Question 05

a). $((p \Rightarrow (q \land r)) \Rightarrow ((s \Rightarrow q) \land (u \Rightarrow v))) \Rightarrow (t \Rightarrow t))$

For this formula, the outermost implication ($\Rightarrow$) implies that the entire formula is true when both the antecedent and consequent are true.

1. **Antecedent**: $(p \Rightarrow (q \land r)) \Rightarrow ((s \Rightarrow q) \land (u \Rightarrow v))$
   - The antecedent will be false if there exists a case where p is true, q is true, r is false, s is true, u is true, and v is false. This is because $p \Rightarrow (q \land r)$ would be false, and consequently, the entire antecedent would be false.
2. **Consequent**: $t \Rightarrow t$
   - The consequent is always true because any statement implies itself.

Based on this analysis, the entire formula could be false if p, q, r, s, u, or v takes specific truth values. Specifically, if p is true, q is true, r is false, s is true, u is true, and v is false, then the formula would be false.

b). $((p \lor \neg p) \Rightarrow (q \land \neg q)) \Rightarrow (((r \Rightarrow q) \land (u \Rightarrow s)) \Rightarrow (t \Rightarrow w))$

For this formula, the same logic applies: the outermost implication implies that the entire formula is true when both the antecedent and consequent are true.

1. **Antecedent**: $(p \lor \neg p) \Rightarrow (q \land \neg q)$
   - The antecedent is always false because q and $\neg q$ cannot both be true simultaneously, regardless of the truth value of p.
2. **Consequent**: $((r \Rightarrow q) \land (u \Rightarrow s)) \Rightarrow (t \Rightarrow w))$
   - The consequent is always true because any statement implies itself.

Therefore, for formula b), the antecedent is always false, and thus, the entire formula is always true. No specific input values can make this formula false because the antecedent is inherently contradictory.

Question 6) part a)

Need to prove that the following implication is wrong.

B is a L.C. of a KB $\rightarrow$ B is a logically consequence of each fact A_i of KB.

It is sufficient to give an example to prove this is wrong.

Example

Let the KB : p,q

From the following truth table p ∧ q is a logically consequence of a kb with p,q facts

p	q	p ∧ q	p∧q → p ∧ q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Table 1

p	q	p ∧ q	p ∧ q → p
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Table 2

p	q	$p \wedge q$	$p \wedge q \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	T

Table 3

From the table 2 p is a logically consequence of $p \wedge q$

From the table 3 q is not a logically consequence of $p \wedge q$

Hence $p \wedge q$ is not a logically consequence of each fact p and q of the KB.
Question 6) b)

Show that the following reasoning is correct: *if B is a logical consequence of at least of one of the components of the knowledge base $A_1, A_2, A_3, \dots A_n$ then B is a logical consequence of this knowledge base.*

Question 6) part b)

Show the following is correct(true).

B is a logical consequence of at least one $A_i \rightarrow$ B is a logical consequence of the KB with $A_i \wedge A_2 \wedge \dots \wedge$

A_n $p \rightarrow q$ is equivalent to $\neg q \rightarrow \neg p$

This means proving the following is equivalent to the above

B is a not a logical consequence of the KB with $A_i \wedge A_2 \wedge \dots \wedge A_n$

$\rightarrow$ B is not a logical consequence of any A_i

Let's assume that B is a not a logical consequence of the KB with $A_i \wedge A_2 \wedge \dots \wedge$ A_n Then $A_i \wedge A_2 \wedge \dots \wedge A_n \rightarrow B$ is not a valid proposition.

This means that there is at least one proposition where $A_i \wedge A_2 \wedge \dots \wedge A_n \rightarrow B$ is False In that case $A_i \wedge A_2 \wedge \dots \wedge A_n$ has to be True and B has to be false.

For $A_i \wedge A_2 \wedge \dots \wedge A_n$ to be true all A_i s has to be True.

Thus $A_i \rightarrow B$ is F for all A_i s

This means B is not a logical consequence of any A_i

Thus $\neg q \rightarrow \neg p$ is correct hence $p \rightarrow q$ is also correct.