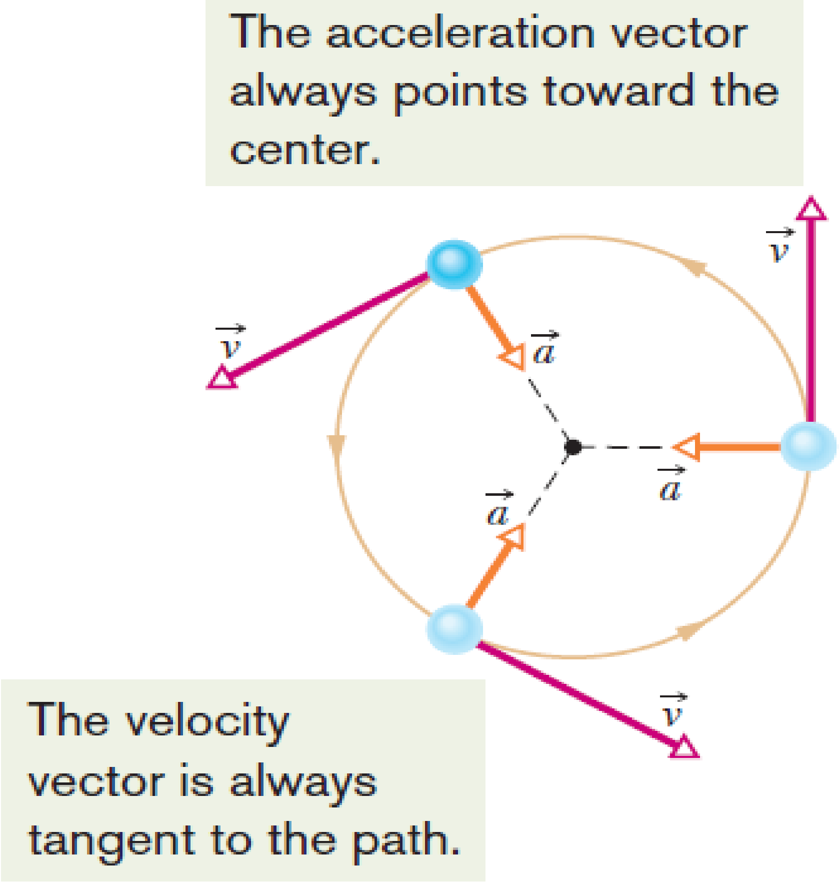
Motion in two and three dimension

Uniform Circular motion

Uniform Circular Motion

- A particle is in uniform circular motion if it travels around a circle or a circular arc at constant (uniform) speed. Although the speed does not vary, the particle is accelerating because the velocity changes in direction.
- Figure shows the relationship between the velocity and acceleration vectors at various stages during uniform circular motion.



Uniform Circular Motion

- Both “velocity & acceleration vectors” have constant magnitude, but their directions change continuously.
- The velocity is always directed **tangent to the circle** in the direction of motion.
- The acceleration is always directed **radially inward.** Because of this, the acceleration associated with uniform circular motion is called a **centripetal acceleration.**
- Centripetal means center seeking.
- Mathematically**,** the magnitude of acceleration is,

$$\boldsymbol{a_c = \frac{v^2}{r}}$$

Uniform Circular Motion(Cont'd)

