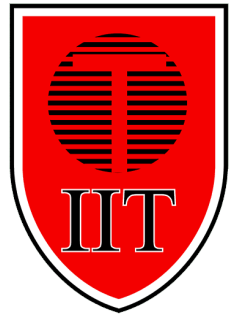
Mathematics in Computing
4COSC007C

Lecture 2: Algorithms

September Intake 2021



Algorithms

- An algorithm is a list of instructions for performing a specific task, or, for solving a particular type of problem.
- We use recipes for cooking: say to make a pizza.
- We use directions from SATNAV.
- We use techniques to add numbers.

What is the Algorithm to Compute Truth Table?

Algorithm to Compute Truth Tables

- Let a formula C consists of the atomic propositions $p_1, p_2, \ldots, p_n$.
- To compute the truth table do the following:
  1. List its atomic propositions $p_1, p_2, \ldots, p_n$ and count their number, n.
  2. Form a table with $m = 2^n$ rows.
  3. List all possible combinations of the truth values for $p_1, p_2, \ldots, p_n$.
  4. Prioritize the logical operations in C, finding main logic operation, etc.
  5. Compute these operations in order.

More on Algorithms

- How to add all natural numbers from 1 to 10?

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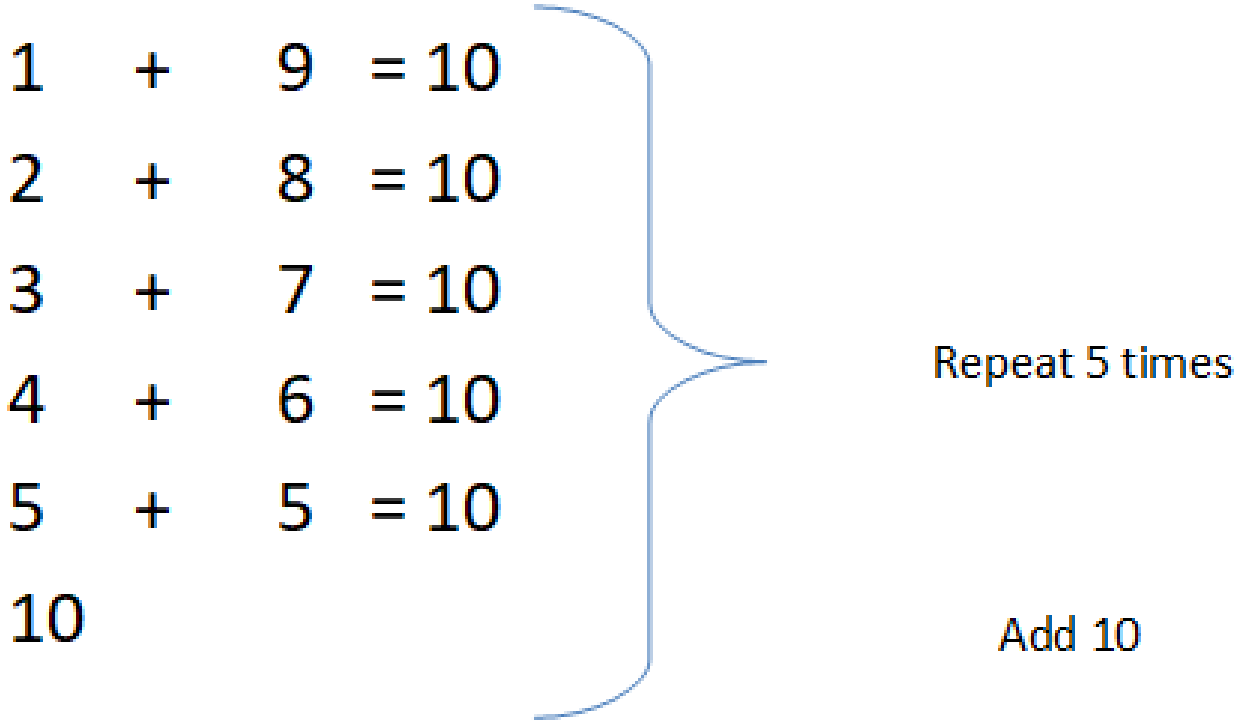
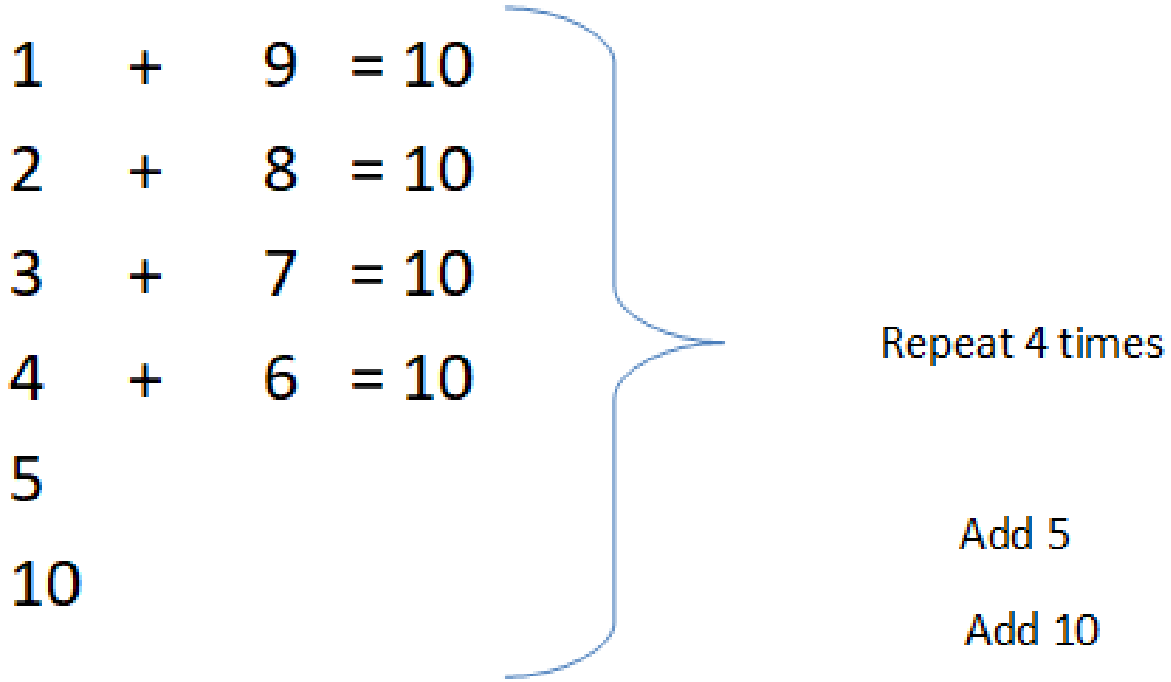


