Electrostatics and Coulomb's Law

Chapter 21, 22 and 23

Fundamental Forces of Nature

- Gravitational Force
  - Weakest force; but infinite range.
- Weak Nuclear Force
  - Next weakest; but short range.
- Electromagnetic Force
  - Stronger, with infinite range.
- Strong Nuclear Force
  - Strongest; but short range.

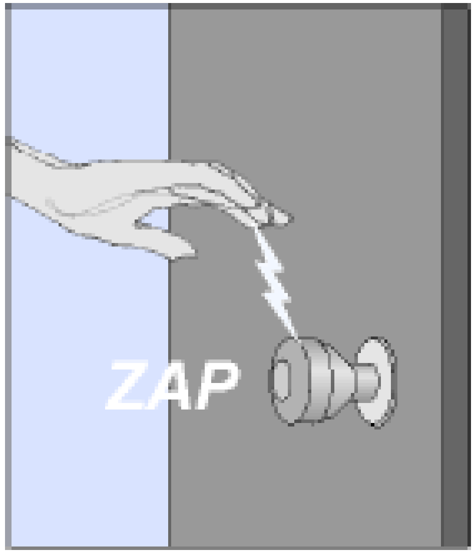
The Electromagnetic Force between charged particles is one of the fundamental forces of nature.

Electrostatics

- Electrostatics is the physics that deals with the interactions of static (non-moving) charges.

Electric Charge

- Objects can lose or gain electric charges.
- The net charge is also sometimes called excess charge because a charged object has an excess of either positive or negative charges.
- A tiny imbalance in either positive or negative charge on an object is the cause of static electricity.



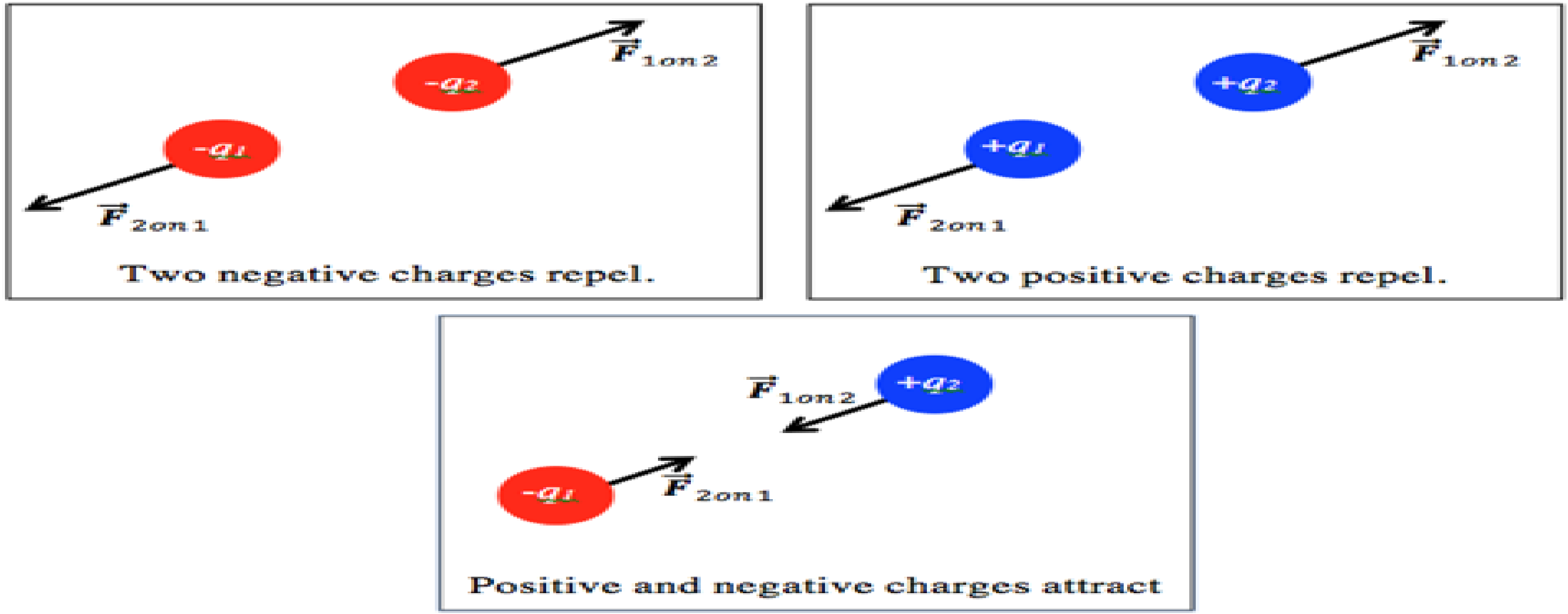
Static electricity

- Electric charge is a property of tiny particles in atoms.
- The unit of electric charge is the coulomb (C).
- A quantity of charge should always be identified with a positive or a negative sign.

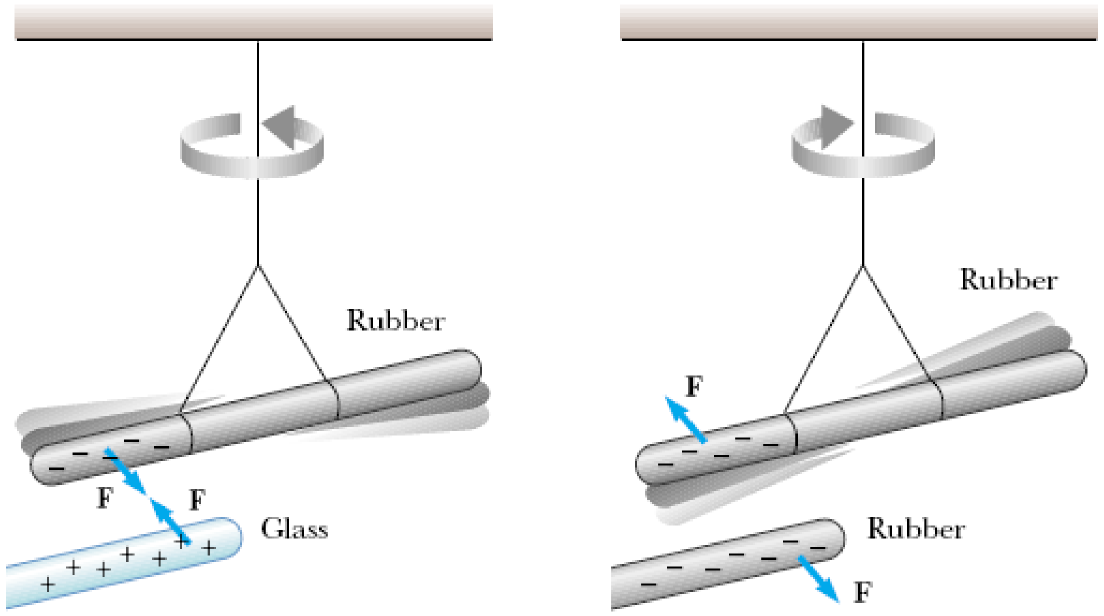
	Mass (kg)	Charge (coulombs)
Electron	9.109×10^{-31}	-1.602×10^{-19}
Proton	1.673×10^{-27}	$+1.602 \times 10^{-19}$
Neutron	1.675×10^{-27}	0

Properties of Electric Charges

- It was found that there are two kinds of electric charges, which were given the names positive and negative by Benjamin Franklin(1706–1790)
- We identify negative charge as that type possessed by **electrons** and positive charge as that possessed by **protons**
- Charges of the same sign repel one another and,
- Charges with opposite signs attract one another.



- Using the convention suggested by **Franklin**, the electric charge **on the glass** rod is called **positive** and that on the **rubber rod** is called **negative**.
- Therefore, any charged object **attracted** to a charged **rubber rod** must have a **positive charge**, and any charged object **repelled** by a charged **rubber rod** must have a **negative** charge.



