Tutorial 3 - Answers

1. Given the set $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$ form a new set B which consists of all elements of set A that are:

i Prime numbers.

Prime Numbers - whole numbers greater than 1 that can not be made by multiplying other whole numbers.

Prime Numbers = {2,3,5,7}

ii Even numbers.

Even Numbers – Numbers that are divisible by 2

Evan numbers = {2,4,6,8}

iii Odd numbers that are greater or equal to 3.

={3,5,7,9}

iv Those numbers that being multiplied by 2 give a number that is also an element of A.

Those numbers that are in set A and also some of its 2s multiples

={2,3,4}

v Those numbers that being multiplied by 2 give a number that is not in the given set A.

Those numbers that are in set A and none of its 2 s multiples are in the set A

={5,6,7,8,9}

vi Those numbers that being squared resulting in a number which also belongs to A.

={2,3}

2. Let N be a set of natural numbers {1, 2, 3 ,....}. For each of the cases below, a new set B is defined by using set builder notation. List all elements of B and establish the cardinality of it.

i $B = \{x : x \in N \text{ and } x^2 = x\}$ = {1}, Cardinality = 1

$x^2 = x$, $x^2 - x = 0$, $x(x-1) = 0$, x = 0 or x = 1, but 0 in not in the set

ii $B = \{x : x \in N \text{ and } x^2 = 2x\}$ = {2}

$x^2 = 2x$, $x^2 - 2x = 0$, $x(x-2) = 0$, x =0 or x = 2

iii $B = \{(x; y) : x \in N \text{ and } y \in N \text{ and } x < y \text{ and } y \leq 3\}$

N = Some definitions, begin the natural numbers with 0, corresponding to the non-negative integers 0, 1, 2, 3, ... , whereas others start with 1, corresponding to the positive integers 1, 2, 3, ... In the questions clearly it will be indicated which definition is used.

={(1,2),(1,3),(2,3)}, cardinality = 3

iv $B = \{(x; y) : x \in N \text{ and } y \in N \text{ and } x = y \text{ and } y < 5\}$

={(1,1),(2,2),(3,3),(4,4)} cardinality = 4

v $B = \{(x; y; z) : x \in N \text{ and } z \in N \text{ and } z = x + 2^y \text{ and } x = 10 \text{ and } 1 < y < 5\}$

{(10,2,14),(10,3,18),(10,4,26)}, cardinality = 3

3. Identify if the following statements are true or false.
i. $3 \in \{3, 4, 5\}$ - T
ii. $3 \in \{3, 4, 5\}$ - T
iii. $\{3\} \in \{\{3\}, \{4\}, \{5\}\}$ - T
iv. $\{3\} \subseteq \{3,4,5\}$ - T
v. $3 \subseteq \{ 3,4,5\}$ - F
vi. $\{] \subseteq \{3, 4,5\}$ - T

4. Let A = [2,9] be a closed interval of all real numbers 2 to 9. Which new interval is introduced by the following set builder notation: $\{x \in A : \sqrt{x} \in A\}$.
=[4,9]

5. Define, using the set builder notation, the set C which is obtained via the following operations on sets A and B:
i. $C = A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
ii. $C = A \cup B = \{x \mid x \in A \text{ or } x \in B$
iii. $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$
iv. $A' = \{x \} x \in U \text{ and } x \notin A\}$

6. Let A and B be the only sets in U and $A = \{5, 6, 10, 12\}$ and $B = \{5, 7, 11\}$ Apply the following operations to sets A and B
i. $A \cup B = \{5,6,7,10,11,12\} = U$
ii. $A \cap B = \{5\}$
iii. $A \setminus B = \{6,10,11\}$
iv. $B \setminus A = \{7,11\}$
v. $A' = \{7,11\}$
vi. $B' = \{6,10,12\}$
vii. $A' \cap B' = \{\}$

Below U is the universal set, {} is the empty set and A is an arbitrary set. Based on the definition of the empty and universal sets establish what should be the resulting set of the following operations:

i. $A \cap U = A$
ii. $A \cup U = U$
iii. $A \setminus U = \{\}$
iv. $(U \setminus A) \cup A = U$
v. $\{\} \cup A = A$
vi. $\{\} \cap A = \{\}$

7. Consider sets $A = \{a\}$, $B = \{g, h, i, j\}$ and $C = \{i, j, k, l\}$. In the Venn diagram below place the elements of the following sets and establish what are the sets resulting in the following operations:

i) $B \cap C = \{i,j\}$
ii) $A \cup B = \{a,g,h,i,j\}$
iii) $A \cap B = \{\}$
iv) $B \cup (B \cap C) = \{g,h,i,j\}$
v) $A \cap (B \setminus U) = \{\}$
vi) $U \cap (A \cup B) = \{a,g,h,i,j\}$

8. Let A = {a, b, c} and B = {1, 0}.

i. Write down all elements of the Power Set of A and Power Set of B

P(A) = {φ,{a},{b},{c},{a,b},{a,c},{b.c},{a,b,c}}

P(B) = {φ,{1},{0},{1,0}}

ii. List all elements of A x B ={(a,1),(a,0),(b,1),(b,0),(c,1),(c,0)}

iii. List all elements of B x A ={(1,a),(0,a),(1,b),(0,b),(1,c),(0,c)}

iv. Calculate |A x B| and |B x A|= 6 and 6

v. What is (A x B) \ (B x A) = A x B as there are no common elements in Ax B and BxA

9. CHALLENGE

i. Show that if A ⊆ B and B ⊆ A then A = B, i.e. that A and B are equivalent, i.e. they have the same elements.

Let's assume that A ⊆ B and B ⊆ A and A # B

Then there should be an element x in A which is not in B

However, A ⊆ B therefor all elements in A should be also in B – Contradiction

Thus A = B

ii. Let A = [2,9] be a closed interval of all real numbers 2 to 9 Which interval is introduced by the following set builder notation: {x ∈ A : $\sqrt{x}$ ∈ A and x < $\sqrt{20}$ } = [4, $\sqrt{20}$)

10. CHALLENGE

Let A = {m : m is an integer satisfying 0 < m < 13} and B = {n : n is an integer satisfying 7 <n < 23}.

Calculate |(A x B) \(B x A)|.

Note that the task is to only calculate the number of elements but not to list the elements of the resulting set!

A = {1,2,3,4,5,6,7,8,9,10,11,12}

B = {8,9,10,......,22}

How many items are common to (A x B) and (B x A) ?

Common elements to both A and B are = {8,9,10,11,12}

Thus, A x B has 12 x 15 = 180 items out of that 5 x5 = 25 items are common

Thus |(A x B) \(B x A)|. = 180 -25 = 155