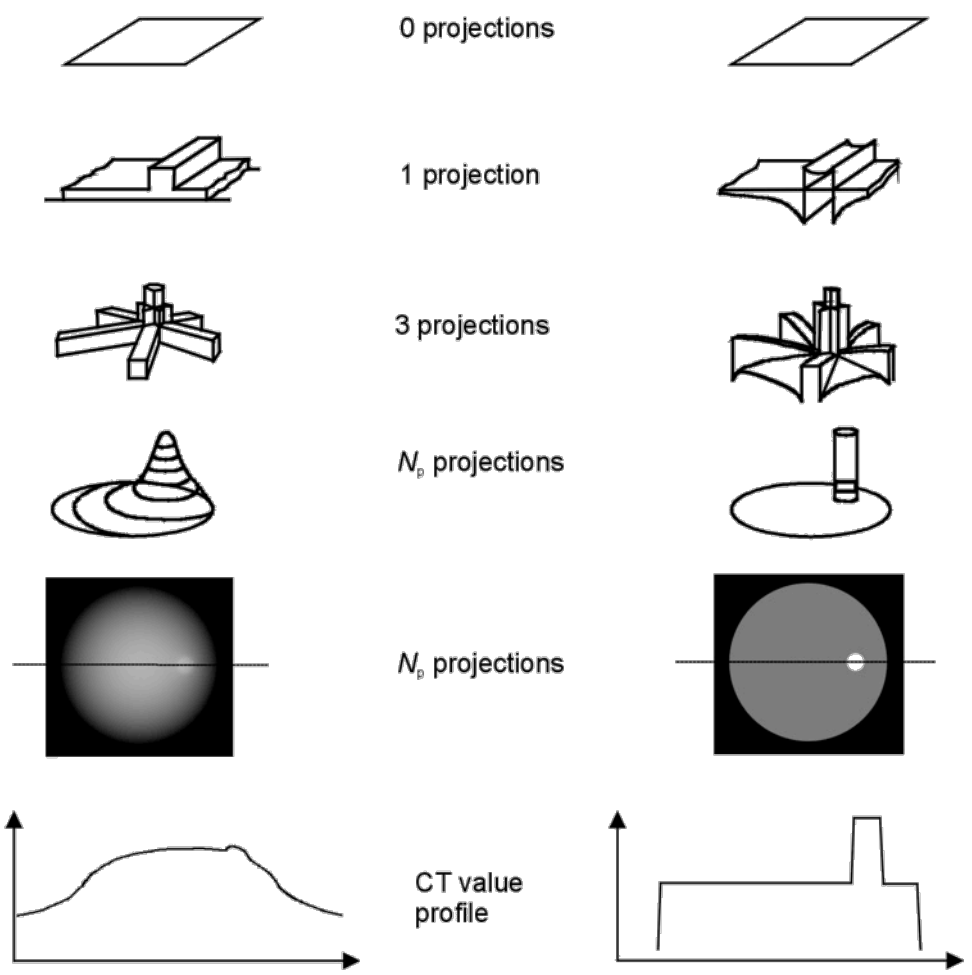
Image function: $$\boxed{\mu(x,y) = \int_0^{\pi} d\theta \int_{-\infty}^{\infty} \tilde{P}(\omega,\theta)\left|\omega\right| e^{j2\pi\omega t}\,d\omega}$$

A. C. Kak, M. Slaney, Principle of Computerized Tomographic Imaging", SIAM Classics in Applied Mathematics 33, New York, 2001, ISBN 0-89871-494-X

# Image reconstruction

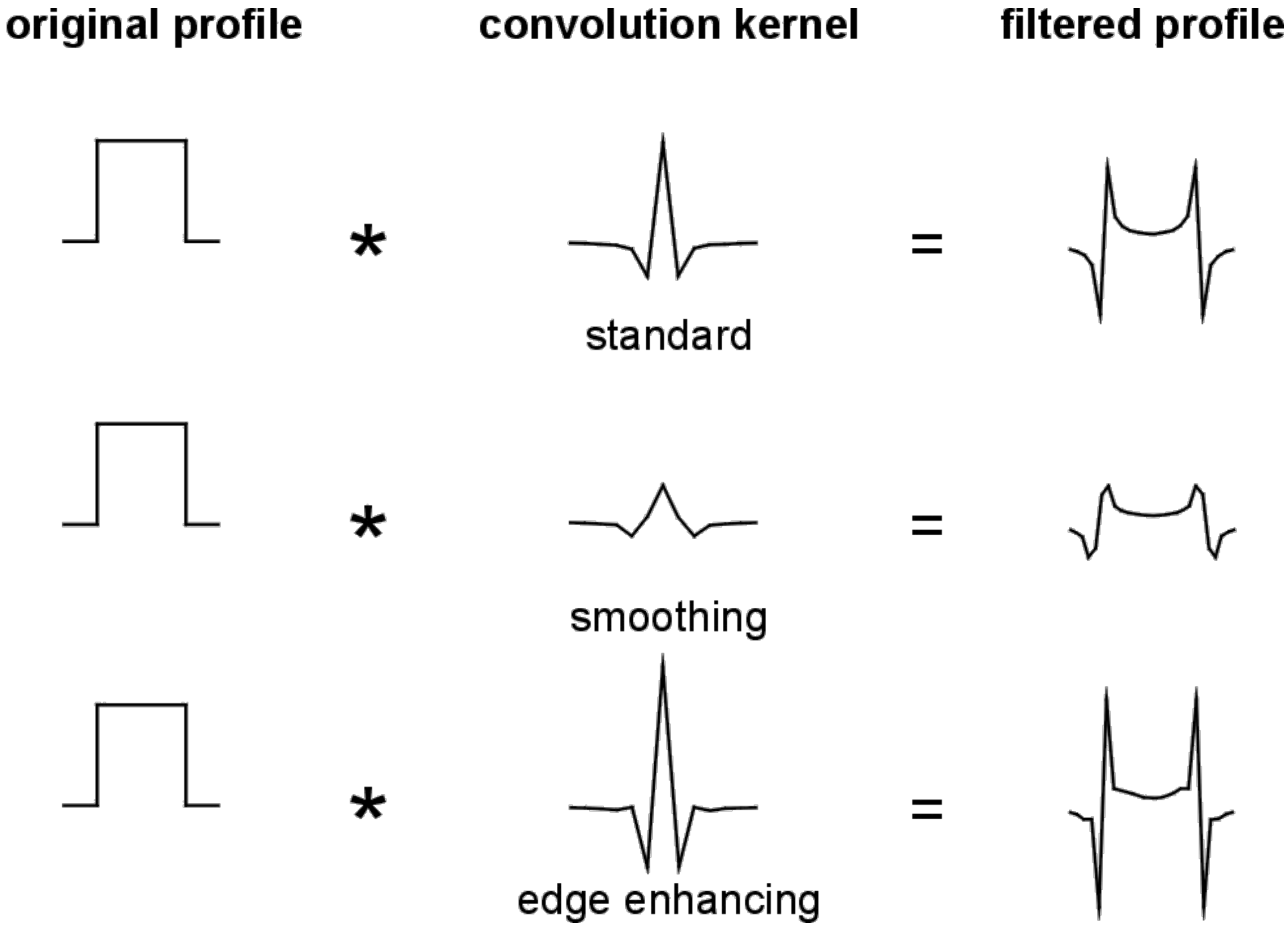
$$f(x,y)=\int_{0}^{\pi}d\theta\int_{-\infty}^{\infty}P(\omega,\theta)|\omega|e^{j2\pi\omega t}d\omega$$

without pre-filtering | **with** filtering



# Filtered backprojection

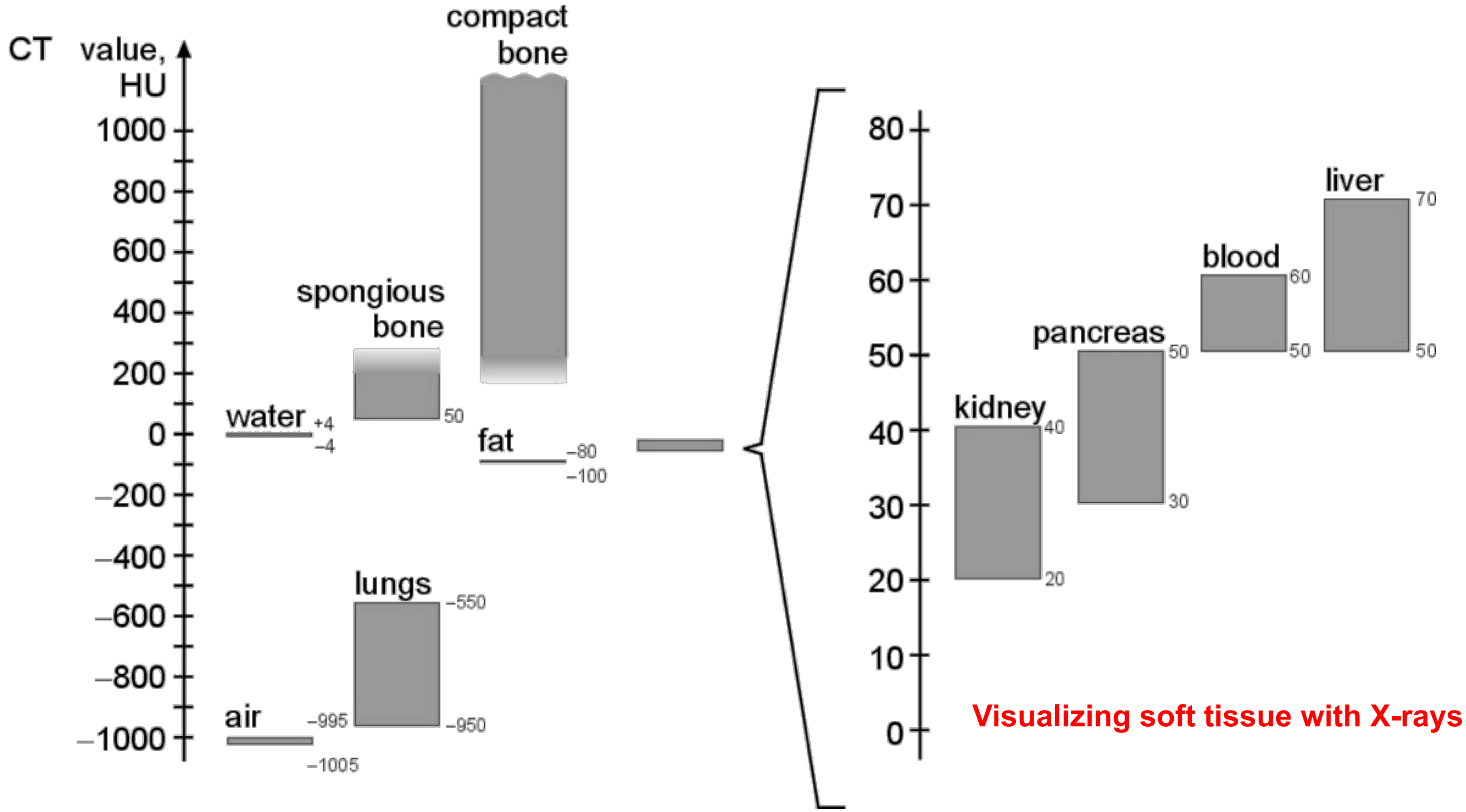
- Image characteristic can be influenced by the choice of a convolution kernel, whereby increasing spatial resolution or edge enhancement also means increasing image noise !



