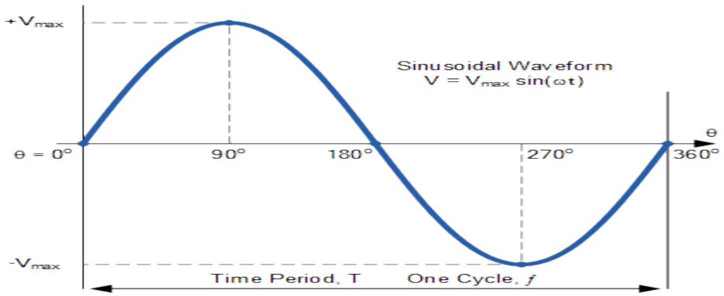
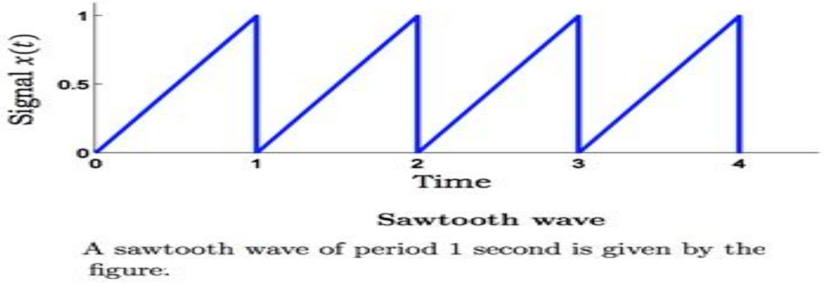
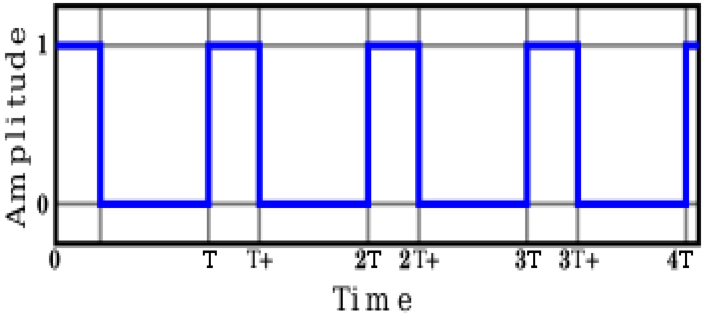
Waves part 3

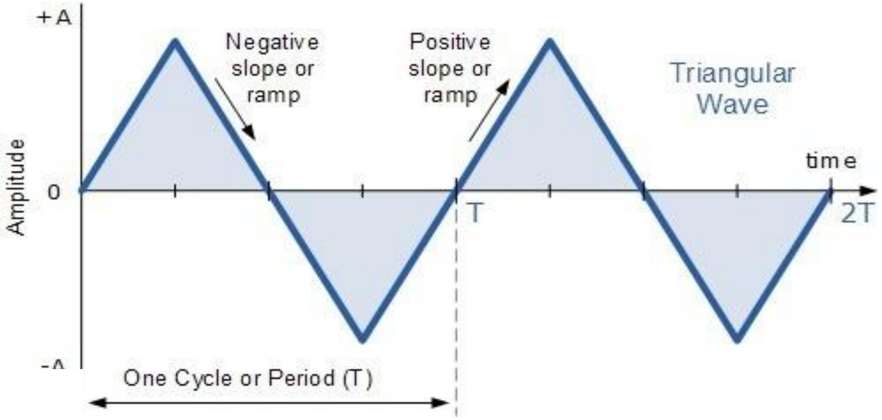
Contents

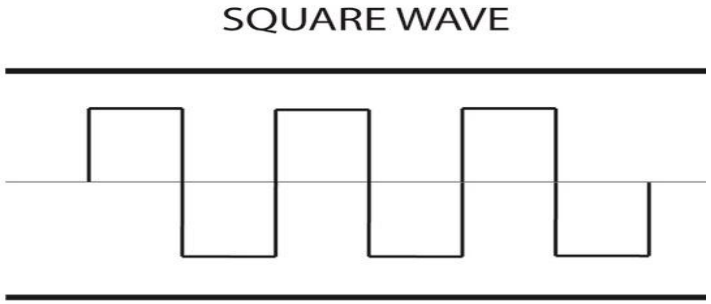
Different types of waves/waveforms

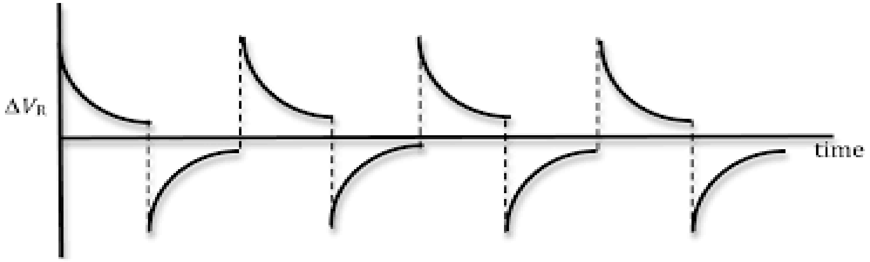












Superposition

- Many interesting wave phenomenon can not be described by single travelling wave. For which we need to analyze some complex waves.
- One can make use of the Superposition Principle
- The term Superposition means “overlapping or combining effect of something”
- In physics we deal with wave phenomenon therefore this term is related to wave.

Superposition Principle

- If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.

OR

- Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$

Superposition Principle(cont'd)

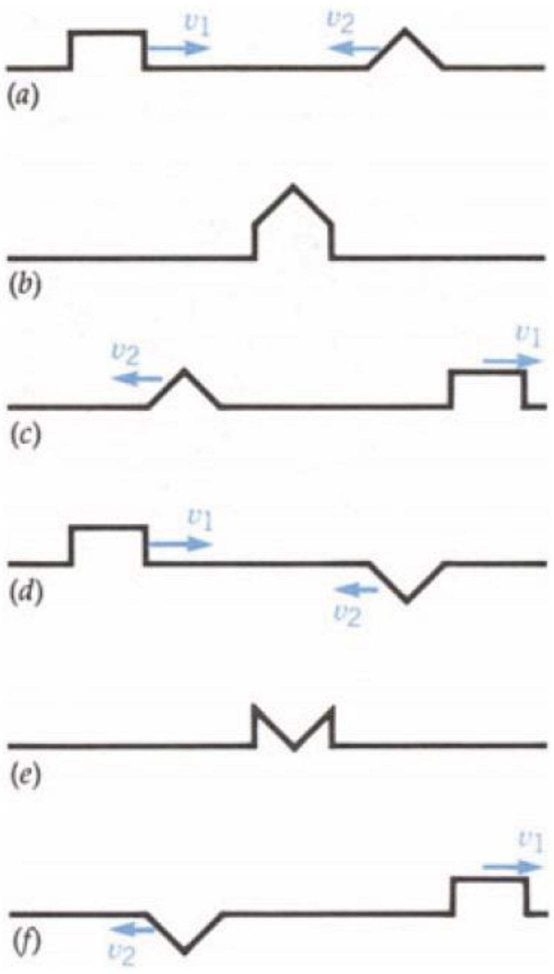
To be more precise….

