

# Electric Fields

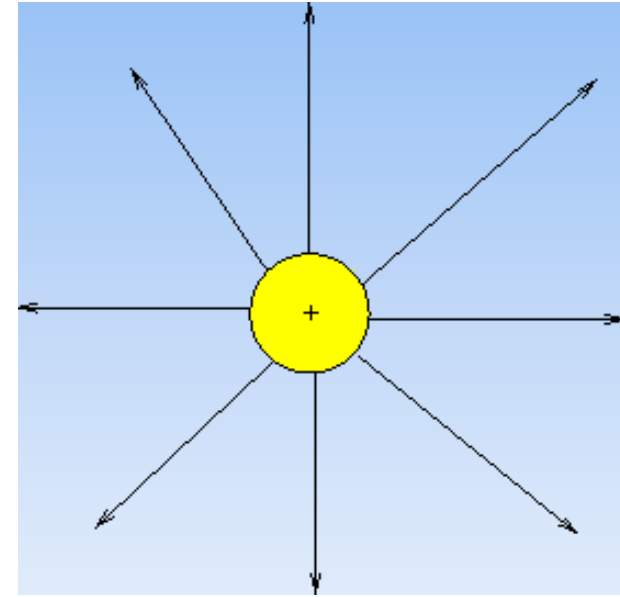
## Chapter 22 Textbook

# Analogy

The electric field is the space around **electrical charge**

just like

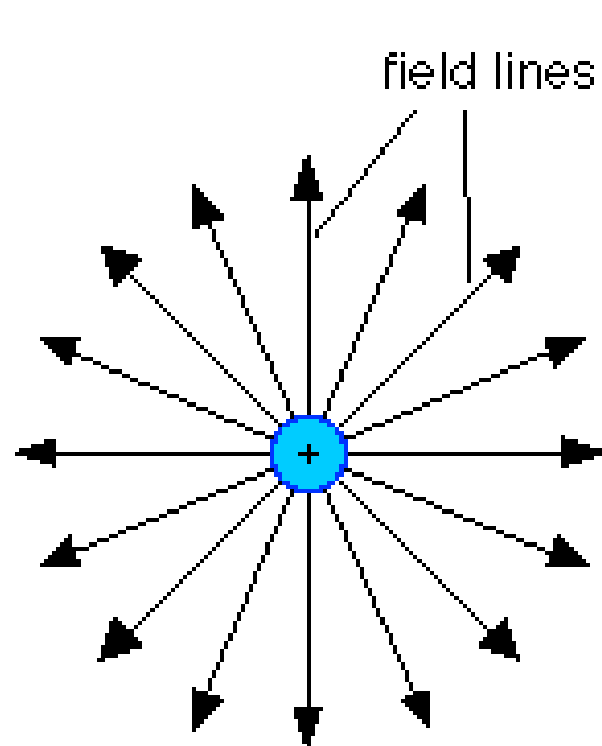
a gravitational field is the space around a **mass**.



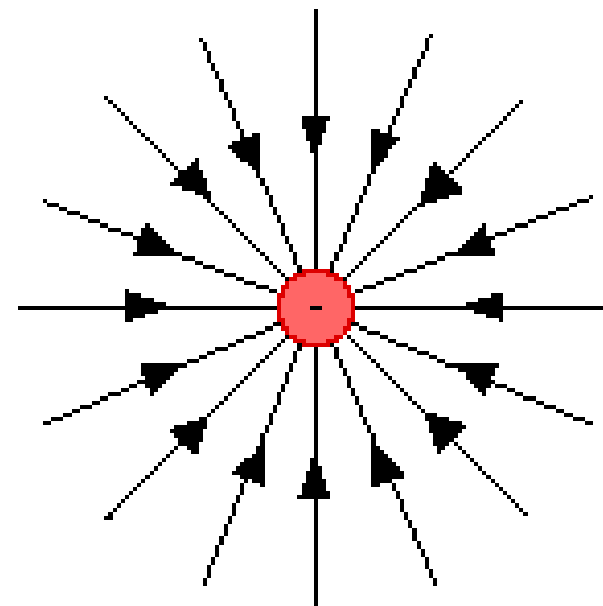
(resourcefulphysics.org)

# Electric Field

- Space around a charge.

