# Force and Motion

## Classical Mechanics

- The field of study in which we study the causes of motion is called "Dynamics".
- The approach to dynamics we consider in this chapter and ahead is known as "Classical Mechanics".
- Often referred to as "Newtonian Mechanics"

# Causes of motion (some examples)

Table: Some accelerated Motions and their Causes		
Object	Change in Motion	Major Cause (Environment)
Apple	Falls from tree	Earth's Gravity
Billiard Ball	Bounces off another	Other ball, table, gravity(earth)
Skier	Slides down hill	Earth's gravity, friction(snow), air resistance
Beam of Electrons	Focusing and deflection	EM fields(Magnets & Voltage Differences)
Comet Halley	Round trip through solar system	Sun's Gravity

## Newtonian Mechanics

- The study of relationship between a force and the acceleration it causes is called Newtonian Mechanics.
- We shall focus on its three primary laws of motion
- 1. First Law of Motion: a qualitative analysis
- 2. Second Law of Motion: a quantitative analysis
- 3. Third Law of Motion

## Limitations of Newtonian Mechanics

It does not apply to all situations.

#### High speeds

• If the speeds are quite high or very large say the body moves with the speed of light then we will replace Newtonian mechanics by Einstein's special theory of relativity, which holds for any speed, including those near the speed of light.

#### Atomic scale

• If the interacting bodies are on the scale of atomic structure(might be electrons in an atom) we must replace Newtonian mechanics by Quantum mechanics.

## Introduction to Force

- Physics is the study of motion
- Physics is also the study of those causes that produces motion in an object.
- That cause is known as FORCE
- A Force is defined generally as a "pull or a push" on the object.
- The force is said to *act* on the object to change its velocity.
- Examples:

car slams on a pole, a Force from the pole causes a car to stop

## Introduction to Force

- Force is measured by the acceleration it produces.
- However, acceleration is a vector quantity, with both magnitude and direction.
- To find force we should have the mass of the object and acceleration produced.

A force  $\vec{F}$  on the standard kilogram gives that body an acceleration  $\vec{a}$ .

# Concept of Force

• The net force acting on an object is defined as the vector sum of all forces acting on the object. (We sometimes refer to the net force as the total force, the resultant force, or the unbalanced force.)

• If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant.

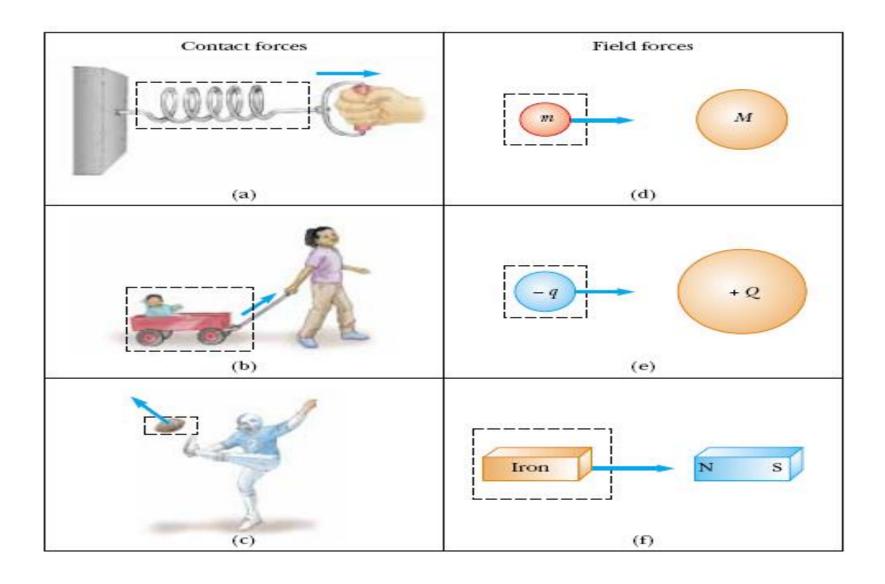
• When the velocity of an object is constant (including when the object is at rest), the object is said to be in equilibrium.

# Principle of superposition for forces:

• Force is a vector quantity and thus has not only magnitude but also direction. So, if two or more forces act on a body, we find the net force (or resultant force) by adding them as vectors.

 A single force that has the same magnitude and direction as the calculated net force would then have the same effect as all the individual forces together. This fact is called the principle of superposition for forces.

