Motion along a straight line

Chapter 2 textbook

One-dimensional motion

- When the object moves along a single axis, such a motion is called one-dimensional motion.
- Example: race car

Introduction to Linear Motion

- The word “Linear” means “Straight” and the word “Motion” means “change in position of a body with respect to frame of reference(surrounding)”

Points to Remember:

- Linear Motion is the motion in One Dimension
- The motion should be in a straight line only.
- The line may be vertical, horizontal, or slanted, but it must be straight.
- All parts of the body move through the same distance in the same direction in same time.
- The motion of the body may not be uniform.

Motion and Rest

- MOTION :

  A body is said to be in Motion if its position changes with respect to its surrounding .

- REST:

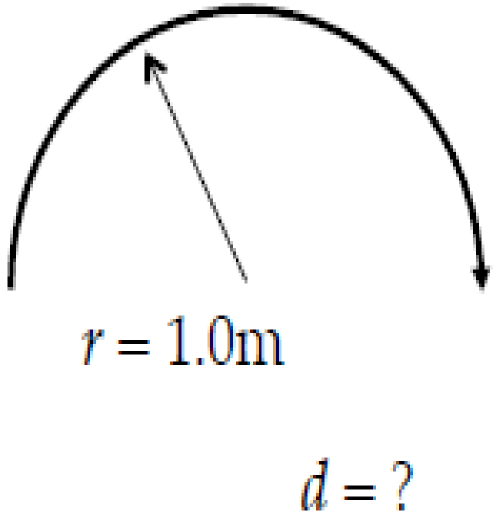
  A body is said to be in Rest if its position does not change with respect to its surrounding.

Time

- Time refers to how long an object is in motion for.
- The SI unit of time is seconds

Distance

Distance is simply how far something travels along its path, whether measured in miles, kilometers, meters, centimeters, feet, or any other unit.



Position and Displacement

- To locate an object means to find its position relative to some reference point, often the origin (or zero point) of an axis such as the x axis in Fig. The positive direction of the axis is in the direction of increasing numbers (coordinates), which is to the right in Fig. The opposite is the negative direction.
- For example, a particle might be located at x = 5 m, which means it is 5 m in the positive direction from the origin. If it were at x = -5 m, it would be just as far from the origin but in the opposite direction. On the axis, a coordinate of -5 m is less than a coordinate of -1 m, and both coordinates are less than a coordinate of + 5 m. A plus sign for a coordinate need not be shown, but a minus sign must always be shown.

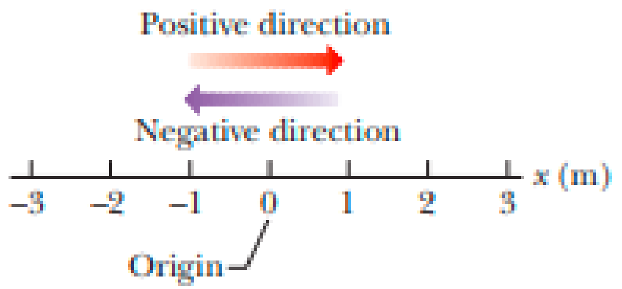


Figure Position is determined on an axis that is marked in units of length (here meters) and that extends indefinitely in opposite directions. The axis name, here x, is always on the positive side of the origin.

Displacement

- Displacement is a measure of how far you have "displaced," or changed your position.
- Displacement is a vector quantity, you need to specify a direction for your displacement.
- A change from position x_1 to position x_2 is called a displacement Δx, where,

$$\Delta x = x_2 - x_1$$

- Displacement involves only the original and final positions.

For instance;

Q# What was your displacement coming to this class?

A# 152 meters, East

Q# How high can you jump?

A# 1 meters up.

Displacement

- Displacement has two features:
- Its magnitude is the distance (such as the number of meters) between the original and final positions.
- Its direction, from an original position to a final position, can be represented by a plus sign or a minus sign if the motion is along a single axis.

Types of Displacement

Mainly two types that we'll go through;

1. Horizontal Displacement

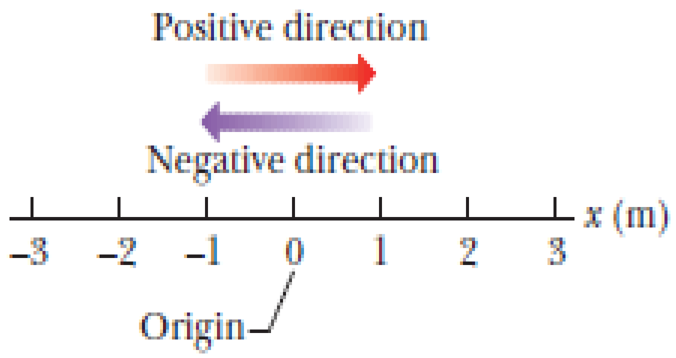
- Example: motion of a car

2. Vertical Displacement

- Example : launching of rocket

Horizontal Displacement

- For horizontal motion, we'll often describe the displacement in regards to an imaginary number line, with "to the right" being the positive-x direction, and "to the left" being the negative-x direction.



Vertical Displacement

- For vertical motion, we'll often describe the displacement in regards to an imaginary number line, with "up" being the positive y-direction, and "down" being the negative-y direction.

Velocity

- It is the rate of change of displacement.
- It is a vector quantity.
- Formula :

$$v = \frac{r}{t}$$

Unit:

It is measured in MKS system of units that is meter per second(m/s)

Speed

- Speed = how fast you're going
- Speed is simply a measure of how quickly an object is moving: how much distance it travels in a given time.
- It is a scalar quantity

Formula:

$$\text{speed} = \frac{distance}{time}$$

Unit: Standard unit in MKS system is meter/sec (m/s)

Average velocity

- When a particle has moved from position x_1 to position x_2 during a time interval $\Delta t = t_2 - t_1$, its average velocity during that interval is:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- It involves only the original and final positions.

Average Velocity

- On a graph of x versus t, v_{avg} is the slope of the straight line that connects two particular points on the x(t) curve: one is the point that corresponds to x_2 and t_2, and the other is the point that corresponds to x_1 and t_1. Like displacement, v_{avg} has both magnitude and direction (it is another vector quantity).
- Its magnitude is the magnitude of the line's slope. A positive v_{avg} (and slope) tells us that the line slants upward to the right; a negative v_{avg} (and slope) tells us that the line slants downward to the right. The average velocity v_{avg} always has the same sign as the displacement Δx because Δt in is always positive

Average Velocity

Consider a particle moving in space. Let the particle be at point P in Fig. 1 at some initial time t_0 and at point P' some later time t_f.

The initial position of the particle can be specified by a *position vector* $\mathbf{r}_0$ obtained by drawing an arrow from the origin of the coordinate system to point P. Similarly, the position at the later time is specified by a second position vector $\mathbf{r}_f$ that results when an arrow is drawn from the origin to point P'. The position at any other point in the motion is specified by a corresponding position vector $\mathbf{r}$. We can now define the *displacement vector* $\Delta\mathbf{r}$ as the vector difference between the final and the initial position vectors, namely, $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_0$ (see Fig. 1). Correspondingly, we define the *average velocity* $\overline{\mathbf{v}}$ as the ratio of the displacement vector to the time taken for the displacement to occur, namely,

$$\overline{\mathbf{v}} = \frac{\mathbf{r}_f - \mathbf{r}_0}{t_f - t_0} = \frac{\Delta\mathbf{r}}{\Delta t} \qquad \textbf{Eq. 2}$$

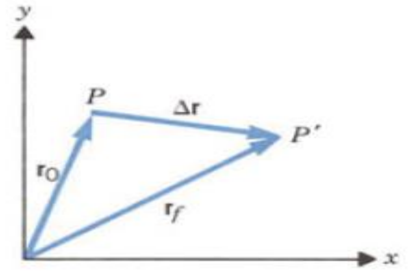


FIGURE 1 The displacement vector $\Delta\mathbf{r}$ is obtained by drawing an arrow from the initial position vector $\mathbf{r}_0$ to the final position vector $\mathbf{r}_f$.

