PHYS2155 Methods in physics II

Tutorial 2

1. Write the system of linear algebraic equations with the following augmented matrix

(a)

$$\begin{pmatrix} 0 & 1 & 5 & -4 \\ 1 & 4 & 3 & -2 \\ 2 & 7 & 1 & -2 \end{pmatrix} \tag{1}$$

(b)

$$\begin{pmatrix} 2 & -2 & -1 & 3 \\ 3 & -2 & 3 & 1 \\ 1 & -1 & 1 & 0 \\ 2 & -1 & 2 & 2 \end{pmatrix}. \tag{2}$$

2. Solve the following systems of linear algebraic equations by performing appropriate elementary row operations on the augmented matrix of the system.

(a)
$$\begin{aligned} x_1 \qquad\quad - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

(b)
$$\begin{aligned} x_1 - \ x_2 - \ x_3 &= 2 \\ 3x_1 - 3x_2 + 2x_3 &= 16 \\ 2x_1 - \ x_2 + \ x_3 &= 9 \end{aligned}$$

3. Show that the given matrices are row equivalent and find a sequence of elementary row operations that will convert A to B.

(a)

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix} \tag{3}$$

(b)

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 1 & -1 \\ 3 & 5 & 1 \\ 2 & 2 & 0 \end{pmatrix} \tag{4}$$

4. Suppose a, b, c, and d are constants such that a is not zero and the linear system below is consistent for all possible values of f and g. What can you say about the numbers a, b, c, and d? Justify your answer.

$$ax_1 + bx_2 = f \tag{5}$$
$$cx_1 + dx_2 = g \tag{6}$$

5. Use elementary row operations to reduce the given matrices to row echelon form and locate the pivot columns of each matrix.
(a)
$$\begin{pmatrix} 0 & 3 & 4 & 1 \\ 3 & 1 & 2 & 2 \\ 1 & 5 & 2 & 1 \end{pmatrix} \tag{7}$$
(b)
$$\begin{pmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 1 \\ -4 & 6 & -7 & 1 \\ 2 & 0 & 1 & 3 \end{pmatrix} \tag{8}$$

6. Determine the rank of the following matrix.
$$\begin{pmatrix} 3 & 2 & 3 & 2 \\ 3 & 7 & 1 & -1 \\ 5 & 1 & 1 & 3 \end{pmatrix} \tag{9}$$

7. Determine the reduced row echelon form of the following matrix.
$$\begin{pmatrix} 0 & 1 & 2 & 0 & 3 \\ 2 & 4 & 8 & 2 & 4 \\ 1 & 2 & 4 & 2 & 2 \\ 1 & 3 & 6 & 1 & 5 \end{pmatrix} \tag{10}$$

8. Use Gaussian elimination to determine the solution set to the given linear system.
$$\begin{aligned} 2x_2 + x_3 + x_4 &= 1 \\ x_1 + 3x_2 + x_3 + 2x_4 &= 1 \\ 3x_1 + 9x_2 + 4x_3 + 3x_4 &= 0 \ . \end{aligned}$$

9. Use Gauss-Jordan elimination to find the solution set to the given linear system.
$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= 10 \\ x_1 + 3x_2 + 6x_3 + 10x_4 &= 20 \\ x_1 + 4x_2 + 10x_3 + 20x_4 &= 35 \end{aligned}$$

10. Use Gauss-Jordan elimination to compute the inverses of the following invertible matrix.
$$\begin{pmatrix} 1 & -1 & 2 & 3 \\ 2 & 0 & 3 & -4 \\ 3 & -1 & 7 & 8 \\ 1 & 0 & 3 & 5 \end{pmatrix} \tag{11}$$

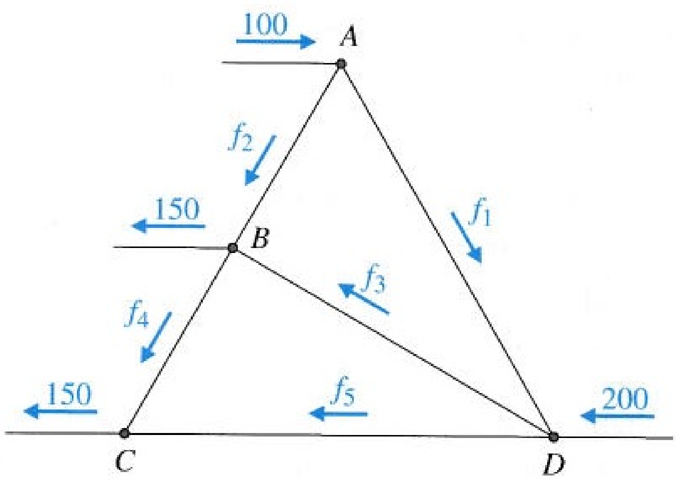
11. Use Cramer's rule to solve the given linear system.