- Is this correct?

More on Algorithms

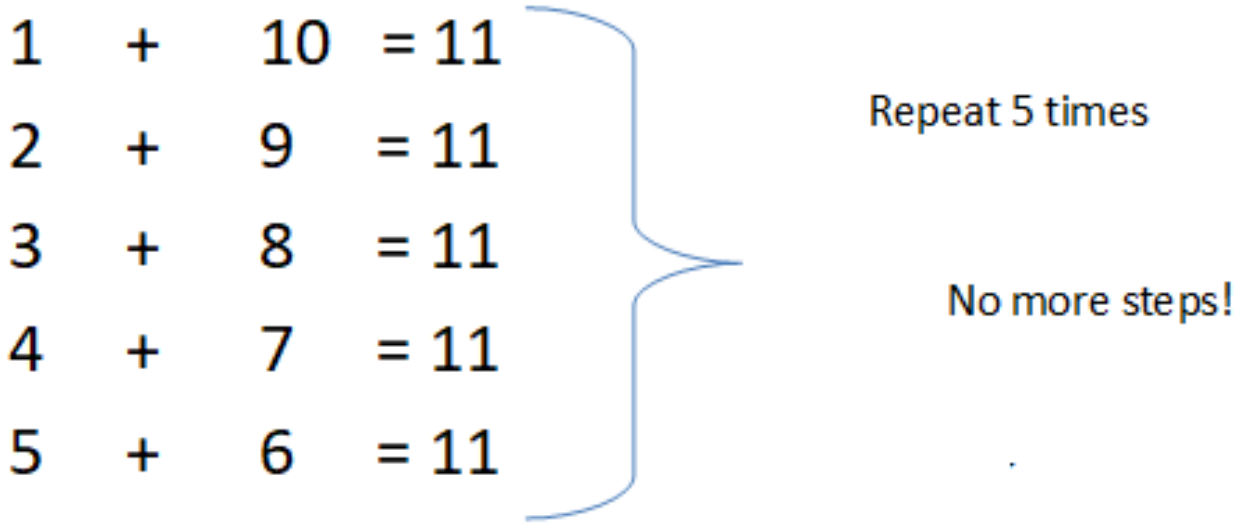
- How to add all natural numbers from 1 to 10?



- Is this correct?

