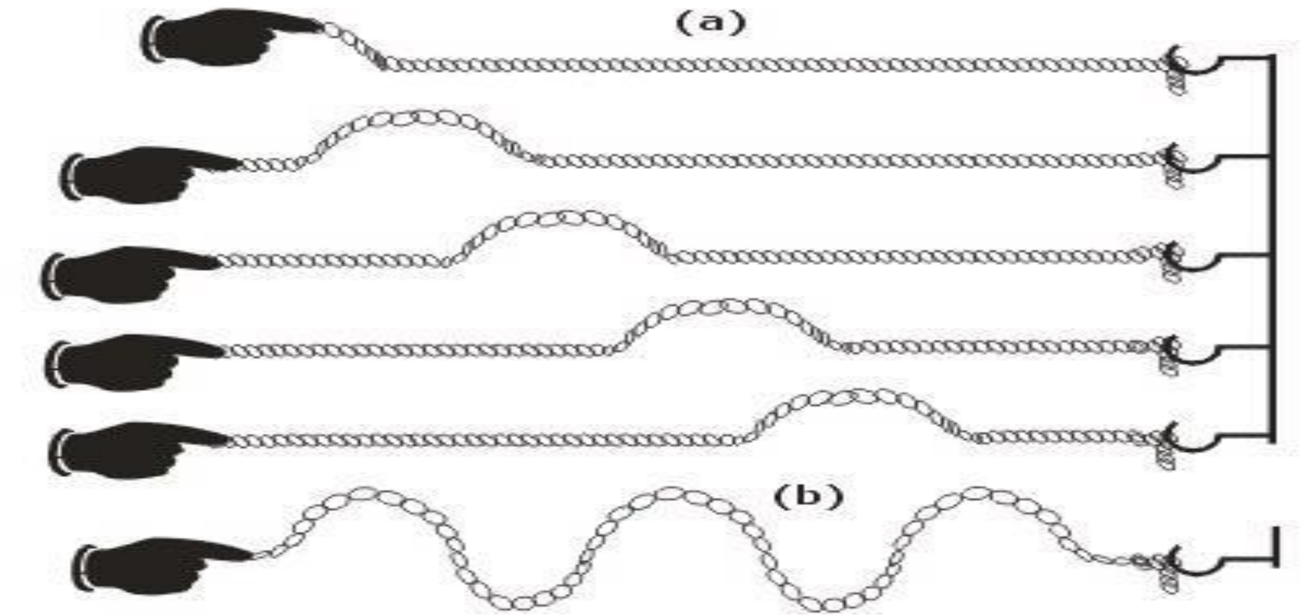
Waves

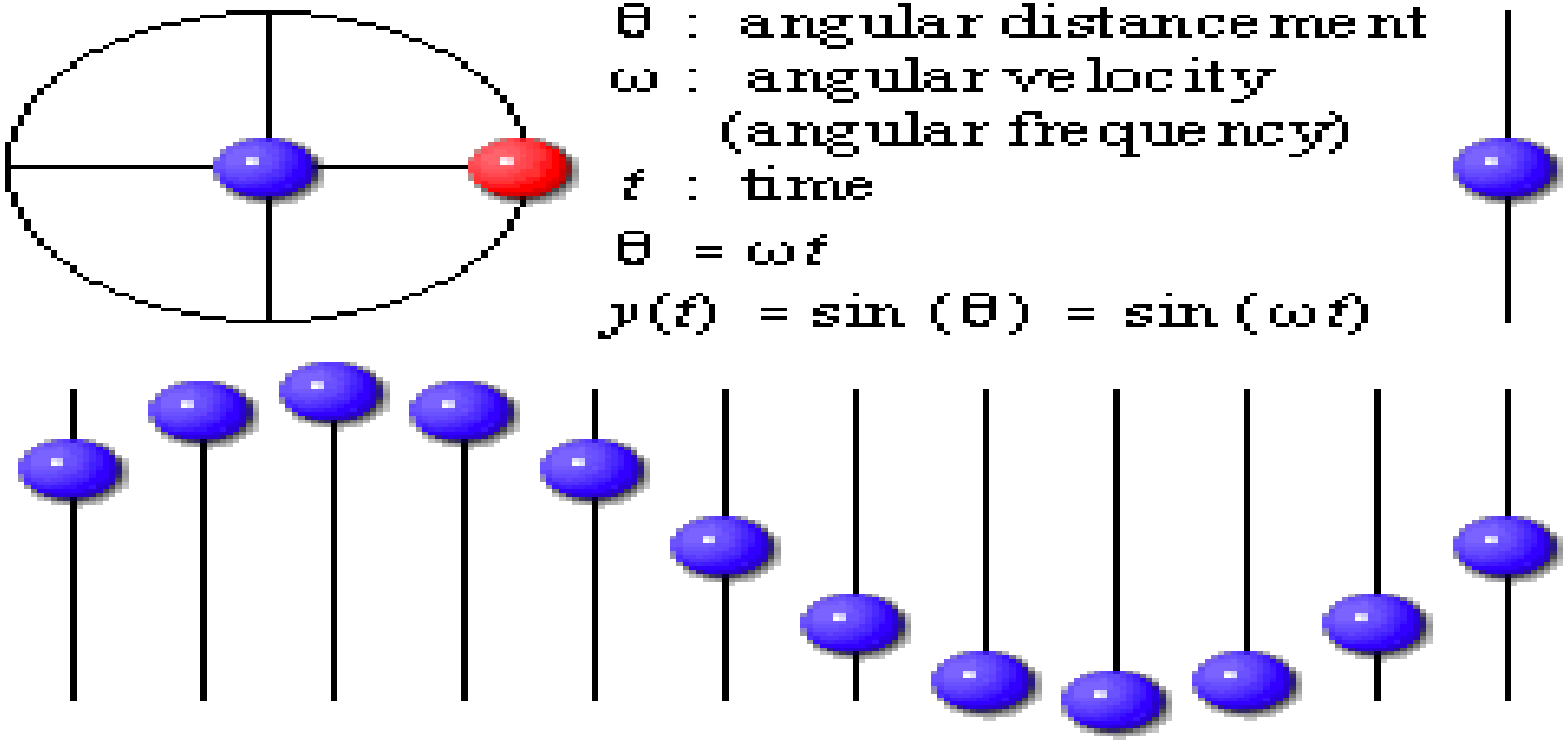
Part 1

Q…What is a Wave?

Answer…Wave is… pattern of motion of particles of the medium which transport disturbance without transporting the medium.

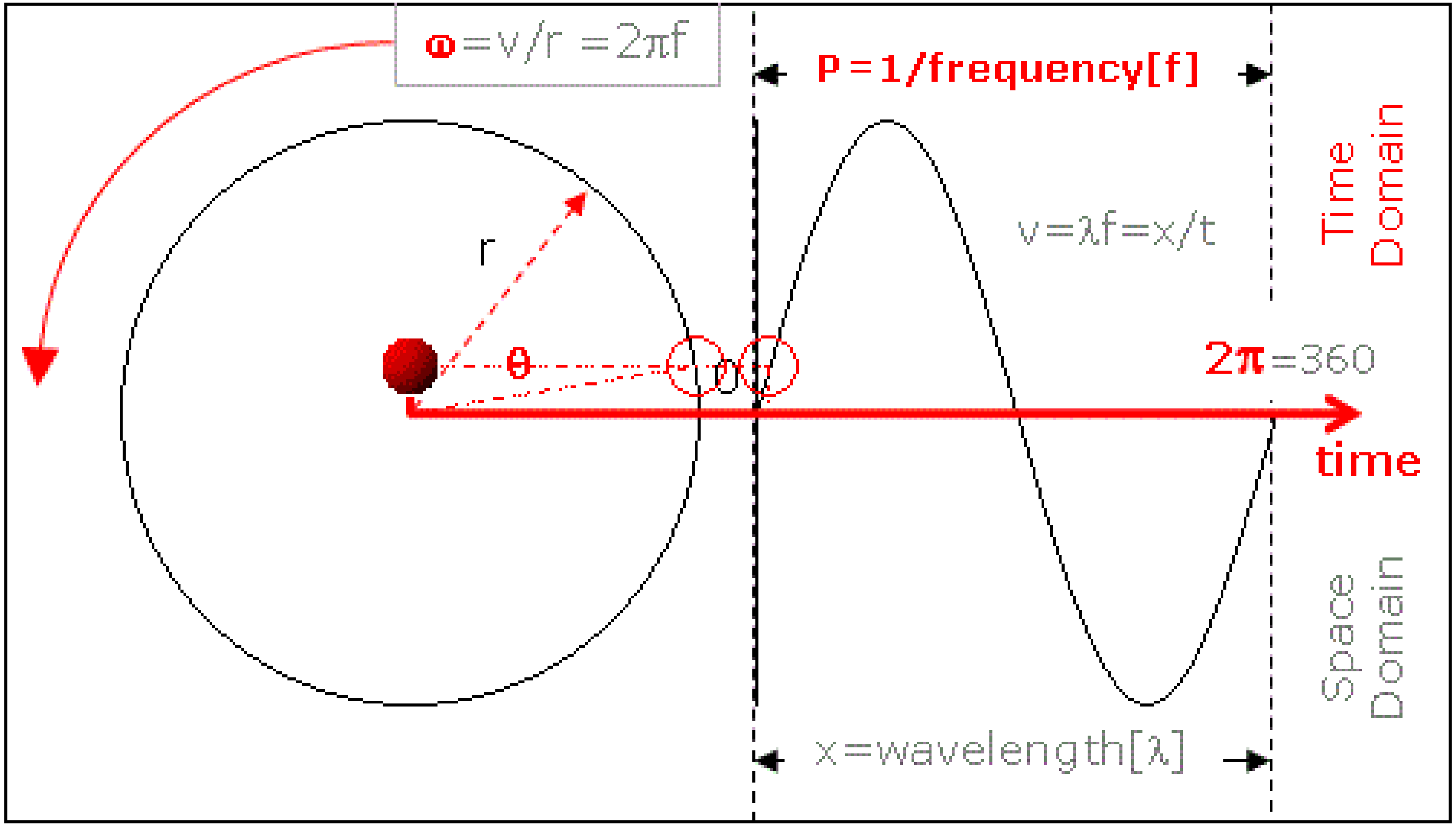


Motion of Particles in the Medium



It is a sinusoidal graph because particles oscillate according to the Sine function

Formation of Sinusoidal Wave



Types of Waves

1. MECHANICAL WAVES

2. ELECTROMAGNETIC WAVES(EMW)

3. MATTER WAVES

Mechanical Waves

- These waves are most familiar because we encounter them almost constantly; common examples include water waves, sound waves, and seismic waves.
- All these waves have **two central features**:

1. They are governed by Newton's laws,
2. and they can exist only within a material medium,

Examples: water, air, and rock.

Electromagnetic Waves

- The disturbance of Electric and Magnetic Field is transported.
- These waves require no material medium.
- Examples include visible and ultraviolet light, radio and television waves, microwaves, x rays, and radar waves.
- All electromagnetic waves travel through a vacuum at speed c = 299 792 458 m/s.