# Hounsfield scale (daily in medical imaging)

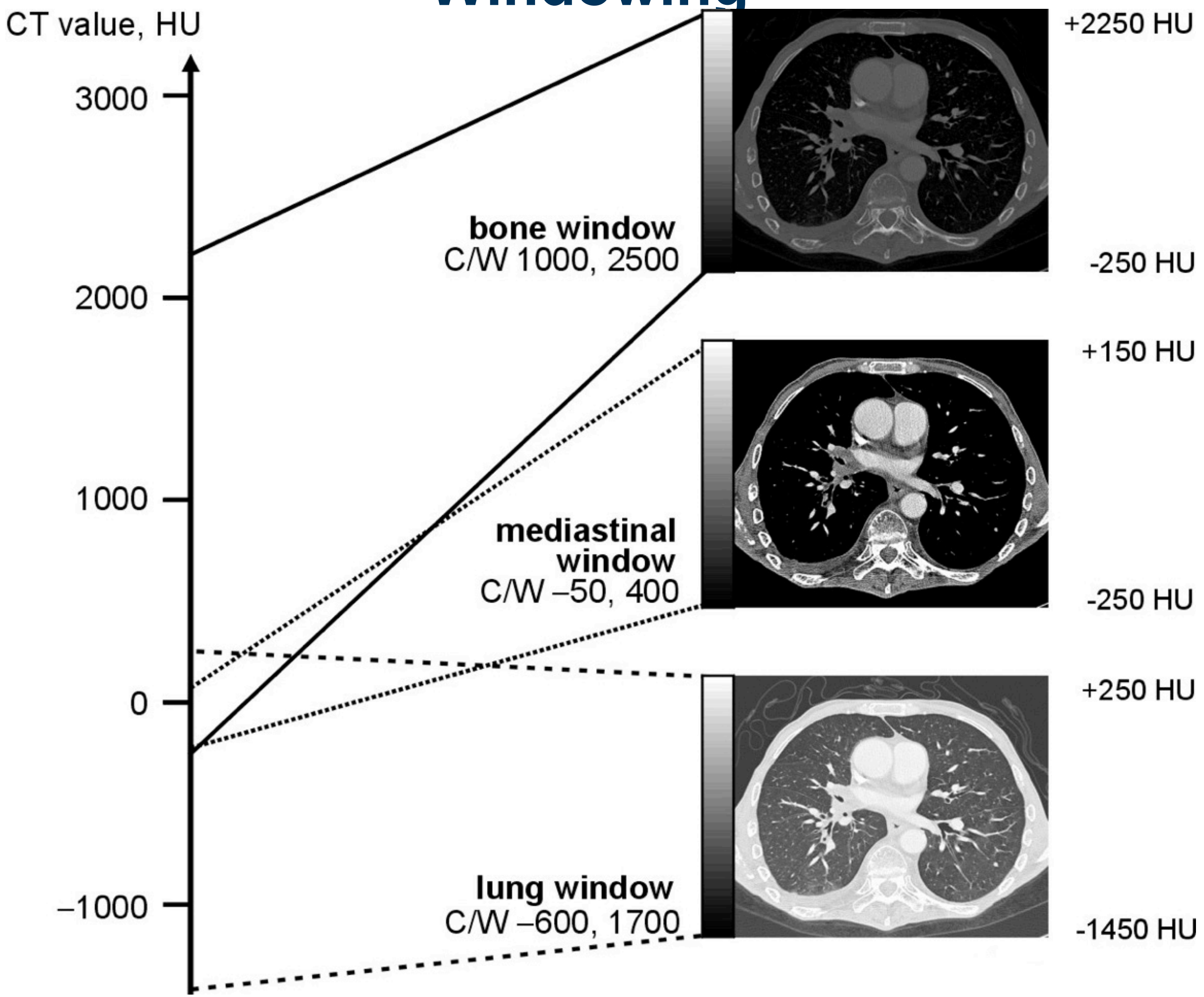
- CT values characterize the linear attenuation coefficient of the tissue in each volume element relative to the μ-value of water.

$$\boxed{CT\ value = \frac{(\mu_{Tissue} - \mu_{water})}{\mu_{water}} \cdot 1000\ HU}$$

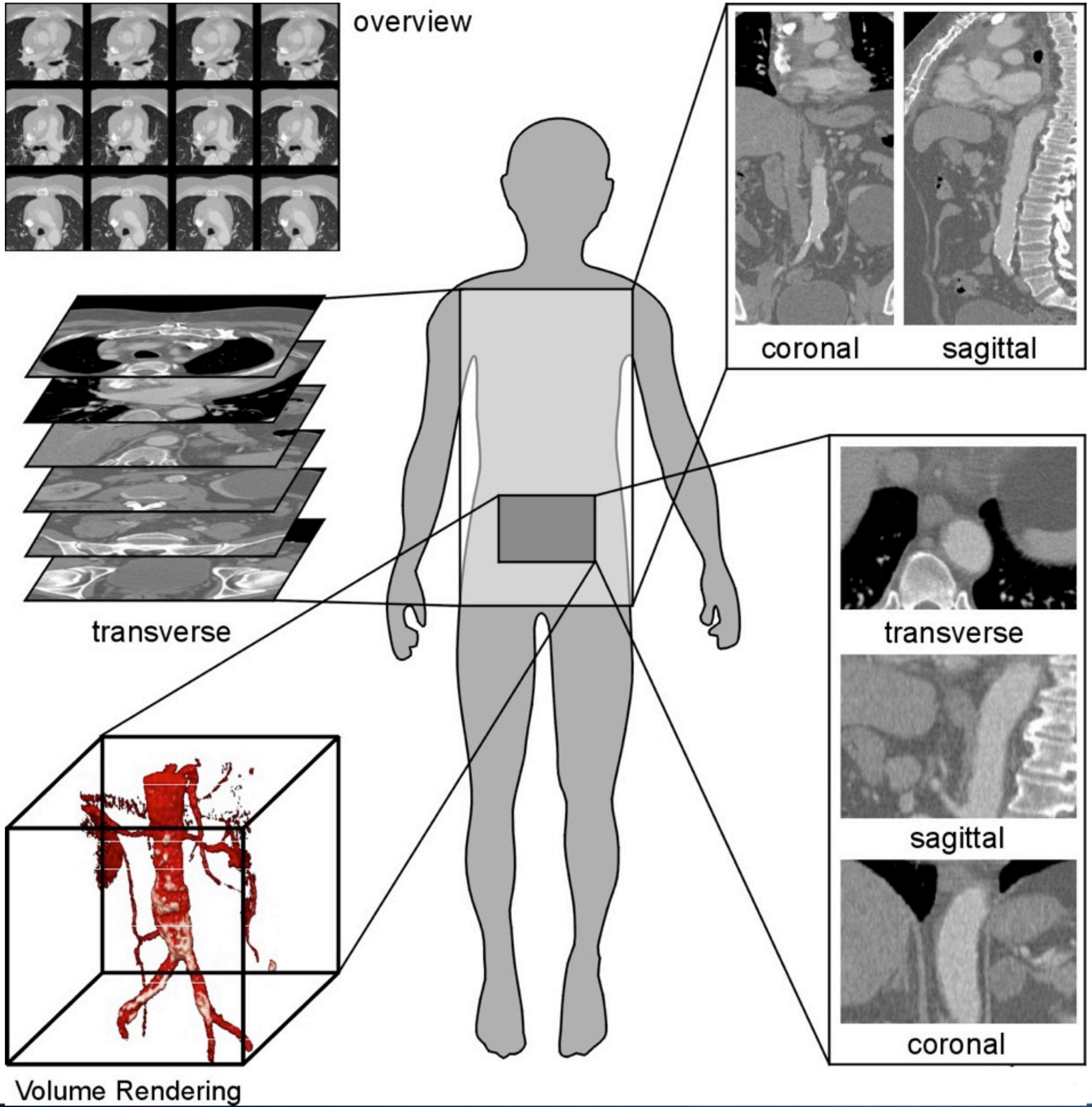



Visualizing soft tissue with X-rays is difficult!