Average speed

The average speed savg of a particle during a time interval Δt depends on the total distance the particle moves in that time interval:

$$\overline{\text{speed}} = \frac{\text{total distance}}{\Delta \text{t}}$$

where,

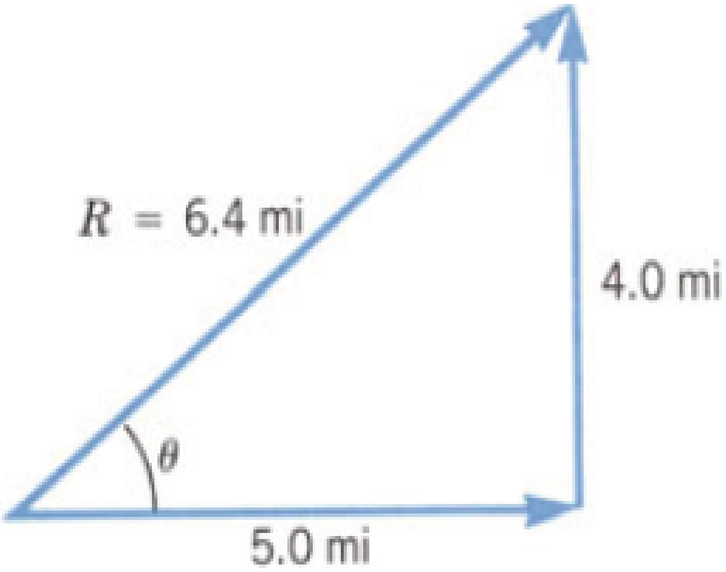
Δ (anything)= final value – initial value

Avg. Speed Vs Avg. Velocity

Consider the walk taken in Fig. Suppose it took 1 h.

Then, by definition, $\overline{\text{speed}} = \dfrac{\Delta s}{\Delta t}$

the average speed of walking was $\Delta s/\Delta t$ $(4.0\,\text{mi} + 5.0\,\text{mi})/1\,\text{h} = 9\,\text{mi/h}$



whereas the average velocity was $\Delta \mathbf{r}/\Delta t = 6.4\,\text{mi}/1\,\text{h}$

6.4 mi/h in the direction 39° north of east.

Average Speed vs Average Velocity

- Average speed s_{avg} is a different way of describing “how fast” a particle moves. Whereas the average velocity involves the particle’s displacement Δx, the average speed involves the total distance covered (for example, the number of meters moved), independent of direction.

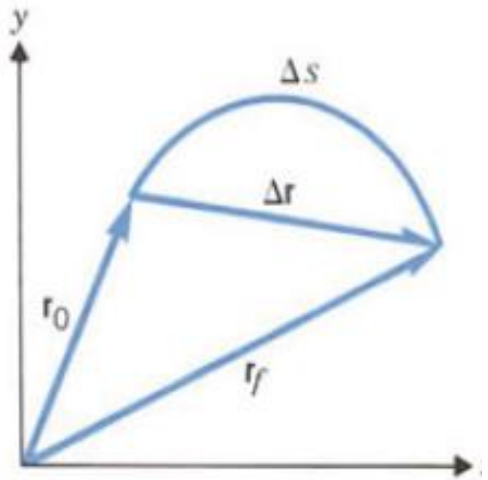


Fig. 2(a)

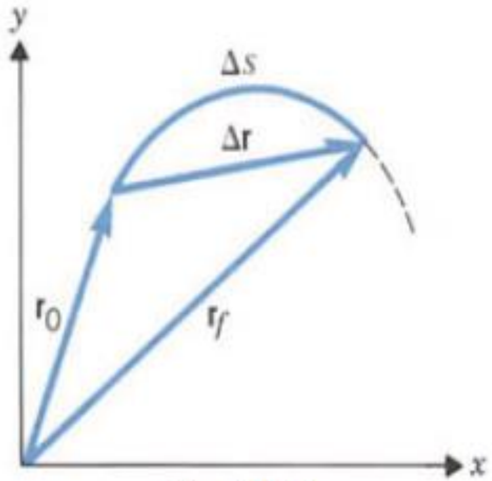


Fig. 2(b)

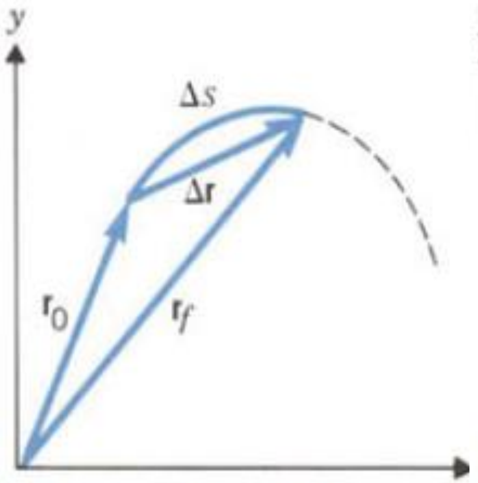


Fig. 2(c)

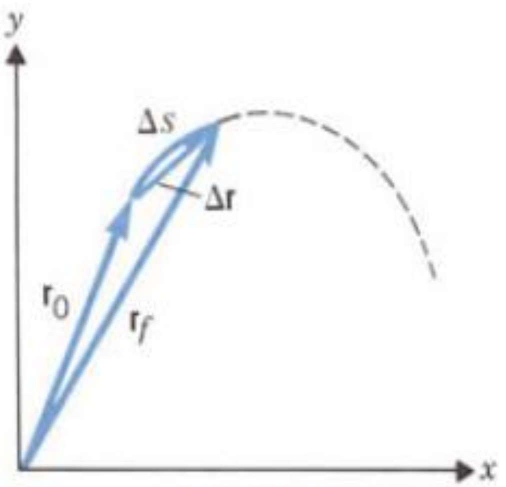


Fig. 2(d)

If we shrink the displacement to a minute amount by taking the limit

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \qquad \textbf{Eq. 3}$$

then the magnitude of the velocity, which is now called the *instantaneous velocity*, at any point on the track will equal the speed. This can be seen in Fig. **2** (a,b,c,d)