Electric Charge is Conserved

- Another important aspect of electricity that arises from experimental observations is that **electric charge is always conserved in an isolated system**.
- That is, when one object is rubbed against another, **charge is not created in the process(electrified state)**.
- The **electrified state** is due to a ***transfer of charge from one object to the other.***

  One object gains some amount of negative charge while the other gains an equal amount of positive charge.

Quantized Electric Charge

- In 1909, Robert Millikan (1868–1953) discovered that electric charge always occurs as some integral multiple of a fundamental amount of charge *e*

- In modern terms, the electric charge *q is said to be* ***quantized***

- where *q is the* standard symbol used for charge as a variable. That is, electric charge exists as discrete "packets," and we can write ***q = Ne, where N is some integer***.

- Magnitude of electron and proton is the same but signs are different.

- Neutrons have no charge

Summarized

Properties of Electric Charges:

- There are two kinds of charges in nature; charges of opposite sign attract one another and charges of the same sign repel one another.
- Total charge in an isolated system is conserved.
- Charge is quantized.

Brainstorming

Question:

If you rub an inflated balloon against your hair, the two materials attract each other,

Is the amount of charge present in the system of the balloon and your hair after rubbing

(a) **less than** amount of charge present before rubbing?

(b) **the same** as amount of charge present before rubbing?

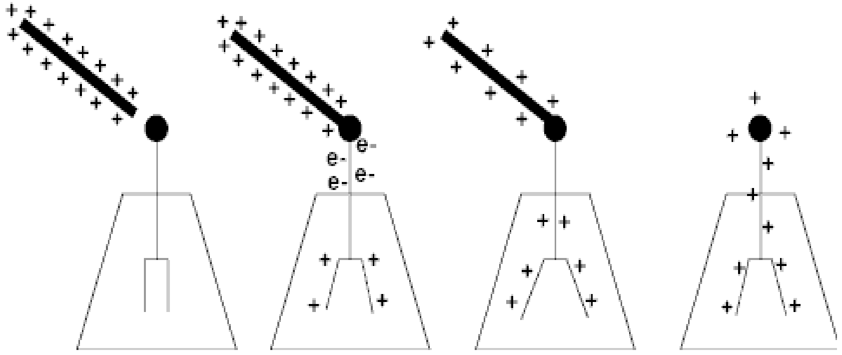
(c) **more than** the amount of charge present before rubbing?

- **Ans: The amount of charge present in the isolated system after rubbing <u>is the same</u> as that before because charge is conserved; it is just distributed differently.**

Methods of Charging

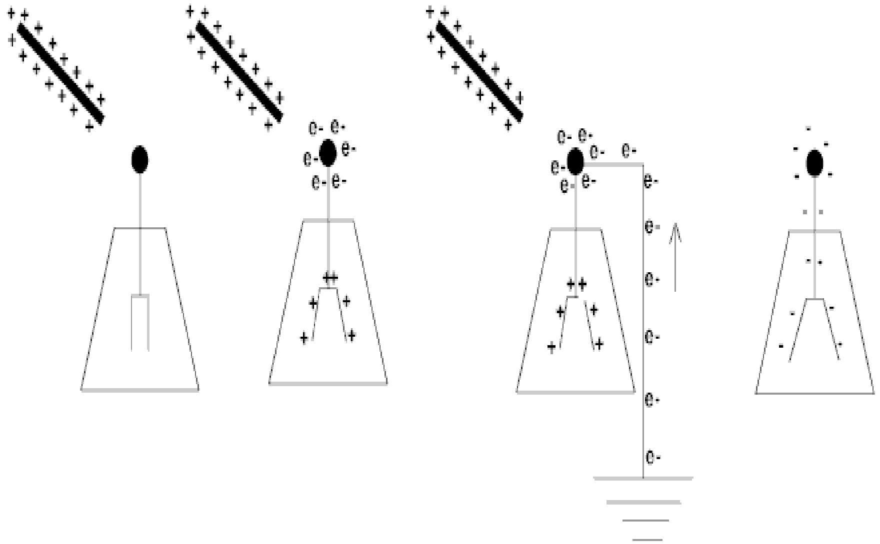
By conduction

- Charges are transferred from one body to another by physical contact.



By induction

- Charges are transferred from one body to another only when charged object comes closer to other object.



Methods of Charging (continued)

By Friction:

- The charging by friction process involves rubbing of one particle on another resulting in electrons moving from one surface to another.

Electrical Conductors & Insulators

- **Electrical conductors** are materials in which some of the electrons are free electrons that are not bound to atoms and can move relatively freely through the material.
- Examples: Materials such as copper, aluminum, and silver are good electrical conductors
- **Electrical Insulators** are materials in which all electrons are bound to atoms and cannot move freely through the material.
- Examples: Materials such as glass, rubber, and wood fall into the category of electrical insulators

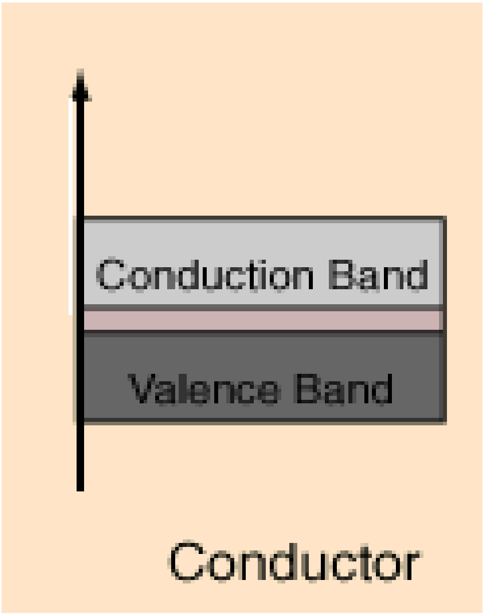
Semi conductors & Super Conductors

- **Semiconductors** are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors.
- Examples: Silicon and germanium are well-known examples commonly used in the fabrication of a variety of electronic chips used in computers, cellular telephones, and stereo systems.

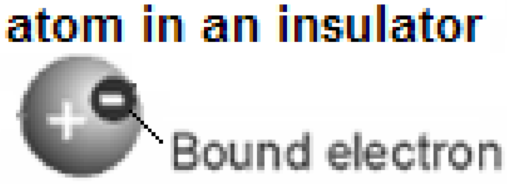
- **Superconductors** are materials that are ***perfect conductors, allowing*** charge to move without *any hindrance.*

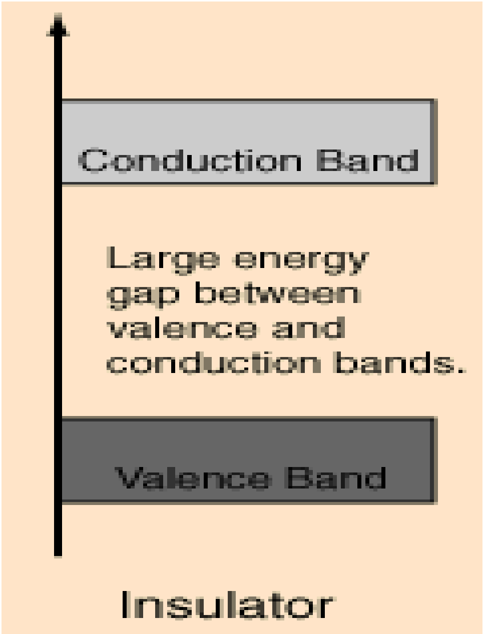
Conductors and insulators

- All materials contain electrons.
- The electrons are what carry the current in a conductor.

