Image Enhancement and Preprocessing continued

Ender Konukoglu

ETH Zürich

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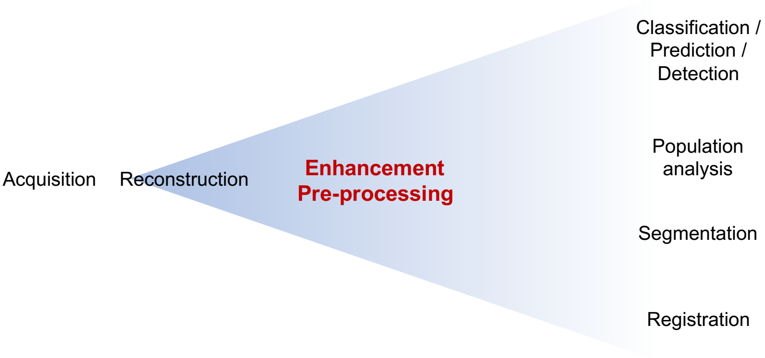
Section 1

Introduction



Enhancement and pre-processing tasks form initial steps for any type of analysis. Same applies to methods that use machine learning, probabilistic, variational or energy-based formulations.

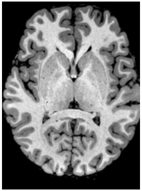
Image enhancement and pre-processing

Enhancement of four conditions

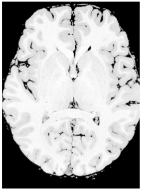
- Bad contrast
- Varying intensity statistics
- Noise
- Bias field
- Variation in pixel-size

Why enhancement?

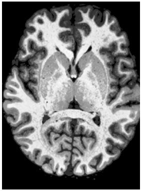
- simplifying interpretation
- better visualization
- normalization for further processing



Original Image

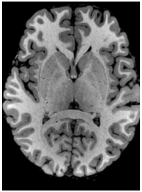


Bad contrast



Intensity variation

Noise



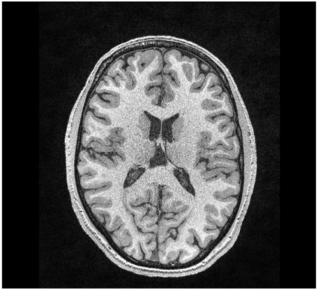
Bias field

Different pixel-size

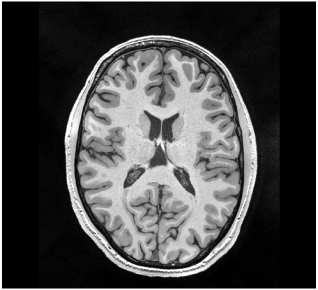
Outline

- Contrast enhancement
- Intensity normalization
- Noise suppression
- Exercise / Challenge
- Bias correction
- Variations in image and pixel-size

Noise suppression challenge: Discussion



Single acquisition



Average of 8

Challenge

High-resolution MRI is hard to acquire. One way is to acquire many images of the same subject and average them. 8 or so images yield good SNR but this leads to long acquisitions. Can you reduce this time with noise suppression?
More details in the moodle platform.

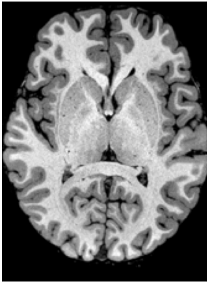
Section 2

Bias Correction

Outline

- Contrast enhancement
- Intensity normalization
- Noise suppression
- Exercise / Challenge
- Bias correction
  - What is bias field?
  - N3 algorithm
- Variations in image and pixel-size

What is bias field?