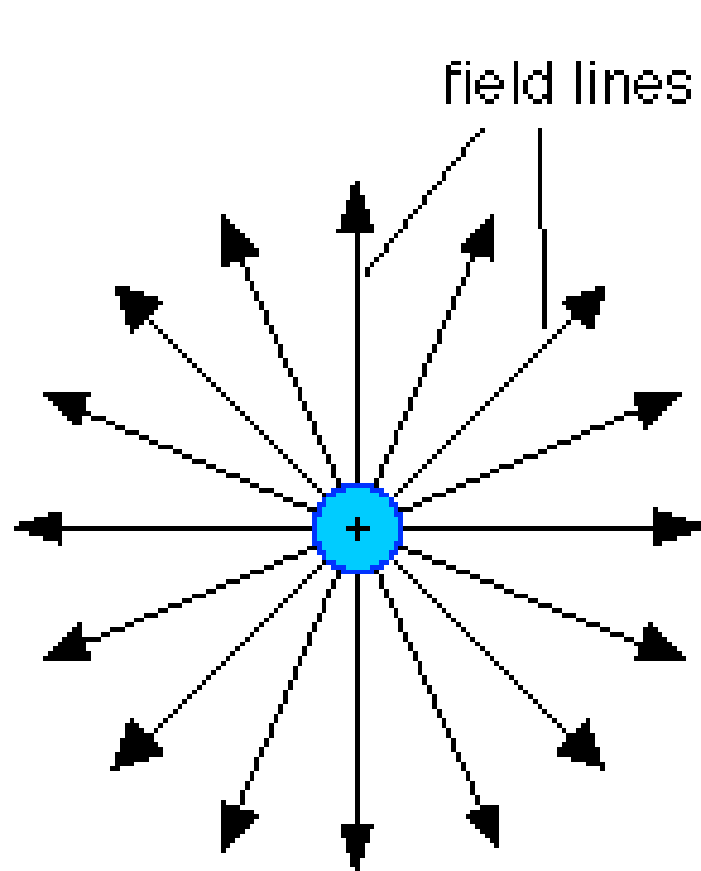


The electric field from an isolated positive charge

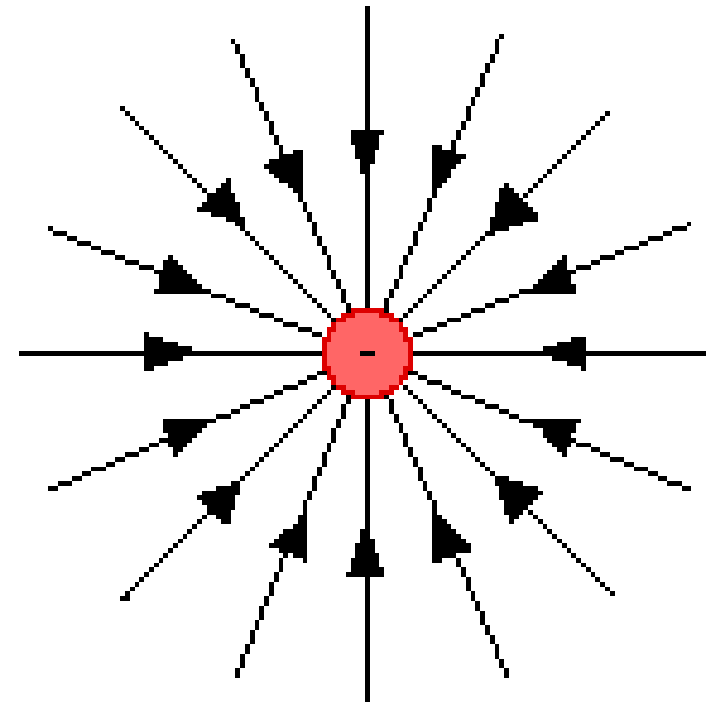


The electric field from an isolated negative charge