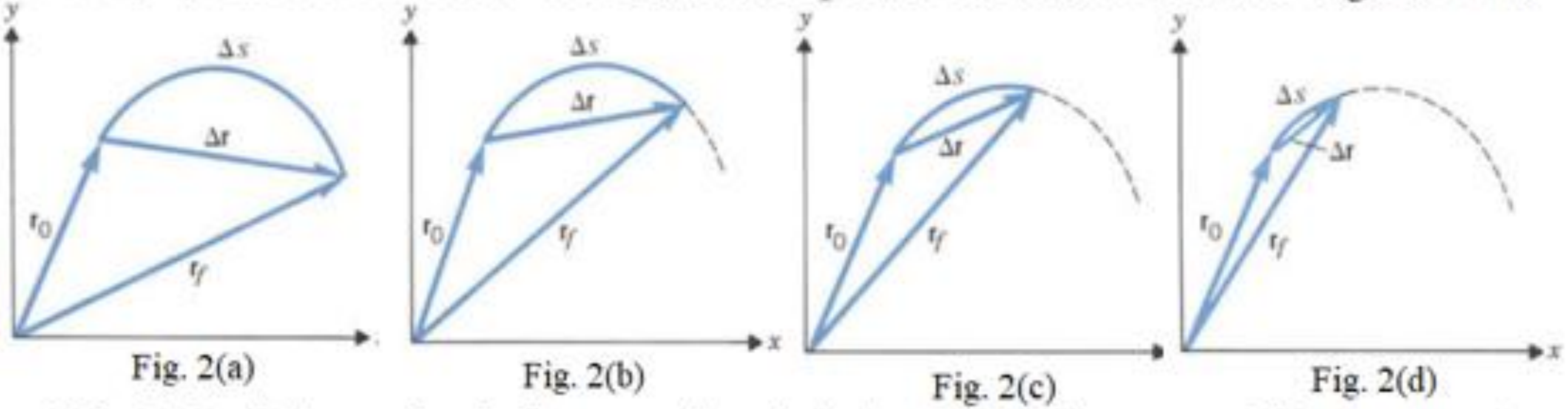


FIG 2 (a,b,c,d) A curved path of a car traveling clockwise. Δs is the distance traveled by the car, and $\Delta \mathbf{r}$ is the displacement vector between the position of the car $\mathbf{r}_f$ at some instant and the position $\mathbf{r}_0$ the initial time. As the distance Δs becomes smaller, its value approaches $\Delta \mathbf{r}$.

we notice that as Δs becomes smaller, the difference between Δs and the corresponding $\Delta \mathbf{r}$ decreases. We should also note that in the limit where $\Delta \mathbf{r}$ becomes infinitesimally small, it becomes tangential to the path, and therefore the direction of the instantaneous velocity is the tangent to the path.

while the magnitude of the instantaneous velocity remains equal to the speed, the direction part of the instantaneous velocity is changing.

Velocity is a vector because it is equal to a vector displacement divided by time, which is scalar, and the division of a vector by a scalar does not remove the vector property.

We should note Eq. 3

instantaneous velocity $\mathbf{v}$ is the first derivative of the position vector with respect to time. It should also be pointed out that because Eq. 3 is a vector equation, it holds for each of the cartesian components of the vectors $\mathbf{v}$ and $\mathbf{r}$, namely,

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}$$

where v_x, v_y, and v_z are the cartesian components of $\mathbf{v}$ and x, y, and z are those of $\mathbf{r}$.

Instantaneous Velocity and Speed

- How fast a particle is moving at a given instant is given by its instantaneous velocity (or simply velocity) v. The velocity at any instant is obtained from the average velocity by shrinking the time interval Δt closer and closer to 0. As Δt dwindles, the average velocity approaches a limiting value, which is the velocity at that instant:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- v is the rate at which position x is changing with time at a given instant; that is, v is the derivative of x with respect to t.
- v at any instant is the slope of the position–time curve at the point representing that instant.
- Velocity is another vector quantity and thus has an associated direction.
- Speed is the magnitude of velocity; that is, speed is velocity that has been stripped of any indication of direction, either in words or via an algebraic sign.

Checkpoint 2:

The following equations give the position x(t) of a particle in four situations (in each equation, x is in meters, t is in seconds, and t>0): (1) x =3t-2; (2) x=$-4t^2 - 2$; (3) x =$\frac{2}{t^2}$; and (4) x = -2. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

Acceleration

- When a particle's velocity changes, the particle is said to undergo acceleration (or to accelerate)
- Unit: Standard unit in MKS system is meter/sec^2 (m/s^2)

Acceleration

If there is a velocity change $\Delta\mathbf{v}$ in a certain time Δt, we define the average acceleration as

$$\bar{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t}$$

or

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_0}{t_f - t_0}$$

where the subscripts f and 0 represent final and initial values, respectively. Usually in a problem we start our stopwatch at $t_0 = 0$, so the elapsed time is simply t_f and we drop the subscript f. We may define an instantaneous acceleration as

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

which is the first derivative of $\mathbf{v}$ with respect to time. Substituting Eq. for $\mathbf{v}$, we write

$$\mathbf{a} = \frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\right) = \frac{d^2\mathbf{r}}{dt^2}$$

which is the second derivative of $\mathbf{r}$ with respect to time.

Important points to remember:

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

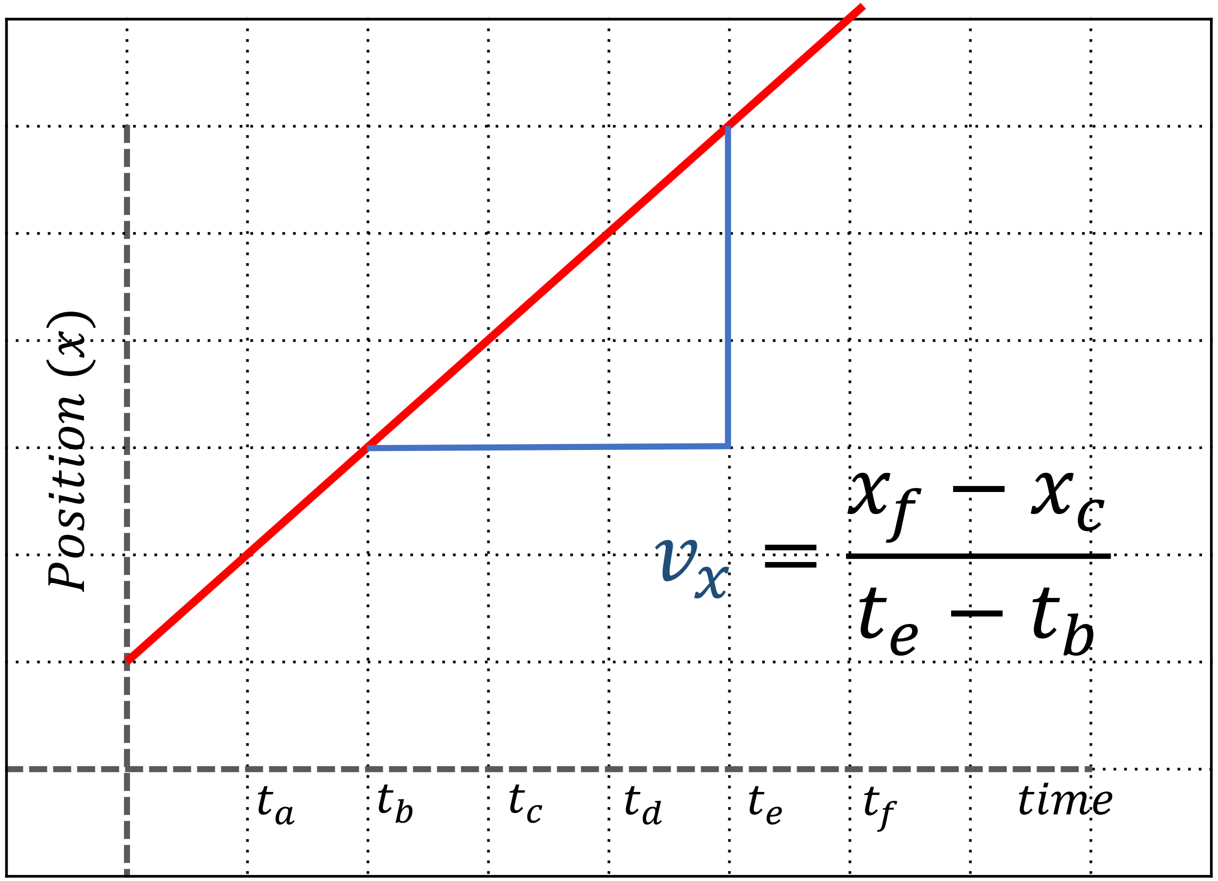
Checkpoint 3

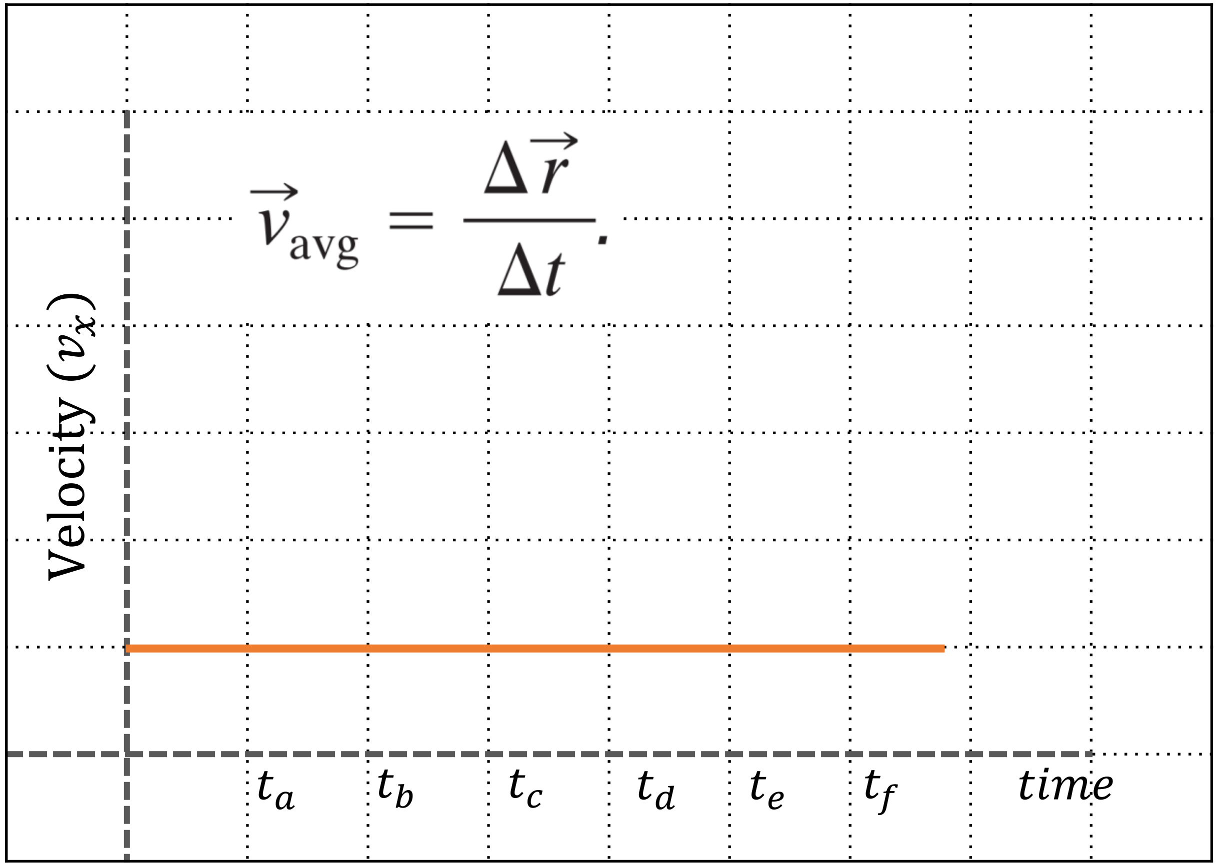
A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Constant Velocities – graphical analysis

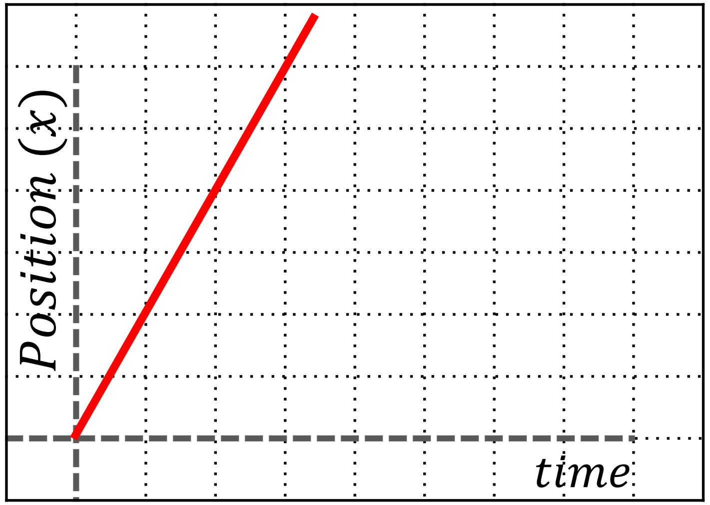
$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

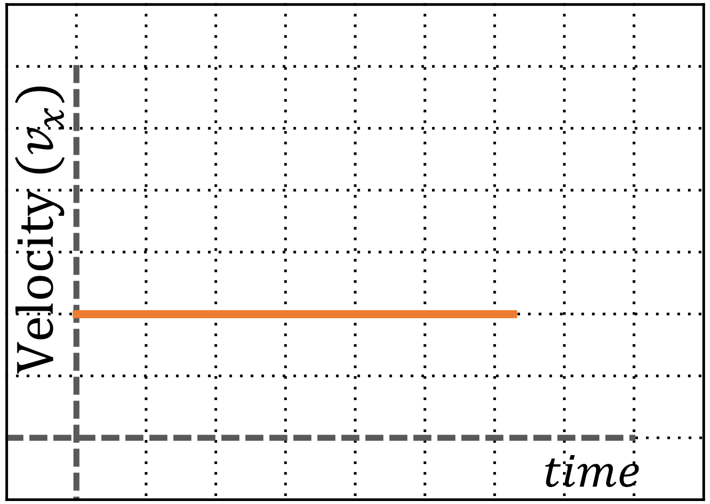
Velocity is "**slope**" in xt graph





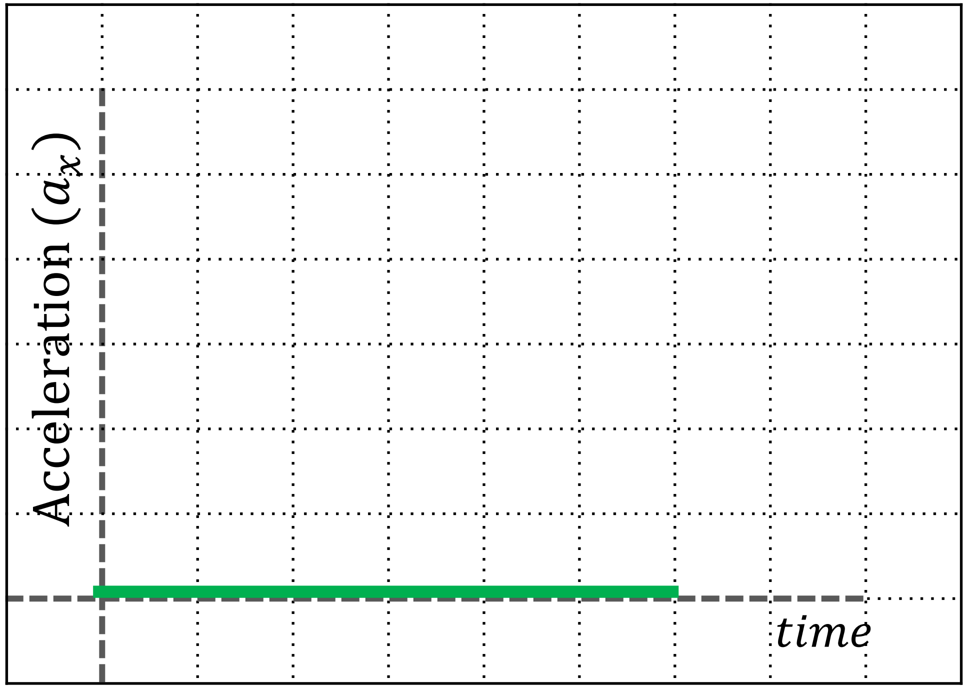
There is no acceleration(a = 0) when velocity is constant