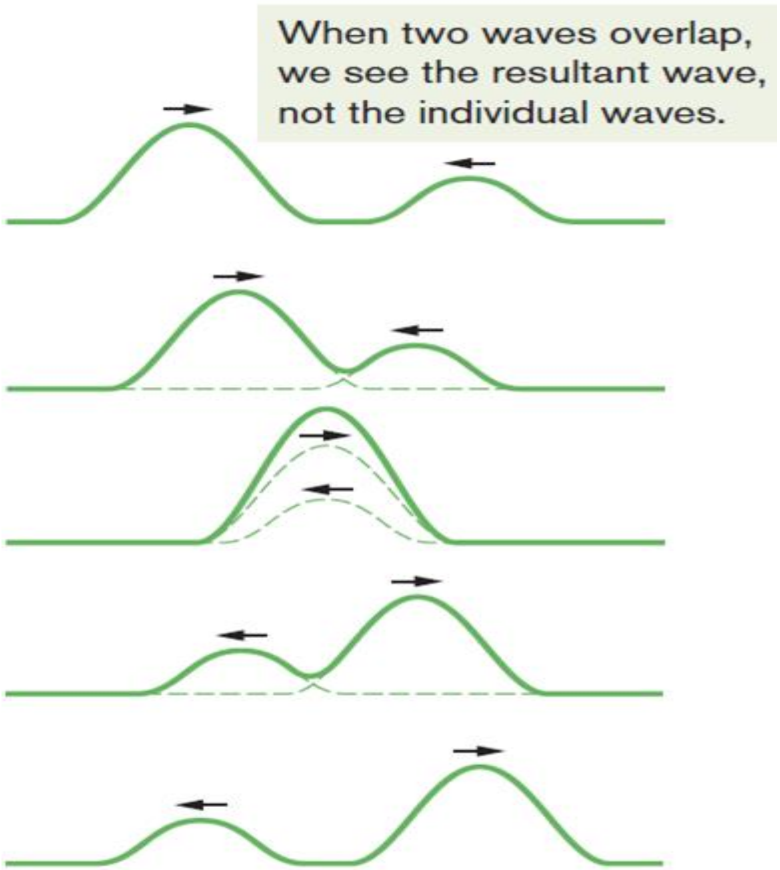
- When particle of one medium is under effect of more than one waves then its displacement at any time is the vector sum of all the displacements which would have been given by individual waves that is,

$$\vec{y} = \overrightarrow{y_1} + \overrightarrow{y_2} + \cdots + \overrightarrow{y_n}$$

- Waves that obey this principle are called ***linear waves***
- Waves that violate the superposition principle are called ***nonlinear waves***

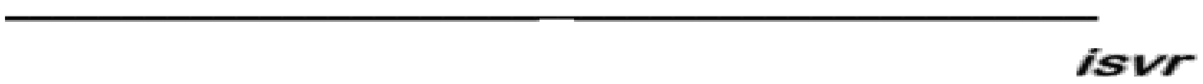
Superposition Principle(cont'd)





A series of snapshots that show two pulses traveling in opposite directions along a stretched string. The superposition principle applies as the pulses move through each other.

Superposition Principle(cont’d)



Note: Overlapping waves do not in any way alter the travel of each other.

Interference & their Types

- The superposition principle leads to a wave phenomenon known as ***interference.***

- The combination of separate waves in the same region of space to produce a resultant wave is called ***interference.***

- There are two types of interference that we usually observe in waves,

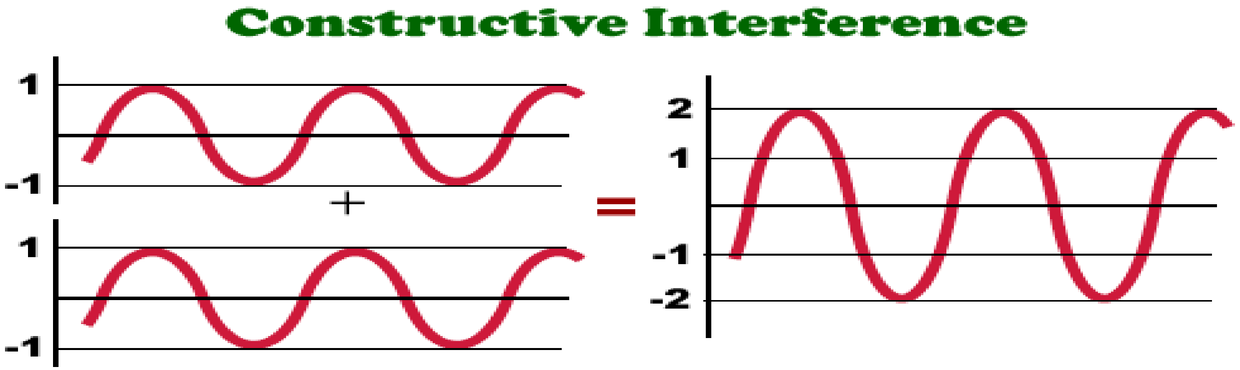
1. Constructive Interference
2. Destructive Interference

Constructive Interference

- It is the type of interference where two interfering waves have a displacement in the same direction at a time.

- **Fully constructive** interference occurs when the phase difference of two waves is a multiple of 2π.

$$\Delta r = \frac{\phi}{2\pi}\lambda \qquad \Delta r = (2n)\frac{\lambda}{2} \quad \text{for constructive interference}$$

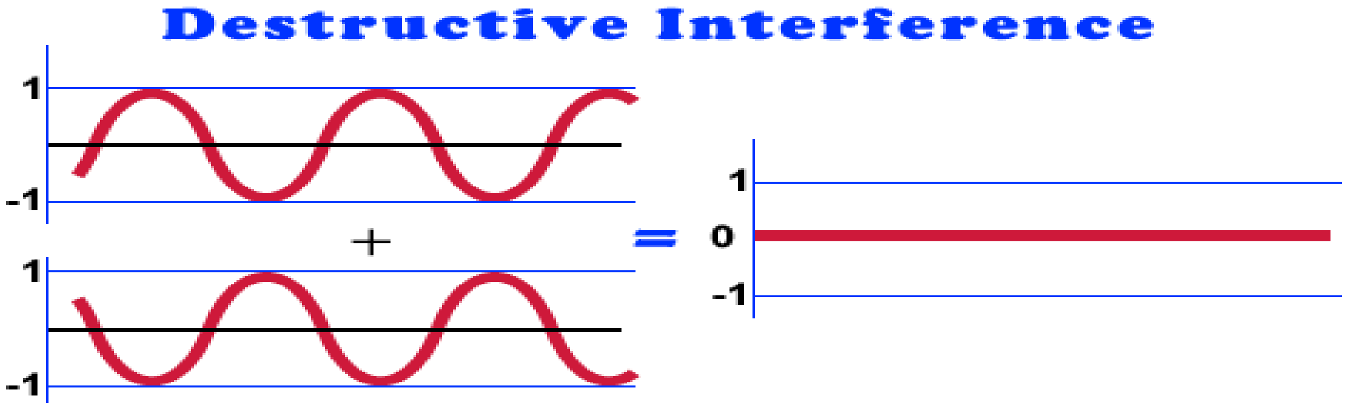


Destructive Interference

- It is the type of interference where two interfering waves have a displacement in the opposite direction simultaneously.

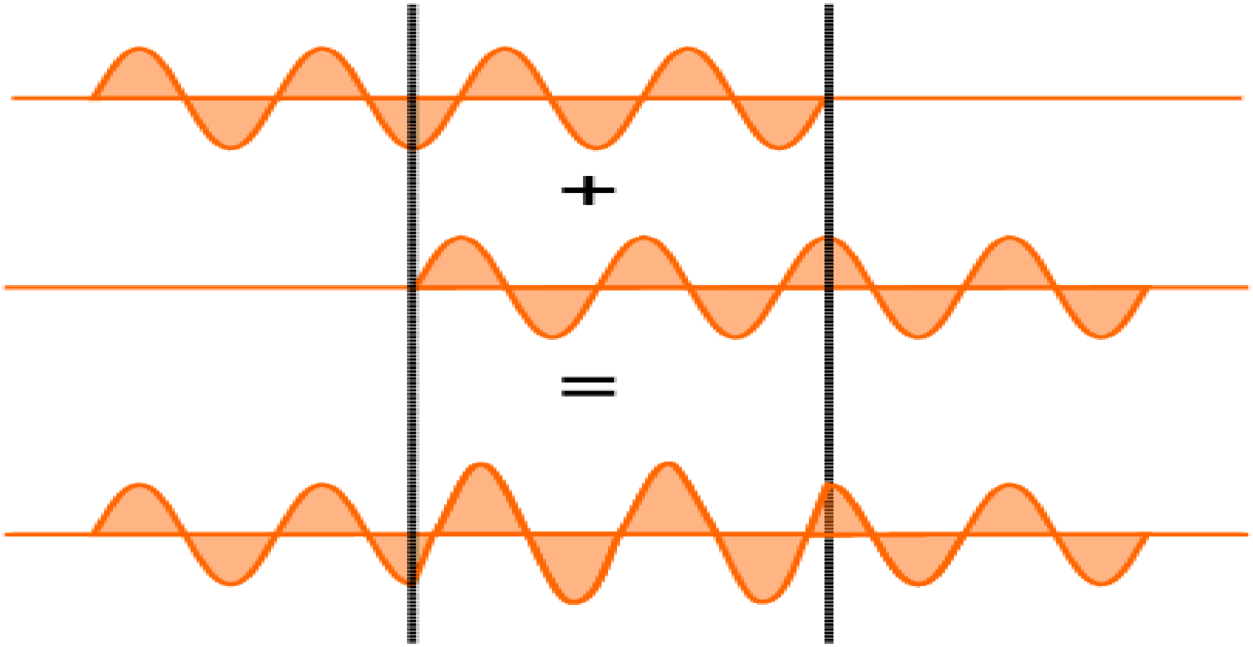
- **Fully destructive** interference is obtained when the phase difference between two waves is an odd multiple of π

$$\Delta r = \frac{\phi}{2\pi}\lambda \qquad \Delta r = (2n+1)\frac{\lambda}{2} \quad \text{for destructive interference}$$



Intermediate Phase Difference

- If the difference between the phases is intermediate between these two extremes (fully constructive or destructive), then the magnitude of displacement of the summed wave lies between the minimum and maximum values.