# What is the difference?



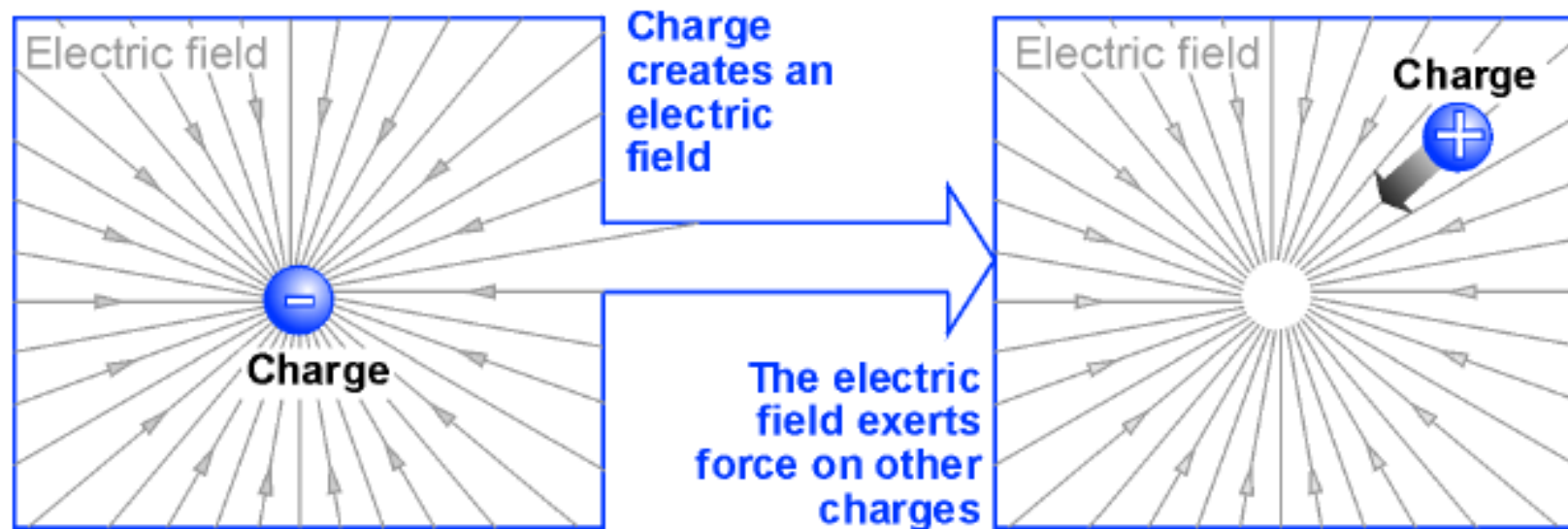
The electric field from an isolated positive charge



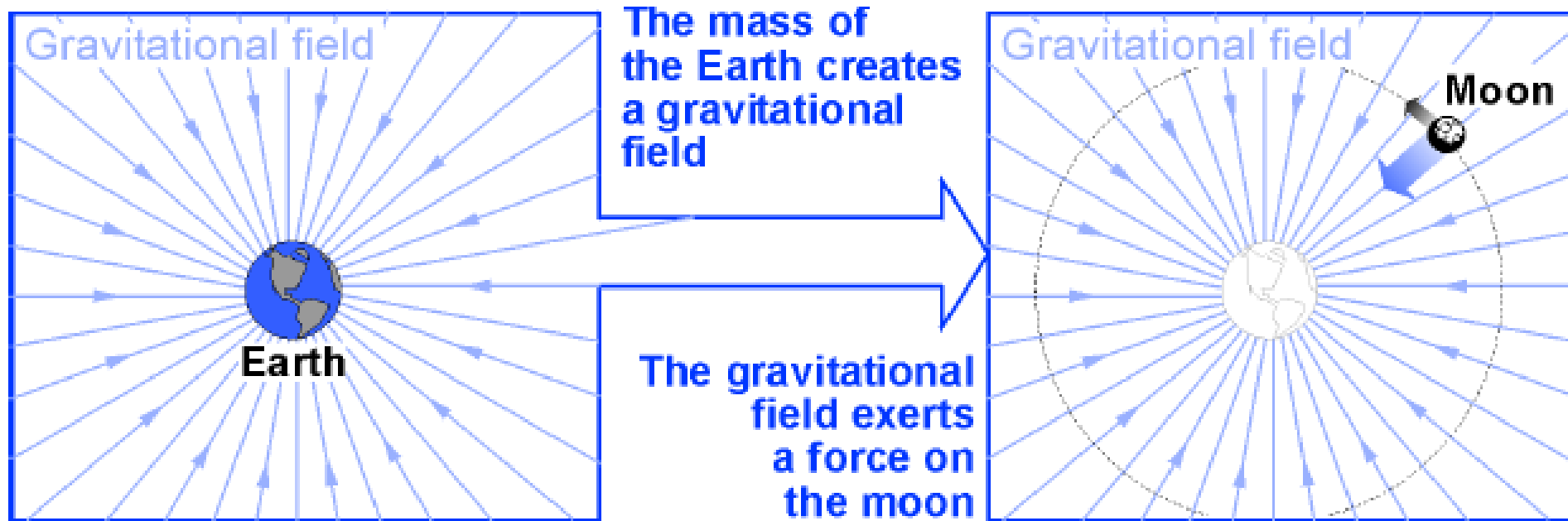
The electric field from an isolated negative charge