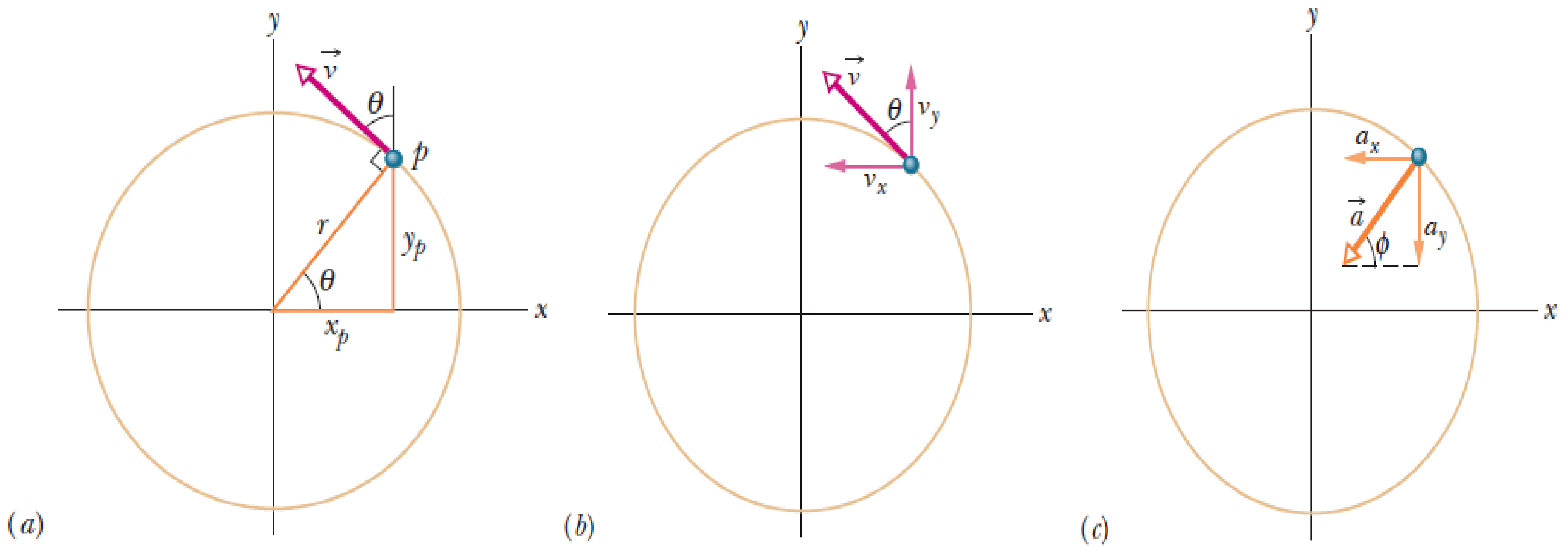
- During this acceleration at constant speed, the particle travels the circumference of the circle (a distance of 2πr) in time t.

$$T = \frac{2\pi r}{v} \text{ (period)}$$

(T is the time for a particle to go around a closed path exactly once. It is known as period of revolution, or simply period.)

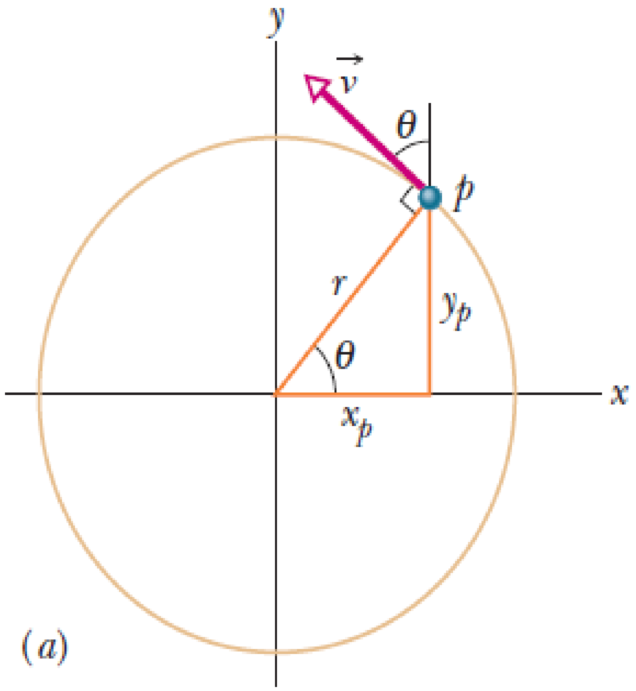
Proof(1) : $\boldsymbol{a_c = \frac{v^2}{r}}$

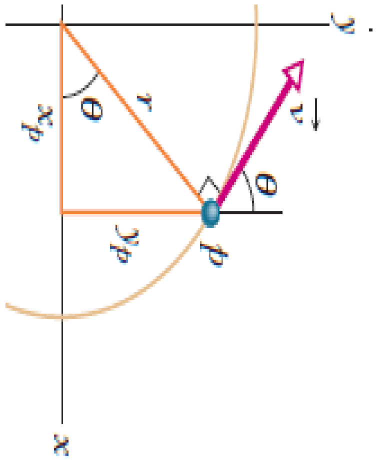
- To find the magnitude and direction of the acceleration for uniform circular motion, we consider the following figures;



Proof (2)

- In Fig. a, particle *P* moves at constant speed v around a circle of radius r.
- At the instant shown, *P* has coordinates x_P and y_P.
- **v** is tangent to the path at the particle's position, hence **v** is perpendicular to a radius **r** drawn to the particle's position. Then the angle θ that **v** makes with a vertical at *P* equals the angle θ that radius r makes with the x axis.

