Mathematics in Computing
4COSC007C

Lecture 1: Logic

September Intake 2021

Basic Rules

Sign in on time

Do not disturb

Ask questions

Take down notes

Answer questions

Module Delivery

- 12 weeks of lectures and tutorials in the semester.
- You will have a 2-hour lecture and a 2-hour tutorial each week.
- Tutorials will follow topics covered in the previous lecture.

Course Contents

- Boolean, Reasoning with Boolean, Logic
- Graphs and Trees
- Probability
- Sets, Relations and Functions
- Statistics
- Matrices
- Vectors
- Problem Solving based on Propositional Logic

Examinations

- You will have TWO in-class tests (ICTs).
- Both ICT's will be carried out online through Blackboard.
- Each ICT will weigh 50% to the final module mark.
- If module mark is ≥ 40 then you will pass the module.
- ICT's will be based on the work done in lectures and tutorials.

Plan

- Primitive and Compound Propositions. Analysis of the logical structure of compound propositions.
- Boolean – introduction, Some Notation.
- Introduction of the Propositional Logic Language. Logical Definitions of Boolean.
- Truth Table Construction.
- Examples.

Propositions

- A proposition (also called as statement) is a sentence that can be true or false. However, not both simultaneously.
- **Examples:**
  1. "Saman is awake" is a proposition because at any given time either Saman is not awake or Saman is awake, and Saman cannot be both awake and asleep at the same time.
  2. "Wake up!" is not a proposition because it cannot be true or false.

Atomic Propositions

- A statement that cannot be broken down into smaller statements, is known as the **atomic proposition (or primitive propositions)**.

  Question: Are all the below statements atomic propositions?

  1. 2 is a prime number.
  2. $6 > 3$.
  3. $3 > 6$.
  4. Kamal is left-handed.

Atomic Propositions

- A statement that cannot be broken down into smaller statements, is known as the **atomic proposition (or primitive propositions)**.

  Question: Are all the below statements atomic propositions?

  1. 2 is a prime number.
  2. $6 > 3$.
  3. $3 > 6$.
  4. Kamal is left-handed.

- **Sentence 1** and **sentence 2** are **true**. **Sentence 3** is **false**. We can't say for certain whether **sentence 4** is true or false without knowing who Kamal is. However, it is **either true or false**. Therefore, it follows that each of the four sentences above are **atomic statements**.

Examples for Atomic Propositions

-
-
-
-
-
-

Compound Propositions

- Many mathematical statements are built using more than one proposition. When we use logical operators to combine two or more propositions, they are called **compound propositions**.

  Question: Are all the below statements compound propositions?

  1. 5 is a prime number and $1 = 0$.
  2. Nimal is holding a pencil or water is a liquid.
  3. If Jack has a dog, then cat have 5 lungs.

Examples for Compound Propositions

-
-
-
-
-
-

Statements with Symbols

- Letters such as p, q, r and s are used to denote atomic statements.
- These letters are known as **propositional variables**.
- Generally a truth value of T (for true) or F (for false) to each propositional variables are assigned.

Operator Symbol	Operator Name	Precedence
$\neg, \sim$	Negation (**Not**)	1
$\wedge, \&$	Conjunction (**And**)	2
$\vee$	Disjunction (**Or**)	3
$\rightarrow, \Rightarrow$	Implication (**If... then**)	4

Table: Table of Logical Operators.

Rules for the Connectives

The truth value of a compound statement is determined by the truth values of its atomic parts together with applying the following rules for the connectives.

- $p \wedge q$ is called the conjunction of p and q. It is pronounced as p and q. $p \wedge q$ is true when both p and q are true. and false otherwise.
- $p \vee q$ called the disjunction of p and q. It is pronounced p or q. $p \vee q$ is true when p or q (or both) are true, and false when p and q are both false.
- $p \to q$ is called implication. It is pronounced if p then q or p implies q. $p \to q$ is true when p is false or q is true (or both), and it is false when p is true and q is false.
- $\neg p$ is called the negation of p. It is pronounced not p. $\neg p$ is true when p s false, and it is false when p is true (p and $\neg p$ have opposite truth values.)