More on Algorithms

- How to add all natural numbers from 1 to 10?



- Is this correct?
- What is the conclusion? **We can have more than one algorithm to solve a problem.**

Algorithms: Efficiency

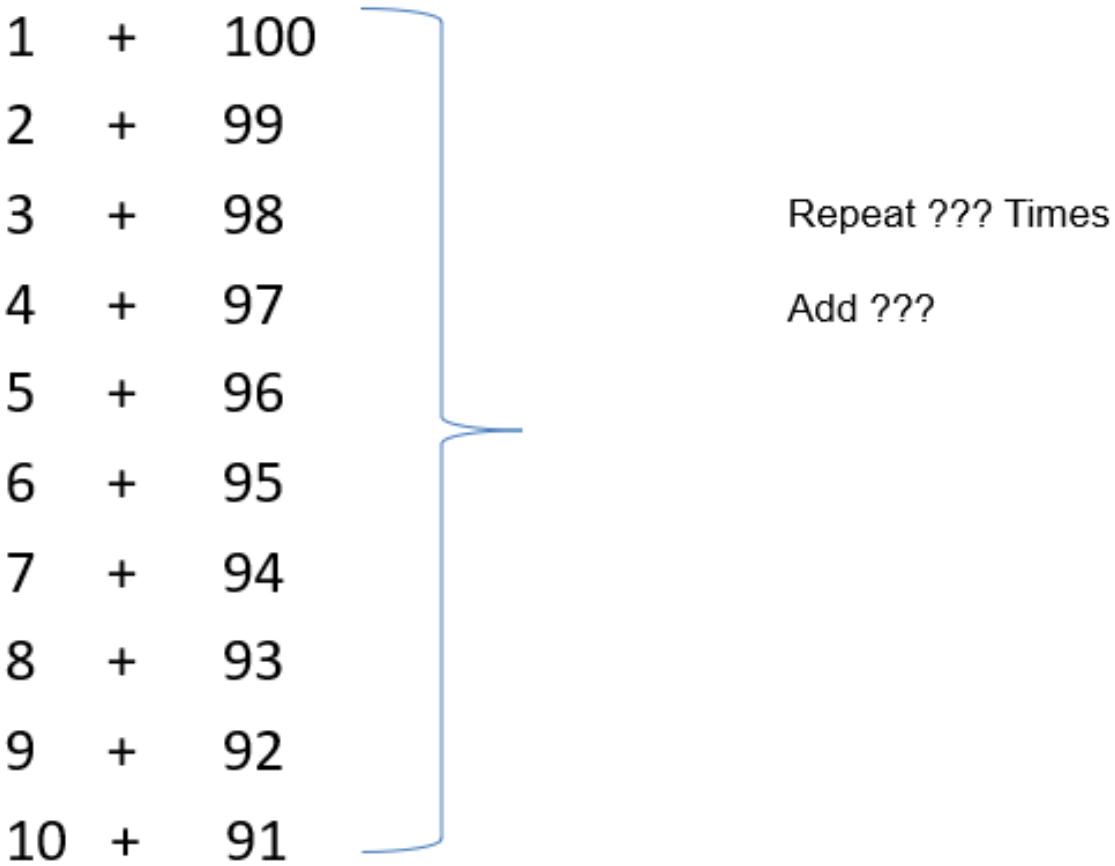
- Always try to find an optimal algorithm.
- Which one was more efficient to find the sum of numbers 1 to 10?

Problem Solving Strategies

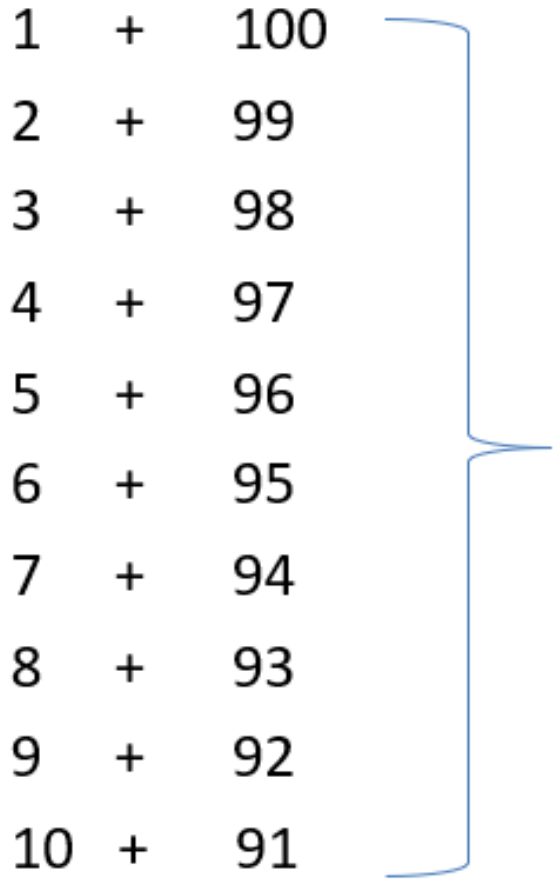
- Always try to find the most efficient way of solving a problem.
- Look for patterns.
- Consider familiar cases with similar patterns or similar ways of identifying patterns.
- Look for possible repetitions of the same operation/method/technique.
- The latter is called recursion.