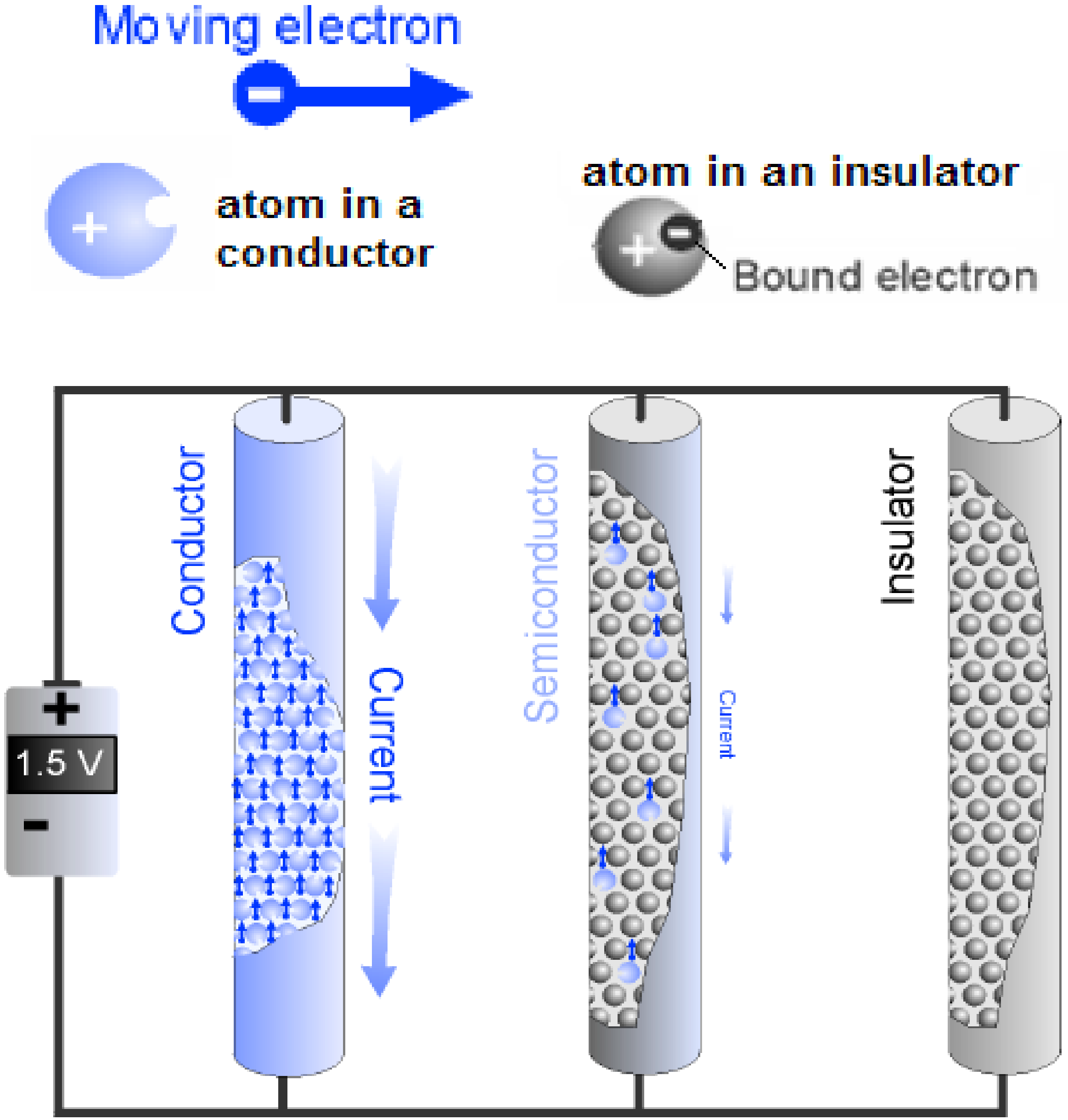


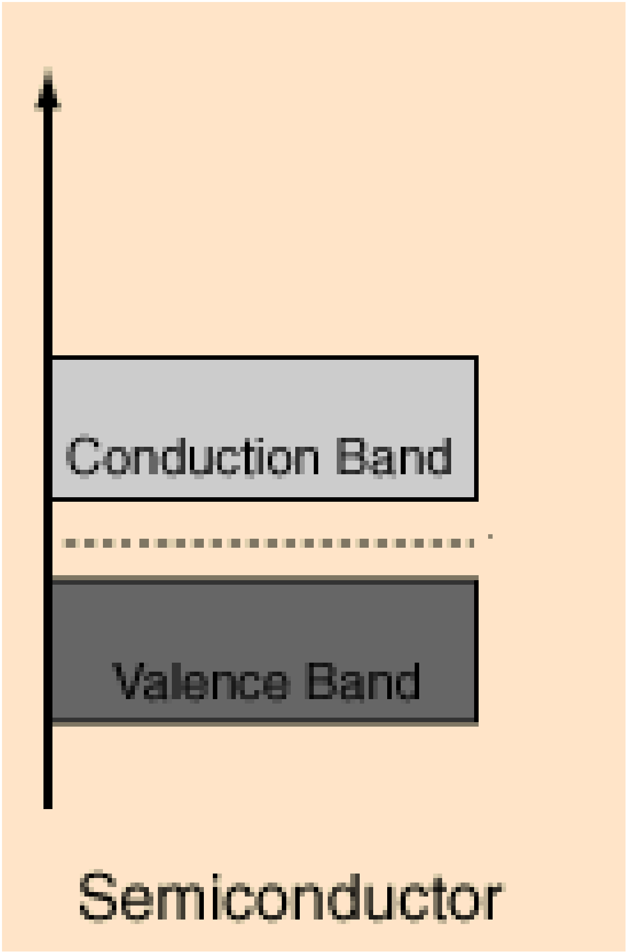
- The electrons in insulators are <u>not</u> free to move—they are tightly bound inside atoms.





- A semiconductor has a few free electrons and atoms with bound electrons that act as insulators.





Coulomb's Law

Coulomb's law relates the force between two single charges separated by a distance.

Force (F) depends on charge (q)

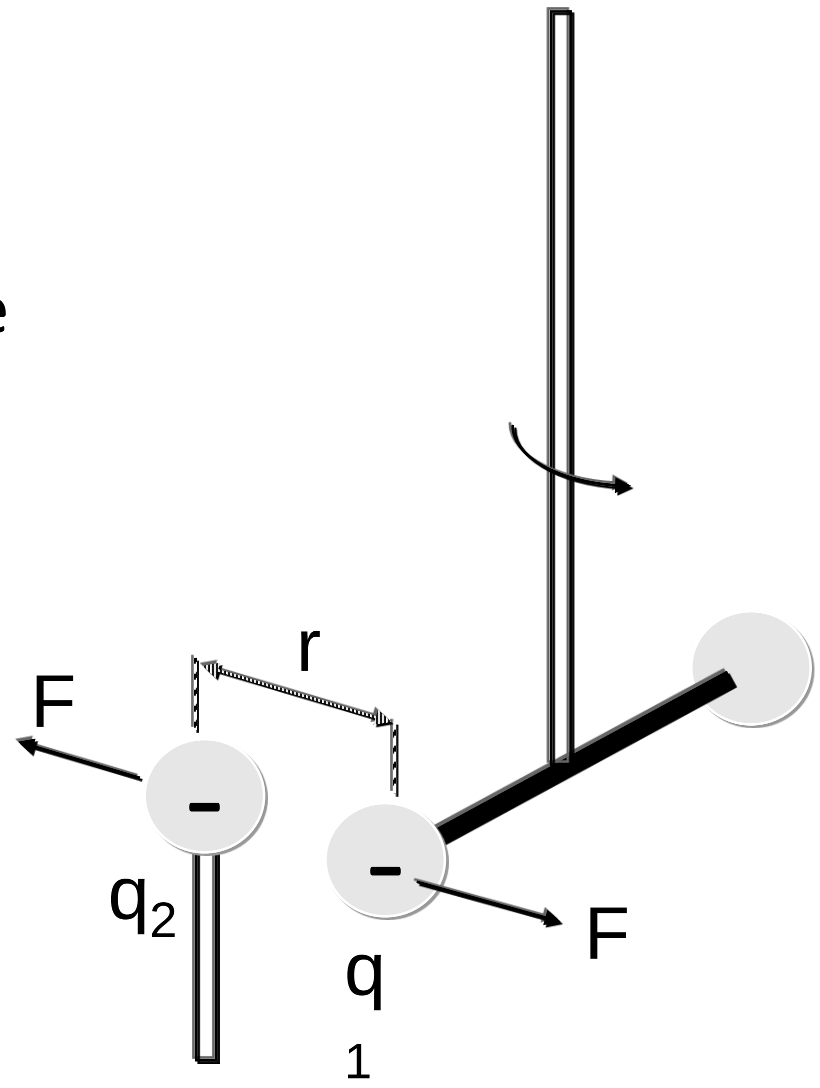
$F \propto q_1$; $F \propto q_2$

Force depends on the inverse square of the distance (r) between the charges

$F \propto 1/r^2$

$F \propto (q_1\, q_2)/r^2$

$$\boxed{F = K\frac{q_1 q_2}{r^2}}$$



Coulomb's Law

From Coulomb's experiments, we can generalize the following properties of the **electric force** between two stationary charged particles. The electric force

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign;
- is a conservative force.

