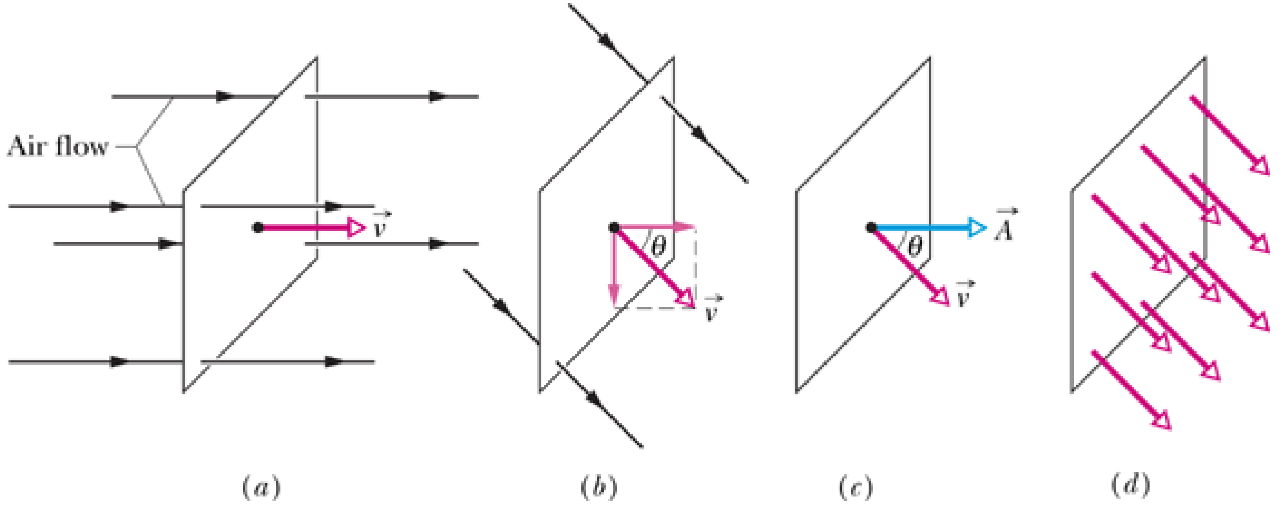
Gauss Law and its applications

Text book chapter 23

Flux



The rate of volume flow through the loop is :

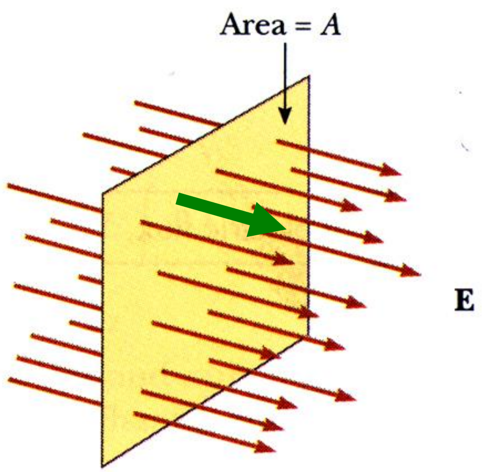
$$\Phi = (v\cos\theta)A\,.$$

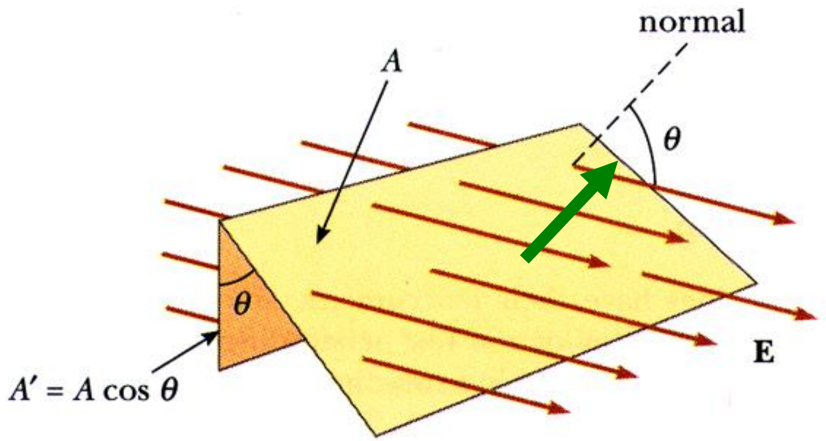
$$\Phi = vA\cos\theta = \overrightarrow{v}\cdot\overrightarrow{A}\,,$$

Definition of Electric Flux

- The amount of field, material or other physical entity passing through a surface.
- Surface area can be represented as vector defined normal to the surface it is describing
- Defined by the equation

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

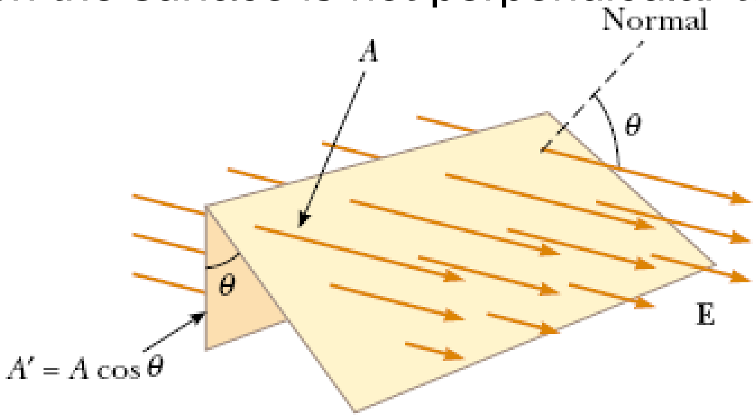




Point to remember:

An inward piercing field is negative flux. An outward piercing field is positive flux. A skimming field is zero flux.

When the surface is not perpendicular to the field



$$\Phi_E = EA' = EA\cos\theta$$

Flux is maximum when surface is perpendicular to the field i.e. θ=0,
and Flux is zero when surface is parallel to the field i.e. θ=90˚.

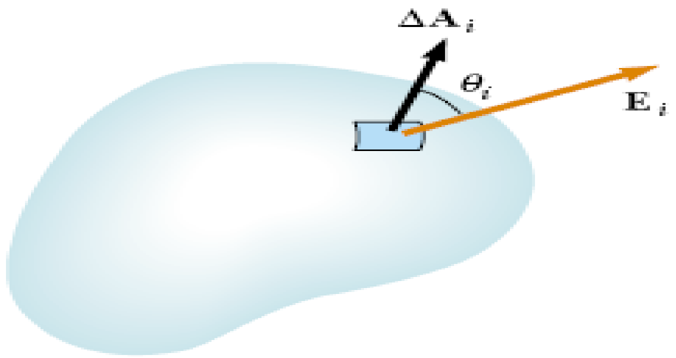


Figure 24.3 A small element of surface area ΔA_i. The electric field makes an angle θ_i with the vector $\Delta\mathbf{A}_i$, defined as being normal to the surface element, and the flux through the element is equal to $E_i\,\Delta A_i \cos\theta_i$.

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \mathbf{E}_i \cdot \Delta\mathbf{A}_i = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$

Electric Flux through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of $+1.00\ \mu C$ at its center?

Solution The magnitude of the electric field 1.00 m from this charge is found using Equation 23.9:

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\ N \cdot m^2/C^2) \frac{1.00 \times 10^{-6}\ C}{(1.00\ m)^2}$$
$$= 8.99 \times 10^3\ N/C$$

The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6\ m^2$) is thus

$$\Phi_E = EA = (8.99 \times 10^3\ N/C)(12.6\ m^2)$$
$$= \boxed{1.13 \times 10^5\ N \cdot m^2/C}$$

Flux of Electric Field

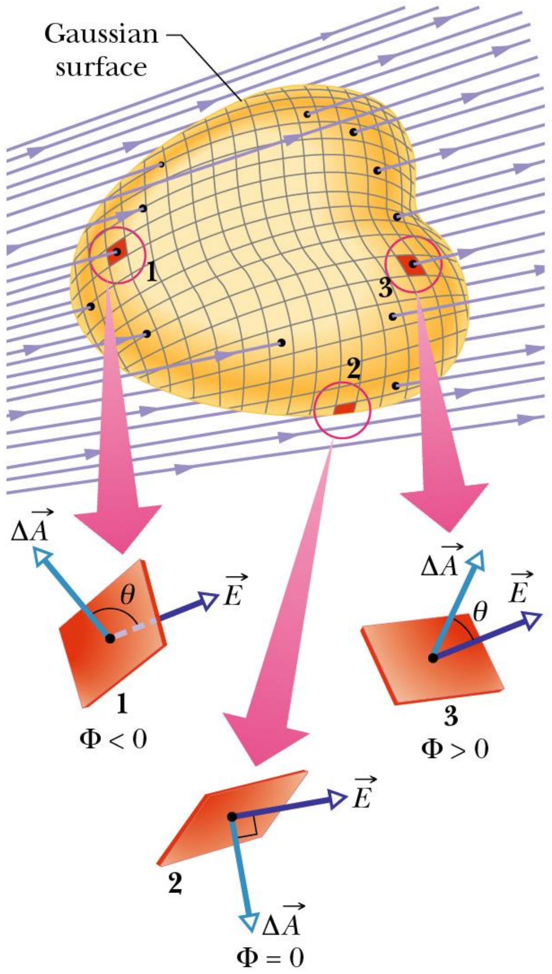
- Like the flow of water, or light energy, we can think of the electric field as flowing through a surface (although in this case nothing is actually moving).
- We represent the flux of electric field as Φ (greek letter phi), so the flux of the electric field through an element of area ΔA is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{A} = E\,\Delta A\cos\theta$$

- When $\theta < 90^\circ$, the flux is positive (out of the surface), and when $\theta > 90^\circ$, the flux is negative.

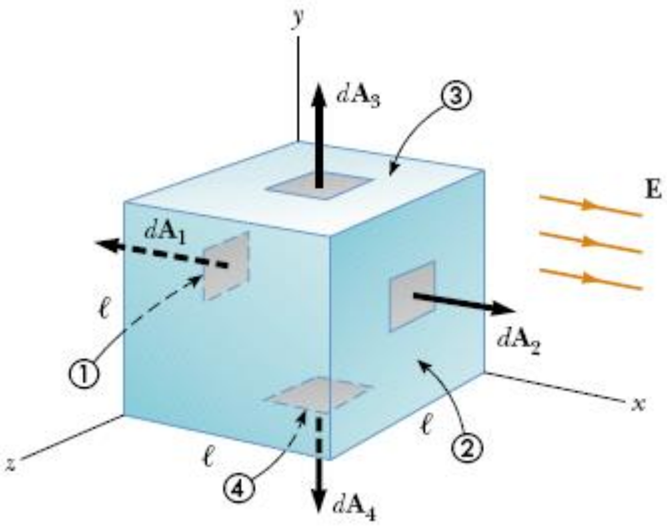
$$d\Phi = \vec{E}\cdot d\vec{A} = E\,dA\cos\theta$$

- When we have a complicated surface, we can divide it up into tiny elemental areas:



Flux Through a Cube

Consider a uniform electric field $\mathbf{E}$ oriented in the x direction. Find the net electric flux through the surface of a cube of edge length ℓ, oriented as shown in Figure 24.5.



