

$$Q.1) a) \int_C (1 + x^2 y^2) ds$$

$$C: r(t) = t\hat{i} + 2\sqrt{2}e^{t/2}\hat{j} + e^t\hat{k}, 0 \leq t \leq 2$$

$$= \int_{t=0}^{t=2} [1 + t^2(8e^t)] \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

$$= \int_0^2 [1 + 8t^2 e^t] (1^2 + (\sqrt{2}e^{t/2})^2 + (e^t)^2)^{1/2} dt$$

$$= \int_0^2 [1 + 8t^2 e^t] (1 + 2e^t + e^{2t})^{1/2} dt$$

$$= \int_0^2 [1 + 8t^2 e^t] (1 + e^t)^{3/2} dt$$

$$= \int_0^2 (1 + 8t^2 e^t + e^t + 8t^2 e^{2t}) dt$$

$$= \int_0^2 (1 + 8t^2 + 1) dt$$

$$= \left[t\right]_0^2 + \left[e^t\right]_0^2 + \int_0^2 8t^2 (e^t + e^{2t}) dt$$

$$= 2 + e^2 - 1 + \left[8t^2 \left(e^t + \frac{e^{2t}}{2}\right)\right]_0^2 - \int_0^2 16t \left(e^t + \frac{e^{2t}}{2}\right) dt$$

$$= 1 + e^2 + \left[32 \left(e^2 + \frac{e^4}{2}\right)\right] - \left[16t \left(e^t + \frac{e^{2t}}{4}\right)\right]_0^2 + \int_0^2 16 \left(e^t + \frac{e^{2t}}{4}\right) dt$$

$$= 1 + 33e^2 + 16e^4 - (32e^2 + 8e^4) + [16e^t + 2e^{2t}]_0^2$$

$$= 1 + e^2 + 8e^4 + 16e^2 + 2e^4 - 16 - 2$$

$$\boxed{= -17 + 17e^2 + 10e^4} \quad \text{Ans}$$

$$b) \int_C (x^4 + y^4) ds$$

$$= \int_0^2 (t^4 + 64e^{2t}) (1 + e^t) dt$$

$$= \int_0^2 t^4 + 64e^{2t} + t^4e^t + 64e^{3t} dt$$

$$= \left[\frac{t^5}{5} \right]_0^2 + \left[32e^{2t} \right]_0^2 + \left[\frac{64e^{3t}}{3} \right]_0^2 + \int_0^2 t^4 e^t dt$$

$$= \frac{32}{5} + 32e^4 - 32 + \frac{64e^6}{3} - \frac{64}{3} + \int_0^2 t^4 e^t dt$$

$$= \frac{96 - 480 - 320 + 32e^4 + \frac{64e^6}{3} + 16e^2 - \left[4t^3 e^t \right]_0^2 + \int_0^2 12t^2 e^t dt$$

$$= \frac{-704}{15} + 32e^4 + \frac{64e^6}{3} + 16e^2 - 32e^2 + \left[12t^2 e^t \right]_0^2 - \int_0^2 24t e^t dt$$

$$= \frac{-704}{15} + 32e^4 + \frac{64e^6}{3} + 16e^2 + 48e^2 - \left[24t e^t \right]_0^2 + \int_0^2 24e^t dt$$

$$= \frac{-704}{15} + 32e^4 + \frac{64e^6}{3} + 32e^2 - 48e^2 + \left[24e^t \right]_0^2$$

$$= \frac{-704}{15} + 32e^4 + \frac{64e^6}{3} - 16e^2 + 24e^2 - 24$$

$$\boxed{= \frac{-1064}{15} + 32e^4 + \frac{64e^6}{3} + 8e^2} \quad \text{Ans}$$

$$x = e^t$$

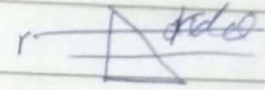
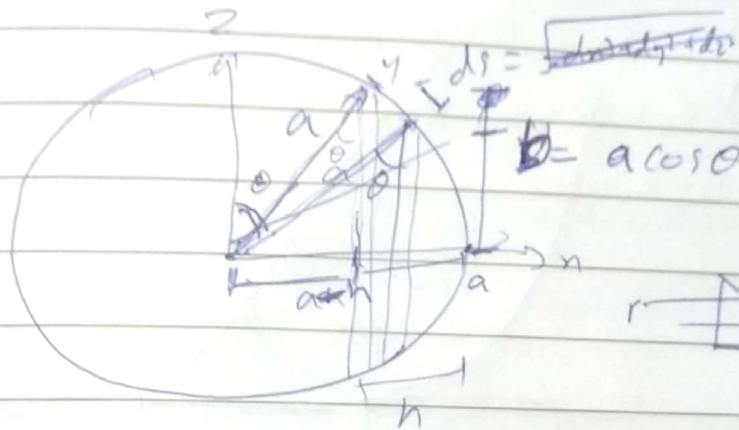
$$x^2 = e^{2t}$$