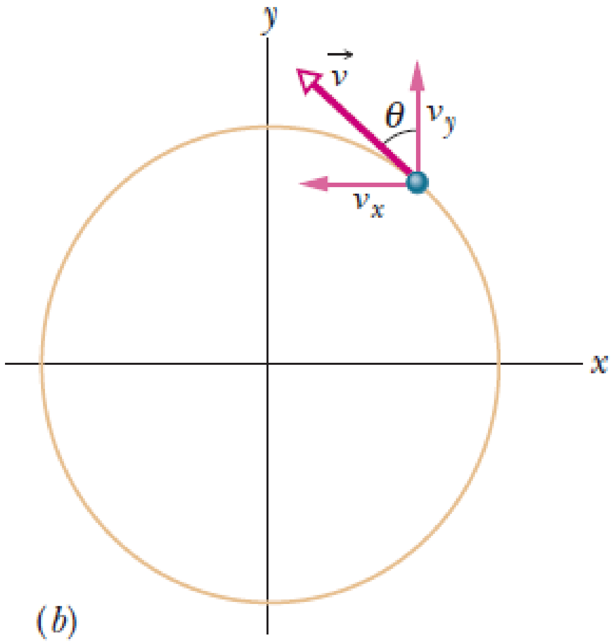


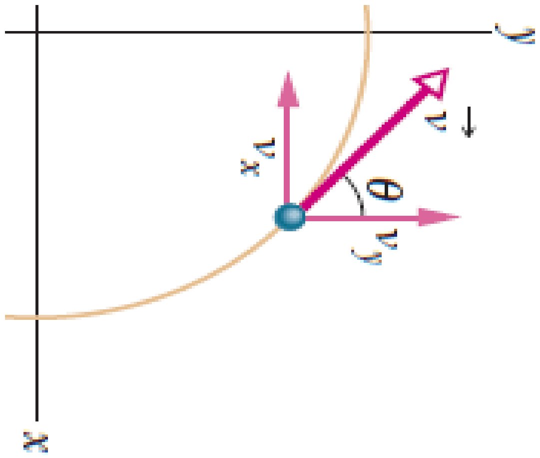


Proof (3)

- The scalar components of v are shown in Fig b. With them, we can write the velocity "v" as

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (-v\sin\theta)\hat{\imath} + (v\cos\theta)\hat{\jmath}$$





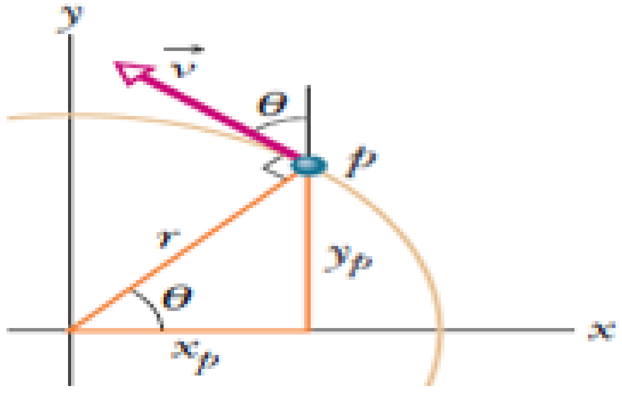
Proof (4)

- Now, using the right triangle in Fig. , we can replace sinθ with y_p/r and cosθ with x_p/r to write

$$\vec{v} = \left(\frac{-vy_p}{r}\right)\hat{i} + \left(\frac{vx_p}{r}\right)\hat{j}$$

To find the acceleration **a** of particle p, we must take the time derivative of this equation. Noting that speed v and radius r do not change with time, we obtain

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\frac{-vy_p}{r}\right)\hat{i} + \left(\frac{vx_p}{r}\right)\hat{j}$$