Normal image

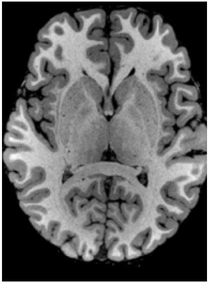


Image with bias

- Acquisition artifact
  - Field inhomogeneity
  - Electrical characteristics of the tissue
  - Poor uniformity of coils
  - Gradients and eddy currents

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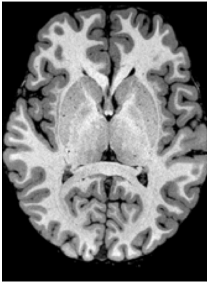

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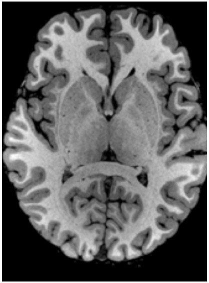

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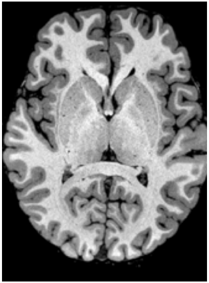

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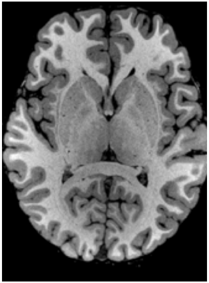

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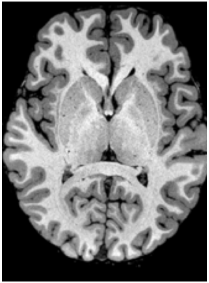

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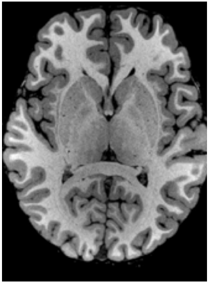

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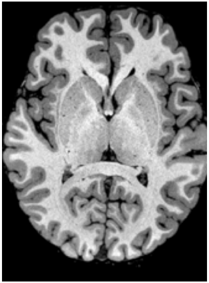

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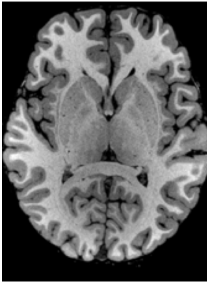

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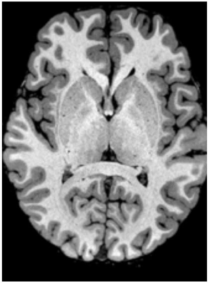

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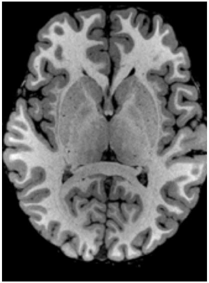

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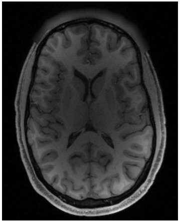
- Remove it as a pre-processing step

What does it look like?



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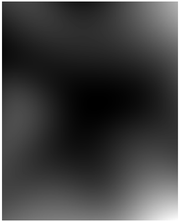
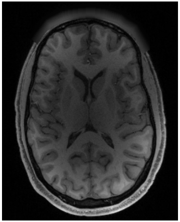


Bias field

What does it look like?

- Smoothly varying



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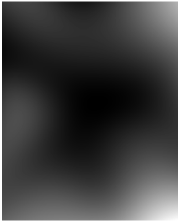

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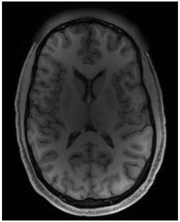

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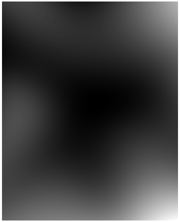

Bias field

- Smoothly varying
- Does not depend on the underlying tissue to the first order approximation

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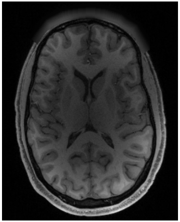

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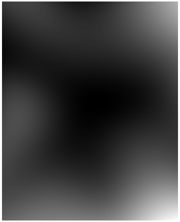

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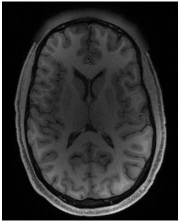

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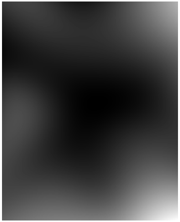