# Fields and forces

- The concept of a **field** is used to describe any quantity that has a value for all points in space.
- You can think of the field as the *way* forces are transmitted between objects.
- Charge creates an **electric field** that creates forces on other charges.




- Mass creates a **gravitational field** that exerts forces on other masses.




Gravitational forces are far weaker than electric

**Gravitational force**

$m_1$    $m_2$

$F = 6.7 \times 10^{-11} \text{ N}$

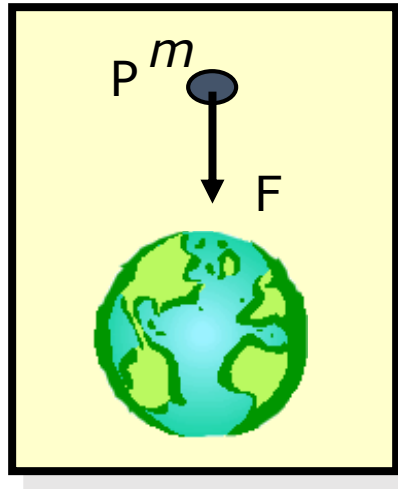
**Electric force**

$q_1$    $q_2$

$F = 1.8 \times 10^{25} \text{ N}$

# The Concept of a Field

A **field** is defined as a **property of space** in which a material object experiences a **force**.



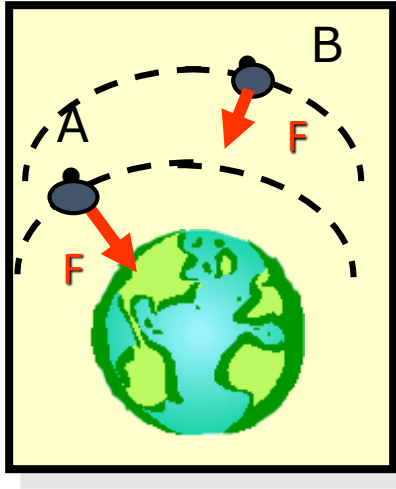
Above earth, we say there is a **gravitational field** at P.

**Because** a mass  $m$  experiences a **downward force** at that point.

The **direction** of the field is determined by the **force**.

# The Gravitational Field

Consider points **A** and **B** above the surface of the earth—just points in **space**.



If **g** is known at every point above the earth then the force **F** on a given mass can be found.

Note that the force **F** is **real**, but the field is just a convenient way of **describing space**.

The field at points A or B might be found from:

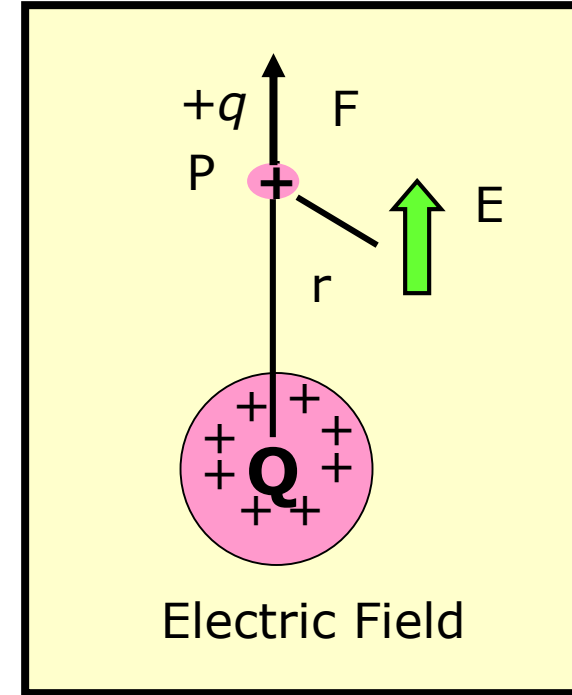
$$g = \frac{F}{m}$$

The **magnitude** and **direction** of the field **g** depends on the weight, which is the force **F**.



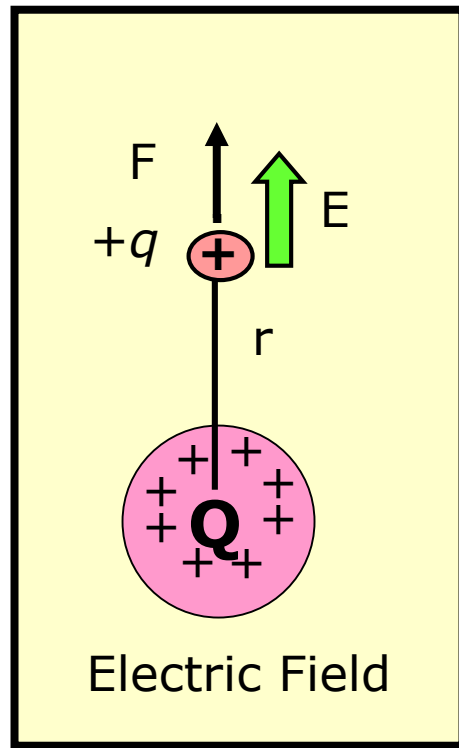
# The Electric Field

1. Now, consider point **P** a distance  $r$  from  $+Q$ .
2. An electric field **E** exists at **P** if a test charge  $+q$  has a force **F** at that point.
3. The **direction** of the **E** is the same as the direction of a **force** on  $+$  (pos) charge.
4. The **magnitude** of **E** is given by the formula:



$$E = \frac{F}{q}; \text{ Units } \frac{\text{N}}{\text{C}}$$

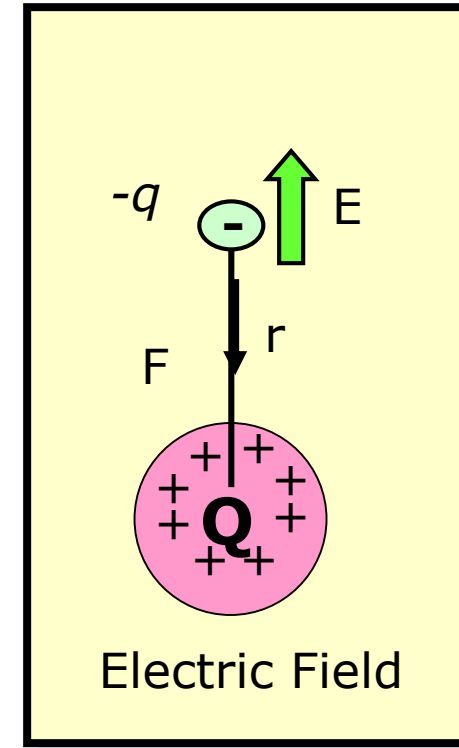
# Field is Property of Space



Force on  $+q$  is with field direction.

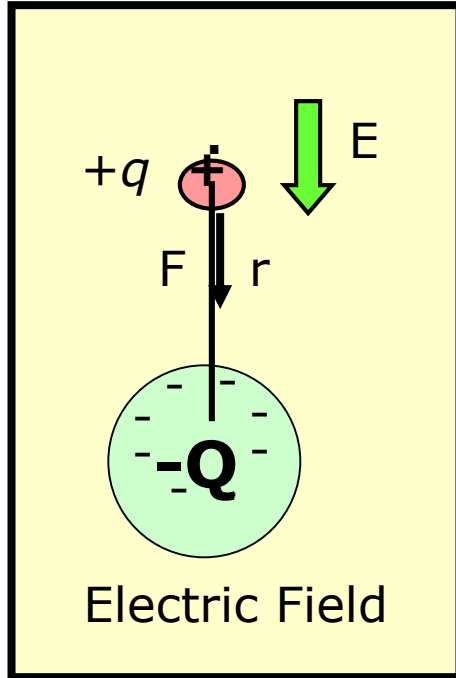


Force on  $-q$  is against field direction.



