



Biomedical Imaging

X-Ray Imaging (II)

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Today's Learning Objectives

- **Describe basic X-ray detection technologies**
- **Relate image contrast to photon scattering**
- **Explain encoding and decoding of projection data**
- **Derive filtered back projection principle**
- **Implement CT encoding and decoding (Exercise)**

Resolution of imaging procedures

- Spatial resolution

- Blurring of object points when projected into image



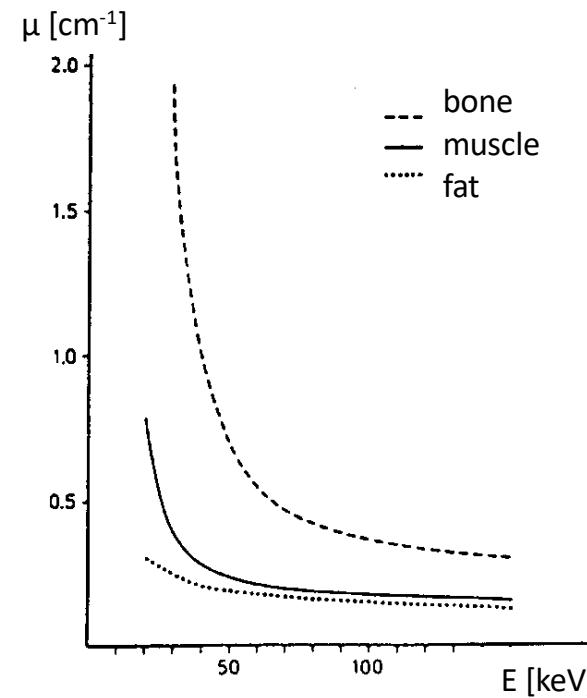
Difference in linear attenuation coefficients between tissues

- Temporal resolution

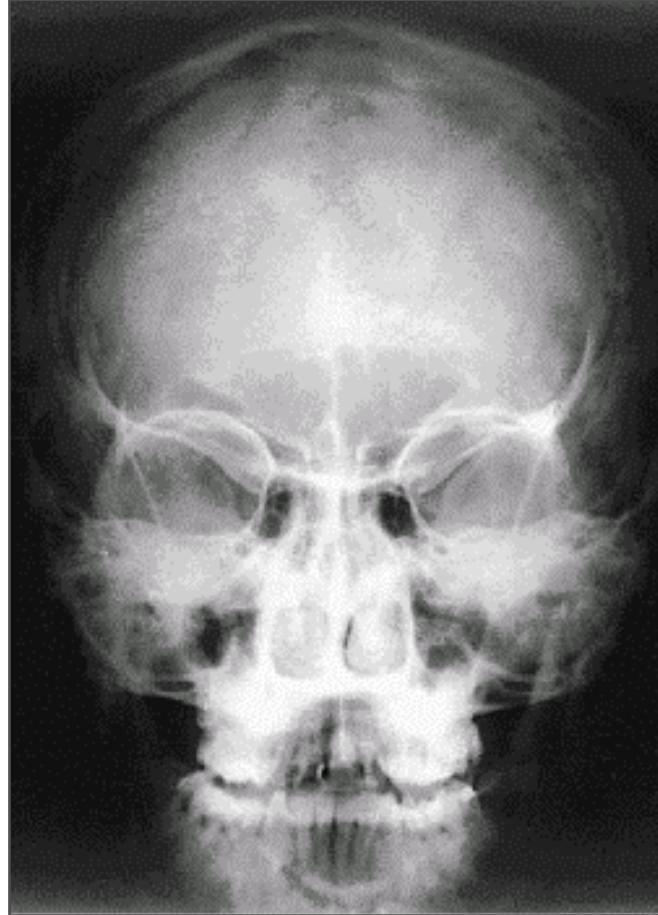
- Sample rate of temporal recording

Resolution – Contrast

$$\text{Contrast} \propto (\mu_1 - \mu_0) d$$



Resolution – Contrast



- Large contrast between bone and tissue/air
- Very little contrast among different tissues

Resolution of imaging procedures

- Spatial resolution

- Blurring of object points when projected into image

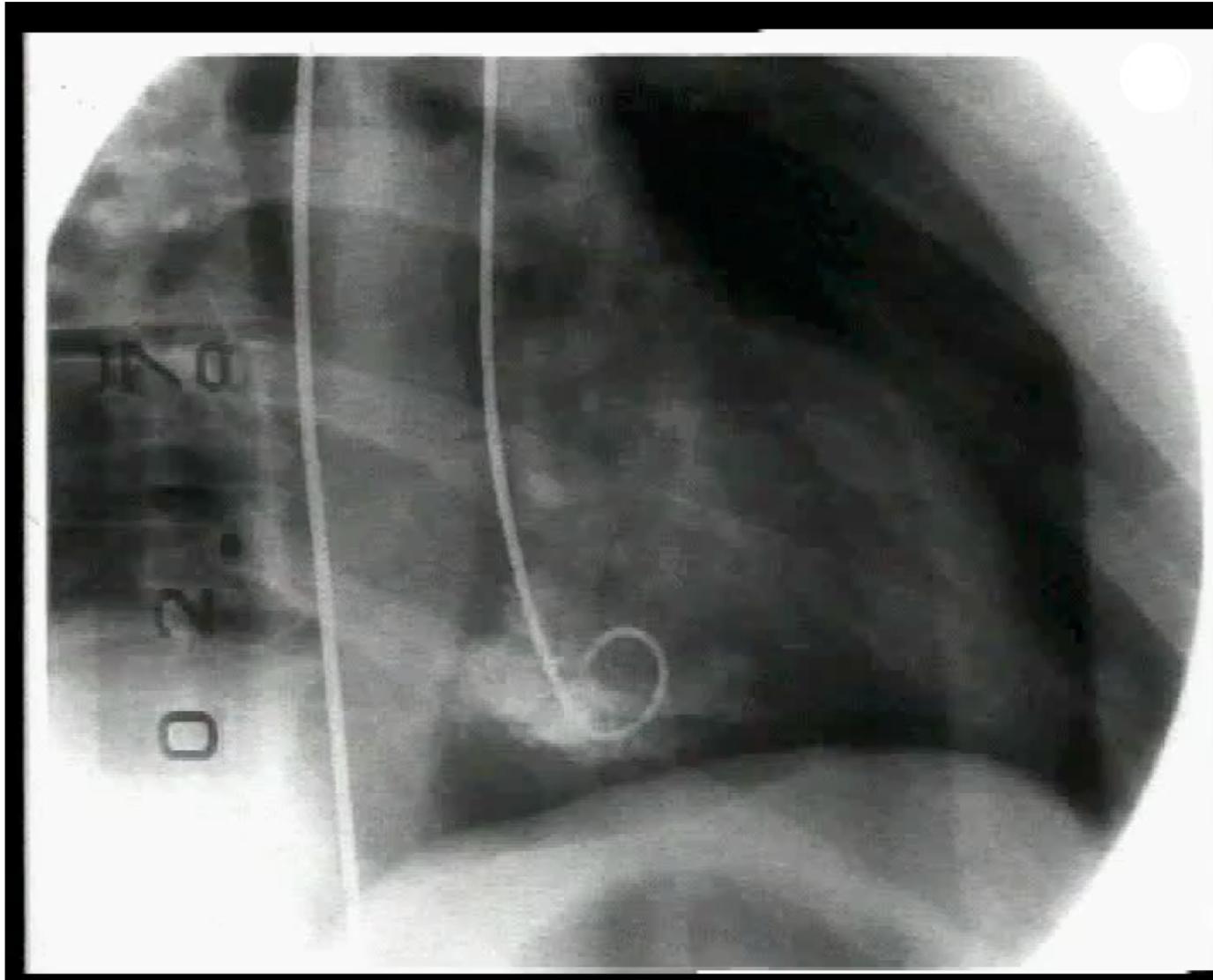
- Contrast

- Difference in linear attenuation coefficients between tissues

- Temporal resolution

- Sample rate of temporal recording

Resolution – Temporal resolution

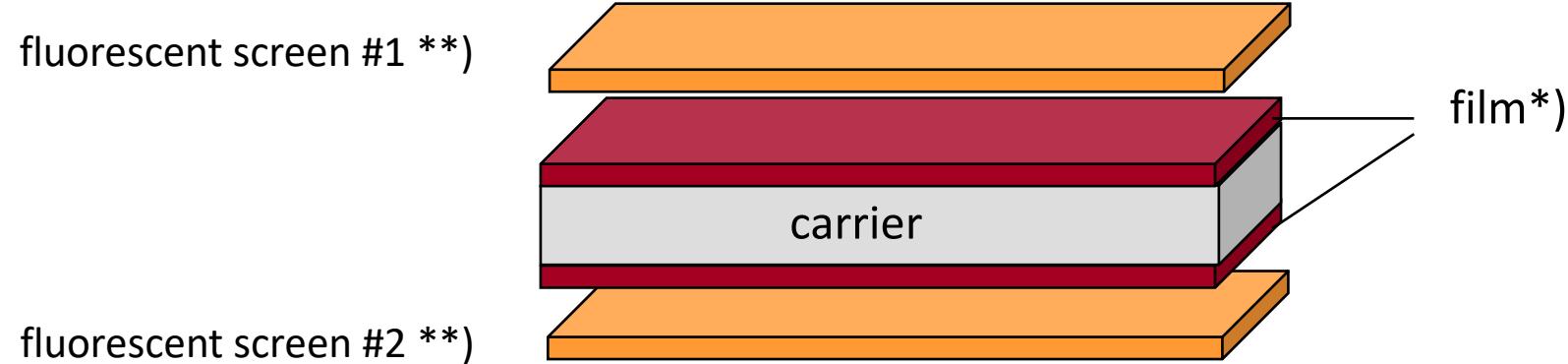




Detector technology

Projection imaging – Photographic film

Double-sided film with intensifying screens



$$\text{Opacity} \propto \frac{I_0}{I}$$

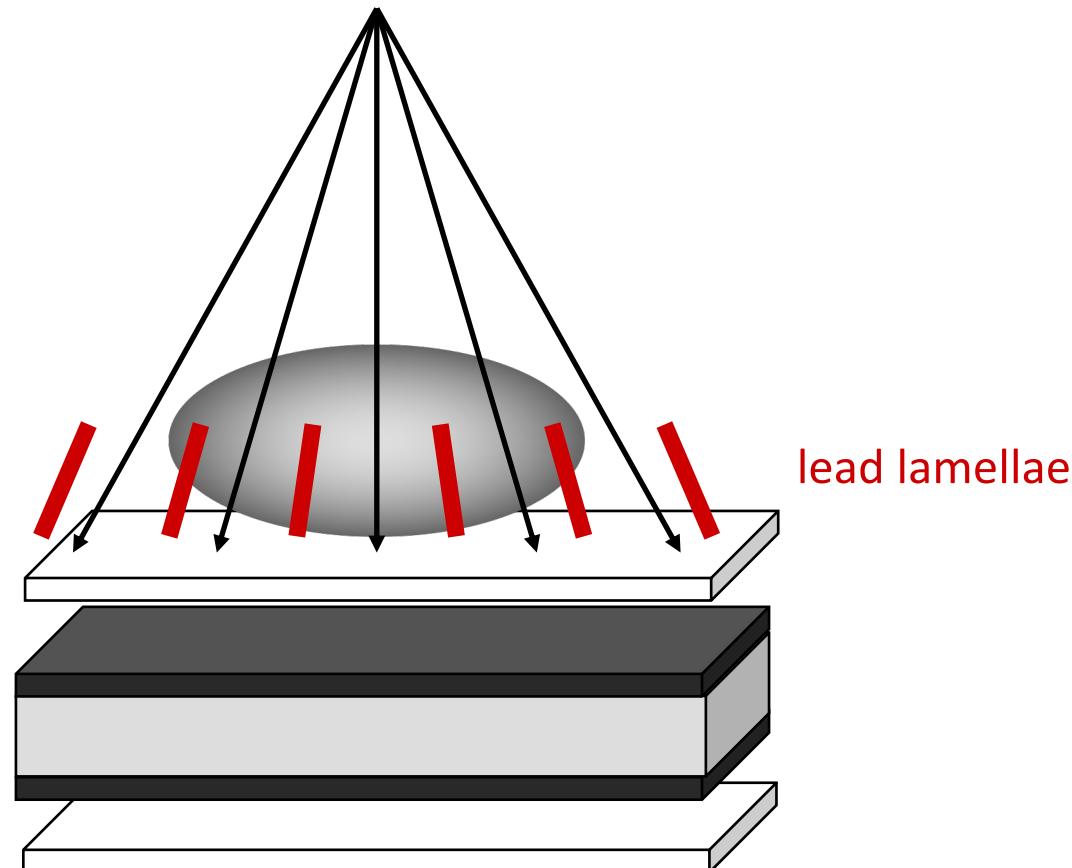
*) Silver halide (AgX): compound formed between silver and silver bromide (AgBr) or silver chloride (AgCl) or silver iodide (AgI).