Coulomb's law as an equation $F_e = k_e \frac{|q_1||q_2|}{r^2}$

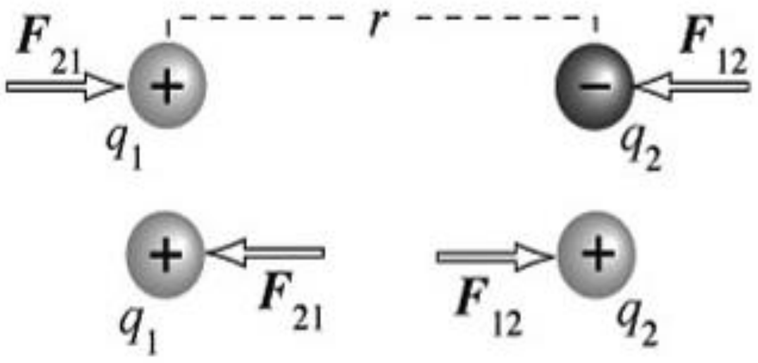
$$k_e = \frac{1}{4\pi\epsilon_0}$$

Coulomb constant $k_e = 8.9875 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$

permittivity of free space $\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$

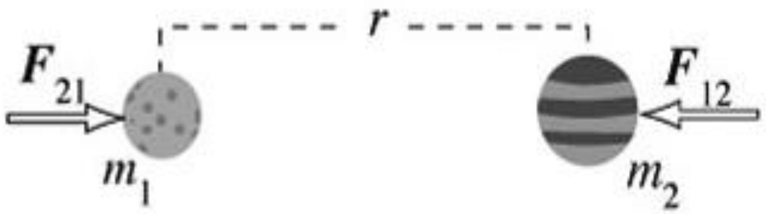
Figure 1 Coulomb's torsion balance, used to establish the inverse-square law for the electric force between two charges.

Electrostatic Force vs. Gravitational Force



$$F = k\frac{q_1 q_2}{r^2}$$

Electrostatic Force



$$F = G\frac{m_1 m_2}{r^2}$$

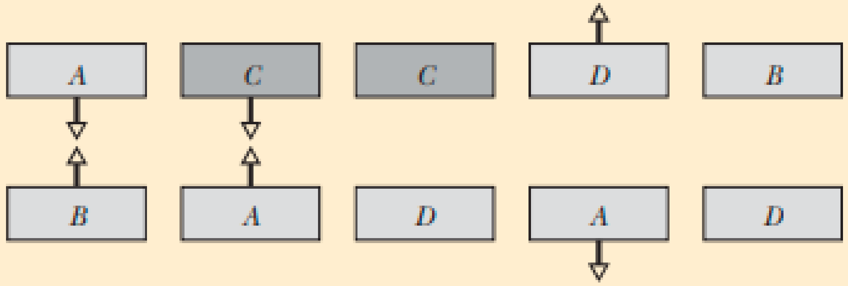
Gravitational Force

F = electrostatic force
q = electric charge
r = distance between centers of charge
k = Coulomb constant
$9.0 \times 10^9\ N \cdot m^2/C^2$

F = gravitational force
m = mass
r = distance between centers of mass
G = gravitational constant
$6.7 \times 10^{-11}\ N \cdot m^2/kg^2$

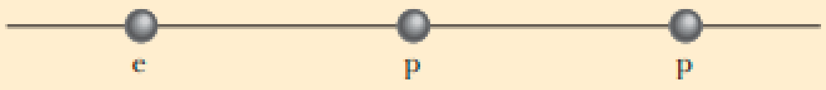
Checkpoint 1

The figure shows five pairs of plates: A, B, and D are charged plastic plates and C is an electrically neutral copper plate. The electrostatic forces between the pairs of plates are shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other?



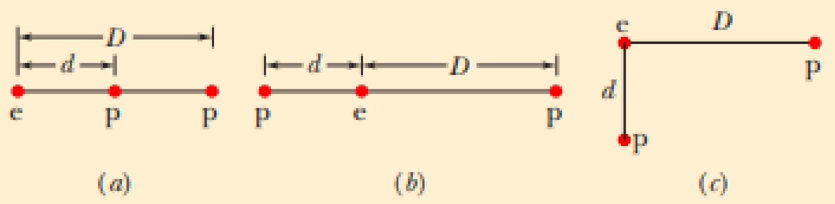
Checkpoint 2

The figure shows two protons (symbol p) and one electron (symbol e) on an axis. On the central proton, what is the direction of (a) the force due to the electron, (b) the force due to the other proton, and (c) the net force?



Checkpoint 3

The figure here shows three arrangements of an electron e and two protons p. (a) Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first. (b) In situation c, is the angle between the net force on the electron and the line labeled d less than or more than 45°?



Checkpoint 4

Initially, sphere A has a charge of $-50e$ and sphere B has a charge of $+20e$. The spheres are made of conducting material and are identical in size. If the spheres then touch, what is the resulting charge on sphere A?

Example 1:

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the magnitude of the electric force is

$$F_e = k_e \frac{|e||-e|}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(1.60 \times 10^{-19}\ \mathrm{C})^2}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\ \mathrm{N}}$$

Using Newton's law of universal gravitation and for the particle masses, we find that the magnitude of the gravitational force is $F_g = G\dfrac{m_e m_p}{r^2}$

$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}) \times \frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.67 \times 10^{-27}\ \mathrm{kg})}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\ \mathrm{N}}$$

The ratio $F_e/F_g \approx 2 \times 10^{39}$.

Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force.

Home Work

Q.. Two protons in an atomic nucleus are typically separated by a distance of 2×10^{-15} m. The electric repulsion force between the protons is huge, but the attractive nuclear force is even stronger and keeps the nucleus from bursting apart. What is the magnitude of the electric force between two protons separated by 2.00×10^{-15} m?

Coulomb's Law in Vector Form

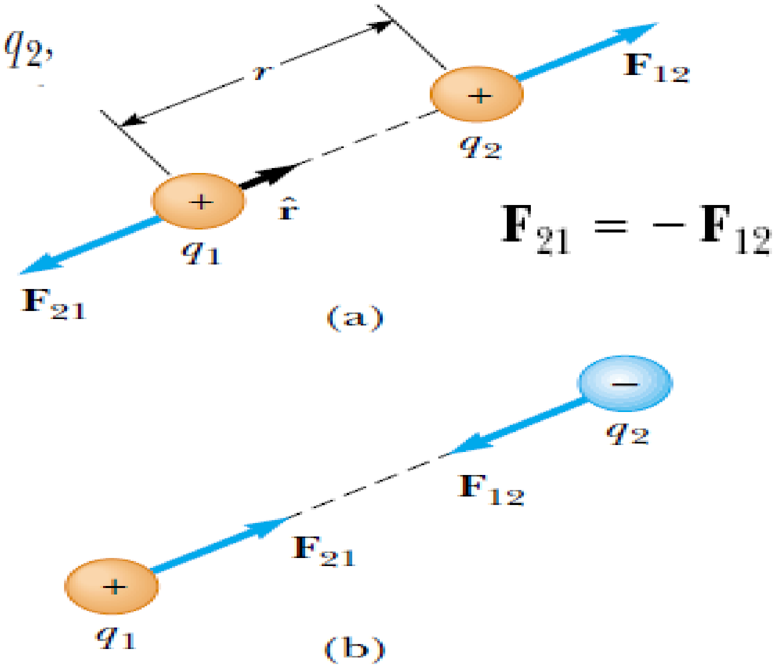
When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. The law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written $\mathbf{F}_{12}$, is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

Vector form of Coulomb's law

where $\hat{\mathbf{r}}$ is a unit vector directed from q_1 toward q_2,

- If charges of same sign the product is positive.
- If charges of different signs the product is negative.
- If product is positive charges repel.
- If product is negative charges attract.