Superposition of Sine Waves

Mathematical Approach

Superposition of Sinusoidal Waves

Let us now apply the principle of superposition to two sinusoidal waves traveling in the same direction in a linear medium. If the two waves are traveling to the right and have the same frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

$$\boxed{y_1 = A\sin(kx - \omega t)} \qquad \boxed{y_2 = A\sin(kx - \omega t + \phi)}$$

where, as usual, $k = 2\pi/\lambda$, $\omega = 2\pi f$, and ϕ is the phase constant,
Hence, the resultant wave function y is

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

To simplify this expression, we use the trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

If we let $a = kx - \omega t$ and $b = kx - \omega t + \phi$, we find that the resultant wave function y reduces to

$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

Superposition of Sinusoidal Waves(Result)

$$\boxed{y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

This result has several important features.

- The resultant wave function **y** also is **sinusoidal**.

- Has the **same frequency** and **wavelength** as the individual waves because the sine function incorporates the same values of k and ω that appear in the original wave functions.

- The amplitude of the resultant wave is **2A cos (Ø/2),** and its phase is Ø/2. If the phase constant Ø = 0, then cos (Ø/2) " cos 0 = 1”, and **the amplitude of the resultant wave is 2A**—twice the amplitude of either individual wave.

- Waves are said to be everywhere **in phase** and thus interfere **constructively**

Superposition of sinusoidal Waves

- Home Work:

By using
$$\boxed{y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

You all need to find the mathematical conclusions for destructive interference as we have done for constructive interference. Also discuss their important features.

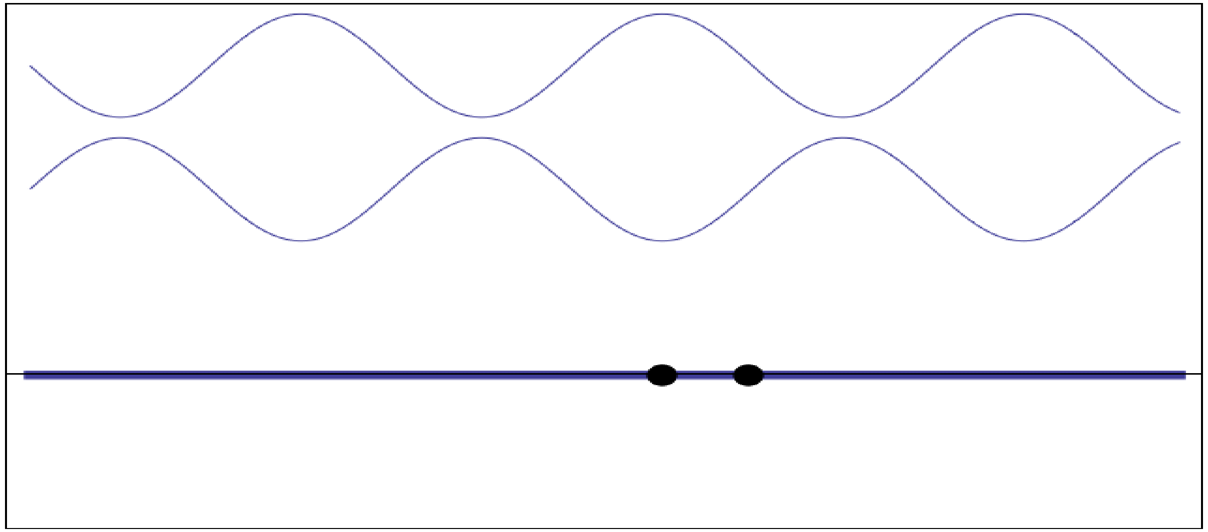
Phase Difference and Resulting Interference Types[a]

Phase Difference, in			Amplitude of Resultant Wave	Type of Interference
Degrees	Radians	Wavelengths		
0	0	0	$2y_m$	Fully constructive
120	$\frac{2}{3}\pi$	0.33	y_m	Intermediate
180	π	0.50	0	Fully destructive
240	$\frac{4}{3}\pi$	0.67	y_m	Intermediate
360	2π	1.00	$2y_m$	Fully constructive
865	15.1	2.40	$0.60y_m$	Intermediate

[a]The phase difference is between two otherwise identical waves, with amplitude y_m, moving in the same direction.

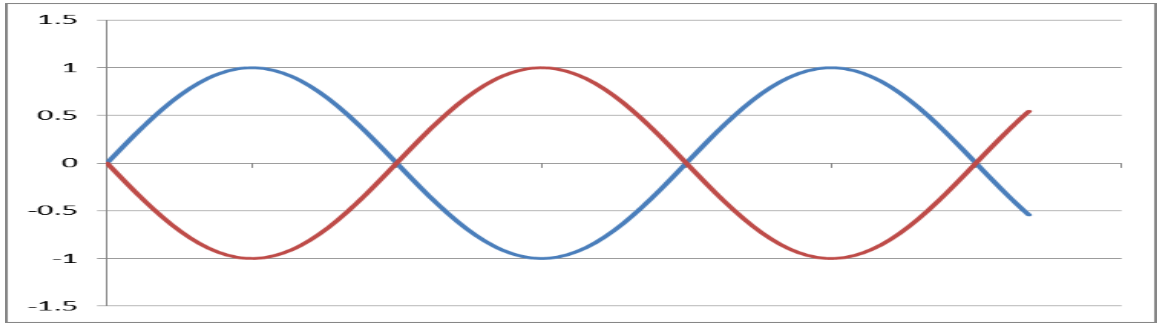
Standing Waves

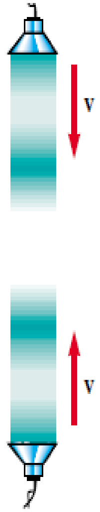
- Another interesting phenomenon resulting from the superposition principle is the formation of ***standing waves.***
- Also known as Stationary wave.



