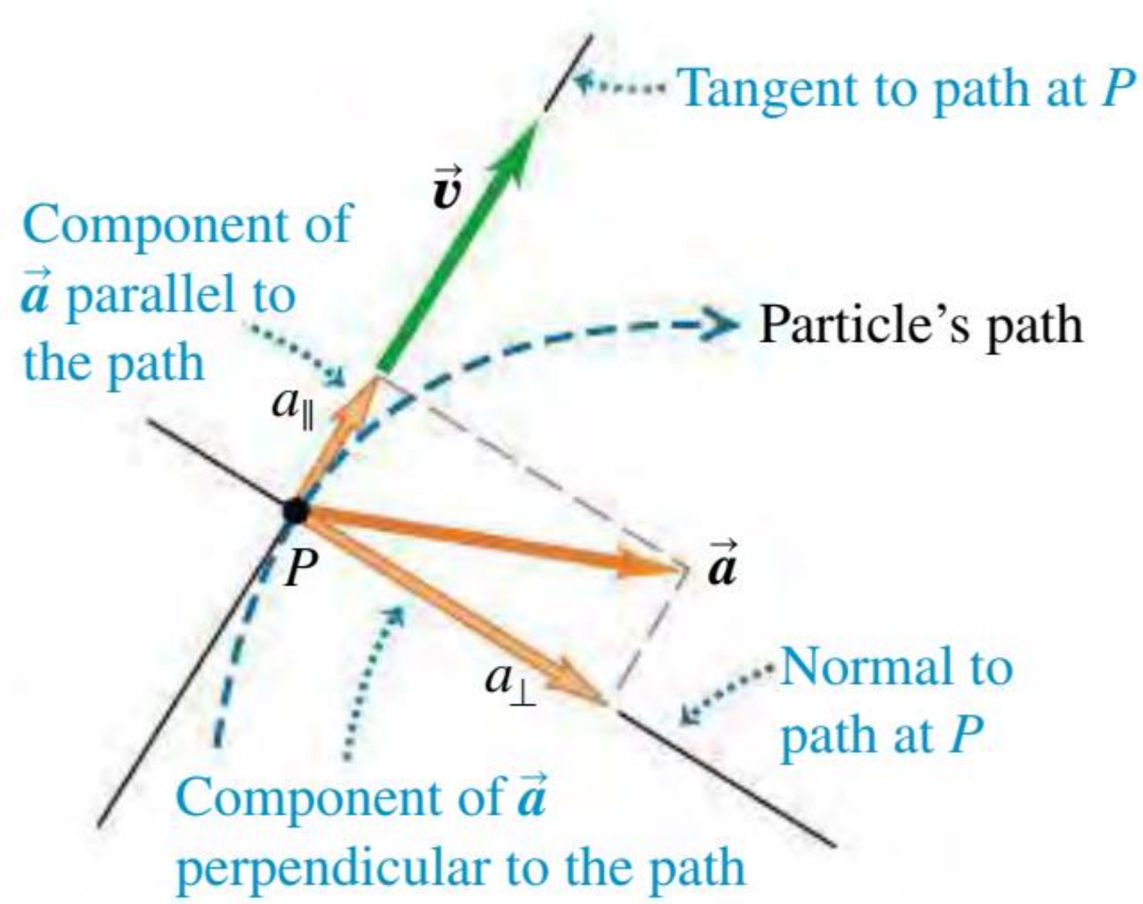
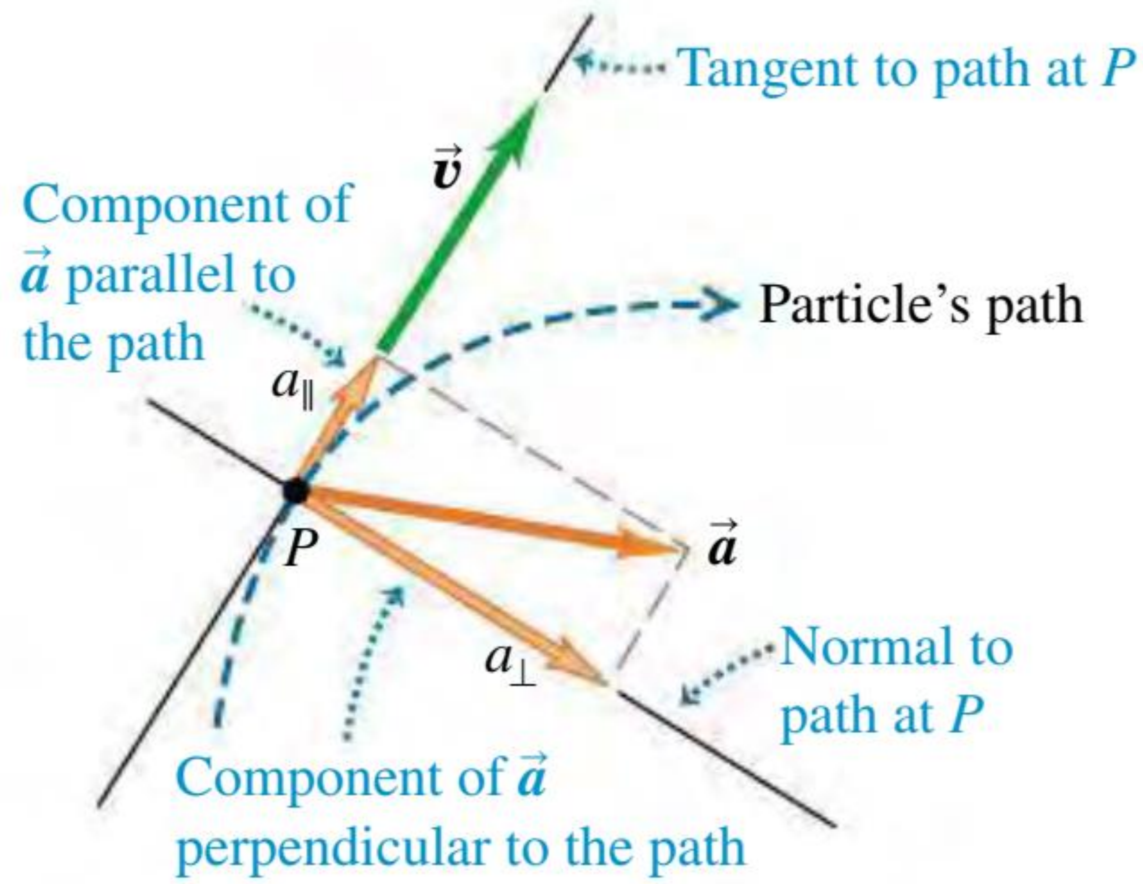
Motion in two and three dimension

Projectile Motion

Acceleration in 2D



Acceleration in 2D



Projectile & Projectile Motion

- A particle moves in a vertical plane with some initial velocity but its acceleration is always the freefall acceleration , which is downward.

