

Mathematics in Computing: 4COSC007C

Tutorial 3

1. Given the set $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$ form a new set B which consists of all elements of set A that are:
 - i Prime numbers.
 - ii Even numbers.
 - iii Odd numbers that are greater or equal to 3.
 - iv Those numbers that being multiplied by 2 give a number that is also an element of A.
 - v Those numbers that being multiplied by 2 give a number that is not in the given set A.
 - vi Those numbers that being squared resulting in a number which also belongs to A.
2. Let \mathbb{N} be a set of natural numbers $\{1, 2, 3 \dots\}$. For each of the cases below we define a new set B using set builder notation. Your task is to list all elements of B and to establish the cardinality of B.
 - i $B = \{x : x \in \mathbb{N} \text{ and } x^2 = x\}$
 - ii $B = \{x : x \in \mathbb{N} \text{ and } x^2 = 2x\}$
 - iii $B = \{(x, y) : x \in \mathbb{N} \text{ and } y \in \mathbb{N} \text{ and } x < y \text{ and } y \leq 3\}$
 - iv $B = \{(x, y) : x \in \mathbb{N} \text{ and } y \in \mathbb{N} \text{ and } x = y \text{ and } y < 5\}$
 - v $B = \{(x, y, z) : x \in \mathbb{N} \text{ and } z \in \mathbb{N} \text{ and } z = x + 2^y \text{ and } x = 10 \text{ and } 1 < y < 5\}$
3. Identify if the following statements are true or false.
 - i. $3 \in \{3, 4, 5\}$
 - ii. $3 \in \{3, 4, 5\}$
 - iii. $\{3\} \in \{\{3\}, \{4\}, \{5\}\}$
 - iv. $\{3\} \subseteq \{3, 4, 5\}$
 - v. $3 \subseteq \{3, 4, 5\}$
 - vi. $\{\} \subseteq \{3, 4, 5\}$
4. Let $A = [2, 9]$ be a closed interval of all real numbers 2 to 9. Which new interval is introduced by the following set builder notation: $\{x \in A : \sqrt{x} \in A\}$.
5. Define, using the set builder notation, the set C which is obtained via the following operations to sets A and B:
 - i. $C = A \cap B$
 - ii. $C = A \cup B$
 - iii. $A \setminus B$
 - iv. A'
6. Let A and B be the only sets in U and $A = \{5, 6, 10, 12\}$ and $B = \{5, 7, 11\}$ Apply the following operations to sets A and B
 - i. $A \cup B$

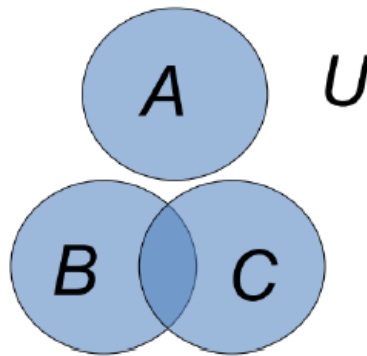
- ii. $A \cap B$
- iii. $A \setminus B$
- iv. $B \setminus A$
- v. A'
- vi. B'
- vii. $A' \cap B'$

7. Below U is the universal set, $\{\}$ is the empty set and A is an arbitrary set. Based on the definition of the empty and universal sets establish what should be the resulting set of the following operations:

- i. $A \cap U$
- ii. $A \cup U$
- iii. $A \setminus U$
- iv. $(U \setminus A) \cup A$
- v. $\{\} \cup A$
- vi. $\{\} \cap A$

8. Consider sets $A = \{a\}$, $B = \{g, h, i, j\}$ and $C = \{i, j, k, l\}$. In the Venn diagram below place the elements of the following sets and establish what are the sets resulting in the following operations:

- i. $B \cap C$
- ii. $A \cup B$
- iii. $A \cap B$
- iv. $B \cup (B \cap C)$
- v. $A \cap (B \setminus U)$
- vi. $U \cap (A \cup B)$



9. Let $A = \{a, b, c\}$ and $B = \{1, 0\}$.

- i. Write down all elements of the Power Set of A and Power Set of B
- ii. List all elements of $A \times B$
- iii. List all elements of $B \times A$
- iv. Calculate $|A \times B|$ and $|B \times A|$
- v. What is $(A \times B) \setminus (B \times A)$

10. **CHALLENGE**

- i. Show that if $A \subseteq B$ and $B \subseteq A$ then $A = B$, i.e. that A and B are equivalent, i.e. they have the same elements.
- ii. Let $A = [2, 9]$ be a closed interval of all real numbers 2 to 9 Which interval is introduced by the following set builder notation: $\{x \in A : \sqrt{x} \in A \text{ \& } x < \sqrt{20}\}$.

11. **CHALLENGE**

Let $A = \{m : m \text{ is an integer satisfying } 0 < m < 13\}$ and $B = \{n : n \text{ is an integer satisfying } 7 < n < 23\}$. Calculate $|(A \times B) \setminus (B \times A)|$.

Note that the task is to only calculate the number of elements but not to list the elements of the resulting set!