Calculate the Force

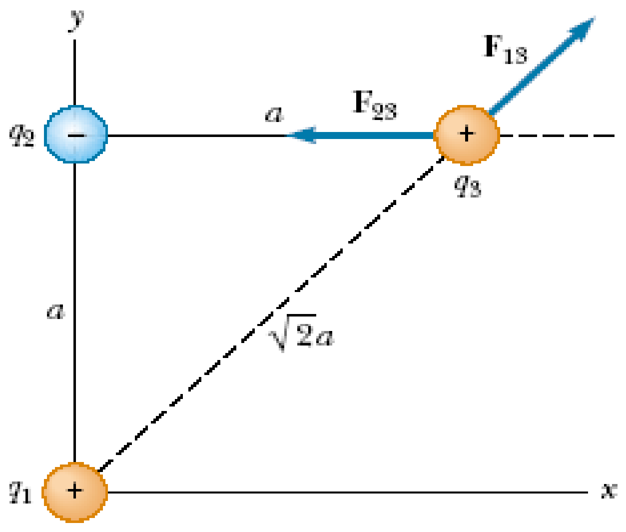
Consider three point charges located at the corners of a right triangle as shown in Figure. Find the resultant force exerted on *q3* .

$q_1 = q_3 = 5.0\,\mu\text{C}, \quad q_2 = -2.0\,\mu\text{C}$, and $a = 0.10$ m.

$$F_{3x} = F_{13x} + F_{23} = 7.9\text{ N} - 9.0\text{ N} = -1.1\text{ N}$$

$$F_{3y} = F_{13y} = 7.9\text{ N}$$

$$\mathbf{F}_3 = \boxed{(-1.1\mathbf{i} + 7.9\mathbf{j})\text{ N}}$$



Find the magnitude and direction of the resultant force $\mathbf{F}_3$.

8.0 N at an angle of 98° with the x axis.

Example: Find the resultant force

Consider three point charges located at the corners of a right triangle as shown in Figure where $q_1 = q_3 = 5.0\ \mu C$, $q_2 = -2.0\ \mu C$, and $a = 0.10$ m. Find the resultant force exerted on q_3.

Solution First, note the direction of the individual forces exerted by q_1 and q_2 on q_3. The force $\mathbf{F}_{23}$ exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs. The force $\mathbf{F}_{13}$ exerted by q_1 on q_3 is repulsive because both charges are positive.

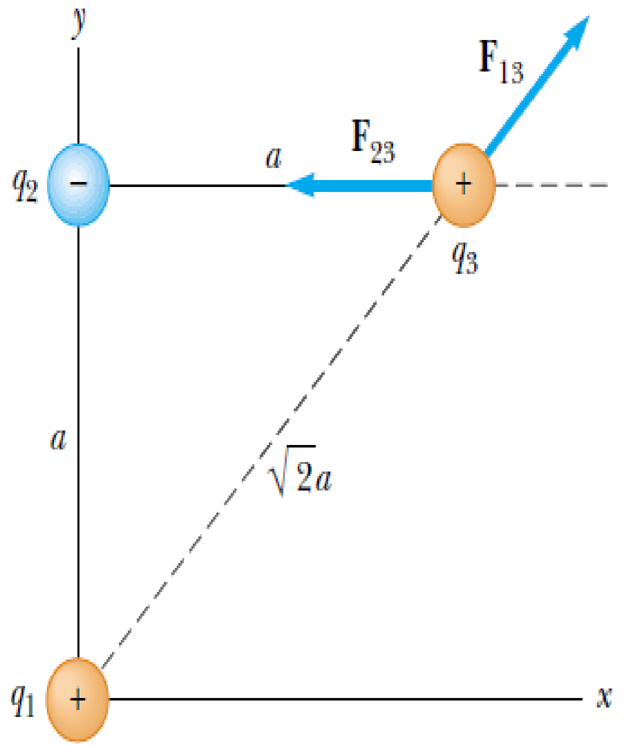
The magnitude of $\mathbf{F}_{23}$ is

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(2.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{(0.10\ \mathrm{m})^2}$$

$$= 9.0\ \mathrm{N}$$

In the coordinate system shown in Figure the attractive force $\mathbf{F}_{23}$ is to the left (in the negative x direction).



The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$. The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$. The resultant force $\mathbf{F}_3$ exerted on q_3 is the vector sum $\mathbf{F}_{13} + \mathbf{F}_{23}$.

The magnitude of the force $\mathbf{F}_{13}$ exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(5.0 \times 10^{-6}\text{ C})(5.0 \times 10^{-6}\text{ C})}{2(0.10\text{ m})^2}$$

$$= 11 \text{ N}$$

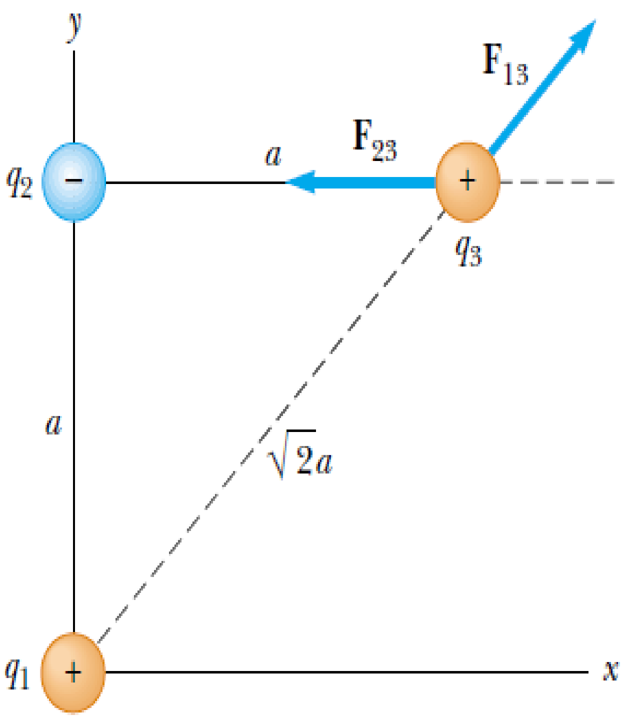
The repulsive force $\mathbf{F}_{13}$ makes an angle of 45° with the x axis. Therefore, the x and y components of $\mathbf{F}_{13}$ are equal, with magnitude given by $F_{13}\cos 45° = 7.9$ N.

