Motion in two and three dimensions

Chapter 4 Textbook

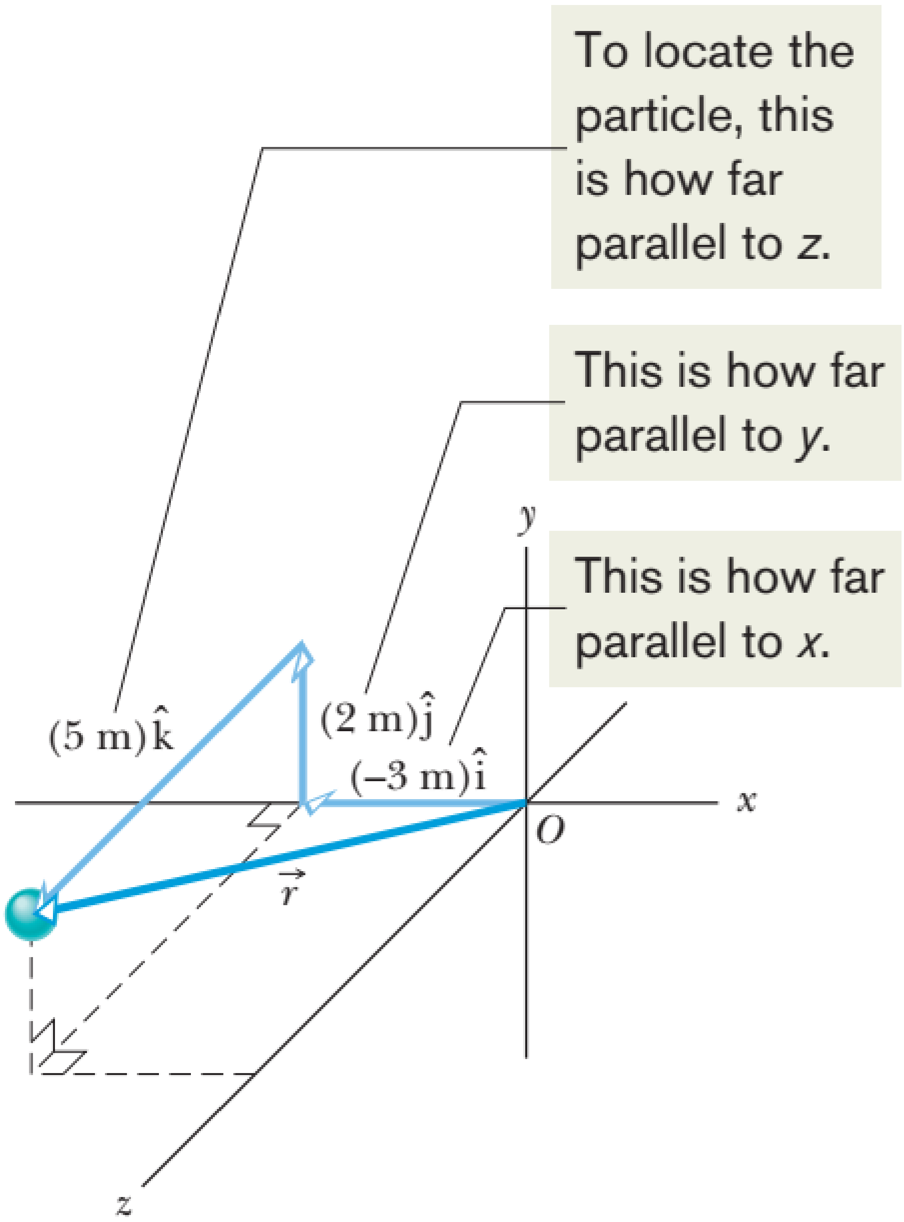
Position and Displacement

Position Vector:

- A vector that extends from a reference point (usually origin) to the particle.

