Functional Dependencies (Armstrong's Axioms)

COMP3278B 2020

Functional Dependencies – quick check

- Which of the following is satisfied in R?
  - $A \rightarrow B$
  - $A \rightarrow C$
  - $A \rightarrow D$
  - $A \rightarrow E$

R

A	B	C	D	E
1	2	3	5	7
1	2	3	4	7
2	3	5	3	7
3	4	2	2	7
4	5	7	1	7

Armstrong's Axioms – basic

R

A	B	C	D	E
1	2	3	5	7
1	2	3	4	7
2	3	5	3	7
3	4	2	2	7
4	5	7	1	7

- Reflexivity – if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$

A	B
1	2
1	2
2	3
3	4
4	5

B
2
2
3
4
5

- Transitivity – if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$

A
1
1
2
3
4

B
2
2
3
4
5

C
3
3
5
2
7

- Augmentation – if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$

D	A
5	1
4	1
3	2
2	3
1	4

D	B
5	2
4	2
3	3
2	4
1	5

Armstrong's Axioms

- Union – if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$
- Decomposition – if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$

A	B
1	2
1	2
2	3
3	4
4	5

A	C
1	3
1	3
2	5
3	2
4	7

A	B	C
1	2	3
1	2	3
2	3	5
3	4	2
4	5	7

- Pseudo-transitivity – if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

A	B
1	2
1	2
2	3
3	4
4	5

D	B	C
5	2	3
4	2	3
3	3	5
2	4	2
1	5	7

D	A	C
5	1	3
4	1	3
3	2	5
2	3	2
1	4	7

Exercise

Estimated time: 15 – 20min

Basic rules

Reflexivity – if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.

Transitivity – if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$

Augmentation – if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$

Secondary rules

Union – if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$

Decomposition – if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$

Pseudo-transitivity – if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

1. Show that the following rules are special cases of a basic rule.
   - Self determination – $\alpha \rightarrow \alpha$ for any α
   - Extensivity – if $\alpha \rightarrow \beta$, then $\alpha \rightarrow \alpha\beta$

Clearly state the rule that you have applied in each step in your proof.

2. Proof the secondary rules using basic rules only.
3. Proof the following composition rule using basic rules only.
   - If $\alpha \rightarrow \beta$ and $\gamma \rightarrow \delta$, then $\alpha\gamma \rightarrow \beta\delta$
4. Given the rules on the right on attributes A, B, C, D, E, F , proof that $AB \rightarrow DE$.
   - You can use any rule.

$AC \rightarrow D$
$AC \rightarrow E$
$B \rightarrow F$
$F \rightarrow C$