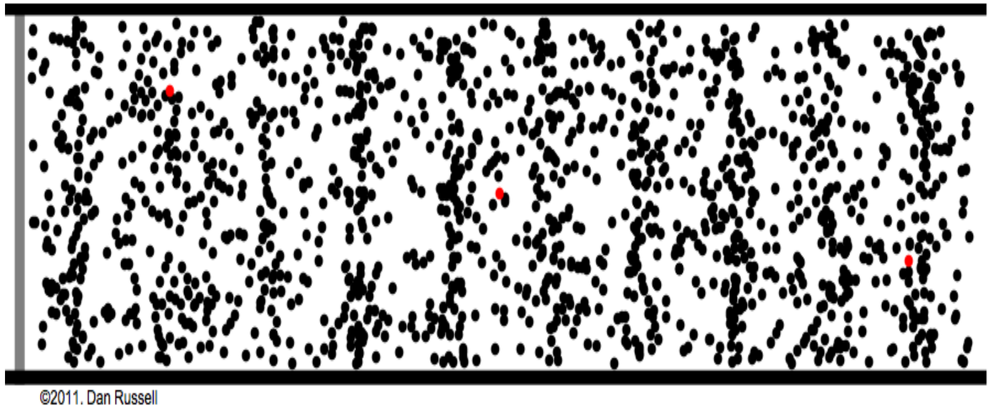
Matter Waves

- Although these waves are commonly used in modern technology, they are probably very unfamiliar to you.
- These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.(study in modern physics)

Classification of Waves

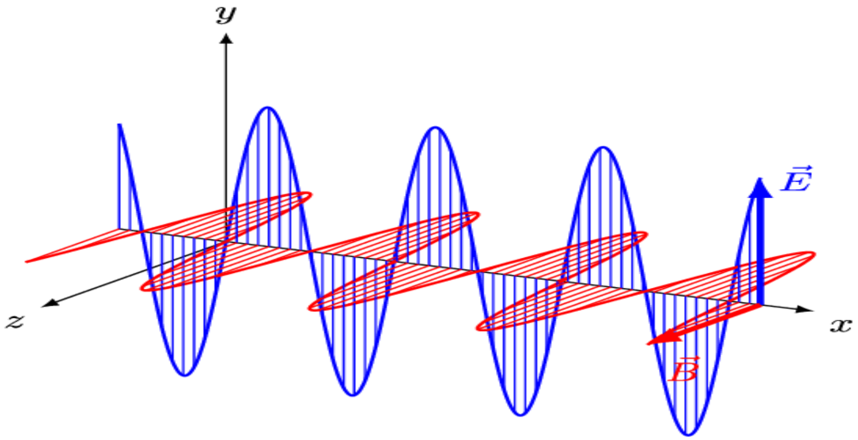
On the basis of Medium

Mechanical waves: require medium to travel



Electromagnetic waves:

- Do not require medium to travel.
- Travel in free space

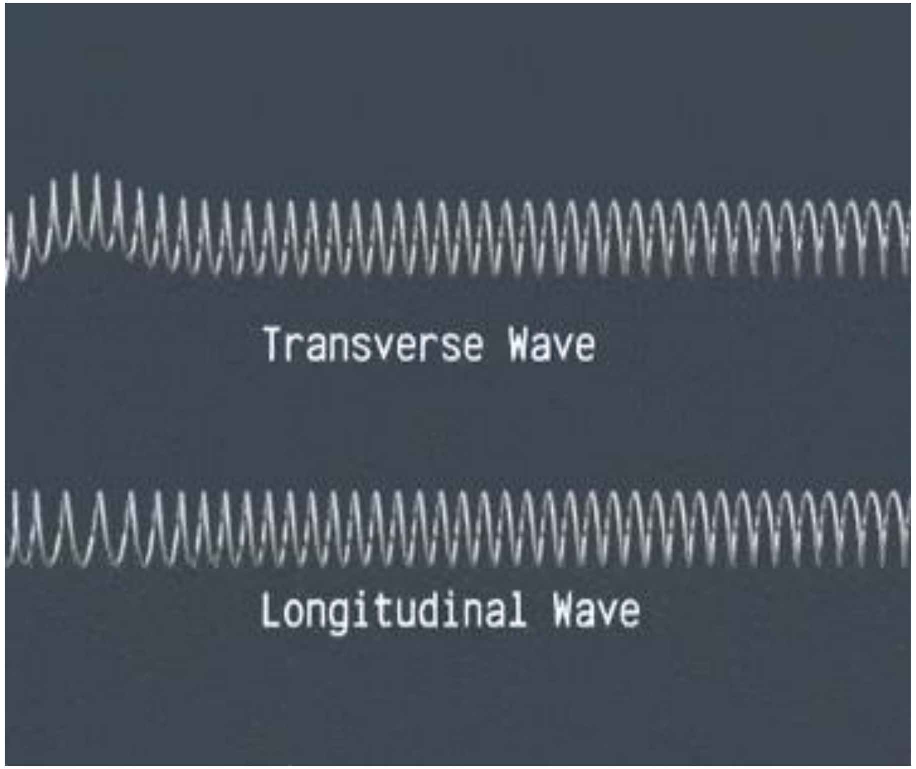


Classification of Waves

On the basis of vibration of particles /wave forms(shapes of the waves)

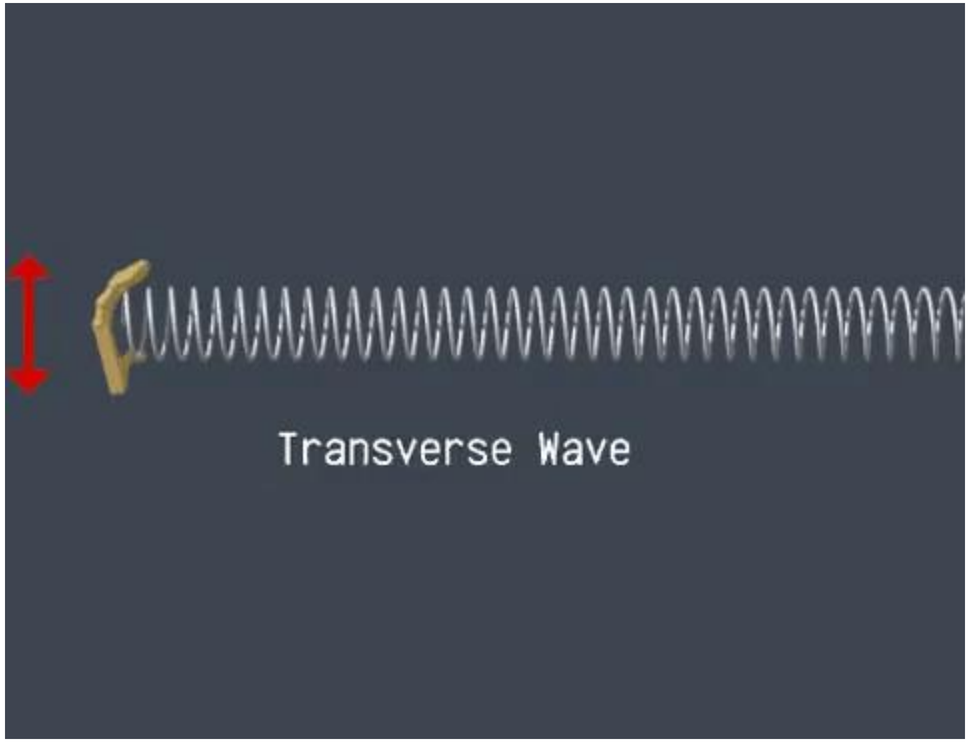
(in material/mechanical wave)

- Transverse Waves
- Longitudinal Waves



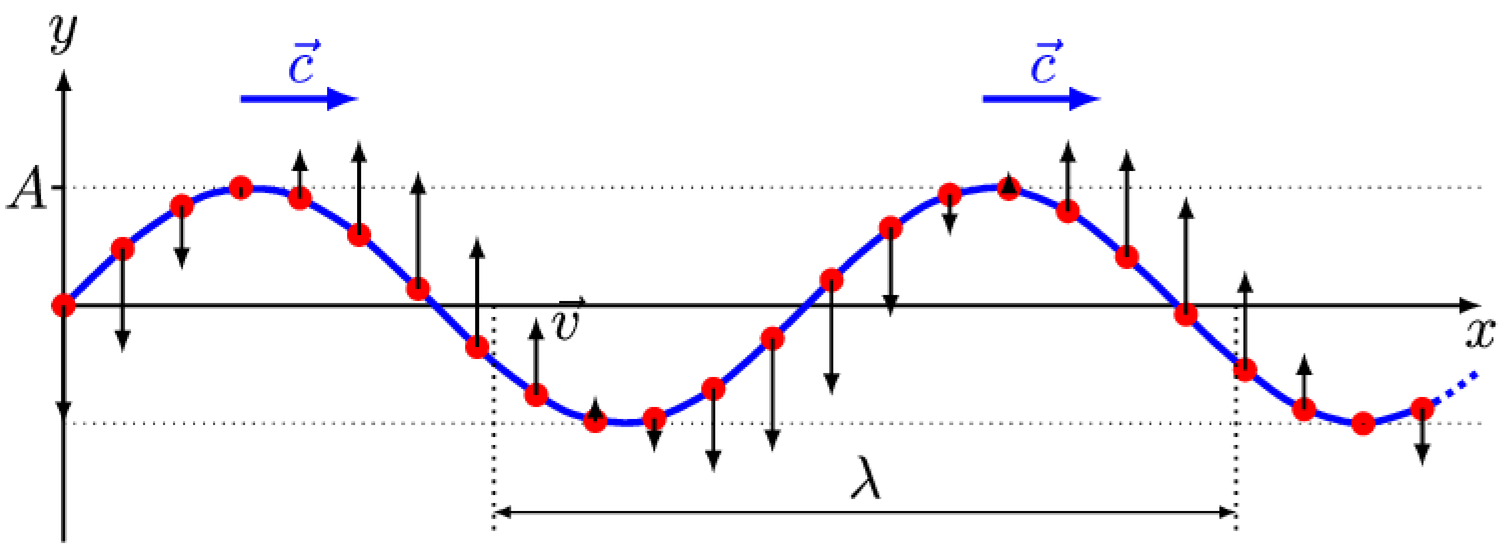
Transverse Waves

- The waves in which particles of the medium vibrates perpendicular to the direction of propagation of wave are said to be transverse waves.
- It travels in the form of crest and trough.



Transverse Waves

Transverse waves vibrate at right angles to the direction of travel of the wave.



Light and radio waves are transverse waves.

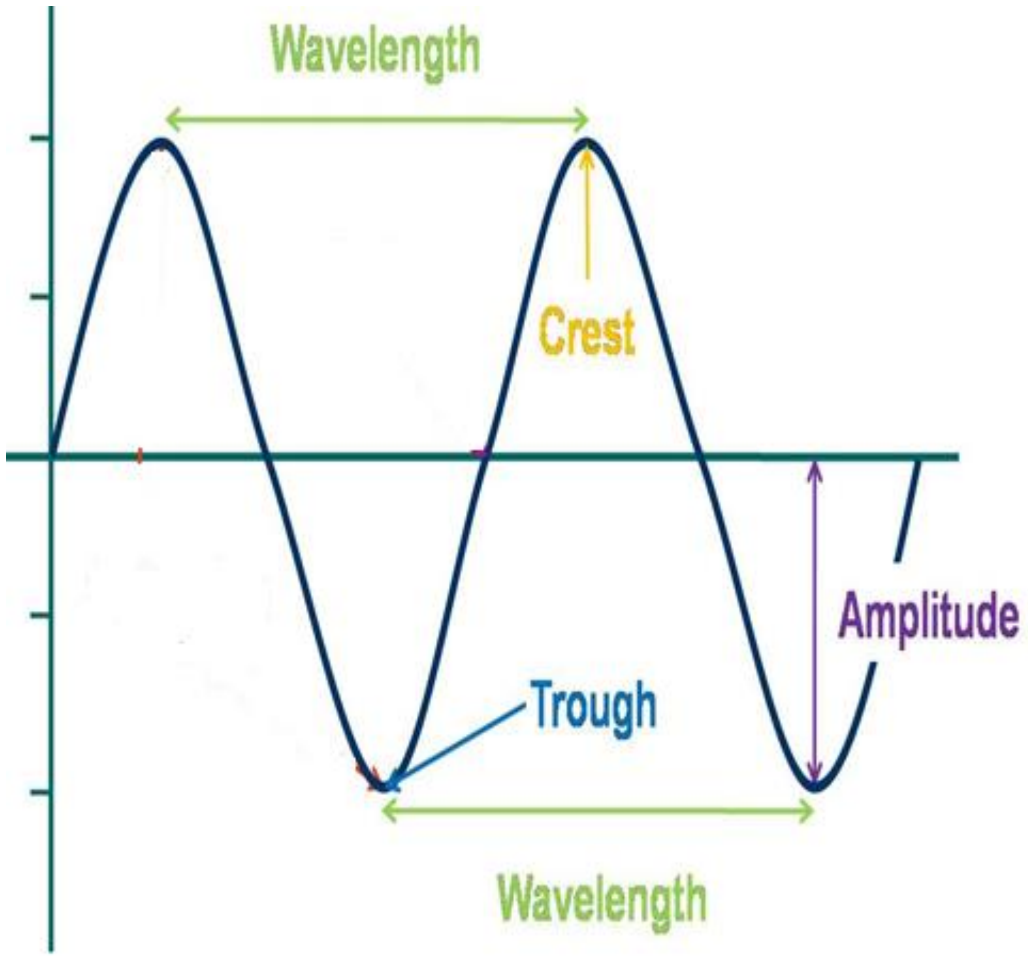
Transverse Waves(TW) …Cont’d

Limitation for the propagation of TW:

- It transmits only in the medium having rigidity , means in solid and on the surface of liquids except gases.
- There is no effect on Temperature and Pressure of the Medium while the transverse waves is being processed on it(medium).