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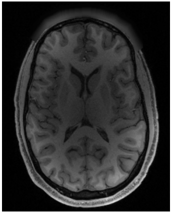
- Does not depend on the underlying tissue to the first order approximation
- In reality it also depends on the tissue but this effect is mostly ignored
- Approximate mathematical model:

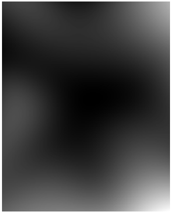
$$j = bi + n,$$

i: real image,
b: bias field,
n: noise and
j: observed image

- Multiplicative factor

Basic methods to correct it





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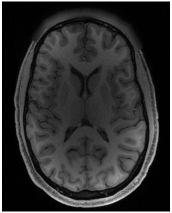

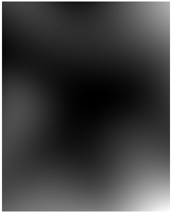

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  - Ideally a perfect solution
  - Additional measurements makes it hard for retrospective data analysis and usually not integrated
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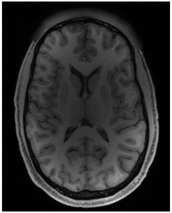

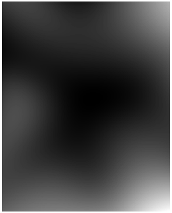

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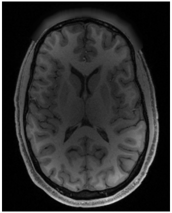

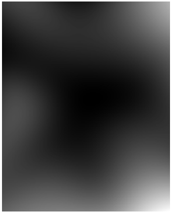

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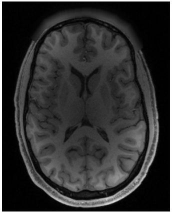

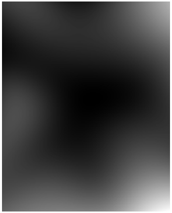

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- N3 algorithm
  - Generic
  - Most popular solution
  - [Sled, Zijdebos and Evans, IEEE TMI 1998]
  - We will study N3 in this lecture.

N3 algorithm - principle

Starting from the main model

$$\tilde{j}(x) = \tilde{b}(x)\tilde{i}(x) + \tilde{n}(x)$$

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Key point

N3 analyzes the distribution of j, b and i to understand the effect of the bias term to understand the effects of the bias term.

Assumption

N3 assumes independence between pixels when considering distributions of j, b and i.

$j(x) = b(x) + i(x)$ is a sum of random variables. Let's study how distribution of j is related to the distribution of i and b.

Sums of random variables

Let a and b be two independent random variables with pdfs denoted by $f_a(a)$ and $f_b(b)$. Let us define another random variable

$$c = a + b$$

Can we compute the distribution of c, i.e. $f_c(c)$, using distributions of a and b?

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Convolution

$c = a + b \Rightarrow f_c = f_a * f_b$ when a are b are independent random variables.

Back to bias problem

$$j(x) = b(x) + i(x),$$

Denote the probability distributions of j, b and i with $J(j)$, $B(b)$ and $I(i)$. The probability distribution J is the following convolution:

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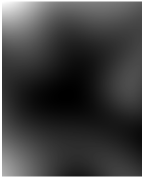

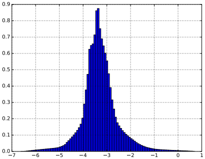

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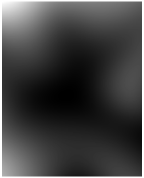

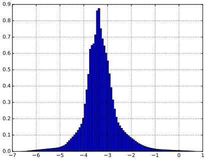

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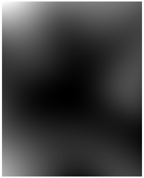

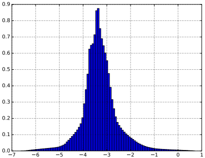

What can we say about B?