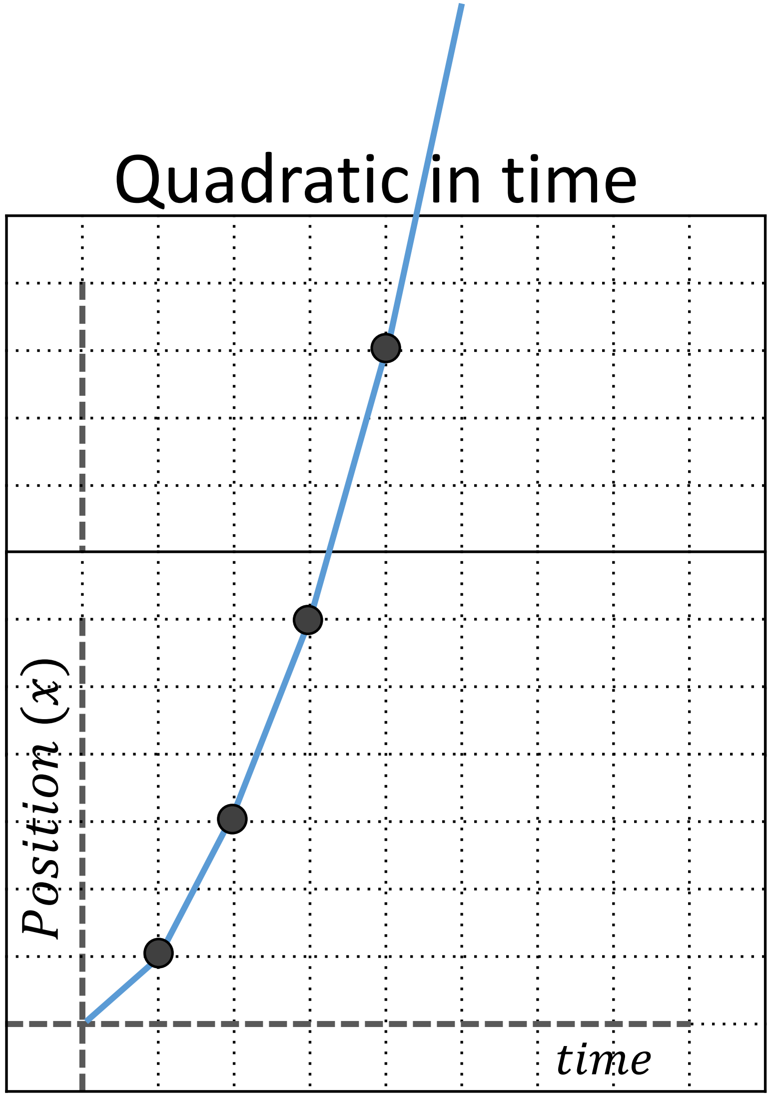
$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}.$$