Solution The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (③, ④, and the unnumbered ones) is zero because $\mathbf{E}$ is perpendicular to $d\mathbf{A}$ on these faces.

The net flux through faces ① and ② is

$$\Phi_E = \int_1 \mathbf{E} \cdot d\mathbf{A} + \int_2 \mathbf{E} \cdot d\mathbf{A}$$

For face ①, $\mathbf{E}$ is constant and directed inward but $d\mathbf{A}_1$ is directed outward ($\theta = 180°$); thus, the flux through this face is

$$\int_1 \mathbf{E} \cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA = -E \int_1 dA = -EA = -E\ell^2$$

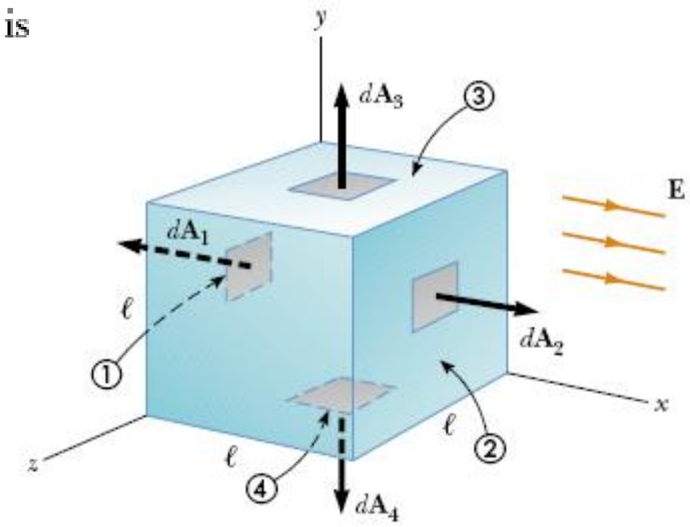
because the area of each face is $A = \ell^2$.

For face ②, $\mathbf{E}$ is constant and outward and in the same direction as $d\mathbf{A}_2$ ($\theta = 0°$); hence, the flux through this face is

$$\int_2 \mathbf{E} \cdot d\mathbf{A} = \int_2 E(\cos 0°)\, dA = E \int_2 dA = +EA = E\ell^2$$

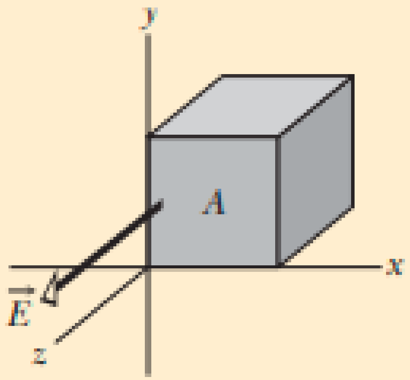
Therefore, the net flux over all six faces is

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$



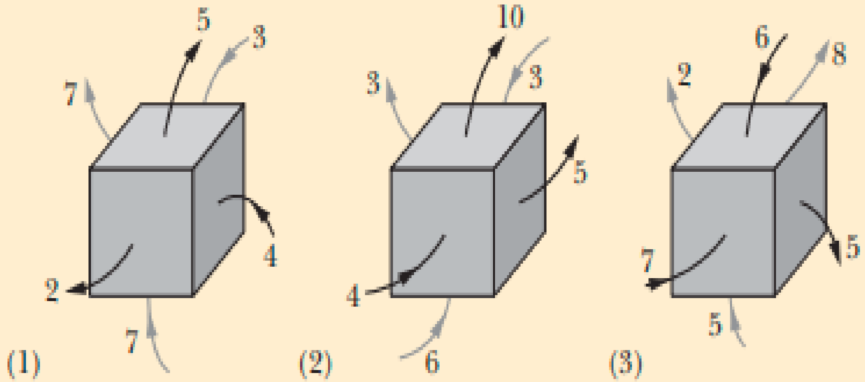
Checkpoint 1

The figure here shows a Gaussian cube of face area A immersed in a uniform electric field $\vec{E}$ that has the positive direction of the z axis. In terms of E and A, what is the flux through (a) the front face (which is in the xy plane), (b) the rear face, (c) the top face, and (d) the whole cube?



Checkpoint 2

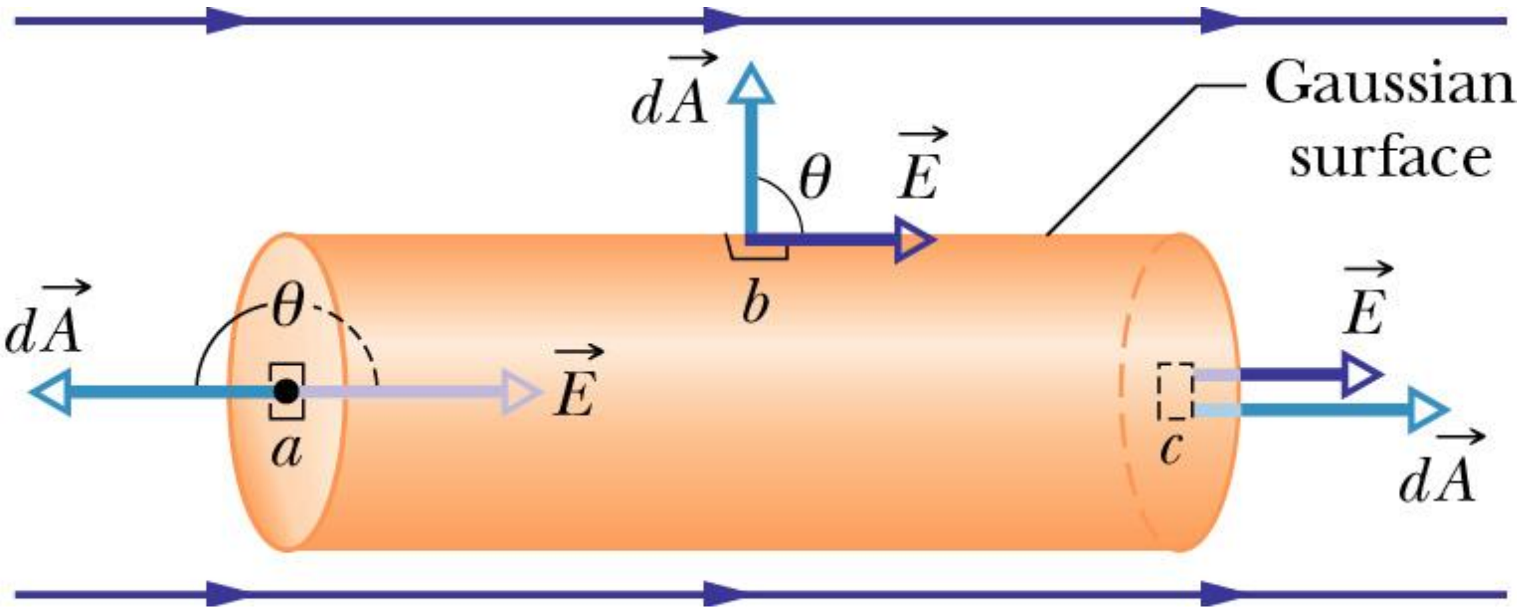
The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in $N \cdot m^2/C$) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.) In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?



Checkpoint 3

There is a certain net flux Φ_i through a Gaussian sphere of radius r enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to r, and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux through the new Gaussian surface greater than, less than, or equal to Φ_i?

Find the electric flux through a cylindrical surface in a uniform electric field **E**



$$\Phi = \oint \vec{E} \bullet d\vec{A} \qquad = \oint E\cos\theta dA \qquad\qquad d\vec{A} = \hat{n} dA$$

a. $\Phi = \int E\cos 180 dA = -\int EdA = -E\pi R^2$

b. $\Phi = \int E\cos 90 dA = 0$

c. $\Phi = \int E\cos 180 dA = \int EdA = E\pi R^2$

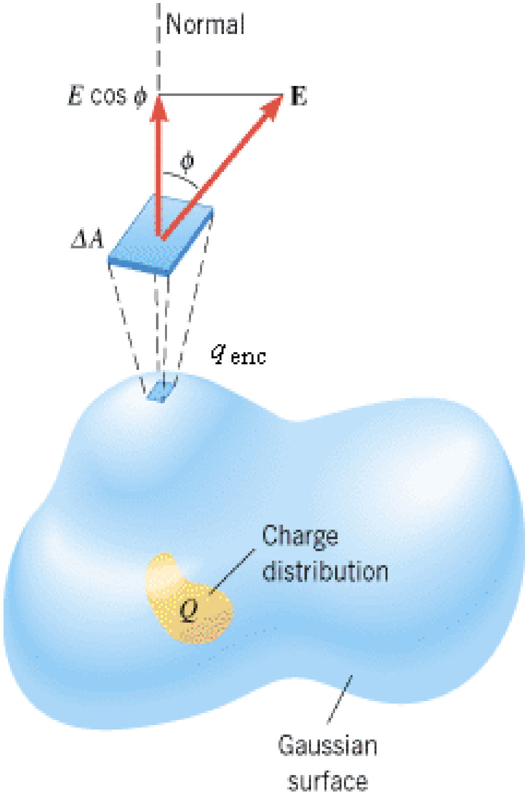
Flux from a. + b. + c. = 0

What is the flux if the cylinder were vertical ?

Suppose it were any shape?

Gauss' Law

For charge distribution Q:



The electric flux through a Gaussian surface times by ε_0 (the permittivity of free space) is equal to the net charge *Q* enclosed :

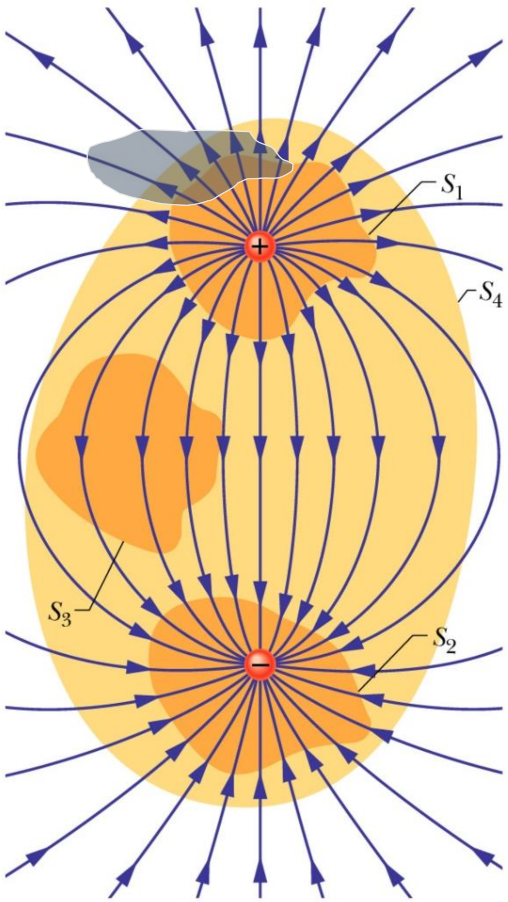
$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law)},$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss ' law)}.$$

- The net charge q_{enc} is the algebraic sum of all the *enclosed* charges.
- Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc}.