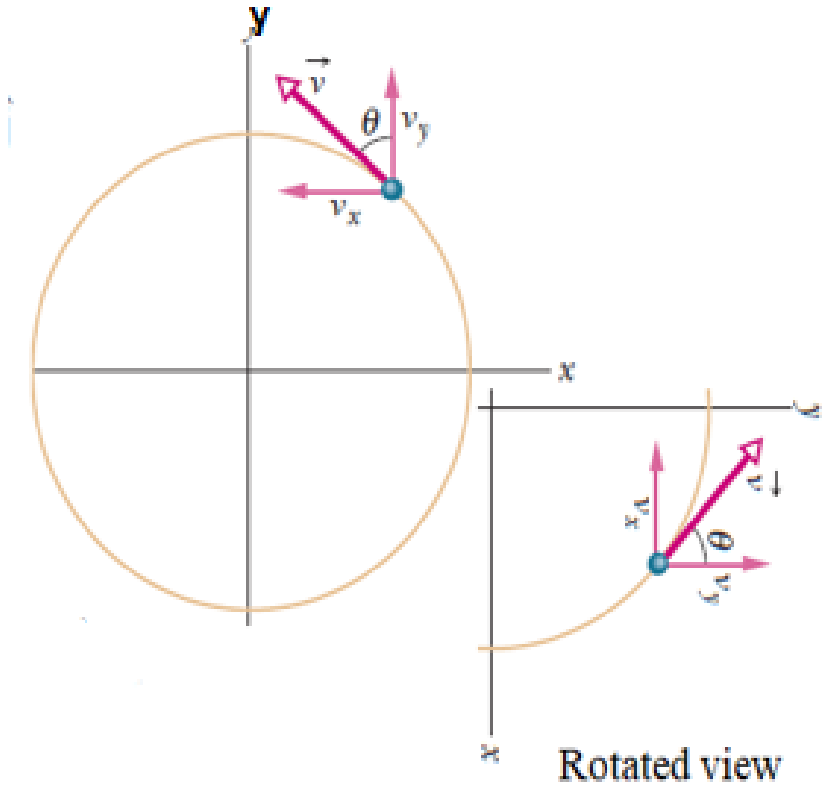
$$\vec{a} = \frac{d}{dt}\left(-\frac{vy_p}{r}\right)\hat{\imath} + \frac{d}{dt}\left(\frac{vx_p}{r}\right)\hat{\jmath}$$

$$= \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}.$$

Proof (5)

- Now note that the rate dy_p/dt at which y_p changes is equal to the velocity component v_y. Similarly, $dx_p/dt = v_x$, and, again from Fig. we see that $v_x = -v\sin\theta$ and $v_y = v\cos\theta$. Making these substitutions in Eq, we find

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\frac{-v^2\cos\theta}{r}\right)\hat{i} + \left(\frac{-v^2\sin\theta}{r}\right)\hat{j}$$



Proof(6)

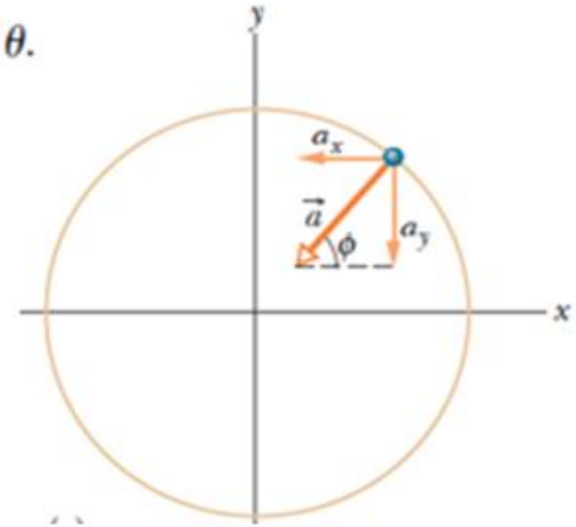
$$\vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}.$$

