PHYS2155 Methods in physics II

Assignment 1
Due date: 5:00pm, February 11, 2021

Give answers and explanations for the following questions.

1. (20 marks) Assuming that A and B are square matrices of same size, either prove the following statements or give counterexamples.
(a) If both matrices A and B are upper triangular, then so is AB.
(b) If both matrices A and B are symmetric, then so is AB.
(c) If both matrices A and B are skew-symmetric, then so is AB.
(d) If both matrices A and B are hermitian and $AB = BA$, then AB is also hermitian.

2. (10 marks) Consider the following matrices

$$A(a,b) \equiv \begin{pmatrix} a & b \\ -b & a \end{pmatrix} , \tag{1}$$

where a and b are real numbers, prove that they are equivalent to complex number $z \equiv a + bi$. In particular, for a, b, c and d real numbers, prove the following.
(a) $A(a,b)A(c,d) = A(ac - bd, ad + bc)$;
(b) $A(a,b)$ has inverse if and only if a and b are not both zero, and the inverse is $A(a,b)^{-1} = A(a/(a^2+b^2), -b/(a^2+b^2))$.

3. (30 marks) (a) Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 3 & -8 \\ -2 & 5 \end{pmatrix} . \tag{2}$$

(b) Find the general solution of the system of differential equations

$$\begin{aligned} \frac{\mathrm{d}y_1}{\mathrm{d}x} &= 3y_1 - 8y_2 + f_1 , \\ \frac{\mathrm{d}y_2}{\mathrm{d}x} &= -2y_1 + 5y_2 + f_2 , \end{aligned} \tag{3}$$

where f_1 and f_2 are constants.

4. (20 marks) (a) Prove by mathematical induction that

$$A^n = \begin{pmatrix} 1 & n\lambda \\ 0 & 1 \end{pmatrix} \tag{4}$$

for integer $n \geq 1$, where

$$A = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} . \tag{5}$$

(b) Hence, find B^n where

$$B = \begin{pmatrix} -41 & 92 \\ -23 & 51 \end{pmatrix} . \tag{6}$$

Hint: Find the eigenvalues and eigenvectors of B.

5. (20 marks) Prove that

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{7}$$

is *not* diagonalizable.