PHYS2155 Methods in physics II

Assignment 2

Due date: 5:00pm, March 4, 2021

Give answers and explanations for the following questions.

1. (20 marks) Calculate the determinant of the following matrix. Show your steps.

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 8 & -4 & 0 \\ 1 & -2 & 7 & -1 \\ 0 & 5 & 3 & 2 \end{pmatrix} \tag{1}$$

2. (20 marks) Determine the reduced row echelon form of the following matrix

$$\begin{pmatrix} 2 & 1 & 10 & 3 \\ 3 & -2 & 0 & 0 \\ -1 & 7 & 4 & -1 \end{pmatrix} . \tag{2}$$

Show your steps.

3. (20 marks) Solve the system of equations

$$\begin{aligned} 3x_1 - 8x_2 + x_3 + x_4 &= 2 \\ -2x_1 + x_2 + 3x_4 &= 1 \\ x_1 - 5x_2 - x_3 &= -3 . \end{aligned} \tag{3}$$

4. (20 marks) Referring to Example 3.32 in lecture note, we consider a two dimensional real vector space with inner product

$$\eta_{ij} = \begin{cases} -1 & \text{if } i = j = 1 \\ 1 & \text{if } i = j = 2 \\ 0 & \text{otherwise} \end{cases} . \tag{4}$$

(a) Define a vector $\mathbf{v} \equiv (x_1, x_2)$ to be time-like if $(\mathbf{v}, \mathbf{v}) < 0$. Prove that the first component x_1 of a time-like vector is non-zero.
(b) A time-like vector $\mathbf{v}$ with $(\mathbf{v}, \mathbf{v}) = -1$ and the first component positive, $x_1 > 0$, is called an observer. For another vector $\mathbf{w}$, prove that if $(\mathbf{w}, \mathbf{w}) > 0$, then there are two observers $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $(\mathbf{w}, \mathbf{v}_1) > 0$ and $(\mathbf{w}, \mathbf{v}_2) < 0$. (In fact, there are infinitely many.)
(c) Prove that if $(\mathbf{w}, \mathbf{w}) < 0$, then $(\mathbf{w}, \mathbf{v})$ has the same sign for all observers (either all positive or all negative).

5. (20 marks) In the Euclidean space $\mathbb{R}^3$, let $\mathbf{e}_1 \equiv (1, 0, 0)^t$, $\mathbf{e}_2 \equiv (0, 1, 0)^t$ and $\mathbf{e}_3 \equiv (0, 0, 1)^t$. Find all orthogonal transformations A such that $A\mathbf{e}_i$ is orthogonal to $\mathbf{e}_i$ for $i = 1, 2, 3$. (Note that $A\mathbf{e}_1$ could be *not* orthogonal to $\mathbf{e}_2$, for example.)