## Contact Forces and Field Forces:



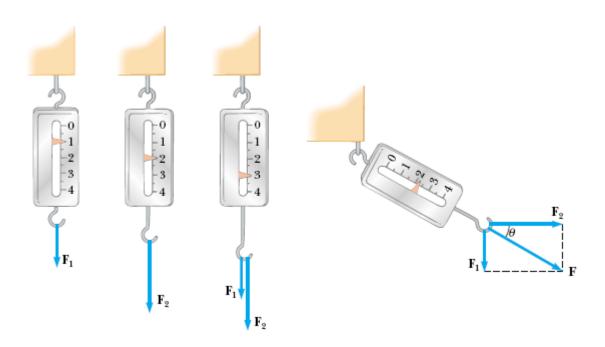
## Points to remember:

All forces can be represented by vector

All forces can be added by finding their vector sum

All components of forces remain independent

# Measuring the strength of force



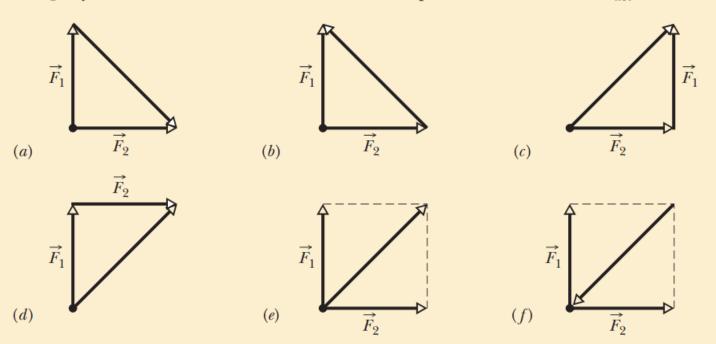
Because forces are vector quantities, you must use the rules of vector addition to obtain the net force acting on an object.

# Checkpoint 1:



#### **Checkpoint 1**

Which of the figure's six arrangements correctly show the vector addition of forces  $\vec{F}_1$  and  $\vec{F}_2$  to yield the third vector, which is meant to represent their net force  $\vec{F}_{net}$ ?



## Reference Frames

• A "frame of reference" is just a set of coordinates: something you use to measure the things that matter in Newtonian problems, that is to say, positions and velocities, so we also need a clock.

An inertial frame is defined as one in which Newton's law are valid.

 In other words we can say that Newton's First law is not true in all reference frames.

## Inertial Frame of Reference

- In classical physics and special relativity, an inertial frame of reference is a frame of reference that is not undergoing acceleration.
- In an inertial frame of reference, a physical object with zero net force acting on it moves with a constant velocity (which might be zero)
- Equivalently, it is a frame of reference in which Newton's first law of motion holds
- A frame where Newton's laws are valid. No accelerations

#### Newton's First Law and Inertial Frames

#### Newton 1<sup>st</sup> Law of Motion

"In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line)".

## Mass

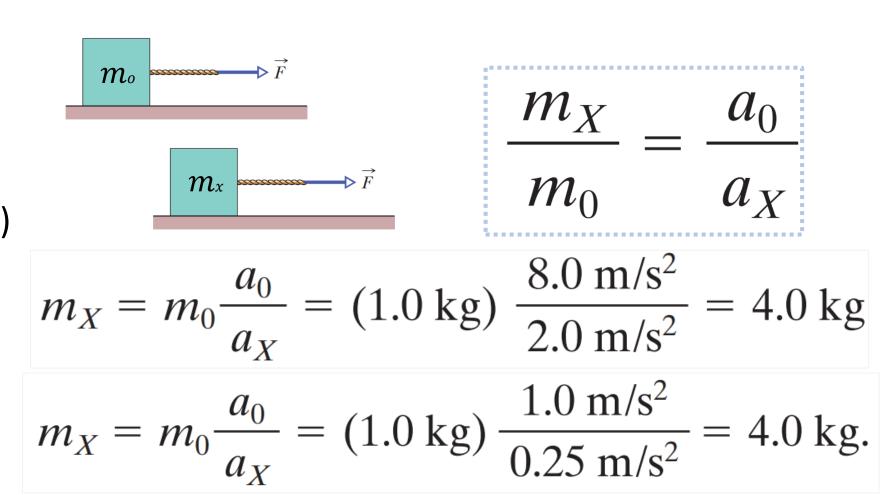
#### Mass

- Mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity.
- Mass of a body is a characteristic that relates a force on the body to the resulting acceleration.
- Mass is an inherent property of an object (intrinsic characteristic) and is independent of the object's surroundings and of the method used to measure it. Also, mass is a scalar quantity and thus obeys the rules of ordinary arithmetic.

#### Note:

Mass and weight are two different quantities

- Mass is scalar
- Mass is intrinsic (it comes with the existence of objects)



Is it body's size, weight, or density? The answer is no, although those characteristics are sometimes confused with mass. We can say only that the mass of a body is the characteristic that relates a force on the body to the resulting acceleration

#### Newton's Second Law

 When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass

• The net force on the body is equal to the product of the body's mass and its acceleration.

$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum F_x = ma_x$$
  $\sum F_y = ma_y$   $\sum F_z = ma_z$ 

## Unit of force

The SI unit of force is the **newton**, which is defined as the force that, when acting on an object of mass 1 kg, produces an acceleration of  $1 \text{ m/s}^2$ . From this definition and Newton's second law, we see that the newton can be expressed in terms of the following fundamental units of mass, length, and time:

$$1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2$$

Newton's second law:  $\sum \vec{F} = m\vec{a}$  the body accelerates in same direction as the net force. Mass of body

The net force 
$$\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$
 acting on a body ...  $\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$  ... is the vector sum, or resultant, of all individual forces acting on that body.