**) Fluorescence increases efficiency about 5-fold but decreases resolution

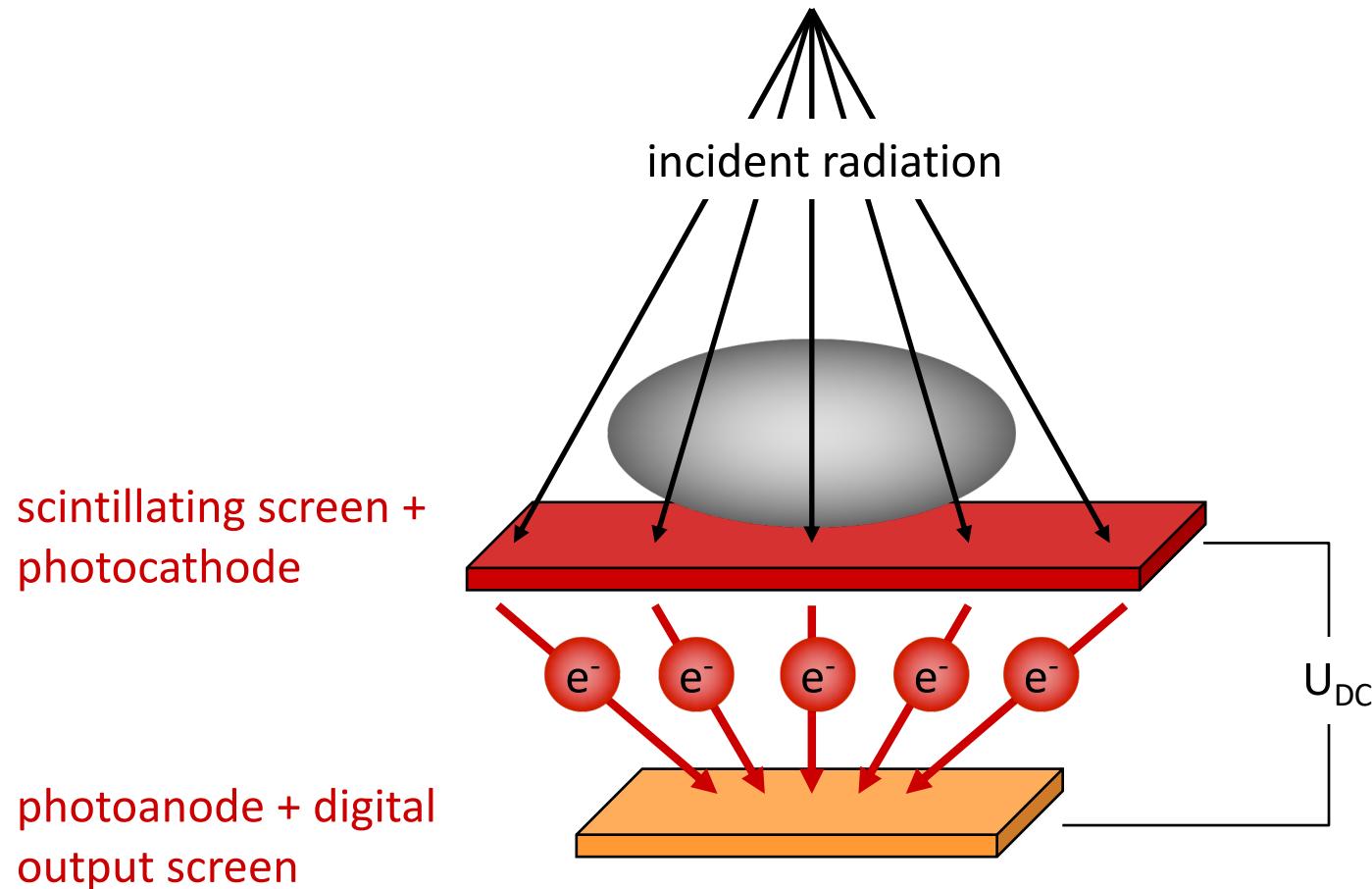


Projection imaging – Anti scatter grids

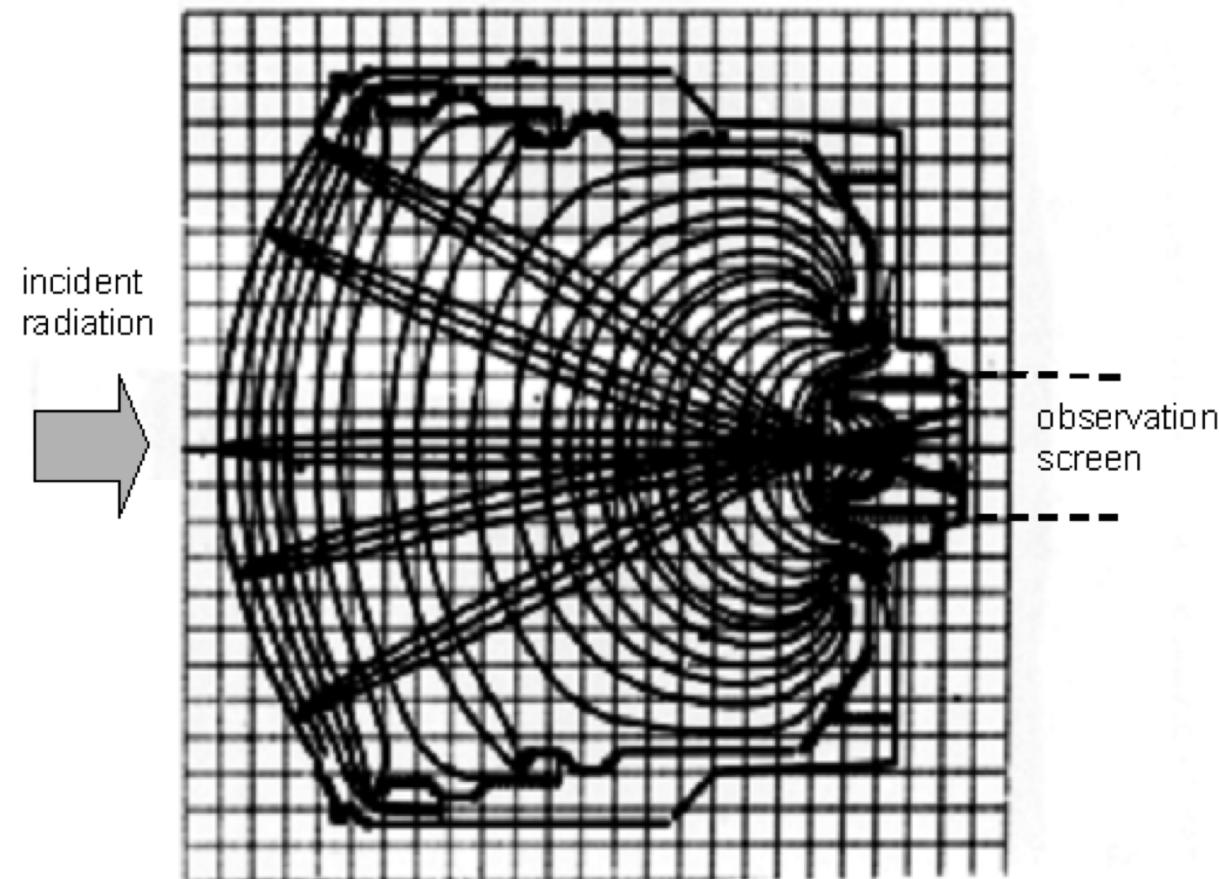
Film with anti-scatter grid



Projection imaging – Image intensifier

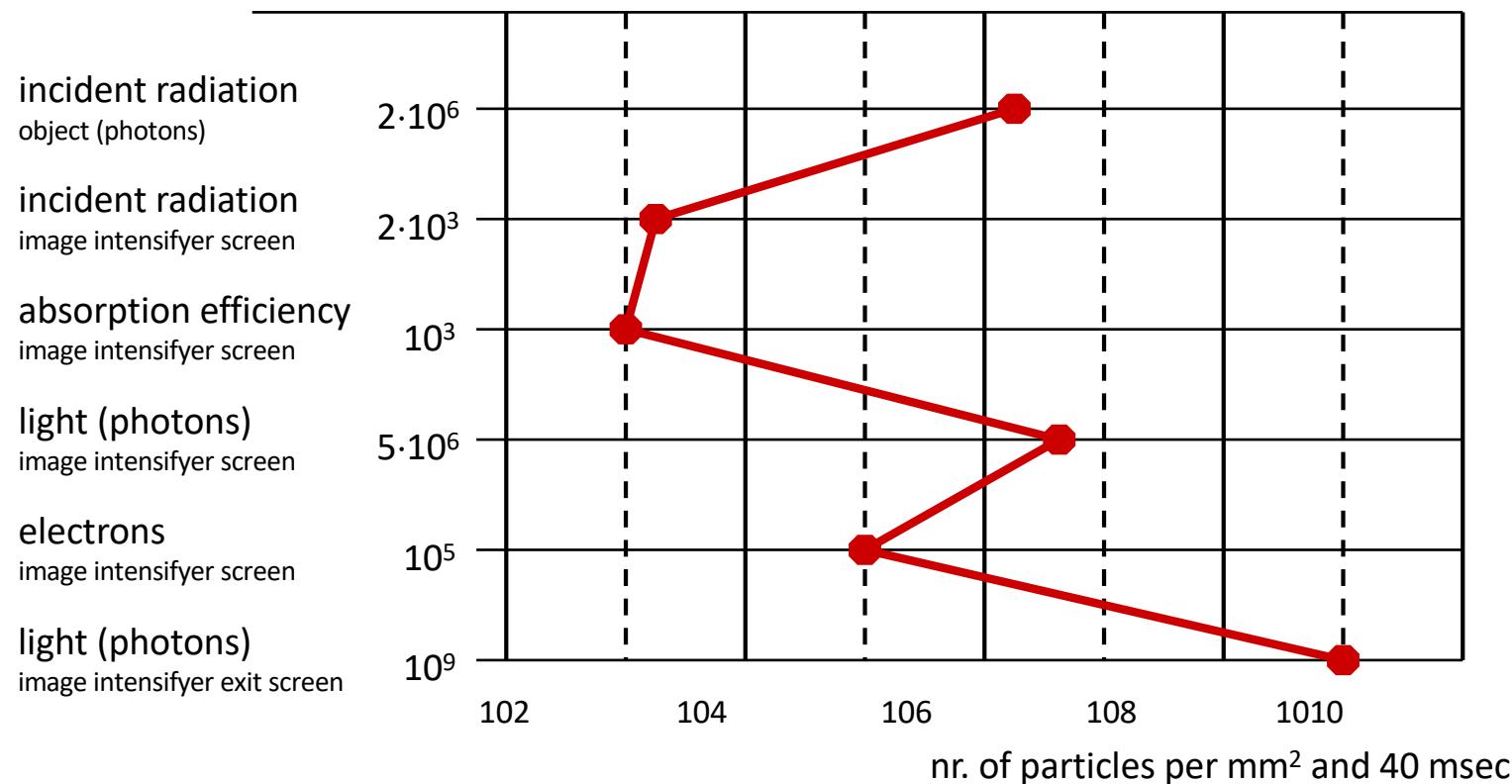


Projection imaging – Image intensifier



Projection imaging – Summary

Particle statistics in a typical X-ray chain

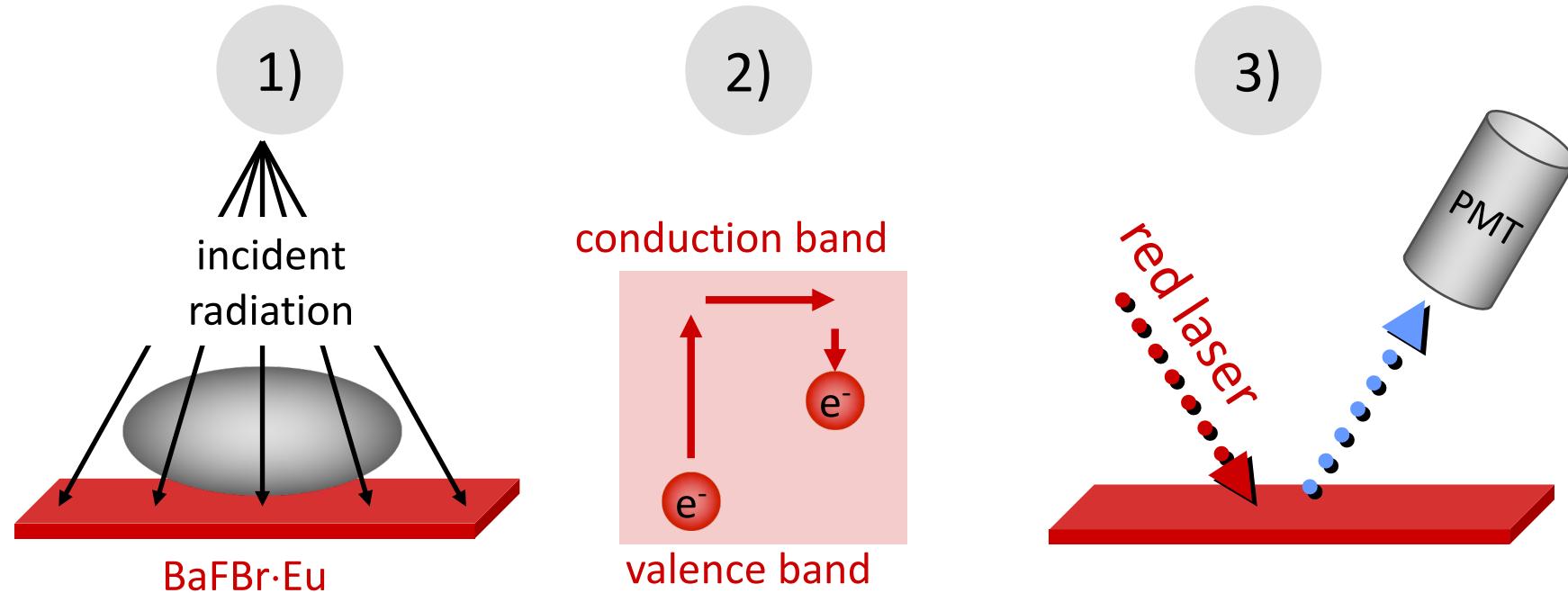




Digital X-ray

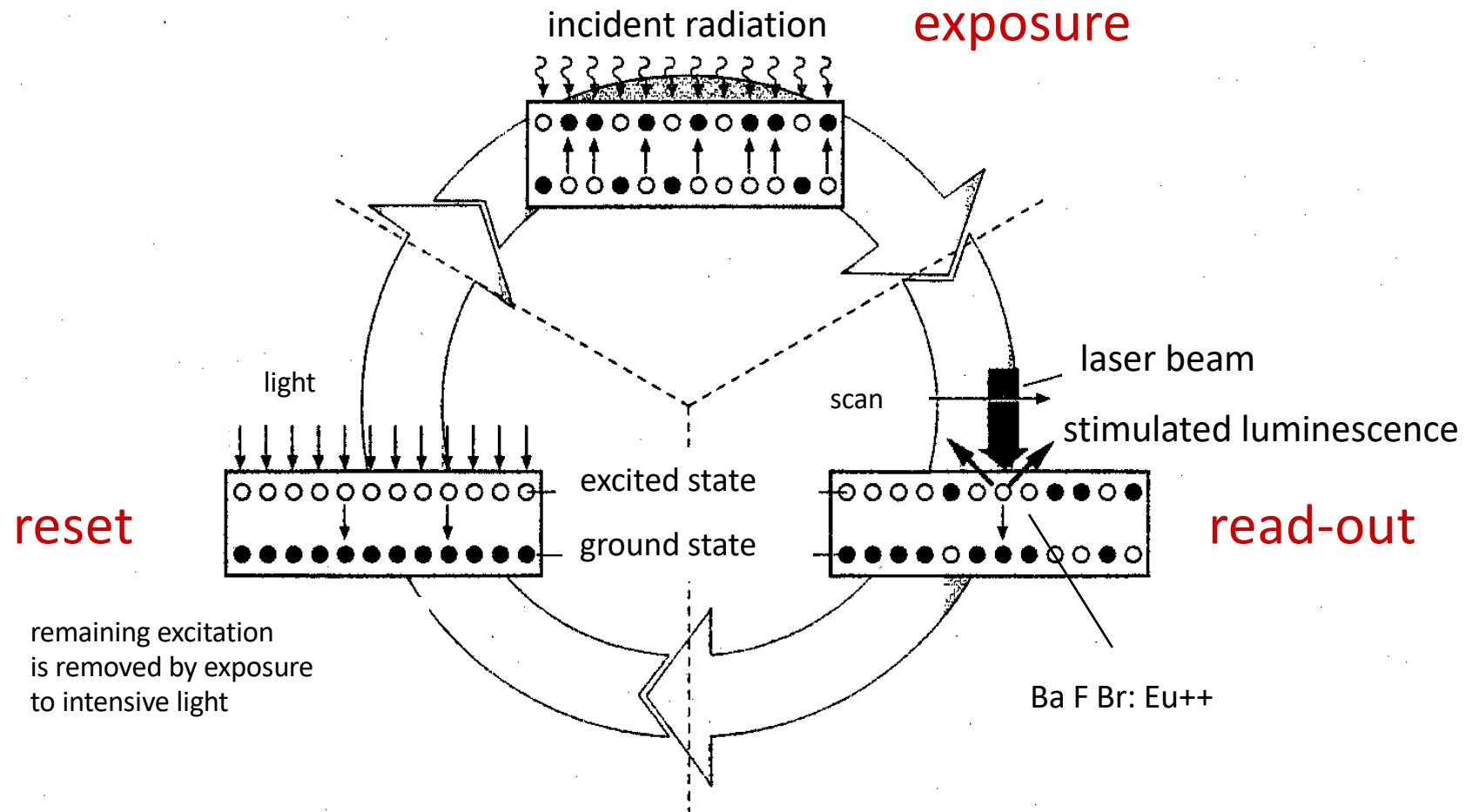
Image conversion

Luminescence using barium fluorohalide (BaFBr:Eu)