$$\frac{d(x^2)}{dt} = 2x \cdot \frac{dx}{dt} = 2xe^{2t}$$

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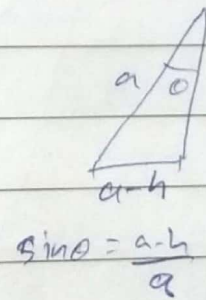
Q.2) a)



$$dA = 2\pi b \cdot ds, \quad ds = r d\theta$$

$$A = \int 2\pi a \cos \theta \cdot a d\theta$$

$$= 2\pi a^2 \int_{\sin \theta = \frac{1-h}{a}}^{\sin \theta = 1} d(\sin \theta)$$



$$\sin \theta = \frac{a-h}{a}$$

$$= 2\pi a^2 \left[1 - 1 + \frac{h}{a} \right]$$

Ans.

$$A_{\text{cap}} = 2\pi a h$$

Q.2b)

$$z = x^2 + y^2$$

$$z = 8 - 2y$$

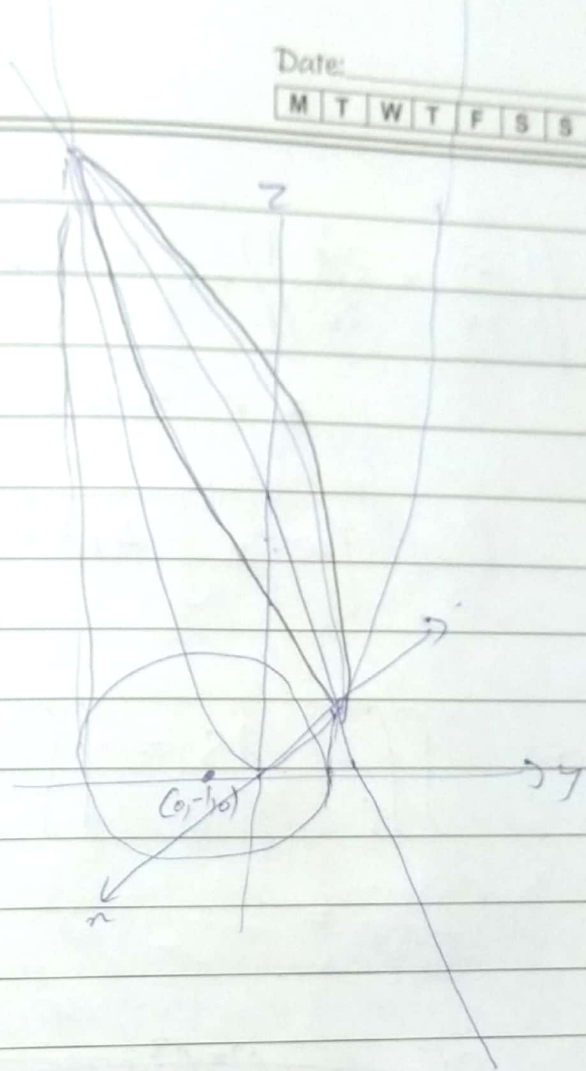
Intersection:-

$$x^2 + y^2 = 8 - 2y$$

$$x^2 + y^2 + 2y + 1 = 9$$

$$x^2 + (y+1)^2 = 9$$

circle of radius 3
centered on $(0, -1, 0)$



Parametrization:-

$$\text{let } x = r \cos \theta$$

$$y = -1 + r \sin \theta$$

$$z = r^2 \cos^2 \theta + (-1 + r \sin \theta)^2$$

$$z = r^2 - 2r \sin \theta + 1$$

$$\text{let } \vec{r} = (x, y, z)$$

$$\frac{\partial \vec{r}}{\partial r} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ 2r - 2 \sin \theta \end{bmatrix}$$

$$\frac{\partial \vec{r}}{\partial \theta} = \begin{bmatrix} -r \sin \theta \\ r \cos \theta \\ -2r \cos \theta \end{bmatrix}$$

$$\begin{aligned} \frac{\partial \vec{r}}{\partial r} \times \frac{\partial \vec{r}}{\partial \theta} &= \begin{bmatrix} -2r \cos \theta \sin \theta - 2r^2 \cos \theta + 2r \cos \theta \sin \theta \\ -2r^2 \sin \theta + 2r \sin \theta + 2r \cos^2 \theta \\ r \cos^2 \theta + r \sin^2 \theta \end{bmatrix} \\ &= r \begin{bmatrix} -2 \cos \theta \sin \theta \\ 2 - 2r \sin \theta \\ 1 \end{bmatrix} \end{aligned}$$