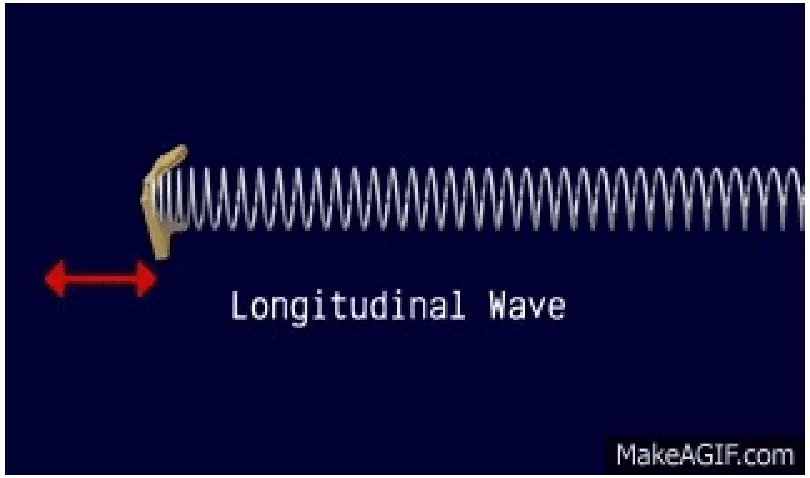
Parameters of TW

- Amplitude (~A)
- Frequency(~ $\nu_{(neu)}$)
- Time Period(~ T)
- Wavelength(~λ)
- Crest & Trough



Longitudinal Waves

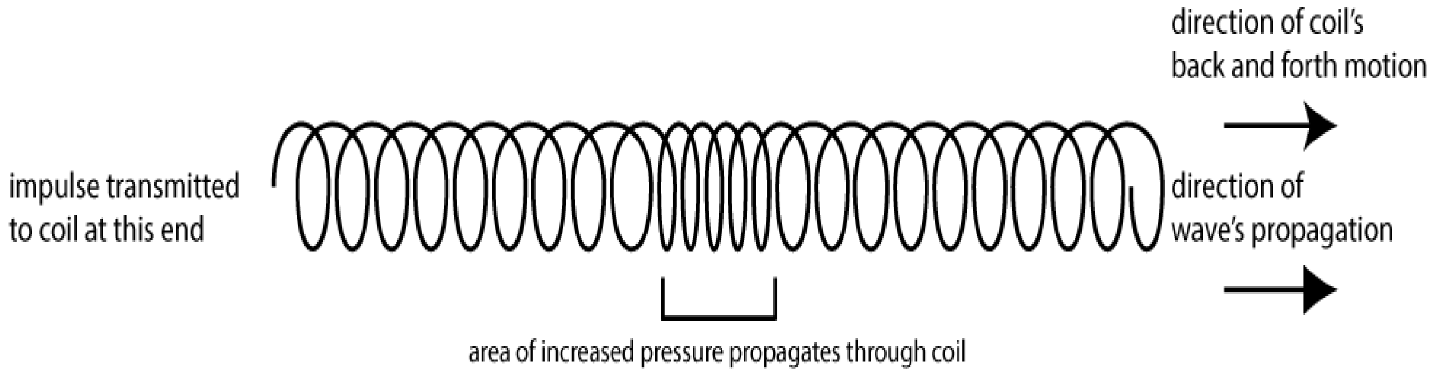
- Waves in which particles of the medium vibrates in the same direction of propagation of the wave are said to be longitudinal waves.
- It travels in the form of compression and rarefaction.



Longitudinal Waves (LW) ...Cont'd

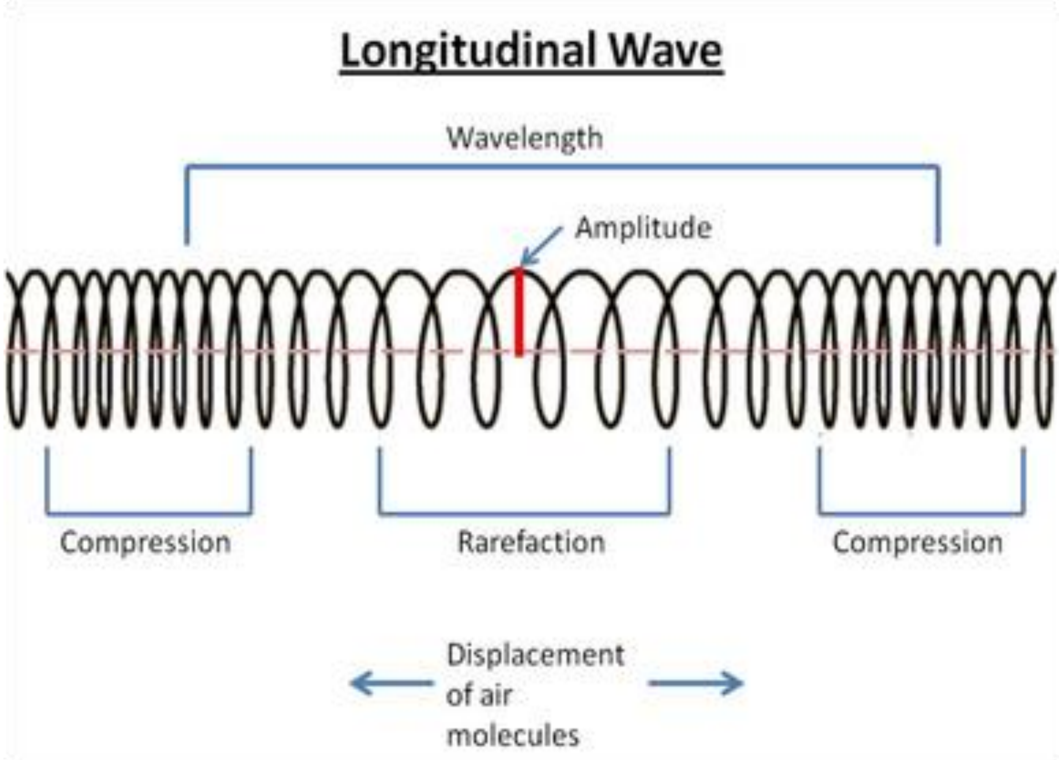
Limitation for the propagation of LW:

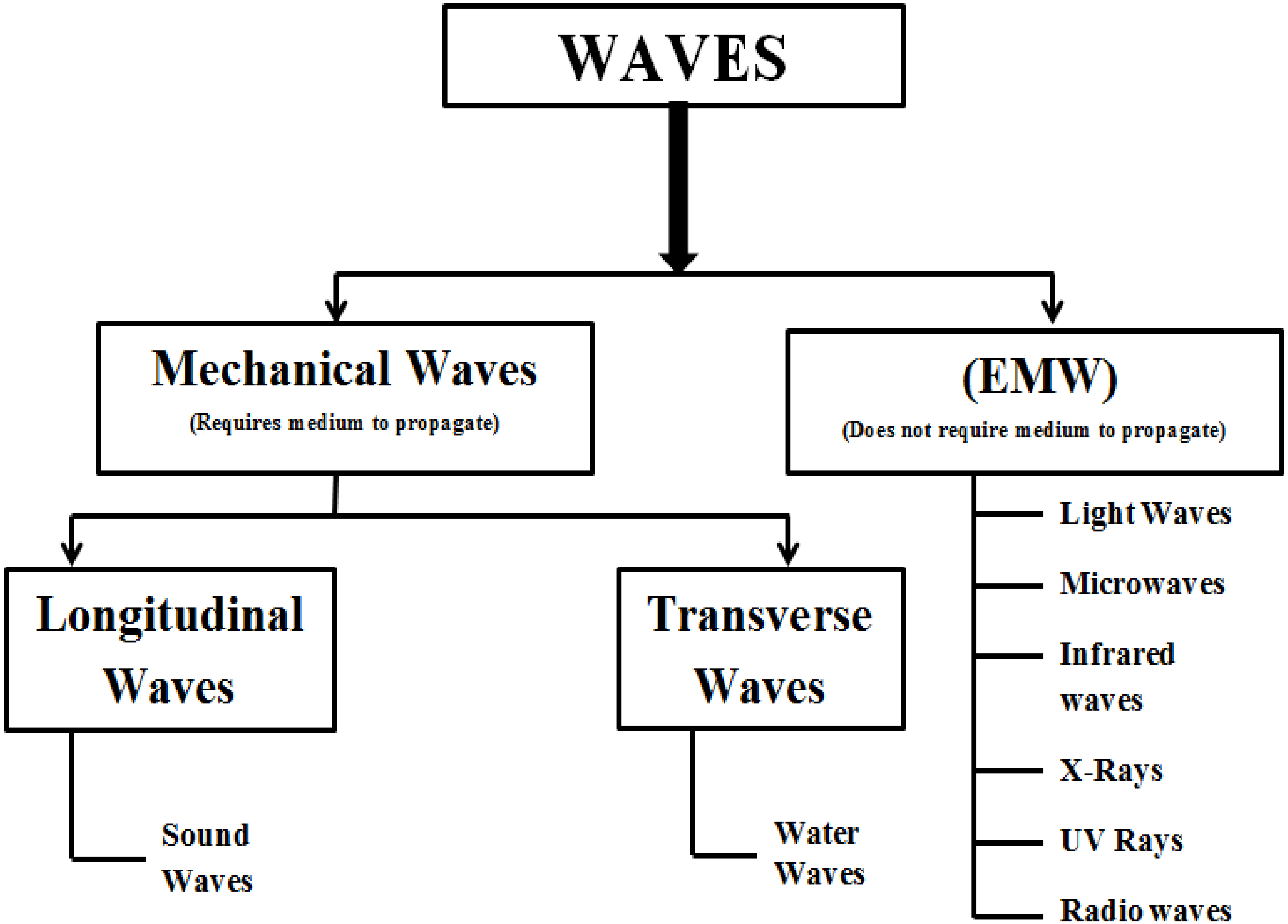
- It travels in all types of medium i.e, solid, liquid and gases.
- Temperature and Pressure of the Medium rises when the longitudinal waves is being processed on it.



Parameters of LW

- Amplitude (~A)
- Frequency(~ $\nu_{(neu)}$)
- Time Period(~ T)
- Wavelength(~λ)
- Compression & Rarefaction




WAVES
Mechanical Waves
(Requires medium to propagate)
(EMW)
(Does not require medium to propagate)
Longitudinal Waves
Transverse Waves
Sound Waves
Water Waves
Light Waves
Microwaves
Infrared waves
X-Rays
UV Rays
Radio waves

Waves

Spatial propagation of oscillations form wave pattern

Mechanical waves.

Electromagnetic waves.

Matter waves.

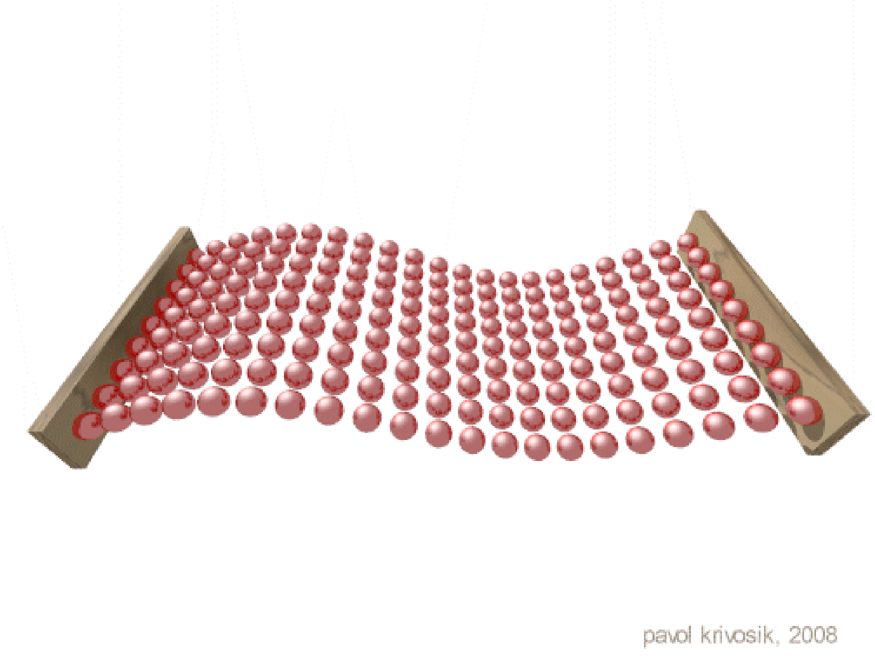
Waves

Spatial propagation of oscillations form wave pattern

Mechanical waves.