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

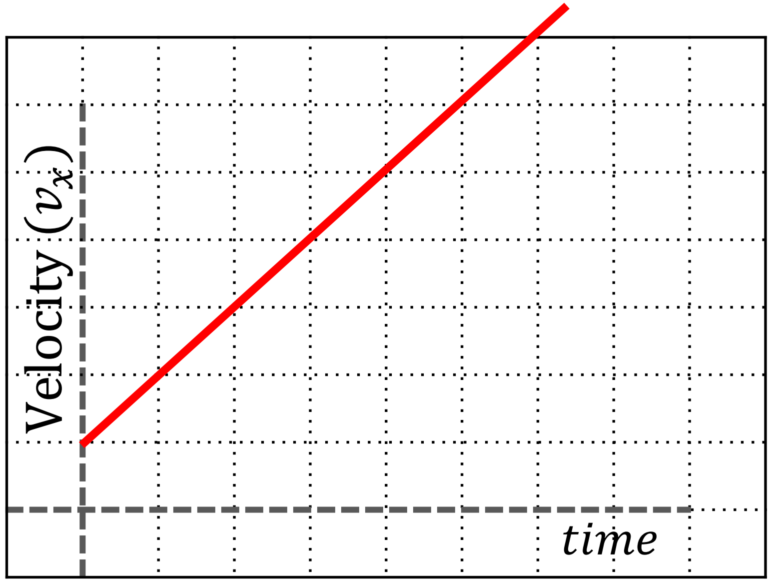


Constant Acceleration

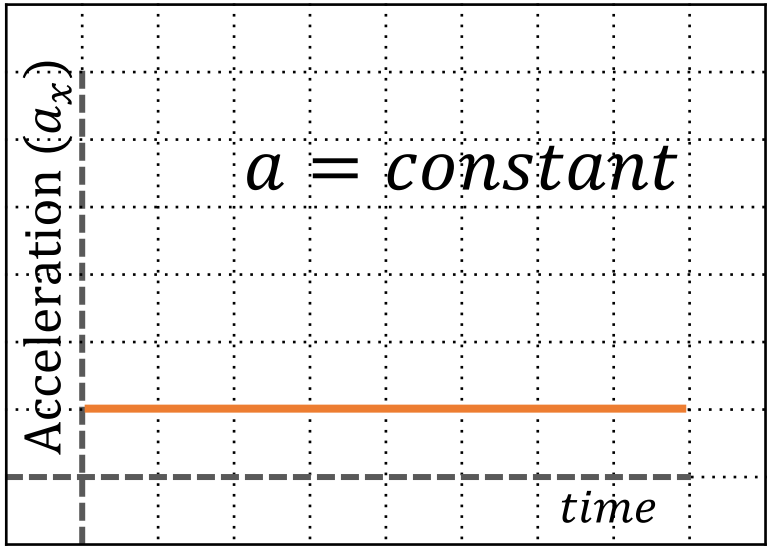
Quadratic in time



Linear in time

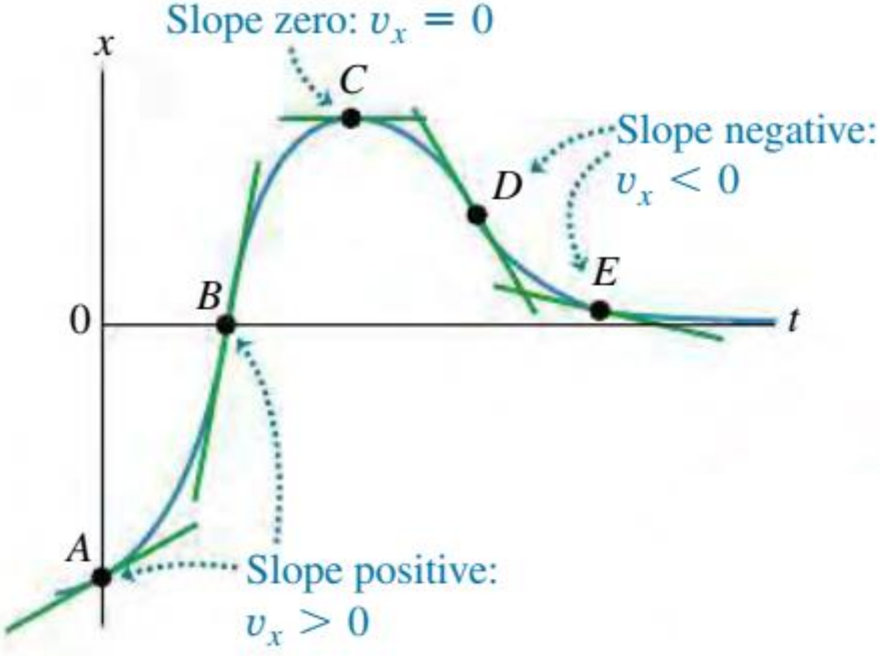


Constant in time

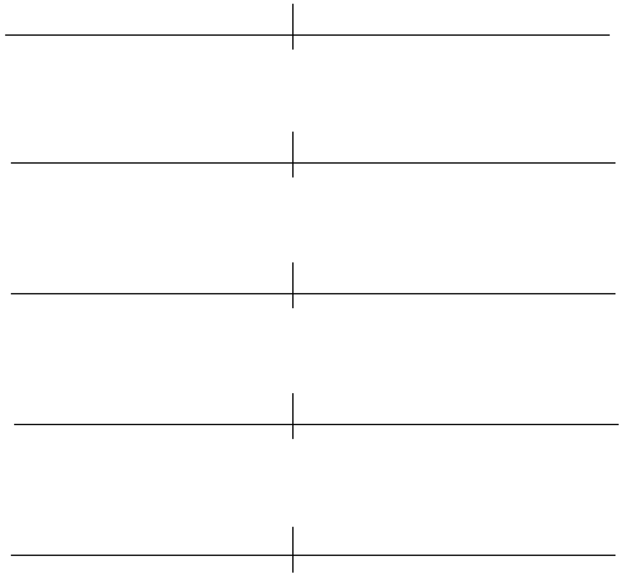


Slope analysis

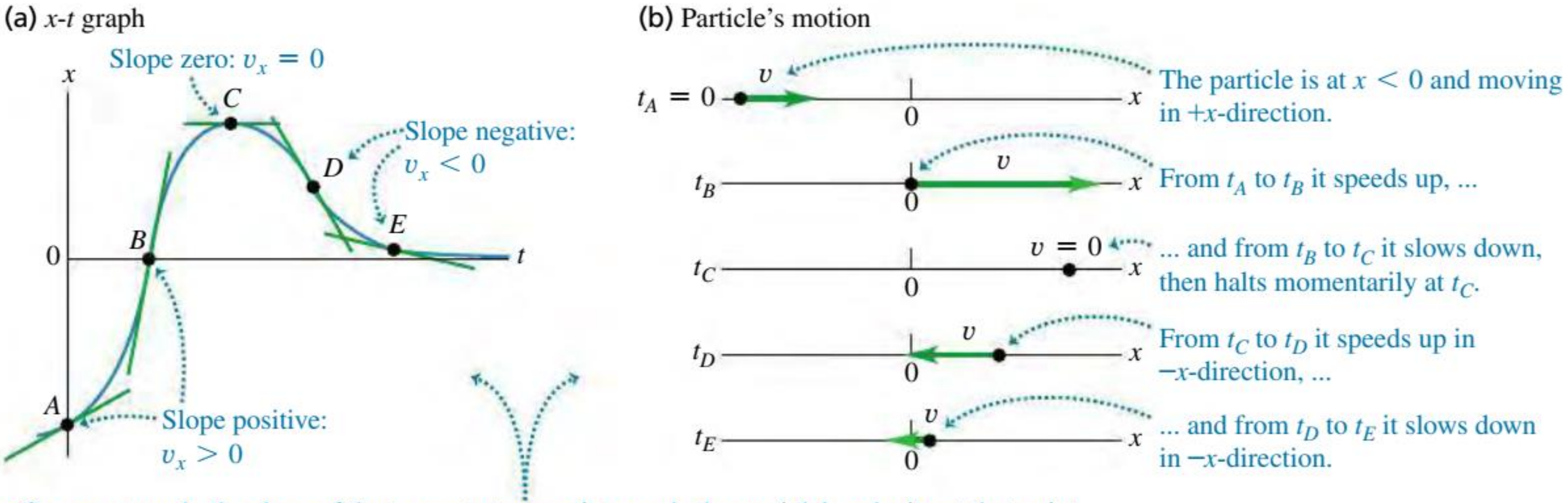
(a) x-t graph



(b) Particle's motion

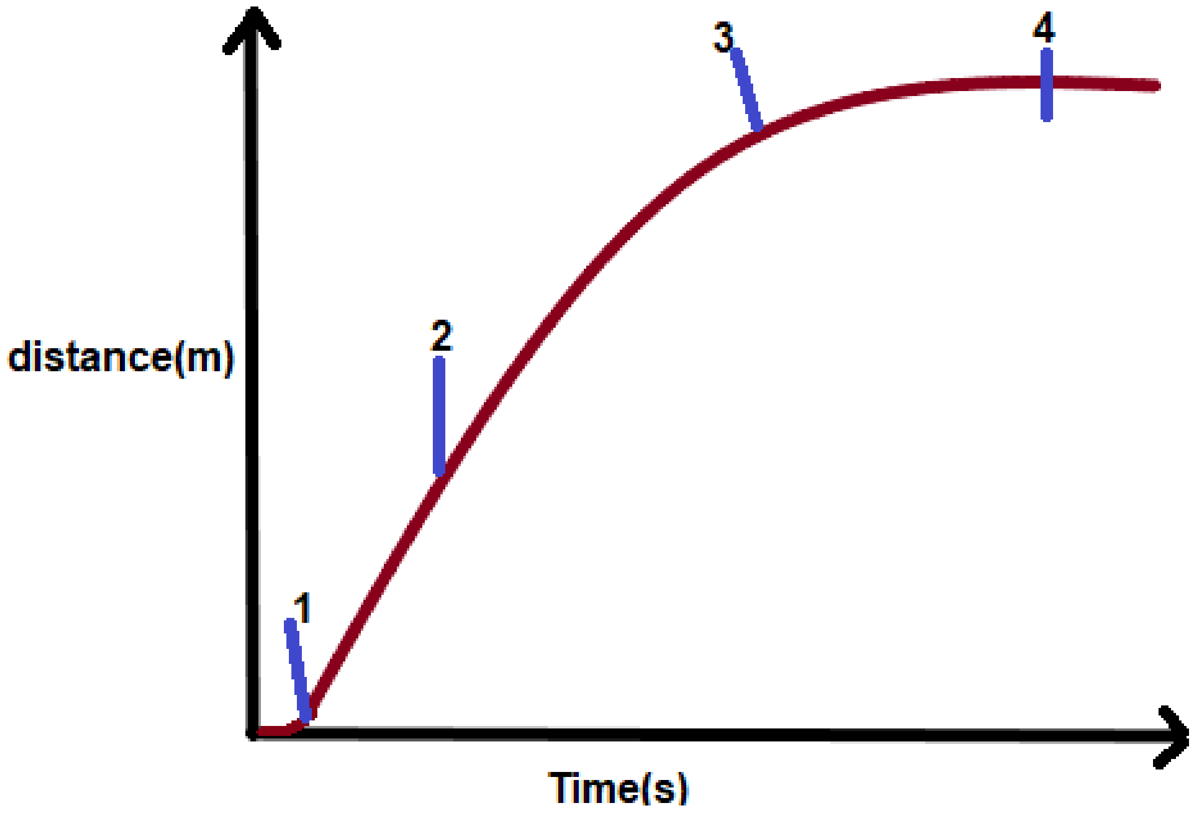


Slope analysis



- On an x-t graph, the slope of the tangent at any point equals the particle's velocity at that point.
- The steeper the slope (positive or negative), the greater the particle's speed in the positive or negative x-direction.