Combining $\mathbf{F}_{13}$ with $\mathbf{F}_{23}$ by the rules of vector addition, we arrive at the x and y components of the resultant force acting on q_3:

$$F_{3x} = F_{13x} + F_{23x} = 7.9 \text{ N} + (-9.0 \text{ N}) = -1.1 \text{ N}$$

$$F_{3y} = F_{13y} + F_{23y} = 7.9 \text{ N} + 0 = 7.9 \text{ N}$$

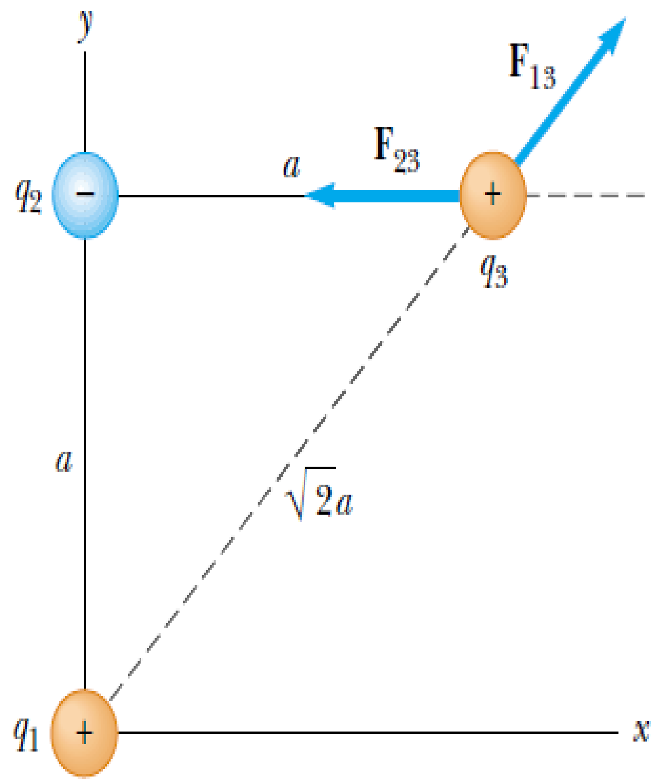
We can also express the resultant force acting on q_3 in unit-vector form as $\mathbf{F}_3 = (-1.1\hat{\mathbf{i}} + 7.9\hat{\mathbf{j}})$ N



The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$. The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$. The resultant force $\mathbf{F}_3$ exerted on q_3 is the vector sum $\mathbf{F}_{13} + \mathbf{F}_{23}$.

What If? What if the signs of all three charges were changed to the opposite signs? How would this affect the result for F_3?

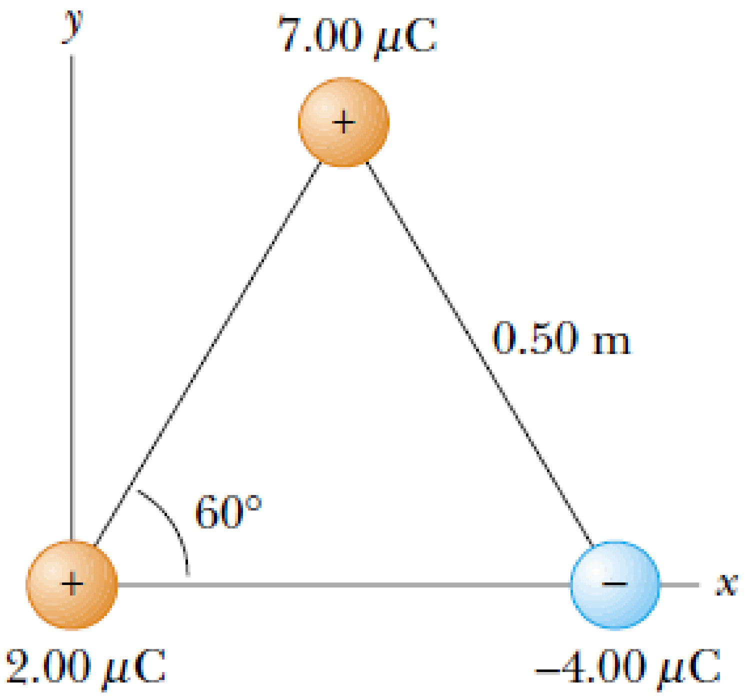
Answer The charge q_3 would still be attracted toward q_2 and repelled from q_1 with forces of the same magnitude. Thus, the final result for $\mathbf{F}_3$ would be exactly the same.



The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$. The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$. The resultant force $\mathbf{F}_3$ exerted on q_3 is the vector sum $\mathbf{F}_{13} + \mathbf{F}_{23}$.

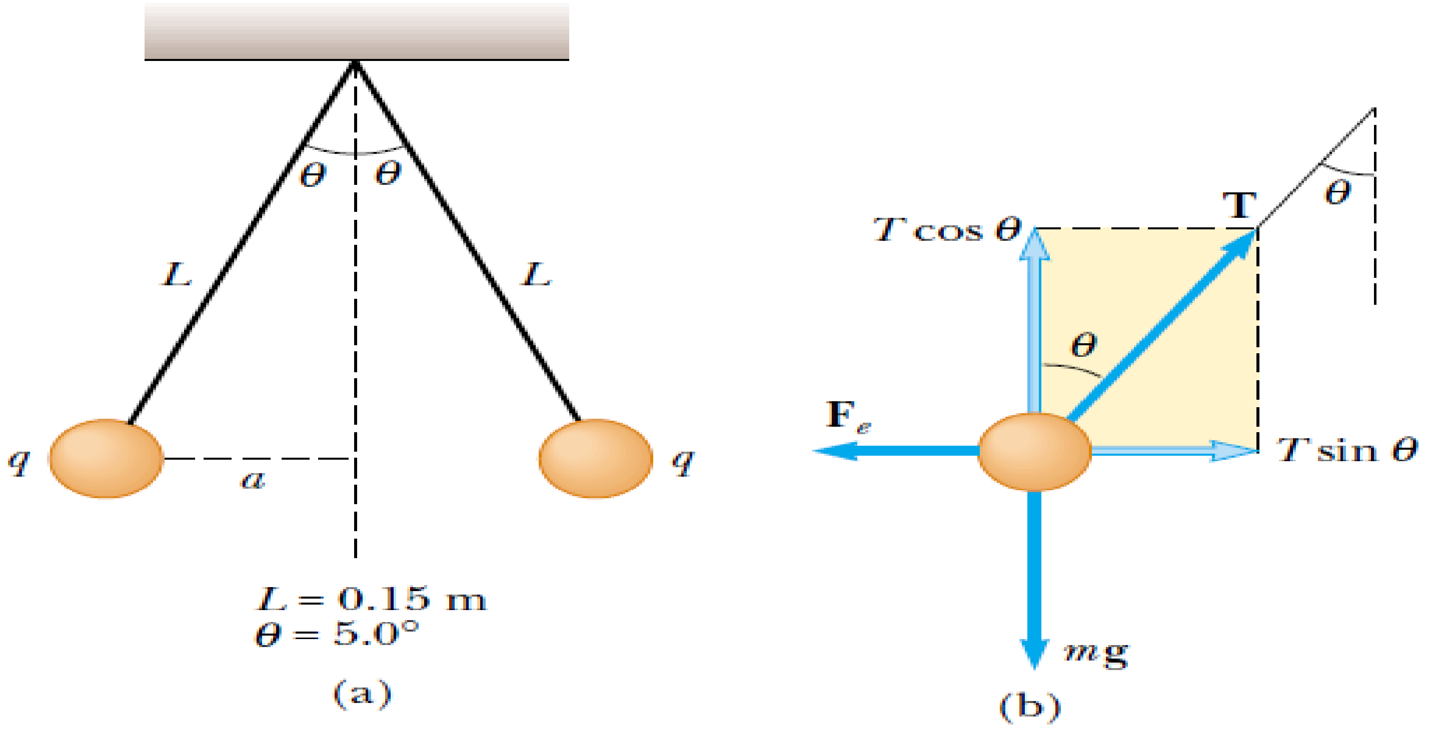
Home Work

Q.1 Three point charges are located at the corners of an equilateral triangle as shown in Figure below Calculate the resultant electric force on the 7.00-μC charge.



Example: Find the Charge on Spheres

Two identical small charged spheres, each having a mass of 3.0×10^{-2} kg, hang in equilibrium as shown in Figure **(a)** The length of each string is 0.15 m, and the angle θ is 5.0°. Find the magnitude of the charge on each sphere.