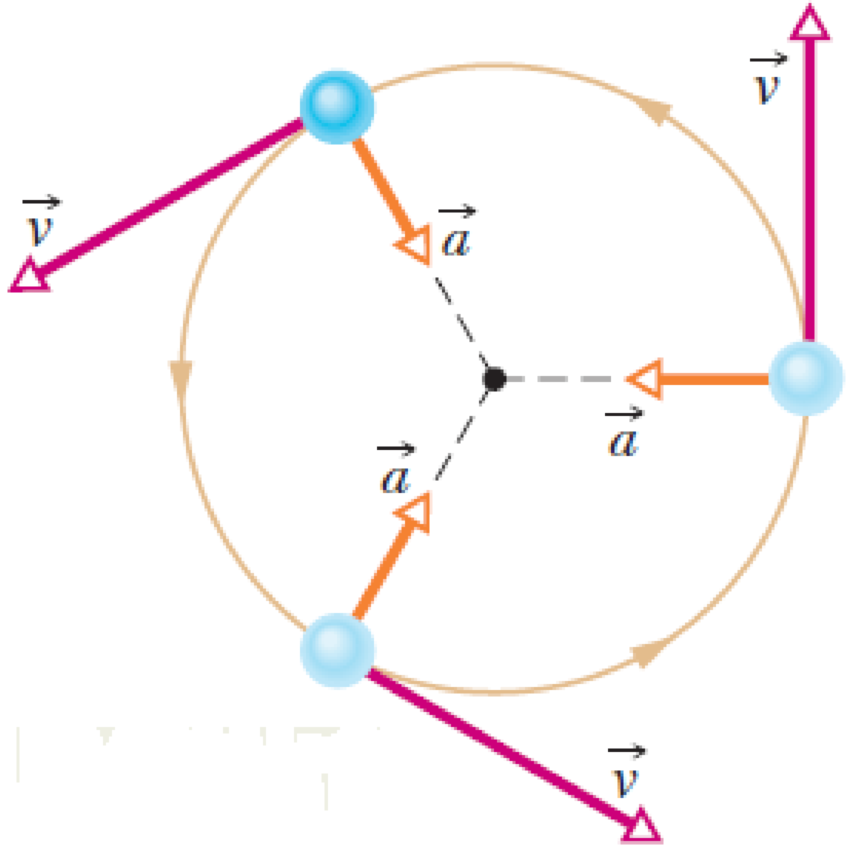
$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r},$$

as we wanted to prove. To orient $\vec{a}$, we find the angle ϕ shown in Fig.

$$\tan\phi = \frac{a_y}{a_x} = \frac{-(v^2/r)\sin\theta}{-(v^2/r)\cos\theta} = \tan\theta.$$

Thus, $\phi = \theta$, which means that $\vec{a}$ is directed along the radius r of Fig. toward the circle's center, as we wanted to prove.





$\vec{v}$
$\vec{a}$
$\vec{v}$
$\vec{a}$
$\vec{a}$
$\vec{v}$

Checkpoint

- An object moves at constant speed along a circular path in a horizontal xy plane, with the center at the origin. When the object is at x = -2 m, its velocity is (-4 m/s) $\hat{j}$. Give the object's (a) velocity and (b) acceleration at y = 2 m.

Practice Problem 1(sample problem 4.06)

”Top gun” pilots have long worried about taking a turn too tightly. As a pilot’s body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is 2g or 3g, the pilot feels heavy. At about 4g, the pilot’s vision switches to black and white and narrows to “tunnel vision.” If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as g-LOC for “g-induced loss of consciousness.” What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\overrightarrow{v_i} = (400\hat{\imath} + 500\hat{\jmath})$ m/s and 24.0 s later leaves the turn with a velocity of $\overrightarrow{v_f} = (-400\hat{\imath} - 500\hat{\jmath})$ m/s?

Answer:

a ≈8.6g

Practice Problem 2: (problem 61, chapter 4)

•61 When a large star becomes a *supernova,* its core may be compressed so tightly that it becomes a *neutron star,* with a radius of about 20 km (about the size of the San Francisco area). If a neutron star rotates once every second, (a) what is the speed of a particle on the star's equator and (b) what is the magnitude of the particle's centripetal acceleration? (c) If the neutron star rotates faster, do the answers to (a) and (b) increase, decrease, or remain the same?

Answers:

a. v = 1.3 x 10^ 5 m/s =126 km/s
b. a = 7.9x 10^5 m/s^2
c. both v and a will increase if T is reduced

Practice Problem 3: (problem 60, chapter 4)

•60 A centripetal-acceleration addict rides in uniform circular motion with radius $r = 3.00$ m. At one instant his acceleration is $\vec{a} = (6.00\ \text{m/s}^2)\hat{\imath} + (-4.00\ \text{m/s}^2)\hat{\jmath}$. At that instant, what are the values of (a) $\vec{v} \cdot \vec{a}$ and (b) $\vec{r} \times \vec{a}$?

Answers:

a. $\vec{v} \cdot \vec{a} = 0.$

b. $\vec{r} \times \vec{a} = 0.$

Homework questions:

- Practice problems:
- End of chapter 4 textbook “ Fundamentals of Physics”by Halliday & Resnick Jearl Walker 10th Edition”

23, 58,67, 85, 92

Answer of even problems:

58: a. c = 0.94 m, b. v = 19m/s, c. a = 2.4x 10^3 m/s^2, d = T = 0.05 s

92: a. v = 19 m/s, b. 35 rev/min, c. T = 1.7 s

References:

Book:

Fundamentals of Physics by Halliday Resnick & Jearl Walker 10th edition