Formation of Standing Wave

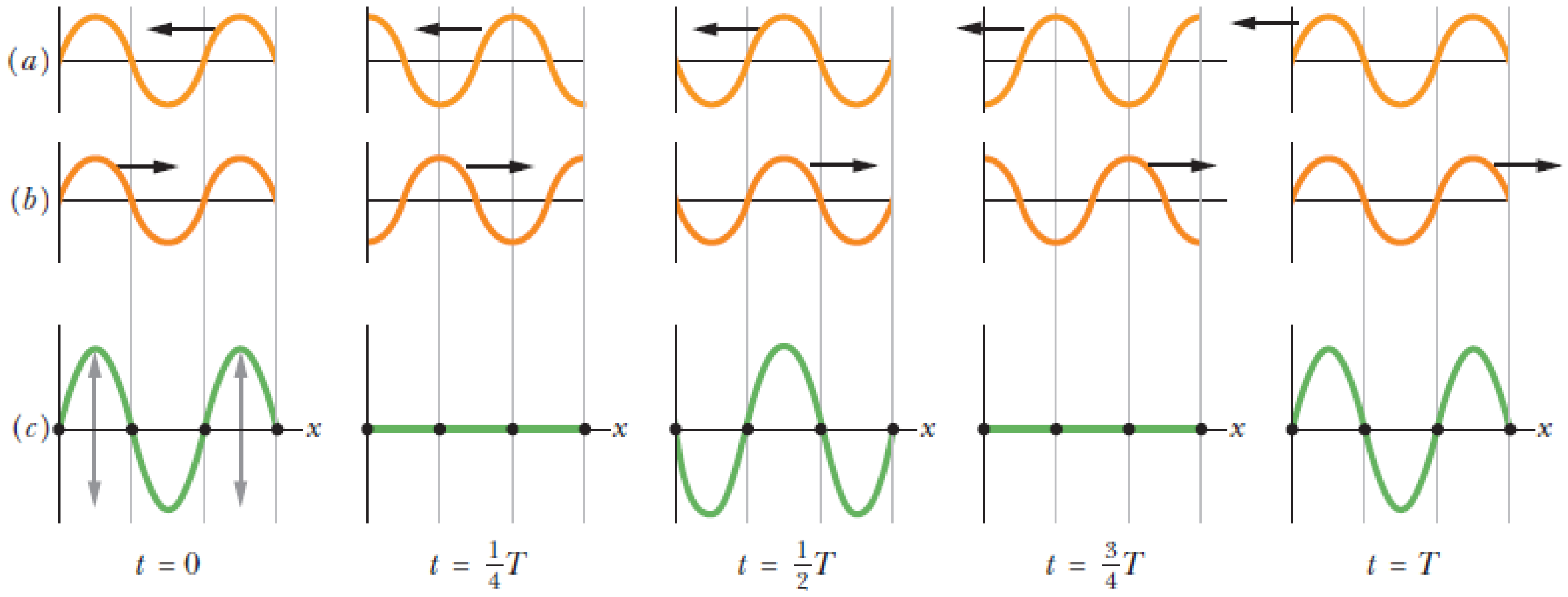
- Consider that the string is of finite length and the other end is clamped to a rigid support. When the wave disturbances reach the fixed end, they will propagate in the opposite direction. The reflected waves will add to the incident waves according to the superposition principle and, under certain conditions, a standing wave pattern will be formed.
- As the waves move through each other some points never move and some move the most.





Two speakers emit sound waves toward each other. When they overlap, identical waves traveling in opposite directions will combine to form standing waves.

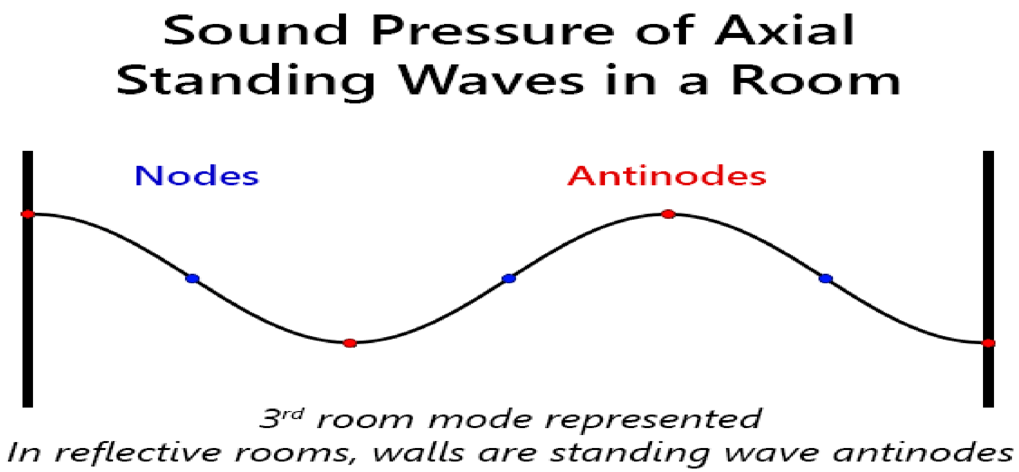
Pictorial View of Standing Wave



(*a*) Five snapshots of a wave traveling to the left, at the times *t* indicated below part (*c*) (*T* is the period of oscillation). (*b*) Five snapshots of a wave identical to that in (*a*) but traveling to the right, at the same times *t*. (*c*) Corresponding snapshots for the superposition of the two waves on the same string. At $t = 0, \frac{1}{2}T$, and T, fully constructive interference occurs because of the alignment of peaks with peaks and valleys with valleys. At $t = \frac{1}{4}T$ and $\frac{3}{4}T$, fully destructive interference occurs because of the alignment of peaks with valleys. Some points (the nodes, marked with dots) never oscillate; some points (the antinodes) oscillate the most.

Nodes & Anti-Nodes

- If two sinusoidal waves of the same amplitude and wavelength travel in *opposite* directions along a stretched string, their interference with each other produces a standing wave.
- Another interesting feature of this is its Nodes and anti-Nodes.
- The **moving part** on string is known as **Anti-Node.**
- The **stationary part** on string is known as **Node**.



Standing Wave(Mathematical Analysis)

To analyze a standing wave, we represent the two combining waves with the equations

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

and

$$y_2(x, t) = y_m \sin(kx + \omega t).$$

The principle of superposition gives, for the combined wave,

$$y'(x, t) = y_1(x, t) + y_2(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t).$$

Applying the trigonometric relation

$$y'(x, t) = [2y_m \sin kx] \cos \omega t.$$

The resultant wave is a standing wave
and is due to the interference of two
sinusoidal waves of the same amplitude
and wavelength that travel
in opposite directions.

Equations for Nodes & Anti-Nodes

Sin(kx) appears zero at kx = nπ $where\ n = 0,1,2,3 \ldots n$

Substituting $k = 2\pi/\lambda$ in this equation and rearranging, we get

$$x = n\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(nodes)},$$

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \sin kx]}_{\substack{\text{Magnitude}\\ \text{gives}\\ \text{amplitude}\\ \text{at position } x}} \underbrace{\cos \omega t}_{\substack{\text{Oscillating}\\ \text{term}}}$$

Sin(kx) appears 1 at kx = n+(½)π $where\ n = 0,1,2,3 \ldots n$

Substituting $k = 2\pi/\lambda$ in $kx = (n + \frac{1}{2})\pi$, and rearranging, we get

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(antinodes)},$$

The distance between adjacent antinodes is equal to $\lambda/2$.

The distance between adjacent nodes is equal to $\lambda/2$.

The distance between a node and an adjacent antinode is $\lambda/4$.

Example Problem 1(includes part A,B)

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = (4.0\ \text{cm})\ \sin(3.0x - 2.0t)$$
$$y_2 = (4.0\ \text{cm})\ \sin(3.0x + 2.0t)$$

where x and y are measured in centimeters.

(A) Find the amplitude of the simple harmonic motion of the element of the medium located at $x = 2.3$ cm.

Solution The standing wave is described by standing wave equation in this problem, we have $A = 4.0$ cm, $k = 3.0$ rad/cm, and $\omega = 2.0$ rad/s. Thus,

$$y = (2A \sin kx) \cos \omega t = [(8.0\ \text{cm}) \sin 3.0x] \cos 2.0t$$

Thus, we obtain the amplitude of the simple harmonic motion of the element at the position $x = 2.3$ cm by evaluating the coefficient of the cosine function at this position:

$$y_{\text{max}} = (8.0\ \text{cm}) \sin 3.0x|_{x=2.3}$$

$$= (8.0\ \text{cm})\ \sin\ (6.9\ \text{rad}) = \boxed{4.6\ \text{cm}}$$

Example Problem 1(cont'd)

(B) Find the positions of the nodes and antinodes if one end of the string is at $x = 0$.

