4COSC007C Mathematics for Computing,
Tutorial ● 7

Task 1 Establish the size (order) of the following matrices:

a) A = (0, 0, 1, 1)

b) B = $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}$

c) $C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix}$

d) $C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Task 2.

In the table below, map the matrices of the sizes given in the left panel to the matrices drawn in the right panel. Which known types of matrices can you identify here?

I. 1x4 II. 2x2 III. 5x1 IV. 3x2 V. 4x4	a) (0, 0, 1, 1) b) $\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 3 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ e) $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$

Task 3. How many elements would the matrices of the following orders have:

a.) 5x5
b.) 1 x 10
c.) 10 x 1
d.) n x n

Task 4
Draw identity matrix A of the size 3 and identity matrix B of the size 4

Task 5.

What are the conditions to add matrices A and B?

Where possible, calculate A + B and A - B for the following matrices, otherwise explain why these operations are not possible

a.) $A = B = (0, 1, 2, 3, 4)$
b.) $A = (0, 1, 2, 3)$ $B = (0, 1, 2, 3, 4)$
c.) $A = B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

d.) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

e.) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

f.) $A = \begin{pmatrix} 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

g.) $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$ $B = \begin{pmatrix} -1 & -4 \\ -2 & -5 \\ -3 & -6 \end{pmatrix}$

Task 6. Multiply the following matrices A and B by the scalar 2

$A = \begin{pmatrix} 1 & -4 \\ 2 & -5 \\ 3 & -6 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 4 \\ -2 & 5 \\ -3 & 6 \end{pmatrix}$

Task 7.

What are the conditions to multiply matrices A of the order k x l and B of the order m x n?

Where possible, multiply the following matrices, otherwise explain why multiplication is not possible

a) Given A = B = (1, 3), multiply A and B

b) Given A = (1, 3) and B = $\begin{pmatrix} 2 & -3 \\ 3 & -2 \end{pmatrix}$ multiply A and B

c) Given A = $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and B = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ multiply A and B

d) Given A = $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and B = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ multiply A^T and B

e) Given A = B = $\begin{pmatrix} 2 & -3 \\ 3 & -2 \end{pmatrix}$ multiply A^T and B

Task 8

Let A be an identity matrix of size 5. Find what are

1. A^2
2. A^{10}
3. A^{100}
4. A^T
5. A^{-1}

Task 9

For which of the following matrices we can compute A^2 and A^3; if the corresponding multiplication is possible then compute the result, otherwise give the reasoning why the computation is not possible

i. A is of the size 4x5
ii. A is a square matrix of the size 2 with all elements 2
iii. A is a row vector with 3 columns
iv. A is a column vector with 6 rows
v. A is a diagonal matrix where the main diagonal has 3 elements 1, 2 and 3
vi. A is a null matrix of the size 2x3

Challenge 1

a) Given A is a symmetric matrix of the sizes 2, 5, and 6 what is A - A^T?
b) Can you generalise this property to a matrix of any size? If you can, formulate the claim and prove it.

Challenge 2.

For the matrices A, B and C given below

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

calculate

a) matrix D such that $2D = AB^T$
b) matrix D such that $5D = A^TC$