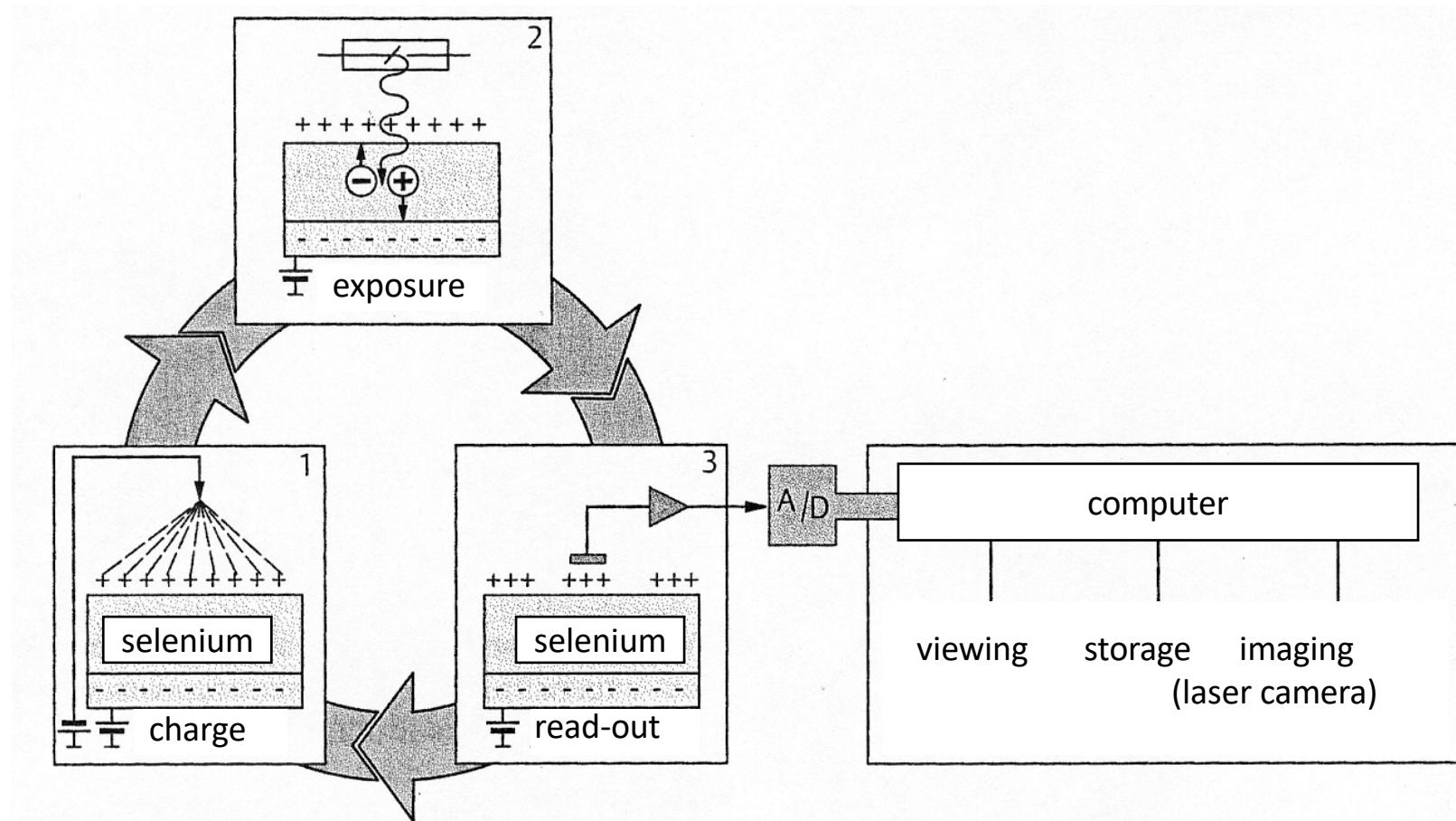


- absorbed x-rays liberate e^- from valence band to conduction band; e^- get trapped
- scanning red laser stimulates trapped electrons
- reflected light is collected in photo multiplier tube + stored as digital image

