3×3 inverse

If a 3×3 matrix A is invertible follow the Steps below.

Step 1: Find Minor matrix (matrix of minors)

Step 2: Find Cofactor matrix. (matrix of cofactors)

Step 3: Find Adjoint matrix.

Step 4: Find the Inverse matrix.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}_{3\times3}$$

Calculation of minors:

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \quad M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}, \quad M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}, \quad M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}, \quad M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}, \quad M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}, \quad M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Minor matrix

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}_{3\times3}$$

Calculation of Cofactors

$$C_{ij} = (-1)^{i+j} M_{ij}$$

eg. $C_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11} = M_{11}$

$C_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12} = -M_{12}$

$C_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13} = M_{13}$

$C_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21} = -M_{21}$

$C_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22} = M_{22}$

$C_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23} = -M_{23}$

$C_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31} = M_{31}$

$C_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32} = -M_{32}$

$C_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33} = M_{33}$

Cofactor matrix

$$C = \begin{pmatrix} \underbrace{M_{11}}_{=C_{11}} & \underbrace{-M_{12}}_{=C_{12}} & \underbrace{M_{13}}_{=C_{13}} \\ \underbrace{-M_{21}}_{=C_{21}} & \underbrace{M_{22}}_{=C_{22}} & \underbrace{-M_{23}}_{=C_{23}} \\ \underbrace{M_{31}}_{=C_{31}} & \underbrace{-M_{32}}_{=C_{32}} & \underbrace{M_{33}}_{=C_{33}} \end{pmatrix}_{3\times 3}$$

note that sign matrix for a 3×3 matrix is $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$$

Adjoint matrix

Adj (A) = Transpose of Cofactor matrix

$$\text{Adj}(A) = C^T$$

Determinant of A (expanding using row 1)

$$\det(A) = |A| = a_{11} M_{11} - a_{12} M_{12} + a_{13} M_{13} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

* You can use any row or column to find the determinant

eg: use column 2

$$\det(A) = |A| = -a_{12} M_{12} + a_{22} M_{22} - a_{32} M_{32}$$

$$|A| = a_{12} C_{12} + a_{22} C_{22} + a_{32} C_{32}$$

Inverse of A

$$\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{Adj}(A)}, \quad \det(A) \neq 0$$

Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}, \quad P(B) \neq 0$$

$$P(B) = P(B|A) \cdot P(A) + P(B|A') \cdot P(A')$$

Eg:

A: event of actually having the disease

B: " " testing positive for the "

$P(A) = 1\%$

$P(B) = ?$

$P(B|A) = 95\%$ = True positive rate

$P(A|B) = ?$

$P(B|A') = 5\%$ = False positive rate

$P(A') = ?$