Example of Gauss' Law

- Consider a dipole with equal positive and negative charges.
- Imagine four surfaces S_1, S_2, S_3, S_4, as shown.
- S_1 encloses the positive charge. Note that the field is everywhere outward, so the flux is positive.
- S_2 encloses the negative charge. Note that the field is everywhere inward, so the flux through the surface is negative.
- S_3 encloses no charge. The flux through the surface is negative at the upper part, and positive at the lower part, but these cancel, and there is no net flux through the surface.
- S_4 encloses both charges. Again there is no net charge enclosed, so there is equal flux going out and coming in—no net flux through the surface.



Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E\cos\theta dA = \oint EdA$ E || n

Cos 0 = 1

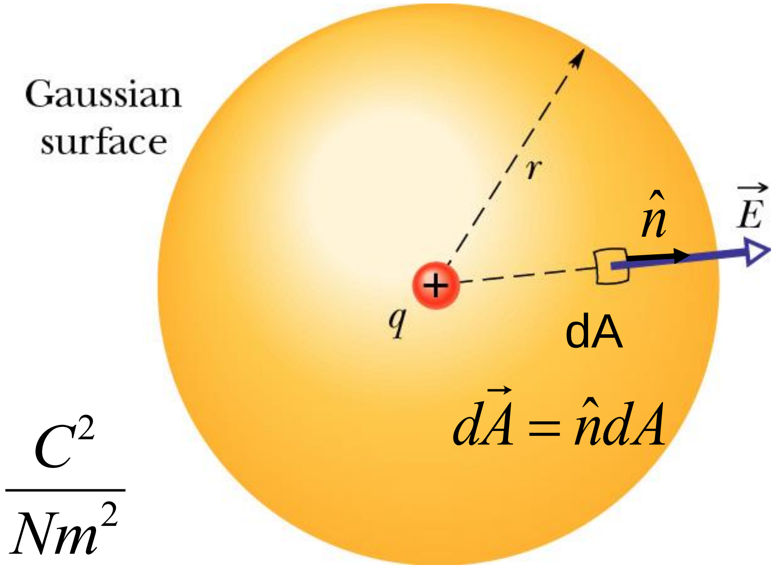
For a Point charge **E=kq/r²**

$$\Phi = \oint EdA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2(4\pi r^2)$$

$$\Phi = 4\pi kq$$

$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12}\frac{C^2}{Nm^2}$$



$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$

Gauss' Law

$k_e = 1/4\pi\epsilon_0$, we can write this equation in the form

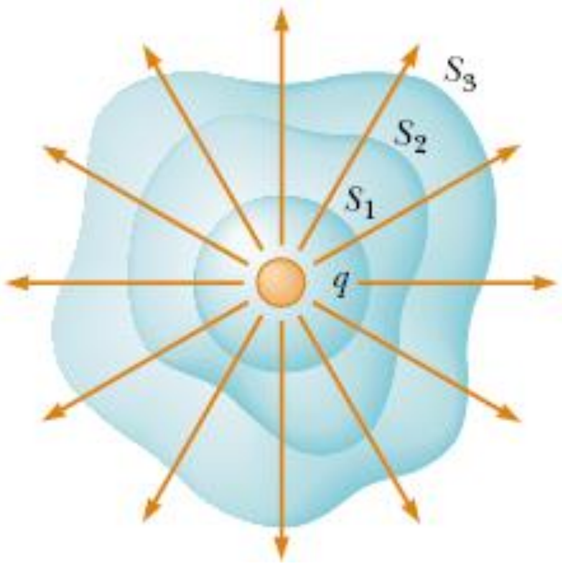
$$\Phi_E = \frac{q}{\epsilon_0}$$

Note from above equation that the net flux through the spherical surface is proportional to he charge inside. The flux is independent of the radius r because the area of the spherical surface is proportional to r^2, whereas the electric field is proportional to $1/r^2$. Thus, in the product of area and electric field, the dependence on r cancels.

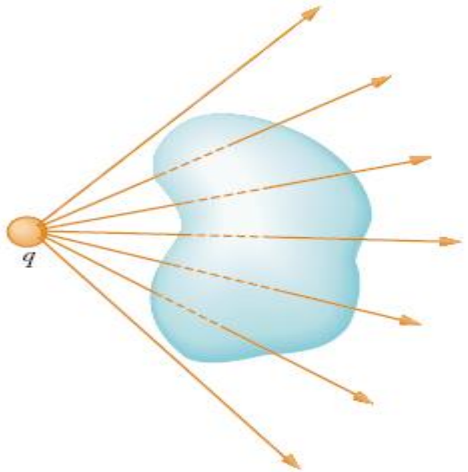
Now consider several closed surfaces surrounding a charge q,

Figure shows that the number of lines through S_1 *is equal to* the number of lines through the non spherical surfaces S_2 *and* S_3.

Therefore, we conclude that the net flux through any closed surface surrounding a point charge q is given by and q/ϵ_0 is independent of the shape of that surface.



Now consider a point charge located outside a closed surface of arbitrary shape, as shown in Figure,



the net electric flux through a closed surface that surrounds no charge is zero.

Let us extend these arguments to two generalized cases: (1) that of many point charges and (2) that of a continuous distribution of charge. We once again use the superposition principle, which states that **the electric field due to many charges is the vector sum of the electric fields produced by the individual charges.** Therefore, we can express the flux through any closed surface as

$$\oint \mathbf{E} \cdot d\mathbf{A} = \oint (\mathbf{E}_1 + \mathbf{E}_2 + \cdots) \cdot d\mathbf{A}$$

Gauss's law, which is a generalization of what we have just described, states that the net flux through *any* closed surface is

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

Flux Due to a Point Charge

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if

(A) the charge is tripled,

(A) The flux through the surface is tripled because flux is proportional to the amount of charge inside the surface.

(B) the radius of the sphere is doubled,

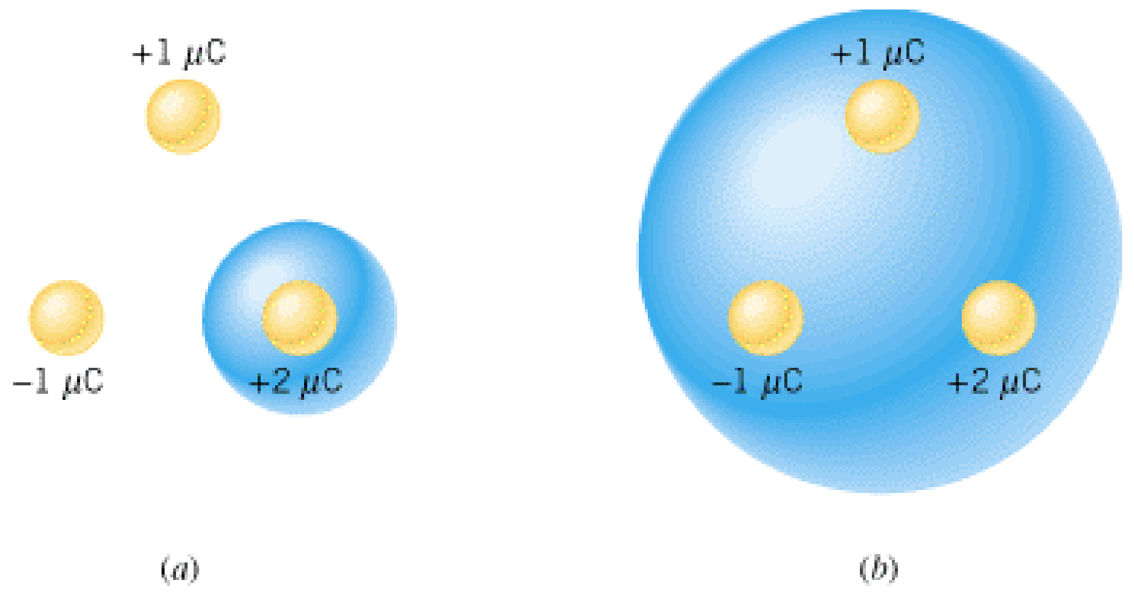
(B) The flux does not change because all electric field lines from the charge pass through the sphere, regardless of its radius.

(C) the surface is changed to a cube,

(C) The flux does not change when the shape of the gaussian surface changes because all electric field lines from the charge pass through the surface, regardless of its shape.

Check Your Understanding

The drawing shows an arrangement of three charges. In parts (*a*) and (*b*) different Gaussian surfaces are shown. Through which surface, if either, does the greater electric flux pass?



$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \tag{24.6}$$

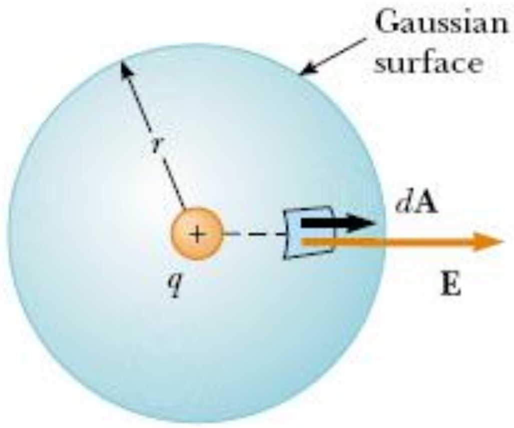
1. The value of the electric field can be argued by symmetry to be constant over the surface.
2. The dot product in Equation 24.6 can be expressed as a simple algebraic product $E\,dA$ because $\mathbf{E}$ and $d\mathbf{A}$ are parallel.
3. The dot product in Equation 24.6 is zero because $\mathbf{E}$ and $d\mathbf{A}$ are perpendicular.
4. The field can be argued to be zero over the surface.

Electric Field due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.