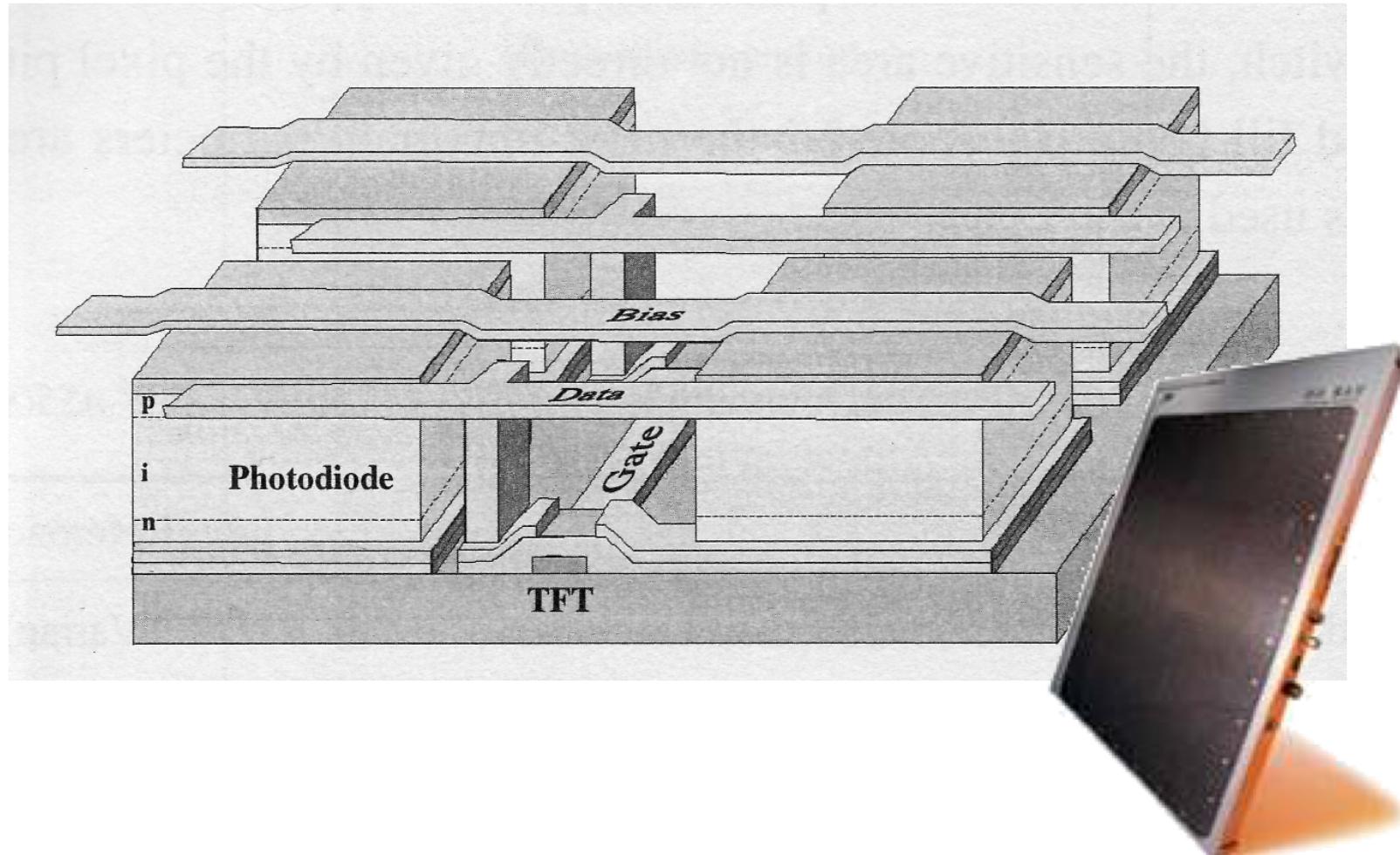
Digital Luminescence



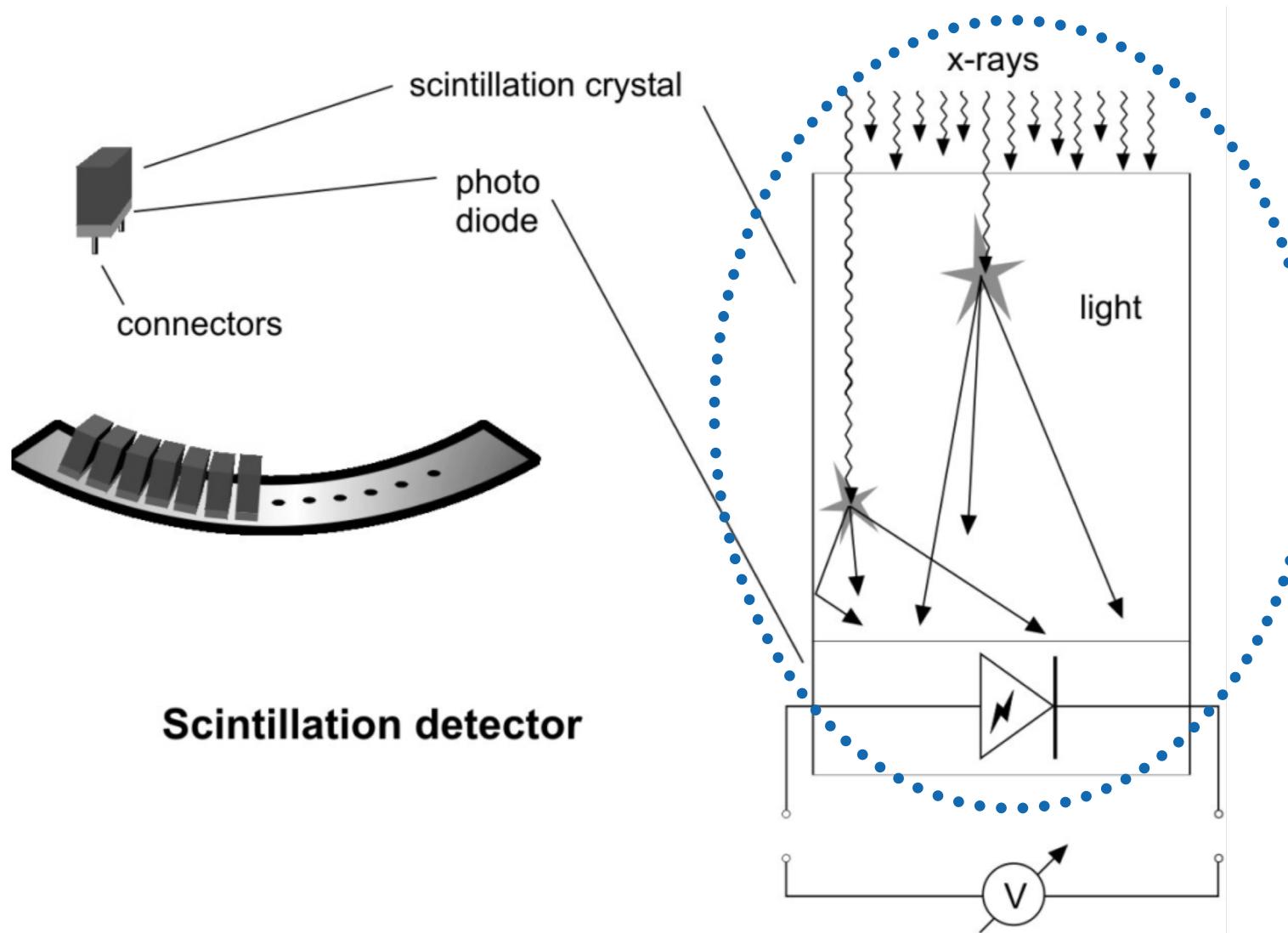
Selenium Detector



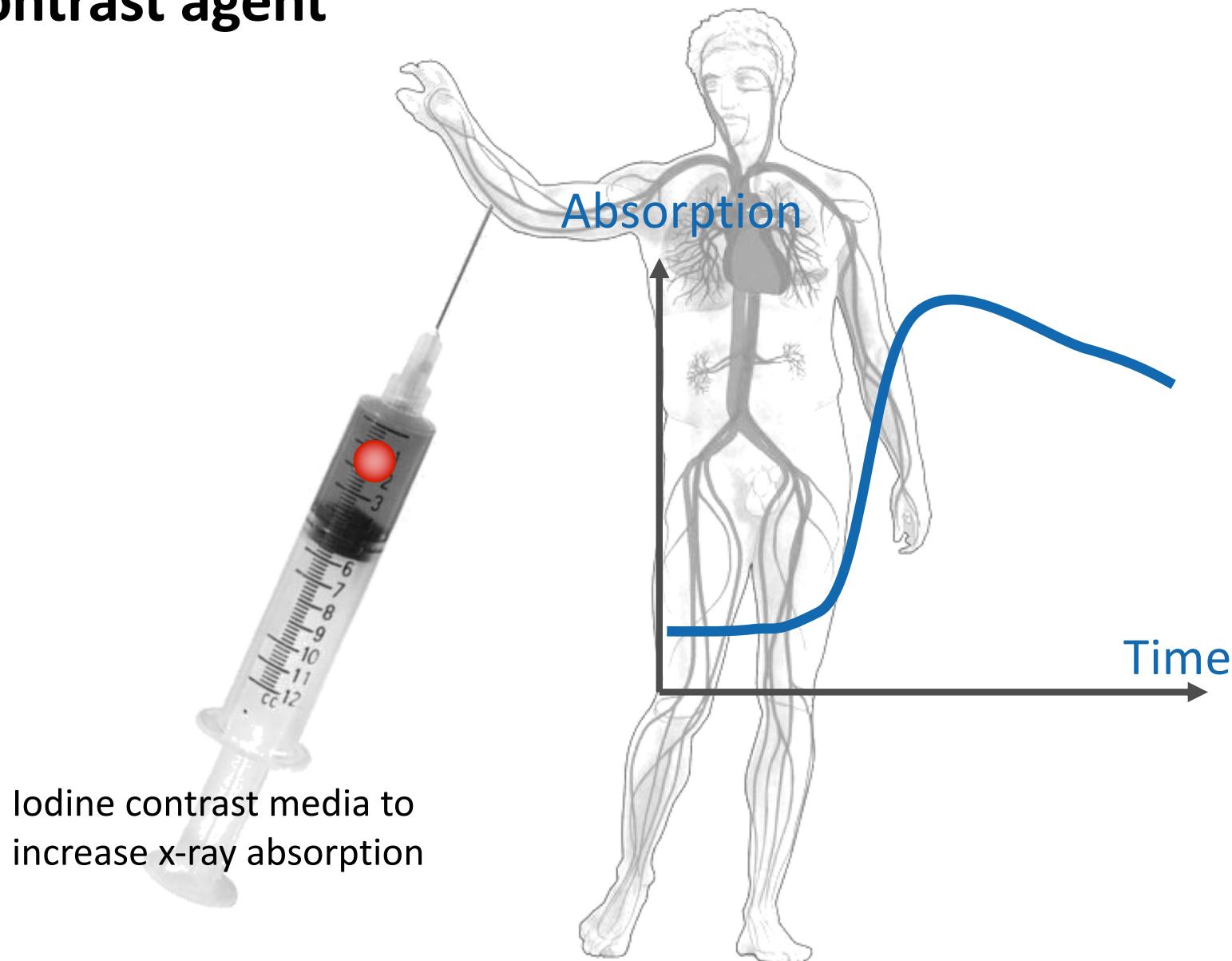
Silicium Detector



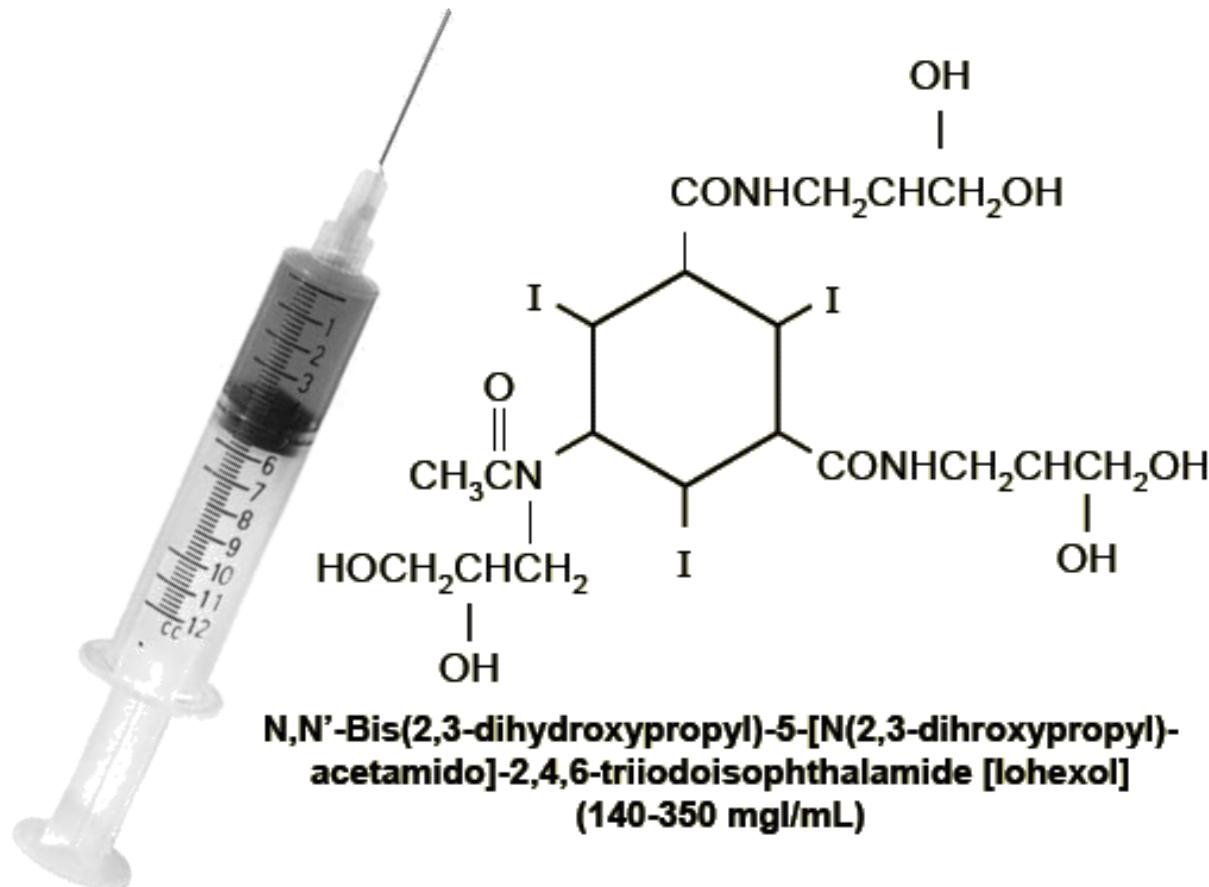
Scintillation Detector



Contrast agent

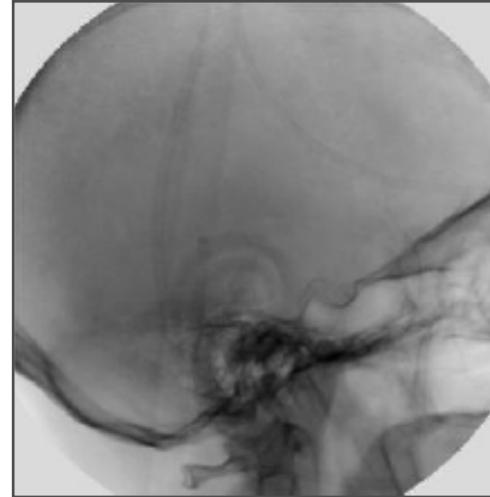


Contrast agent

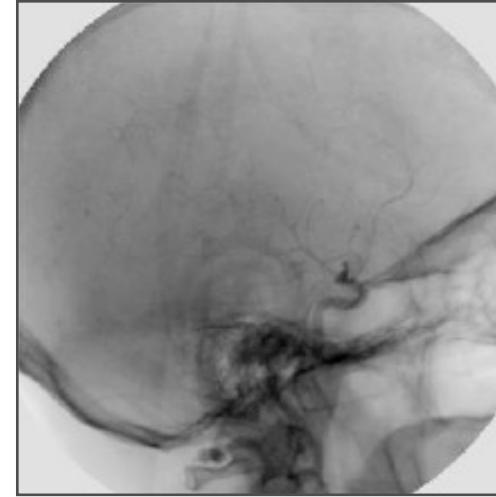


Digital Subtraction

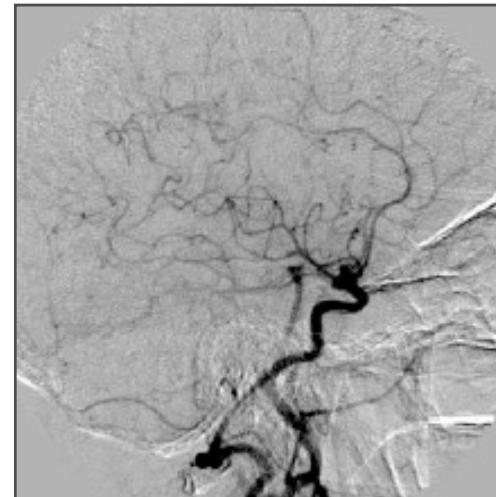
pre-contrast



post-contrast



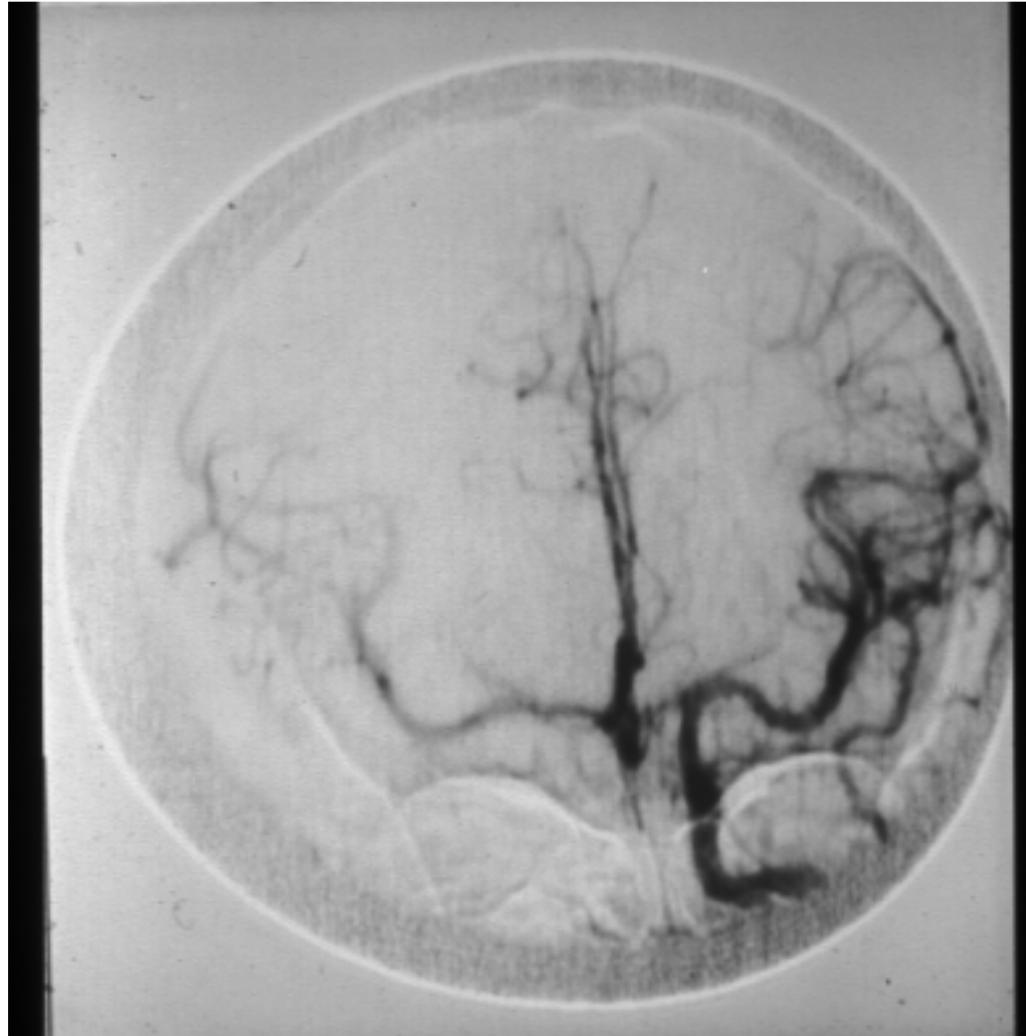
difference+enhancement



DSA Angiography



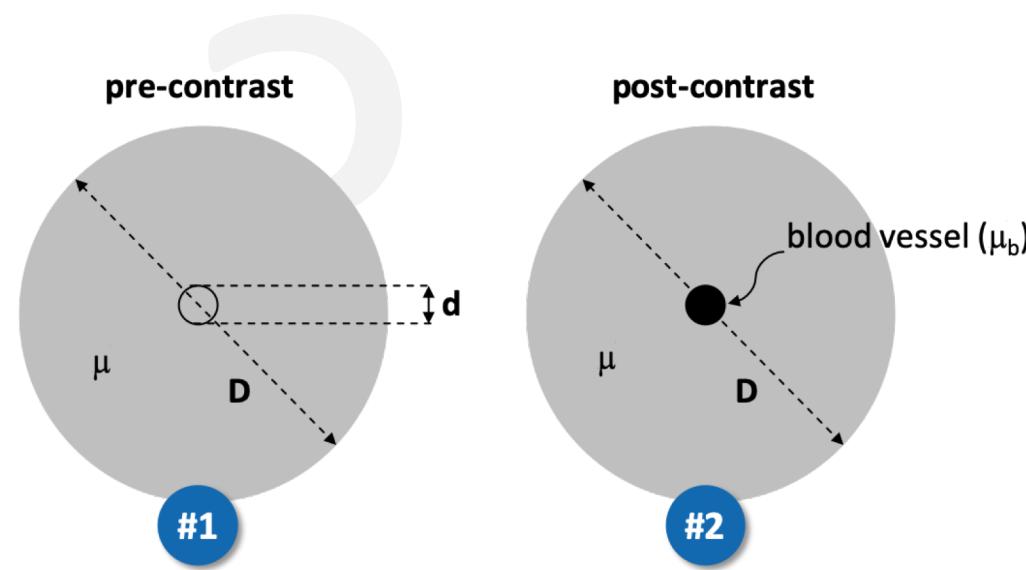
Real-time DSA Angiography



Clicker Activity (5 min)

Digital Subtraction Angiography is performed including a pre-contrast measurement #1 and a post-contrast measurement #2. The contrast agent increases the linear attenuation coefficient in blood. What is the image contrast of projection through the center point of the setup?

- Contrast $\sim (\mu_b * d - \mu * D)$
- Contrast $\sim (\mu_b * D - \mu * d)$
- Contrast $\sim (\mu_b - \mu) * d$



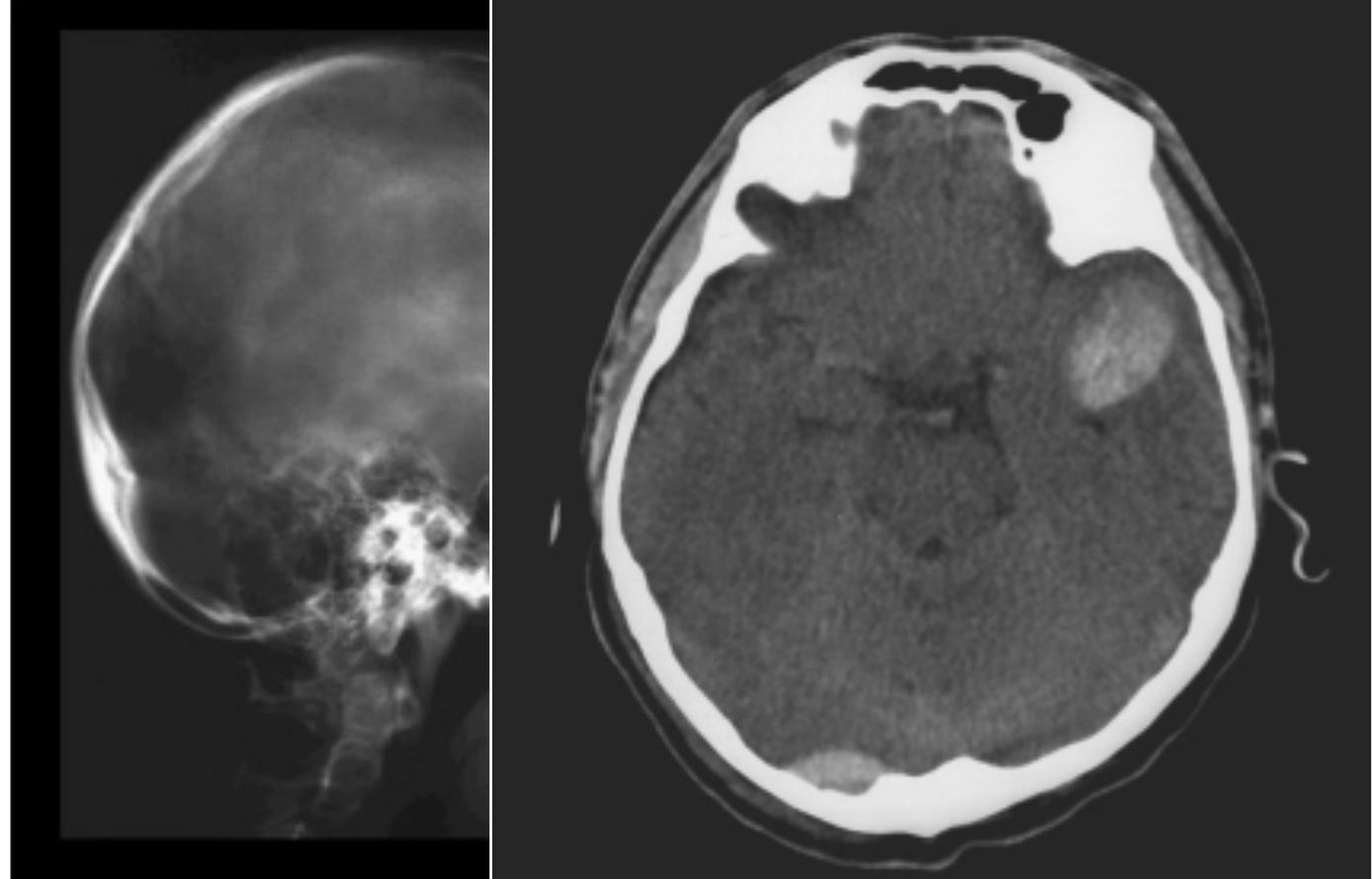
Clicker Activity (Notes)





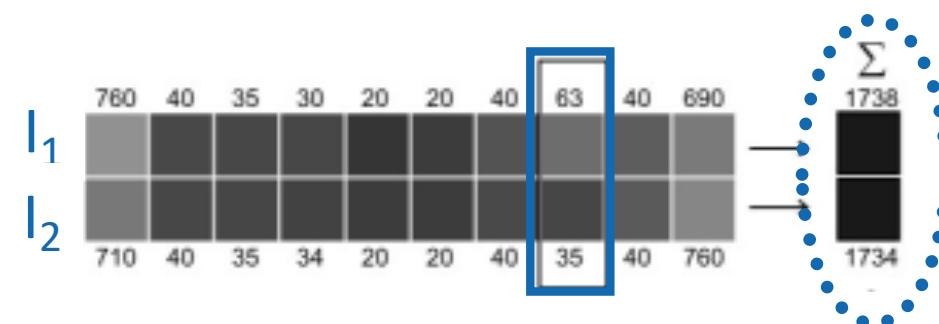
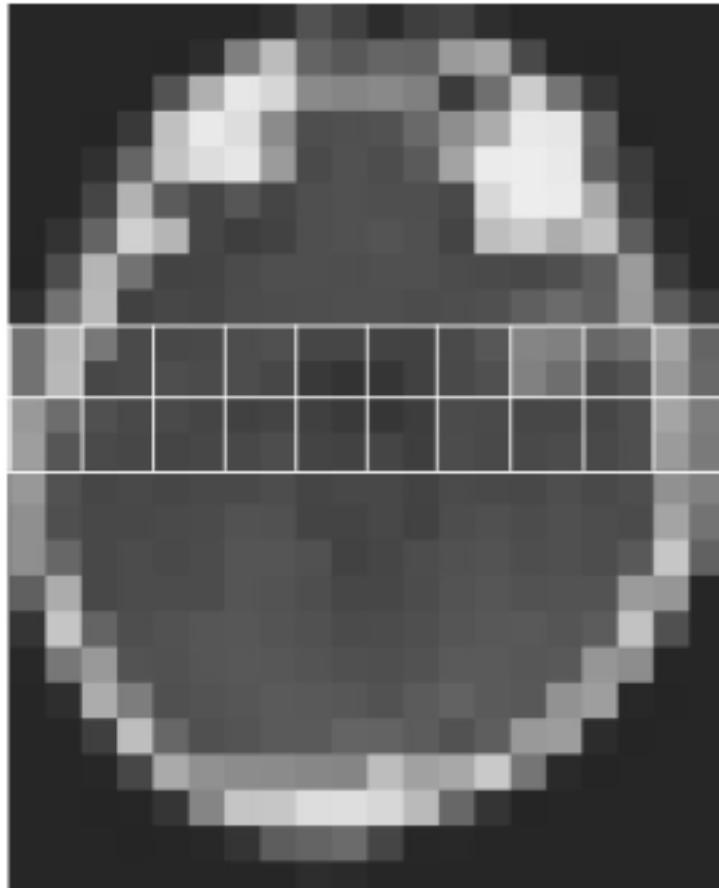
Computed Tomography

Problem



Adapted from Kalender WA, Computed Tomography, ISBN 3-89578-216-5

Problem



$$\text{Contrast} = \frac{\ln(I_1) - \ln(I_2)}{\ln(I_1) + \ln(I_2)}$$

Projection versus Tomographic imaging

- Projection imaging

- = 3D structures are collapsed onto 2D image:

- details are obscured
 - low soft-tissue contrast
 - not quantitative

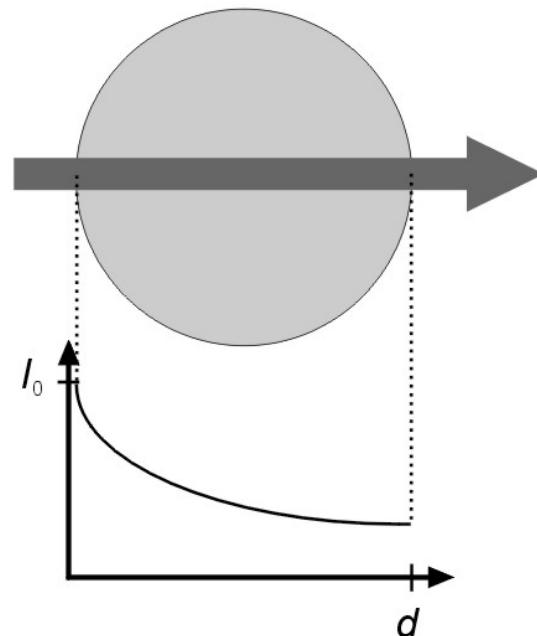
- Tomographic imaging

- = 3D structure is resolved:

- slices (“gr: Tomos = slice”)
 - Computed image reconstruction
 - Projection imaging and CT share same principles of x-ray generation, interaction, detection

CT – Reconstruction problem

1) Homogeneous object, monochromatic radiation



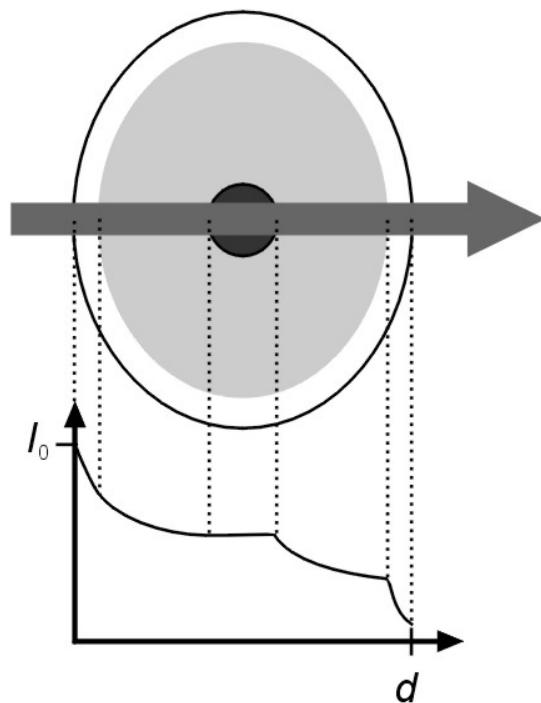
$$I = I_0 \cdot e^{-\mu \cdot d}$$

$$P = \ln \frac{I_0}{I} = \mu \cdot d$$

$$\mu = \frac{1}{d} \cdot \ln \frac{I_0}{I}$$

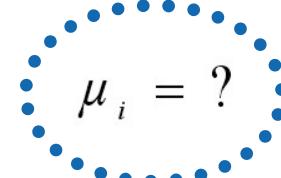
CT – Reconstruction problem

2) Inhomogeneous object, monochromatic radiation



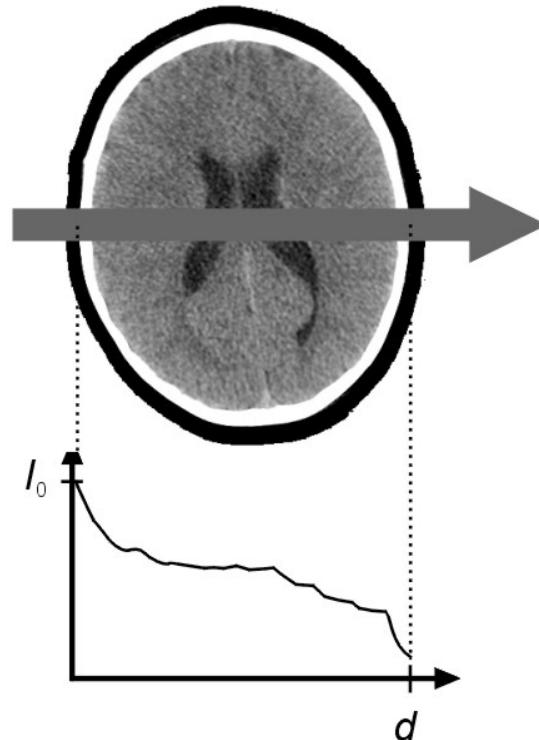
$$\begin{aligned}
 I &= I_0 \cdot e^{-\mu_1 \cdot d_1 - \mu_2 \cdot d_2 - \mu_3 \cdot d_3 - \dots} = \\
 &= I_0 \cdot e^{-\left[\sum_{i=1}^n \mu_i d_i \right]} = I_0 \cdot e^{-\int_0^d \mu \, ds}
 \end{aligned}$$

$$P = \ln \frac{I_0}{I} = \sum \mu_i d_i$$



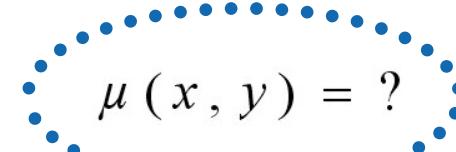
CT – Reconstruction problem

3) Inhomogeneous object, polychromatic radiation



$$I = \int_0^{E_{\max}} I_0(E) \cdot e^{-\int_0^d \mu(E) ds} dE$$

$$P = \ln \frac{I_0}{I}$$

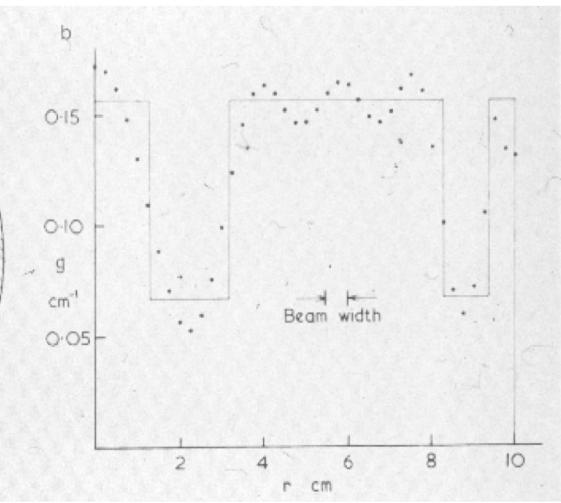
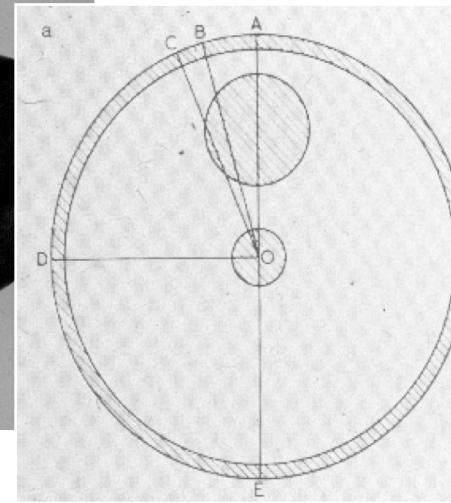
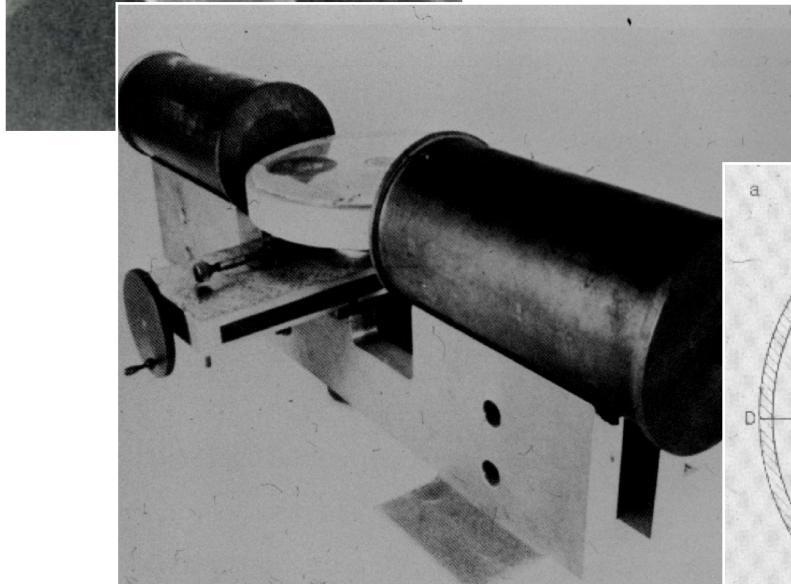


CT – History

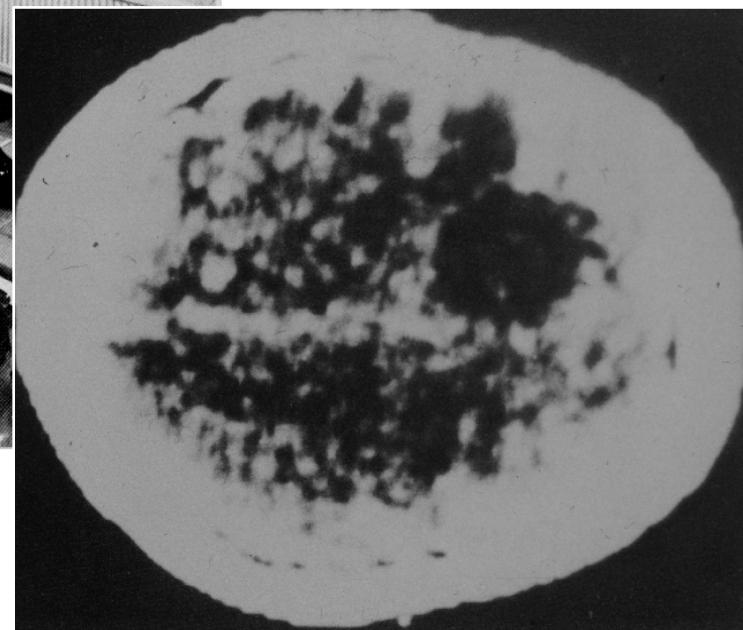
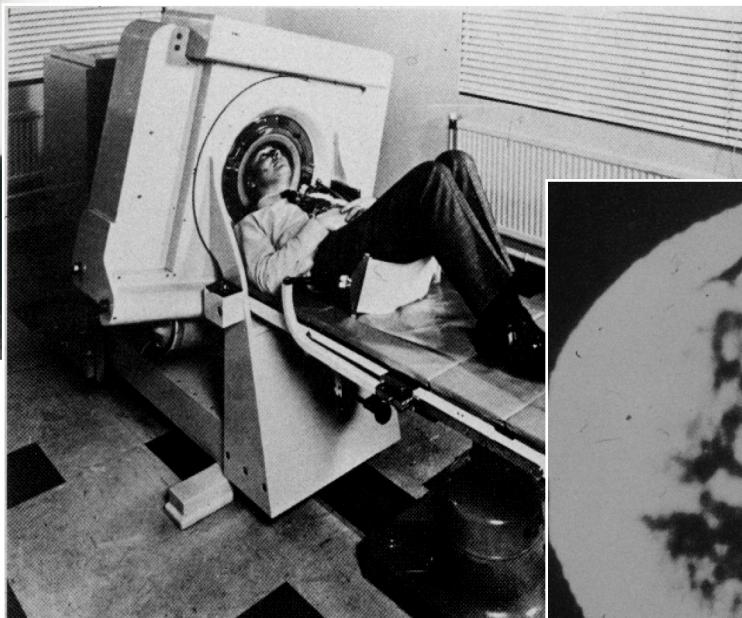
- Johann Radon, 1917
 - mathematical foundation for tomographic image reconstruction
- Allan Cormack, 1963
 - first experiments with axial tomography
- Godfrey Hounsfield, 1972
 - first clinical prototype; manufactured at EMI Ltd., England

(Cormack and Hounsfield shared Nobel prize in 1979)

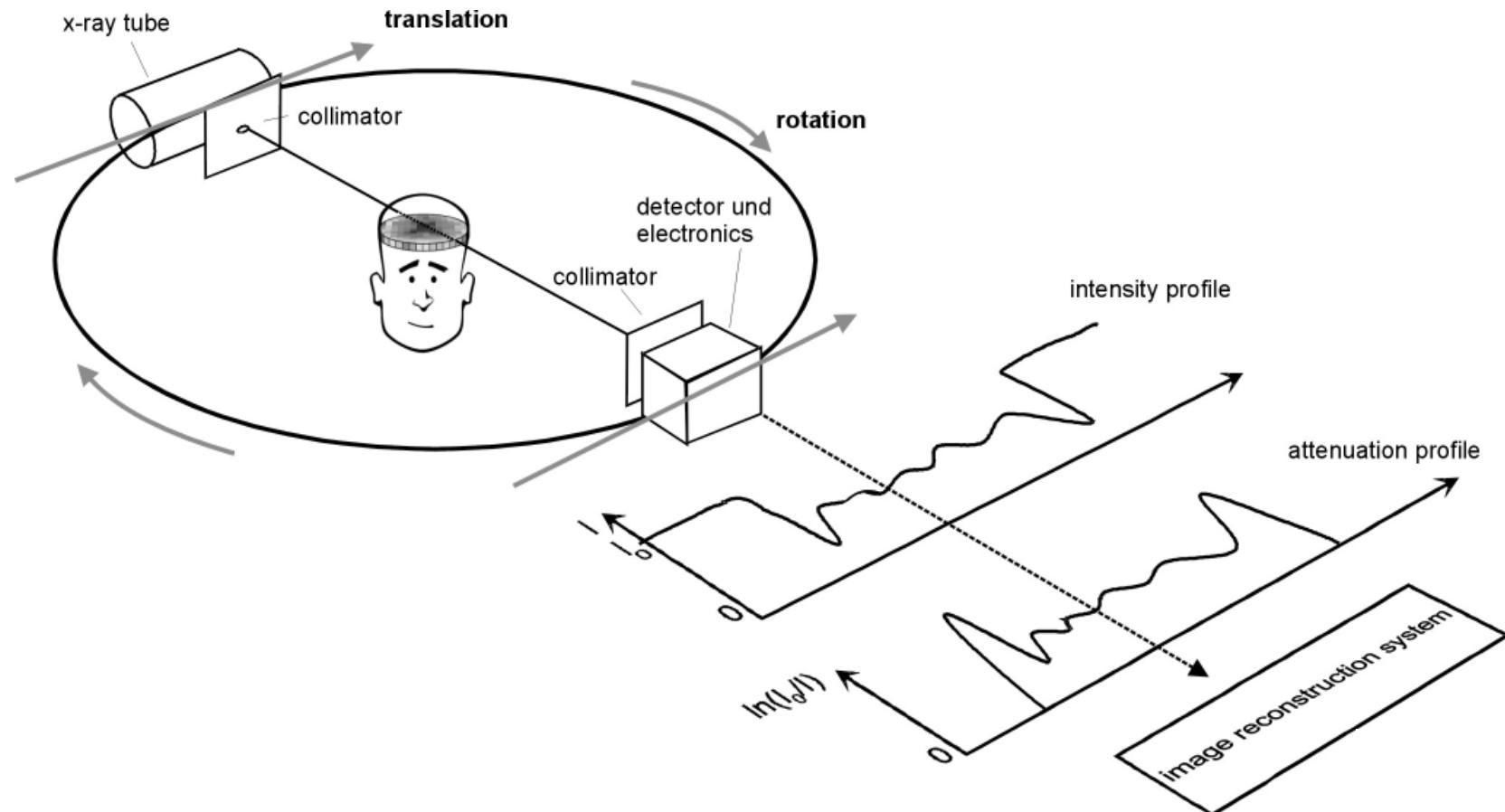
CT – Allan Cormack



CT – Godfrey Hounsfield



CT – Principle



Adapted from Kalender WA, Computed Tomography, ISBN 3-89578-216-5

CT – Image reconstruction

- Direct inversion (solving N unknowns from at least N equations)

$$\begin{array}{|c|c|} \hline \mu_1 & \mu_2 \\ \hline \mu_3 & \mu_4 \\ \hline \end{array} \rightarrow p_1 = \mu_1 + \mu_2$$