Position Vector in 3-D

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



Displacement vector

As a particle moves, its position vector changes in such a way that the vector always extends to the particle from the reference point (the origin). If the position vector changes—say, from $\vec{r}_1$ to $\vec{r}_2$ during a certain time interval—then the particle's **displacement** $\Delta\vec{r}$ during that time interval is

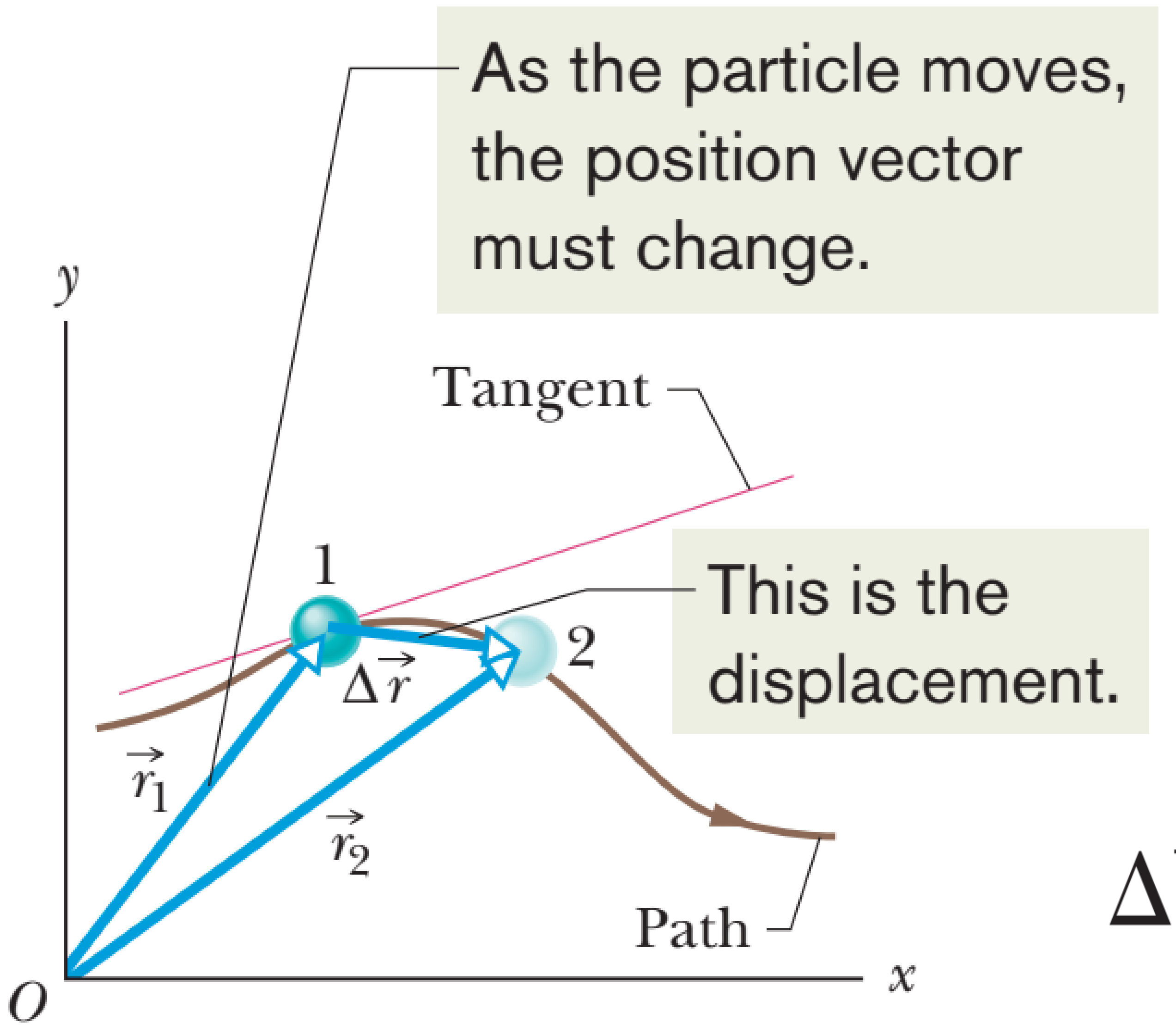
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