- Such a particle is called a **projectile (meaning** that it is projected or launched), and its motion is called **projectile motion.**

- In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

Examples:

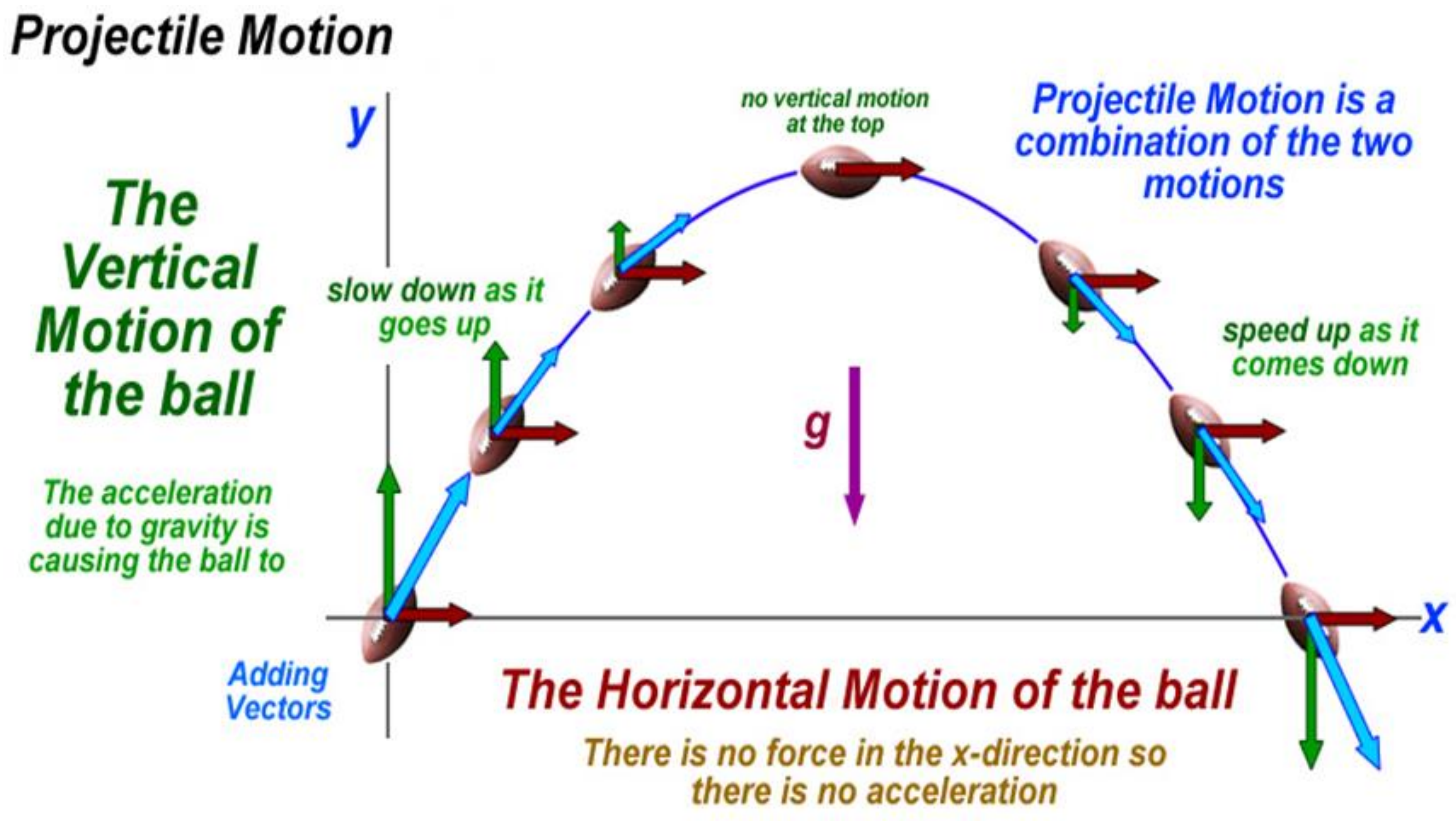
Projectile might be ;

- a tennis ball
- a baseball in flight
- Cannon ball shot from a cannon
- A basket ball thrown in a basket
- A kicked football