Solution A single charge represents the simplest possible charge distribution, and we use this familiar case to show how to solve for the electric field with Gauss's law. Figure 24.10 and our discussion of the electric field due to a point charge in Chapter 23 help us to conceptualize the physical situation. Because the space around the single charge has spherical symmetry, we categorize this problem as one in which there is enough symmetry to apply Gauss's law. To analyze any Gauss's law problem, we consider the details of the electric field and choose a gaussian surface that satisfies some or all of the conditions that we have listed above. We choose a spherical gaussian surface of radius r centered on the point charge, as shown in Figure 24.10. The electric field due to a positive point charge is directed radially outward by symmetry and is therefore normal to the surface at every point. Thus, as in condition (2), $\mathbf{E}$ is parallel to $d\mathbf{A}$ at each point. Therefore, $\mathbf{E} \cdot d\mathbf{A} = E\,dA$ and Gauss's law gives

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

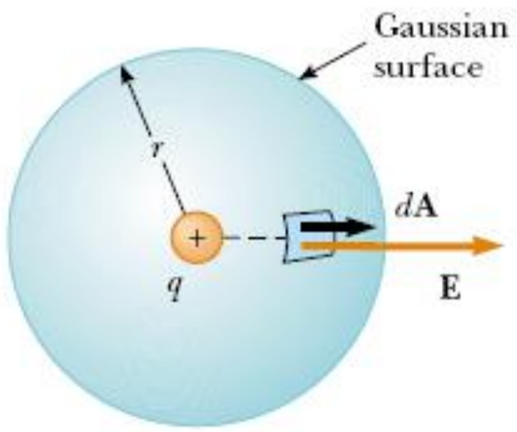


By symmetry, E is constant everywhere on the surface, which satisfies condition (1), so it can be removed from the integral. Therefore,

$$\oint E\,dA = E \oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

where we have used the fact that the surface area of a sphere is $4\pi r^2$. Now, we solve for the electric field:

$$E = \frac{q}{4\pi\epsilon_0 r^2} = \boxed{k_e \frac{q}{r^2}}$$

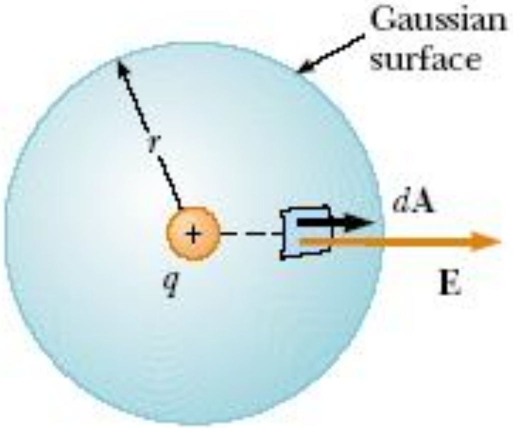


What If? **What if the charge in Figure 24.10 were not at the center of the spherical gaussian surface?**

Answer In this case, while Gauss's law would still be valid, the situation would not possess enough symmetry to evaluate the electric field. Because the charge is not at the center, the magnitude of **E** would vary over the surface of the sphere and the vector **E** would not be everywhere perpendicular to the surface.

The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.



$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\, dA = \frac{q}{\epsilon_0}$$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2} = \boxed{k_e \frac{q}{r^2}}$$

A spherically symmetric Charge Distribution

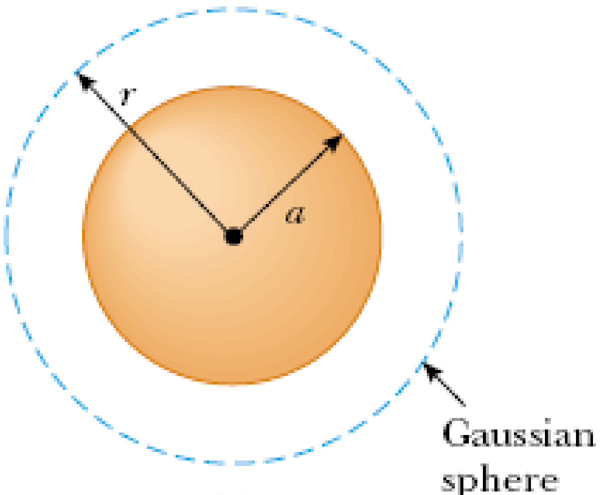
An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q (Fig. 24.11).

(A) Calculate the magnitude of the electric field at a point outside the sphere.

Solution Because the charge distribution is spherically symmetric, we again select a spherical gaussian surface of radius r, concentric with the sphere, as shown in Figure 24.11a. For this choice, conditions (1) and (2) are satisfied, as they were for the point charge in Example 24.4. Following the line of reasoning given in Example 24.4, we find that

$$(1) \qquad E = \boxed{k_e \frac{Q}{r^2}} \qquad (\text{for } r > a)$$

Note that this result is identical to the one we obtained for a point charge. Therefore, we conclude that, **for a uniformly charged sphere, the field in the region external to the sphere is *equivalent* to that of a point charge located at the center of the sphere.**



(a)

(B) Find the magnitude of the electric field at a point inside the sphere.

Solution In this case we select a spherical gaussian surface having radius $r < a$, concentric with the insulating sphere (Fig. 24.11b). Let us denote the volume of this smaller sphere by V'. To apply Gauss's law in this situation, it is important to recognize that the charge q_{in} within the gaussian surface of volume V' is less than Q. To calculate q_{in}, we use the fact that $q_{\text{in}} = \rho V'$:

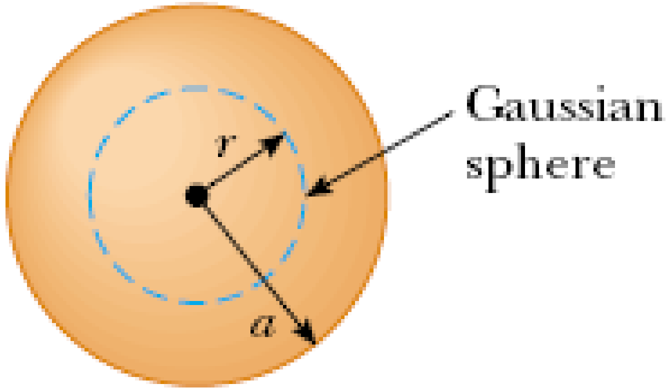
$$q_{\text{in}} = \rho V' = \rho \left(\tfrac{4}{3}\pi r^3\right)$$

By symmetry, the magnitude of the electric field is constant everywhere on the spherical gaussian surface and is normal to the surface at each point—both conditions (1) and (2) are satisfied. Therefore, Gauss's law in the region $r < a$ gives

$$\oint E\,dA = E\oint dA = E\,(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

Solving for E gives

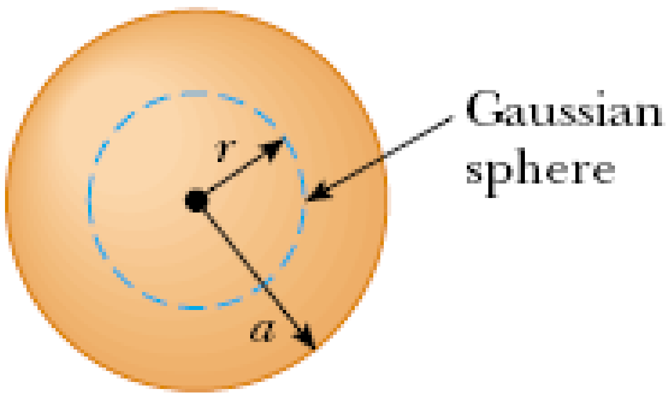
$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}\,r$$



(b)

Because $\rho = Q/\frac{4}{3}\pi a^3$ by definition and because $k_e = 1/4\pi\epsilon_0$, this expression for E can be written as

$$(2) \qquad E = \frac{Qr}{4\pi\epsilon_0 a^3} = \boxed{k_e \frac{Q}{a^3}\, r} \qquad (\text{for } r < a)$$



(b)

Note that this result for E differs from the one we obtained in part (A). It shows that $E \rightarrow 0$ as $r \rightarrow 0$. Therefore, the result eliminates the problem that would exist at $r = 0$ if E varied as $1/r^2$ inside the sphere as it does outside the sphere. That is, if $E \propto 1/r^2$ for $r < a$, the field would be infinite at $r = 0$, which is physically impossible.

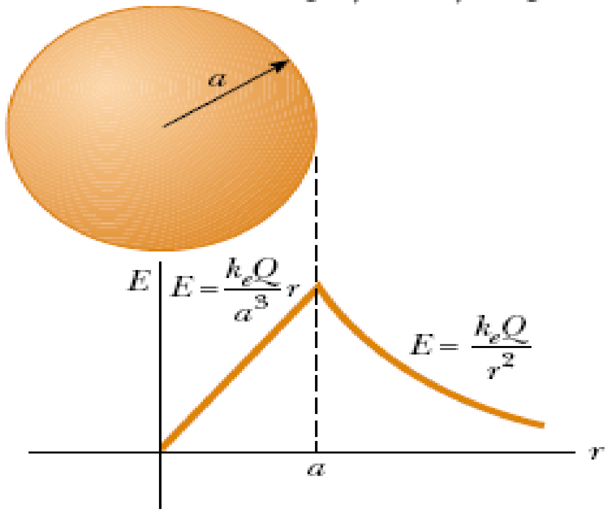


Figure 24.12 (Example 24.5) A plot of E versus r for a uniformly charged insulating sphere. The electric field inside the sphere ($r < a$) varies linearly with r. The field outside the sphere ($r > a$) is the same as that of a point charge Q located at $r = 0$.

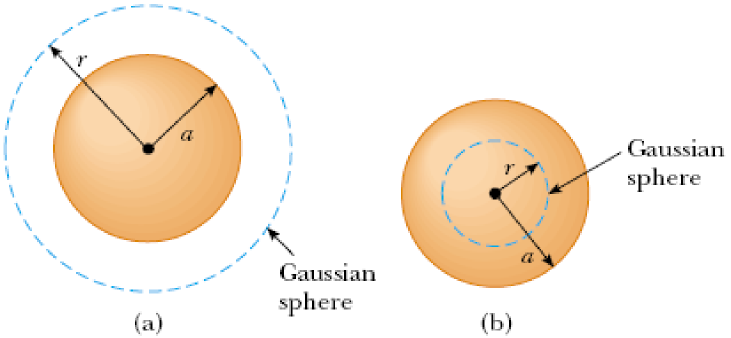
What If? **Suppose we approach the radial position $r = a$ from inside the sphere and from outside. Do we measure the same value of the electric field from both directions?**

Answer From Equation (1), we see that the field approaches a value from the outside given by

$$E = \lim_{r \to a} \left(k_e \frac{Q}{r^2} \right) = k_e \frac{Q}{a^2}$$

From the inside, Equation (2) gives us

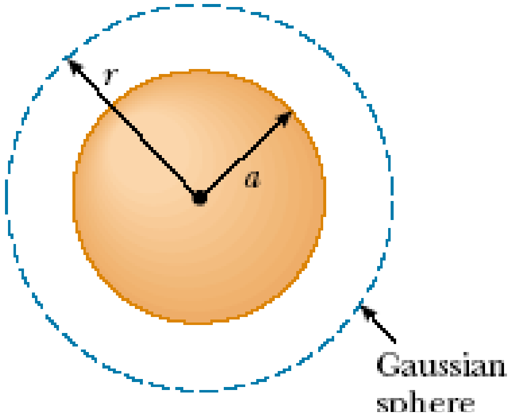
$$E = \lim_{r \to a} \left(k_e \frac{Q}{a^3} r \right) = k_e \frac{Q}{a^3} a = k_e \frac{Q}{a^2}$$



Thus, the value of the field is the same as we approach the surface from both directions. A plot of E versus r is shown in Figure 24.12. Note that the magnitude of the field is continuous, but the derivative of the field magnitude is not.