Finding velocity from distance-time graph



Section	Gradient	velocity
1.	increasing	increasing
2.	constant	constant
3.	decreasing	decreasing
4.	zero	zero

- Now we'll discuss velocity, displacement and acceleration in terms of Cartesian components as they all are vector quantities.
- Considering motion only in the direction of a single component, for example, the x direction, that is, motion in a straight line.
- If we start timing an object moving in the x direction when it starts from (x_o =0) point, we may write

$$\overrightarrow{v_x} = \frac{\vec{x} - \overrightarrow{x_0}}{t - t_0} \; ; (x_0 = 0)$$

$$\overline{v}_x = \frac{x - 0}{t - 0}$$

$$\boxed{x = \overline{v}_x t} \longrightarrow$$

Equation results from the definition of average velocity; thus it holds in all cases whether or not the acceleration is constant

Constant Acceleration: a special case

In many types of motion, the acceleration is either constant or approximately so. Such cases are so common that a special set of equations has been derived for dealing with them. The graphs for such case are shown in the diagram:

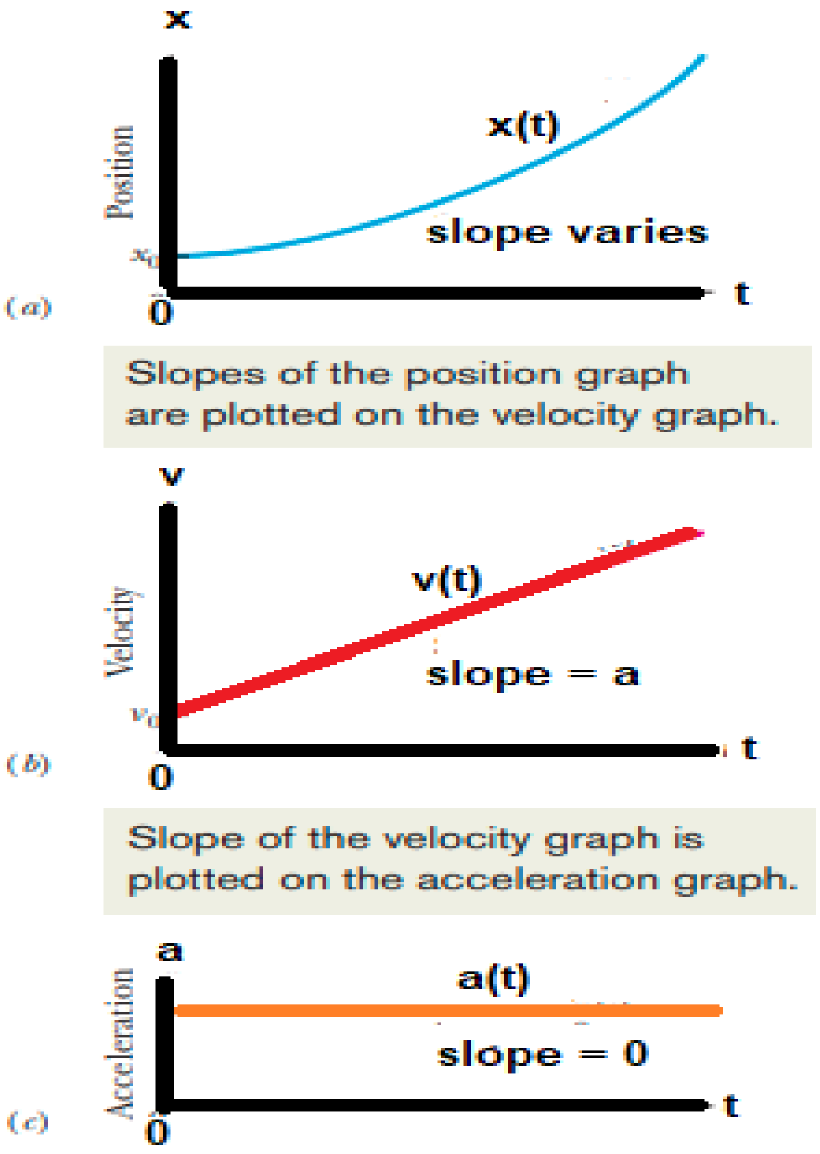


Figure (a) The position $x(t)$ of a particle moving with constant acceleration. (b) Its velocity $v(t)$, given at each point by the slope of the curve of $x(t)$. (c) Its (constant) acceleration, equal to the (constant) slope of the curve of $v(t)$.

Equations of motion for uniformly accelerated motion

$$v = v_0 + at$$

$$v^2 - v_0^2 = 2ax$$

$$x = v_0 t + \frac{1}{2} a t^2$$

1^{st} Equation of Motion

- The acceleration is defined as the rate of change of the velocity.

- If the acceleration is constant, the change in the velocity during the first, second, third, and all succeeding seconds of the motion will be the same and equal to the acceleration

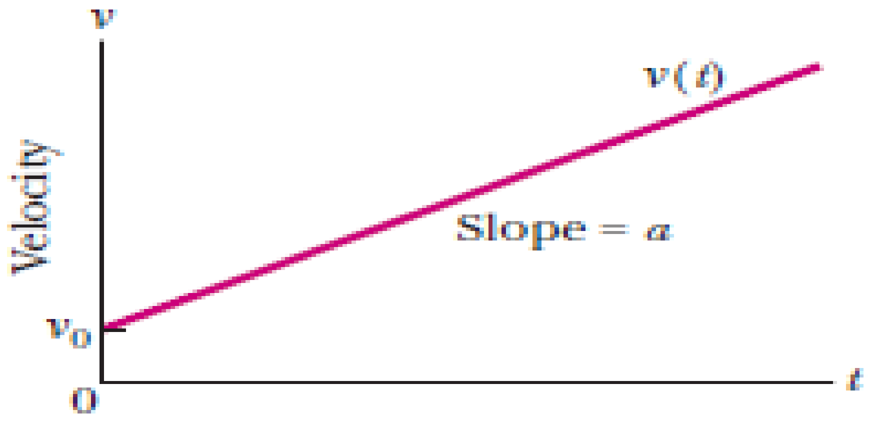
- if the motion lasts t seconds, the change in the velocity
  $\Delta v = v - v_0 = at$, where v is the final velocity and v_0 is the initial velocity.

- We can rewrite this result as

$$\boxed{v = v_0 + at} \longrightarrow \textbf{Eq.1.2}$$

Velocity – time Graph

- If we plot equation 1.2 on graph we will obtain a straight line, as indicated in the Figure below.
- The slope of this line is the constant acceleration *a.*



2^{nd} Equation of Motion

- Another important relation that we can have when the velocity increases at a constant rate the average velocity is one half the sum of the initial velocity v_0 and the final velocity namely, v

$$\overline{v} = \frac{v + v_0}{2} \longrightarrow \textbf{Eq. 1.3}$$

- Previously we have,

$$x = \overline{v}\, t \longrightarrow \textbf{Eq. 1.1}$$

- On substituting value of $\overline{v}$ in eq 1.1, we get

$$x = \frac{v + v_0}{2} t$$

2nd Equation of Motion

$$x = \frac{v + v_0}{2} t \longrightarrow \textbf{Eq. 1.4}$$

If we substitute the value of "v" from 1st equation of motion in eq.1.4 then we will have

$$x = \frac{v_0 + at + v_0}{2} t$$

Or we can have,

$$\boxed{x = v_0 t + \frac{1}{2} a t^2}$$

3^{rd} Equation of Motion

- Considering equation 1.4,

$$x = \frac{v + v_0}{2} t \longrightarrow \textbf{Eq. 1.4}$$

- We can find out the value of “t” from 1^{st} equation of motion that is,

$$t = \frac{v - v_0}{a}$$

- On substituting the above value of “t” in eq. 1.4 we will have the following results

$$x = \frac{(v + v_0)}{2} \frac{(v - v_0)}{a}$$

$$\boxed{v^2 - v_0^2 = 2ax}$$

Derivations by Integration

We may derive these equations more formally by integration.