How to add all natural numbers from 1 to 100?

- We just saw a similar problem. Its solution was based on grouping numbers. Can we do the same here?



How to add all natural numbers from 1 to 100?



10 x 101 = 1010

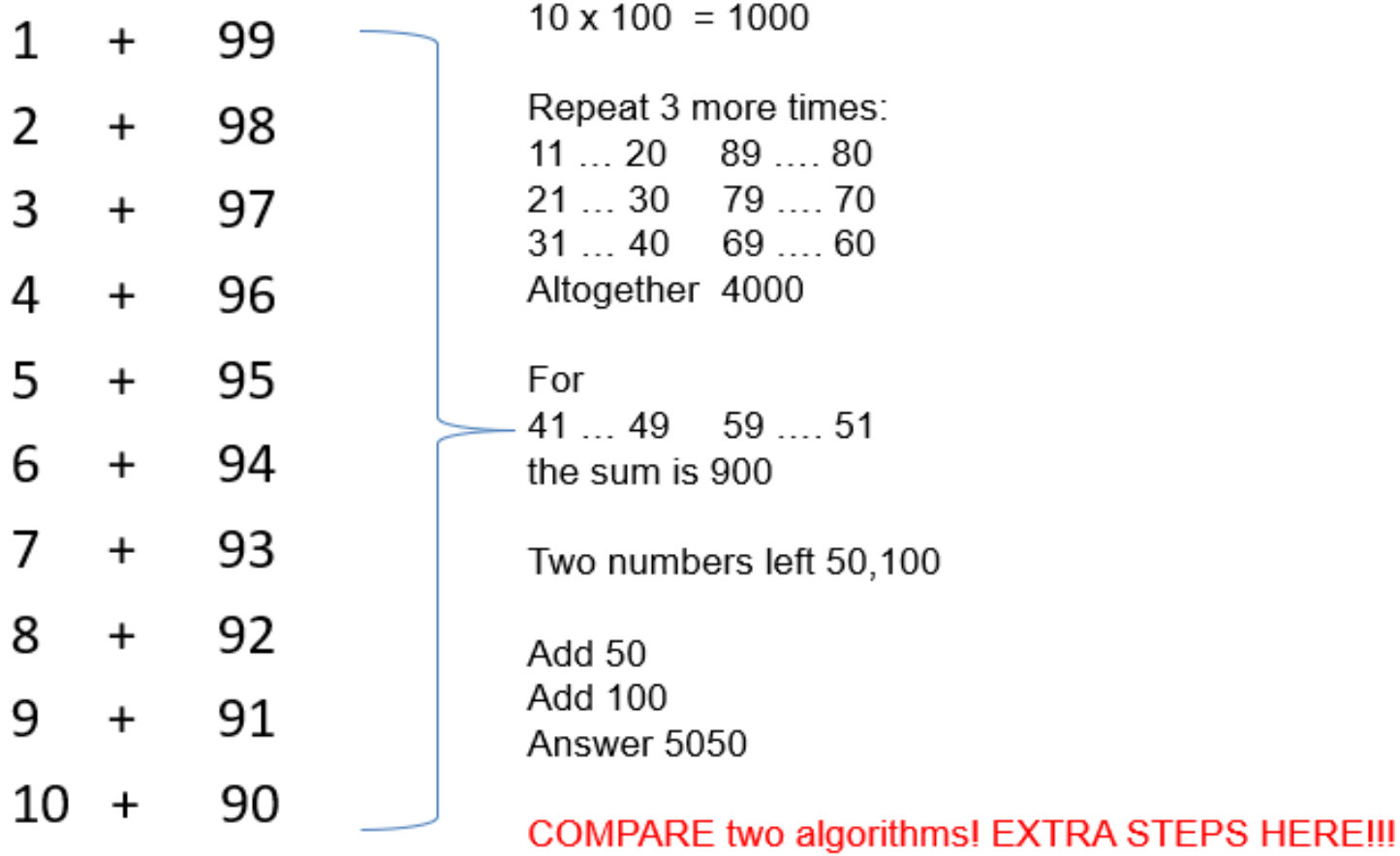
Repeat 4 more times:

11 … 20	90 …. 81
21 … 30	80 …. 71
31 … 40	70 …. 61
41 … 50	60 …. 51

Altogether 5 times 1010
= 5050

How to add all natural numbers from 1 to 100?