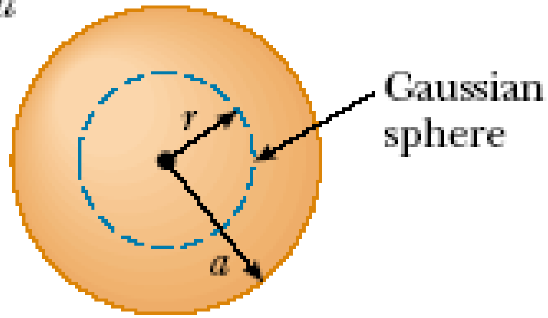
A Spherically Symmetric Charge Distribution

$$E = \boxed{k_e \frac{Q}{r^2}} \qquad (\text{for } r > a)$$

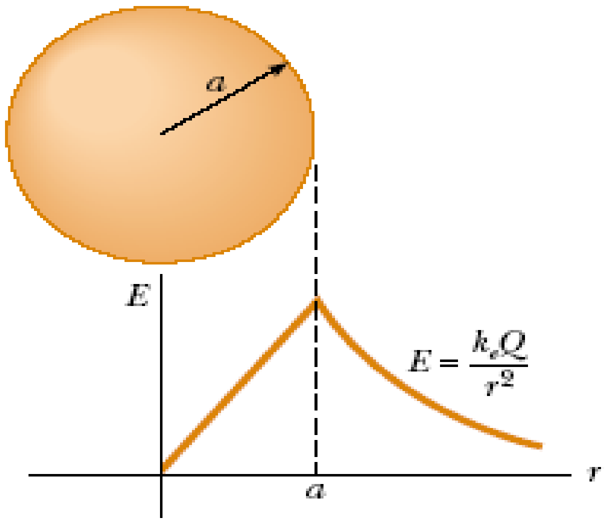


$$q_{\text{in}} = \rho V' = \rho\left(\tfrac{4}{3}\pi r^3\right)$$

$$\rho = Q/\tfrac{4}{3}\pi a^3$$



$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

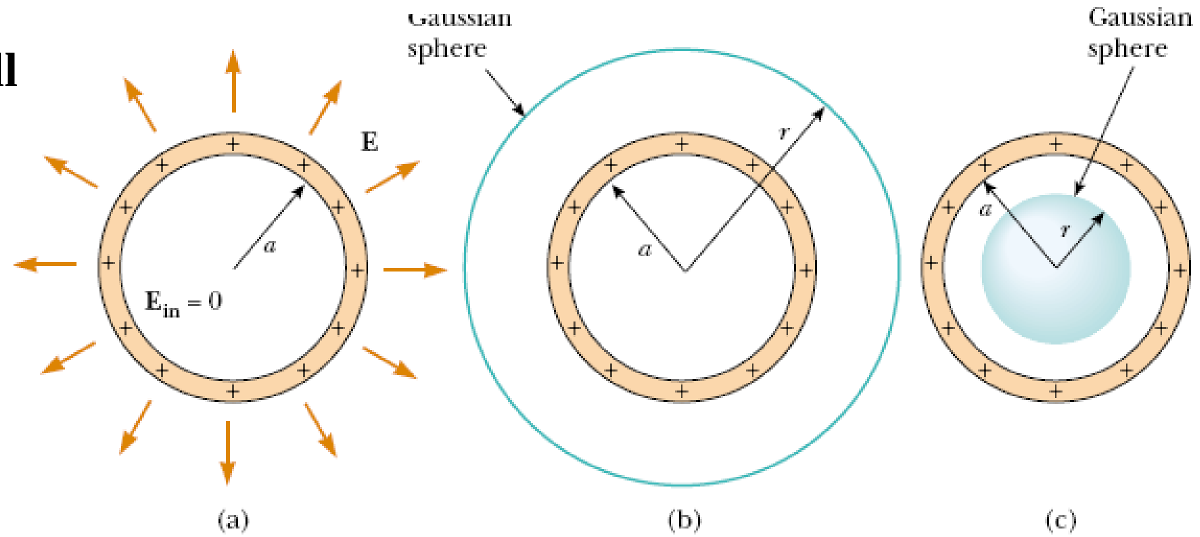


$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \boxed{\frac{k_e Q}{a^3}\,r} \qquad (\text{for } r < a)$$

Electric Field due to Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Fig. 24.13a). Find the electric field at points

(A) Outside the Shell



Solution

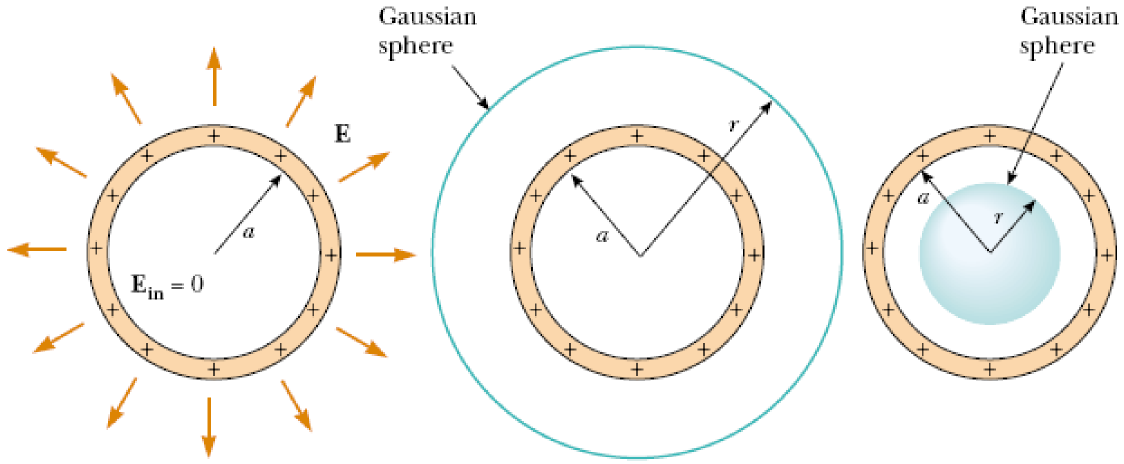
(A) The calculation for the field outside the shell is identical to that for the solid sphere shown in Example 24.5a. If we construct a spherical gaussian surface of radius $r > a$ concentric with the shell (Fig. 24.13b), the charge inside this surface is Q. Therefore, the field at a point outside the shell is equivalent to that due to a point charge Q located at the center:

$$E = \boxed{k_e \frac{Q}{r^2}} \qquad (\text{for } r > a)$$

(B) inside the shell.

(B) The electric field inside the spherical shell is zero. This follows from Gauss's law applied to a spherical surface of radius $r < a$ concentric with the shell (Fig. 24.13c). Because of the spherical symmetry of the charge distribution and because the net charge inside the surface is zero—satisfaction of conditions (1) and (2) again—application of Gauss's law shows that $E = 0$ in the region $r < a$. We obtain the same results using Equation 23.11 and integrating over the charge distribution. This calculation is rather complicated. Gauss's law allows us to determine these results in a much simpler way.

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \qquad (23.11)$$

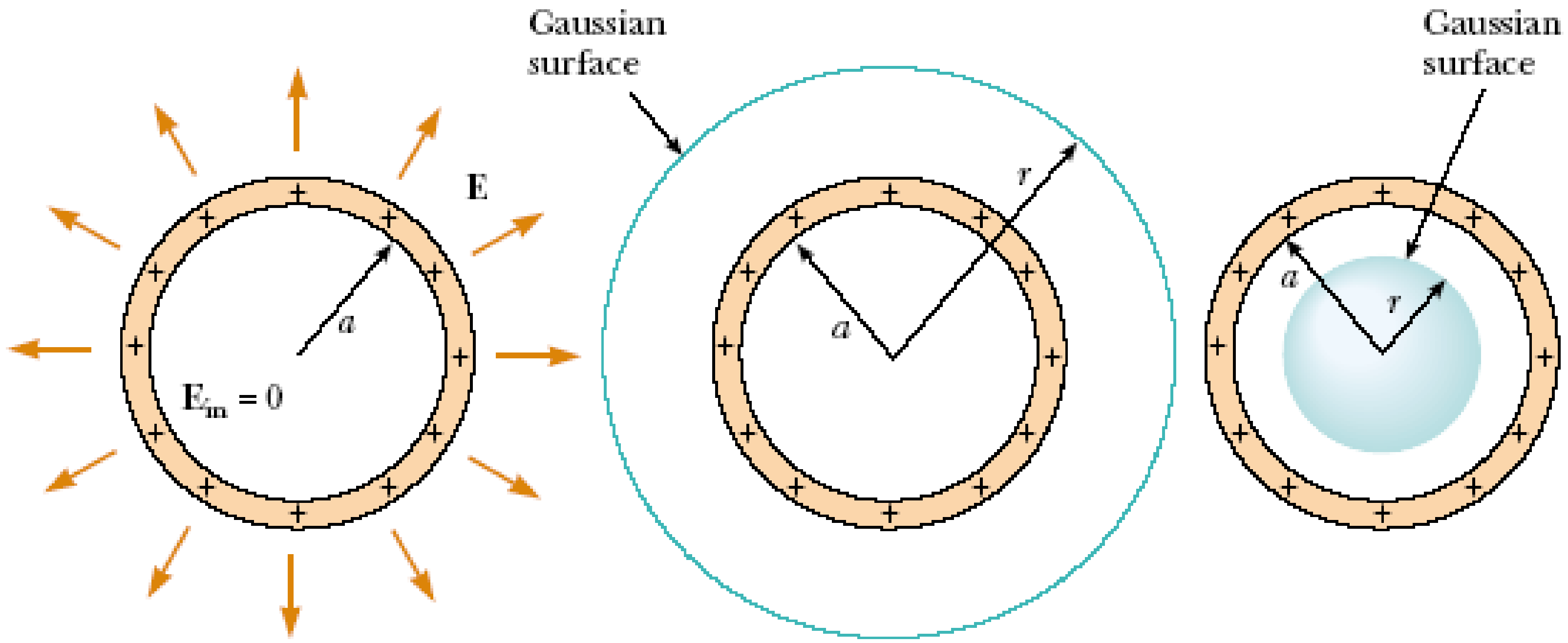


(a) (b) (c)

The Electric Field Due to a Thin Spherical Shell

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

$$E = k_e \frac{Q}{r^2} \quad (\text{for } r > a) \qquad E = 0 \text{ in the region } r < a.$$

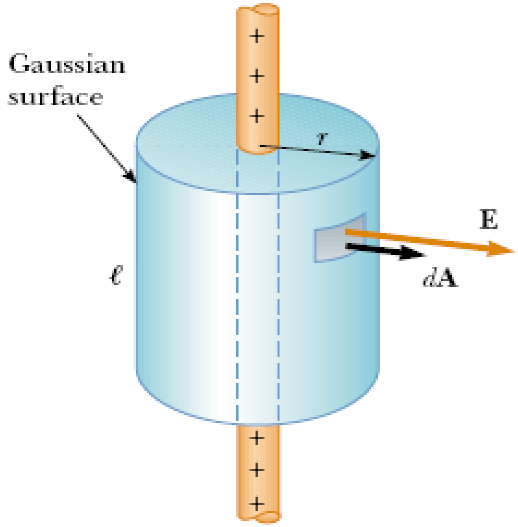


Electric Field due to a Cylindrically Symmetric Charge Distribution

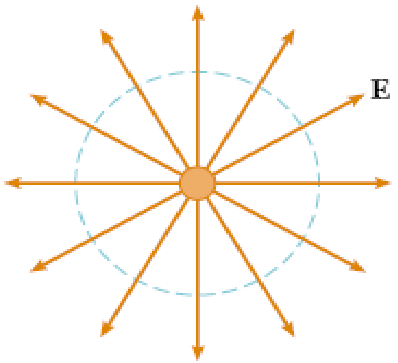
Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ (Fig. 24.14a).