• Equilibrium,  $|\vec{R}| = \sum \vec{F} = 0$ 

(a) A constant net force  $\Sigma \vec{F}$  causes a constant acceleration  $\vec{a}$ .



(b) Doubling the net force doubles the acceleration.



(c) Halving the force halves the acceleration.



# Types of Forces

#### The Gravitational Force and Weight:

• We are well aware that all objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the gravitational force  $\mathbf{F}_g$ . This force is directed toward the center of the Earth, and its magnitude is called the weight of the object.

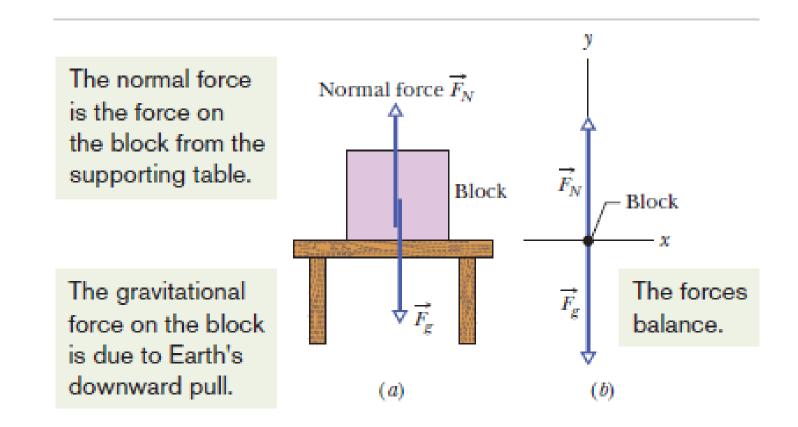
$$\mathbf{F}_g = m\mathbf{g}$$

#### The Normal Force(F<sub>N</sub>):

 When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force that is perpendicular to the surface.

#### Free-Body Diagram

• To solve problems with Newton's second law, we often draw a free-body diagram in which the only body shown is the one for which we are summing forces.



## Points to remember

- The weight of a body must be measured when the body is not accelerating vertically relative to the ground.
- For example, you can measure your weight on a scale in your bathroom or on a fast train. However, if you repeat the measurement with the scale in an accelerating elevator, the reading differs from your weight because of the acceleration. Such a measurement is called an apparent weight.

# How much do you weigh in an elevator

You have most likely had the experience of standing in an elevator that accelerates upward as it moves toward a higher floor. In this case, you feel heavier. In fact, if you are standing on a bathroom scale at the time, the scale measures a force magnitude that is greater than your weight. Thus, you have tactile and measured evidence that leads you to believe you are heavier in this situation. *Are* you heavier?

#### Answer:

No, your weight is unchanged. To provide the acceleration upward, the floor or scale must exert on your feet an upward force that is greater in magnitude than your weight. It is this greater force that you feel, which you interpret as feeling heavier. The scale reads this upward force, not your weight, and so its reading increases.

#### Tension:

When a rope attached to an object is pulling on the object, the rope exerts a force T on the object, and the magnitude of that force is called the tension in the rope.

#### Objects in Equilibrium

$$\sum F_{y} = T - F_{g} = 0$$
 or  $T = F_{g}$ 

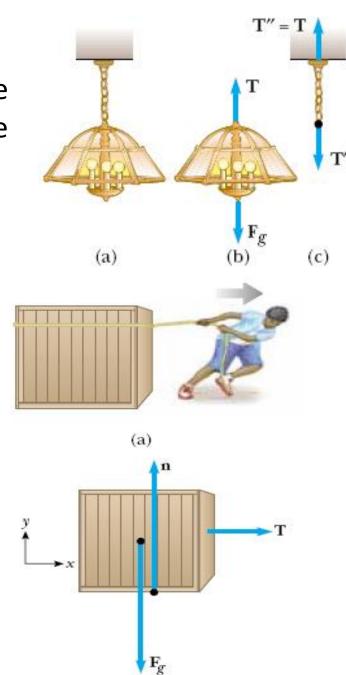
#### Objects Experiencing a Net Force

$$\sum F_x = T = ma_x$$
 or  $a_x = \frac{T}{m}$ 

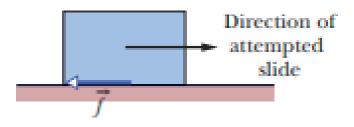
$$n + (-F_g) = 0$$
 or  $n = F_g$ 

$$v_{xf} = v_{xi} + \left(\frac{T}{m}\right)t$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}\left(\frac{T}{m}\right)t^2$$