- The bias field is a smooth field - b is smooth
- $\tilde{j}(x) = \tilde{i}(x)\tilde{b}(x)$
- $\tilde{b}$ is around 1 so b is around 0

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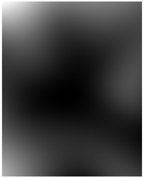

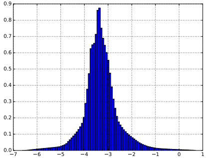

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- The bias field is a smooth field - b is smooth
- $\tilde{j}(x) = \tilde{i}(x)\tilde{b}(x)$
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Back to bias problem

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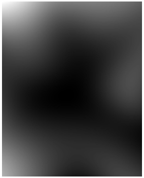

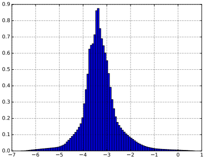

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- J is a smoothed version of I

Two components in the problem

$$j(x) = b(x) + i(x),$$

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The estimation must determine $I(i)$ and b given only j. We divide this into two parts:

- Field estimation: Estimation of b given $I(i)$.
- $I(i)$ estimation: Estimation of $I(i)$.

Always assuming B is a Gaussian with small standard deviation.

Component 1: Field estimation

One component is field estimation. **If** we are given the distribution $I(i)$ we can estimate the bias field at any point by

$$\mathbb{E}[i|j] = \int_{-\infty}^{\infty} ip(i|j)di$$

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We know all the terms in this equation and i is one dimensional, so, we can solve it numerically with ease.

Component 2: $I(i)$ estimation

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$J(j)$ is a smoothed version of $I(i)$, and $B(b)$ is roughly Gaussian with a small standard deviation.

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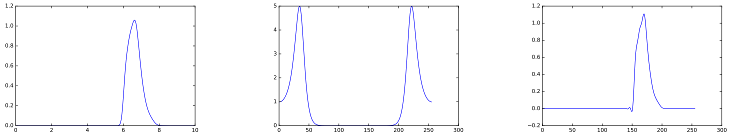
Note: This is inverse filtering!

Intermediate summary

- Estimate of the unbiased image's distribution: $I(i) \approx \mathcal{F}^{-1}\left(\hat{F}\mathcal{F}\{J\}\right)$



$J(j)$ $\quad$ $|\hat{F}|$ $\quad$ $I \approx \mathcal{F}^{-1}(\hat{F}\mathcal{F}\{J\})$

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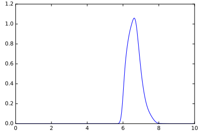

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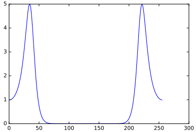

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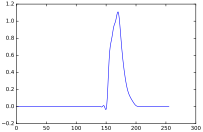

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- Solution: Smooth approximation based on anchor points using Splines or Radial Basis Function approximation:

$$b(x) = \sum_{n=1}^{N} b_e(x_n) K(\|x - x_n\|)$$

with K smoothing kernel such as $\exp(\|x - x_n\|_2^2/\sigma^2)$.

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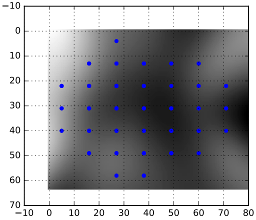
- Example



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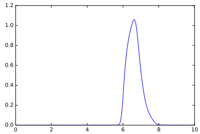

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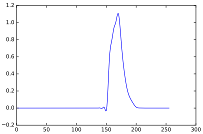
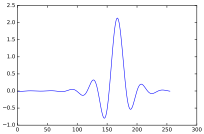


std $= 0.1$



with std $= 0.5$

Remember they have to be distributions (> 0). Deconvolution is hard.

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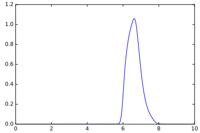

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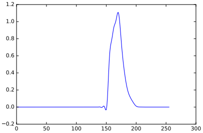

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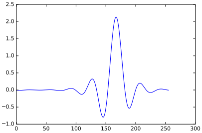

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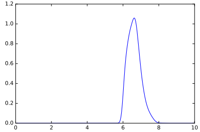

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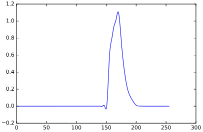

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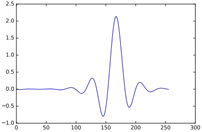

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Remember they have to be distributions (> 0). Deconvolution is hard.