The field  $E$  at a point exists whether there is a charge at that point or not.  
The direction of the field is away from the  $+Q$  charge.

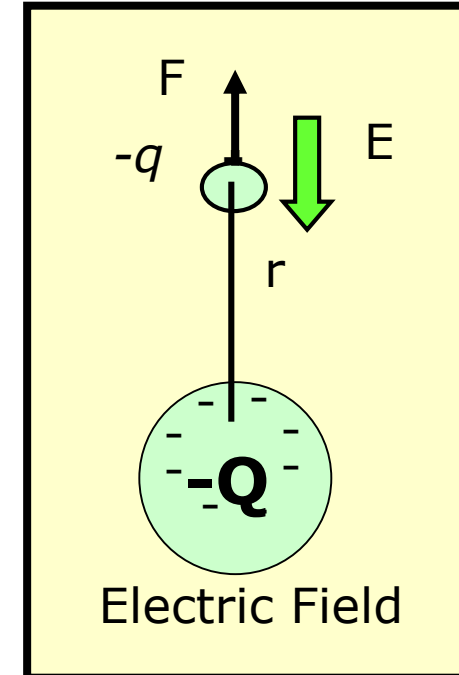
# Field Near a Negative Charge



Force on  $+q$  is with field direction.



Force on  $-q$  is against field direction.



Note that the field  $E$  in the vicinity of a negative charge  $-Q$  is toward the charge—the direction that a  $+q$  test charge would move.

# The Magnitude of E-Field

The **magnitude** of the electric field intensity at a point in space is defined as the **force per unit charge (N/C)** that would be experienced by any test charge placed at that point.

Electric Field  
Intensity  $E$

$$E = \frac{F}{q}; \text{ Units } \left( \frac{\text{N}}{\text{C}} \right)$$

The **direction** of  $E$  at a point is the same as the direction that a **positive** charge would move **IF** placed at that point.

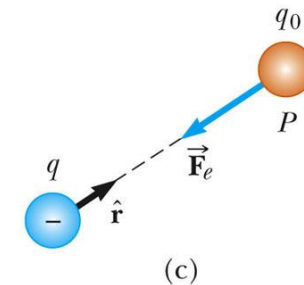
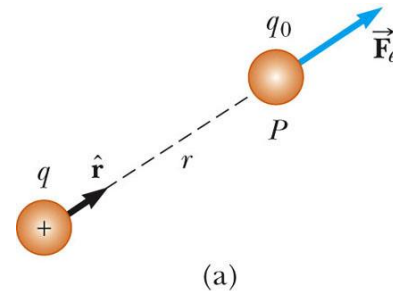
# Relationship Between F and E

- If  $q$  is placed in electric field , then we have  $\vec{F}_e = q\vec{E}$ 
  - This is valid for a point charge only
  - For larger objects, the field may vary over the size of the object
- If  $q$  is positive, the force and the field are in the same direction
- If  $q$  is negative, the force and the field are in opposite directions

# Electric Field, Vector Form

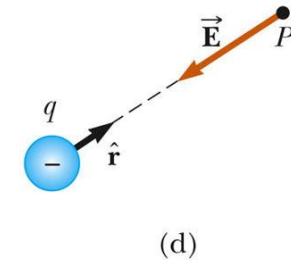
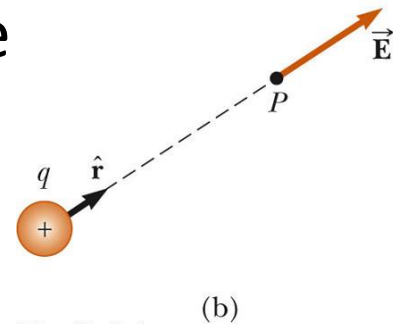
- From Coulomb's law, force between the source and test charges, can be expressed as

$$\vec{\mathbf{F}}_e = k_e \frac{qq_o}{r^2} \hat{\mathbf{r}}$$



- Then, the electric field will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_o} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$