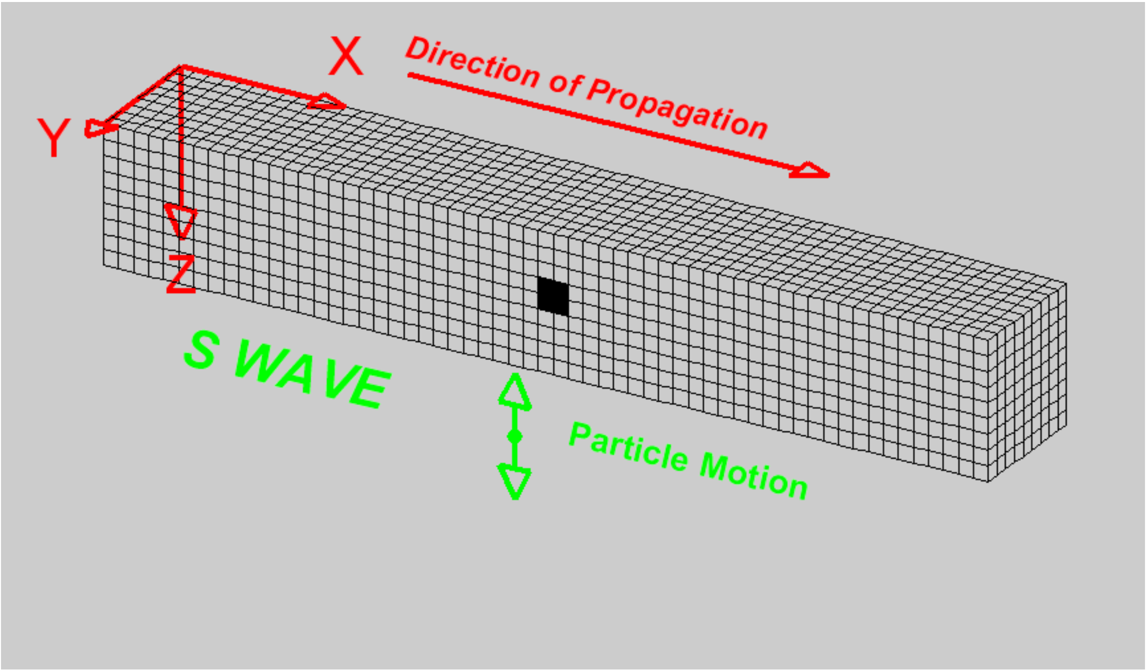
Electromagnetic waves.

Matter waves.



Waves

Spatial propagation of oscillations form wave pattern



Mechanical waves.

Electromagnetic waves.

Matter waves.

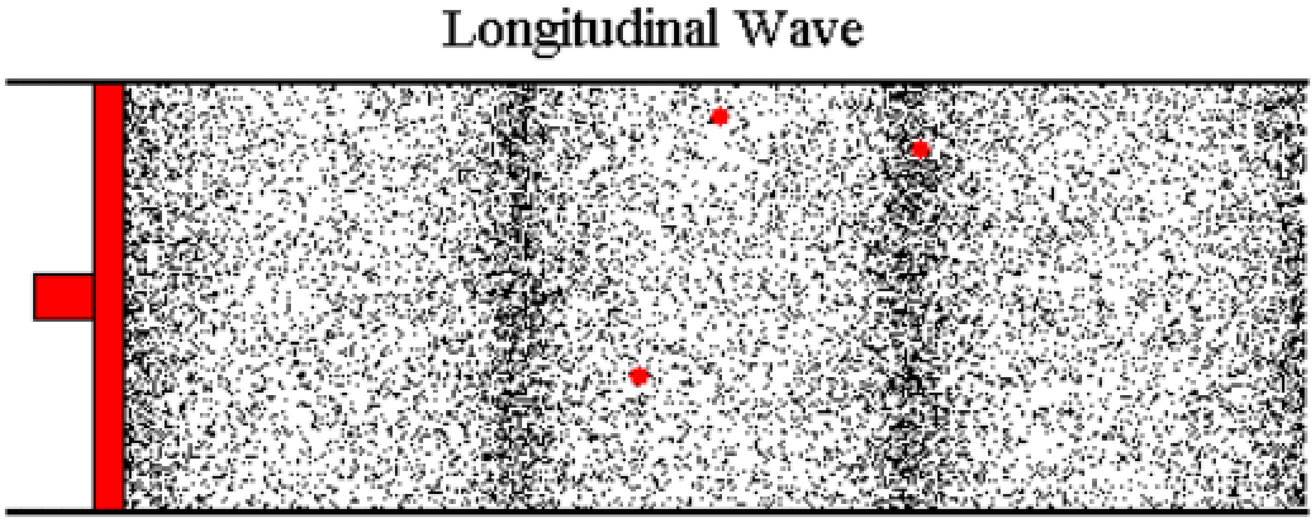
Waves

Spatial propagation of oscillations form wave pattern

Mechanical waves.

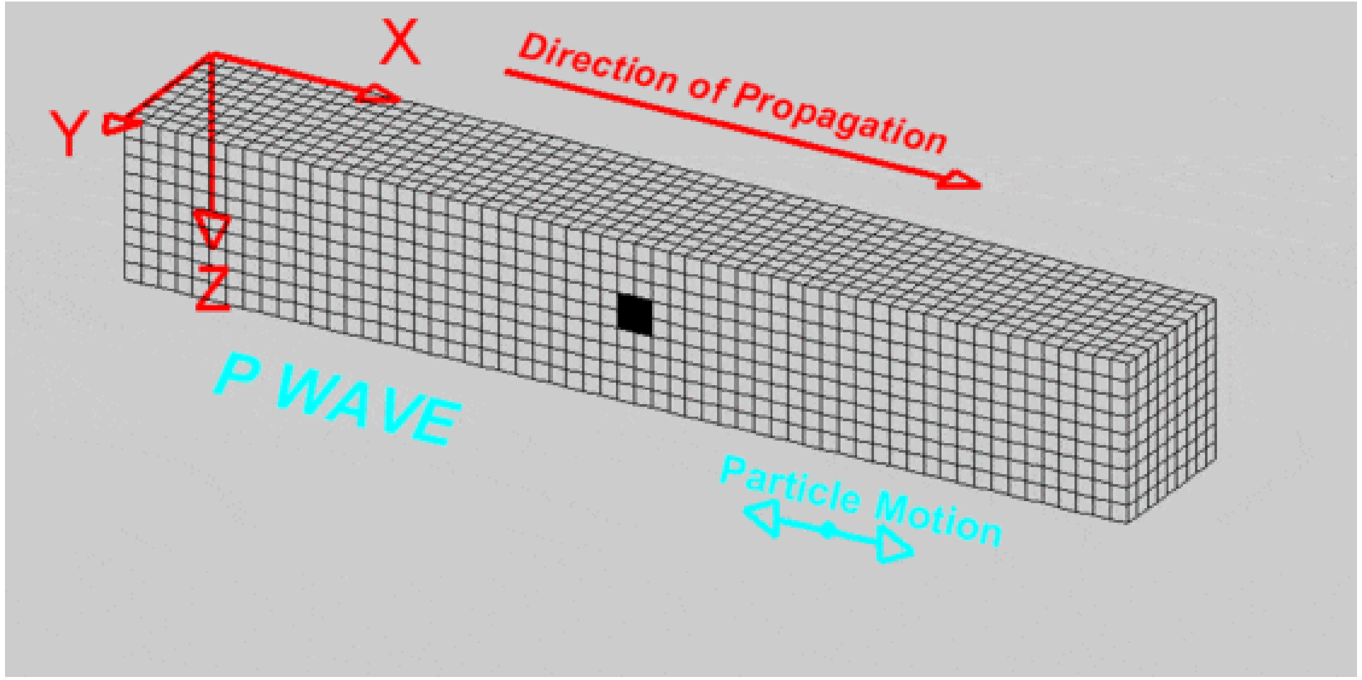
Electromagnetic waves.

Matter waves.



Waves

Spatial propagation of oscillations form wave pattern



Mechanical waves.

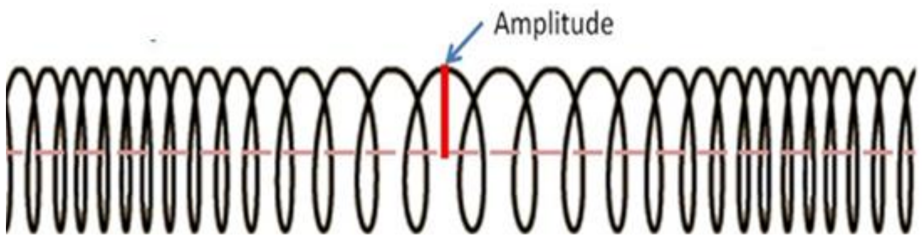
Electromagnetic waves.

Matter waves.

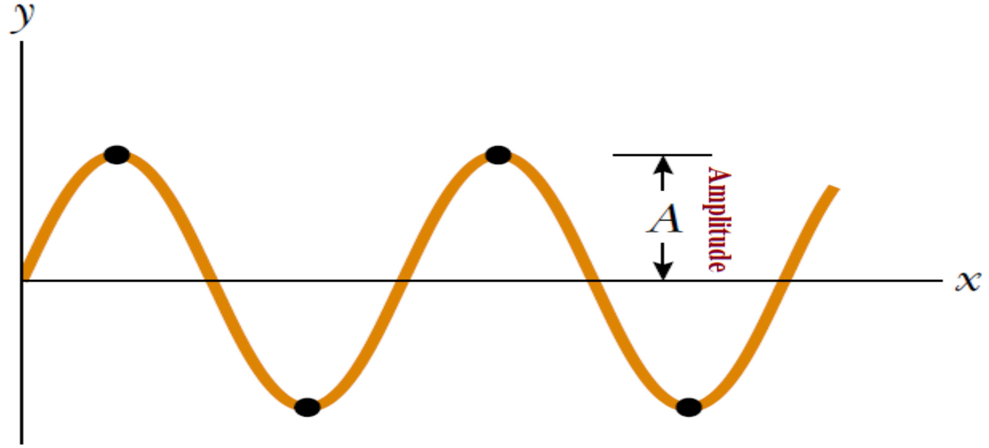
Amplitude (A)

- The maximum displacement from equilibrium of an element of the medium is called the amplitude *A of the wave.*
- Larger amplitude more energy *(A* αE^2)

In case of Longitudinal Wave



In case of Transverse Wave



Time Period (T) & Frequency $\nu_{(neu)}$:

- The period is the time interval required for two identical points (such as the crests) of adjacent waves to pass by a point.

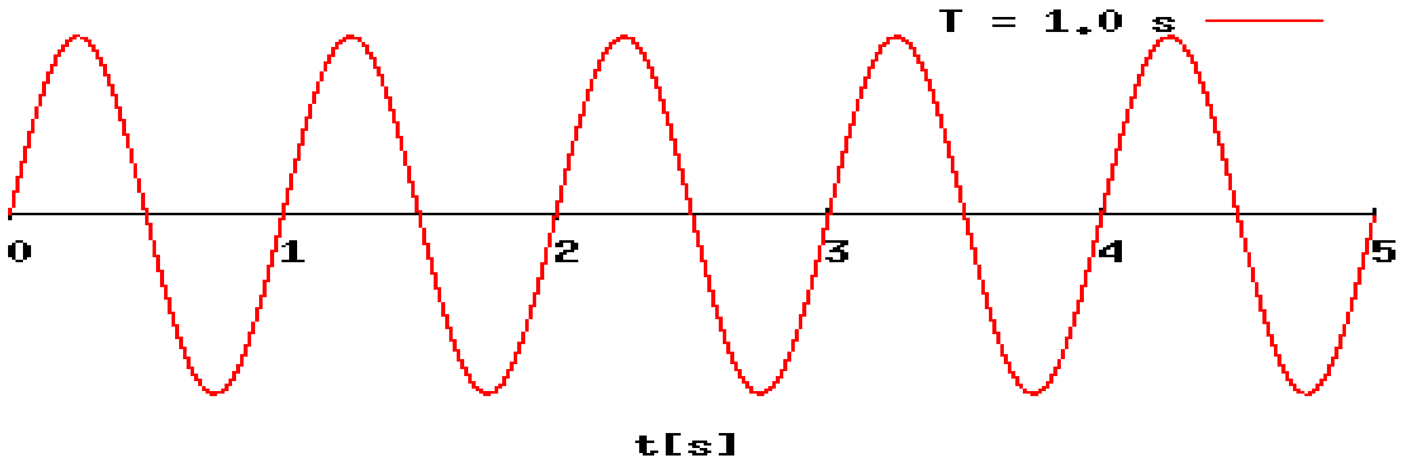
- The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium.