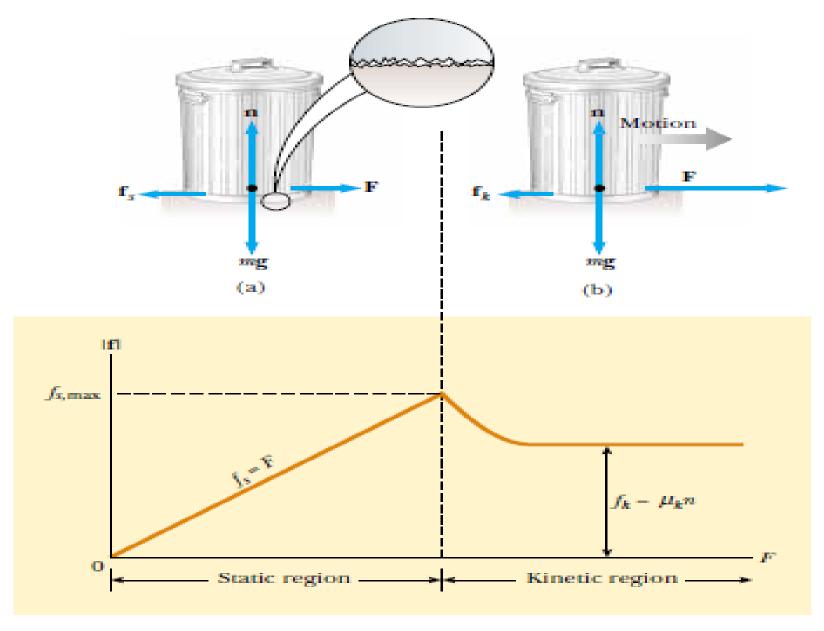
#### Forces of Friction



A frictional force  $\vec{f}$  opposes the attempted slide of a body over a surface.

- If we either slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. The resistance is considered to be a single force called either the frictional force or simply friction.
- This force is directed along the surface, opposite the direction of the intended motion.
- Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

## Static and Kinetic Friction



 The magnitude of the force of static friction between any two surfaces in contact can have the values.

$$f_s \leq \mu_s n$$

 The magnitude of the force of kinetic friction acting between two surfaces is

$$f_k = \mu_k n$$

• The values of  $\mu_s$  and  $\mu_k$  depend on the nature of the surfaces, but  $\mu_k$  is generally less than  $\mu_s$ .

• The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.

# Experimental Method of determining $\mu_{s}$ and $\mu_{k}$

The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure 5.19. The incline angle is increased until the block starts to move. Show that by measuring the critical angle  $\theta_c$  at which this slipping just occurs, we can obtain  $\mu_s$ .

**Solution** Conceptualizing from the free body diagram in Figure 5.19, we see that we can categorize this as a Newton's second law problem. To analyze the problem, note that the only forces acting on the block are the gravitational force  $m\mathbf{g}$ , the normal force  $\mathbf{n}$ , and the force of static friction  $\mathbf{f}_s$ . These forces balance when the block is not moving. When we choose x to be parallel to the plane and y perpendicular to it, Newton's second law applied to the block for this balanced situation gives

(1) 
$$\sum F_x = mg \sin \theta - f_s = ma_x = 0$$

(2) 
$$\sum F_y = n - mg \cos \theta = ma_y = 0$$

We can eliminate mg by substituting  $mg = n/\cos \theta$  from (2) into (1) to find

(3) 
$$f_s = mg \sin \theta = \left(\frac{n}{\cos \theta}\right) \sin \theta = n \tan \theta$$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value  $\mu_s n$ . The angle  $\theta$  in this situation is the critical angle  $\theta_c$ , and (3) becomes

$$\mu_s n = n \tan \theta_c$$

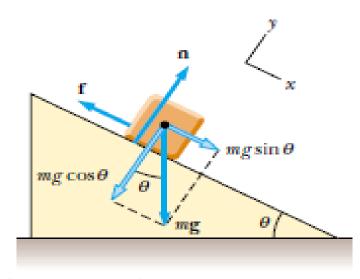


Figure 5.19 (Example 5.12) The external forces exerted on a block lying on a rough incline are the gravitational force mg, the normal force n, and the force of friction f. For convenience, the gravitational force is resolved into a component along the incline  $mg \sin \theta$  and a component perpendicular to the incline  $mg \cos \theta$ .

$$\mu_s = \tan \theta_c$$

For example, if the block just slips at  $\theta_c = 20.0^{\circ}$ , then we find that  $\mu_s = \tan 20.0^{\circ} = 0.364$ .