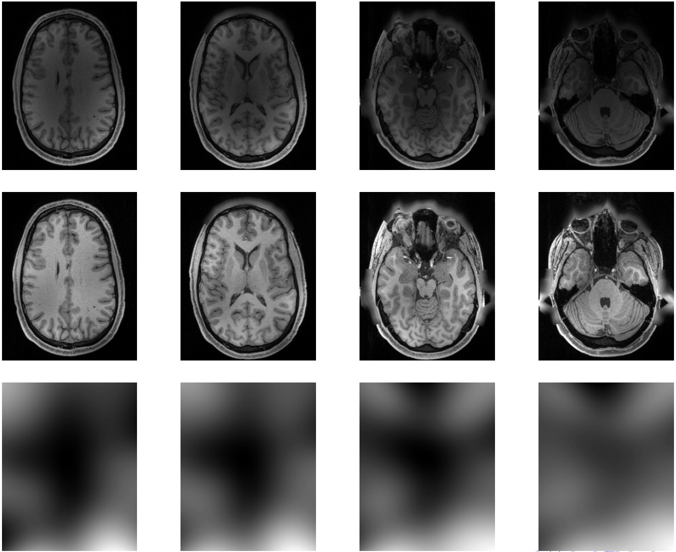
- Solution: Iterate many times with small Gaussians (small stds)
- Not a bad approximation: large Gaussians are convolutions of smaller ones

Algorithm

Algorithm 1 Bias correction with N3

$b = 0$, $j = \log(\tilde{j})$ observed image
$\hat{F} = \frac{\hat{B}^*}{|\hat{B}|^2 + Z^2}$
while $\|\Delta b\| < eps$ **do**
  Estimate $J(j)$ with kernel density estimation or simple histogram
  Compute $I(i) = \mathcal{F}^{-1}(\hat{F}\mathcal{F}\{J\})$
  Compute $\Delta b_e = j - \mathbb{E}(i|j)$ at x_n anchor points.
  Get the smooth estimate $\Delta b(x) = \sum_{n=1}^{N} \Delta b_e(x_n) K(\|x - x_n\|)$ for all voxels
  Update $b = b + \Delta b$
  Update $j = j - \Delta b$
end while

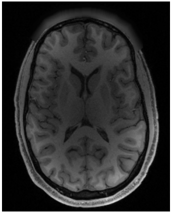
Examples

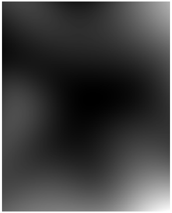


Analysis

- Bias correction with N3 is routinely used for MRI today
- There are variations available, e.g. N4ITK
- In the method there are multiple approximations
- Multiresolution solutions exist and perform better
- May need to run the tool a few times to get better results

Question - Group discussion





- Manually placed landmarks
  - Voxels expected to have similar intensity
  - Human in the loop - a very good solution
  - Need for human interaction
- How would this exactly work?
- Let's only concentrate on structural brain MRI, e.g. like the images on the left hand-side.

Section 3

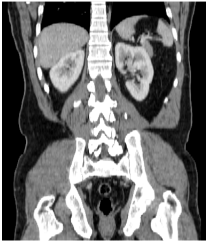
Variations in image and pixel-size

Outline

- Contrast enhancement
- Intensity normalization
- Noise suppression
- Exercise / Challenge
- Bias correction
- Variations in image and pixel-size
  - What is the problem?
  - How to tackle it.
  - How **not** to tackle it.

Different image and pixel size and FOV

Fine pixel-size, small FOV



Coarse pixel-size, large FOV

What is it?

- Images may come with different pixel and/or image size.
- They may have different Field-of-View (FOV).
- You may need to compare such "heterogeneous" images.

Why does it happen?

- Differences in acquisitions
- in FOV
- in pixel size

In which applications does it matter?

- Comparing measurements
- Registration
- Machine learning applications

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- Comparing measurements
  - measurements taken in an image have a correspondence in real life.
  - real life measurements rely on pixel size.
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- Machine learning applications
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  - applying the same model to an image with different image/pixel size or FOV requires attention
- Effectively important for all applications

How do we tackle this?

