Waves

Part 2

Contents

Speed of wave on a stretched string

- The speed of a wave is related to the wave's wavelength and frequency, but it is set by the properties of the medium.

- If a wave is to travel through a medium such as water, air, steel, or a stretched string, it must cause the particles of that medium to oscillate as it passes, which requires both mass (for kinetic energy) and elasticity (for potential energy). Thus, the mass and elasticity determine how fast the wave can travel.

Speed of wave on a stretched string

Let us first conceptually predict the parameters that determine the speed.
If a string under tension is pulled sideways and then released, **the tension is responsible for accelerating a particular element of the string back toward its equilibrium position.**

According to Newton's second law, the acceleration of the element increases with increasing tension. If the element returns to equilibrium more rapidly due to this increased acceleration, we would intuitively argue that the wave speed is greater.

Thus, we expect the wave speed to increase with increasing tension.

Likewise, **the wave speed should decrease as the mass per unit length(linear density) of the string increases. This is because it is more difficult to accelerate a massive element of the string than a light element**. If the tension in the string is T and its mass per unit length is μ(Greek mu), then as we shall show, the wave speed is

$$v = \sqrt{\frac{T}{\mu}}$$

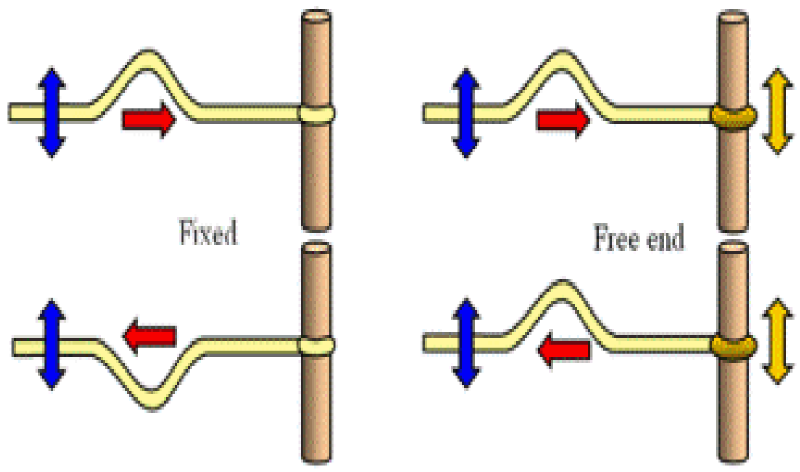
Where, $\mu = \frac{m}{\ell}$

The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

Reflection of Wave

Two type of reflection of waves

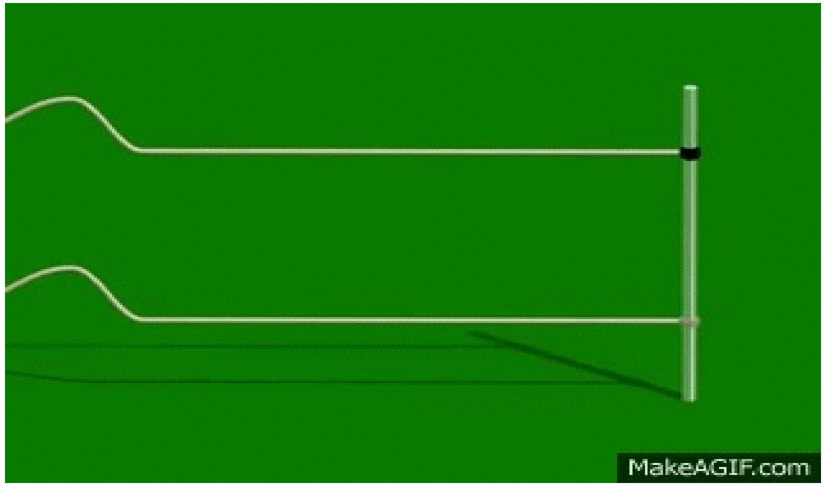
1. Fixed End Reflection



2. Free End Reflection

Reflection of Wave

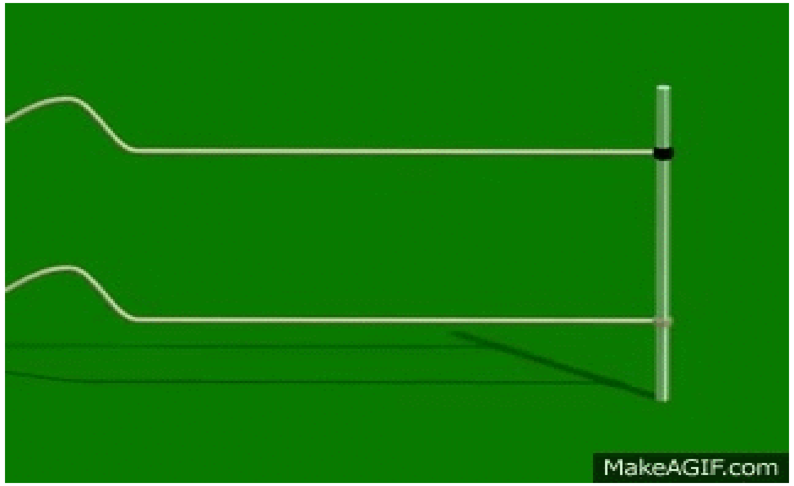
Fixed End Reflection



- Medium plays an important role while travelling of wave from one point to another.
- When the pulse reaches the support, a severe change in the medium occurs—the string ends.
- The result of this change is that the pulse undergoes reflection.
- Note that the reflected pulse is *inverted*.

Reflection of Wave

Fixed End Reflection



- When the pulse reaches the fixed end of the string, the string produces an upward force on the support.

- By Newton’s third law, the support must exert an equal magnitude and oppositely directed (downward) reaction force on the string.

- This downward force causes the pulse to invert upon reflection.

Reflection of Wave

Free End Reflection: this time, the pulse arrives at the end of a string that is free to move vertically, as in Figure below.

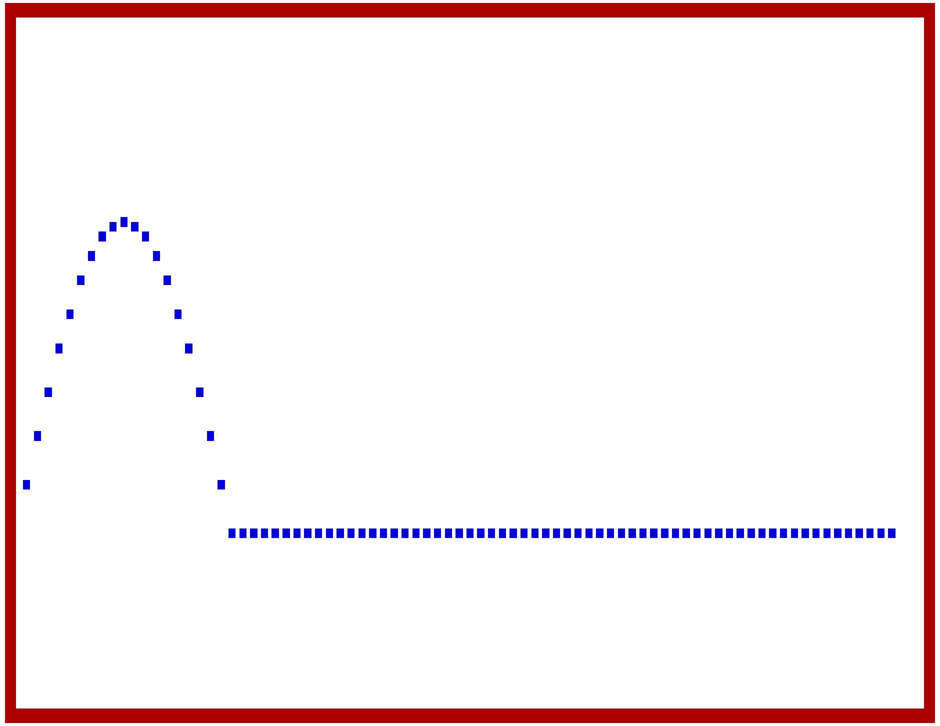
The tension at the free end is maintained because the string is tied to a ring of negligible mass that is free to slide vertically on a smooth post without friction.

