Set Theory – Introduction
(Supplement for Lecture 3)

<u>Definition</u>**:**

- Any well-defined collection of distinct objects is called a set.
- **Example 1**: A = {'Amal', 'Bimal', 'Carol', 'Dias', 'Emma', 'Farook', 'Ganesh'} is a set of names.
- **Example 2**: B = {1, 3, 5, 7, 9} is the set of <u>odd natural numbers</u> between 1 and 10.
- **Example 3**: C = {2, 4, 6, 8, 10} is the set of <u>even natural numbers</u> between 1 and 10.

Null Set: (Empty set)

- Is the set with no elements.

Notation: Φ, { }

(Note {0} is not an empty set)

Equality of Sets:

- The sets A and B are equal if and only if A and B both contain the same elements.

Subsets:

- A is a subset of B if every element of A is also in B. (A is a subset of itself). Notation:

$$A \subseteq B$$

If A is a subset of B and A≠ B, then A is called a proper subset of B.

- **Example 4**: D = {'Amal', 'Bimal', 'Emma'}. Then D is a subset of A of
- <u>Universal Set</u>**:**
- **Denoted by** $\mathbb{U}$, universal set is the set of all elements in a given discussion.
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- **Example 5**: If A of Example 1 contains names from a given class, then $\mathbb{U}$
- is the set of all names in the class.

Set Operations:

Union of Sets:

- If A and B are two subsets of U, then ‘union of A and B’ is the set whose elements belong to either A or B (or both).

Notation: $A \cup B$

Example 6: In Examples 2 and 3, let U = {all integers from 1 to 10}. Then,

$B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the union of B and C.

Example 7: Let U = {all integers from 1 to 10}; B = {1, 3, 5, 7, 9} and

- E = {1, 2, 3, 4, 5}. Then, $B \cup E = \{1, 2, 3, 4, 5, 7, 9\}$.

Intersection of Sets:

- If A and B are two subsets of U, the ‘intersection of A and B’ is the set consisting of elements that belong to both A and B.

Notation: A ∩ B

Example 8: Consider Example 7. Then B ∩ E = {1, 3, 5}

Complement of a Set:

- If A is a subset of U, the set of all elements of U not in A, is called the ‘complement of A’.
- Notation: A’, $\overline{A}$, A^c

Example 9:

- Let B = {1, 3, 5, 7, 9} and U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Then
- Bc = {2, 4, 6, 8, 10}.

Difference between B and A:

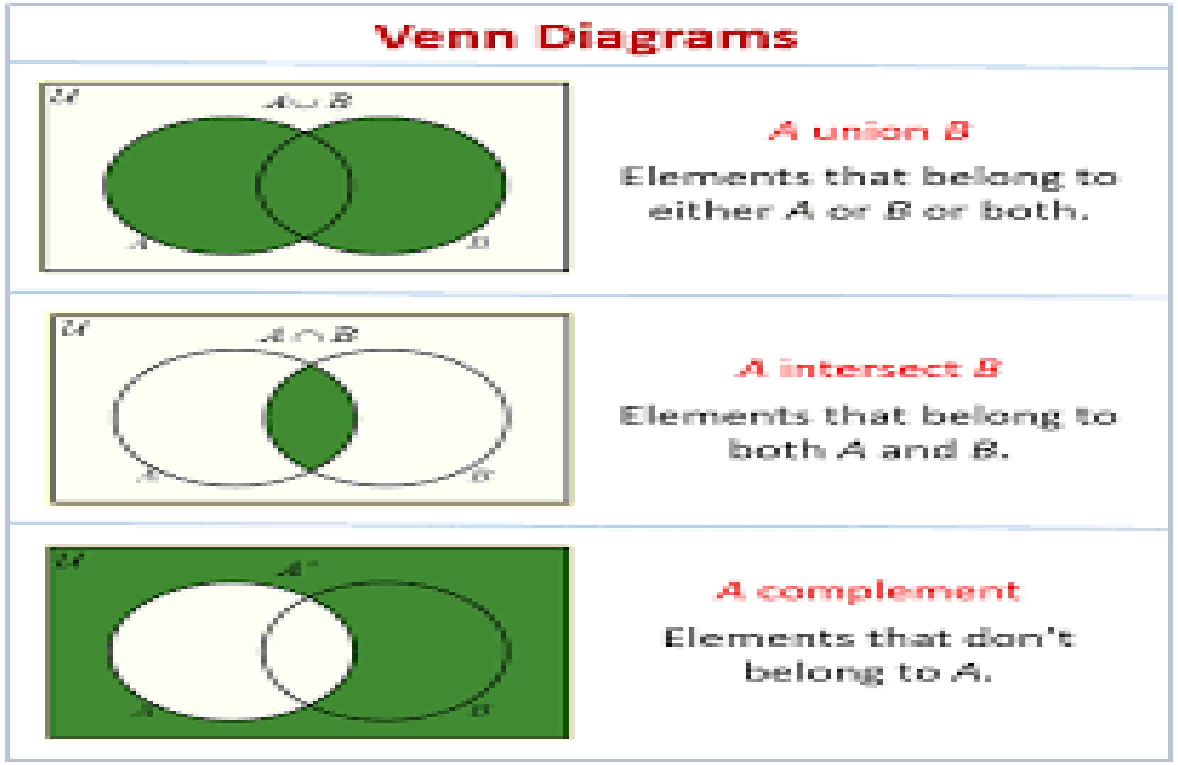
- If A and B are two subsets of U, 'B-A' is the set that consists of elements of B that is not A. (i.e $B – A = B \cap A^c$; Alternative notation: $B\setminus A$)

Example 10: Consider Example 7. Then $B - E = \{7, 9\}$

Venn Diagram:

- Graphical presentation of U and its subsets: Usually, U is represented by a rectangle and its subsets, by different shapes (regions) inside U.

- A ∪ B; A ∩ B; Ac:


Venn Diagrams
A union B
Elements that belong to either A or B or both.
A intersect B
Elements that belong to both A and B.
A complement
Elements that don't belong to A.

Finite, Infinite, and Countable Sets:

- A finite set contains either no elements or a natural number (n) of elements.
- An infinite set is a set which is not finite (eg. Set of natural numbers, N).
- A countable set is either finite, or it can be put in one-to-one correspondence with N.
- Eg. {1, 3, 5, 7, ….} is countably infinite.

Power sets:

- Let A be any set in U. The power set of A, denoted by P(A), is the set with its elements being all subsets of A.

Example 11:

- A = {a, b, c} ➔ P(A) = { Φ, {a}, {b}, {c}, {a,b}, {a,c}, {b, c}, A}.

(P(A) has 2^n elements where n= number of elements of A).

Laws of the Sets:

Identity Laws

- $A \cup \Phi = A$;
- $A \cap \Phi = \Phi$;
- $A \cup U = U$;
- $A \cap U = A$.

Idempotent Laws

- $A \cup A = A$;
- $A \cap A = A$.

Complement Laws

- $A \cup Ac = U$;
- $A \cap Ac = \Phi$;
- $(Ac)c = A$;
- $Uc = \Phi$;
- $\Phi c = U$.

Commutative Laws

- $A \cup B = B \cup A$;
- $A \cap B = B \cap A$.

Associative Laws

- $A \cup (B \cup C) = (A \cup B) \cup C$;
- $A \cap (B \cap C) = (A \cap B) \cap C$.

Distributive Laws

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$;
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

- DeMorgan's Laws
- $(A \cup B)^c = A^c \cap B^c$;
- $(A \cap B)^c = A^c \cup B^c$.