# Windowing

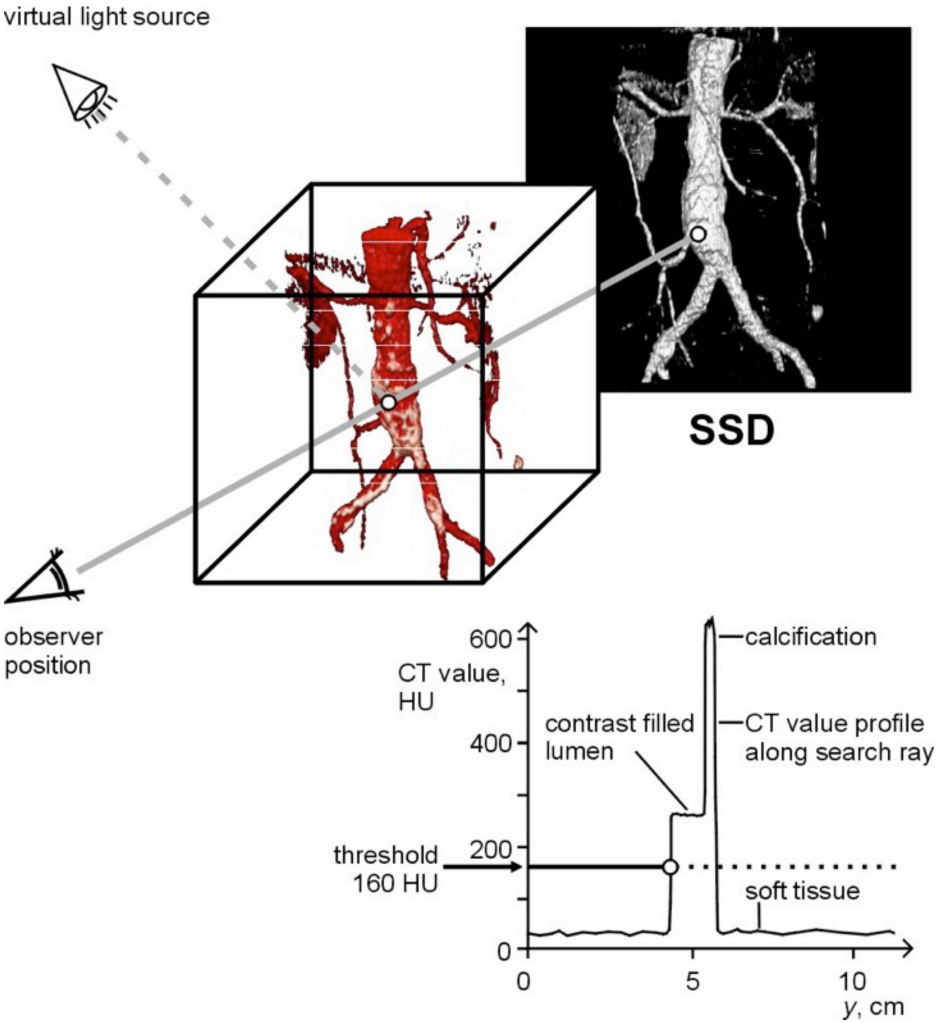

CT value, HU
3000
2000
1000
0
–1000
bone window
C/W 1000, 2500
+2250 HU
-250 HU
mediastinal
window
C/W –50, 400
+150 HU
-250 HU
lung window
C/W –600, 1700
+250 HU
-1450 HU

# Visualization of CT images



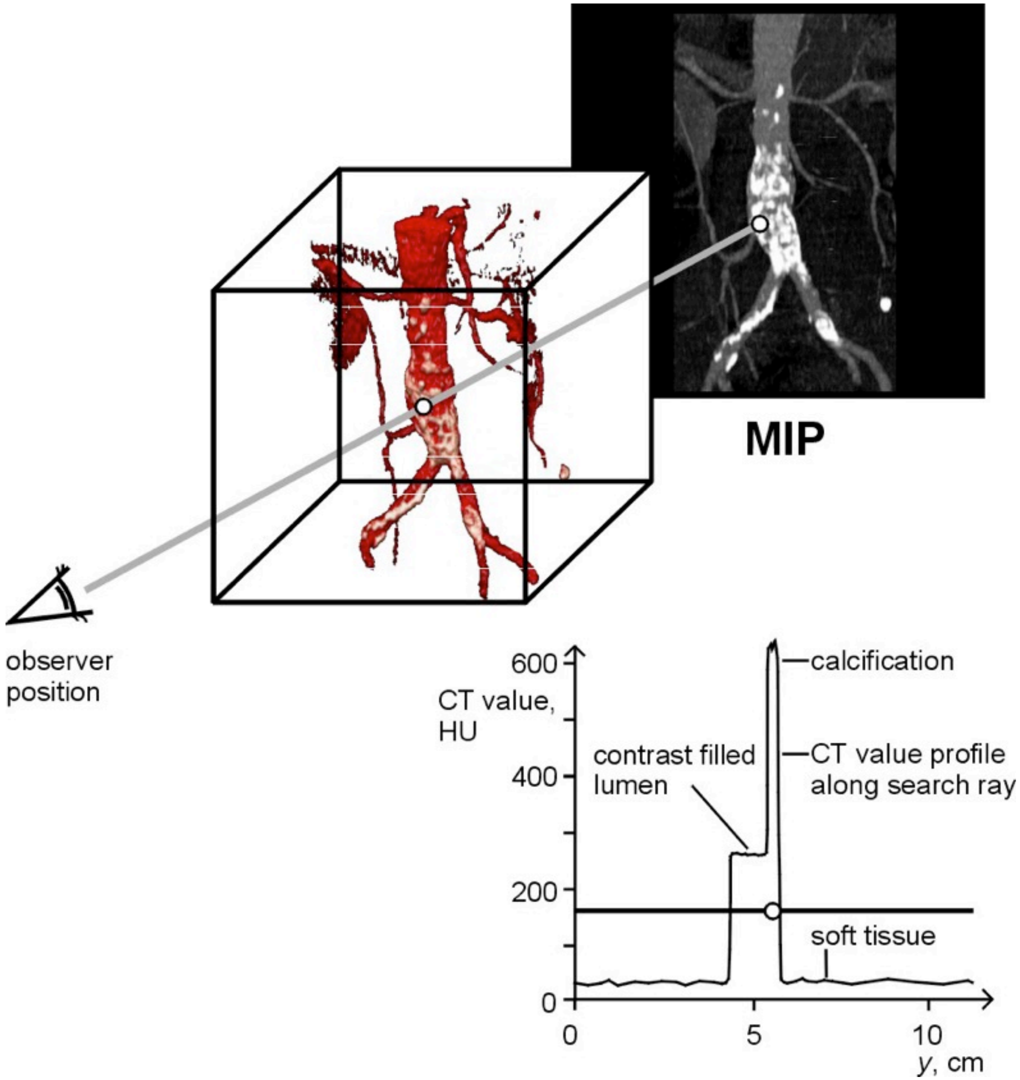

- Single/multiple images
- Arbitrary reformation:
  - Transverse
  - Sagittal
  - Coronal
  - → CT values OK!!
- 3D

# Shaded Surface Displays (SSD)



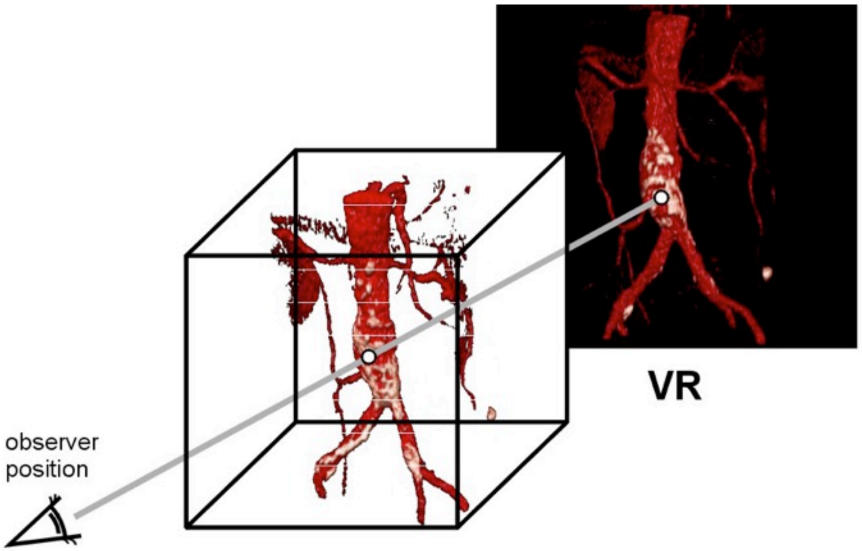

- Principle:
  - The voxel along a search ray from an assumed observer position through the 3D data volume is determined for which a predefined CT value is reached for the first time
  - A surface is computed taking all determined points into account and is illuminated to generate shading effects.
- Pos:
  - Largely realistic 3D impression
- Neg:
  - Only structure close to the observer will be visible
  - Strong dipendence from pre-defined threshold value!

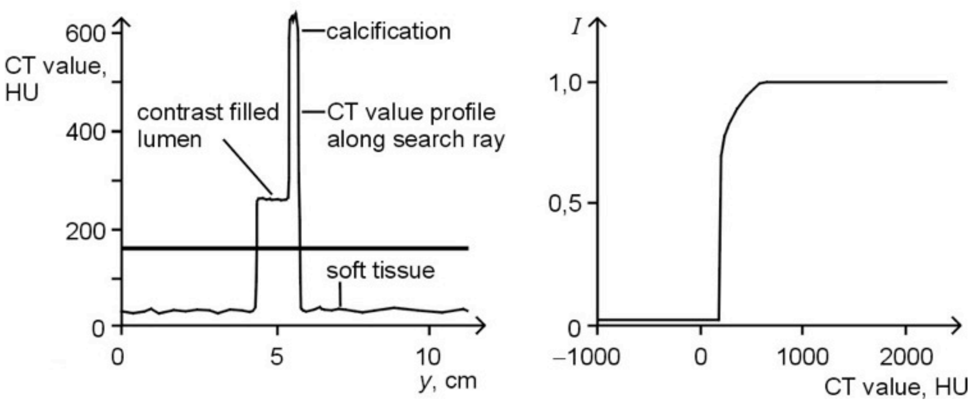
# Maximum Intensity Projections (MIP)




- **Principle:**
  - The maximum CT value found along each search ray from the observer through the 3D data volume to the fictitious image plane is used directly for display.
- **Pos:**
  - A minimum of CT value information is preserved
  - In principle a MIP image is a 2D projection image!
- **Neg:**
  - The 3D impression (generated non-intentionally) can be deceptive!