Solution The symmetry of the charge distribution requires that **E** be perpendicular to the line charge and directed outward, as shown in Figure 24.14a and b. To reflect the symmetry of the charge distribution, we select a cylindrical gaussian surface of radius r and length ℓ that is coaxial with the line charge. For the curved part of this surface, **E** is constant in magnitude and perpendicular to the surface at each point—satisfaction of conditions (1) and (2). Furthermore, the flux through the ends of the gaussian cylinder is zero because **E** is parallel to these surfaces—the first application we have seen of condition (3).

We take the surface integral in Gauss's law over the entire gaussian surface. Because of the zero value of $\mathbf{E} \cdot d\mathbf{A}$ for the ends of the cylinder, however, we can restrict our attention to only the curved surface of the cylinder.



(a)



(b)

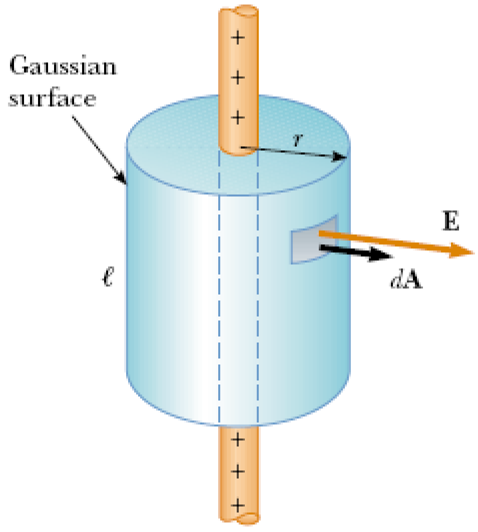
The total charge inside our gaussian surface is $\lambda\ell$. Applying Gauss's law and conditions (1) and (2), we find that for the curved surface

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda\ell}{\epsilon_0}$$

The area of the curved surface is $A = 2\pi r\ell$; therefore,

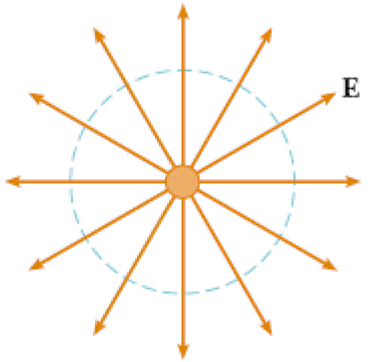
$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2k_e \frac{\lambda}{r}} \qquad (24.7)$$



(a)

Thus, we see that the electric field due to a cylindrically symmetric charge distribution varies as 1/r,
whereas the field external to a spherically symmetric charge distribution varies as $1/r^2$.

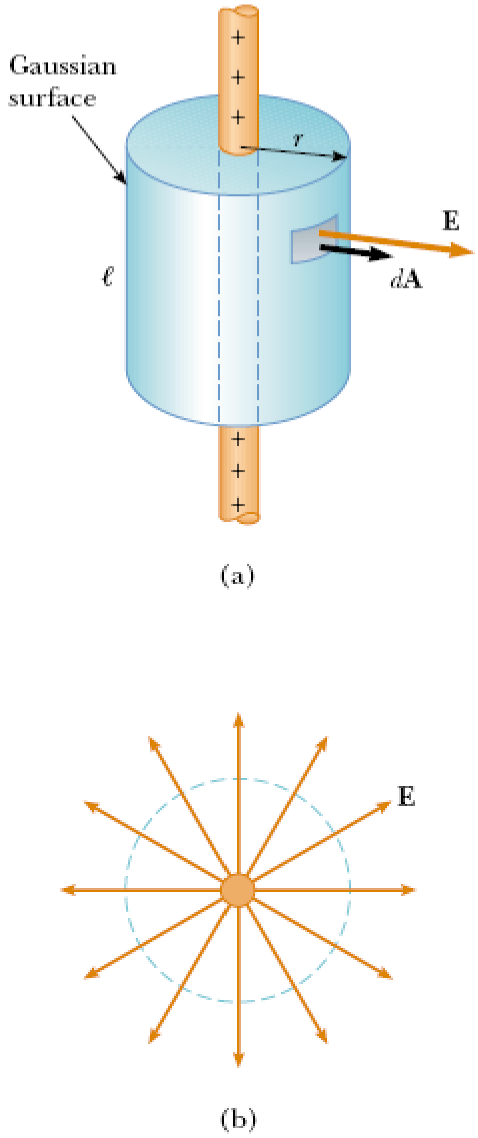


(b)

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2k_e \frac{\lambda}{r}}$$

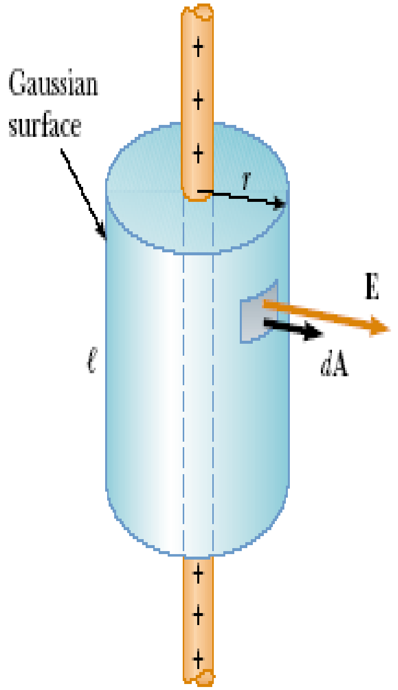
What If? What if the line segment in this example were not infinitely long?

Answer If the line charge in this example were of finite length, the result for E would not be that given by Equation 24.7. A finite line charge does not possess sufficient symmetry for us to make use of Gauss's law. This is because the magnitude of the electric field is no longer constant over the surface of the gaussian cylinder—the field near the ends of the line would be different from that far from the ends. Thus, condition (1) would not be satisfied in this situation. Furthermore, **E** is not perpendicular to the cylindrical surface at all points—the field vectors near the ends would have a component parallel to the line. Thus, condition (2) would not be satisfied. For points close to a finite line charge and far from the ends, Equation 24.7 gives a good approximation of the value of the field.



A Cylindrically Symmetric Charge Distribution

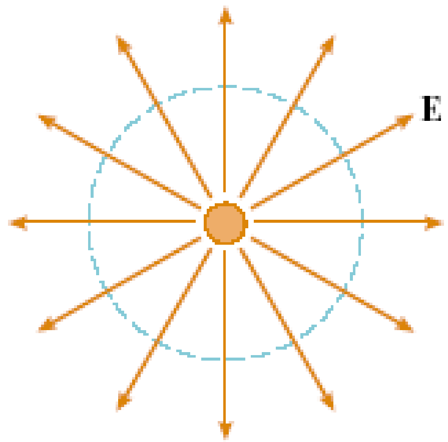
$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$



$$A = 2\pi r \ell$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2k_e \frac{\lambda}{r}}$$



Electric Field at a point near infinite line of charge

Key Idea

- The electric field at a point near an infinite line of charge (or charged rod) with uniform linear charge density λ is perpendicular to the line and has magnitude

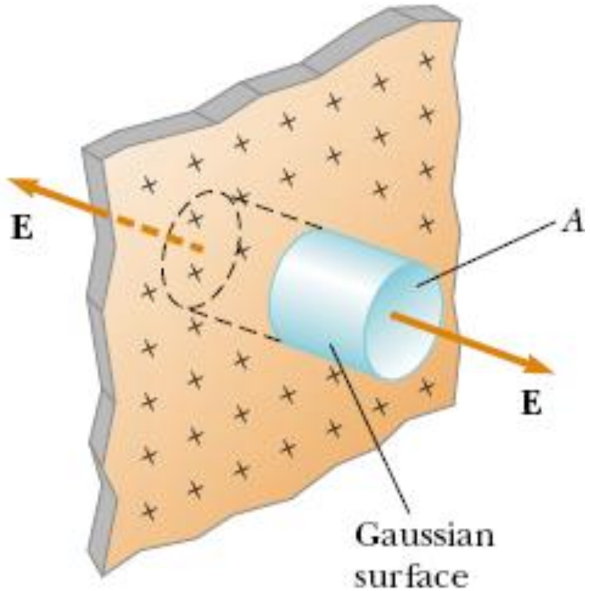
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge)},$$

where r is the perpendicular distance from the line to the point.

Electric Field due to a Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.

Solution By symmetry, **E** must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane. The fact that the direction of **E** is away from positive charges indicates that the direction of **E** on one side of the plane must be opposite its direction on the other side, as shown in Figure 24.15. A gaussian surface that reflects the symmetry is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area A and are equidistant from the plane. Because **E** is parallel to the curved surface—and, therefore, perpendicular to $d\mathbf{A}$ everywhere on the surface—condition (3) is satisfied and there is no contribution to the surface integral from this surface. For the flat ends of the cylinder, conditions (1) and (2) are satisfied. The flux through each end of the cylinder is EA; hence, the total flux through the entire gaussian surface is just that through the ends, $\Phi_E = 2EA$.