Solution With $k = 2\pi/\lambda = 3.0$ rad/cm, we see that the wavelength is $\lambda = (2\pi/3.0)$ cm. Therefore, from Equation 18.4 we find that the nodes are located at

$$x = n\frac{\lambda}{2} = n\left(\frac{\pi}{3}\right) \text{ cm} \qquad n = \boxed{0, 1, 2, 3, \ldots}$$

and from standing wave eq. we find that the antinodes are located at

$$x = n\frac{\lambda}{4} = n\left(\frac{\pi}{6}\right) \text{ cm} \qquad n = \boxed{1, 3, 5, \ldots}$$

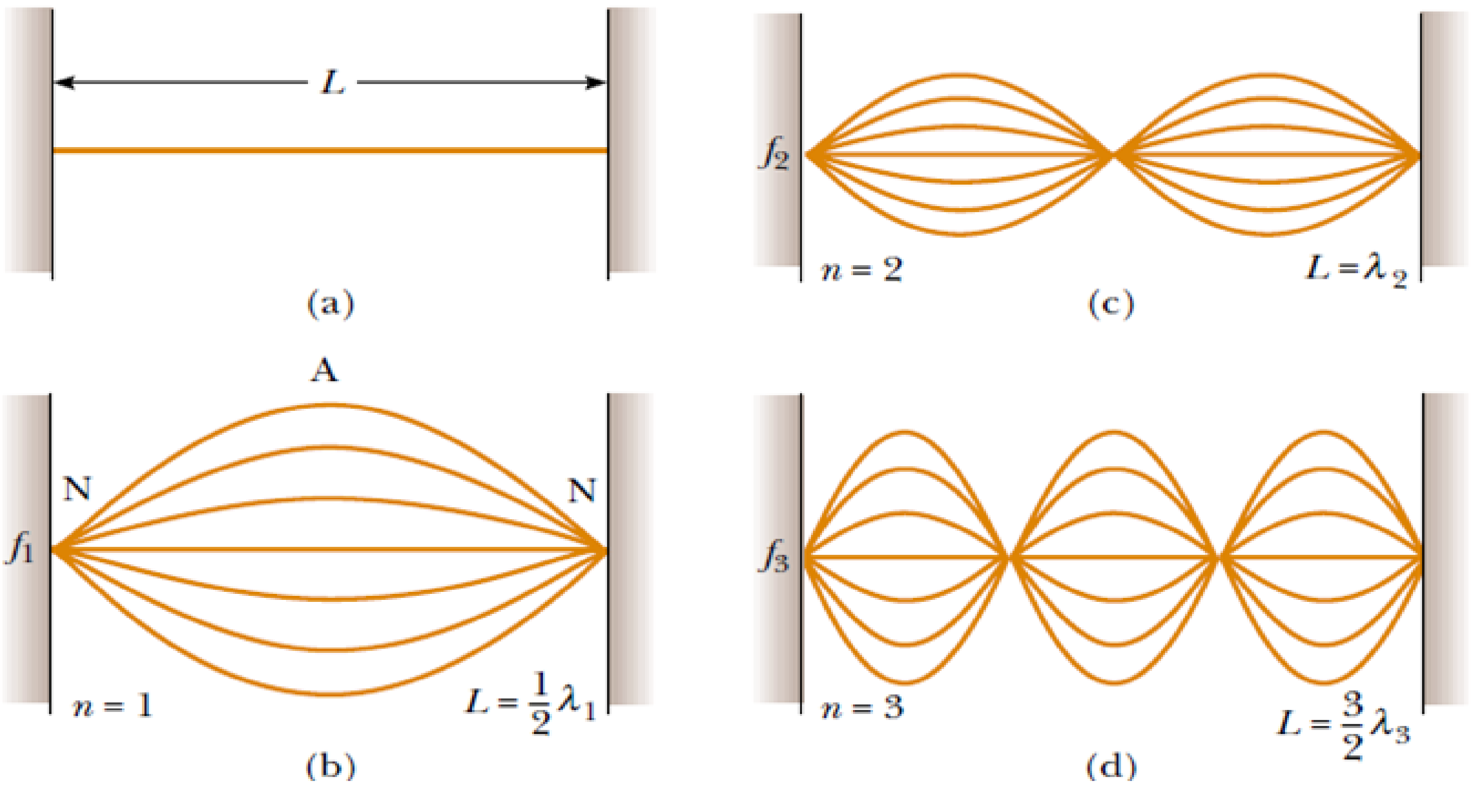
Standing Waves in a String Fixed at Both Ends

- We have two conditions on which standing waves are observed,

1. Boundary Condition
2. With out Boundary

- We cover the boundary condition in this section of study, or standing wave in a string fixed at both ends.

Fixed ends standing wave

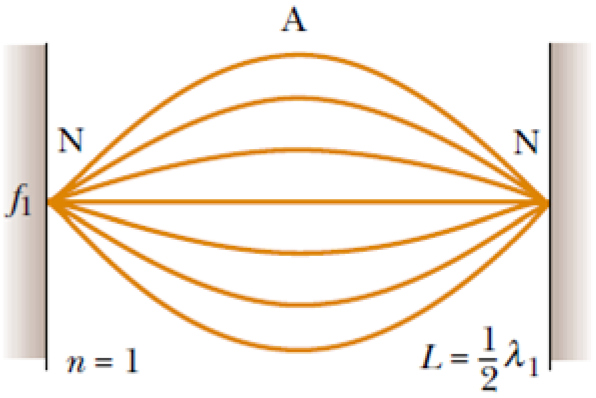


(a) A string of length L fixed at both ends. The normal modes of vibration form a harmonic series: (b) the fundamental, or first harmonic; (c) the second harmonic; (d) the third harmonic.

Harmonics

- The boundary condition results in the string having a number of natural patterns of oscillation, called **normal modes**, each of which has a characteristic frequency that is easily calculated.
- Normal modes are referred to as **harmonics**
- The first normal mode that is consistent with the boundary conditions, shown in Figure(b) has nodes at its ends and one antinode in the middle
- This is the longest-wavelength mode

  that is consistent with our requirements



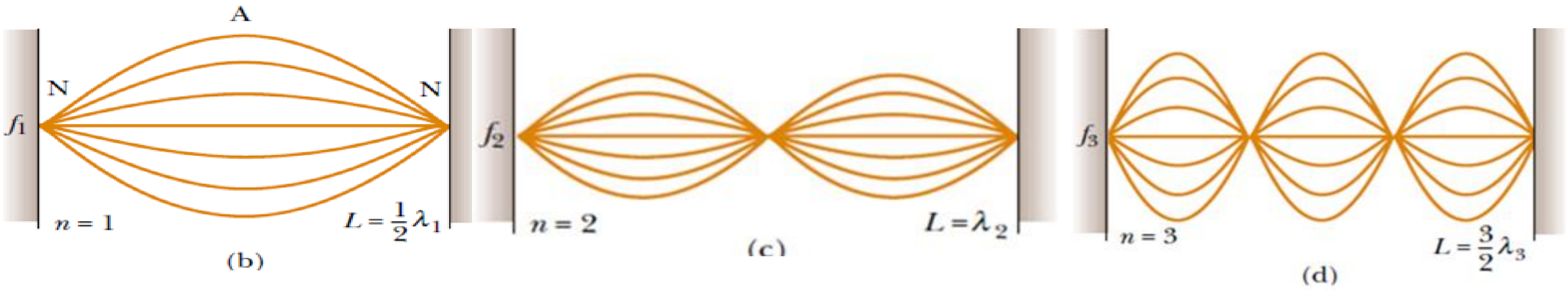
(b)

Harmonics

- The first normal mode occurs when the length of the string is half the wavelength. λ_1, as indicated in Figure or $\lambda_1 = 2L$.
- The next normal mode of wavelength

λ_2 occurs when the wavelength equals the length of the string, that is, when $\lambda_2 = L$.

- The third normal mode corresponds to the case in which $\lambda_3 = 2L/3$.



Harmonics: Natural Frequency (fn)

- From normal modes we get a general equation i.e,

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, \ldots$$

- These are the *possible* modes of oscillation for the string.
- The natural frequencies associated with these modes are obtained from the relationship $f = v/\lambda$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} \qquad n = 1, 2, 3, \ldots$$

- These natural frequencies are also called the *quantized frequencies associated with the vibrating* string fixed at both ends.