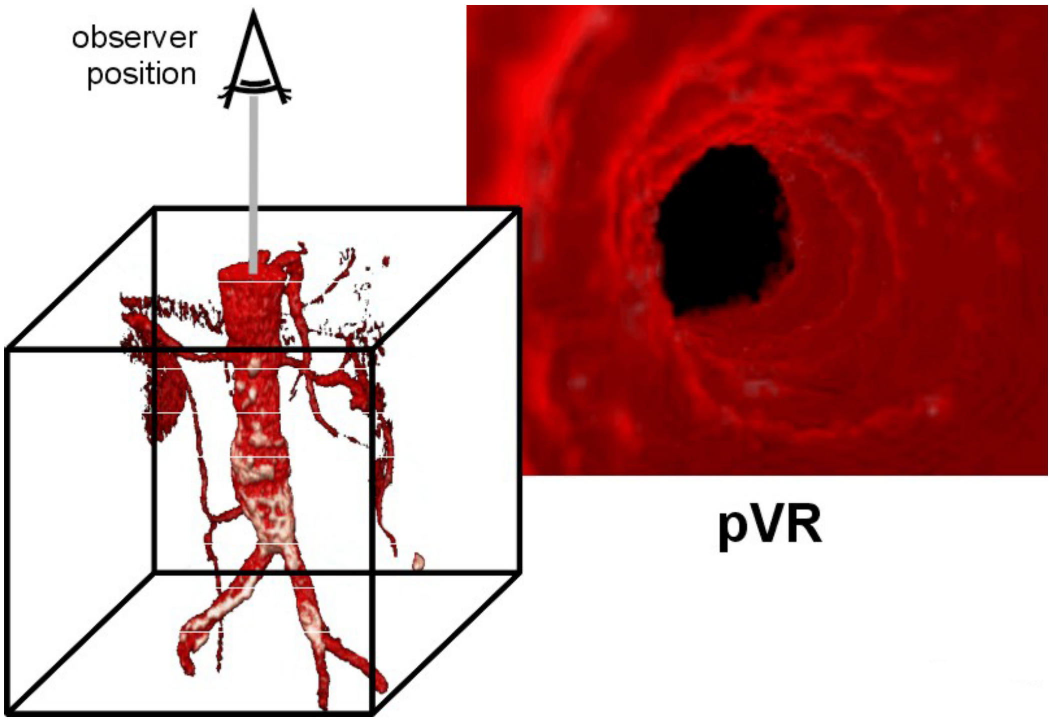
# Volume Rendering (VR)




- <u>Principle</u>:
  - Opacity and color are assigned to each CT value interval via so-called transfer functions. The sum of all CT values each search ray from the observer through the 3D data volume, weighted by the transfer functions, is displayed
  - Color intensities are used to enhance the 3D impression
- <u>Pos:</u>
  - VR is considered the method of choice among the 3D rendering approaches.
  - Great variability of displays possibilities
- <u>Neg:</u>
  - Extremely hardware/software demanding

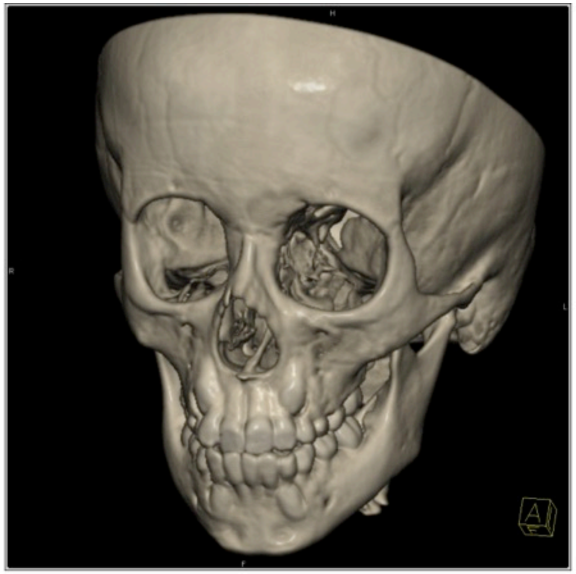
# Perspective Volume Rendering (pVR)




- Principle:
  - Display is generated similarly to volume rendering.
  - A perspective view as seen by a camera placed close-by is generated using specially adapted opacity functions
- Pos:
  - Useful for virtual endoscopy

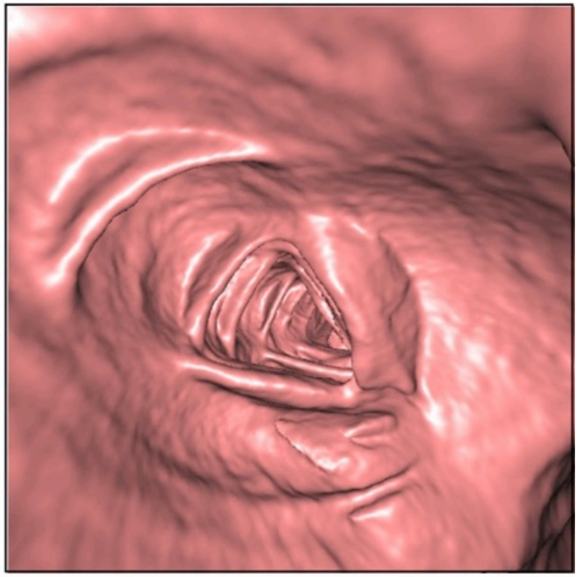
# 3D-displays

Shaded Surface



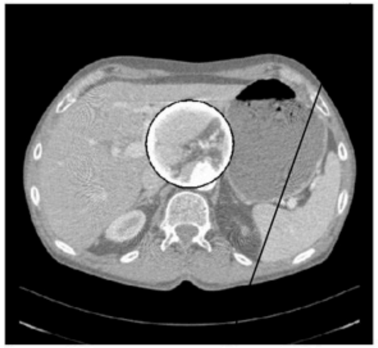
Maximum Intensity Projection

Volume Rendering

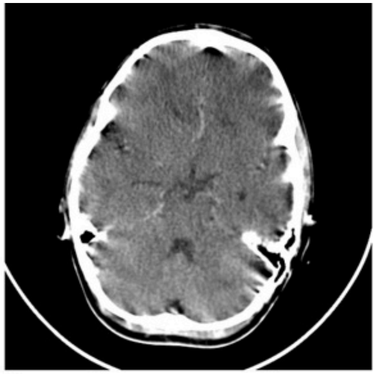


Perspective Volume Rendering

# Artifacts in CT images

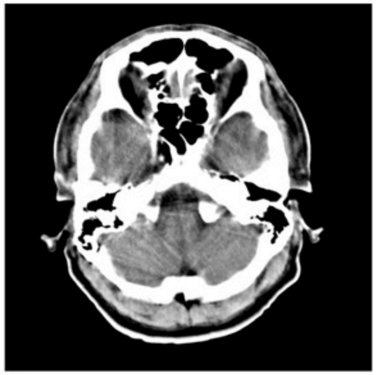


a)

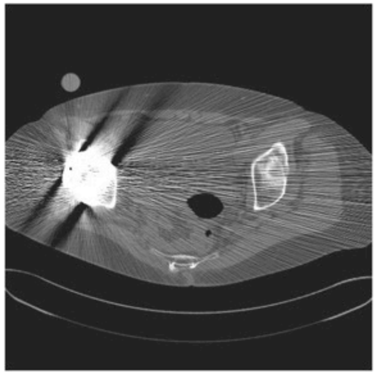


b)

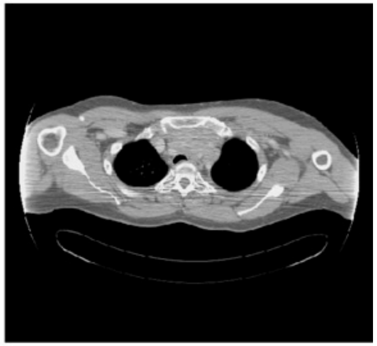
c)



d)



e)



f)

a) Failure of detector electronics
(drop out of individual measured values, difference in sensitivities of individual detector channels)
→ Streaks and ring artifacts!

b) Sample motion
→ Local blurring or more subtle falsified CT values

c) Beam hardening
(Usually detected as dark zones of streaks between hard absorbing regions) → see exercise.

d) Partial volume effects
(when high-contrast structures extend only partly into the examined slice) → see image analysis part

e) Metallic implants
(they enhance the previous artifacts and can completely extinguish the image contents)

f) Patient exceeding the field of view
→ Hyperdense areas near the exceeding peripheral regions

# **FLOW DIAGRAM for** CT Image formation

Raw data acquisition

Pre-processing and calibration
(compensate for x-ray source, detector, mechanical and sample induced imperfections)

Minus logarithm

Tomographic reconstruction

Post Processing
(correction for image artifacts, image enhancement)

Reconstructed CT data

Visualization of CT data

# References – Computed tomography


- A. C. Kak, M. Slaney, Principle of Computerized Tomographic Imaging", SIAM Classics in Applied Mathematics 33, New York, 2001, ISBN 0-89871-494-X

- F. Natterer, *"The Mathematics of Computerized Tomography",* SIAM Classics in Applied Mathematics 32, Philadelphia 2001, ISBN 0-89871-493-1

- W. A. Kalender, *"Computed Tomography – Fundamentals, System Technology, Image Quality, Applications",* 2nd revised edition, 2005, Publicis Corporate Publishing, ISBN 3-89578-216-5

- J. Hsieh, *Computed Tomography: Principles, Design, Artifacts and Recent Advances,* SPIE Press Monograph, ISBN 0-8194-4425-1

# Appendix 1.1 : Image Quality

- Image quality is of central importance for the evaluation of an imaging system.
- Frequently subjectively assessed !
- Try to be objective and give quantitative determination of individual parameters for image quality.
- As we have seen up to now:

$$I(x,y,z) = K(E) \cdot O(x,y,z) \times PSF(x,y,z) + noise + artifacts$$

Image function

Energy dependent contrast factor (detector type, filtration, etc)

Object function

Point Spread Function

# Appendix 1.2: Pixel Noise

- Pixel noise σ is determined as the standard deviation of the values $P_i$ from $N$ pixels of a given region of interest in a homogeneous image section relative to their mean value $P$:

$$\sigma^2 = \frac{1}{N-1} \cdot \sum_{i=1}^{N} \left(P_i - P\right)^2$$

- The measurement is normally performed using water phantoms.
- The noise increases when the detector registers less quanta, i.e for:
  - *high attenuation $I_0/I$, due to strongly absorbing objects*
  - *(low flux * scan time) product Q*
  - *small slice thickness S (see later)*

$$\sigma = r_F \cdot \sqrt{\frac{I_0 / I}{\varepsilon \cdot Q \cdot S}}$$

→ The dose (Q) must be increased by a factor 4 to reduce the noise by a factor 2 !!