$$\begin{aligned} 3x_1 - 2x_2 + x_3 &= 4 \\ x_1 + x_2 - x_3 &= 2 \\ x_1 \qquad + x_3 &= 1 \end{aligned}$$

12. A florist offers three sizes of flower arrangements containing roses, daisies, and chrysanthemums. Each small arrangement contains one rose, three daisies, and three chrysanthemums. Each medium arrangement contains two roses, four daisies, and six chrysanthemums. Each large arrangement contains four roses, eight daisies, and six chrysanthemums. One day, the florist noted that she used a total of 24 roses, 50 daisies, and 48 chrysanthemums in filling orders for these three arrangements. How many arrangements of each type did she make?

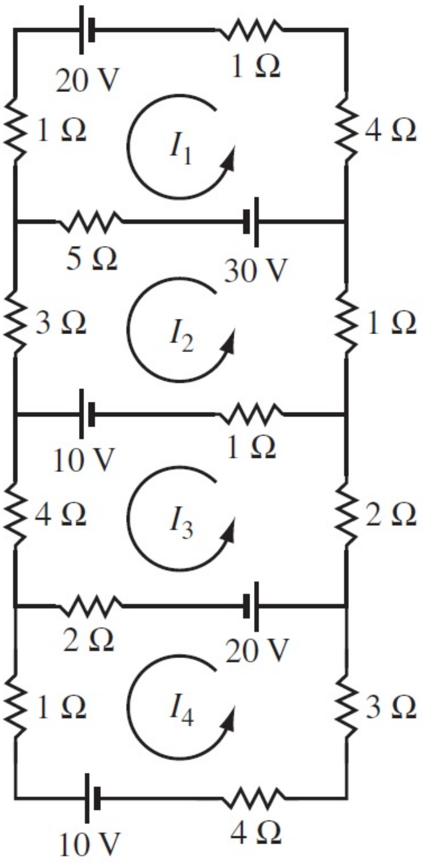
13.

14. A network of irrigation ditches is shown in the figure, where flows are measured in thousands of liters per day.
(a) Set up and solve a system of linear equations to find the possible flows $f_1, f_2, \ldots, f_5$.



(b) Suppose the ditch DC is closed. What range of flow will need to be maintained through the ditch DB?
(c) Why can the ditch DB not be closed?
(d) From your solution in part (a), determine the minimum and maximum flows through the ditch DB.

15. Figure below shows an electric circuits composed of five power sources and thirteen resistors. Determine the currents I_1, I_2, I_3, and I_4.



16. A systematic method for balancing chemical equations is to set up a vector equation that describes the numbers of atoms of each type presented in a reaction. Alka-Seltzer contains sodium bicarbonate ($NaHCO_3$) and citric acid ($H_3C_6H_5O_7$). When a tablet of Alka-Seltzer is dissolved in water, sodium citrate ($Na_3C_6H_5O_7$), water (H_2O), and carbon dioxide (CO_2) are produced according to the equation of the form

$$(x_1)\mathrm{NaHCO_3} + (x_2)\mathrm{H_3C_6H_5O_7} \to (x_3)\mathrm{Na_3C_6H_5O_7} + (x_4)\mathrm{H_2O} + (x_5)\mathrm{CO_2}\,.$$

Balance this chemical equation using the vector equation approach. That is to say, find the smallest whole numbers $x_1, x_2, \ldots, x_5$ such that the total numbers of each type of atoms on the left match the corresponding number of atoms on the right (because atoms are neither created or destroyed in the reaction).

17. Let $W = \mathrm{Span}((1,1,0),(1,0,-1))$ in $\mathbb{R}^3$. Determine whether $\mathbf{v}_1 = (0,1,-2)$ and $\mathbf{v}_2 = (1,-1,-2)$ are in W.

18. Are $\mathbf{v}_1 = (0,1,-2)$ and $\mathbf{v}_2 = (1,-1,-2)$ linearly independent in $\mathbb{R}^3$?

19. Find out the subspace of $\mathbb{R}^3$ that is orthogonal to both $\mathbf{v}_1 = (0,1,-2)$ and $\mathbf{v}_2 = (1,-1,-2)$.

20. Find an orthonormal basis of $\mathbb{R}^3$ from

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \qquad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \qquad \mathbf{v}_3 = \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix}. \tag{12}$$