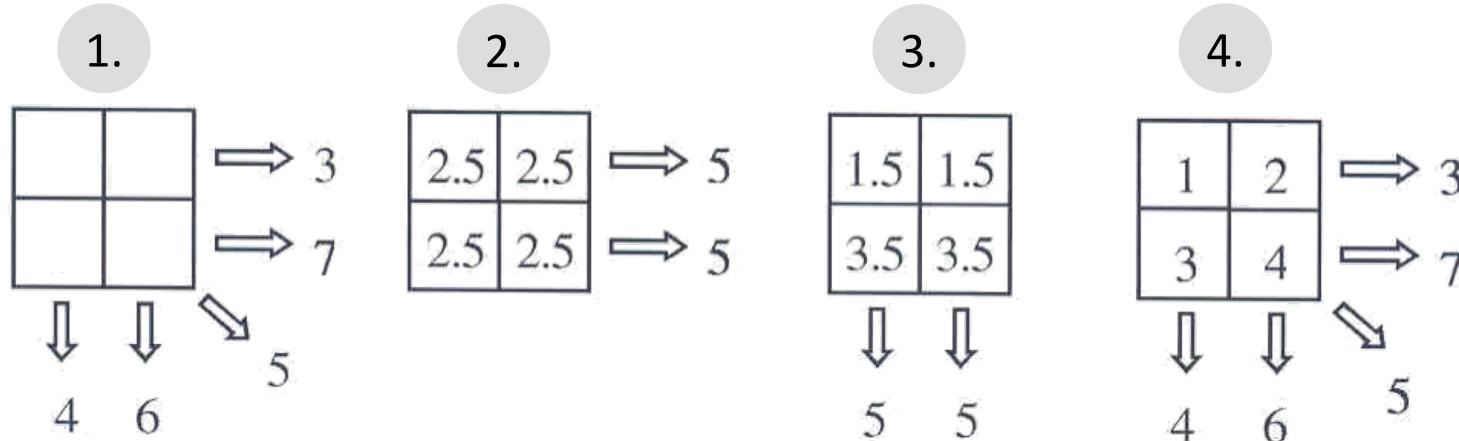
$$\begin{array}{|c|c|} \hline \mu_1 & \mu_2 \\ \hline \mu_3 & \mu_4 \\ \hline \end{array} \rightarrow p_2 = \mu_3 + \mu_4$$

$$\begin{array}{|c|c|} \hline \mu_1 & \mu_2 \\ \hline \mu_3 & \mu_4 \\ \hline \end{array} \rightarrow p_4 = \mu_1 + \mu_4$$

$$\begin{array}{|c|c|} \hline \mu_1 & \mu_2 \\ \hline \mu_3 & \mu_4 \\ \hline \end{array} \rightarrow p_3 = \mu_1 + \mu_3$$

$$\begin{array}{|c|c|} \hline \mu_1 & \mu_2 \\ \hline \mu_3 & \mu_4 \\ \hline \end{array} \rightarrow p_5 = \mu_2 + \mu_4$$

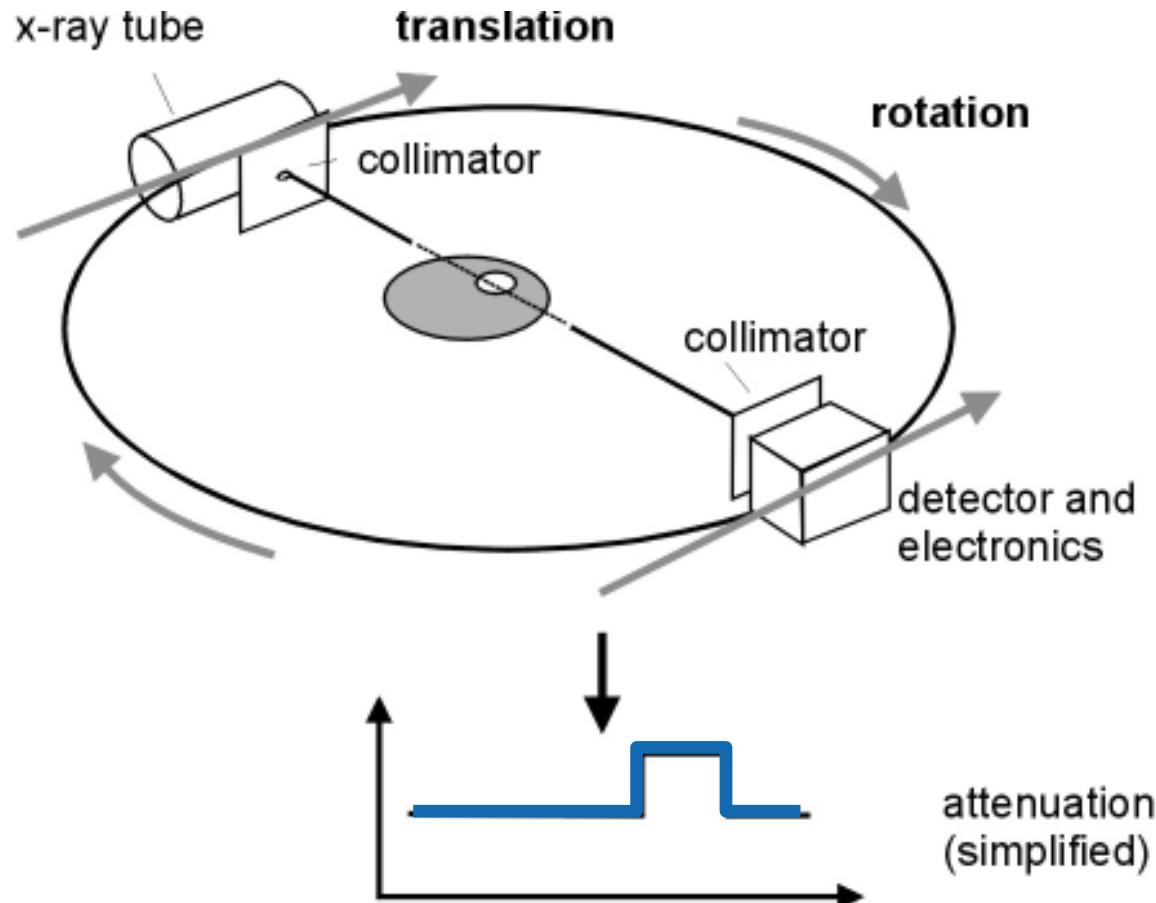
- Iterative reconstruction



(image estimation, pseudo-projection, minimization of residuum)

CT – Image reconstruction

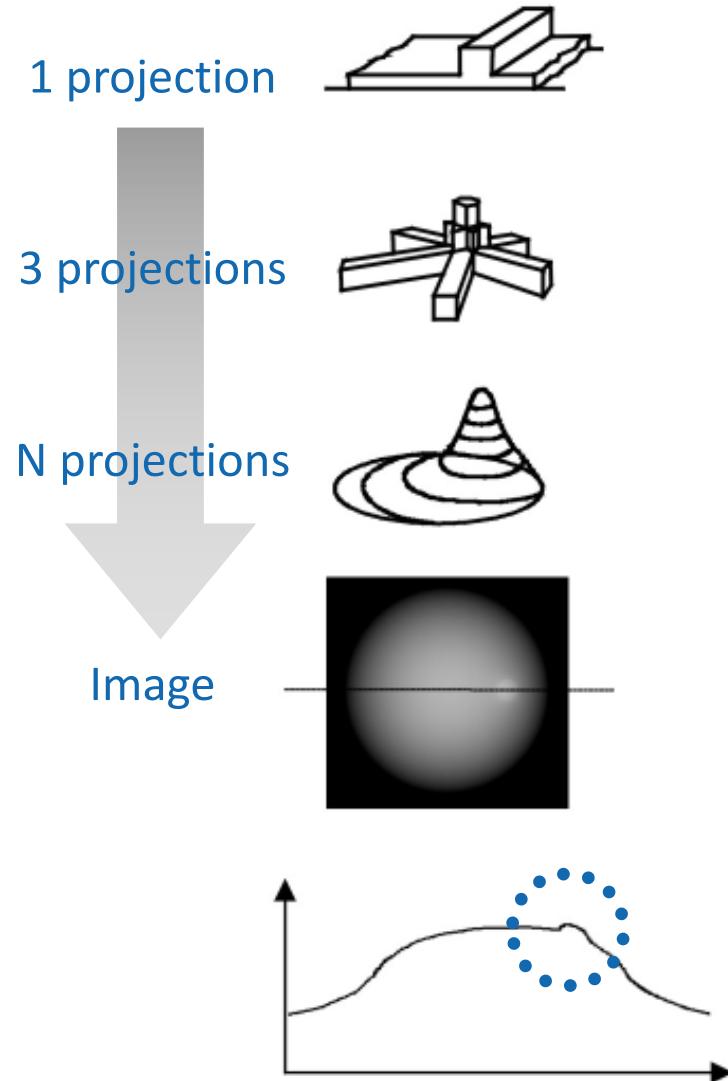
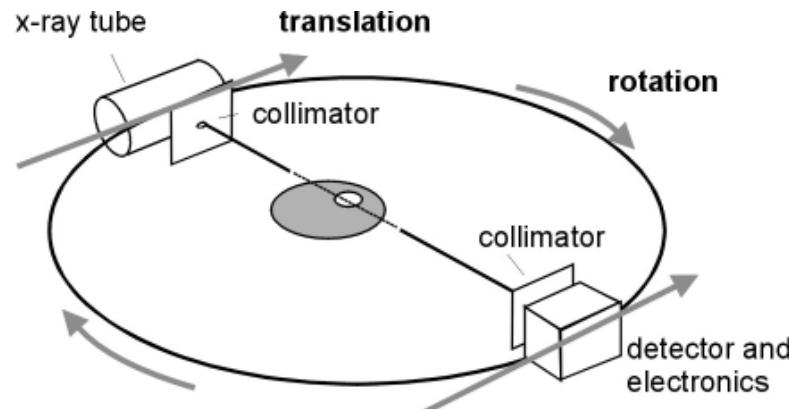
- Backprojection



Adapted from Kalender WA, Computed Tomography, ISBN 3-89578-216-5

CT – Image reconstruction

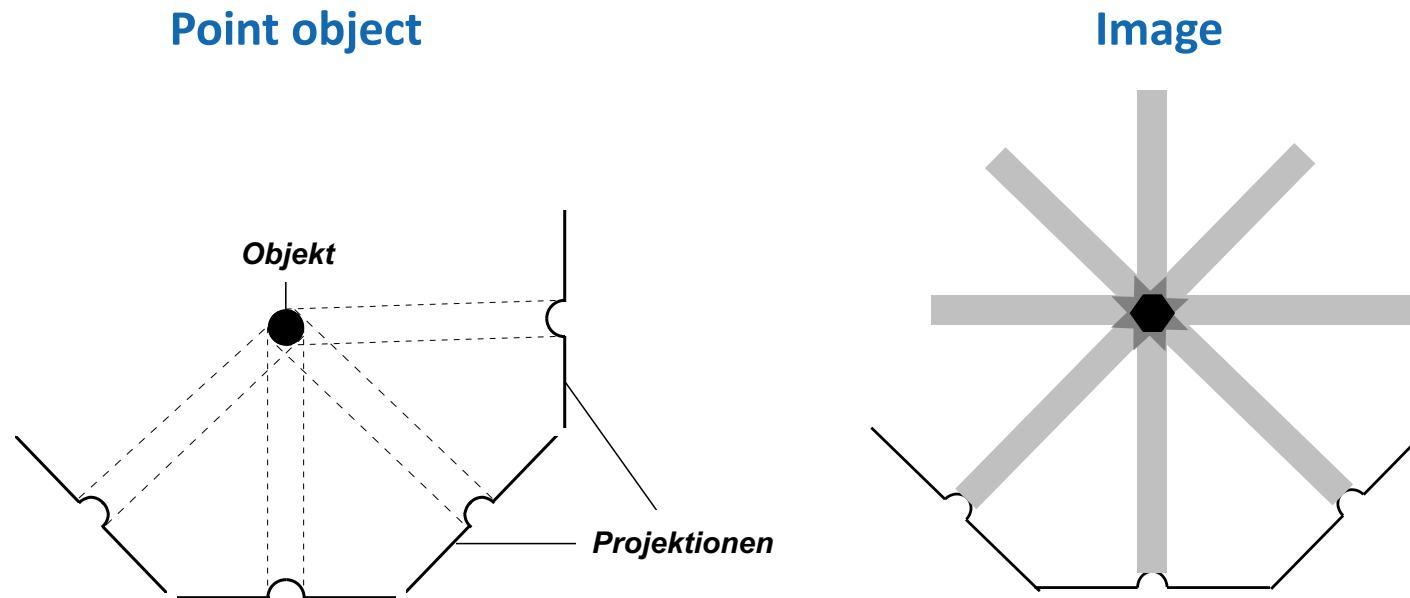
- Backprojection



Adapted from Kalender WA, Computed Tomography, ISBN 3-89578-216-5

CT – Image reconstruction

- Backprojection – Point Spread Function (PSF)



PSF is obtained if

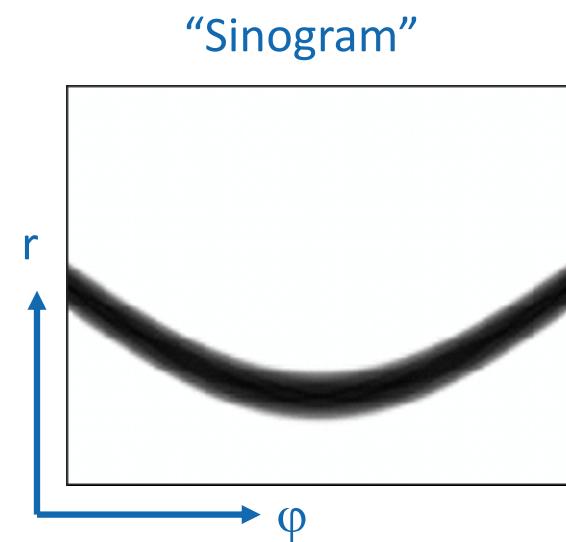
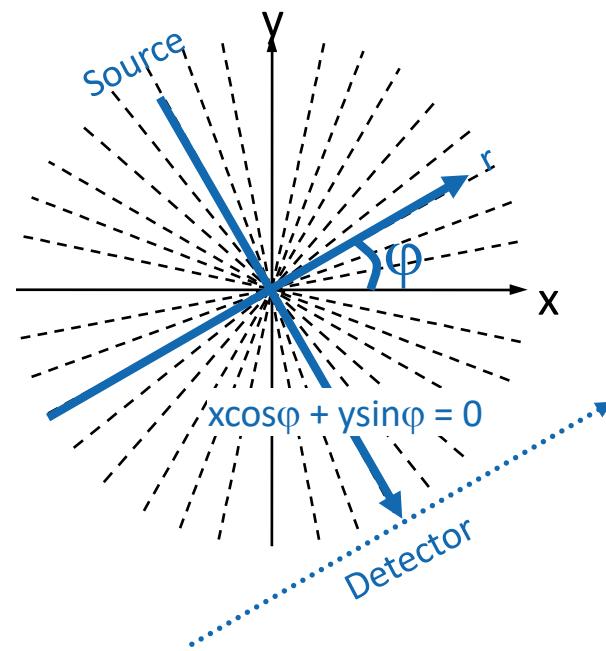
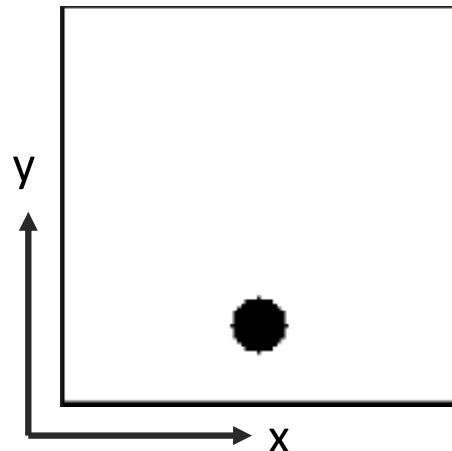
- 1) $\text{radius}_{\text{object}} \rightarrow 0$
- 2) number of projections $\rightarrow \infty$
- 3) number samples per projection $\rightarrow \infty$

$$\text{PSF} \propto \frac{1}{r}$$

CT – Image reconstruction

- Radon transform

$$P_\varphi(r) = \mathcal{R}[\mu](r, \varphi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) \delta(x \cos \varphi + y \sin \varphi - r) dx dy$$



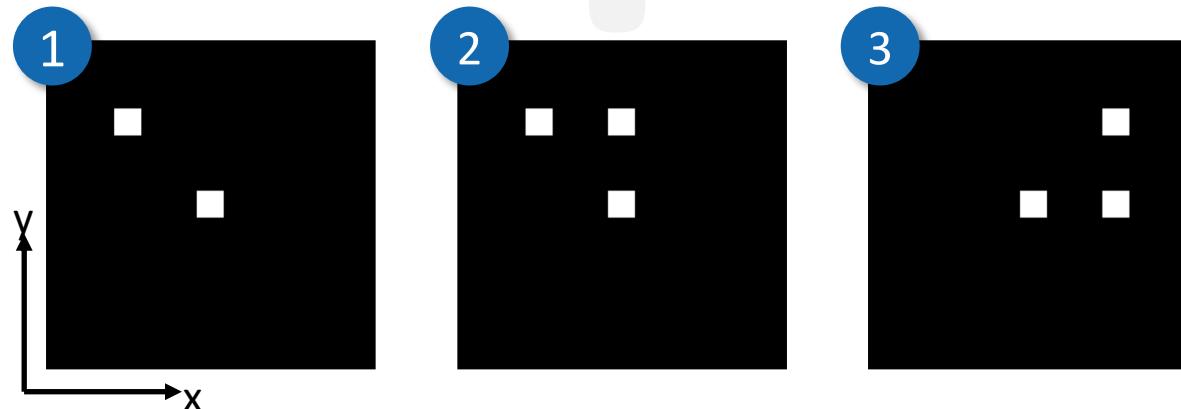
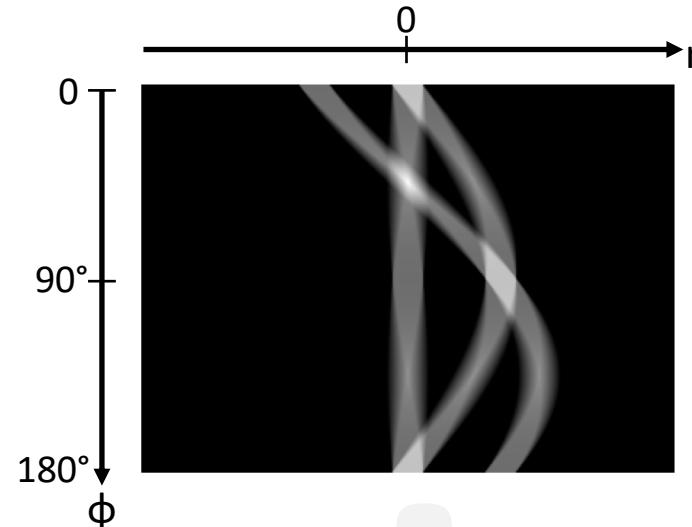
Radon transform = Image encoding principle in CT



Clicker Activity (5 min)

Which object does the sinogram below correspond to?