# Appendix 1.3: System Performance

## INTERDEPENDENCE OF NOISE (σ), DOSE (D) AND RESOLUTION (Δx)

- We know:

$$\sigma = r_F \cdot \sqrt{\frac{I_0 / I}{\varepsilon \cdot Q \cdot S}} \longrightarrow \sigma \propto \sqrt{\frac{1}{D}}$$

- It can be shown that:

$$\begin{aligned} &\sigma \propto 1/\Delta x^3 \quad \text{for 2D} \\ &\sigma \propto 1/\Delta x^4 \quad \text{for 3D} \end{aligned}$$

where Δx is the resolution element!

- Implication:
  - The product of noise variance $\sigma^2$ and $\Delta x^4$ should be constant
  - The system which offers the lower product $\sigma^2 \Delta x \Delta^4$ is the best.

- → Figure of merit:

$$FigOfMerit = c \cdot \frac{1}{\sigma^2 \cdot MTF_{10\%} \cdot D}$$

# Appendix 2.1 : Dose Issues - Basics

- X-rays penetrating an object, transfer part of their energy to that object causing changes in the material.
- X-rays can produce ion pairs in the tissue during energy transfer and the ion pairs react with other chemical systems causing **indirect radiation damage**
- X-rays can strike and break molecular bonds, such as those in DNA, and cause **direct damage**

- **ABSORBED DOSE** (energy deposited per unit mass)
  - **1 Gray = 1 Gy = 1 J/kg**

- **EQUIVALENT DOSE** (reflects biological in contrast to physical effect)
  - **1 Sievert = 1 Sv = 1 J/kg**

# Appendix 2.2 : Equivalent Dose

$$1\ [Sv] = Q \cdot N\ [Gy]$$

**<u>Quality factor Q</u>**

| | |
|---|---|
| Photons, all energies | Q=1 |
| Electrons and muons, all energies | Q=1 |
| Neutrons (energy dependent) | Q=5-20 |
| Protons, energy > 2 MeV | Q=5 |
| Alpha particles and others | Q=20 |

**<u>Pertinent factor N</u>**

| | |
|---|---|
| Bone marrow, colon,lung, stomach | Q=0.12 |
| Bladder, brain, breat, kidney,liver | Q=0.05 |
| Muscles, pancreas, intestine,uterus | Q=0.05 |
| Bone surface, skin | Q=0.01 |

# Appendix 2.3: Biological effects of radiation damage

- Reaction mechanisms of a cell hit by radiation:
  - Cell repair
  - Interaction with nearby cells (bystander effect)
  - Genomic instability
  - Cell death
- Cells growing out of control becomes cancers
- Non-cancers effects includes:
  - Blood disorders
  - Circulatory problems
  - Liver disease, thyroid disease, …

# Appendix 3.1: Noise – Dose and ALARA



$$\text{signal} = (I_1 - I_2) \times A \tag{4.7}$$

Where $A$ is the area that our small target volume presents to the X-ray beam. We compare this with an identical area in the background just adjacent to the target. This reference area receives an intensity, $I_2A$ and it is the fluctuations in this count that we are trying to beat with our signal. Thus the image noise in the background just next to our target is given by noise $= \sqrt{I_2 A}$. The signal to noise ratio is then:

$$\frac{\text{Signal}}{\text{Noise}} = K = \frac{(I_1 - I_2) \times A}{\sqrt{I_2 \times A}} \tag{4.8}$$

If we now use the expressions that we obtained above for $C$, $I_1$ we can write

$$K = \frac{CI_2A}{\sqrt{I_2 A}} = C\sqrt{I_2 A}$$

but since $I_2 = Ne^{-\mu_2 T}$ and $C = \dfrac{\Delta\mu x}{1+R}$

$$\text{we have } K^2 = \frac{(\Delta\mu x)^2 \times A \times Ne^{-\mu_2 T}}{(1+R)^2} \tag{4.9}$$

if we chose $A = x^2$ and solve for $N$ we get

$$N = \frac{K^2 (1+R)^2 e^{\mu_2 T}}{(\Delta\mu)^2 x^4}$$

The number of photons for a given contrast and a given signal to noise ratio, K, increases:

-- the **inverse fourth power** of the size and of the target object
-- the **inverse square** of the difference in attenuation coefficients between target and background.

The dose delivered (D [Gy]) is directly proportional to the number of particles impinging on the sample.

For photons, the dose equivalent factor H = D x Q [Sv] equals the absorbed dose since the quality factor Q (determined by the LET) is = 1.

For any imaging techniques involving radiation sensitive “samples” the ALARA principle should be adopted.

ALARA = “As Low As Reasonably Achievable”

C. Guy, The Principles of Medical Imaging, 2005

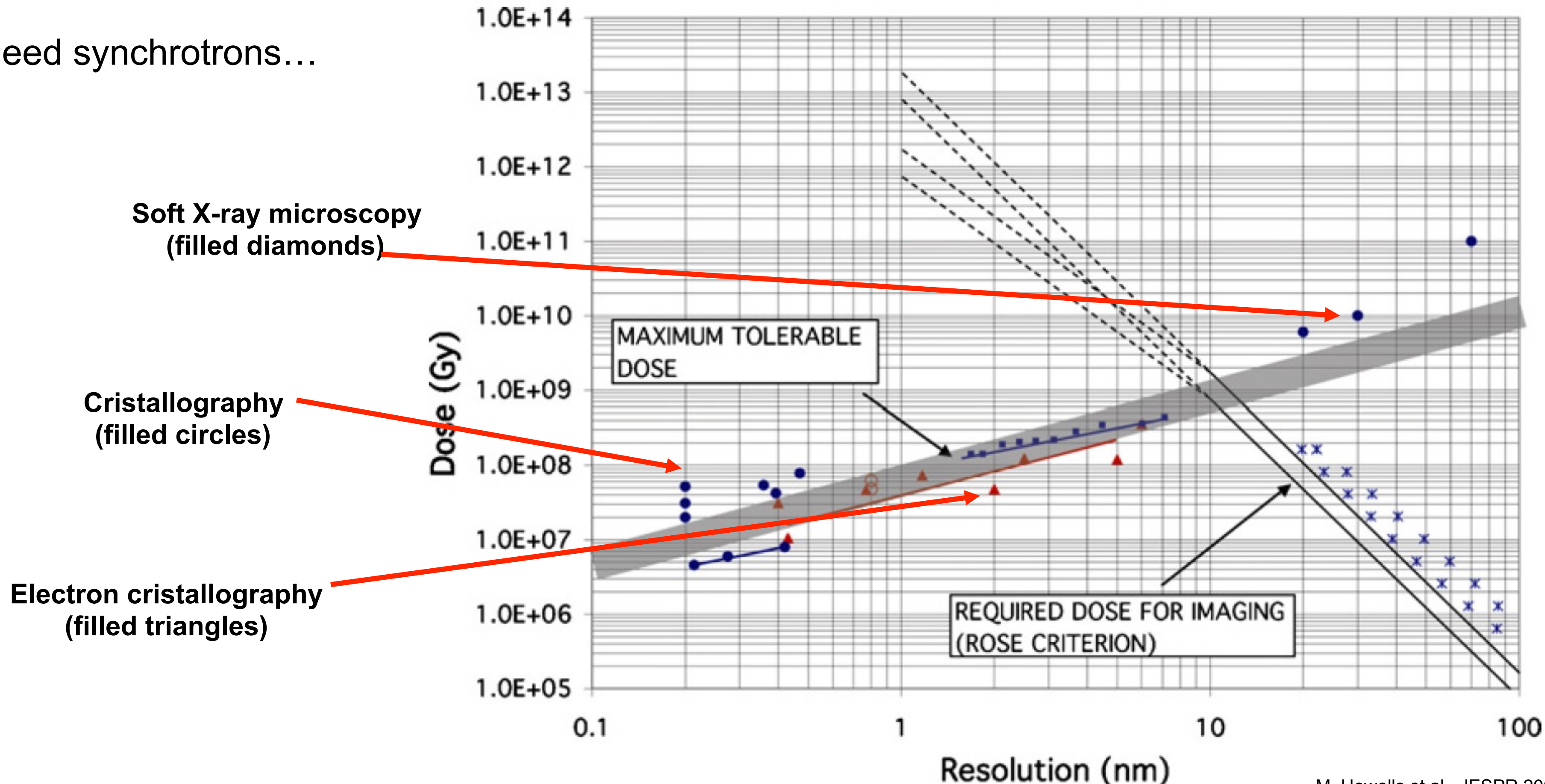
# Appendix 3.2: Dose issues

$$N = \frac{K^2 (1+R)^2 e^{\mu_2 T}}{(\Delta\mu)^2 x^4}$$

So:
1 mm visible with N
1 micron visible with N^12
1 nanometer visible with N^24

Rose Criterion: K >= 5 !