Harmonics: Fundamental Frequency

$v = \sqrt{T/\mu}$, where T is the tension in the string and μ is its linear mass density, we can also express the natural frequencies of a taut string as

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad n = 1, 2, 3, \ldots$$

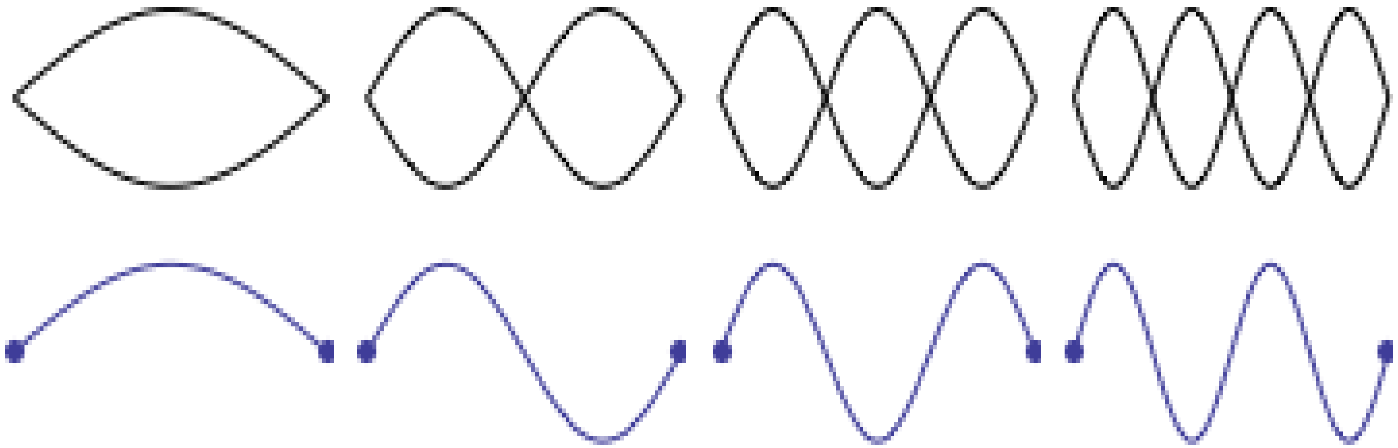
The lowest frequency f_1, which corresponds to $n = 1$, is called either the **fundamental** or the **fundamental frequency** and is given by

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

The frequencies of the remaining normal modes are integer multiples of the fundamental frequency. Frequencies of normal modes that exhibit an integer-multiple relationship such as this form a **harmonic series,** and the normal modes are called **harmonics.**

Harmonics

harmonics. The fundamental frequency f_1 is the frequency of the first harmonic; the frequency $f_2 = 2f_1$ is the frequency of the second harmonic; and the frequency $f_n = nf_1$ is the frequency of the nth harmonic.



Example Problem 2

Middle C on a piano has a fundamental frequency of 262 Hz, and the first A above middle C has a fundamental frequency of 440 Hz.

(A) Calculate the frequencies of the next two harmonics of the C string.

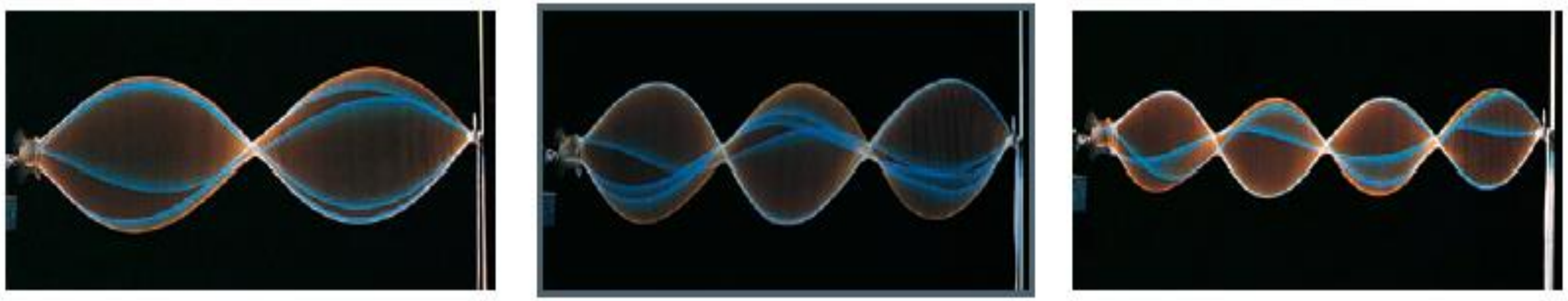
Solution Knowing that the frequencies of higher harmonics are integer multiples of the fundamental frequency $f_1 = 262$ Hz, we find that

$$f_2 = 2f_1 = \boxed{524\ \text{Hz}}$$

$$f_3 = 3f_1 = \boxed{786\ \text{Hz}}$$

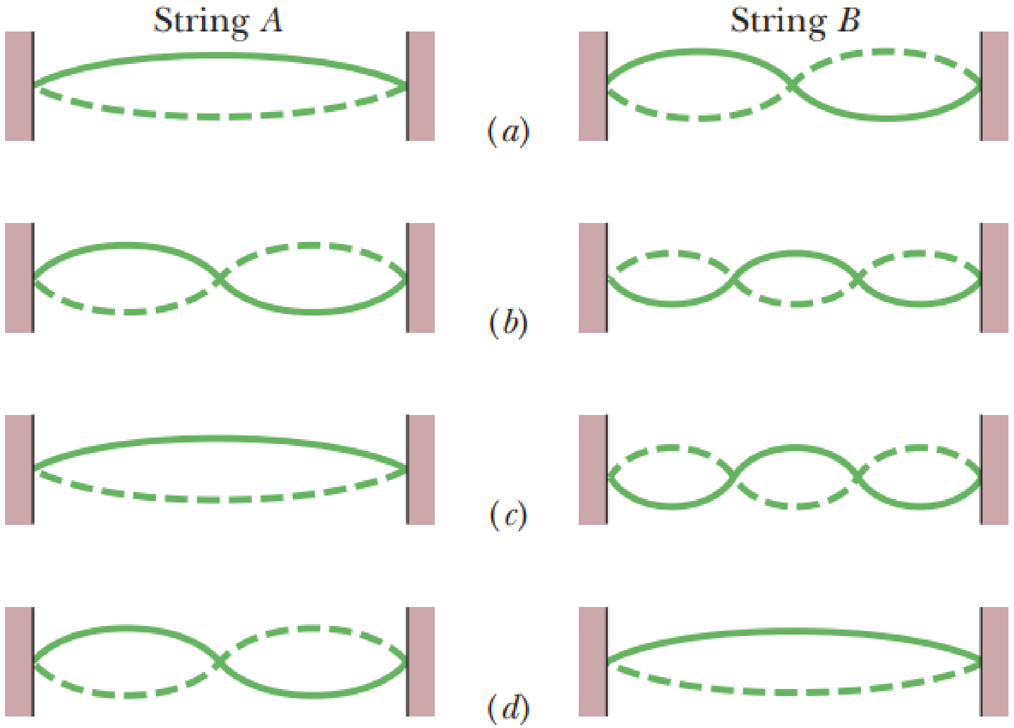
Resonance & Standing Waves

- An oscillating system is in resonance with some driving force whenever the frequency of the driving force matches one of the natural frequencies of the system.
- When the system is resonating, it responds by oscillating with a relatively large amplitude.
- For certain frequencies, the interference produces a standing wave pattern (or **oscillation mode**) with nodes and large antinodes like those in Fig. shown
- Such a standing wave is said to be produced at **resonance,** and the string is said to *resonate* at these certain frequencies, called **resonant frequencies.**



Stroboscopic photographs reveal (imperfect) standing wave patterns on a string being made to oscillate by an oscillator at the left end. The patterns occur at certain frequencies of oscillation.

9 Strings A and B have identical lengths and linear densities, but string B is under greater tension than string A. Figure 16-27 shows four situations, (a) through (d), in which standing wave patterns exist on the two strings. In which situations is there the possibility that strings A and B are oscillating at the same resonant frequency?



•1 If a wave $y(x, t) = (6.0\text{ mm})\sin(kx + (600\text{ rad/s})t + \phi)$ travels along a string, how much time does any given point on the string take to move between displacements $y = +2.0$ mm and $y = -2.0$ mm?

••6 GO A sinusoidal wave travels along a string under tension. Figure 16-31 gives the slopes along the string at time $t = 0$. The scale of the x axis is set by $x_s = 0.80$ m. What is the amplitude of the wave?