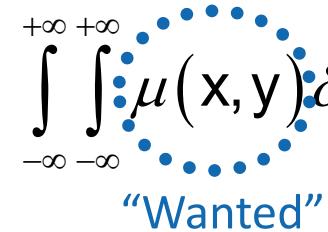
- Object #1
- Object #2
- Object #3



CT – Image reconstruction

- Inverse transform (I)

$$P_\varphi(r) = R[\mu](r, \varphi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) \delta(x \cos \varphi + y \sin \varphi - r) dx dy$$



Radon found analytical solution but it's numerically impractical



“Use Fourier transform”

$$F\{P_\varphi(r)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(s, r) e^{-iur} dr ds$$

CT – Image reconstruction

- Inverse transform (II)

$$F\{P_\varphi(r)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(s, r) e^{-iur} dr ds$$



$$F\{P_\varphi(u)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) e^{-iu(x \cos \varphi + y \sin \varphi)} dx dy$$



$$F\{P_\varphi(p, q)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) e^{-ixp} e^{-iyq} dx dy$$



Reconstruction formula

$$\mu(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F\{P_\varphi(p, q)\} e^{ixp} e^{iyq} dp dq$$



CT – Image reconstruction

- Inverse transform (III)

$$\mu(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F\{P_\varphi(p, q)\} e^{ixp} e^{iyq} dp dq$$



$$\mu(x, y) = \frac{1}{4\pi^2} \int_0^{\pi} \int_{-\infty}^{+\infty} F\{P_\varphi(u)\} e^{iux} |u| du d\varphi$$



1. $F\{P_\varphi(u)\}$ is weighted by $|u|$
2. $F\{P_\varphi(u)\} |u|$ is inversely transformed
3. and summed over all angles φ (=backprojection)



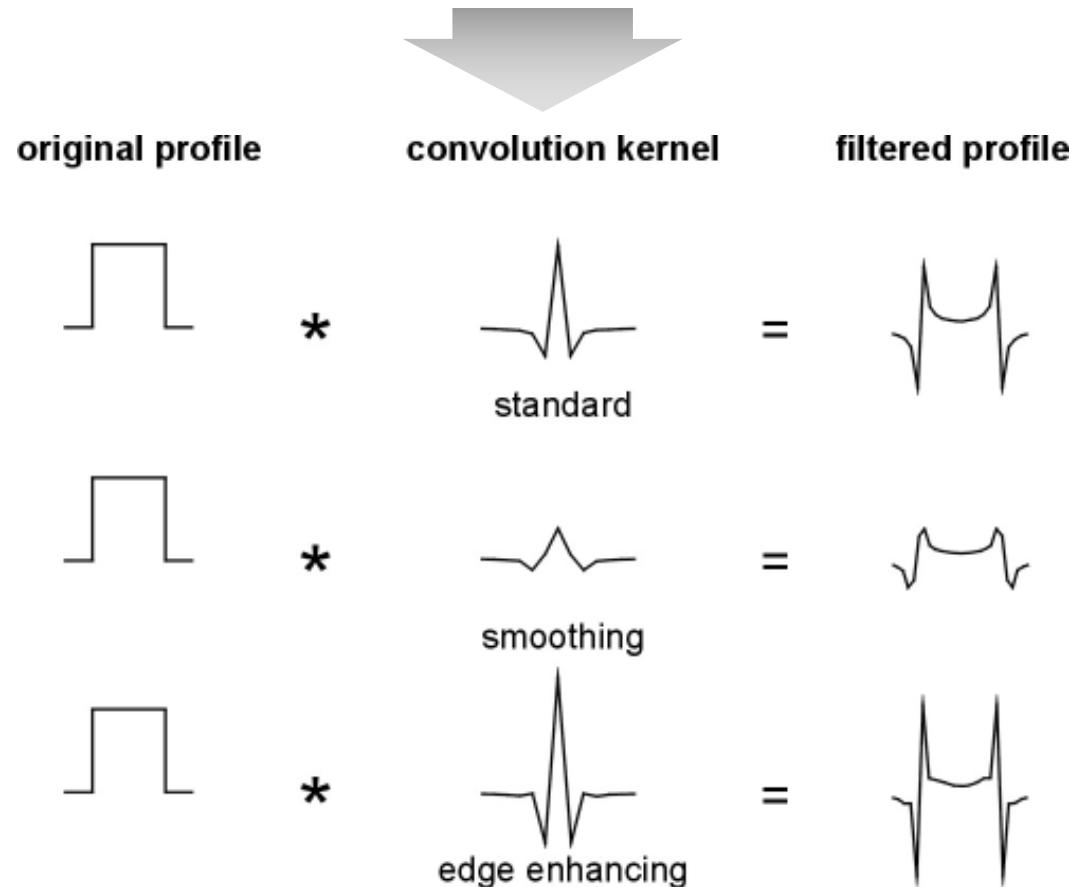
$$\text{Image} = \text{Filter} * \text{Object} \leftrightarrow F\{\text{Image}\} = F\{\text{Filter}\} \cdot F\{\text{Object}\}$$



CT – Image reconstruction

- Convolution backprojection

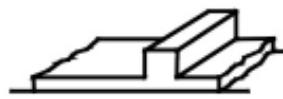
$$\text{Image} = \text{Filter} * \text{Object} \leftrightarrow F\{\text{Image}\} = F\{\text{Filter}\} \cdot F\{\text{Object}\}$$



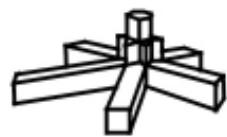
Adapted from Kalender WA, Computed Tomography, ISBN 3-89578-216-5

CT – Image reconstruction

without pre-filtering



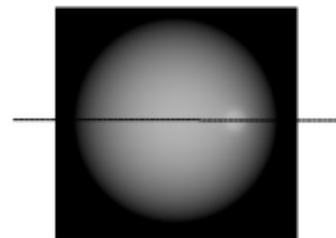
1 projection



3 projections



N_p projections

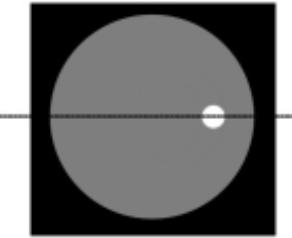
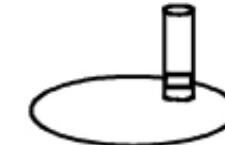