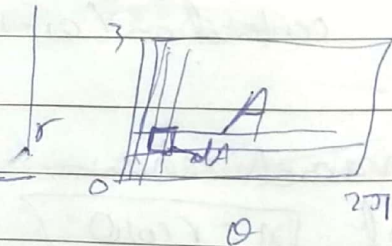
$$\left\| \frac{\partial \vec{r}}{\partial r} \times \frac{\partial \vec{r}}{\partial \theta} \right\| = r \sqrt{4r^2 \cos^2 \theta + (2 - 2r \sin \theta)^2 + 1^2}$$

$$= r \sqrt{4r^2 \cos^2 \theta + 4r^2 \sin^2 \theta - 8r \sin \theta + 4 + 1}$$

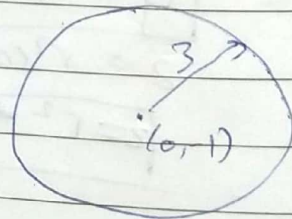
$$r = r \sqrt{4r^2 - 8r \sin \theta + 5}$$

$$\iint_S dS = \iint \left\| \frac{\partial \vec{r}}{\partial r} \times \frac{\partial \vec{r}}{\partial \theta} \right\| \frac{dA}{dr d\theta}$$

$$\iint_S dS = \int_0^{2\pi} \int_0^3 4$$



$$\Rightarrow \iint_S dS = \int_0^{2\pi} \int_0^3$$



$$\Rightarrow \iint_S dS = \int_0^{2\pi} \int_0^3 \sqrt{4r^2 - 8r \sin \theta + 5} r dr d\theta$$

Shawn!

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$$Q.3) a) F = \frac{1}{r} \hat{\theta}$$

$$\int_C F \cdot dl$$

$$x = L, \quad y = L \tan \theta$$

$$\frac{\partial x}{\partial \theta} = 0, \quad \frac{\partial y}{\partial \theta} = L \sec^2 \theta$$

$$\int_{-\pi/8}^{\pi/8} F \cdot \begin{bmatrix} 0 \\ L \sec^2 \theta \end{bmatrix} d\theta$$

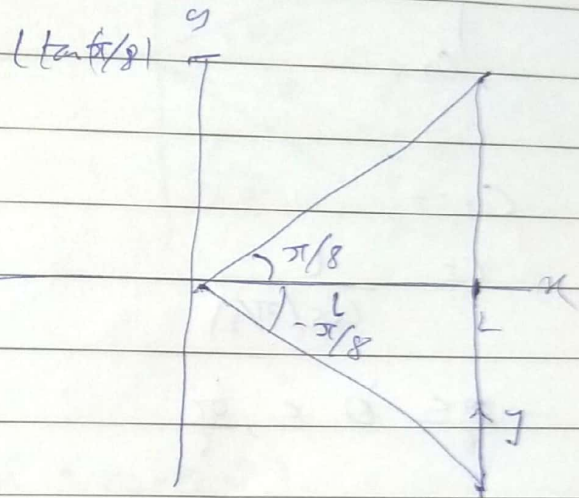
$$F = \frac{1}{\sqrt{x^2 + y^2}} (-\sin \theta \hat{x} + \cos \theta \hat{y})$$

$$\Rightarrow \int_{-\pi/8}^{\pi/8} \frac{1}{\sqrt{L^2 + L^2 \tan^2 \theta}} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \cdot \begin{bmatrix} 0 \\ L \sec^2 \theta \end{bmatrix} d\theta$$

