PHYS2155 Methods in physics II

Assignment 4

Due date: 5:00pm, April 22, 2021

Give answers and explanations for the following questions.

1. (20 marks) Verify Green's theorem for the region D, bounded by the two parabolas, $y = x^2 - 4$ and $y = 4 - x^2$ and the two functions $P(x, y) = x^2 y$ and $Q(x, y) = x + y$.

2. (20 marks) Use the Divergence Theorem to find the outward flux of the vector field $\mathbf{F} = (5x^3 + 12xy^2)\,\hat{\imath} + (y^3 + e^y \sin z)\,\hat{\jmath} + (5z^3 + e^y \cos z)\,\hat{k}$ across the surface S, which is the surface of the solid region E bounded between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 2$.

3. (20 marks) Verify Stokes's theorem for the vector field $\mathbf{F} = 5x^2 y\,\hat{\imath} + x^3\,\hat{\jmath} + 5z^3\,\hat{k}$ and the surface S, which is the northern hemisphere of the sphere $x^2 + y^2 + z^2 = 1$ and $z \geq 0$.

4. (40 marks) (a) Use MATLAB to find out the two intersection points of the two curves $y = 5e^{-x^2}$ and $y = x^2$, up to 4 significant figures.
   (b) A surface S is the graph of the function $z = g(x, y) = e^{y \sin x}$, with parameter domain D the region bounded by the two curves in (a). A vector field is given $\mathbf{F} = z\,\hat{\imath} + z^2\,\hat{\jmath} + z^3\,\hat{k}$. What is the surface integral of $\mathbf{F}$ over S, up to 4 significant figures?
   (c) Verify Stokes' theorem for these vector field and region numerically.
   (Hint: Students might find `fzero` and `integral2` in MATLAB useful.)