N_p projections



CT value profile

with pre-filtering



Adapted from Kalender WA, Computed Tomography, ISBN 3-89578-216-5

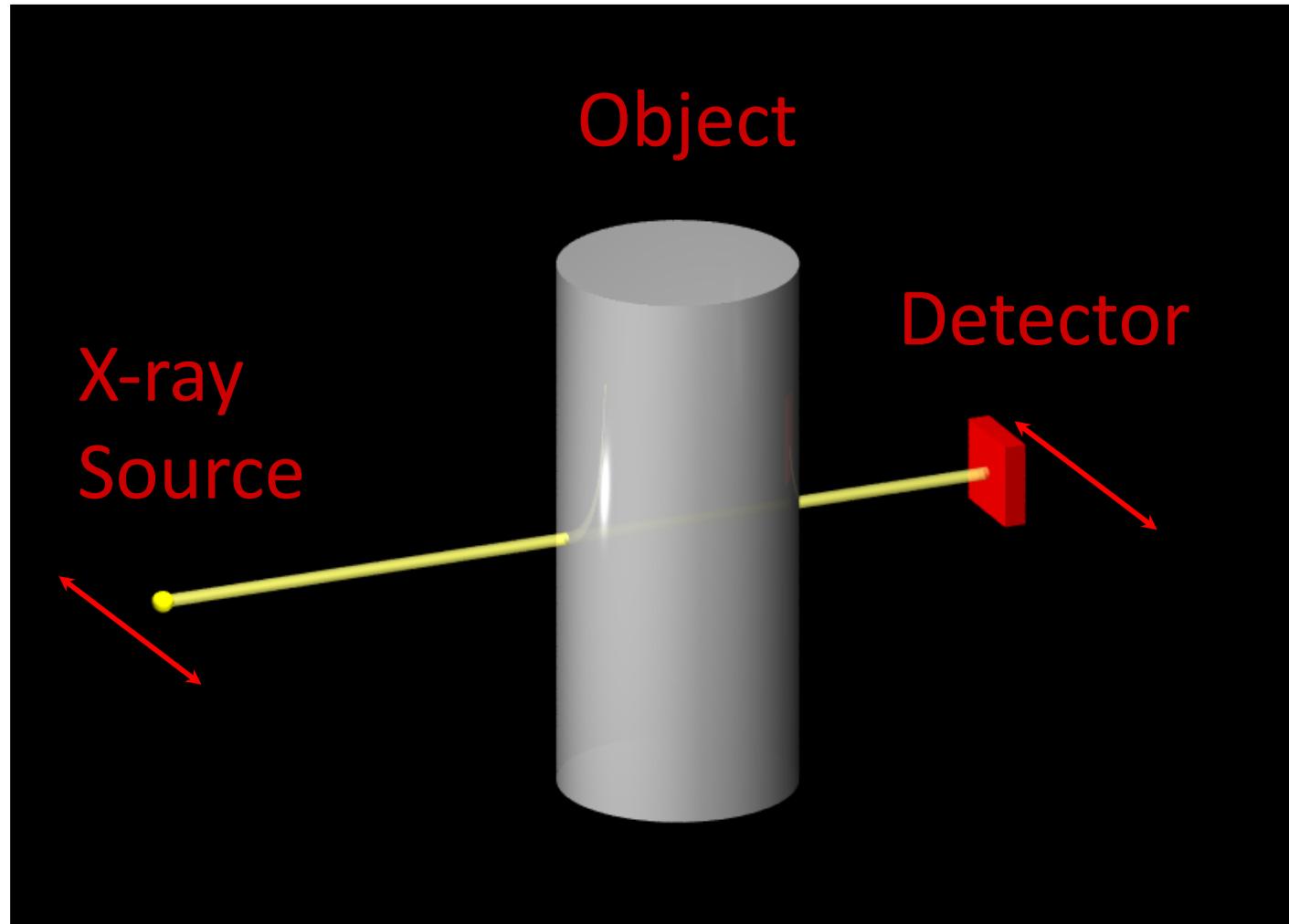


CT – Image reconstruction

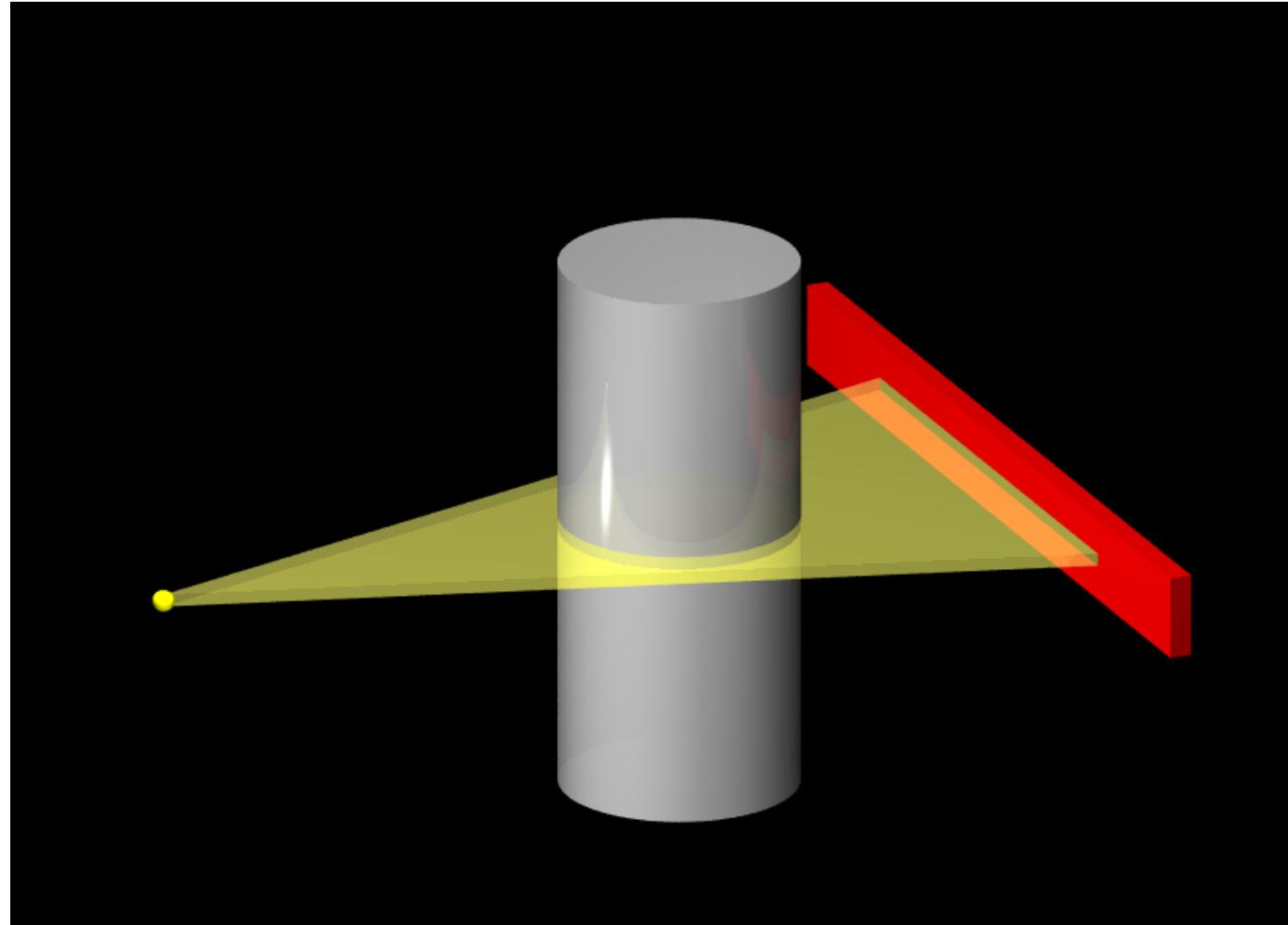
- Convolution backprojection process



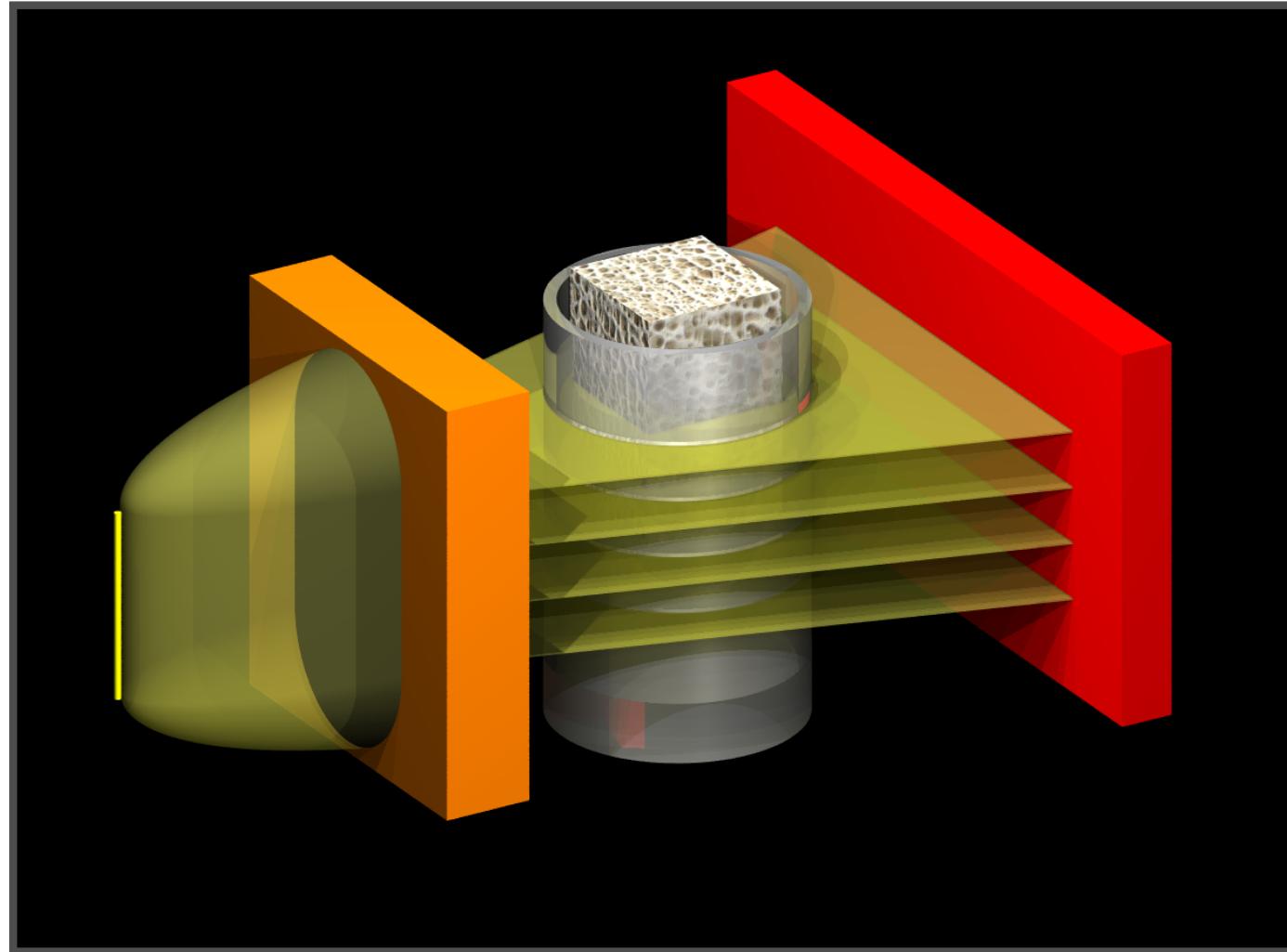
CT – Pencil beam



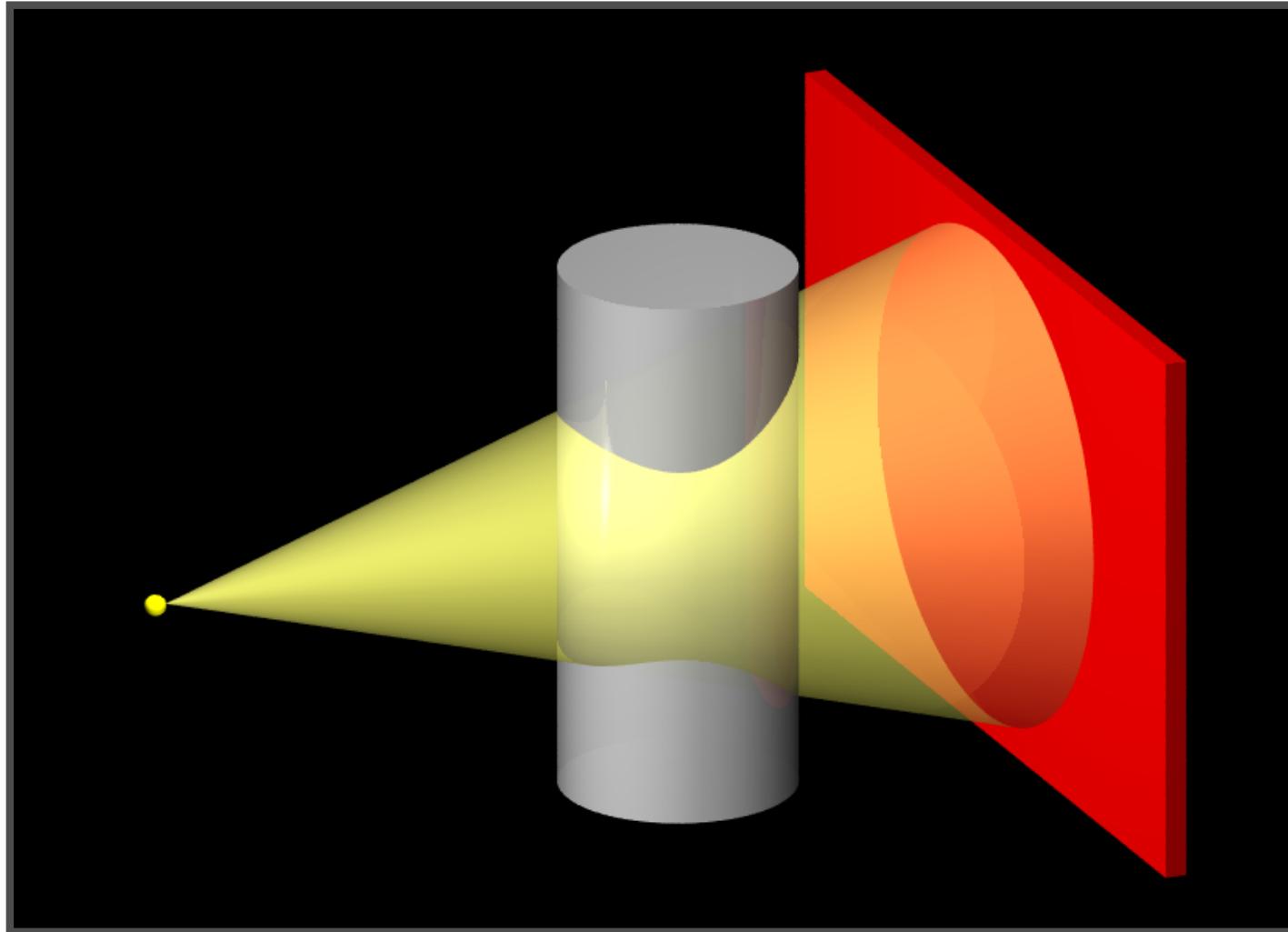
CT – Fan beam



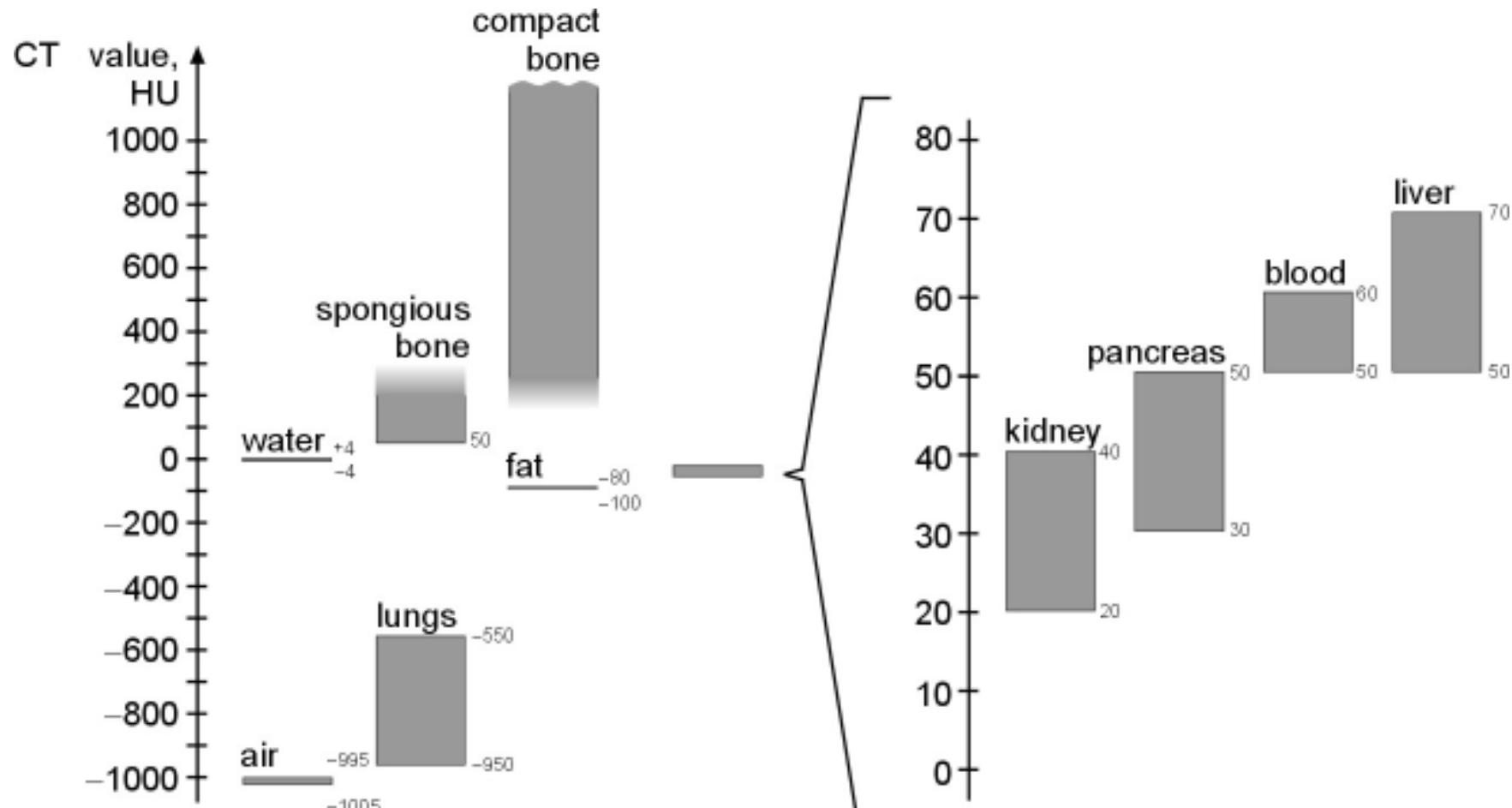
CT – Parallel fan beam



CT – Cone beam

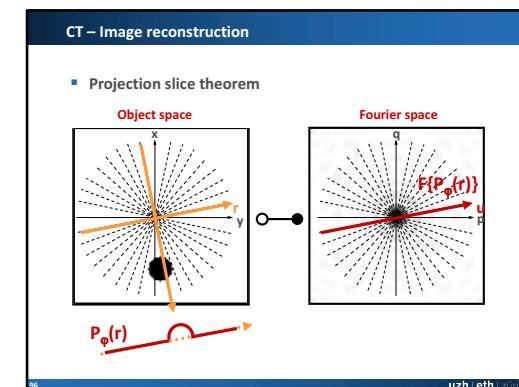
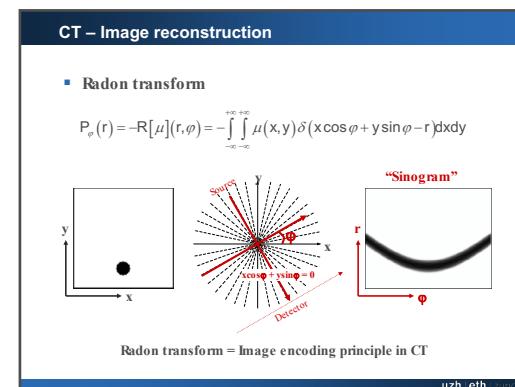
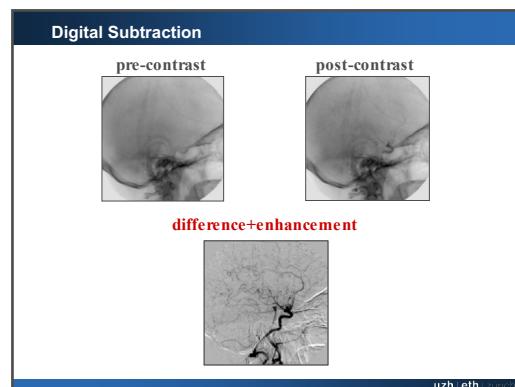
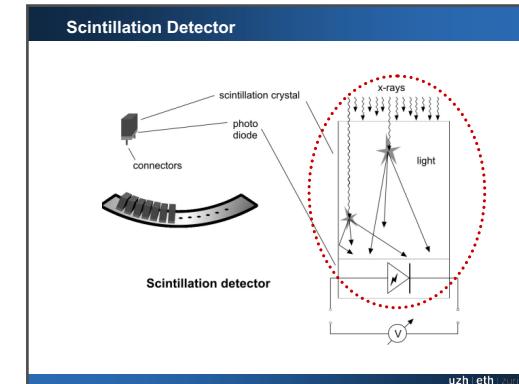
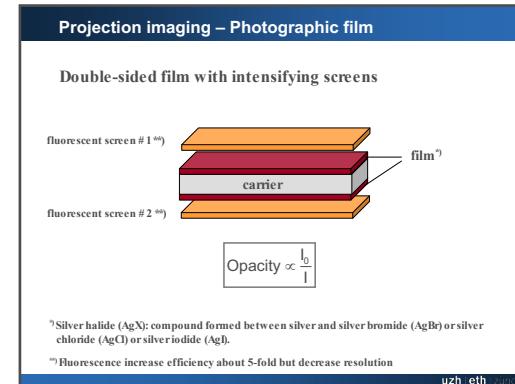
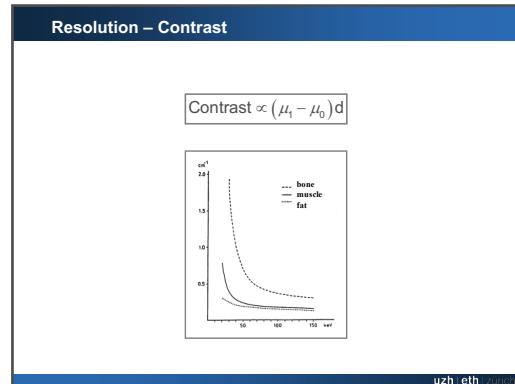


CT – Hounsfield unit



$$\text{CT value} = \frac{(\mu - \mu_{\text{water}})}{\mu_{\text{water}}} 1000 \text{HU}$$

“The most important slides”



CT – Image reconstruction

- Inverse transform (II)

$$F\{P_\varphi(r)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(s, r) e^{-iur} ds dr$$

$$F\{P_\varphi(u)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) e^{-i[u \cos \varphi + y \sin \varphi]} dx dy$$

$$F\{P_\varphi(p, q)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) e^{-ipx} e^{-iqy} dx dy$$

Reconstruction formula

$$\mu(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F\{P_\varphi(p, q)\} e^{ipx} e^{iqy} dp dq$$

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CT – Image reconstruction

- Inverse transform (III)

$$\mu(x, y) = -\frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F\{P_\varphi(p, q)\} e^{ipx} e^{iqy} dp dq$$

$$\mu(x, y) = -\frac{1}{4\pi^2} \int_0^{\pi/2} \int_0^{\pi/2} F\{P_\varphi(u)\} e^{iu|x|} e^{iv|y|} du dv$$

1. $F\{P_\varphi(u)\}$ is weighted by $|u|$
 2. $F\{P_\varphi(u)\} |u|$ is inversely transformed
 3. and summed over all angles φ (=backprojection)

Image = Filter * Object $\leftrightarrow F\{\text{Image}\} = F\{\text{Filter}\} \cdot F\{\text{Object}\}$

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