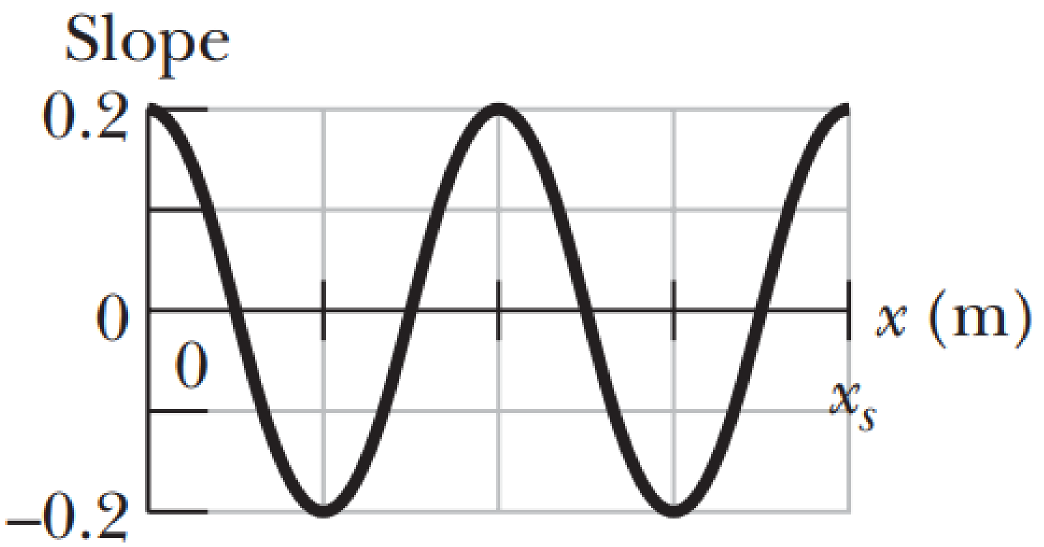


Figure 16-31 Problem 6.

Answer: The $k = 15.7$ rad/m

$A = y_m = 1.3$ cm

••7 A transverse sinusoidal wave is moving along a string in the positive direction of an x axis with a speed of 80 m/s. At $t = 0$, the string particle at $x = 0$ has a transverse displacement of 4.0 cm from its equilibrium position and is not moving. The maximum transverse speed of the string particle at $x = 0$ is 16 m/s. (a) What is the frequency of the wave? (b) What is the wavelength of the wave? If $y(x, t) = y_m \sin(kx \pm \omega t + \phi)$ is the form of the wave equation, what are (c) y_m, (d) k, (e) ω, (f) ϕ, and (g) the correct choice of sign in front of ω?

••22 A sinusoidal wave is traveling on a string with speed 40 cm/s. The displacement of the particles of the string at $x = 10$ cm varies with time according to $y = (5.0\ \text{cm})\sin[1.0 - (4.0\ \text{s}^{-1})t]$. The linear density of the string is 4.0 g/cm. What are (a) the frequency and (b) the wavelength of the wave? If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (c) y_m, (d) k, (e) ω, and (f) the correct choice of sign in front of ω? (g) What is the tension in the string?

22. (a) T

$f = \omega/2\pi = 0.64$ Hz.

(b) Since $k(10\ \text{cm}) = 1.0$, the wave number is $k = 0.10$/cm. Consequently, the wavelength is $\lambda = 2\pi/k = 63$ cm.

(c) The amplitude is $y_m = 5.0$ cm.

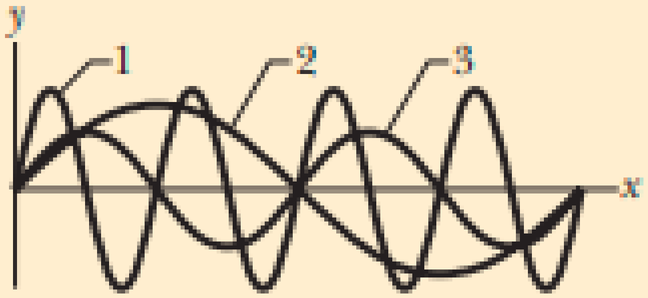
(d) In part (b), we have shown that the angular wave number is $k = 0.10$/cm.

(e) The angular frequency is $\omega = 4.0$ rad/s.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

Checkpoint 1

The figure is a composite of three snapshots, each of a wave traveling along a particular string. The phases for the waves are given by (a) $2x - 4t$, (b) $4x - 8t$, and (c) $8x - 16t$. Which phase corresponds to which wave in the figure?



Checkpoint 2

Here are the equations of three waves:

(1) $y(x,t) = 2\sin(4x - 2t)$, (2) $y(x,t) = \sin(3x - 4t)$, (3) $y(x,t) = 2\sin(3x - 3t)$.

Rank the waves according to their (a) wave speed and (b) maximum speed perpendicular to the wave's direction of travel (the transverse speed), greatest first.

Checkpoint 3

You send a traveling wave along a particular string by oscillating one end. If you increase the frequency of the oscillations, do (a) the speed of the wave and (b) the wavelength of the wave increase, decrease, or remain the same? If, instead, you increase the tension in the string, do (c) the speed of the wave and (d) the wavelength of the wave increase, decrease, or remain the same?

Checkpoint 4

Here are four possible phase differences between two identical waves, expressed in wavelengths: 0.20, 0.45, 0.60, and 0.80. Rank them according to the amplitude of the resultant wave, greatest first.

Checkpoint 5

Two waves with the same amplitude and wavelength interfere in three different situations to produce resultant waves with the following equations:

(1) $y'(x, t) = 4\sin(5x - 4t)$

(2) $y'(x, t) = 4\sin(5x)\cos(4t)$

(3) $y'(x, t) = 4\sin(5x + 4t)$

In which situation are the two combining waves traveling (a) toward positive x, (b) toward negative x, and (c) in opposite directions?

Checkpoint 6

In the following series of resonant frequencies, one frequency (lower than 400 Hz) is missing: 150, 225, 300, 375 Hz. (a) What is the missing frequency? (b) What is the frequency of the seventh harmonic?

Sample Problem 16.01:

Sample Problem 16.01 **Determining the quantities in an equation for a transverse wave**

A transverse wave traveling along an x axis has the form given by

$$y = y_m \sin(kx \pm \omega t + \phi). \qquad (16\text{-}18)$$

Figure 16-8*a* gives the displacements of string elements as a function of x, all at time $t = 0$. Figure 16-8*b* gives the displacements of the element at $x = 0$ as a function of t. Find the values of the quantities shown in Eq. 16-18, including the correct choice of sign.

KEY IDEAS

(1) Figure 16-8*a* is effectively a snapshot of reality (something that we can see), showing us motion spread out over the x axis. From it we can determine the wavelength λ of the wave along that axis, and then we can find the angular wave number k $(= 2\pi/\lambda)$ in Eq. 16-18. (2) Figure 16-8*b* is an abstraction, showing us motion spread out over time. From it we can determine the period T of the string element in its SHM and thus also of the wave itself. From T we can then find angular frequency ω $(= 2\pi/T)$ in Eq. 16-18. (3) The phase constant ϕ is set by the displacement of the string at $x = 0$ and $t = 0$.

Amplitude: From either Fig. 16-8*a* or 16-8*b* we see that the maximum displacement is 3.0 mm. Thus, the wave's amplitude $x_m = 3.0$ mm.

Wavelength: In Fig. 16-8*a*, the wavelength λ is the distance along the x axis between repetitions in the pattern. The easiest way to measure λ is to find the distance from one crossing point to the next crossing point where the string has the same slope. Visually we can roughly measure that distance with the scale on the axis. Instead, we can lay the edge of a