Using the unit-vector notation we can rewrite this displacement as

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k},$$

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}.$$



$$\vec{r_2} = \vec{r_1} + \Delta\vec{r}$$

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}$$

Avg. Velocity in 3-Dimensions

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}.$$

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}.$$

Put in the formula of average velocity we'll get

$$\vec{v}_{avg} = \frac{\Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathrm{i}} + \frac{\Delta y}{\Delta t}\hat{\mathrm{j}} + \frac{\Delta z}{\Delta t}\hat{\mathrm{k}}.$$

Instantaneous Velocity in 3-D

When we speak of the **velocity** of a particle, we usually mean the particle's **instantaneous velocity** $\vec{v}$ at some instant. This $\vec{v}$ is the value that $\vec{v}_{avg}$ approaches in the limit as we shrink the time interval Δt to 0 about that instant. Using the language of calculus, we may write $\vec{v}$ as the derivative

$$\vec{v} = \frac{d\vec{r}}{dt}.$$

- Substitute the value of unit vector

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}},$$

$$\vec{v} = \frac{d}{dt}(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

Instantaneous Velocity in 3-D

- Simply we can

$$\vec{v} = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

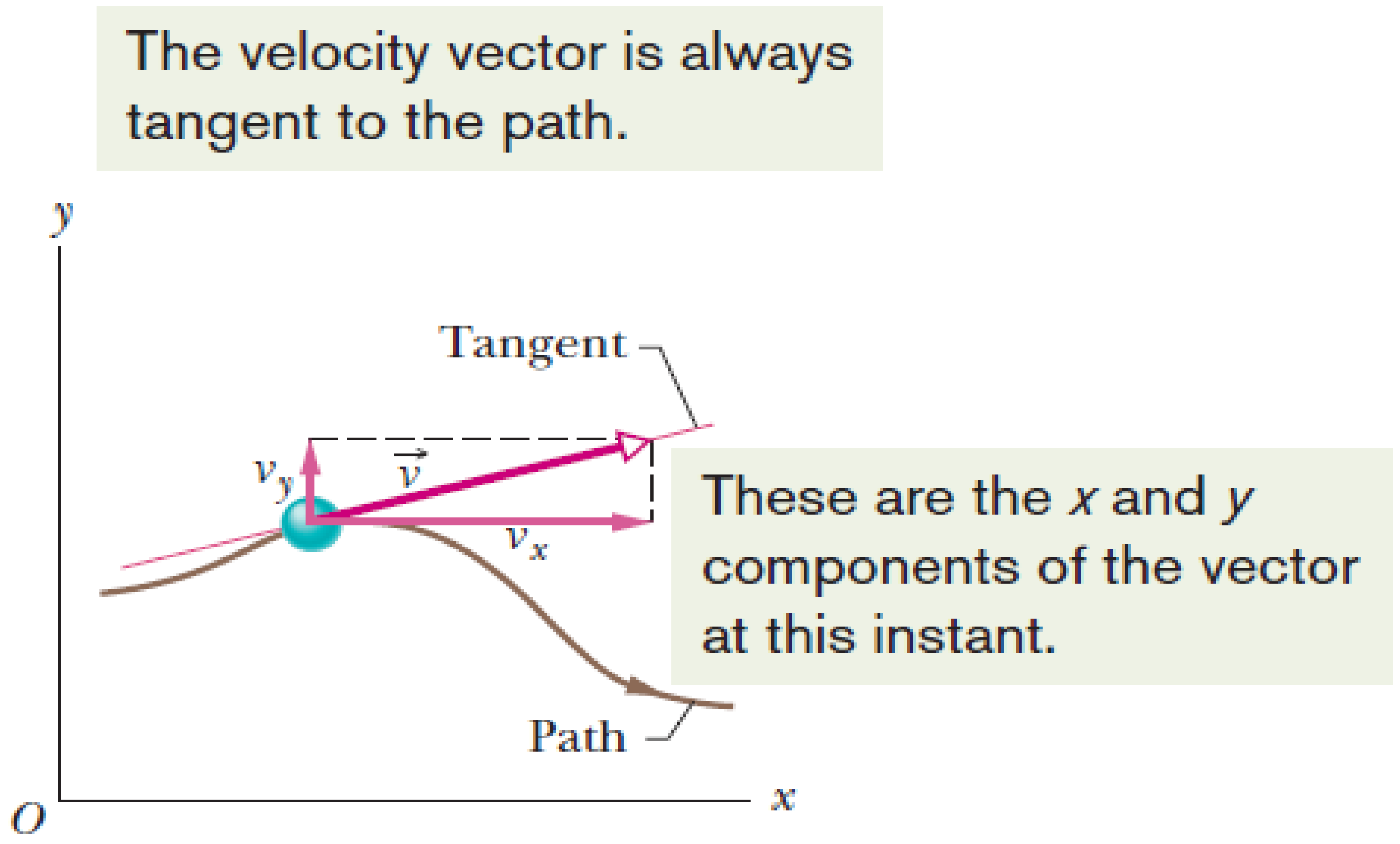
$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$