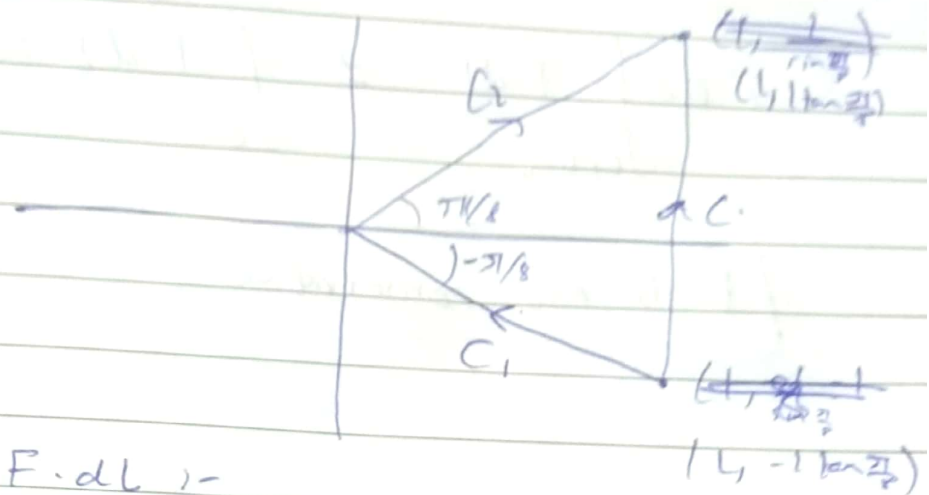
$$= \int_{-\pi/8}^{\pi/8} \frac{1}{\sqrt{L^2 \sec^2 \theta}} (L \sec \theta) d\theta$$

$$= \int_{-\pi/8}^{\pi/8} \frac{1}{L \sec \theta} (L \sec \theta) d\theta \quad \left(\sec \theta \text{ is positive for } -\pi/8 \leq \theta \leq \pi/8 \right)$$

$$= \frac{\pi}{8} + \frac{\pi}{8} \quad \boxed{= \frac{\pi}{4}}$$



Q.3b)



$$\int_{C_1 \cup C_2} F \cdot d\mathbf{l} =$$

$$= \int_{C_1} F \cdot d\mathbf{l} + \int_{C_2} F \cdot d\mathbf{l}$$

$$C_1: \theta = -\frac{\pi}{8}, \quad r = L \cos(\frac{\pi}{8}) - t \quad 0 \leq t \leq L \cos(\frac{\pi}{8})$$

$$\frac{d\vec{r}}{dt} = r \frac{d\hat{r}}{dt} + \hat{r} \frac{dr}{dt}$$

$$\int_{C_1} F \cdot d\mathbf{l} =$$

$$= r \frac{dr}{dt} \frac{d\theta}{dt} + \hat{r} (-1)$$

$$\int_{C_1} F \cdot d\mathbf{l} = \int_{t=0}^{L \cos(\pi/8)} \frac{1}{r} \hat{\theta} \cdot (-\hat{r}) dt \quad \boxed{= 0}$$

$$C_2: \theta = \frac{\pi}{8}, \quad r = t \quad 0 \leq t \leq \frac{L}{\cos(\pi/8)}$$

$$\frac{d\vec{r}}{dt} = \hat{r}$$

$$\Rightarrow \int_{C_2} F \cdot d\mathbf{l} =$$

$$\Rightarrow \int_{C_2} F \cdot d\mathbf{l} = \int_0^{\frac{L}{\cos(\pi/8)}} \frac{1}{r} \hat{\theta} \cdot \hat{r} dt = 0$$

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Since $0 = \int_{\text{closed}} \vec{F} \cdot d\vec{l} \neq \int_C \vec{F} \cdot d\vec{l} = \frac{\pi}{4}$,

\vec{F} is not conservative

$$Q.4) F(n, y) = \left[\frac{\ln(\ln y)}{n} + \frac{2}{3} n y^3 \right] n + \left(\frac{\ln n}{y \ln y} + n^2 y \right) y$$

$$\frac{\partial P}{\partial y} = \frac{1}{n} \cdot \frac{1}{\ln y} \cdot \frac{1}{y} + 2 n y^2$$

$$\frac{\partial Q}{\partial n} = \frac{1}{n y \ln y} + 2 n y^2$$

$$\frac{\partial Q}{\partial n} = \frac{1}{n y \ln y} + 2 n y^2$$

$$\Rightarrow \frac{\partial Q}{\partial n} = \frac{\partial P}{\partial y} \Rightarrow \text{conservative}$$

Note:- its conservative on the region where its defined i.e. $n > 0, y > 0$.