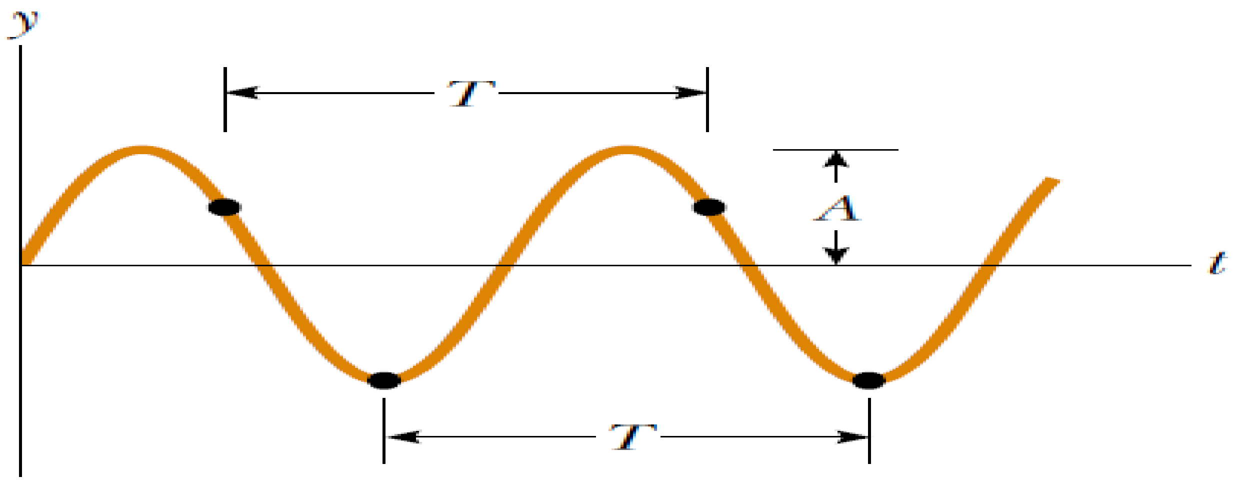
- The inverse of the period, is called the frequency *f*.

- In general, the frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval.

$$f = 1/T \text{ (hertz)}$$

Visual Concept of Time Period & Frequency

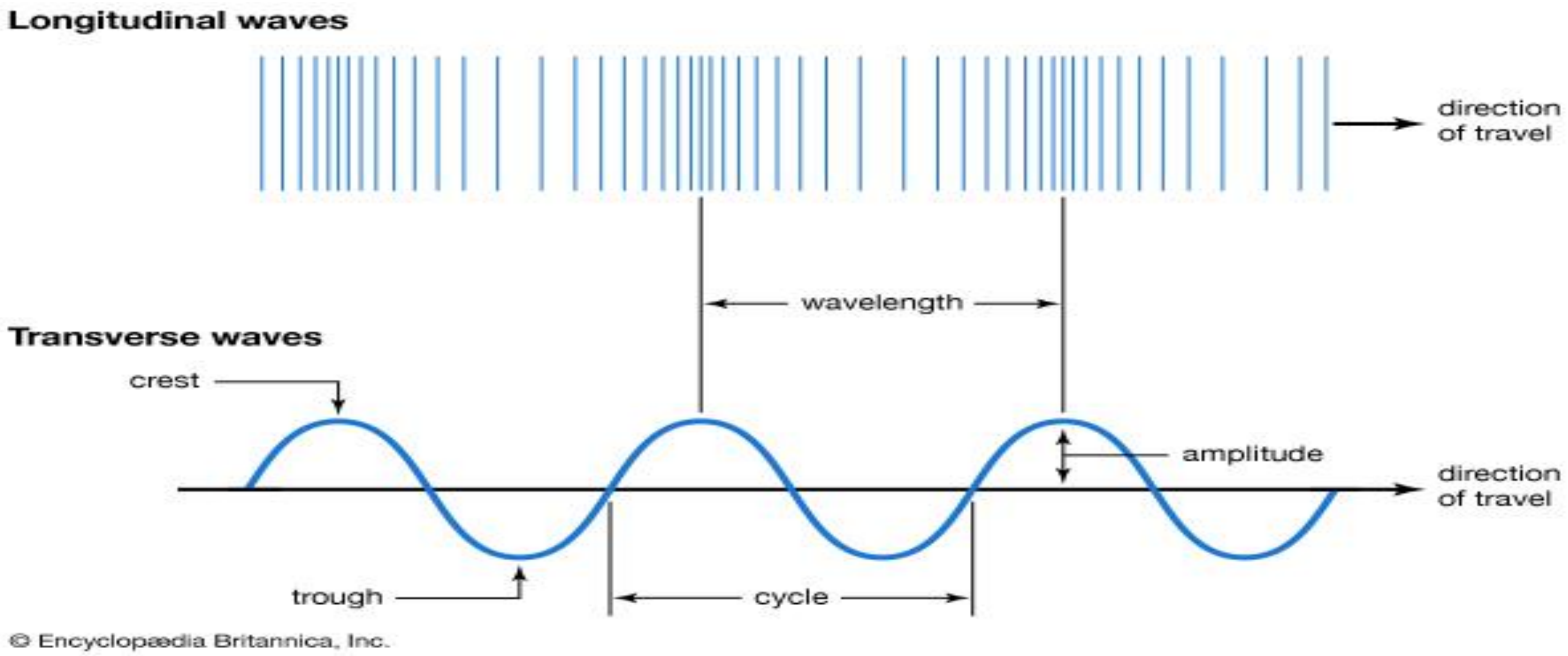




Wave length (λ)

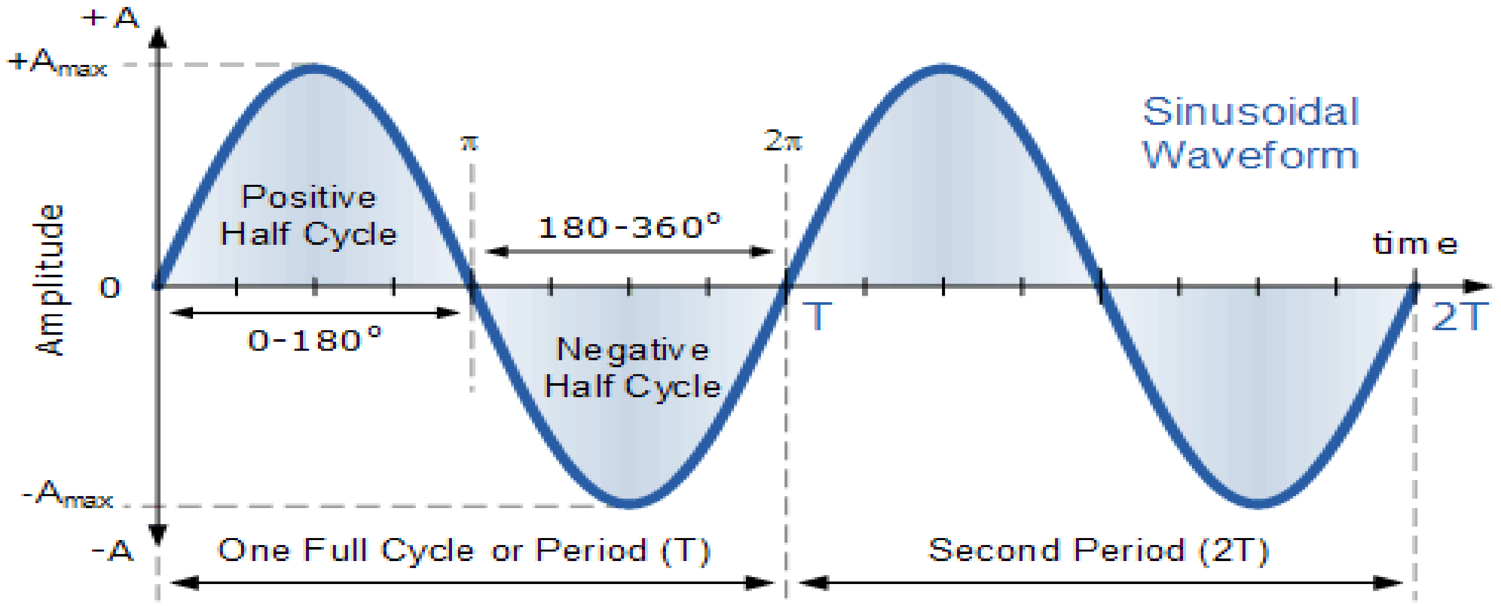
- (In terms of TW) The wavelength is the minimum distance between any two identical points (such as the crests) on adjacent waves.

- (In terms of LW) The Wavelength is the minimum distance between any two identical points (such as the compressions) on adjacent waves.



Phase of Waveform

- Phase is the position of a point in time (an instant) on a waveform cycle. A complete cycle is defined as the interval required for the waveform to return to its arbitrary initial value.
- The graph below shows how one cycle constitutes 360° of phase.



Comparison of LW & TW

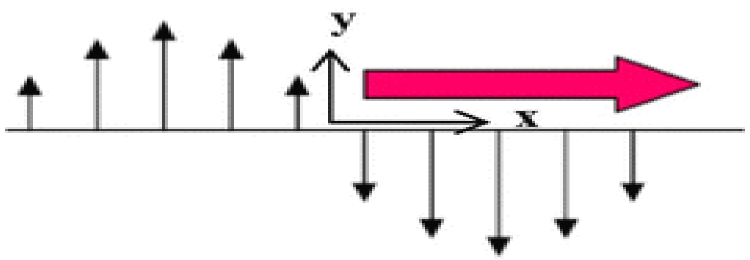
Transverse Waves

- Travels perpendicular to direction of propagation.
- Travels in the form of crests and troughs.
- It does not effect medium’s temperature and pressure.
- Travels through only in solid and surface of liquids except gases.

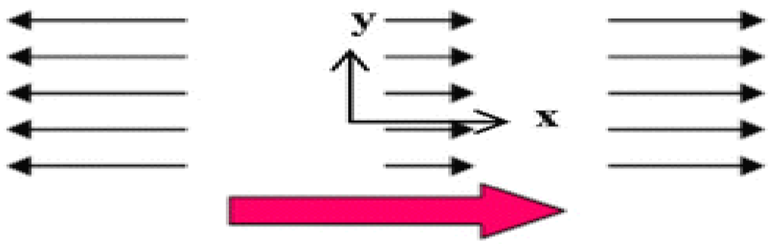
Longitudinal Waves

- Travels parallel to direction of propagation.
- Travels in the form of compression and rarefactions.
- Temperature and pressure rises in its propagation.
- Travels through all type of medium except plasma.