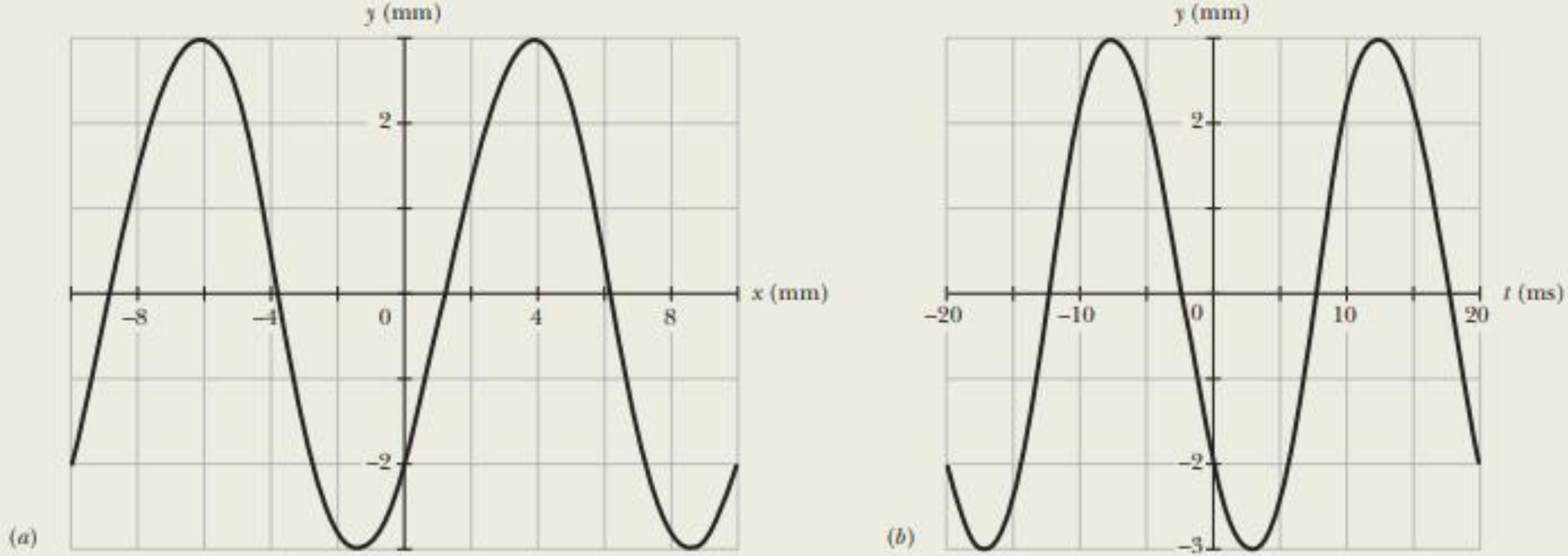


Figure 16-8 (*a*) A snapshot of the displacement y versus position x along a string, at time $t = 0$. (*b*) A graph of displacement y versus time t for the string element at $x = 0$.

paper sheet on the graph, mark those crossing points, slide the sheet to align the left-hand mark with the origin, and then read off the location of the right-hand mark. Either way we find $\lambda = 10$ mm. From Eq. 16-5, we then have

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.010\text{ m}} = 200\pi\text{ rad/m}.$$

Period: The period T is the time interval that a string element's SHM takes to begin repeating itself. In Fig. 16-8*b*, T is the distance along the t axis from one crossing point to the next crossing point where the plot has the same slope. Measuring the distance visually or with the aid of a sheet of paper, we find $T = 20$ ms. From Eq. 16-8, we then have

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.020\text{ s}} = 100\pi\text{ rad/s}.$$

Direction of travel: To find the direction, we apply a bit of reasoning to the figures. In the snapshot at $t = 0$ given in Fig. 16-8*a*, note that if the wave is moving rightward, then just after the snapshot, the depth of the wave at $x = 0$ should increase (mentally slide the curve slightly rightward). If, instead, the wave is moving leftward, then just after the snapshot, the depth at $x = 0$ should decrease. Now let's check the graph in Fig. 16-8*b*. It tells us that just after $t = 0$, the depth increases. Thus, the wave is moving rightward, in the positive direction of x, and we choose the minus sign in Eq. 16-18.

Phase constant: The value of ϕ is set by the conditions at $x = 0$ at the instant $t = 0$. From either figure we see that at that location and time, $y = -2.0$ mm. Substituting these three values and also $y_m = 3.0$ mm into Eq. 16-18 gives us

$$-2.0\text{ mm} = (3.0\text{ mm})\sin(0 + 0 + \phi)$$

or

$$\phi = \sin^{-1}(-\tfrac{2}{3}) = -0.73\text{ rad}.$$

Note that this is consistent with the rule that on a plot of y versus x, a negative phase constant shifts the normal sine function rightward, which is what we see in Fig. 16-8*a*.

Equation: Now we can fill out Eq. 16-18:

$$y = (3.0\text{ mm})\sin(200\pi x - 100\pi t - 0.73\text{ rad}), \quad \text{(Answer)}$$

with x in meters and t in seconds.

Sample Problem 16.02:

Sample Problem 16.02 **Transverse velocity and transverse acceleration of a string element**

A wave traveling along a string is described by

$$y(x, t) = (0.00327 \text{ m}) \sin(72.1x - 2.72t),$$

in which the numerical constants are in SI units (72.1 rad/m and 2.72 rad/s).

(a) What is the transverse velocity u of the string element at $x = 22.5$ cm at time $t = 18.9$ s? (This velocity, which is associated with the transverse oscillation of a string element, is parallel to the y axis. Don't confuse it with v, the constant velocity at which the wave form moves along the x axis.)

KEY IDEAS

The transverse velocity u is the rate at which the displacement y of the element is changing. In general, that displacement is given by

$$y(x, t) = y_m \sin(kx - \omega t). \qquad (16\text{-}19)$$

For an element at a certain location x, we find the rate of change of y by taking the derivative of Eq. 16-19 with respect to t while treating x as a constant. A derivative taken while one (or more) of the variables is treated as a constant is called a partial derivative and is represented by a symbol such as $\partial/\partial t$ rather than d/dt.

Calculations: Here we have

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t). \qquad (16\text{-}20)$$

Next, substituting numerical values but suppressing the units, which are SI, we write

$$u = (-2.72)(0.00327) \cos[(72.1)(0.225) - (2.72)(18.9)]$$
$$= 0.00720 \text{ m/s} = 7.20 \text{ mm/s}. \qquad \text{(Answer)}$$

Thus, at $t = 18.9$ s our string element is moving in the positive direction of y with a speed of 7.20 mm/s. (*Caution:* In evaluating the cosine function, we keep all the significant figures in the argument or the calculation can be off considerably. For example, round off the numbers to two significant figures and then see what you get for u.)

(b) What is the transverse acceleration a_y of our string element at $t = 18.9$ s?

KEY IDEA

The transverse acceleration a_y is the rate at which the element's transverse velocity is changing.

Calculations: From Eq. 16-20, again treating x as a constant but allowing t to vary, we find

$$a_y = \frac{\partial u}{\partial t} = -\omega^2 y_m \sin(kx - \omega t). \qquad (16\text{-}21)$$

Substituting numerical values but suppressing the units, which are SI, we have

$$a_y = -(2.72)^2(0.00327) \sin[(72.1)(0.225) - (2.72)(18.9)]$$
$$= -0.0142 \text{ m/s}^2 = -14.2 \text{ mm/s}^2. \qquad \text{(Answer)}$$

From part (a) we learn that at $t = 18.9$ s our string element is moving in the positive direction of y, and here we learn that it is slowing because its acceleration is in the opposite direction of u.

Sample Problem 16.06:

Figure 16-23 shows resonant oscillation of a string of mass $m = 2.500$ g and length $L = 0.800$ m and that is under tension $\tau = 325.0$ N. What is the wavelength λ of the transverse waves producing the standing wave pattern, and what is the harmonic number n? What is the frequency f of the transverse waves and of the oscillations of the moving string elements? What is the maximum magnitude of the transverse velocity u_m of the element oscillating at coordinate $x = 0.180$ m? At what point during the element's oscillation is the transverse velocity maximum?

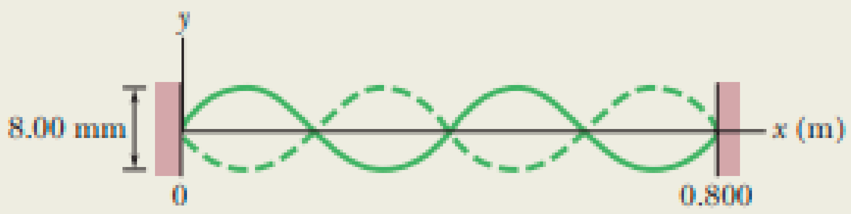


Figure 16-23 Resonant oscillation of a string under tension.

Homework questions:

- Practice problems:
- End of chapter 16 textbook " Fundamentals of Physics" by Halliday & Resnick Jearl Walker 10th Edition"
- Questions:

Go through all the questions

- Problems:
- **3,5,9,13,23,26,31,33,43,49**

Answer of even problems:

26: $f = 198$ Hz

Adapted from:

Book:

Fundamentals of Physics by Halliday Resnick & Jearl Walker 10^{th} edition