Example 1 - Strike off the inappropriate answer

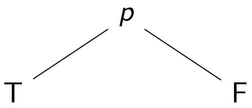
- Let p represent the statement Whale can swim, and let q represent the statement $8 < 4$. Note that p is true and q is false.
    1. $p \wedge q$ represents Whale can swim and $8 < 4$. Since q is false, it follows that $p \wedge q$ is false/true.
    2. $p \vee q$ represents Whale can swim or $8 < 4$. Since p is true, it follows that $p \vee q$ is true/false.
    3. $p \to q$ represents If Whale can swim, then $8 < 4$. Since p is true and q is false. $p \to q$ is false/true.
    4. $\neg q$ represents the statement 8 is not less than 4. This is equivalent to 8 is greater than or equal to 4. Since q is false, $\neg q$ is true/false.
    5. $\neg p \vee q$ represents the statement Whales cannot swim or $8 < 4$. Since $\neg p$ and q are both false, $\neg p \vee q$ is false/true.
    6. $\neg(p \vee q)$ represents the statement It is not the case that either Whale can swim or $8 < 4$. Since $(p \vee q)$ is true $\neg(p \vee q)$ is false/true.

Truth Tables

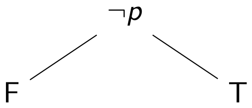
- Every Proposition is either true (T) or false (F).
- Assume that an atomic proposition (primitive proposition) is denoted by p.
- It could abbreviate any primitive proposition.
- Today is Thursday (true).
- Today is Friday (false).
- $2 \times 2 = 5$ (false).

Truth Tables-Negation

- So, for every proposition p,



- What would happen if we negate it?



- Negation is so called **UNARY** operation as it only applies to one proposition.

p	$\neg p$
T	F
F	T

Table: Truth Table of Negation.

Truth tables for the Binary Boolean operations

- We have defined the truth table for the unary negation.
- Other Boolean operations are binary.
- In binary Boolean operations there are two atomic propositions.
- Each of the propositions p and q can be either true or false.

- What would be an input for the truth table?
- First let's look at the truth tables of conjunction, disjunction and implication.

Truth Table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The conjunction of a Boolean expression p with a second Boolean expression q is True when both p and q are True and is False otherwise.

Truth Table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The disjunction of a Boolean expression p with a second Boolean expression q is False when both p and q are False and is True otherwise.

Truth Table for $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The implication of a Boolean expression p with a second Boolean expression q is False when p is true and q is false and is True otherwise.

Computing Truth Tables

- We consider formulae computationally, i.e. given a formula C, we want to compute its truth table.
- Let a formula C consist of the atomic propositions $p_1, p_2, \ldots, p_n$.
- To compute the truth table do the following:
  1. List its atomic propositions $p_1, p_2, \ldots, p_n$ and count their number, n.
  2. Form a table with $m = 2^n$ rows.
  3. List all possible combinations of the truth values for $p_1, p_2, \ldots, p_n$.
  4. Prioritize the logical operations in C, finding main logic operation, etc.
  5. Compute these operations in order.

Example 2: Computing Truth Tables

Apply the algorithm to C: $p \to p$

1. List its atomic propositions of C and count their number, $n = 1$.
2. Form a table with $m = 2^1 = 2$ rows.
3. List all possible combinations of the truth values for $p : T, F$.
4. Prioritize the logical operations in C – only one operator
5. Compute $p \to p$.

Example 2: Construct the truth table for $p \to p$

Example 3: Computing Truth Tables

Apply the algorithm to C: $(p \wedge q) \to p$.

1. List its atomic propositions of C and count their number, $n = 2$.
2. Form a table with $m = 2^2 = 4$ rows.
3. List all possible combinations of the truth values for *pandq*.
4. Prioritize the logical operations in $(p \overset{1}{\wedge} q) \overset{2}{\to} p$.
5. Compute first 1 and then 2.

Example 3: Construct the truth table for $(p \wedge q) \rightarrow p$.

Example 4: Computing Truth Tables

Apply the algorithm to C: $(p \vee q) \rightarrow (p \wedge r)$.

1. List its atomic propositions of C and count their number, $n = 3$.
2. Form a table with $m = 2^3 = 8$ rows.
3. List all possible combinations of the truth values for $p, qandr$.
4. Prioritize the logical operations in $(p \overset{1}{\wedge} q) \overset{3}{\rightarrow} (p \overset{2}{\vee} r)$.
5. First compute 1, 2 and then 3.

Example 4: Construct the truth table for $(p \vee q) \rightarrow (p \wedge r)$.

Exercise

1. Consider the following proposition **If X is divisible by 4 then X is divisible by 2.**
    1. What are the atomic propositions?
    2. What is the logical formula for this?
    3. How many rows should be there in the table?
    4. Construct the truth table for this.

Exercise

2. Construct the truth tables for the following:

   1. $(p \to p) \to p$.
   2. $p \to (p \to p)$.
   3. $(p \vee q) \to (p \wedge q)$.
   4. $p \vee (q \wedge r) \to (p \wedge r) \vee q$.
   5. $p \to (q \to p)$.

THANK YOU

Any Questions?