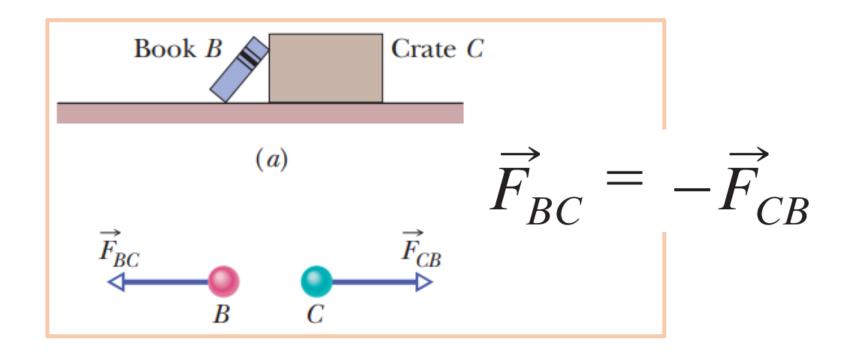
To finalize the problem, note that once the block starts to move at  $\theta \ge \theta_c$ , it accelerates down the incline and the force of friction is  $f_k = \mu_k n$ . However, if  $\theta$  is reduced to a value less than  $\theta_c$ , it may be possible to find an angle  $\theta_c$  such that the block moves down the incline with constant speed  $(a_x = 0)$ . In this case, using (1) and (2) with  $f_s$  replaced by  $f_k$  gives

$$\mu_k = \tan \theta_c'$$

where  $\theta_c' < \theta_c$ .

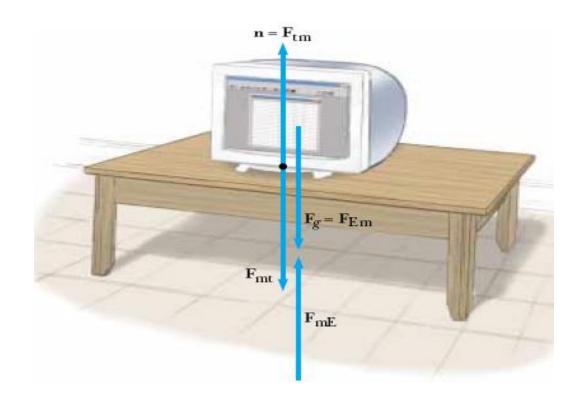
## Newton's Third Law of Motion

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.



## Newton's third law of motion:

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$



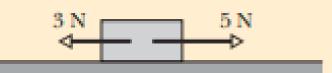
This is equivalent to stating that a single isolated force cannot exist.

# Checkpoint



#### Checkpoint 2

The figure here shows two horizontal forces acting on a block on a frictionless floor. If a third horizon-



tal force  $\vec{F}_3$  also acts on the block, what are the magnitude and direction of  $\vec{F}_3$  when the block is (a) stationary and (b) moving to the left with a constant speed of 5 m/s?

# Checkpoint

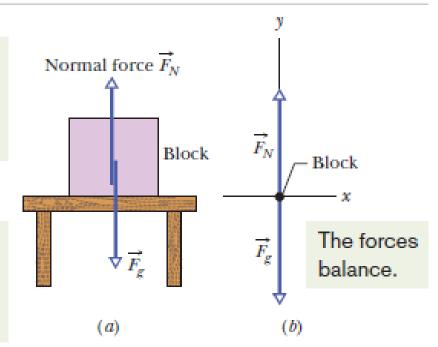


#### Checkpoint 3

In Fig. 5-7, is the magnitude of the normal force  $\vec{F}_N$  greater than, less than, or equal to mg if the block and table are in an elevator moving upward (a) at constant speed and (b) at increasing speed?

The normal force is the force on the block from the supporting table.

The gravitational force on the block is due to Earth's downward pull.

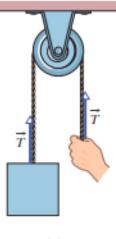


## Checkpoint



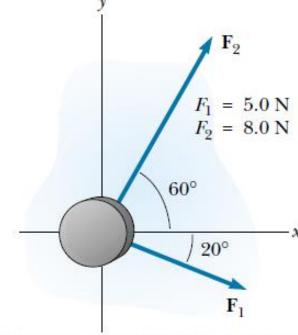
#### Checkpoint 4

The suspended body in Fig. 5-9c weighs 75 N. Is T equal to, greater than, or less than 75 N when the body is moving upward (a) at constant speed, (b) at increasing speed, and (c) at decreasing speed?



#### Practice Problem 1:

A hockey puck having a mass of 0.30 kg slides on the horizontal, frictionless surface of an ice rink. Two forces act on the puck, as shown in Figure  $\,$ . The force  $\mathbf{F}_1$  has a magnitude of 5.0 N, and the force  $\mathbf{F}_2$  has a magnitude of 8.0 N. Determine both the magnitude and the direction of the puck's acceleration.



A hockey puck moving on a frictionless surface accelerates in the direction of the resultant force  $\mathbf{F}_1 + \mathbf{F}_2$ .

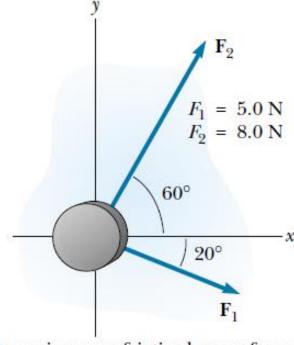
#### Practice Problem 1(Solution):

**Solution** The resultant force in the x direction is

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos(-20^\circ) + F_2 \cos 60^\circ$$
  
= (5.0 N)(0.940) + (8.0 N)(0.500) = 8.7

The resultant force in the y direction is

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin(-20^\circ) + F_2 \sin 60^\circ$$
  
= (5.0 N)(-0.342) + (8.0 N)(0.866) = 5.2 N



A hockey puck moving on a frictionless surface accelerates in the direction of the resultant force  $\mathbf{F}_1 + \mathbf{F}_2$ .