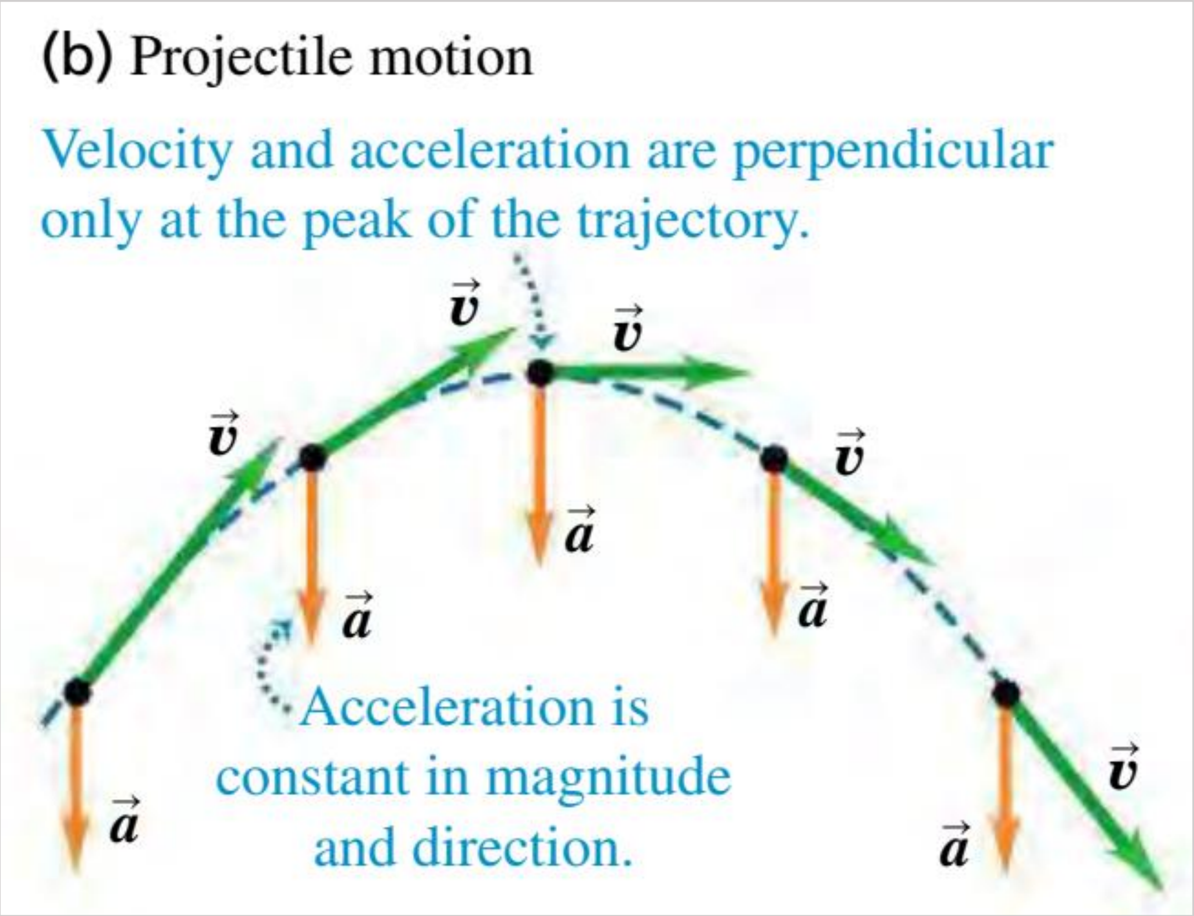
Many sports (from golf and football to lacrosse and Racquetball)involve the projectile motion of a ball, and much effort is spent in trying to control that motion for an advantage.

