4COSC007 Mathematics for Computing

Tutorial 1: Logic

1. For each of the examples of reasoning below, answer the following questions:
   a) Find all Boolean expressions identifying all primitive propositions and the structure of the compound proposition.
   b) Is the conclusion correct?
   c) If it is correct, explain why.
   d) If it is not correct, give a counter example.

   a. If X is divisible by 4 then it is divisible by 2.
   b. If X is divisible by 4 or 6 then it is divisible by 2 or 3.
   c. If X is divisible by 2 then it is divisible by 4.
   d. If X is not divisible by 2 then it is not divisible by 4.
   e. If X is not divisible by 2 and not divisible by 3 then it is not divisible by 4 and it is not divisible by 6.

2. For the following formal expressions establish if they have a good logical grammar or indicate a problem in their logical structure otherwise.

		If NO what is a problem?
p	YES/NO	
$\Rightarrow p$	YES/NO	
$\neg\Rightarrow p$	YES/NO	
$p\ q$	YES/NO	
$p \wedge q$	YES/NO	
$p \neg q$	YES/NO	
$p \wedge \neg q$	YES/NO	
$(\neg p \vee q$	YES/NO	
$(\neg p \vee) q$	YES/NO	
$\neg(\neg p \wedge \neg q)$	YES/NO	
$(p \Rightarrow q) \wedge (q \Rightarrow p)$	YES/NO	
$(p \Rightarrow (q \wedge r)) \Rightarrow ((p \Rightarrow q) \wedge (p \Rightarrow r))$	YES/NO	

3. Build truth tables for the formulae below. You should follow the algorithm described in the lecture.

   a. $\neg (p \Rightarrow p)$
   b. $\neg (p \vee \neg p)$
   c. $(p \wedge q) \Rightarrow p$
   d. $\neg (q \vee \neg q)$
   e. $p \Rightarrow (p \vee q)$
   f. $(s \wedge t) \Rightarrow s$
   g. $u \Rightarrow (u \vee w)$

   Look at the structure of the given formulae and their truth tables – can you see some patterns here? Can you identify those expressions that have similar logical structures, compare their truth tables.

4. Build truth tables for the formulae below following the algorithm described in the lecture.

   a. $\neg p \vee \neg q$
   b. $\neg (p \wedge q)$
   c. $\neg p \wedge \neg q$
   d. $\neg (p \vee q)$

   Compare the resulting columns for these expressions and identify the cases where the same input values written on the left of the truth table give the same resulting value.

5. Draw up the truth tables for

   a) $\neg(\neg P)$
   b) $(P \wedge Q) \vee R$
   c) $P \wedge (Q \vee R)$

6. Compare the truth tables of $P \rightarrow Q$ and $\neg Q \rightarrow \neg P$. State whether the expressions are logically equivalent.

7. CHALLENGE

a) Build a truth table for the following expression following the algorithm:

$$\neg((s \wedge t) \Rightarrow (s \vee r))$$

Can you find input values for which the resulting value is true?

b) Consider the following two expressions:

$$\neg\neg((s \wedge t) \Rightarrow (s \vee r))$$

$$\neg\neg\neg((s \wedge t) \Rightarrow (s \vee r))$$

Can you obtain the truth tables for the logical expressions in b) by only looking at the case in a)?

8. CHALLENGE

Three boxes are presented to you. One contains gold, the other two are empty. Each box has clue to its contents; the clues are

(Box 1) “The gold is not here”,

(Box 2) “The gold is not here”,

(Box 3) “the gold is in Box 2”.

Only one of these clues is true; the other two are false. Which box has the gold?