Mathematics in Computing
4COSC007C

Lecture 3: Set Theory

Set Builder Notation

- Question: how would you indicate a set which has a large number of elements or even infinitely many numbers?
- We cannot list all elements.
- Set builder notation – we say that we collect in a set A all elements of a dedicated domain D which also satisfy some rule R:

$$A = \{x \in D \mid x \text{ satisfies rule } R\}$$

- Let $\mathbb{N}$ abbreviate a set of Natural numbers.

We can then introduce a set, of even natural numbers (note there are infinitely many numbers) as follows

$$\text{Even Numbers} = \{x \in \mathbb{N} \mid x \text{ is an even number}\}$$

- And a set of even natural numbers greater than 10 (let's call it A) can be introduced as follows

$$A = \{x \in \mathbb{N} \mid x \text{ Even Numbers } \& \; x > 10\}$$

Using Set Builder Notation

- Question: how would you indicate a set A which is identified by the following:
each element x of A is a natural number such that $10 < x < 14$

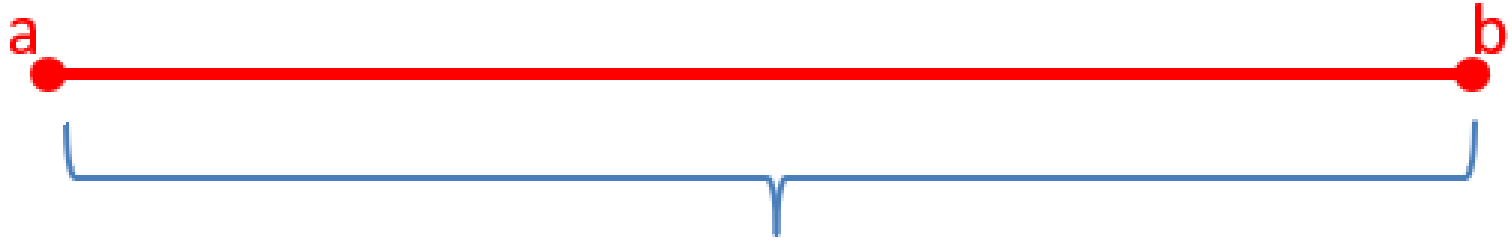
- Option 1:
We can list all natural numbers - elements of A: $A = \{11, 12, 13\}$
- Option 2:
  Use Set builder notation – let $\mathbb{N}$ be a set of Natural Numbers

$$A = \{x \in \mathbb{N} \mid 10 < x < 14\}$$

Intervals - Closed Interval [a,b]

Let $\mathbb{R}$ be a set of real numbers.

• $[a,b]$ abbreviate a closed interval between numbers a and b (That is all those numbers that are between a and b including a and b themselves).

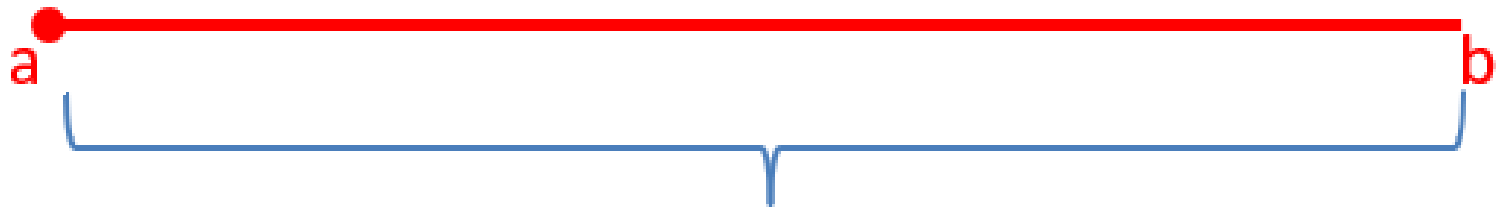


any $x \in \mathbb{R}$ such that $a \leq x \leq b$, *and in set builder notation this is*

$$\{x \in \mathbb{R} : a \leq x \leq b\}$$

Intervals - open ended interval [a,b)

- Let $\mathbb{R}$ be a set of real numbers.
- $[a, b)$ denotes an open ended interval, i.e. a belongs to the interval but now b does not belong to it

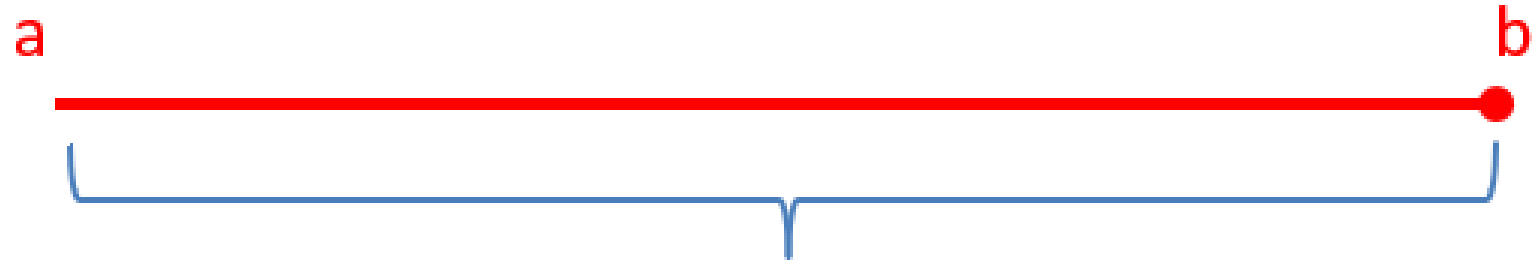


any $x \in \mathbb{R}$ such that $a \leq x < b$

$\{x \in \mathbb{R} : a \leq x < b\}$

Intervals - open beginning interval (a, b]

- Let $\mathbb{R}$ be a set of real numbers.
- (a,b] denotes an open beginning interval, i.e. *a* does not belong to it while *b* does.