Note: Projectile is not an airplane in flight

Vector Perspective

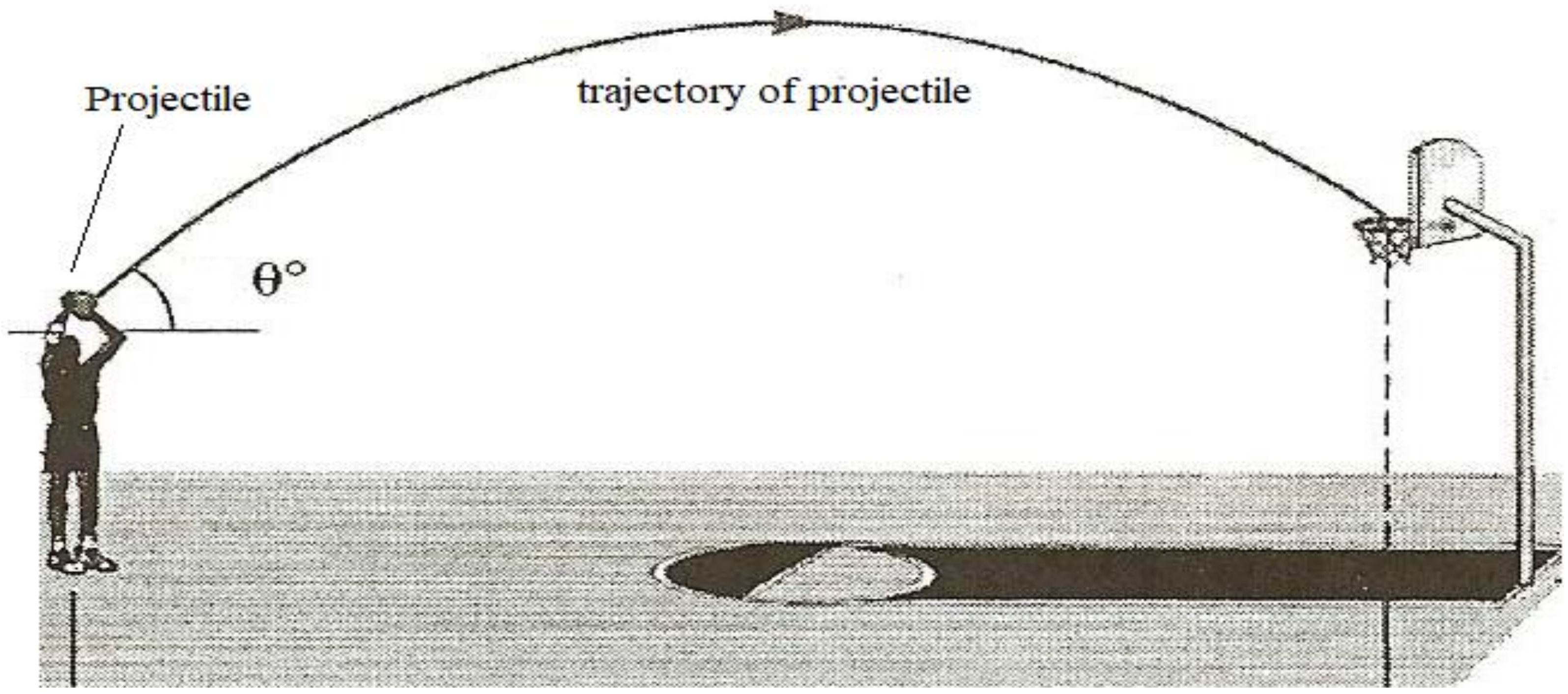


Vector Perspective

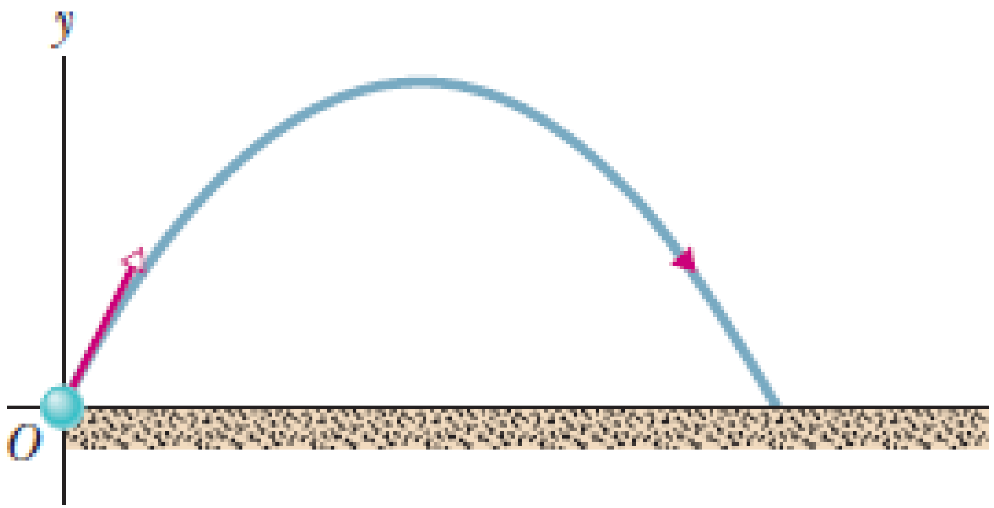


Trajectory of projectile

- Path followed by the projectile is called as **trajectory of that projectile.**



Pictorial Representation

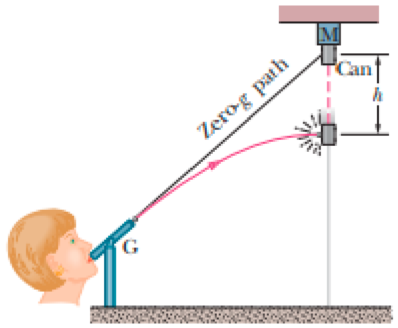


The *projectile motion* of an object launched into the air at the origin of a coordinate system and with launch velocity $\vec{v}_0$ at angle θ_0. The motion is a combination of vertical motion (constant acceleration) and horizontal motion (constant velocity), as shown by the velocity components.

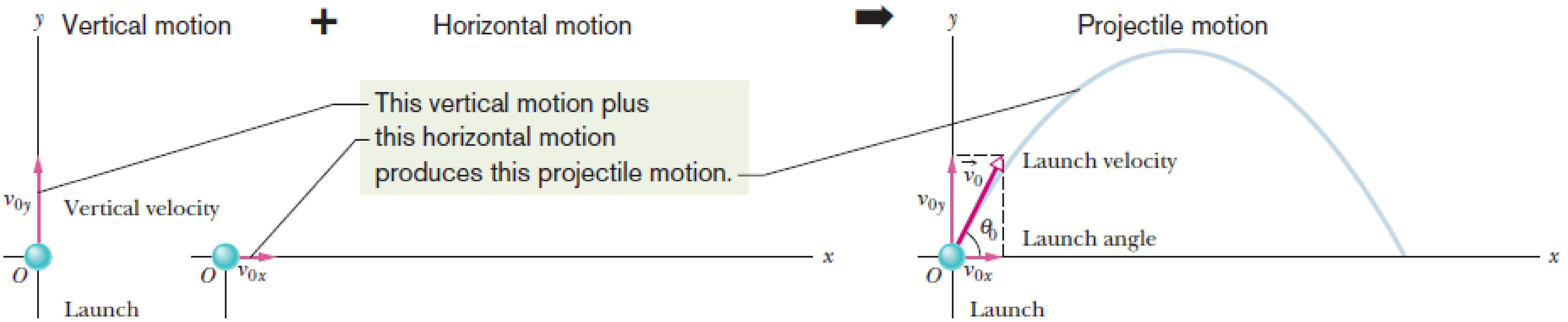
A great student rouser

- In Fig, a blowgun G using a ball as a projectile is aimed directly at a can suspended from a magnet M. Just as the ball leaves the blowgun, the can is released. If g (the magnitude of the free-fall acceleration) were zero, the ball would follow the straight-line path shown in Fig and the can would float in place after the magnet released it. The ball would certainly hit the can. However, g is not zero, but the ball still hits the can! As Fig. shows, during the time of flight of the ball, both ball and can fall the same distance h from their zero-g locations. The harder the demonstrator blows, the greater is the ball's initial speed, the shorter the flight time, and the smaller the value of h.

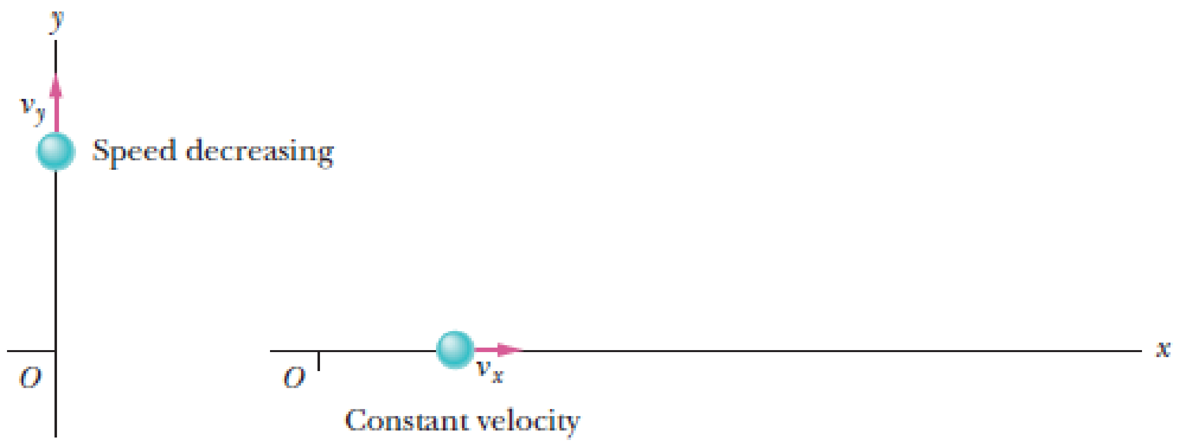
The ball and the can fall the same distance *h*.

