

4COSCOO2W MATHEMATICS FOR COMPUTING

Week 4 Seminar Tasks

Relations and Functions

READING

Lecture 3 Notes (available on Blackboard)

Chapters 16-20. *Croft, T and Davison R (2016) Foundation maths, 6th ed. Harlow: Pearson.*

Chapter 6,7. *Grossman, P., 2017. Discrete mathematics for computing. Bloomsbury Publishing.*

TASK 1. FUNCTIONS VS RELATIONS

See Lectures 3 Notes -Slides 15-20

Let $A = \{\text{Mary, John, Peter, Chris}\}$ and $B = \{\text{Accountant, Lawyer, Programmer, Lecturer}\}$.

Which of the following relations is a function with input A and output B:

1. $\{ \langle \text{Mary, Accountant} \rangle, \langle \text{John, Lawyer} \rangle, \langle \text{Peter, Programmer} \rangle, \langle \text{Chris, Lecturer} \rangle \}$
2. $\{ \langle \text{Mary, Accountant} \rangle, \langle \text{John, Lawyer} \rangle, \langle \text{Peter, Programmer} \rangle, \langle \text{Chris, Programmer} \rangle \}$
3. $\{ \langle \text{Mary, Accountant} \rangle, \langle \text{John, Lawyer} \rangle, \langle \text{Peter, Programmer} \rangle \}$
4. $\{ \langle \text{Mary, Programmer} \rangle, \langle \text{John, Programmer} \rangle, \langle \text{Peter, Programmer} \rangle, \langle \text{Chris, Programmer} \rangle \}$
5. $\{ \langle \text{Mary, Accountant} \rangle, \langle \text{John, Lawyer} \rangle, \langle \text{Peter, Lecturer} \rangle, \langle \text{Peter, Programmer} \rangle, \langle \text{Chris, Programmer} \rangle \}$

TASK 2. Analysis of a Function

See Lectures 3 Notes -Slides 16, 23-26

Given the function:

$$h(x) = \frac{1}{2}x - \frac{3}{4}$$

1. Identify the type of the function.
2. Evaluate the output for the input values: -2, -1, 0, 1, 2. Form the *ordered pairs of x-y values*.
3. Create a *table of x-y values*.
4. Find the points at which the function intersects the x-axis and y-axis.
5. Draw the graph of the function on paper using the information you got in the previous tasks.
6. Plot the function in <https://www.desmos.com/calculator>, and check if you got the same function.

TASK 3. SLOPE OF A LINE

See Lectures 3 Notes -Slides 24

Find the equation of the straight line that passes through the points (6, -1) and (-2, 7) and sketch it by hand. Check your plot on <https://www.desmos.com/calculator>.

TASK 4. TYPES OF FUNCTIONS

See Lectures 3 Notes -Slides 23, 31-33

Consider the 5 standard functions below:

1. $f(x)=e^x$
2. $g(x)=1/x$
3. $h(x)=x^2$
4. $u(x)=x$
5. $v(x)=\ln(x)$

Plot the curves of the above functions on the same graph on <https://www.desmos.com/calculator>. Identify the curves of each function on the graph.

TASK 5. THE INTERSECTION OF TWO FUNCTIONS

See Lectures 3 Notes -Slides 27-30

Given the functions p and q:

$$p(x)=x^2-4x-5$$

$$q(x)=x+1$$

Find the intersection points of these functions (using the factorisation method).

Check your answer using <https://www.desmos.com/calculator>.

TASK 6. THE INVERSE OF A FUNCTION

See Lectures 3 Notes -Slides 21-22

Given the function:

$$h(x) = \frac{3x - 4}{2}$$

1. Find the inverse of the function $h(x)$.
2. Confirm that the original function and its inverse are reflections across the line $y = x$ using algebraic verification.
3. Plot the graphs of $h(x)$, $h^{-1}(x)$, and $y = x$ to visually confirm that h and h^{-1} are reflections of each other across the line $y = x$.

TASK 7. RECURSIVE FUNCTIONS

See Lectures 3 Notes -Slides 34-36

Task 7.1. Explore a sequence $p(n)$ defined for all non-negative integers using the following recursive definition:

Base Case (BC): $p(0) = 2$

Recursive Case (RC): $p(n+1) = p(n)^2 + 1$, for $n \geq 0$.

Calculate the value of $p(4)$.

Note: You may need to use a calculator for this task.

Task 7.2. Let $t(n)$ be a sequence defined for all non-negative integers by the following recursive definition:

Base Case (BC): $t(0) = 1$; $t(1) = 3$

Recursive Case (RC): $t(n) = t(n-2) + t(n-1)$, for $n \geq 2$

Find out the value of $t(7)$.

Task 7.3. Investigate a sequence $f(n)$ that employs the *Factorial function* in its definition for all non-negative integers:

Base Case (BC): $f(0)=1$

Recursive Case (RC): $f(n+1)=f(n)+n!$ for $n \geq 0$

Determine the value of $f(4)$.