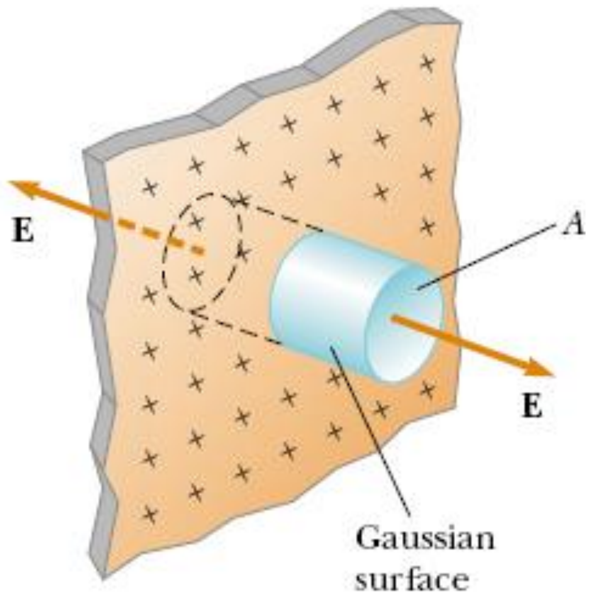
Noting that the total charge inside the surface is $q_{in} = \sigma A$, we use Gauss's law and find that the total flux through the gaussian surface is

$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

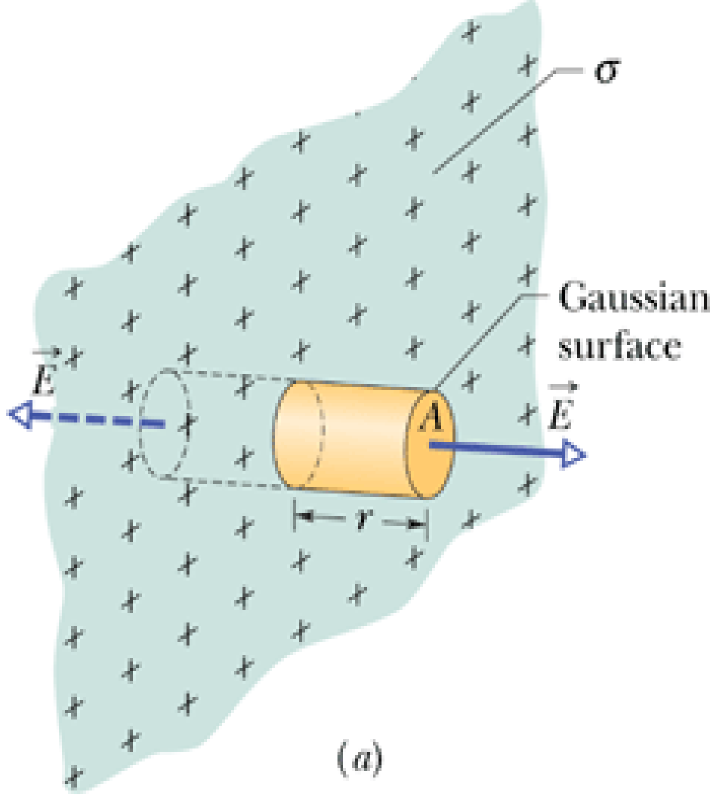
leading to

$$\boxed{E = \frac{\sigma}{2\epsilon_0}} \qquad (24.8)$$

Because the distance from each flat end of the cylinder to the plane does not appear in Equation 24.8, we conclude that $E = \sigma/2\epsilon_0$ at *any* distance from the plane. That is, the field is uniform everywhere.



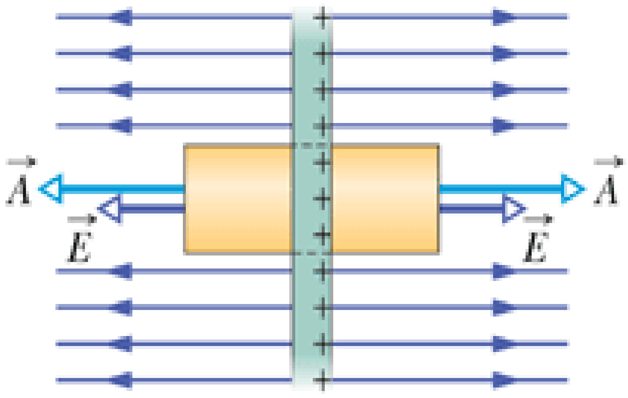
A Nonconducting Plane of Charge



(a)

$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \boxed{\frac{\sigma}{2\epsilon_0}}$$



(b)

Key Ideas

- The electric field due to an infinite nonconducting sheet with uniform surface charge density σ is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(nonconducting sheet of charge).}$$

- The external electric field just outside the surface of an isolated charged conductor with surface charge density σ is perpendicular to the surface and has magnitude

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(external, charged conductor).}$$

Inside the conductor, the electric field is zero.

What If? Suppose we place two infinite planes of charge parallel to each other, one positively charged and the other negatively charged. Both planes have the same surface charge density. What does the electric field look like now?

Answer In this situation, the electric fields due to the two planes add in the region between the planes, resulting in a uniform field of magnitude σ/ϵ_0, and cancel elsewhere to give a field of zero.

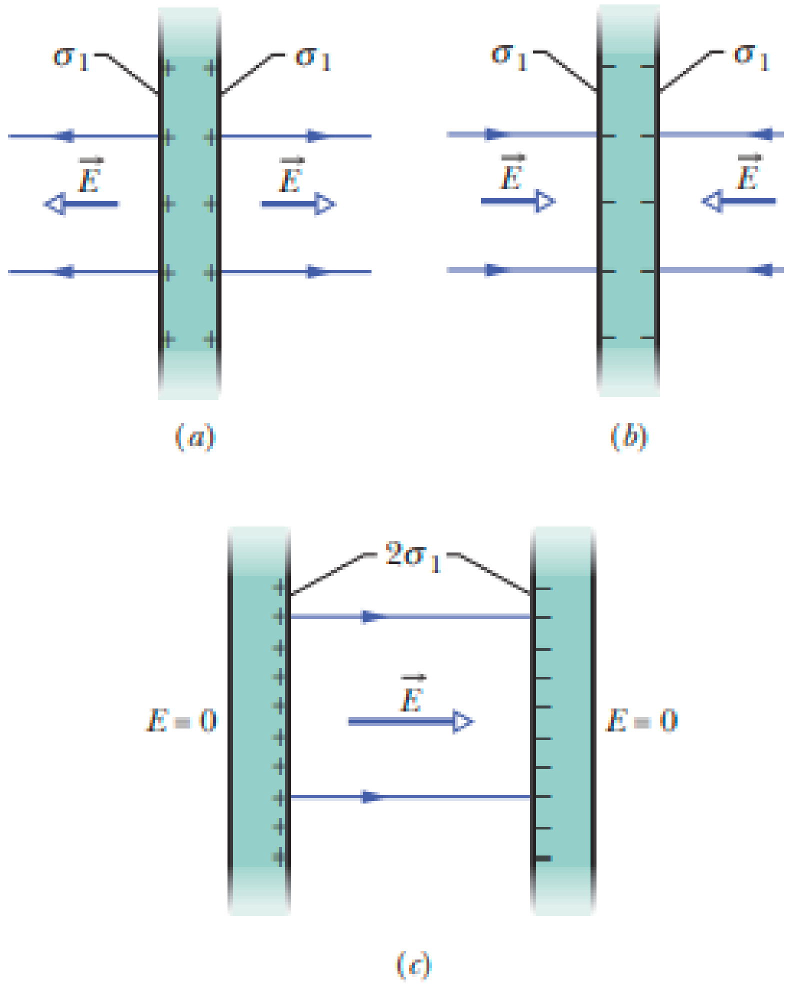
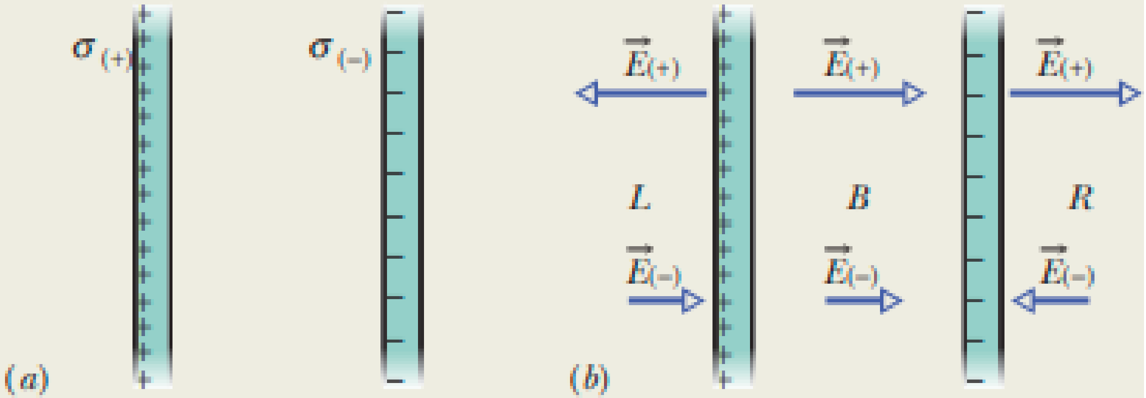


Figure 23-18 (*a*) A thin, very large conducting plate with excess positive charge. (*b*) An identical plate with excess negative charge. (*c*) The two plates arranged so they are parallel and close.

Sample Problem 23.07 Electric field near two parallel nonconducting sheets with charge

Figure 23-19a shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu\text{C/m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu\text{C/m}^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.



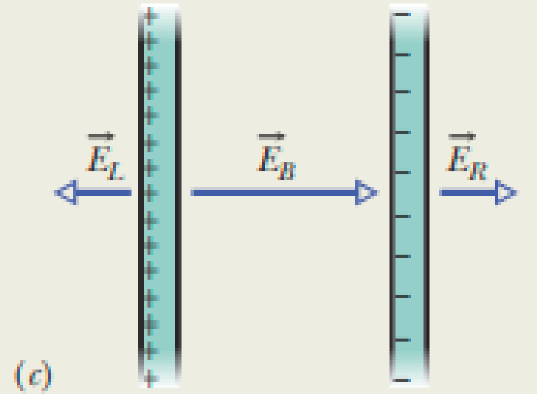


Figure 23-19 (a) Two large, parallel sheets, uniformly charged on one side. (b) The individual electric fields resulting from the two charged sheets. (c) The net field due to both charged sheets, found by superposition.

KEY IDEA

With the charges fixed in place (they are on nonconductors), we can find the electric field of the sheets in Fig. 23-19a by (1) finding the field of each sheet as if that sheet were isolated and (2) algebraically adding the fields of the isolated sheets via the superposition principle. (We can add the fields algebraically because they are parallel to each other.)

Calculations: At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$E_{(+)} = \frac{\sigma_{(+)}}{2\varepsilon_0} = \frac{6.8 \times 10^{-6}\ \mathrm{C/m^2}}{(2)(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})}$$
$$= 3.84 \times 10^5\ \mathrm{N/C}.$$

Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed *toward* that sheet and has the magnitude

$$E_{(-)} = \frac{\sigma_{(-)}}{2\varepsilon_0} = \frac{4.3 \times 10^{-6}\ \mathrm{C/m^2}}{(2)(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})}$$
$$= 2.43 \times 10^5\ \mathrm{N/C}.$$

Figure 23-19*b* shows the fields set up by the sheets to the left of the sheets (L), between them (B), and to their right (R).