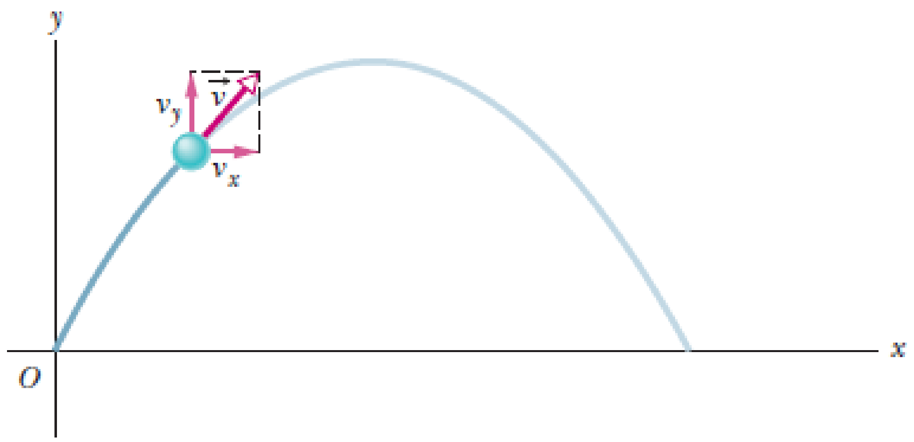


Pictorial Representation(1)

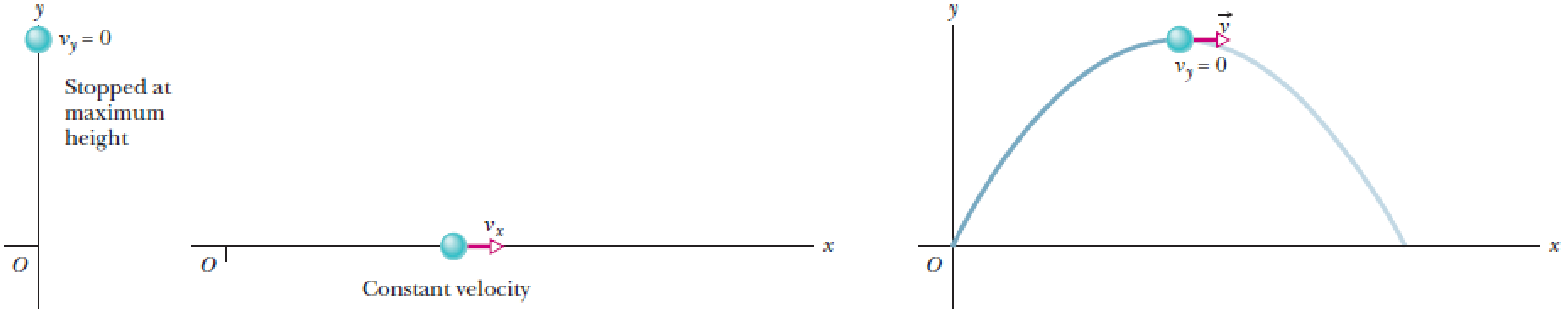


Pictorial Representation(2)

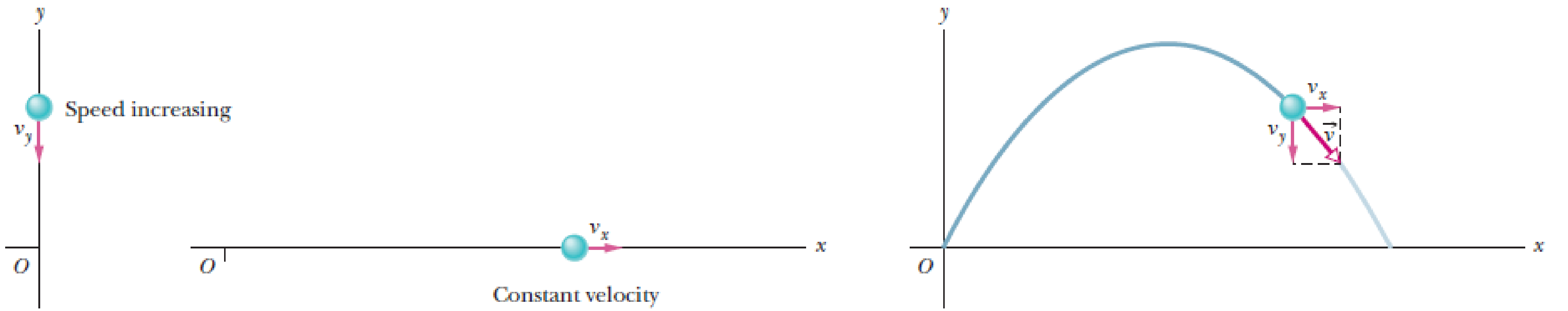




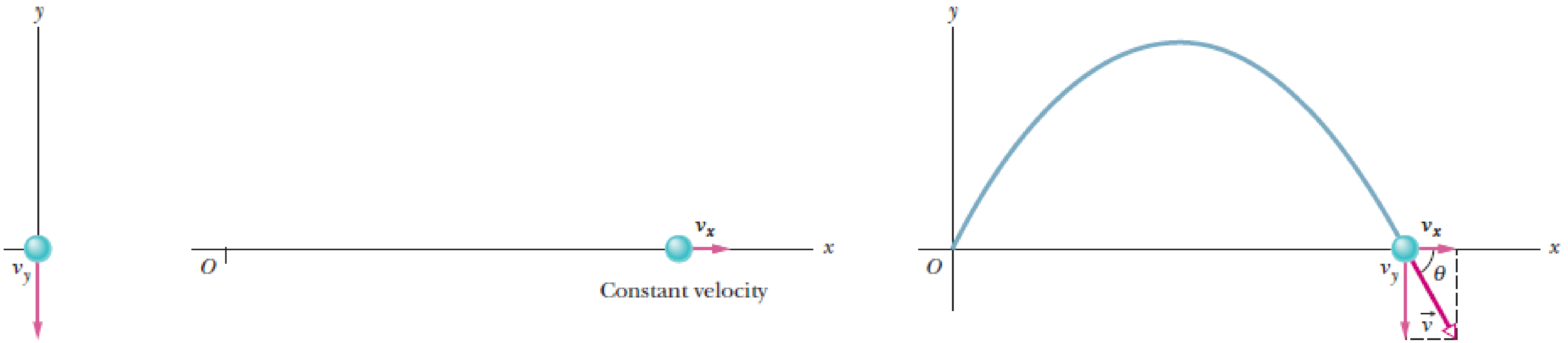
Pictorial Representation(3)



Pictorial Representation(4)



Pictorial Representation(5)