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

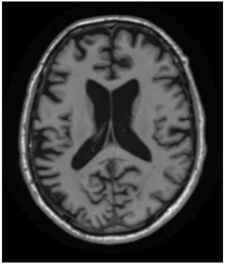
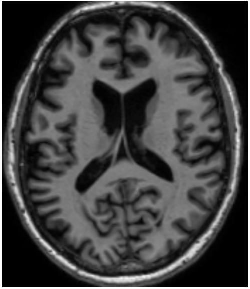


Image size: [454, 384]
Pixel size: [1.25, 1.25]



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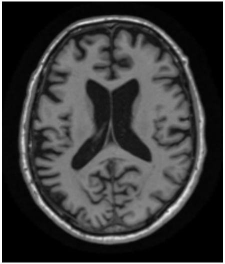

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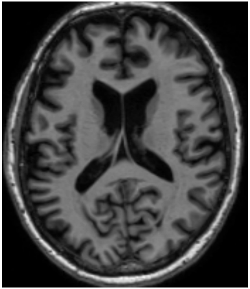

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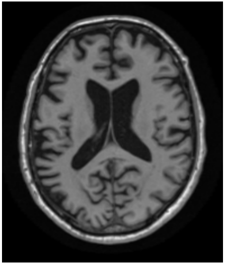

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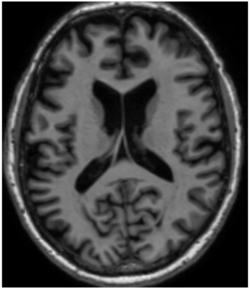

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- Measurements taken in the two images should be comparable.
- What is the important factor to match?

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Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

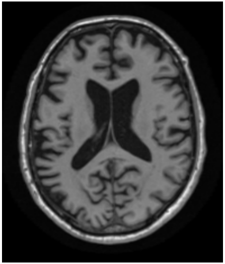

Image size: [454, 384]
Pixel size: [1.25, 1.25]

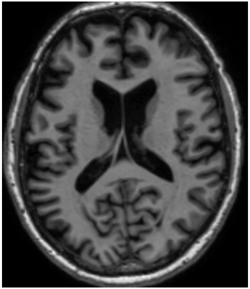

Image size: [494, 424]
Pixel size: [1.0, 1.0]

- How do we compare these images?
- Measurements taken in the two images should be comparable.
- What is the important factor to match? **Pixel size**

Match the pixel size

Images of the same person one year apart

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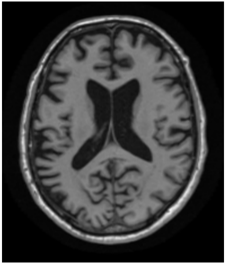

Image size: [454, 384]
Pixel size: [1.25, 1.25]

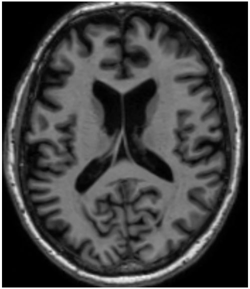

Image size: [494, 424]
Pixel size: [1.0, 1.0]

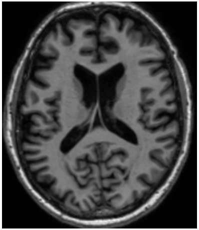

Image size: [395, 339]
Pixel size: [1.25, 1.25]

- Resample one image to match the *pixel-size* of the other.
- Resize an image with a factor determined by the pixel-size. One example tool in python "scipy.ndimage.zoom".