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$\begin{aligned} E_L &= E_{(+)} - E_{(-)} \\ &= 3.84 \times 10^5\ \mathrm{N/C} - 2.43 \times 10^5\ \mathrm{N/C} \\ &= 1.4 \times 10^5\ \mathrm{N/C}. \end{aligned} \qquad \text{(Answer)}$$

Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field $\vec{E}_L$ in this region is directed to the left, as Fig. 23-19*c* shows. To the right of the sheets, the net electric field has the same magnitude but is directed to the right, as Fig. 23-19*c* shows.

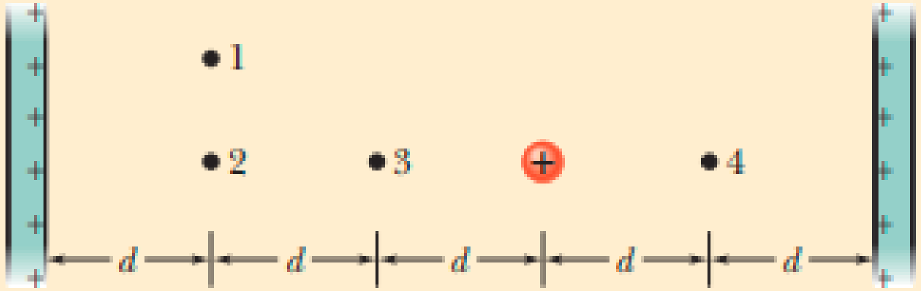
Between the sheets, the two fields add and we have

$$\begin{aligned} E_B &= E_{(+)} + E_{(-)} \\ &= 3.84 \times 10^5\ \mathrm{N/C} + 2.43 \times 10^5\ \mathrm{N/C} \\ &= 6.3 \times 10^5\ \mathrm{N/C}. \end{aligned} \qquad \text{(Answer)}$$

The electric field $\vec{E}_B$ is directed to the right.

Checkpoint 4

The figure shows two large, parallel, nonconducting sheets with identical (positive) uniform surface charge densities, and a sphere with a uniform (positive) volume charge density. Rank the four numbered points according to the magnitude of the net electric field there, greatest first.

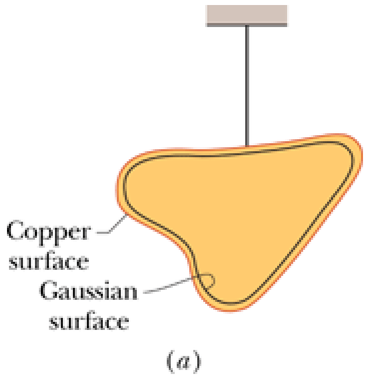


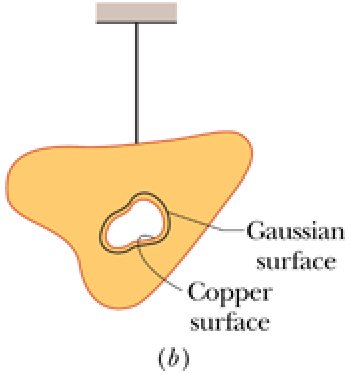
Question:

What is the relation between distance and electric field in case of :

1. Point charge
2. Infinite line charge
3. Infinite sheet of charge
4. Electric Dipole

A Charged Isolated Conductor

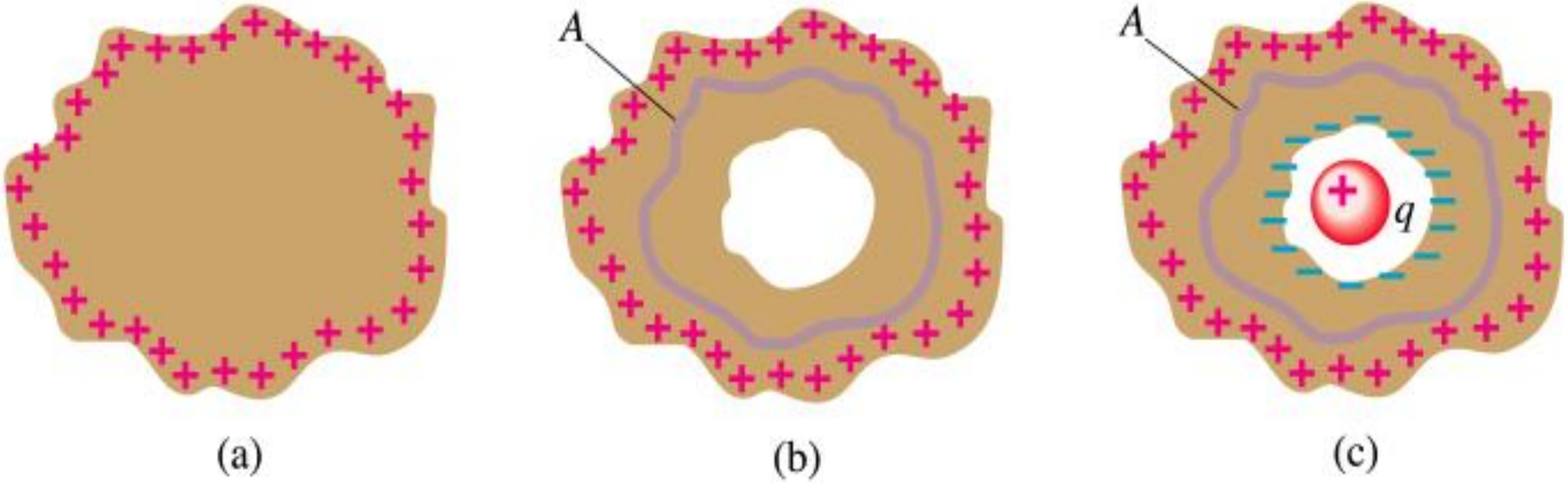




- If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.
- For an Isolated Conductor with a Cavity, There is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

- The electric field inside this conductor must be zero. If this were not so, the field would exert forces on the conduction (free) electrons, which are always present in a conductor, and thus current would always exist within a conductor. (That is, charge would flow from place to place within the conductor.) Of course, there is no such perpetual current in an isolated conductor, and so the internal electric field is zero.
- (An internal electric field does appear as a conductor is being charged. However, the added charge quickly distributes itself in such a way that the net internal electric field—the vector sum of the electric fields due to all the charges, both inside and outside — is zero. The movement of charge then ceases, because the net force on each charge is zero; the charges are then in electrostatic equilibrium.)



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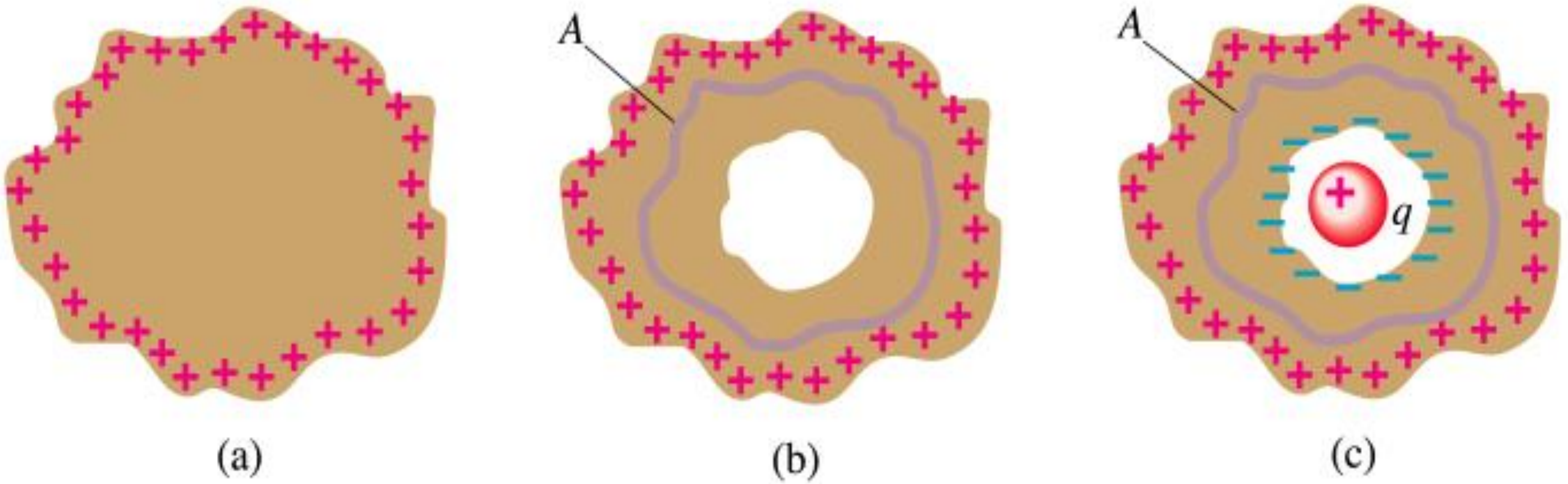
Find electric charge q on surface of hole in the charged conductor.



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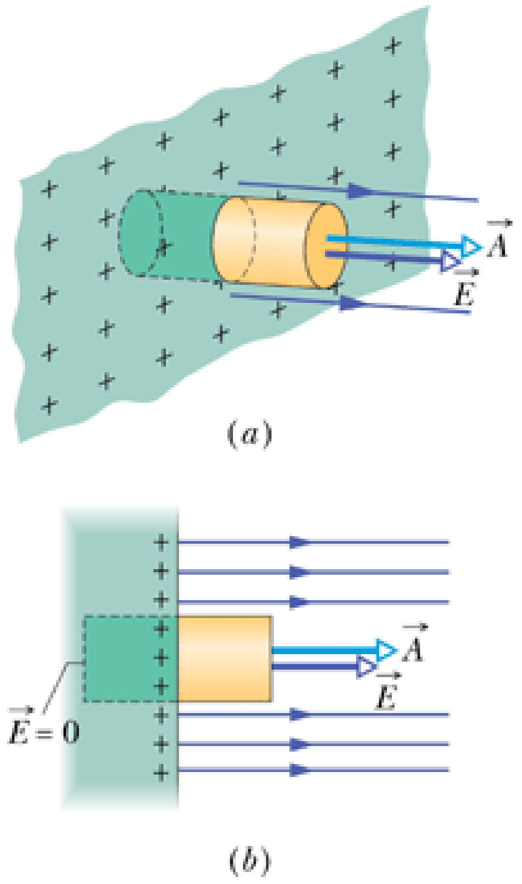
The solution of this problem lies in the fact that the electric field inside a conductor is zero and if we place our Gaussian surface inside the conductor (where the field is zero), the charge enclosed must be zero (+ q – q) = 0.

Find electric charge q on surface of hole in the charged conductor.

The External Electric Field of a Conductor



If σ is the charge per unit area,

according to Gauss' law,

$$\varepsilon_0 EA = \sigma A,$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface)}.$$

Adapted from:

Book:

- Fundamentals of Physics by Halliday Resnick & Jearl Walker 10^{th} edition