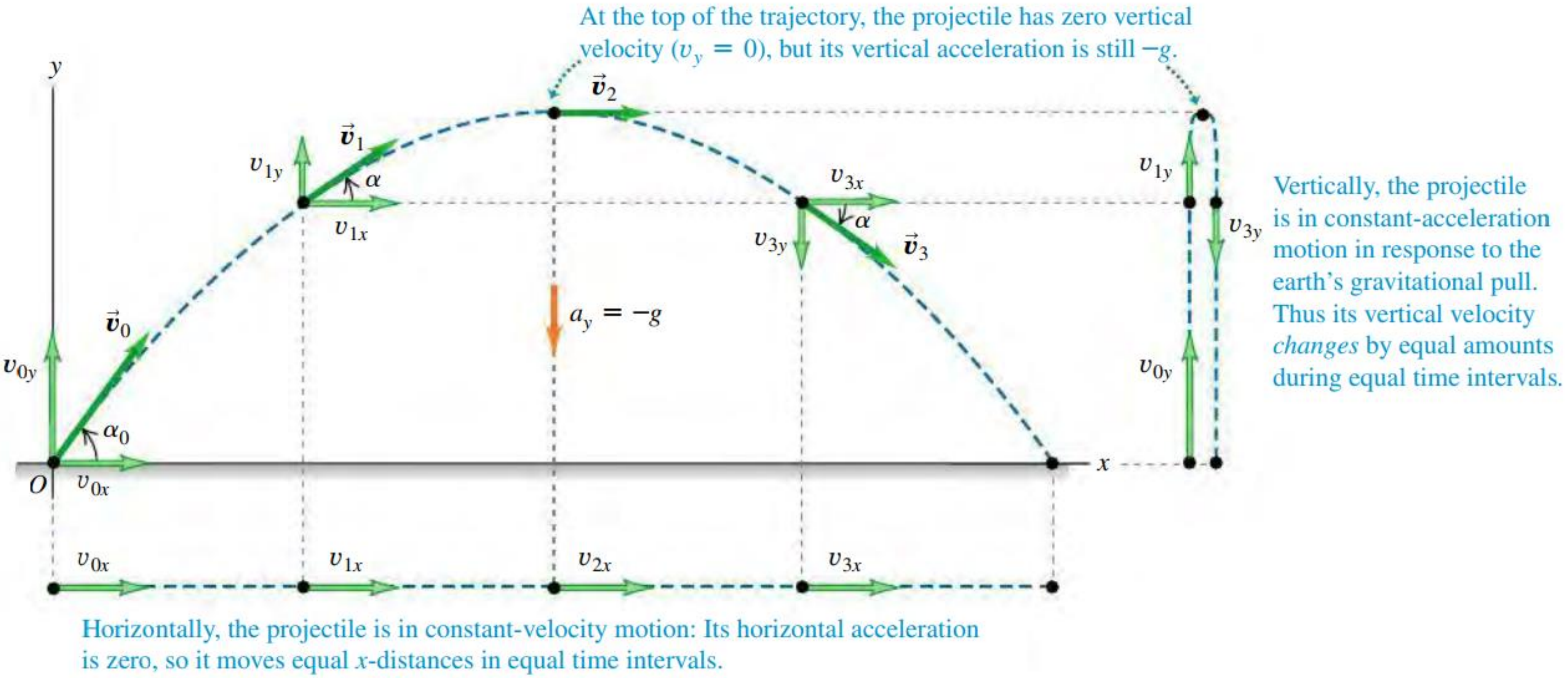


Checkpoint

At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{\imath} - 4.9\,\hat{\jmath}$ the x- axis is horizontal, the y- axis is upward, and $\vec{v}$ is in meters per second).Has the ball passed its highest point?

Analysis of Projectile Motion

- The Horizontal Motion
- The Vertical Motion



The Horizontal Motion

Because there is no acceleration in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion, as shown earlier,

At any time t, the projectile's horizontal displacement $x - x_0$ from an initial position x_0 is given by

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2.$$

where a=0

$$x - x_0 = v_{0x} t.$$

Because $v_{0x} = v_0 \cos \theta_0$, this becomes

$$\boxed{x - x_0 = (v_0 \cos \theta_0) t.}$$

The Vertical Motion(1)

- The vertical motion is a motion of a particle in a free fall.
- The acceleration in free fall remains constant
- We substitute “negative acceleration” because the direction of motion of the body is against the gravity.
- As the motion is vertical or in y-axis we switch to the y-component notation, therefore

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$
$$y - y_0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2$$

The Vertical Motion(2)

where the initial vertical velocity component v_{0y} is replaced with the equivalent $v_0 \sin \theta_0$.

- By using first equation of motion in y-direction

$$v_y = v_{oy} + (-gt)$$

$$v_y = v_{oy} - gt$$

$$v_y = v_o \sin \theta_o - gt$$

The Vertical Motion(3)

$$v^2{}_y = (v_o \sin\theta_o - gt)^2$$

$$v^2{}_y = (v_o \sin\theta_o)^2 - 2(v_o \sin\theta_o)(gt) + (gt)^2$$

$$v^2{}_y = (v_o \sin\theta_o)^2 - 2g(v_o \sin\theta_o\, t + \frac{1}{2}gt^2)$$

$$\boxed{v^2{}_y = (v_o \sin\theta_o)^2 - 2g(y - y_o)}$$

The Vertical Motion(4)

$$\boxed{v_y = v_0 \sin\theta_0 - gt}$$

$$\boxed{v^2{}_y = (v_o \sin\theta_o)^2 - 2g(y - y_o)}$$

Conclusion:

The vertical velocity component behaves just as for a ball thrown vertically upward. It is directed upward initially, and its magnitude steadily decreases to zero, which marks the maximum height of the path. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

Some important points

Coordinates at time t of a **projectile** (positive y-direction is upward, and $x = y = 0$ at $t = 0$)

$$x = (v_0 \cos \alpha_0)t$$

Speed at $t = 0$

Direction at $t = 0$

Time

$$y = (v_0 \sin \alpha_0)t - \tfrac{1}{2}gt^2$$

Velocity components at time t of a **projectile** (positive y-direction is upward)

$$v_x = v_0 \cos \alpha_0$$

Speed at $t = 0$

Direction at $t = 0$

Acceleration due to gravity: Note $g > 0$.

