4COSCOO2W MATHEMATICS FOR COMPUTING

Week 9 Seminar Tasks

Matrices. Part 1

READING

Lecture 7 Notes (available on Blackboard)

Chapter 27. *Croft, T and Davison R (2016) Foundation maths, 6th ed. Harlow: Pearson.*

TASK 1. Matrix Order

See Lectures 7 Notes -Slides 2-5

Task 1.1

Establish the size (order) of the following matrices:

$A = (0, 0, 1, 1)$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Task 1.2

How many elements would the matrices of the following orders have:

5x5;

1 x 10;

10 x 1;

n x n;

Task 1.3

In the table below, map the matrices of the sizes given in the left panel to the matrices drawn in the right panel. Which known types of matrices can you identify here?

I. 1x4
II. 2x2
III. 5x1
IV. 3x2
V. 4x4

a) (0, 0, 1, 1)

b) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

e) $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$

TASK 2. Identity Matrices

See Lectures 7 Notes -Slides 7-8

Provide a definition and the features of the Identity matrix. Draw Identity matrix A of the size 3 and identity matrix B of the size 4.

TASK 3. Operations with Matrices

See Lectures 7 Notes -Slides 15-35

Task 3.1

What are the conditions for adding matrices A and B?

Where possible, calculate A + B and A - B for the following matrices otherwise explain why these operations are not possible (15 min)

1) $A = B = (0, 1, 2, 3, 4)$
2) $A = (0, 1, 2, 3)$ $B = (0, 1, 2, 3, 4)$
3) $A = B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
4) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
5) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
6) $A = (0 \quad 0)$ $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
7) $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$ $B = \begin{pmatrix} -1 & -4 \\ -2 & -5 \\ -3 & -6 \end{pmatrix}$

Task 3.2.

Multiply the following matrices A and B by the scalar 2.

$A = \begin{pmatrix} 1 & -4 \\ 2 & -5 \\ 3 & -6 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 4 \\ -2 & 5 \\ -3 & 6 \end{pmatrix}$

Task 3.3

What are the conditions for multiplying matrices A of the order k x l and B of the order m x n?

Where possible, multiply the following matrices, otherwise explain why multiplication is not possible

a) Given A = B = (1, 3), multiply A and B
b) Given A = (1, 3) and B = $\begin{pmatrix} 2 & -3 \\ 3 & -2 \end{pmatrix}$ multiply A and B
c) Given A = $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and B = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ multiply A and B
d) Given A = $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and B = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ multiply AT and B
e) Given A = B = $\begin{pmatrix} 2 & -3 \\ 3 & -2 \end{pmatrix}$ multiply A^T and B

Task 3.4

Let A be an identity matrix of size 5. Find what are

a) A^2
b) A^{10}
c) A^{100}
d) A^T
e) A^{-1}

Task 3.5

For which of the following matrices we can compute A^2 and A^3 ; If the corresponding multiplication is possible then compute the result, otherwise give the reasoning why the computation is not possible

i. A is of the size 4x5
ii. A is a square matrix of the size 2 with all elements 2
iii. A is a row vector with 3 columns
iv. A is a column vector with 6 rows
v. A is a diagonal matrix where the main diagonal has 3 elements 1, 2 and 3
vi. A is a null matrix of the size 2x3

TASK 4. Matrix Equations

See Lectures 7 Notes -Slides 28-29

For the matrices A, B and C given below

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

calculate

a) matrix D such that $2D = AB^T$
b) matrix D such that $5D = A^TC$