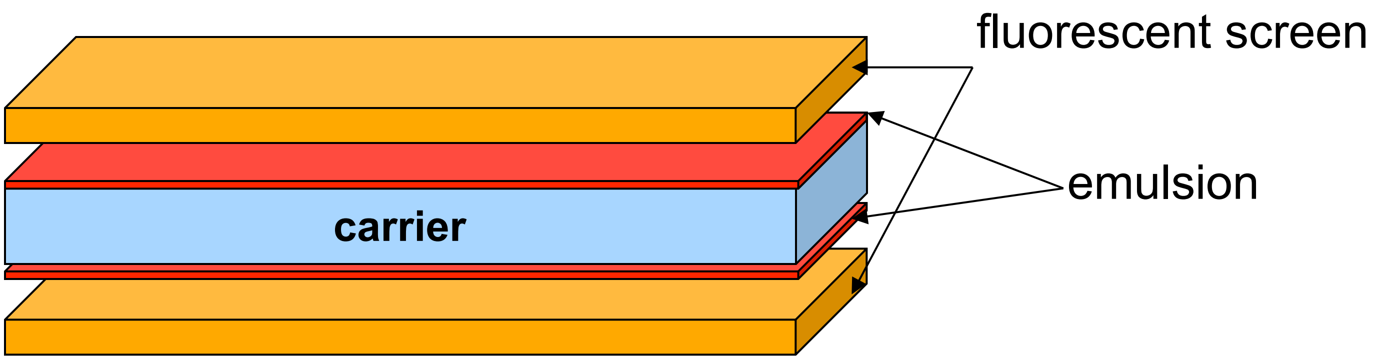
Need synchrotrons…




M. Howells et al., JESPR 2009

# Appendix 4: X-ray film

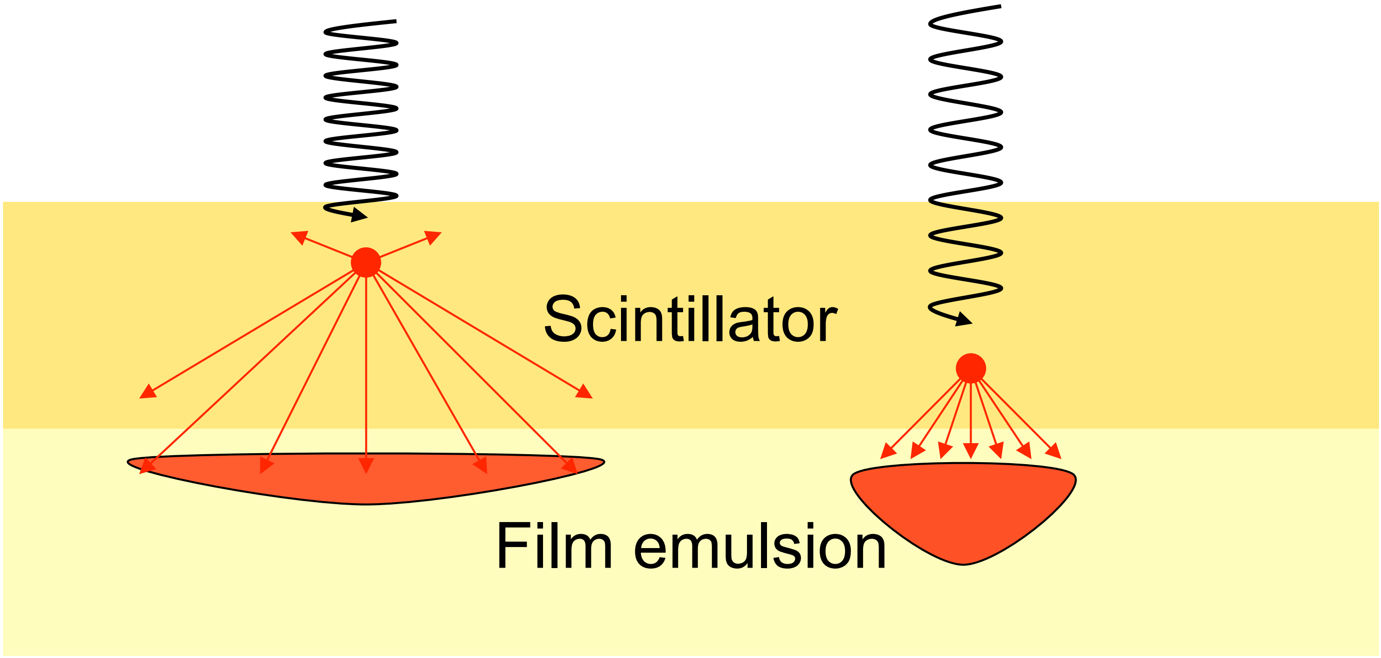
- The goal of each X-ray detector is to record position and intensity of X-rays as much as efficient and with the best resolution and contrast as possible.
- The oldest and still widely used type of X-ray detector is the **photographic plate**. The local color level in a developed film is determined by chemical reactions stimulated by the absorbed photons and depends on the local absorbed photon beam intensity




- A carrier foil made of polyester is usually coated by 1 micron thick AgBr (silverbromide) crystals in gelatine form.
- Typical density are 1 mg/cm$^2$. Efficiency only 1%!

# Appendix 4: Scintillator

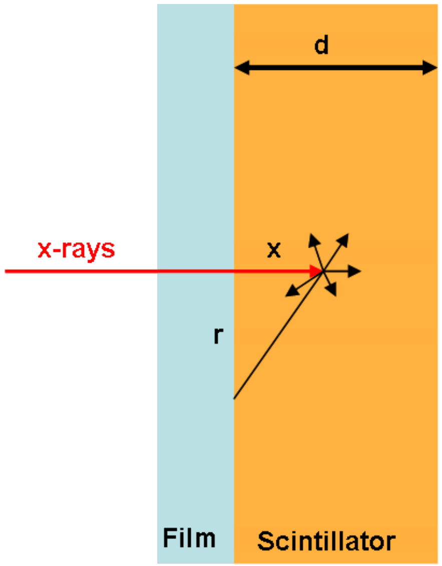
- To increase efficiency, a scintillator is usually combined with the film.
- The efficiency can be increased up to 10% but the price to pay is a loss in resolution



# Appendix 4: Resolution vs efficiency

- The light intensity per unit surface on the film is proportional to the product from the cosinus of the incident direction and inversely proportional to the square of the distance (inverse square law).
  It means:

$$h(r) = \frac{x}{\sqrt{(r^2 + x^2)}} \cdot \frac{1}{r^2 + x^2}$$




- It can be shown that the mean depth of the photons is:

$$\bar{x} = \frac{1}{\mu} - \frac{d}{\exp(\mu \cdot d) - 1}$$

- Performing the Fourier Transform of h(r) one obtains the frequency response of the detection system to a point-like irradiation. The width of the distribution function in Fourier space is called bandwidth:

$$B = \frac{1}{2\pi \cdot \left( \frac{1}{\mu} - \frac{d}{\exp(\mu \cdot d) - 1} \right)}$$

See Exercise!

# Appendix 4: X-ray film pro and cont

(+) Well established, especially in medical radiology

(-) Limited sensitivity at low exposure level

Non linear response

Resolution limited by the grain size

Comparison is difficult since gray level depends on development

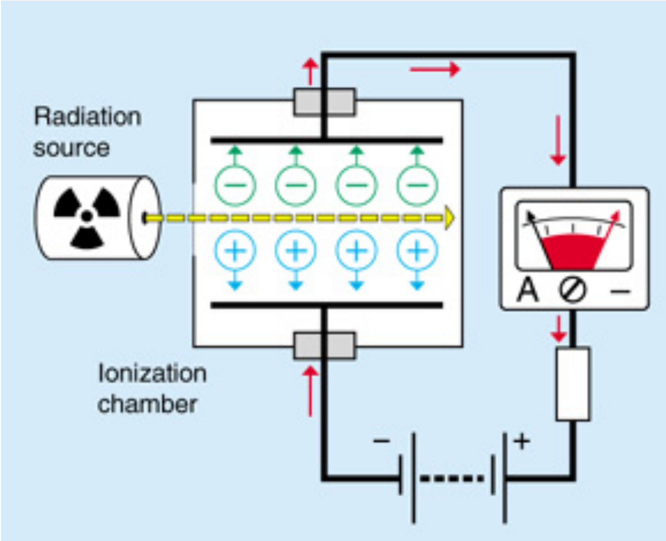
Difficult to store

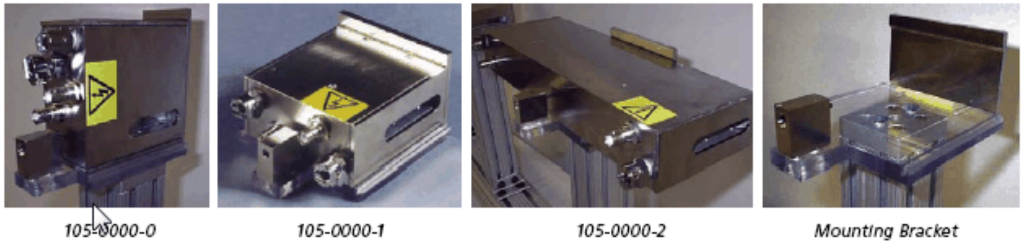
Off line

→ Move towards detectors that directly produce electronic data suitable for digitalization and online post-processing.

# Appendix 4: Ionization chamber (online beam monitoring)

- In a ionization chamber, X-rays pass through a window and reach a gas-filled chamber (no spatial information available)
- The X-ray photons are absorbed and cause the ionization of the gas molecules (which lose negatively charged electrons and therefore become positively charged)
- The photon-ionized molecules and the corresponding free electrons are separated by the voltage difference between the anode and the cathode.



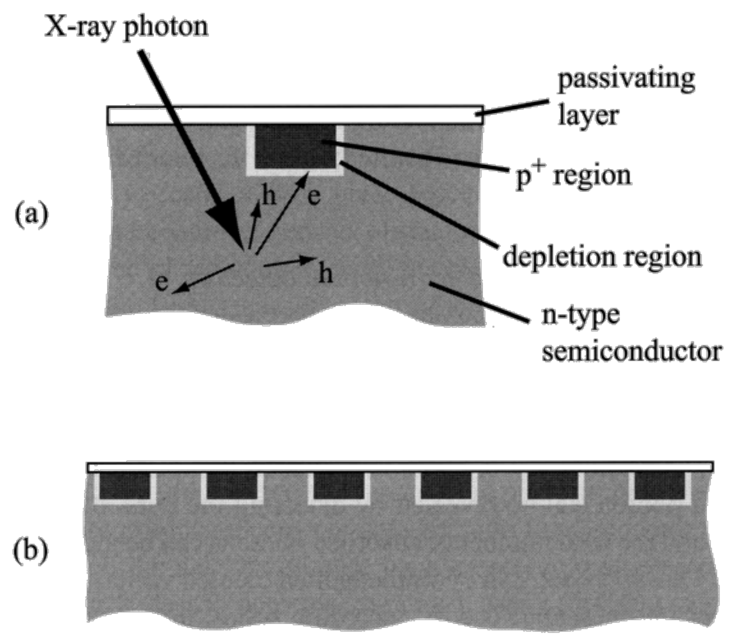




| Part Number | Electrode (HV) In. (mm) | Electrode (Collector) In. (mm) | Electrode Width In. (mm) | Electrode Separation In. (mm) |
|---|---|---|---|---|
| 105-0000-0 | 1.50 (38.10) | 0.875 (22.22) | 2.50 (63.50) | 0.438 (11.11) |
| 105-0000-1 | 3.00 (76.20) | 2.375 (60.32) | 2.50 (63.50) | 0.438 (11.11) |
| 105-0000-2 | 11.25 (285.75) | 10.62 (269.87) | 2.50 (63.50) | 0.438 (11.11) |