Match the pixel size

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

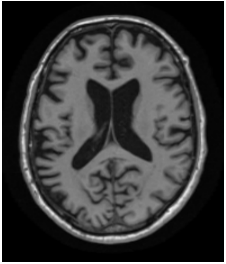

Image size: [454, 384]
Pixel size: [1.25, 1.25]

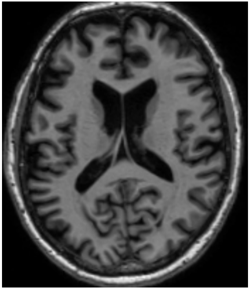

Image size: [494, 424]
Pixel size: [1.0, 1.0]

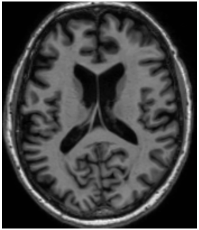

Image size: [395, 339]
Pixel size: [1.25, 1.25]

- Resample one image to match the *pixel-size* of the other.
- Resize an image with a factor determined by the pixel-size. One example tool in python "scipy.ndimage.zoom".
- If volumetric images in 3D, then match the *voxel-size*.

Match the pixel size

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

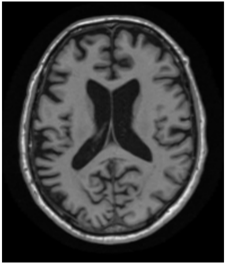

Image size: [454, 384]
Pixel size: [1.25, 1.25]

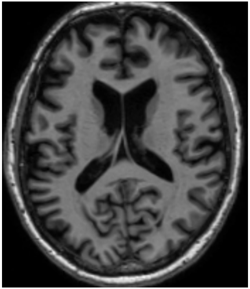

Image size: [494, 424]
Pixel size: [1.0, 1.0]

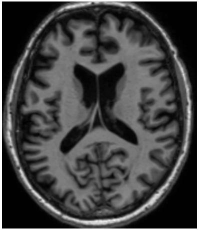

Image size: [395, 339]
Pixel size: [1.25, 1.25]

- Resample one image to match the *pixel-size* of the other.
- Resize an image with a factor determined by the pixel-size. One example tool in python "scipy.ndimage.zoom".
- If volumetric images in 3D, then match the *voxel-size*.
- Crop or pad if necessary to make the image sizes the same.

Exercise

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

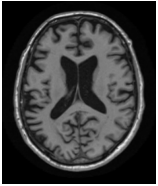

Image size: [454, 384]
Pixel size: [1.25, 1.25]

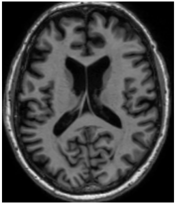

Image size: [494, 424]
Pixel size: [1.0, 1.0]

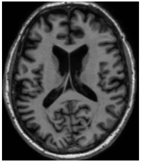

Image size: [395, 339]
Pixel size: [1.25, 1.25]

- If we are matching the image in the second column to the one in the first one, what are the resize ratios in x- and y-directions?

Exercise

Images of the same person one year apart
Sizes of the images on the slides are proportional to their number of pixels.

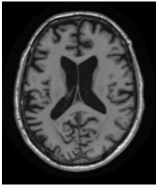

Image size: [454, 384]
Pixel size: [1.25, 1.25]

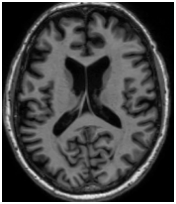

Image size: [494, 424]
Pixel size: [1.0, 1.0]

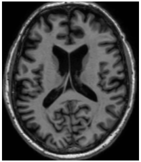

Image size: [395, 339]
Pixel size: [1.25, 1.25]

- If we are matching the image in the second column to the one in the first one, what are the resize ratios in x- and y-directions? 0.8, 0.8

Exercise

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

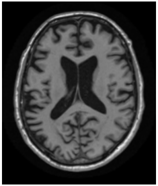

Image size: [454, 384]
Pixel size: [1.25, 1.25]

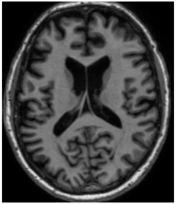

Image size: [494, 424]
Pixel size: [1.0, 1.0]

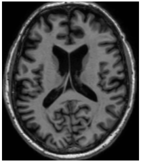

Image size: [395, 339]
Pixel size: [1.25, 1.25]

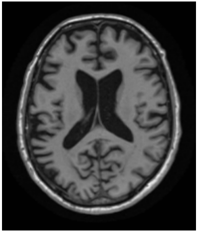

Image size: [568, 480]
Pixel size: [1.0, 1.0]

- If we are matching the image in the second column to the one in the first one, what are the resize ratios in x- and y-directions? 0.8, 0.8
- If we are matching the image in the first column to the one in the second one, what are the resize ratios in x- and y-directions?