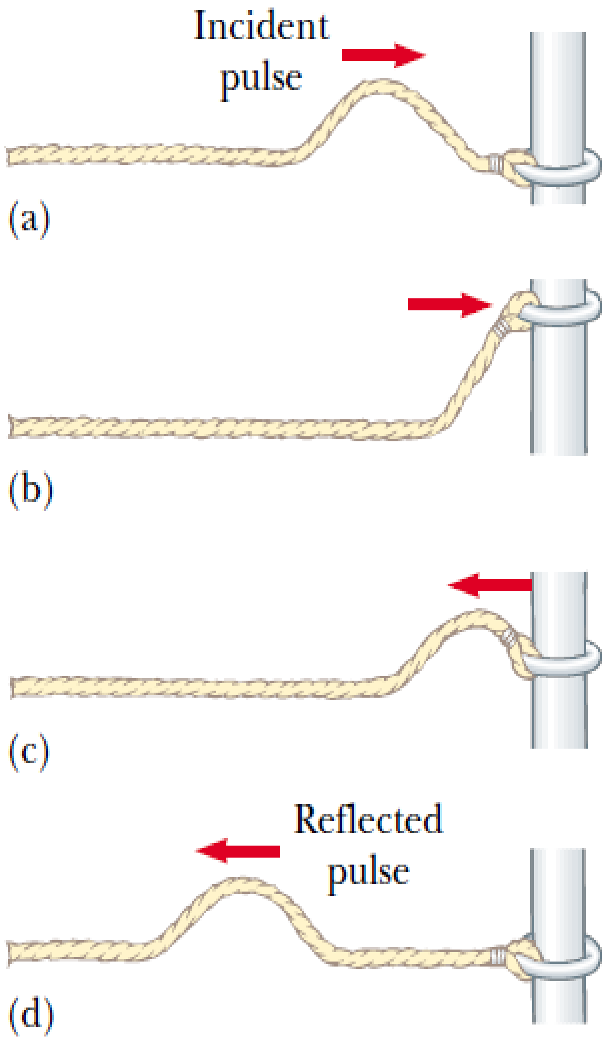
Again, the pulse is reflected, but this time it is not inverted.

When it reaches the post, the pulse exerts a force on the free end of the string, causing the ring to accelerate upward. The ring rises as high as the incoming pulse, and then the downward component of the tension force pulls the ring back down.

This movement of the ring produces a reflected pulse that is not inverted and that has the same amplitude as the incoming pulse.

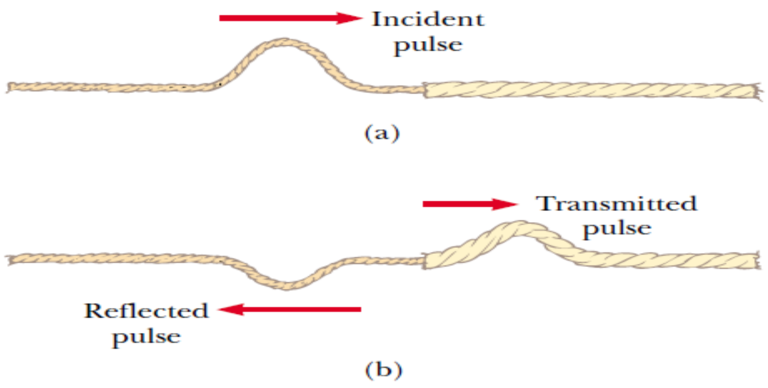
Free End Reflection:

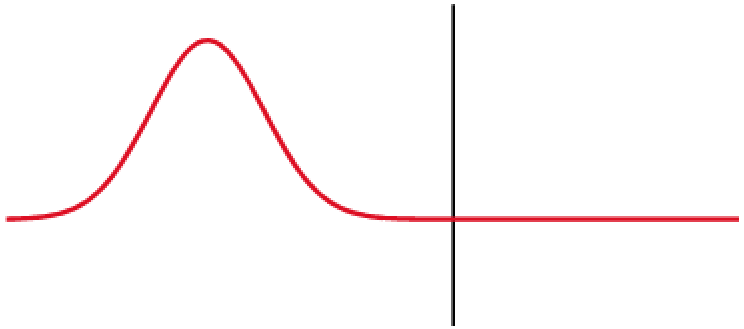




Transmission of Wave

- Case 1: Now we have a situation in which the boundary is intermediate between these two extremes.
- In this case, part of the energy in the incident pulse is reflected and part undergoes transmission—that is, some of the energy passes through the boundary.



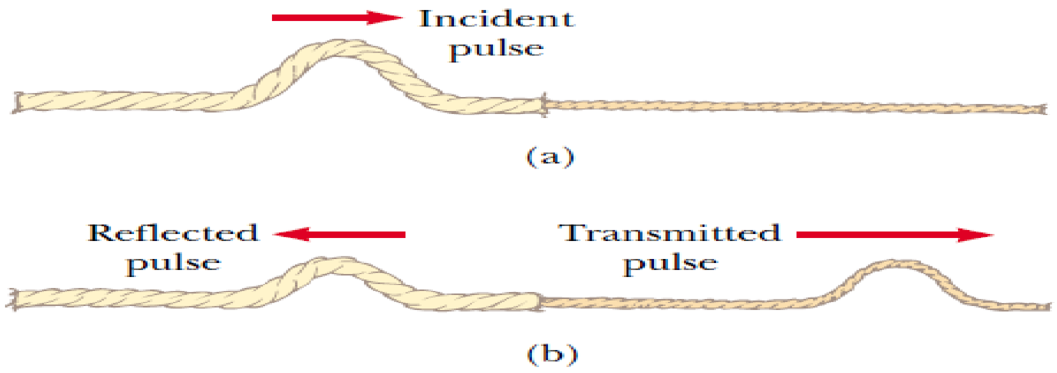


Transmission of Wave

- Case 2: When a pulse traveling on a heavy string strikes the boundary between the heavy string and a lighter one, as in Figure shown below, again part is reflected and part is transmitted.