Some history: http://www.orau.org/ptp/collection/ionchamber/ionizationchambers.htm

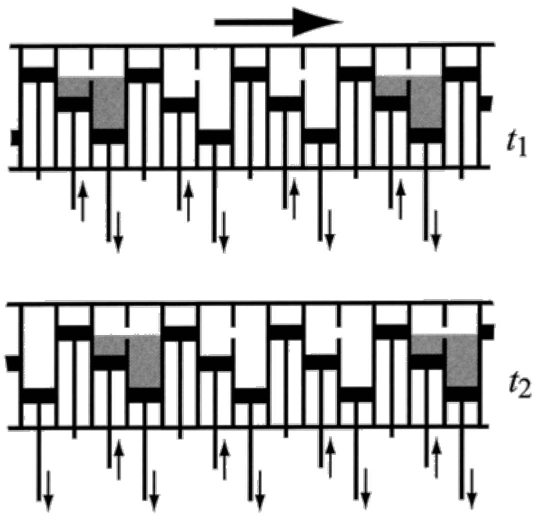
# Appendix 4: Semiconductor detectors (visible light)

- The absorbed photons create pairs of positive and negative charges that can produce an electric signal in the external circuit.
- Instead of creating electron-ion pair in gas (like the ionization chamber) the absorbed photon produces pairs of “free electrons” and “free holes” in a semiconducting material.
- By creating a series of electron-hole pairs, each absorbed photon produces a current pulse in the external circuit.
- The pulse intensity is proportional to the number of electrons-hole pairs created by the photons, which in turn increases with the photon energy
- The spatial resolution of semiconductor detectors is excellent.

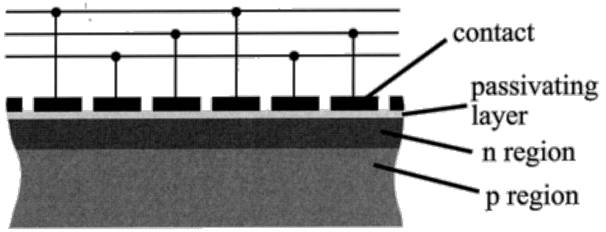


# Appendix 4: Charge coupled device (CCD) detectors

- It is an highly sophisticated semiconductor device, used nowadays in almost all camcorders and digital cameras.
- A CCD consists of MOS (metal oxide semiconductor) microelements. The operation starts with an accumulation phase, where a voltage bias transforms the MOS elements into localized charge traps.



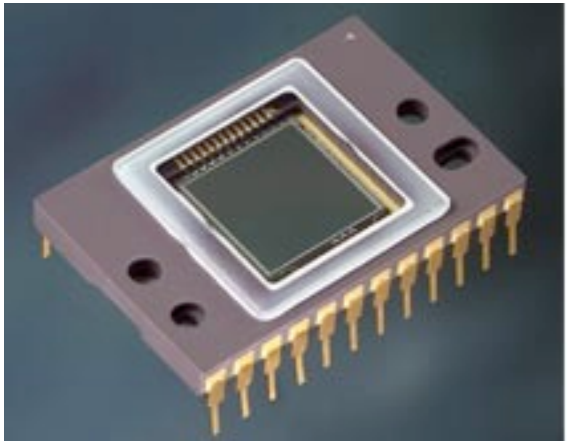

- When a photon is absorbed, it creates a “bucket” of charge that is captured by one of the MOS elements.
- After accumulation ends, the charge in each bucket is individually moved to the end of the MOS array.




- Transfer is done by dynamically and synchronously changing the bias voltages. The charge is transferred to its nearest neighbor, to the next and so on until the photon-generated pulse reaches the array edge and is read out.

# Appendix 4: CCD on the shelf

This KODAK KLI-14403 Image Sensor is designed for color scanning applications where ultra-high resolution is required. The imager consists of three parallel linear photodiode arrays, each with 14404 (!) active photosites for the output of R, G, and B signals.



The KODAK KAF-0261 Image Sensor is a (512x512) pixel, full-frame Blue Plus image sensor. It has 20 µm square pixels for high dynamic range

The Kodak KAF-39000 is a dual output, high performance CCD image sensor with 7216(H) x 5412(V) **(39 MP !! )**photoactive pixels

http://www.kodak.com/US/en/dpq/site/SENSORS/name/ISSProductFamiliesRoot_product

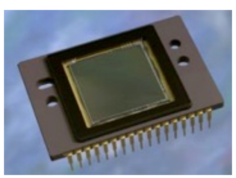
# Appendix 4: Detector at SLS-TOMCAT

## PCO 2000 (CCD)

- 2048 x 2048pixel, 7.4 microns
- 14bit dynamic range
- up to 2 fps
- camRAM up to 4GB
- low noise of 10 e- rms @ 10MHz
- IEEE1394, camera link

## PCO DIMAX (CMOS)

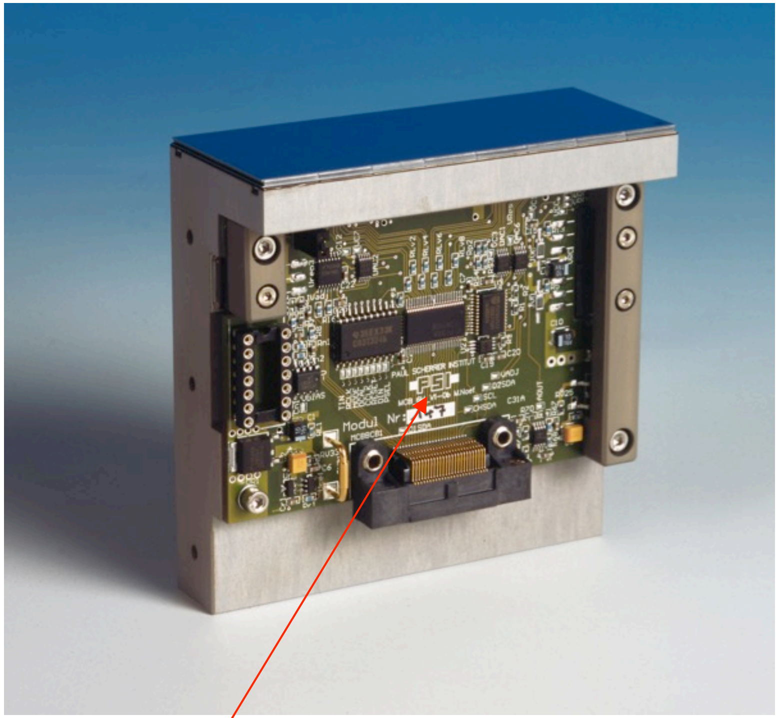
- 2016x2016, 11 microns
- 12bit dynamic range
- 1236 fps (!)
- camRAM up to 32GB
- r-noise of 23 e- rms @ 62.5 MHz
- IEEE1394, camera link




# Appendix 4: Next generation detectors

The next generation detector should provide the following properties:

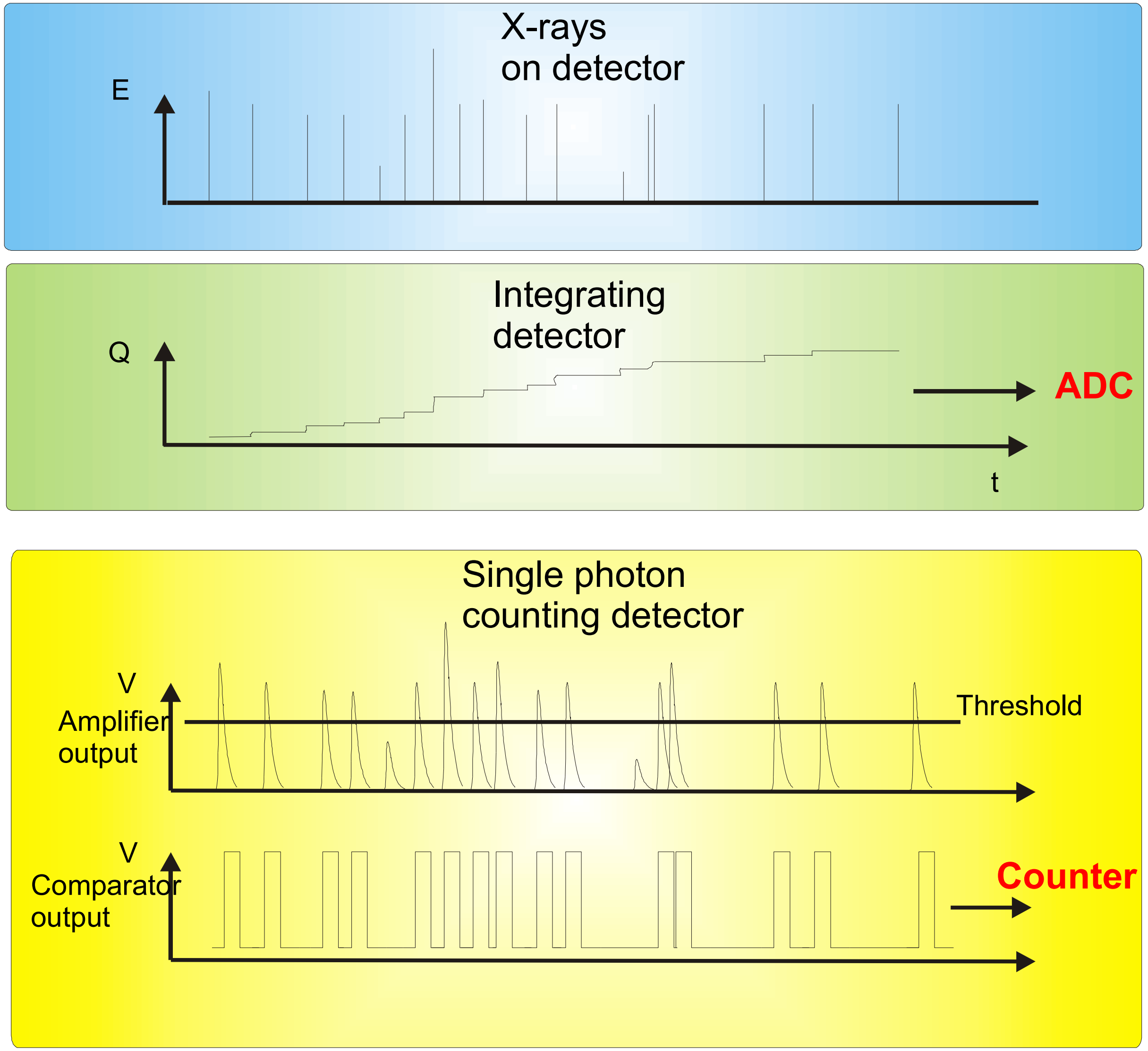
- No dark current
- No readout noise
- Excellent point spread function
- Short readout times (CMOS-like)
- Suppression of fluorescent background
- Very good signal/noise ratio
- Broad energy range
- High resolution