Exercise

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

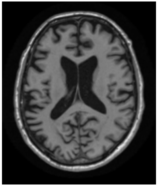

Image size: [454, 384]
Pixel size: [1.25, 1.25]

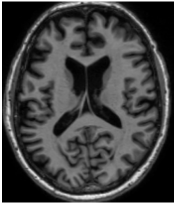

Image size: [494, 424]
Pixel size: [1.0, 1.0]

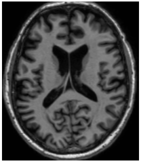

Image size: [395, 339]
Pixel size: [1.25, 1.25]

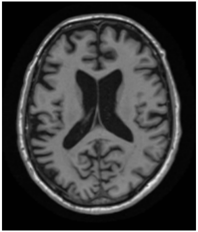

Image size: [568, 480]
Pixel size: [1.0, 1.0]

- If we are matching the image in the second column to the one in the first one, what are the resize ratios in x- and y-directions? 0.8, 0.8
- If we are matching the image in the first column to the one in the second one, what are the resize ratios in x- and y-directions? 1.25, 1.25

Where do I find this information?

- We need the pixel-size of each image for the matching.
- Such information are stored in the header of the DICOM files.
- It is also stored in the headers of many volumetric medical image formats.
- One way to reach them is to use the “nibabel” package in python.

How **not** to tackle it.

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

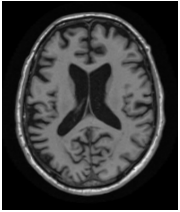

Image size: [454, 384]
Pixel size: [1.25, 1.25]

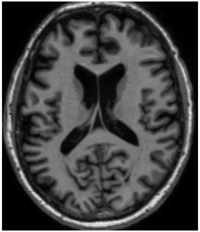

Image size: [494, 424]
Pixel size: [1.0, 1.0]

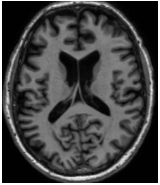

Image size: [395, 339]
Pixel size: [1.25, 1.25]

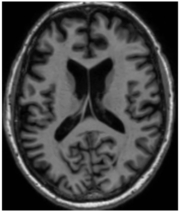

Image size: [454, 384]
Pixel size: [1.09, 1.11]

- A very naive way of doing the matching is through matching the image sizes.
- Last column on the right is obtained by matching the size of the image on the second column to that in the first one.
- Notice that the images are not comparable any more.
- Even the aspect ratio changed.

How **not** to tackle it.

Images of the same person one year apart

Sizes of the images on the slides are proportional to their number of pixels.

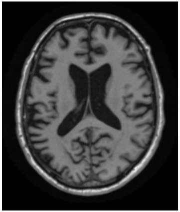

Image size: [454, 384]
Pixel size: [1.25, 1.25]

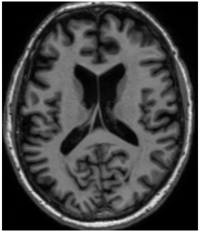

Image size: [494, 424]
Pixel size: [1.0, 1.0]

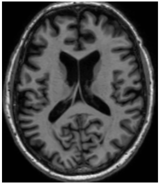

Image size: [395, 339]
Pixel size: [1.25, 1.25]

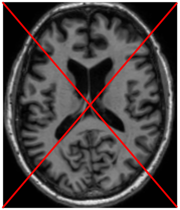

Image size: [454, 384]
Pixel size: [1.09, 1.11]

- A very naive way of doing the matching is through matching the image sizes.
- Last column on the right is obtained by matching the size of the image on the second column to that in the first one.
- Notice that the images are not comparable any more.
- Even the aspect ratio changed.

Important

Always match the pixel / voxel-size. Do not match the image size, it may lead to wrong normalization.