Since $\ln n$ and $\ln y$ are not defined for non positive ~~negative~~ n and y and $\frac{1}{n}$ and $\frac{1}{y}$ are not

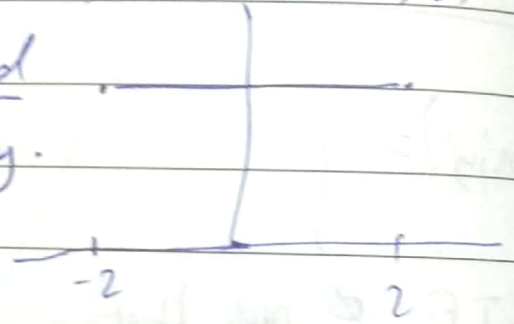
$$\phi = \int \left[\frac{\ln(\ln y)}{n} + \frac{2}{3} n y^3 \right] dn$$

$$\boxed{\phi = \ln n \cdot \ln(\ln y) + \frac{n^2 y^3}{3} + C}$$

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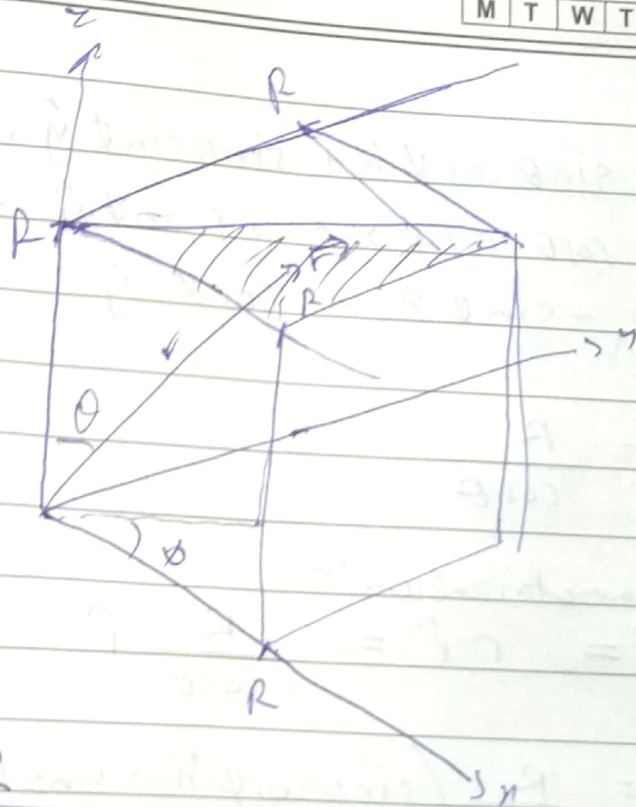
b) The line integral of F along the straight line from $(-2, 2)$ to $(2, 2)$ is NOT equal to $\phi(2, 2) - \phi(-2, 2)$ since F is not defined for nonpositive x and y .



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Q.5)



$$r \cos \theta = R$$

$$\Rightarrow \boxed{r = \frac{R}{\cos \theta}}$$

$$0 \leq r \sin \theta \sin \phi \leq r \sin \theta \cos \phi$$

$$\Rightarrow 0 \leq \sin \phi \leq \cos \phi \Rightarrow \boxed{0 \leq \phi \leq \pi/4}$$

$$0 \leq r \sin \theta \cos \phi \leq r \cos \theta$$

$$\Rightarrow \boxed{\tan \theta \leq \sec \phi} \Rightarrow 0 \leq \tan^{-1}(\sec \phi)$$

$$\Rightarrow \tan \theta \leq \sec \phi \Rightarrow \boxed{\theta \leq \tan^{-1}(\sec \phi)}$$

Q.5b)

$$\iint_r \sin \theta \, d\theta \, d\phi$$

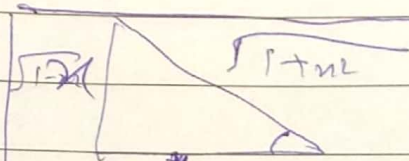
$$r = \frac{R}{\cos \theta}$$

$$= \int_0^{\pi/4} \int_0^{\tan^{-1}(\sec \theta)} \sin \theta \, d\theta \, d\phi$$



$$= \int_0^{\pi/4} \left[-\cos \theta \right]_0^{\tan^{-1}(\sec \theta)} d\phi$$

$$= \int_0^{\pi/4} \left[\frac{-1}{1 + \sec^2 \theta} + 1 \right] d\phi$$



$$= \int_0^{\pi/4} \left[\frac{1 + \sec^2 \theta - 1}{1 + \sec^2 \theta} \right] d\phi$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$= \frac{1}{\sqrt{1 + \tan^2 \theta}}$$

$$= \int_0^{\pi/4} \left[\frac{1}{\cos^2 \theta + 1} \right] d\phi$$

$$= 0.435$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sqrt{1 - u^2}}{u}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\sqrt{1 - u^2}}$$