- Different solution – similar to the one for the case of 10 numbers but different grouping.



Question?

- What would you do when you need to reuse a word in the text which you have already typed in? Is Copy and Paste always more efficient?
- Assume that the word we are using is **TEXT**. There are two ways to reuse the word.
  - Typing again the word **TEXT**,
  - Copy/Paste.
- Algorithm for Copy/Paste with the mouse:

Question?

- What would you do when you need to reuse a word in the text which you have already typed in? Is Copy and Paste always more efficient?
- Assume that the word we are using is **TEXT**. There are two ways to reuse the word.
  - Typing again the word **TEXT**,
  - Copy/Paste.
- Algorithm for Copy/Paste with the mouse:
  1. Move the cursor to the beginning of the word
  2. Highlight the word **TEXT**
  3. Copy
  4. Move the cursor to the new position where this word should be typed again
  5. Paste the saved word

Is it more efficient than just typing 4 characters?

Classification of Formulae - Satisfiable and Unsatisfiable

Definition

In Logic a formula C is called SATISFIABLE if there exists at least one output value T (true) in the truth table for C.

Examples

- p, $\neg p$, $p \wedge q$

Definition

In Logic a formula C is called UNSATISFIABLE if there are no output values T (true) in the truth table for C.

Examples

- $p \wedge \neg p$

Classification of Formulae - Valid

Definition

In Logic a formula C is called VALID if its output values in the truth table are all T (true).

Are theses valid formulas?

- $p \vee \neg p$, $\neg(p \wedge \neg p)$, $(p \wedge q) \rightarrow p$

Typical Simple Reasoning

- Logical Consequence from a Knowledge Base.
- Consider the following knowledge base:
  - File 'X' is either a binary file or a text file.
  - If file 'X' is a binary file then program 'P' does not accept it.
  - If file 'X' is a text file then program 'P' accepts it.
  - Program 'P' accepts file 'X'.
  - What can we conclude from here?
  - In particular, can we conclude that 'File 'X' is a text file'?

Logical Consequence from a Knowledge Base

- Having defined the vocabulary, the knowledge base is formalized as:
  - File ‘X’ is either a binary file or a text file
  - If file ‘X’ is a binary file then program ‘P’ does not accept it
  - If file ‘X’ is a text file then program ‘P’ accepts it
  - Program ‘P’ accepts file ‘X’

Logical Consequence from a Knowledge Base

- Having defined the vocabulary, the knowledge base is formalized as:
  - File 'X' is either a binary file or a text file $p \vee q$
  - If file 'X' is a binary file then program 'P' does not accept it $p \implies \neg r$
  - If file 'X' is a text file then program 'P' accepts it $q \implies r$
  - Program 'P' accepts file 'X' r

How Do We Reason

- What are the rules to apply when we reason?
- Many of these are based on Boolean.
- What can you conclude from:
  - A and $A \implies B$?
  - $\neg A$ and $A \implies B$?
  - $\neg B$ and $A \implies B$?
  - A and $A \vee B$?
  - $\neg A$ and $A \vee B$?

Reasoning and Logical Consequence

Definition

B is a logical consequence of a knowledge base $A_1, A_2, A_3, ...A_n$ if the following formula is valid: $(A_1 \wedge (A_2 \wedge (A_3 \wedge ...A_n))) \implies B$.

Informally: we form an implicative statement with all members of the knowledge base joint by conjunction on the left hand side and B on the right hand side.

Reasoning and Logical Consequence

- From p and $p \Longrightarrow q$ we conclude q.
  - Knowledge Base:
  - Conclusion:
  - Apply the definition of the logical consequence, i.e form the implicative formula
  - Form the truth table

Good Tenants Problem

- A landlady has a room to let.
- Two conditions: tenants should not drink and should not smoke. Two students came.
  - A came and said: If I drink then I smoke, but I do not drink.
  - B came and said: If I drink then I smoke, but I do not smoke.
- Who is the one to get this room?


THANK YOU
Any Questions?