Now we use Newton's second law in component form to find the x and y components of acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{8.7 \text{ N}}{0.30 \text{ kg}} = 29 \text{ m/s}^2$$

$$a_y = \frac{\sum F_y}{m} = \frac{5.2 \text{ N}}{0.30 \text{ kg}} = 17 \text{ m/s}^2$$

The acceleration has a magnitude of

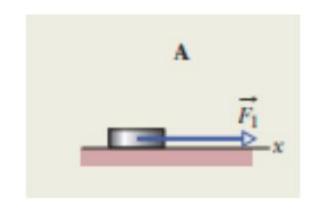
$$a = \sqrt{(29)^2 + (17)^2} \,\text{m/s}^2 = 34 \,\text{m/s}^2$$

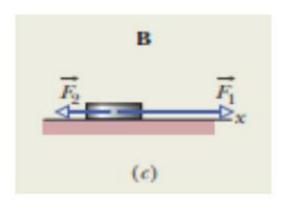
and its direction relative to the positive x axis is

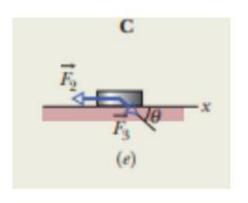
$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

## Practice Problem 2(Sample Problem 5.01)

Parts A, B, and C of Fig. show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is m = 0.20 kg. Forces  $F_1$  and  $F_2$  are directed along the axis and have magnitudes  $F_1 = 4.0$  N and  $F_2 = 2.0$  N. Force  $F_3$  is directed at an angle  $\theta = 30^\circ$  and has magnitude  $F_3 = 1.0$  N. In each situation, what is the acceleration of the puck? Also draw the free-body diagram.



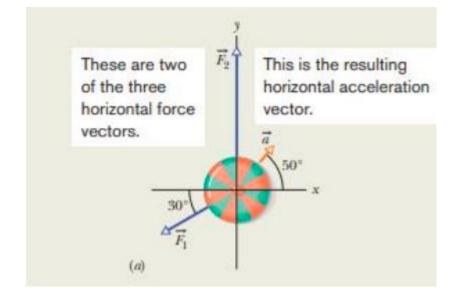




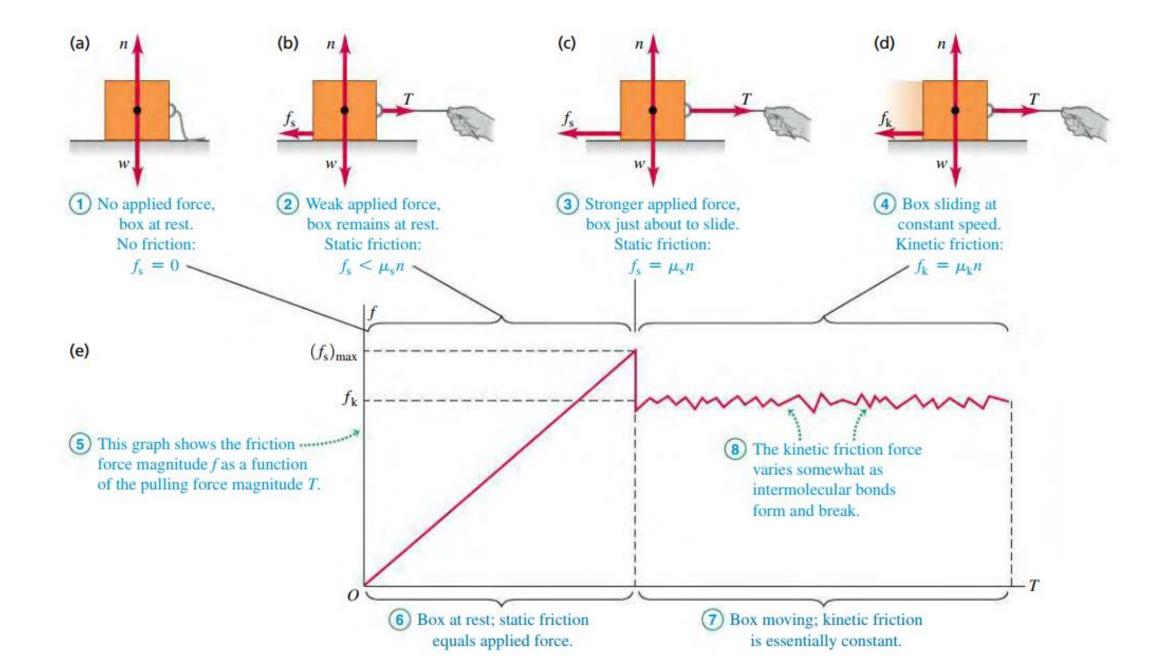
# Practice Problem 3(Sample Problem 5.02)