Time

$$v_y = v_0 \sin \alpha_0 - gt$$

Equation of the trajectory of the projectile(1)

We can find the equation of the projectile's path (its **trajectory**) by eliminating time t

$$x - x_0 = (v_0 \cos \theta_0)t. \qquad \text{........(Eq iii)}$$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

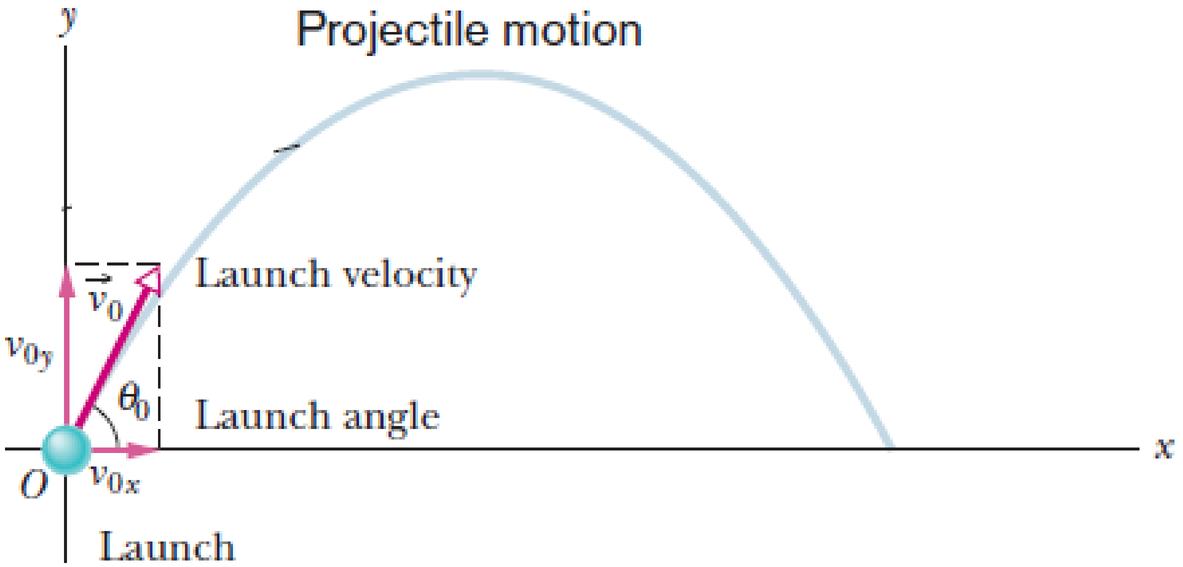
$$= (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2 \text{........(Eq iv)}$$

Solving Eq. iii for t and substituting into

Eq. iv we obtain, after a little rearrangement.

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \qquad \text{(trajectory)} \longrightarrow \text{(Eq v)}$$

Equation of the trajectory of the projectile(2)



This is the equation of the path shown in Fig. In deriving it, for simplicity we let x0 =0 and y0 0 in Eqs derived earlier. Because g, θo, and vo are constants, Eq. v is of the form $y = ax + bx^2$, in which a and b are constants. This is the equation of a parabola, so the path is parabolic.

The Horizontal range

The *horizontal range* R of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). To find range R, let us put

$$x - x_0 = R$$

in Eq. $x - x_0 = (v_0 \cos \theta_0)t$.

$$R = (v_0 \cos \theta_0)t \quad \text{......(Eq vi)}$$

and $y - y_0 = 0$ in Eq. $y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$.

obtaining $$0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2. \quad \text{........(Eq vii)}$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$

Using the identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$
we obtain

$$R = \frac{v_0^2}{g} \sin 2\theta_0. \quad \text{......(Eq viii)}$$

The Horizontal Range(2)

Caution: This equation does *not* give the horizontal distance traveled by a projectile when the final height is not the launch height.

Note that R in Eq viii has its maximum value when $\sin 2\theta_0 = 1$, which corresponds to $2\theta_0 = 90°$ or $\theta_0 = 45°$.

The horizontal range R is maximum for a launch angle of 45°.

However, when the launch and landing heights differ, as in shot put, hammer throw, and basketball, a launch angle of 45° does not yield the maximum horizontal distance.

Effects of Air on a Projectile

- We have assumed that the air through which the projectile moves has no effect on its motion.
- However, in many situations, the disagreement between our calculations and the actual motion of the projectile can be large because the air resists the motion.
- As an example figure on the next page shows two paths for a fly ball that leaves the bat at an angle of 60° with the horizontal and an initial speed of 44.7 m/s.

Graphical view

- Path I (the baseball player’s fly ball) is a calculated path that
- approximates normal conditions of play, in air. Path II (the physics professor’s fly ball) is the path the ball would follow in a vacuum.
- Air reduces height and range.

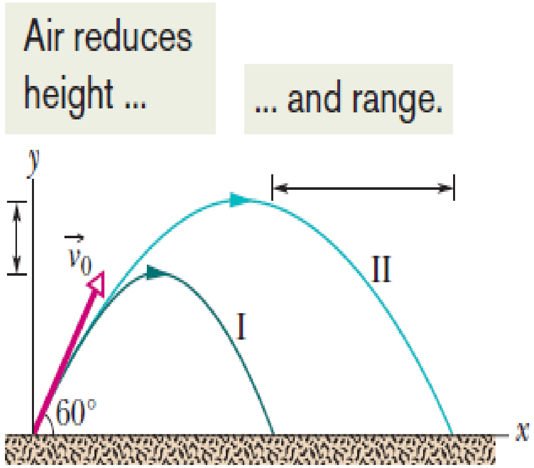


Figure (I) The path of a fly ball calculated by taking air resistance into account. (II) The path the ball would follow in a vacuum, calculated by the methods of this chapter.

Corresponding data of the fly balls

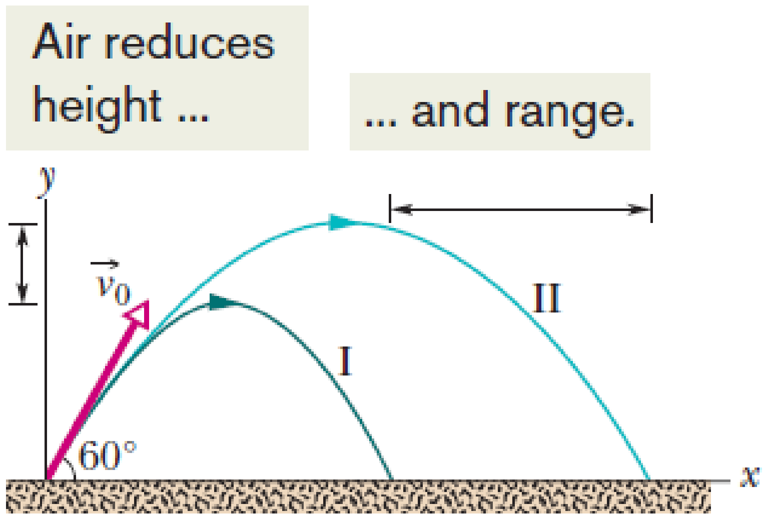


Figure (I) The path of a fly ball calculated by taking air resistance into account. (II) The path the ball would follow in a vacuum, calculated by the methods of this chapter.