**Developed at PSI !!**

# Appendix 4: Single photon counting vs integrating systems

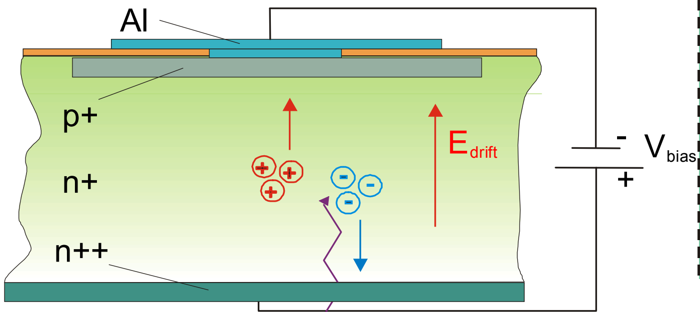
CCD-Like…




Slide courtesy of C. Brönimann, PSI-DECTRIS

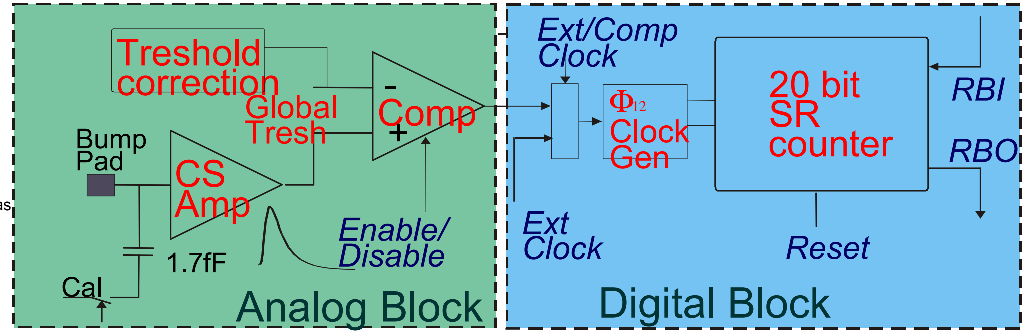
# Appendix 4: Detection and readout on chip

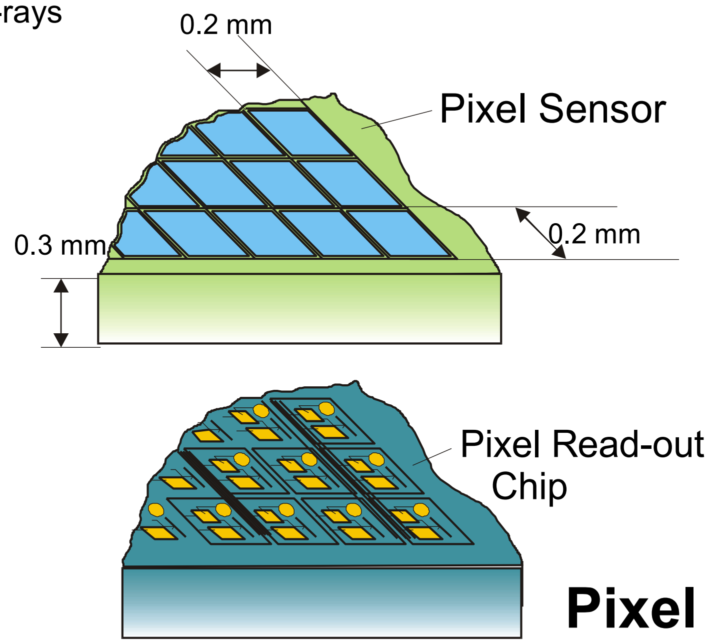
## Sensor: Si pn-junction

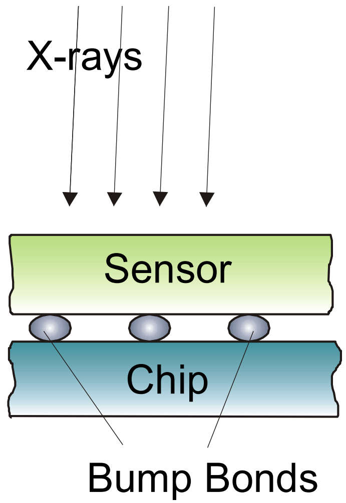



3.6 eV to create
1 eh-pair
0.3mm,
75% @ 12keV

## Readout Chips: Single Photon Counting Electronics




**Pixel Detector (2D)**

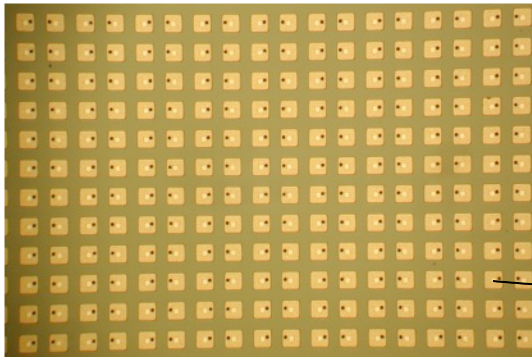
Slide courtesy of C. Brönimann, PSI-DECTRIS

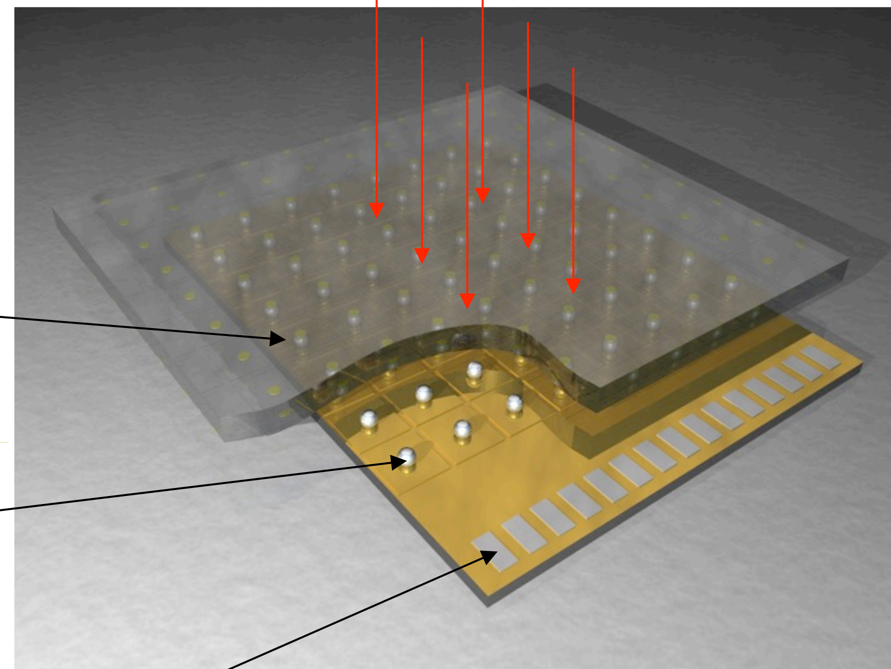
# Appendix 4: Single-photon counting device

## Hybrid pixel technology

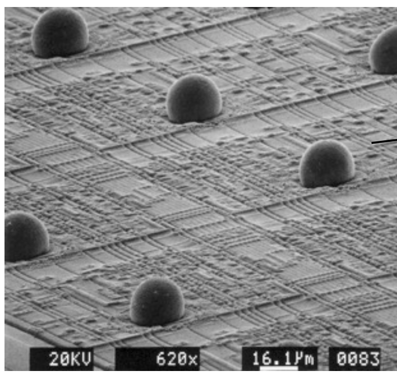
X-rays

Si-sensor with pixelated pn-diode structure

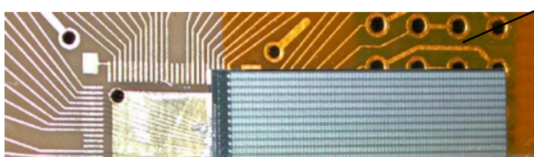




Indium bump-bonding technology (18um bumps)




CMOS chip technology



**Direct Detection of X-rays**

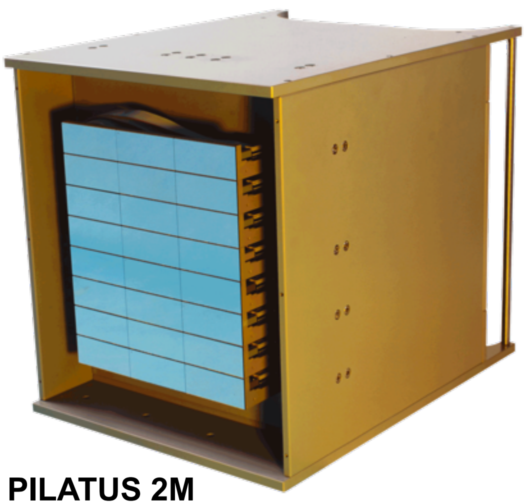
**Single Photon Counting**

Slide courtesy of C. Brönimann, PSI-DECTRIS

# Appendix 4: Modular concept

**PILATUS 100K**
1 module
Area: 83.8 x 33.5 mm$^2$
Frame Rate: 300Hz

**PILATUS 500K**
1 x 5 modules
Area: 33.5 x 431 mm$^2$
Frame Rate: 12Hz



**PILATUS 2M**
3 x 8 modules
Area: 254 x 289 mm$^2$
Frame Rate: 12Hz

**PILATUS 6M**
5 x 12 modules
Area: 431 x 448 mm$^2$
Frame Rate: 12Hz