Transverse wave

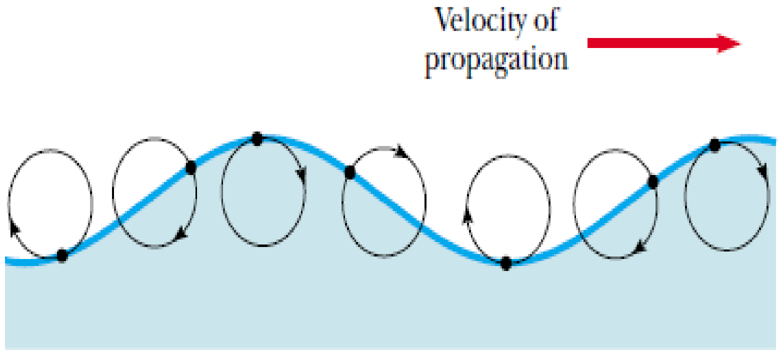


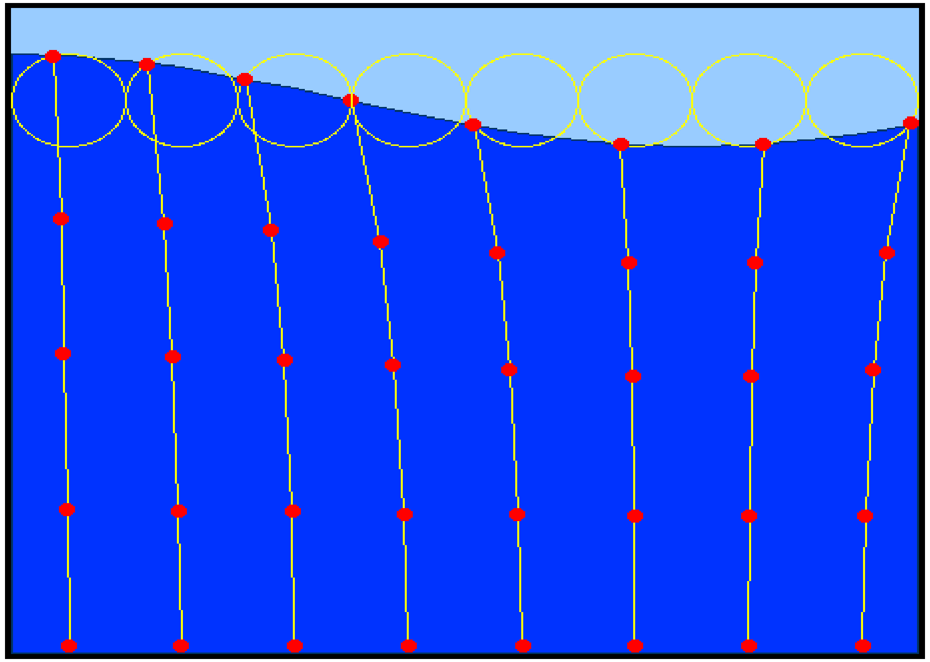
Longitudinal wave



Combination of Transverse and Longitudinal Waves

- Some waves in nature exhibit a combination of transverse and longitudinal displacements.
- Surface water waves are a good example.
- Note that in this disturbance has both transverse and longitudinal waves.





Question :

Think of some other examples in nature in which they exhibits both the features (TW & LW) at a time.

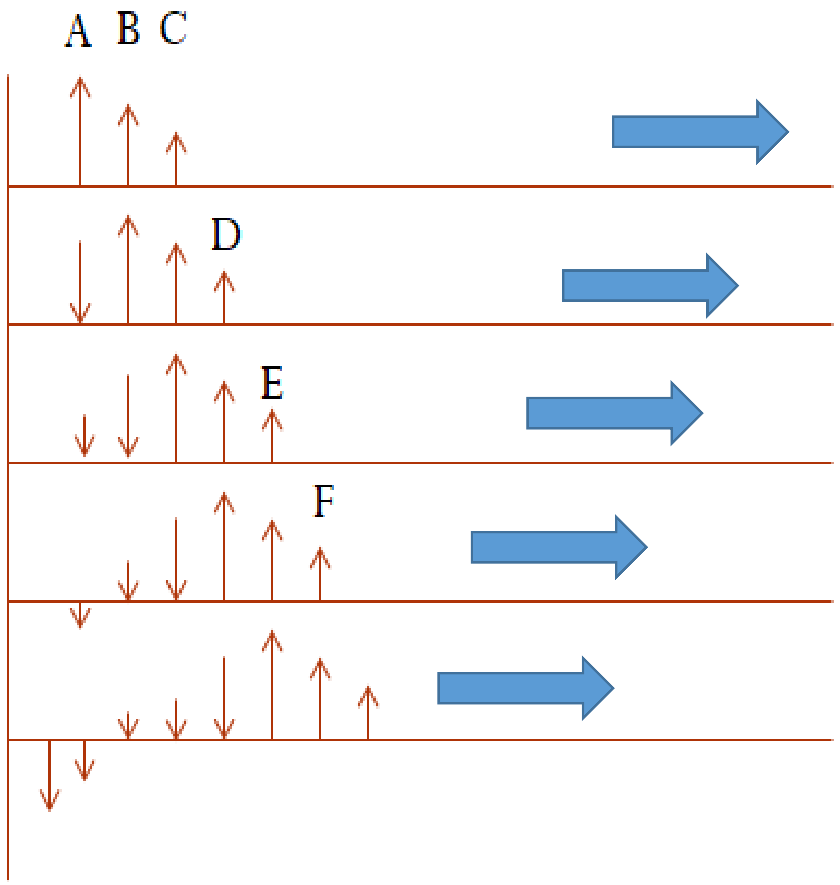
Formation of Wave

- In order to have a wave, our medium should have two properties

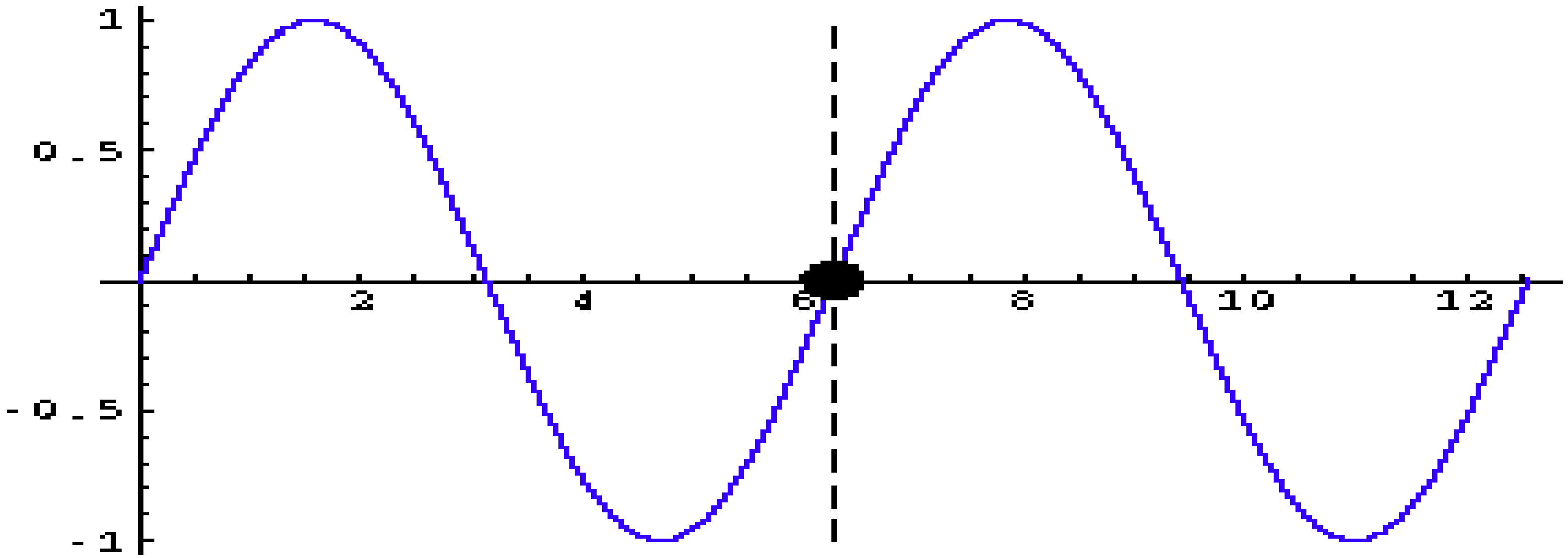
1. Elasticity
2. Inertia(measure of mass)

Or

we can say that .....for oscillation our particle should have inertia and elasticity.

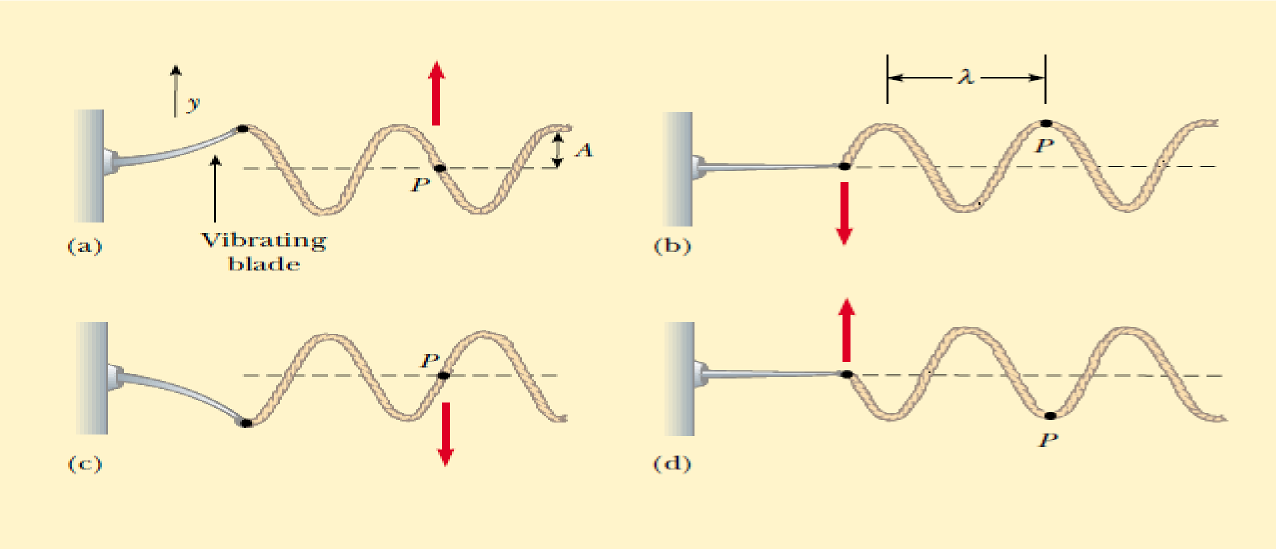


Effect of travelling wave on Particle



Representation of a Wave

Mathematical Approach



$$y = A\sin(kx - \omega t)$$

Wave Velocity

- Below is the fundamental wave velocity equation where υ is the wave velocity , λ is the wavelength and ν is the frequency of wave.

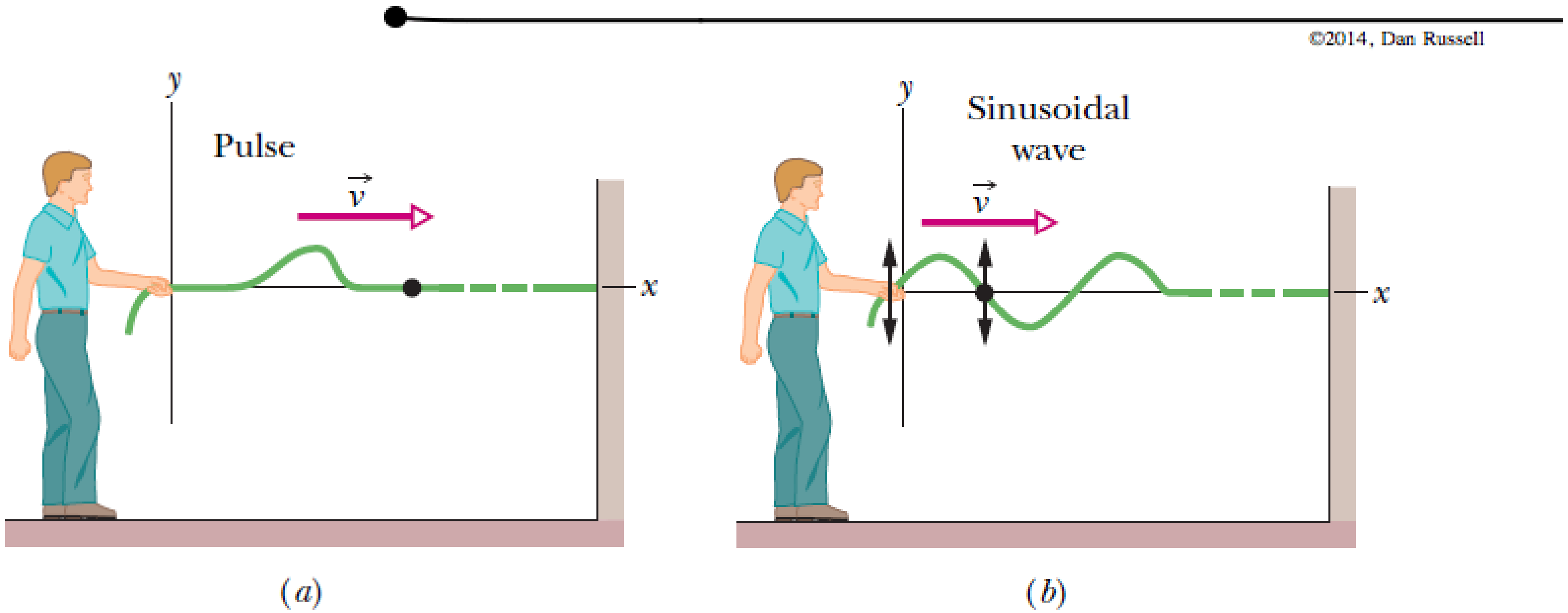
- Frequency is the number of consecutive risings of wave /cycles per second measured in hertz.