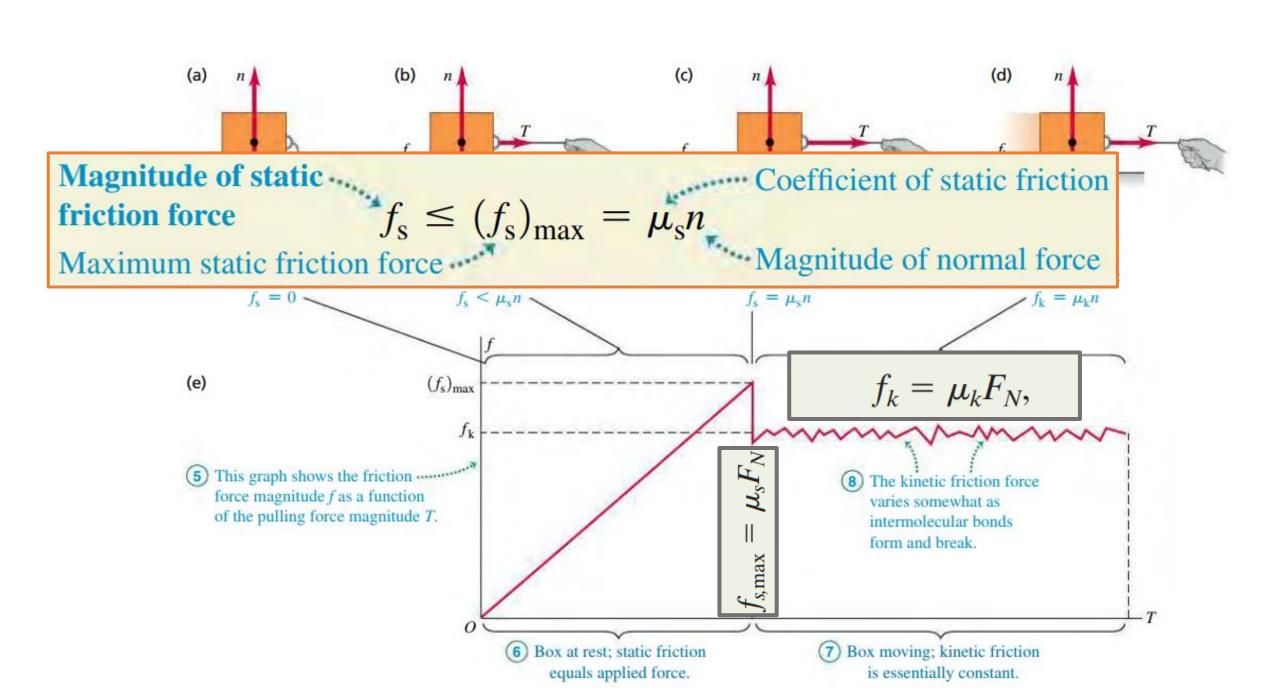
In the overhead view of Fig., a 2.0 kg cookie tin is accelerated at 3.0 m/s^2 in the direction shown by  $\bf a$ , over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown:  $\bf F_1$  of magnitude 10 N and  $\bf F_2$  of magnitude 20 N. What is the third force in unit-vector notation and in magnitude-angle

notation?



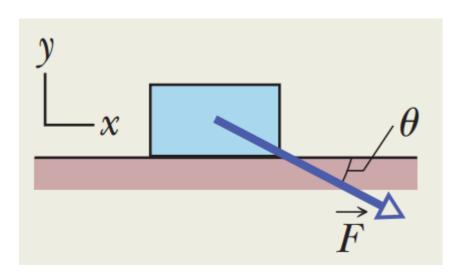
#### Friction

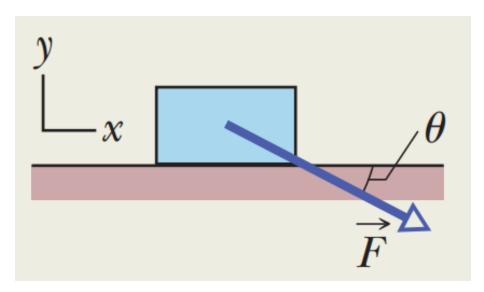


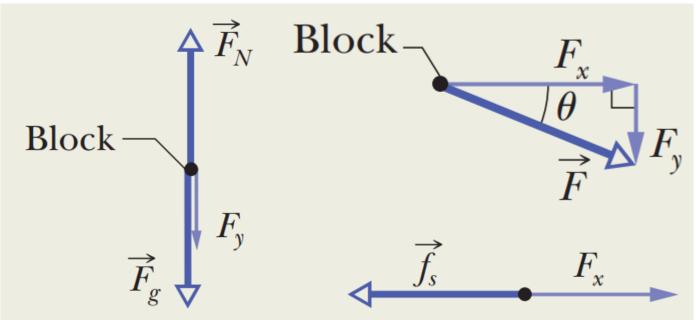


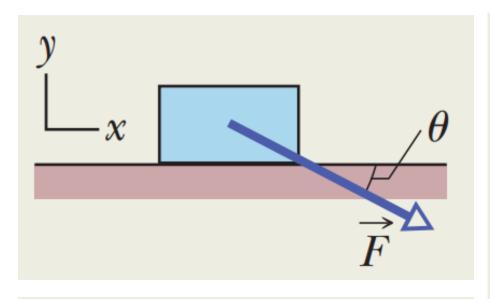
#### Practice Problem 4(Sample problem 6.01)