By definition

$$a = \frac{dv}{dt}$$

Rearranging terms and integrating, we write

$$\int_{v_0}^{v} dv = \int_0^t a\, dt$$

acceleration is taken as constant, so a can be taken out of the integral and we write

Derivations by Integration

$$\int_{v_0}^{v} dv = a \int_0^t dt$$

This integrates to

$$v - v_0 = at$$

and

$$v = v_0 + at$$ **1st equation of motion**

Derivations by Integration

- From definition we know,

$$v = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_0^t v\,dt$$

- Rearranging terms

$$\int_{x_0}^{x} dx = \int_0^t (v_0 + at)dt$$

$$\int_{x_0}^{x} dx = v_0 \int_0^t dt + a \int_0^t t\,dt$$

Derivations by Integration

- After applying limits we will have the following result,

$$\boxed{x - x_0 = v_0 t + \frac{1}{2} a t^2}$$

- Above is the 2nd Equation of motion

Note that in this formulation we have not required that $x = 0$ at $t = 0$ as in the previous algebraic derivations.

Derivations by Integration

For 3rd equation of motion

We may use thc chain rule to write

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

$$\int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = a(x - x_0)$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

Results by Integration

- Following are the results found by integration,

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \frac{1}{2}at^2$$
$$v^2 - v_0^2 = 2a(x - x_0)$$

- All the equations that we have derived for motion are in the x-direction. Similar equations can simply be written for motion in the y and z directions when the components of the acceleration in these directions are also constant.

Constant Acceleration

$$v = v_0 + at,$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2,$$

$$v^2 = v_0^2 + 2a(x - x_0),$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t,$$

$$x - x_0 = vt - \tfrac{1}{2}at^2.$$

These are *not* valid when the acceleration is not constant.

Free Fall Motion/Acceleration

- There is one important thing to be noted here. In the solution of motion problems we must assign vector directions.
- If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate.
- The acceleration is independent of the object's characteristics, such as mass, density, or shape; it is the same for all objects
- That rate is called the **free-fall acceleration, and its magnitude** is represented by **g**.
- The value of g varies slightly with latitude and with elevation.
- At sea level in Earth's mid latitudes the value is 9.8 m/s^2

Important points to remember

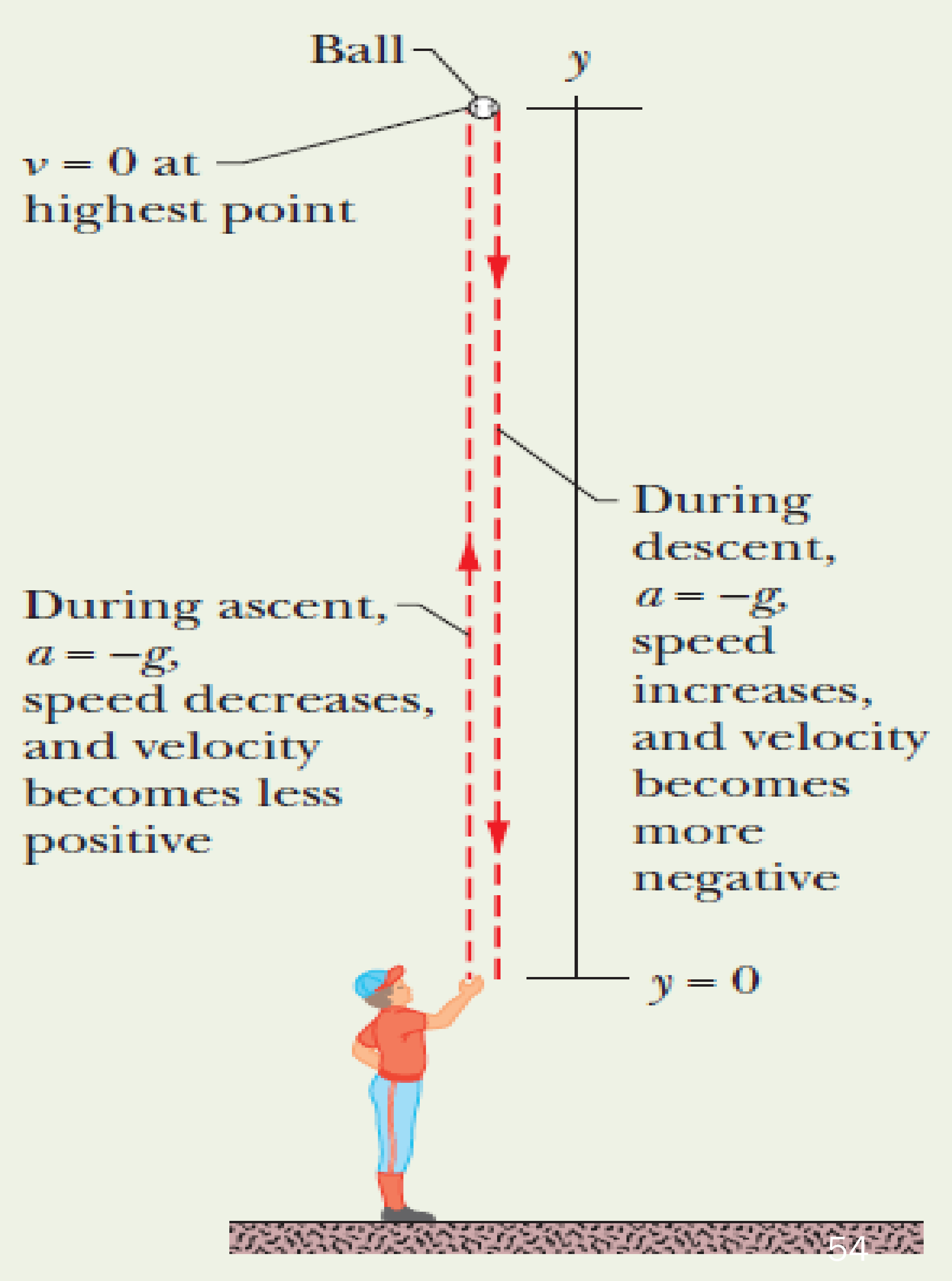
- The equations of motion for constant acceleration also apply to free fall near Earth's surface; that is, they apply to an object in vertical flight, either up or down, when the effects of the air can be neglected

- Points to be noted:
  1. The directions of motion are now along a vertical y axis instead of the x axis, with the positive direction of y upward.
  2. The free-fall acceleration is negative—that is, downward on the y axis, toward Earth's center—and so it has the value - g in the equations.

Direction of Velocity in free fall

- In order to work with free fall problems choosing a particular coordinate system is a matter of personal convenience
- Consider a boy throwing a ball vertically upwards then we will take the motion of ball is along “y-axis” and we will take the upward motion of ball as positive.
- It is important to note that once we choose a coordinate system, all parameters have their vector direction controlled by it.

The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected



Direction of Velocity in free fall

- If we choose the positive y direction as up and the boy throws the ball straight up, then the vector displacement from the ground to its highest position is positive.

- During its upward travel, because velocity is the displacement divided by the scalar time, it too is positive

- The only motion is in the "y direction", so we therefore use y, v_y, a_y in the equations previously derived

$$\boxed{v_y = v_{0y} + a_y t}$$

Checkpoint 5

a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point?

b) What is it for the descent, from the highest point back to the release point?

c) What is the ball's acceleration at its highest point?

Adapted from:

Book:

Fundamentals of Physics by Halliday Resnick & Jearl Walker 10^{th} edition