where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}.$$

The direction of the instantaneous velocity of a particle is always tangent to the particle’s path at the particle’s position

Instantaneous Velocity in 3-D



Instantaneous acceleration in 3-D

instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$

$$\vec{a} = \frac{d}{dt}(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}})$$

$$= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}$$

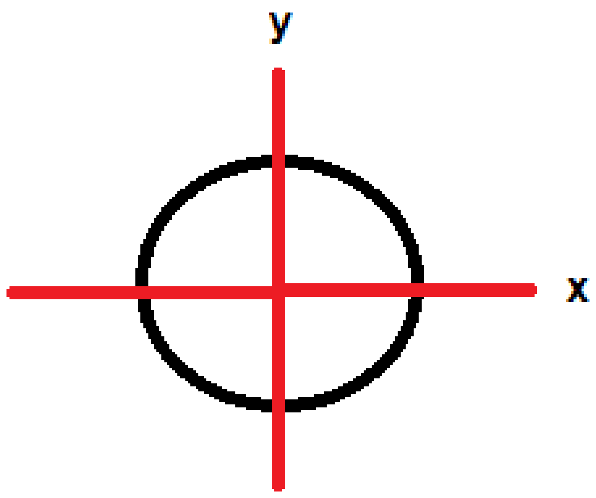
$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}},$$

where the scalar components of $\vec{a}$ are

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}.$$

Checkpoint 1:

The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2\text{ m/s})\hat{\imath} - (2\text{ m/s})\hat{\jmath}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.



Checkpoint 2

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{\text{i}} - (4t + 3)\hat{\text{j}}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{\text{i}} + 3\hat{\text{j}}$

Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

Sample Problem 4.01

A rabbit runs across a parking lot on which a set of coordin7ate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30.$$

At t =15 s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

Answers:

$\vec{r} = (66m)\hat{\imath} - (57m)\hat{j}$

r (magnitude) = 87 m

θ = -41°

Sample Problem 4.02

For the rabbit in the preceding sample problem, find the velocity $\vec{v}$ at time t = 15 s.

Answers:

$$\vec{v} = (-2.1\ m/s)\hat{\imath} + (-2.5\ m/s)\hat{\jmath}$$

v (magnitude) = 3.3 m/s

θ = -130°

Sample Problem 4.03

For the rabbit in the preceding two sample problems, find the acceleration $\vec{a}$ at time t = 15 s.

Answers:

$$\vec{a} = (-0.62\ m/s^2)\hat{\imath} + (0.44\ m/s^2)\hat{\jmath}$$

a (magnitude) = 0.76 m/s^2

θ = -35°

Homework questions:

- Practice problems:
- End of chapter 4 textbook “ Fundamentals of Physics”by Halliday & Resnick Jearl Walker 10^{th} Edition”

Problem 17

Adapted from:

Book:

Fundamentals of Physics by Halliday Resnick & Jearl Walker 10^{th} edition