12.0 N applied to an 8.00 kg block at a downward angle of  $\theta = 30.0^{\circ}$ . The coefficient of static friction between block and floor is  $\mu_s = 0.700$ ; the coefficient of kinetic friction is  $\mu_k = 0.400$ . Does the block begin to slide or does it remain stationary? What is the magnitude of the frictional force on the block?

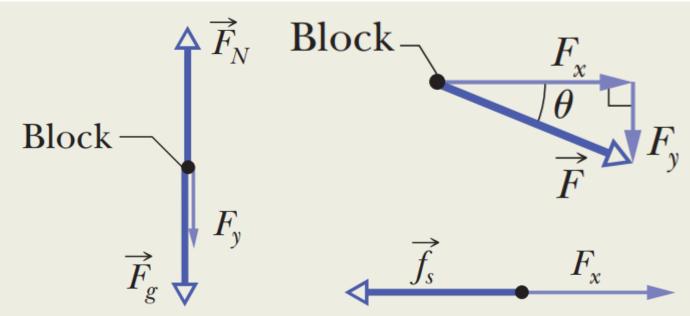


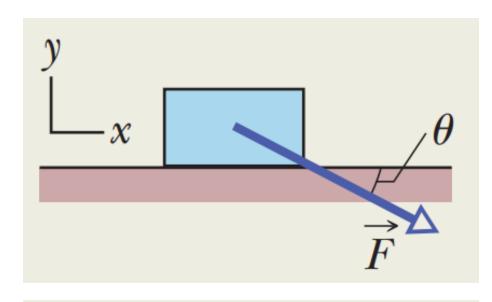




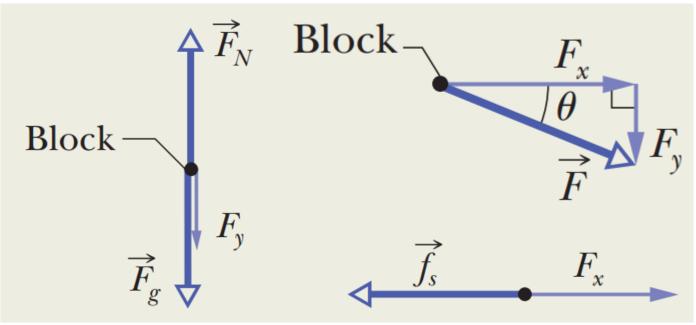


$$F_x = F \cos \theta$$
  
= (12.0 N) cos 30° = 10.39 N.

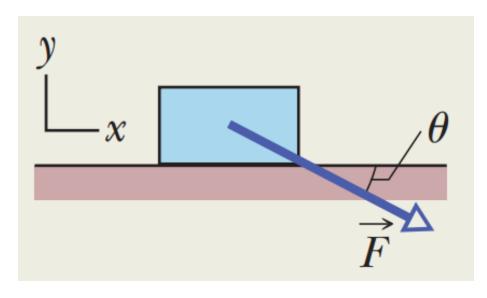




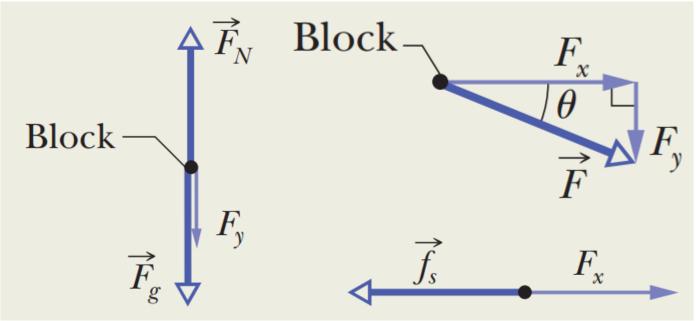
$$F_x = F \cos \theta$$
  
= (12.0 N) cos 30° = 10.39 N.



$$F_N - mg - F\sin\theta = m(0),$$



$$F_x = F \cos \theta$$
  
= (12.0 N) cos 30° = 10.39 N.



$$F_N - mg - F\sin\theta = m(0),$$

$$f_{s,\text{max}} = \mu_s (mg + F \sin \theta)$$
= (0.700)((8.00 kg)(9.8 m/s<sup>2</sup>) + (12.0 N)(sin 30°))
= 59.08 N. (6

## Adapted from:

#### Book:

Fundamentals of Physics by Halliday Resnick & Jearl Walker 10<sup>th</sup> edition