Table 1 **Two Fly Balls**[a]

	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s

The launch angle is 60° and the launch speed is 44.7m/s.

Practice Problem 1(sample problem 4.04)

In figure, a rescue plane flies at 198 km/h (=55.0 m/s) and constant height h = 500 m toward a point directly over a victim, where a rescue capsule is to land.

a) What should be the angle f of the pilot's line of sight to the victim when the capsule release is made?

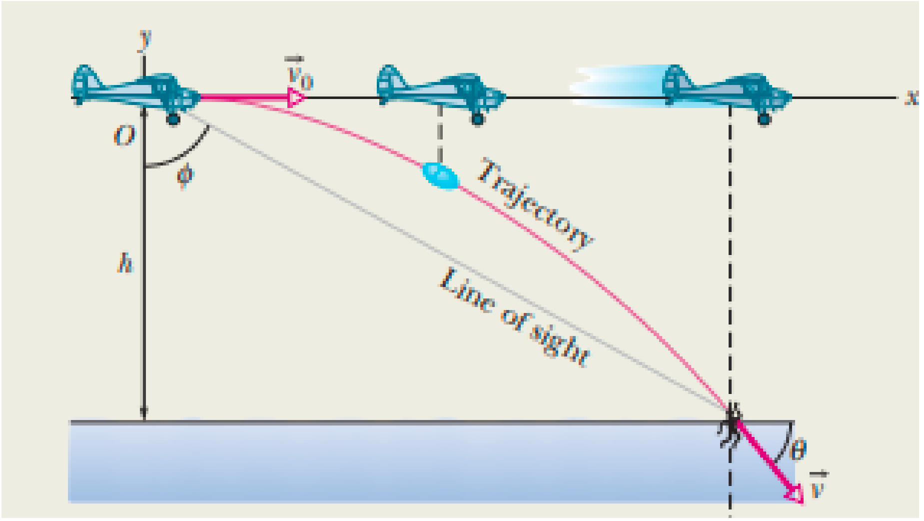
b) As the capsule reaches the water, what is its velocity ?

Answers:

a. $\phi = 48.0°$

b. $\vec{v} = \left(55.0\ \frac{m}{s}\right)\hat{\imath} - \left(99.0\frac{m}{s}\right)\hat{\jmath}$

v = 113 m/s and θ = -60.9°



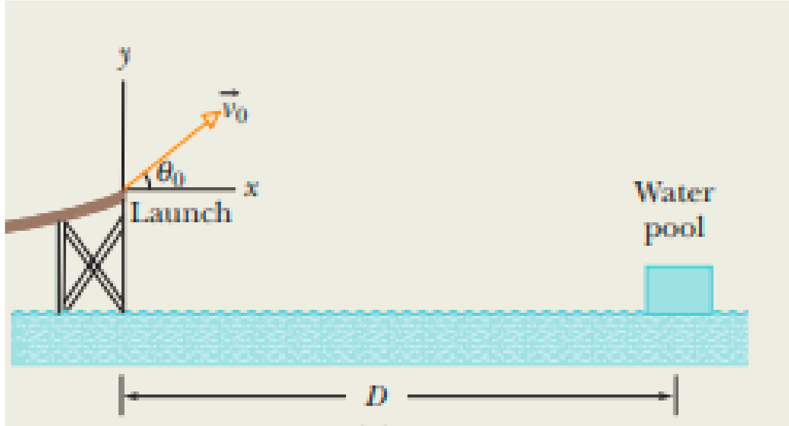
Practice Problem 2(sample problem 4.05)

One of the most dramatic videos on the web (but entirely fictitious) supposedly shows a man sliding along a long water slide and then being launched into the air to land in a water pool. Let's attach some reasonable numbers to such a flight to calculate the velocity with which the man would have hit the water. Figure indicates the launch and landing sites and includes a superimposed coordinate system with its origin conveniently located at the launch site. From the video we take the horizontal flight distance as D = 20.0 m, the flight time as t = 2.50 s, and the launch angle as θ_o = 40.0°. Find the magnitude of the velocity at launch and at landing.

Answers:

v_o = 10.4 m/s

v = 19.5 m/s



Practice Problem 3

••43 ILW A ball is shot from the ground into the air. At a height of 9.1 m, its velocity is $\vec{v} = (7.6\hat{i} + 6.1\hat{j})$ m/s, with $\hat{i}$ horizontal and $\hat{j}$ upward. (a) To what maximum height does the ball rise? (b) What total horizontal distance does the ball travel? What are the (c) magnitude and (d) angle (below the horizontal) of the ball's velocity just before it hits the ground?

Answers:

a. 11 m

b. 23 m

c. 17 m/s

d. -63°

Practice Problem 4: (problem 35, chap 4)

A rifle that shoots bullets at 460 m/s is to be aimed at a target 45.7 m away. If the center of the target is level with the rifle, how high above the target must the rifle barrel be pointed so that the bullet hits dead center?

Answer:

4.84 cm

Homework questions:

- Practice problems:
- End of chapter 4 textbook " Fundamentals of Physics"by Halliday & Resnick Jearl Walker 10th Edition"

35, 42, 48, 90

Answer of even problems:

42: vo = 26.5 m/s, y = 23.3 m, $\Delta y = 5.3$ m

b. $\Delta y = 7.9$ m

c. $x = 69.0$ m

48: a. d= 33.7 m

b. 26 m/s, c = angle $\theta = -71.1°$

90. $v_0 = 23$ ft/s for $g = 32$ ft/s^2, $x = 13$ ft, $y = 3$ ft and $\theta_0 = 55°$.

Checkpoint:

- A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity? What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent, and at the topmost point of the flight?

References:

Book:

Fundamentals of Physics by Halliday Resnick & Jearl Walker 10^{th} edition