$$\upsilon = \lambda \nu$$

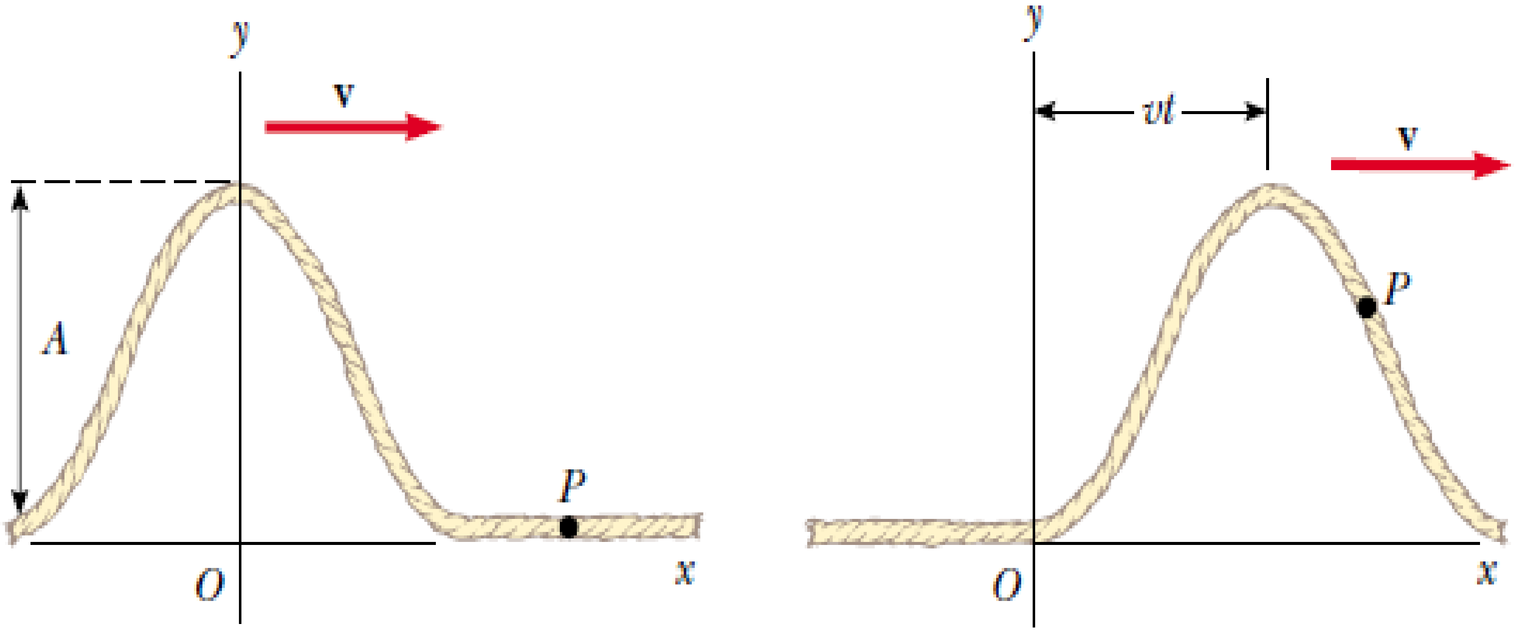
- This is a fundamental equation obeyed by all waves.

One Dimensional Pulse and a Wave Function

- We need a function that gives the shape of the wave



One Dimensional Pulse and a Wave Function



(a) Pulse at $t = 0$ (b) Pulse at time t

A one-dimensional pulse traveling to the right with a speed v. (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$. (b) At some later time t, the shape remains unchanged and the vertical position of an element of the medium any point P is given by $y = f(x - vt)$.

Wave Function

- To completely describe a wave on a string (and the motion of any element along its length), we need a function that gives the shape of the wave.
- y is the perpendicular displacement , x is the horizontal distance covered by wave in time t.

Wave Function

Consequently, an element of the string at x at this time has the same y position as an element located at $x - vt$ had at time $t = 0$:

$$y(x, t) = y(x - vt, 0)$$

In general, then, we can represent the transverse position y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x, t) = f(x - vt)$$ **Pulse traveling to the right**

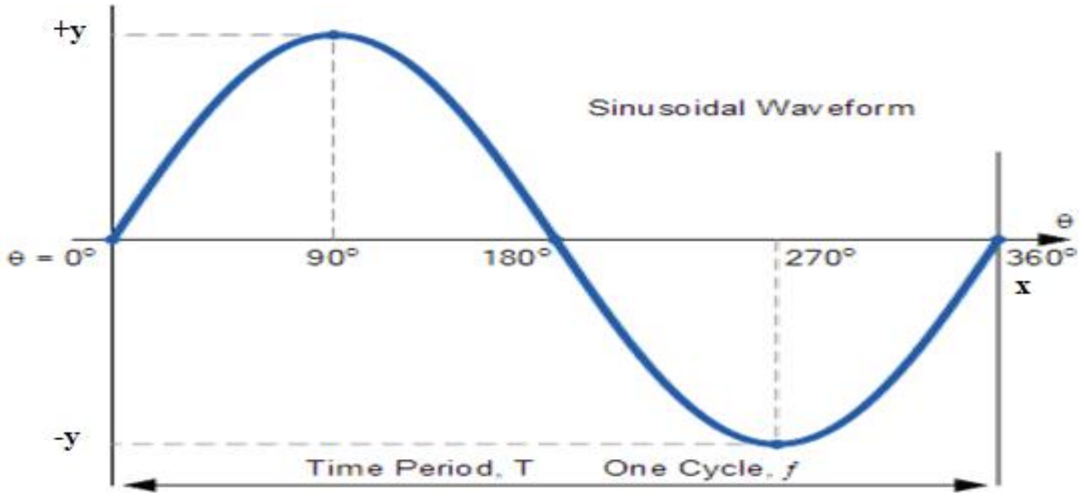
Similarly, if the pulse travels to the left, the transverse positions of elements of the string are described by

$$y(x, t) = f(x + vt)$$ **Pulse traveling to the left**

The function y, sometimes called the **wave function,** depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t."

Wave Form

It is important to understand the meaning of y. Consider an element of the string at point P, identified by a particular value of its x coordinate. As the pulse passes through P, the y coordinate of this element increases, reaches a maximum, and then decreases to zero. **The wave function $y(x, t)$ represents the y coordinate—the transverse position—of any element located at position x at any time t.** Furthermore, if t is fixed (as, for example, in the case of taking a snapshot of the pulse), then the wave function $y(x)$, sometimes called the **waveform,** defines a curve representing the actual geometric shape of the pulse at that time.



Example Problem 1

- Pulse moving to the Right:

A pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is measured in seconds. Plot the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

Solution First, note that this function is of the form $y = f(x - vt)$. By inspection, we see that the wave speed is $v = 3.0$ cm/s. Furthermore, the maximum value of y is given by $A = 2.0$ cm. (We find the maximum value of the function representing y by letting $x - 3.0t = 0$.) The wave function expressions are

$$y(x, 0) = \frac{2}{x^2 + 1} \qquad \text{at } t = 0$$

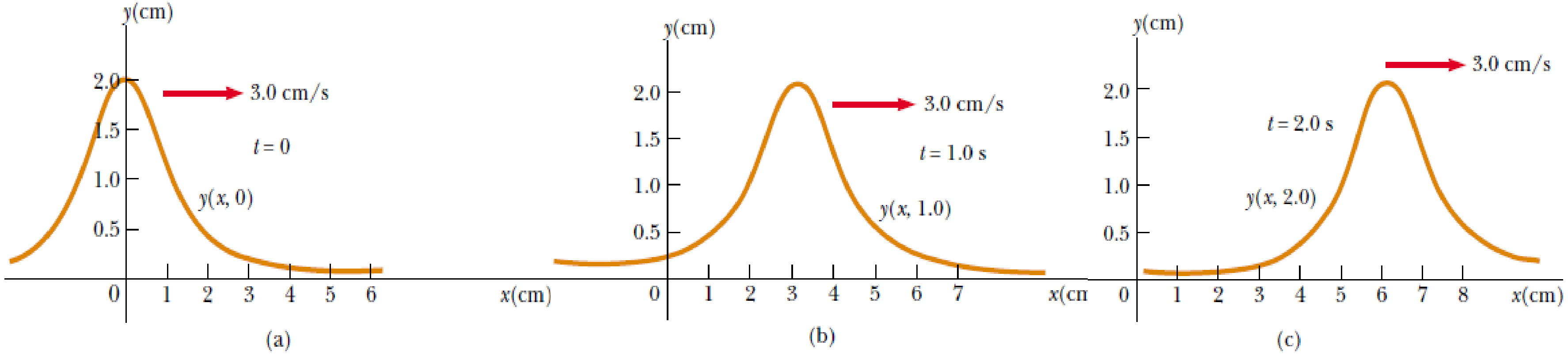
$$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1} \qquad \text{at } t = 1.0 \text{ s}$$

$$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1} \qquad \text{at } t = 2.0 \text{ s}$$

We now use these expressions to plot the wave function versus x at these times. For example, let us evaluate $y(x, 0)$ at $x = 0.50$ cm:

$$y(0.50, 0) = \frac{2}{(0.50)^2 + 1} = 1.6 \text{ cm}$$

Likewise, at $x = 1.0$ cm, $y(1.0, 0) = 1.0$ cm, and at $\mathbf{x} = 2.0$ cm, $y(2.0, 0) = 0.40$ cm. Continuing this procedure for other values of x yields the wave function shown in Figure **a**. In a similar manner, we obtain the graphs of $y(x, 1.0)$ and $y(x, 2.0)$, shown in Figure b and c respectively. These snapshots show that the pulse moves to the right without changing its shape and that it has a constant speed of 3.0 cm/s.



What If? (A) What if the wave function were

$$y(x, t) = \frac{2}{(x + 3.0t)^2 + 1}$$

How would this change the situation?

Answer (A) The new feature in this expression is the plus sign in the denominator rather than the minus sign. This results in a pulse with the same shape as that in Figure **a,b,c** but moving to the left as time progresses.

(B) What if the wave function were

$$y(x, t) = \frac{4}{(x - 3.0t)^2 + 1}$$

How would this change the situation?