- In this case, the reflected pulse is not inverted.

- In either case, the relative heights of the reflected and transmitted pulses depend on the relative densities of the two strings. If the strings are identical, there is no discontinuity at the boundary and no reflection takes place.



Remember:

According to Equation $\sqrt{T/\mu}$ the speed of a wave on a string increases as the mass per unit length of the string decreases. In other words, a wave travels more slowly on a heavy string than on a light string if both are under the same tension. The following general rules apply to reflected waves: **when a wave or pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), it is inverted upon reflection. When a wave or pulse travels from medium A to medium B and $v_A < v_B$ (that is, when A is denser than B), it is not inverted upon reflection.**

Rate of Energy Transfer or Power transmitted by Sine wave

- The expression below shows that the rate of energy transfer by a sinusoidal wave on a string is proportional to

(a) the square of the frequency,

(b) the square of the amplitude, and

(c) the wave speed

$$\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 \left(\frac{\lambda}{T}\right)$$

or it may be,

$$\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v$$

- In fact: the rate of energy transfer(power) in any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.

- The factors μ and v in this equation depend on the material and tension of the string. The factors ω and y_m depend on the process that generates the wave.
- The dependence of the average power of a wave on the square of its amplitude and also on the square of its angular frequency is a general result, true for waves of all types.

Example Problem 4

A taut string for which $\mu = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution The wave speed on the string is,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80.0 \text{ N}}{5.00 \times 10^{-2} \text{ kg/m}}} = 40.0 \text{ m/s}$$

Because $f = 60.0$ Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0 \text{ Hz}) = 377 \text{ s}^{-1}$$

Using these values in Equation 16.21 for the power, with $A = 6.00 \times 10^{-2}$ m, we obtain

$$\begin{aligned} \mathcal{P} &= \tfrac{1}{2}\mu\omega^2 A^2 v \\ &= \tfrac{1}{2}(5.00 \times 10^{-2} \text{ kg/m})(377 \text{ s}^{-1})^2 \\ &\quad \times (6.00 \times 10^{-2} \text{ m})^2 (40.0 \text{ m/s}) \\ &= 512 \text{ W} \end{aligned}$$

Exercise: (Sample problem 16.03)

Sample Problem 16.03 **Average power of a transverse wave**

A string has linear density $\mu = 525$ g/m and is under tension $\tau = 45$ N. We send a sinusoidal wave with frequency $f = 120$ Hz and amplitude $y_m = 8.5$ mm along the string. At what average rate does the wave transport energy?

KEY IDEA

The average rate of energy transport is the average power P_{avg} as given by Eq. 16-33.

Calculations: To use Eq. 16-33, we first must calculate angular frequency ω and wave speed v. From Eq. 16-9,

$$\omega = 2\pi f = (2\pi)(120 \text{ Hz}) = 754 \text{ rad/s}.$$

From Eq. 16-26 we have

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{45 \text{ N}}{0.525 \text{ kg/m}}} = 9.26 \text{ m/s}.$$

Equation 16-33 then yields

$$\begin{aligned} P_{avg} &= \tfrac{1}{2}\mu v \omega^2 y_m^2 \\ &= (\tfrac{1}{2})(0.525 \text{ kg/m})(9.26 \text{ m/s})(754 \text{ rad/s})^2(0.0085 \text{ m})^2 \\ &\approx 100 \text{ W}. \qquad \text{(Answer)} \end{aligned}$$

Linear Wave Equation & Power transmitted by Sine Wave

The **power** transmitted by a sinusoidal wave on a stretched string is

$$\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v$$

Wave functions are solutions to a differential equation called the **linear wave equation:**

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

This equation gives a complete description of the wave motion, and from it one can derive an expression for the wave speed.

Furthermore, the linear wave equation is basic to many forms of wave motion.

Answer these:

1. By what factor would you have to multiply the tension in a stretched string in order to double the wave speed?

2. When traveling on a taut string, does a pulse always invert upon reflection? Explain.

3. If you shake one end of a taut rope steadily three times each second, what would be the period of the sinusoidal wave set up in the rope?

4. A vibrating source generates a sinusoidal wave on a string under constant tension. If the power delivered to the string is doubled, by what factor does the amplitude change? Does the wave speed change under these circumstances?

Adapted from:

Book:

Fundamentals of Physics by Halliday Resnick & Jearl Walker 10th edition