(a) Two identical spheres, each carrying the same charge q, suspended in equilibrium. (b) The free-body diagram for the sphere on the left.

Solution Figure a helps us conceptualize this problem—the two spheres exert repulsive forces on each other. If they are held close to each other and released, they will move outward from the center and settle into the configuration in Figure a after the damped oscillations due to air resistance have vanished. The key phrase "in equilibrium" helps us categorize this as an equilibrium problem, with the added feature that one of the forces on a sphere is an electric force.

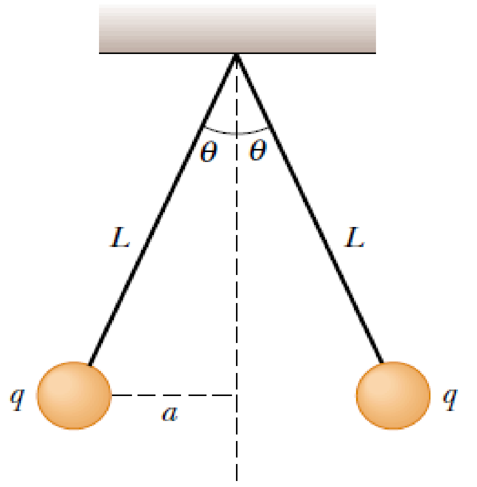
We analyze this problem by drawing the free-body diagram for the left-hand sphere in Figure b.

The sphere is in equilibrium under the application of the forces **T** from the string, the electric force $\mathbf{F}_e$ from the other sphere, and the gravitational force $m\mathbf{g}$.

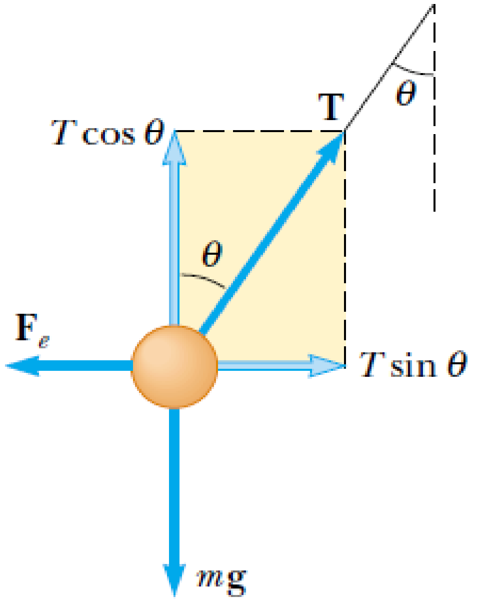
Because the sphere is in equilibrium, the forces in the horizontal and vertical directions must separately add up to zero:

$$(1) \qquad \sum F_x = T\sin\theta - F_e = 0$$

$$(2) \qquad \sum F_y = T\cos\theta - mg = 0$$



(a)

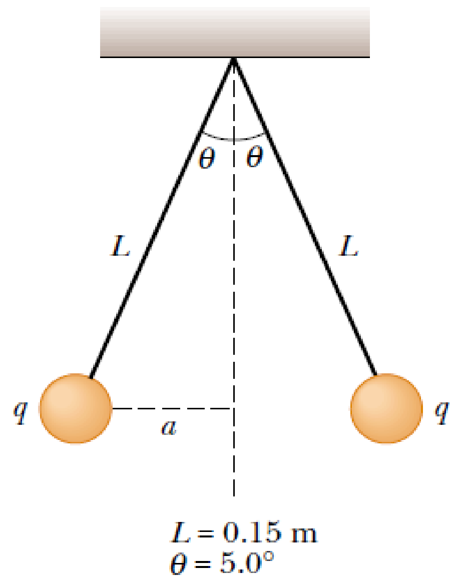


(b)

$$(2) \qquad \sum F_y = T\cos\theta - mg = 0$$

From Equation (2), we see that $T = mg/\cos\theta$; thus, T can be eliminated from Equation (1) if we make this substitution. This gives a value for the magnitude of the electric force F_e:

$$F_e = mg\tan\theta = (3.0\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)\tan(5.0^\circ)$$
$$= 2.6\times10^{-2}\text{ N}$$



(a)

Considering the geometry of the right triangle in Figure a, we see that $\sin\theta = a/L$. Therefore,

$$a = L\sin\theta = (0.15\text{ m})\sin(5.0^\circ) = 0.013\text{ m}$$

The separation of the spheres is $2a = 0.026$ m.

From Coulomb's law, the magnitude of the electric force is

$$F_e = k_e\frac{|q|^2}{r^2}$$

where $r = 2a = 0.026$ m

$$|q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6\times10^{-2}\text{ N})(0.026\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 1.96\times10^{-15}\text{ C}^2$$

$$|q| = \boxed{4.4\times10^{-8}\text{ C}}$$

Sample Problem 21.02 Equilibrium of two forces on a particle

Figure 21-8*a* shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)

KEY IDEA

If $\vec{F}_1$ is the force on the proton due to charge q_1 and $\vec{F}_2$ is the force on the proton due to charge q_2, then the point we seek is where $\vec{F}_1 + \vec{F}_2 = 0$. Thus,

$$\vec{F}_1 = -\vec{F}_2. \tag{21-8}$$

This tells us that at the point we seek, the forces acting on the proton due to the other two particles must be of equal magnitudes,

$$F_1 = F_2, \tag{21-9}$$

and that the forces must have opposite directions.

Reasoning: Because a proton has a positive charge, the proton and the particle of charge q_1 are of the same sign, and force $\vec{F}_1$ on the proton must point away from q_1. Also, the proton and the particle of charge q_2 are of opposite signs, so force $\vec{F}_2$ on the proton must point toward q_2. "Away from q_1" and "toward q_2" can be in opposite directions only if the proton is located on the x axis.

If the proton is on the x axis at any point between q_1 and q_2, such as point P in Fig. 21-8*b*, then $\vec{F}_1$ and $\vec{F}_2$ are in the same direction and not in opposite directions as required. If the proton is at any point on the x axis to the left of q_1, such as point S in Fig. 21-8*c*, then $\vec{F}_1$ and $\vec{F}_2$ are in opposite directions. However, Eq. 21-4 tells us that $\vec{F}_1$ and $\vec{F}_2$ cannot have equal magnitudes there: F_1 must be greater than F_2, because F_1 is produced by a closer charge (with lesser r) of greater magnitude ($8q$ versus $2q$).

Finally, if the proton is at any point on the x axis to the right of q_2, such as point R in Fig. 21-8*d*, then $\vec{F}_1$ and $\vec{F}_2$ are again in opposite directions. However, because now the charge of greater magnitude (q_1) is *farther* away from the proton than the charge of lesser magnitude, there is a point at which F_1 is equal to F_2. Let x be the coordinate of this point, and let q_p be the charge of the proton.

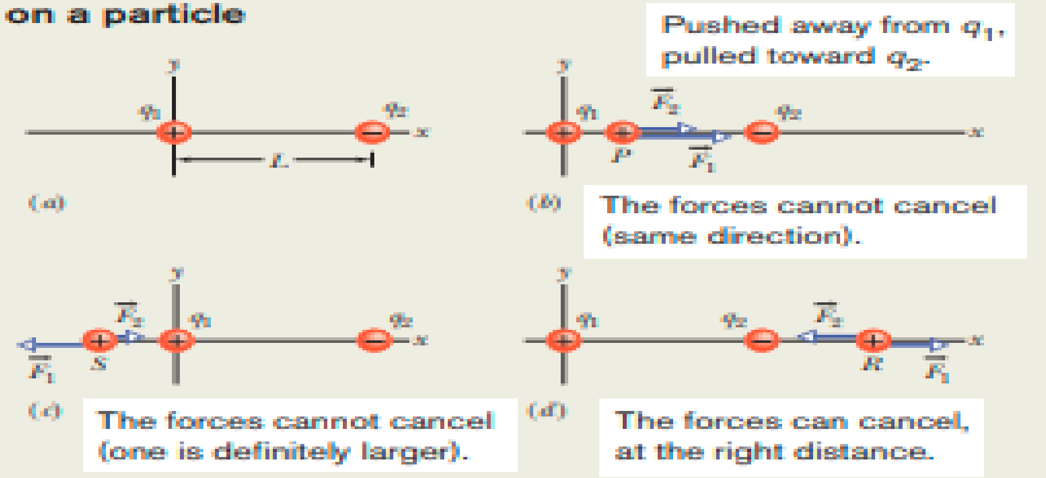


Figure 21-8 (*a*) Two particles of charges q_1 and q_2 are fixed in place on an x axis, with separation L. (*b*)–(*d*) Three possible locations P, S, and R for a proton. At each location, $\vec{F}_1$ is the force on the proton from particle 1 and $\vec{F}_2$ is the force on the proton from particle 2.