(B) The new feature here is the numerator of 4 rather than 2. This results in a pulse moving to the right, but with twice the height of that in Figure **a,b,c**

Sinusoidal Wave

Consider the sinusoidal wave in Figure below, which shows the position of the wave at $t = 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as $y(x, 0) = A \sin ax$, where A is the amplitude and a is a constant to be determined. At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Figure The next value of x for which y is zero is $x = \lambda/2$. Thus,

$$y(x, 0) = A \sin ax,$$

$$y\left(\frac{\lambda}{2}, 0\right) = A \sin a\left(\frac{\lambda}{2}\right) = 0$$

For this to be true, we must have

$$a(\lambda/2) = \pi,$$

$$\text{or } a = 2\pi/\lambda.$$

therefore

$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda} x\right)$$

Wave function

To take snapshot,

$$y(x,0) = y_m \sin kx.$$

To plot a graph,

$$\begin{aligned} y(0,t) &= y_m \sin(-\omega t) \\ &= -y_m \sin \omega t \end{aligned}$$

The generalized wave function

$$\boxed{y = y_m \sin(kx \pm \omega t + \phi).}$$

$$\underbrace{y(x,t)}_{\text{Displacement}} = \underbrace{y_m}_{\text{Amplitude}}\ \underbrace{\sin\,(\underbrace{kx - \omega t}_{\text{Phase}})}_{\text{Oscillating term}}$$

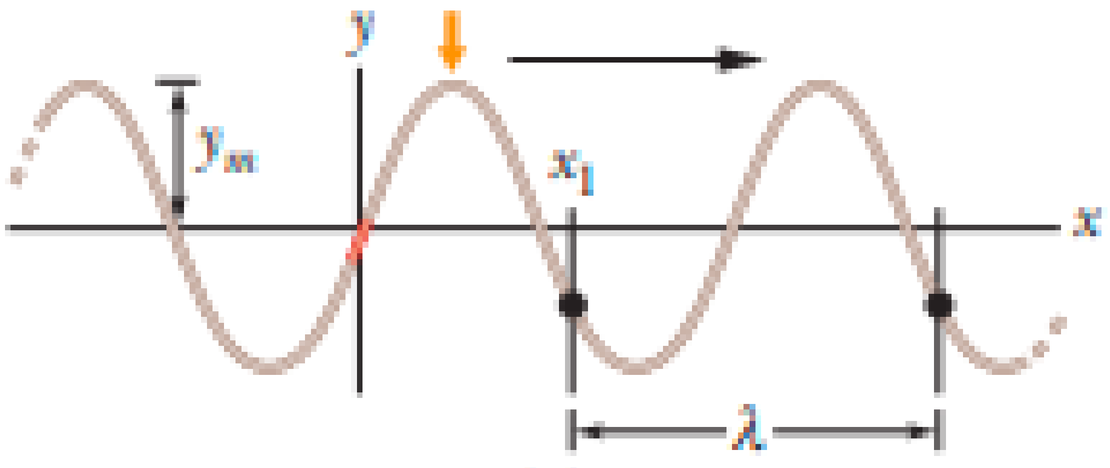
Angular wave number (k)

Position (x)

Angular frequency (ω)

Time (t)

Watch this spot in this series of snapshots.



(a)

Amplitude and Phase

- The amplitude y_m of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them. (The subscript m stands for maximum.) Because y_m is a magnitude, it is always a positive quantity, even if it is measured downward instead of upward.
- The phase of the wave is the argument $kx - \omega t$ of the sine in Eq. As the wave sweeps through a string element at a particular position x, the phase changes linearly with time t.

$$y(x, t) = y_m \sin(kx - \omega t).$$

Wavelength and Angular Wave number

- The wavelength λ of a wave is the distance (parallel to the direction of the wave's travel) between repetitions of the shape of the wave (or wave shape).

$$y(x, 0) = y_m \sin kx. \tag{16-3}$$

By definition, the displacement y is the same at both ends of this wavelength—that is, at $x = x_1$ and $x = x_1 + \lambda$. Thus, by Eq. 16-3,

$$\begin{aligned} y_m \sin kx_1 &= y_m \sin k(x_1 + \lambda) \\ &= y_m \sin(kx_1 + k\lambda). \end{aligned} \tag{16-4}$$

A sine function begins to repeat itself when its angle (or argument) is increased by 2π rad, so in Eq. 16-4 we must have $k\lambda = 2\pi$, or

$$k = \frac{2\pi}{\lambda} \quad \text{(angular wave number).} \tag{16-5}$$

We call k the **angular wave number** of the wave; its SI unit is the radian per meter, or the inverse meter. (Note that the symbol k here does *not* represent a spring constant as previously.)

$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda} x\right)$$ at t=0

$$y(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$ at time "t" .....equation 1

we know $v = \frac{\lambda}{T}$

Substituting this expression for v into Equation 1

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

We can express the wave function in a convenient form by defining two other quantities, the **angular wave number** k (usually called simply the **wave number**) and the **angular frequency** ω:

$$k \equiv \frac{2\pi}{\lambda}$$ Angular wave number

$$\omega \equiv \frac{2\pi}{T}$$ Angular frequency

$$k \equiv \frac{2\pi}{\lambda} \qquad \omega \equiv \frac{2\pi}{T}$$

$$y = A \sin(kx - \omega t)$$

Wave function for a sinusoidal wave ...equation 2

we can express the wave speed v originally

in the alternative forms $v = \frac{\omega}{k}$

$$v = \lambda f$$

The wave function given by Equation 2 assumes that the vertical position y of an element of the medium is zero at $x = 0$ and $t = 0$. This need not be the case. If it is not, we generally express the wave function in the form

$$y = A \sin(kx - \omega t + \phi)$$

...Equation 3 General expression for a sinusoidal wave

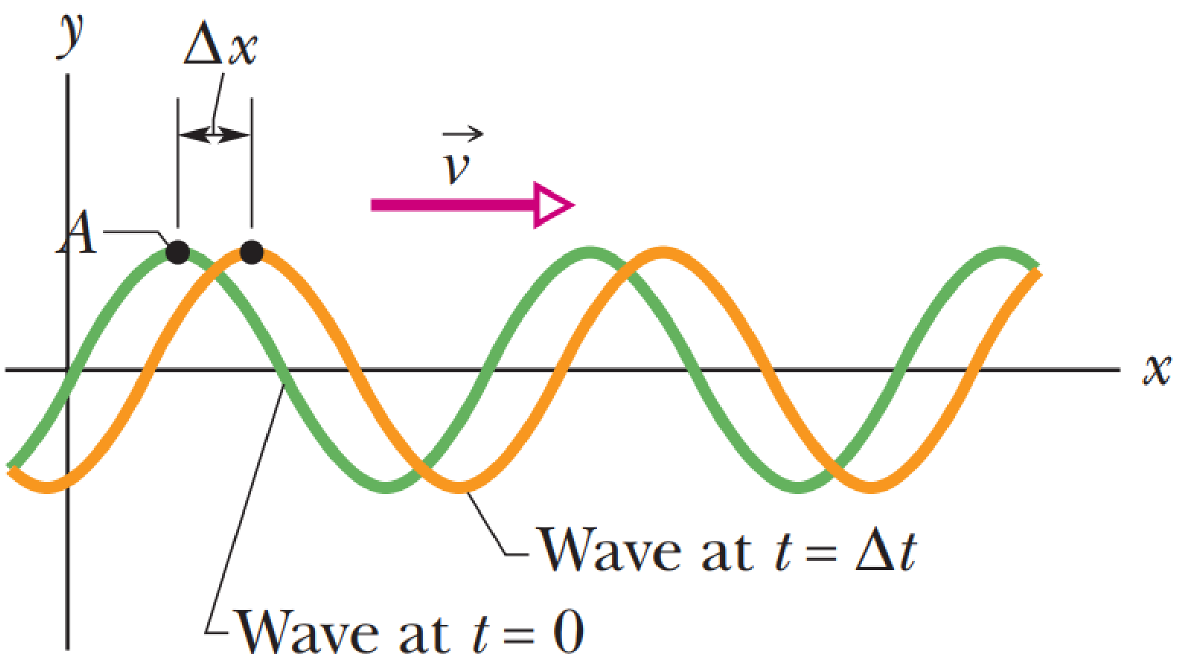
where ϕ is the **phase constant,**

This constant can be determined from the initial conditions.

Wave Velocity

point A retains its displacement

$$kx - \omega t = \text{a constant.}$$



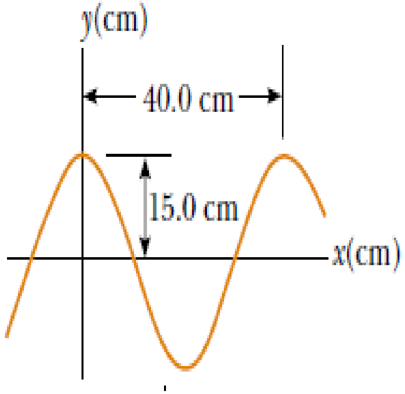
$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = v = \frac{\omega}{k}.$$

$$\boxed{v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f} \quad \text{(wave speed)}$$

Example Problem 2

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t = 0$ and $x = 0$ is also 15.0 cm, as shown in Figure below



A sinusoidal wave of wavelength $\lambda = 40.0$ cm and amplitude $A = 15.0$ cm. The wave function can be written in the form $y = A\cos(kx - \omega t)$.

(A) Find the wave number k, period T, angular frequency ω, and speed v of the wave.

(B) Determine the phase constant ϕ, and write a general expression for the wave function.

***Solution* A** Using Equations below we find the following:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = \boxed{0.157 \text{ rad/cm}}$$

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = \boxed{0.125 \text{ s}}$$

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = \boxed{50.3 \text{ rad/s}}$$

$$v = \lambda f = (40.0 \text{ cm})(8.00 \text{ s}^{-1}) = \boxed{320 \text{ cm/s}}$$

***Solution* B** Because $A = 15.0$ cm and because $y = 15.0$ cm at $x = 0$ and $t = 0$, substitution into

Equation $\quad y = A\sin(kx - \omega t + \phi)$

$$15.0 = (15.0)\sin\phi \quad \text{or} \quad \sin\phi = 1$$

We may take the principal value $\phi = \pi/2$ rad (or 90°). Hence, the wave function is of the form

$$y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right) = A\cos(kx - \omega t)$$

By inspection, we can see that the wave function must have this form, noting that the cosine function has the same shape as the sine function displaced by 90°. Substituting the values for A, k, and ω into this expression, we obtain

$$y = \boxed{(15.0 \text{ cm})\cos(0.157x - 50.3t)}$$

Example Problem 3

The string shown in Figure 16.10 is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Determine the angular frequency ω and wave number k for this wave, and write an expression for the wave function.

Solution Using Equations below we find that

$$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00\text{ Hz}) = \boxed{31.4\text{ rad/s}}$$

$$k = \frac{\omega}{v} = \frac{31.4\text{ rad/s}}{20.0\text{ m/s}} = \boxed{1.57\text{ rad/m}}$$

Because $A = 12.0\text{ cm} = 0.120\text{ m}$, we have

$$y = A\sin(kx - \omega t)$$

$$= \boxed{(0.120\text{ m})\sin(1.57x - 31.4t)}$$

Practice Problems

$\boxed{1.}$ At $t = 0$, a transverse pulse in a wire is described by the function

$$y = \frac{6}{x^2 + 3}$$

where x and y are in meters. Write the function $y(x, t)$ that describes this pulse if it is traveling in the positive x direction with a speed of 4.50 m/s.

2. Ocean waves with a crest-to-crest distance of 10.0 m can be described by the wave function

$$y(x, t) = (0.800 \text{ m}) \sin[0.628(x - vt)]$$

where $v = 1.20$ m/s. (a) Sketch $y(x, t)$ at $t = 0$. (b) Sketch $y(x, t)$ at $t = 2.00$ s. Note that the entire wave form has shifted 2.40 m in the positive x direction in this time interval.

3. A pulse moving along the x axis is described by

$$y(x, t) = 5.00e^{-(x+5.00t)^2}$$

where x is in meters and t is in seconds. Determine (a) the direction of the wave motion, and (b) the speed of the pulse.

4. For a certain transverse wave, the distance between two successive crests is 1.20 m, and eight crests pass a given point along the direction of travel every 12.0 s. Calculate the wave speed

5 A sinusoidal wave on a string is described by

$$y = (0.51 \text{ cm}) \sin(kx - \omega t)$$

where $k = 3.10$ rad/cm and $\omega = 9.30$ rad/s. How far does a wave crest move in 10.0 s? Does it move in the positive or negative x direction?

Sinusoidal Wave on String

We can use this expression to describe the motion of any element of the string. An element at point P (or any other element of the string) moves only vertically, and so its x coordinate remains constant. Therefore, the **transverse speed** v_y (not to be confused with the wave speed v) and the **transverse acceleration** a_y of elements of the string are

$$v_y = \frac{dy}{dt}\bigg]_{x\,=\,\text{constant}} = \frac{\partial y}{\partial t} = -\omega A \cos(kx - \omega t)$$

$$a_y = \frac{dv_y}{dt}\bigg]_{x\,=\,\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A \sin(kx - \omega t)$$

In these expressions, we must use partial derivatives because y depends on both x and t. In the operation $\partial y/\partial t$, for example, we take a derivative with respect to t while holding x constant. The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

$$v_{y,\,\max} = \omega A$$

$$a_{y,\,\max} = \omega^2 A$$

The transverse speed and transverse acceleration of elements of the string do not reach their maximum values simultaneously. The transverse speed reaches its maximum value (ωA) when $y = 0$, whereas the magnitude of the transverse acceleration reaches its maximum value ($\omega^2 A$) when $y = \pm A$.