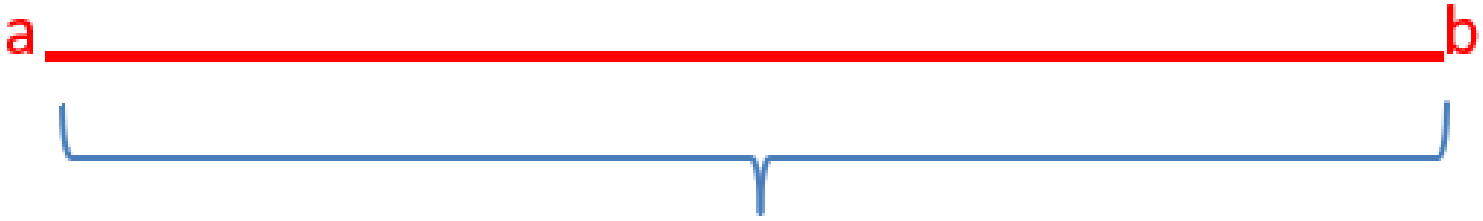
any $x \in \mathbb{R}$ such that $a < x \leq b$

$\{x \in \mathbb{R} : a < x \leq b\}$

Intervals - open beginning and end interval (a,b)

- Let $\mathbb{R}$ be a set of real numbers.
- (a,b) denotes an interval which is both sides open – open beginning and open ended.

i.e. both *a* and *b* do not belong to it



any $x \in \mathbb{R}$ such that $a < x < b$

$\{x \in \mathbb{R} : a < x < b\}$

Intervals - Summary

Let $x \in \mathbb{R}$ where $\mathbb{R}$ is a set of real numbers.

- $[a,b]$ abbreviate a closed interval between numbers a and b, i.e. all those numbers that are between a and b including a and b themselves.
- $[a,b)$ denotes an open ended interval, i.e. now b does not belong to the interval
- (a,b] denotes an open beginning interval, i.e. a does not belong to it while b does.
- (a,b) denotes an interval which is both open beginning and open ended.

Solve the following problems

- $[3,4]$ what is it?
- $\{3,4\}$ what is it?
- $[3,4)$ what is it?

Solve the following problems

- [3,4] what is it? Closed interval
- {3,4} what is it? A set of only two numbers
- [3,4) what is it? Open ended Interval

Interval

$[3,4]$

What are the elements of the corresponding set?

$[3,4]$ abbreviates a closed interval between numbers 3 and 4, i.e. all those numbers that are between 3 and 4 including 3 and 4 themselves.

Is it true that $3 \in [3,4]$?

Is it true that $3.5 \in [3,4]$?

Is it true that $4 \in [3,4]$?

Interval

What are the elements of the corresponding set?

[*3*,*4*] abbreviates a closed interval between numbers *3* and *4*, i.e. all those numbers that are between *3* and *4* including *3* and *4* themselves.

Is it true that $3 \in [3,4]$? YES

Is it true that $3.5 \in [3,4]$? YES

Is it true that $4 \in [3,4]$? YES

NOTE: {3,4} is a set of only two numbers 3 and 4

Solve the following problems

$\{3,4\}$ what is it?

What are the elements of the corresponding set?

There are only two of them – 3 and 4

Is it true that $3 \in \{3,4\}$?

Is it true that $3.5 \in \{3,4\}$?

Is it true that $4 \in \{3,4\}$?

Solve the following problems

$\{3,4\}$ what is it?

What are the elements of the corresponding set?

There are only two of them – 3 and 4

Is it true that $3 \in \{3,4\}$? YES

Is it true that $3.5 \in \{3,4\}$? NO

Is it true that $4 \in \{3,4\}$? YES

Interval

$[3,4)$

What are the elements of the corresponding set?

$[3,4)$ abbreviates a closed interval between numbers 3 and 4, i.e. all those numbers that are between 3 and 4 including 3 and excluding 4.

Is it true that $3 \in [3,4)$?

Is it true that $3.5 \in [3,4)$?

Is it true that $4 \in [3,4)$?

Interval

$[3,4)$

What are the elements of the corresponding set?

$[3,4)$ abbreviates a closed interval between numbers 3 and 4, i.e. all those numbers that are between 3 and 4 including 3 and excluding 4.

Is it true that $3 \in [3,4)$? YES

Is it true that $3.5 \in [3,4)$? YES

Is it true that $4 \in [3,4)$? NO

Cardinality of a Set

- What is the cardinality of the sets we just considered?
- In other words – how many elements are there in these sets:

$|[3,4]| = ?$

$|\{3,4\}| = ?$

$|[3,4)| = ?$

Cardinality of a Set

- What is the cardinality of the sets we just considered?
- In other words – how many elements are there in these sets:

$|[3,4]| = \infty$, i.e this set has infinitely many elements

$|\{3,4\}| = 2$, i.e this set has only 2 elements

$|[3,4)| = \infty$, i.e this set has infinitely many elements

How to define a Power Set using Set Builder Notation

Given a set A, a powerset of A is

$$\mathbb{P}(A) = \{C \text{ is a set: } C \subseteq A\}$$

Cartesian Product

$$A \times B \qquad A \times B \times C$$

Creates a new set from sets A, B and C, which consists of ordered tuples of elements (one from each set) in all possible combinations.

Set Builder Notation in Cartesian Products

Definition:

A Cartesian Product of sets $A1 \times A2 \times \ldots \times An$

$\{x,y,\ldots,z \mid x \in A1, y \in A2, \ldots z \in An\}$