Calculations: With Eq. 21-4, we can now rewrite Eq. 21-9:

$$\frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}. \tag{21-10}$$

(Note that only the charge magnitudes appear in Eq. 21-10. We already decided about the directions of the forces in drawing Fig. 21-8*d* and do not want to include any positive or negative signs here.) Rearranging Eq. 21-10 gives us

$$\left(\frac{x-L}{x}\right)^2 = \frac{1}{4}.$$

After taking the square roots of both sides, we find

$$\frac{x-L}{x} = \frac{1}{2}$$

and

$$x = 2L. \qquad \text{(Answer)}$$

The equilibrium at $x = 2L$ is unstable; that is, if the proton is displaced leftward from point R, then F_1 and F_2 both increase but F_2 increases more (because q_2 is closer than q_1), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both F_1 and F_2 decrease but F_2 decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.

End of chapter Problems:

••13 GO In Fig. 21-26, particle 1 of charge $+1.0\ \mu C$ and particle 2 of charge $-3.0\ \mu C$ are held at separation $L = 10.0$ cm on an x axis. If particle 3 of unknown charge q_3 is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) x and (b) y coordinates of particle 3?

Figure 21-26 Problems 13, 19, 30, 58, and 67.

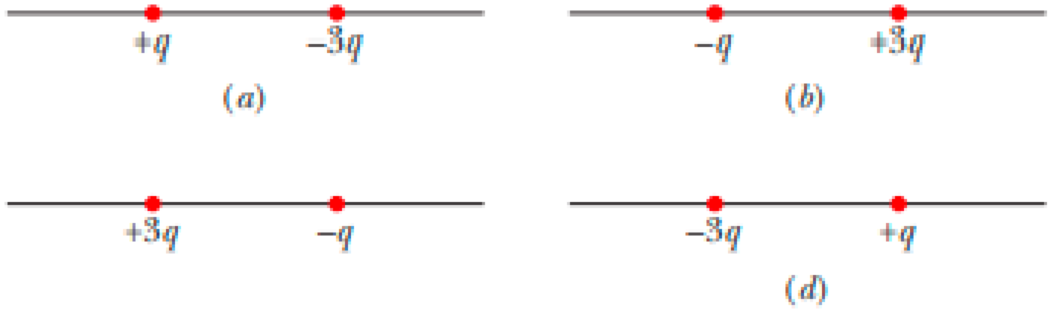
•3 SSM What must be the distance between point charge $q_1 = 26.0\ \mu C$ and point charge $q_2 = -47.0\ \mu C$ for the electrostatic force between them to have a magnitude of 5.70 N?

•4 In the return stroke of a typical lightning bolt, a current of 2.5×10^4 A exists for 20 μs. How much charge is transferred in this event?

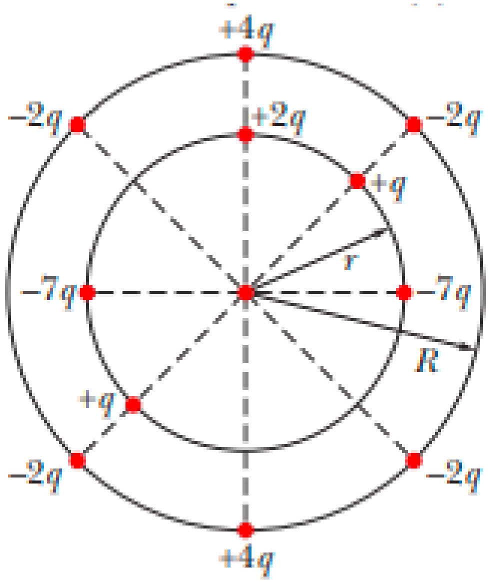
Solution: $q = it = (2.5\times10^4\ \text{A})(20\times10^{-6}\ \text{s}) = 0.50\ \text{C}.$

Questions (textbook)

3 Figure 21-13 shows four situations in which charged particles are fixed in place on an axis. In which situations is there a point to the left of the particles where an electron will be in equilibrium?



5 In Fig. 21-15, a central particle of charge $-q$ is surrounded by two circular rings of charged particles. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consider symmetry.)



7 Figure 21-16 shows three situations involving a charged particle and a uniformly charged spherical shell. The charges are given, and the radii of the shells are indicated. Rank the situations according to the magnitude of the force on the particle due to the presence of the shell, greatest first.

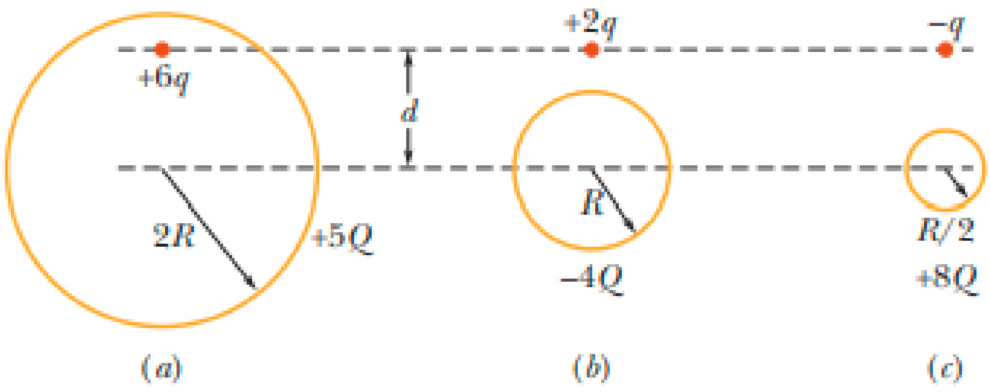
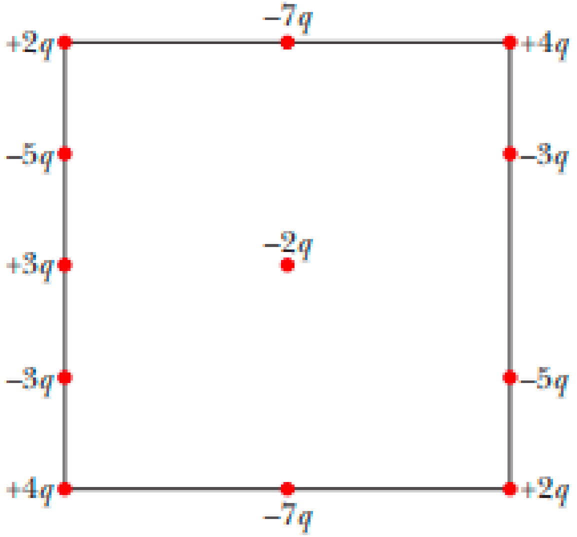


Figure 21-16 Question 7.

10 In Fig. 21-19, a central particle of charge $-2q$ is surrounded by a square array of charged particles, separated by either distance d or $d/2$ along the perimeter of the square. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consideration of symmetry can greatly reduce the amount of work required here.)



Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

Adapted from:

Book:

- Fundamentals of Physics by Halliday Resnick & Jearl Walker 10th edition