# Superposition with Electric Fields

- At any point  $P$ , the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

## Definition of electric field

**the electric field  $\mathbf{E}$**  at a point in space is defined as the electric force  $\mathbf{F}_e$  acting on a positive test charge  $q_0$  placed at that point divided by the magnitude of the test charge:

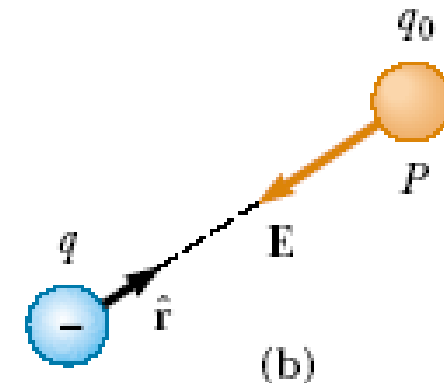
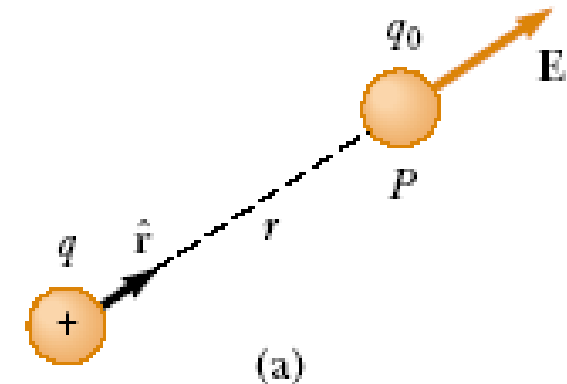
$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$

To determine the direction of an electric field, consider a point charge  $q$  located a distance  $r$  from a test charge  $q_0$  located at a point  $P$ . According to Coulomb's law, the force exerted by  $q$  on the test charge is

$$\mathbf{F}_e = k_e \frac{qq_0}{r^2} \hat{\mathbf{r}}$$

The electric field created by  $q$  is

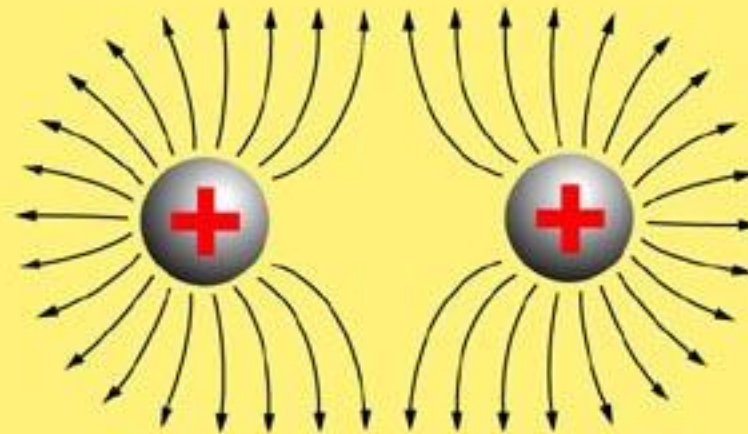
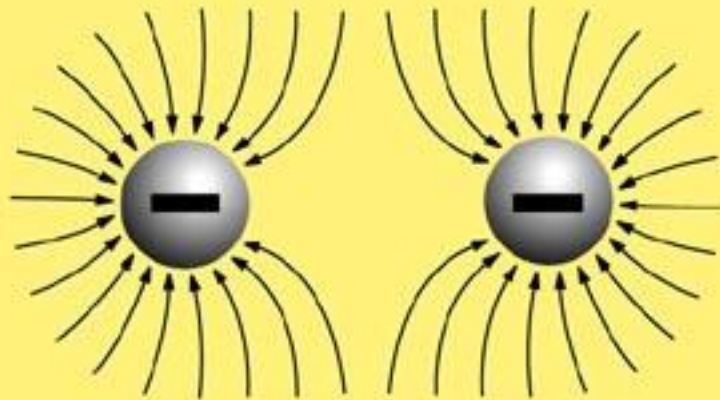
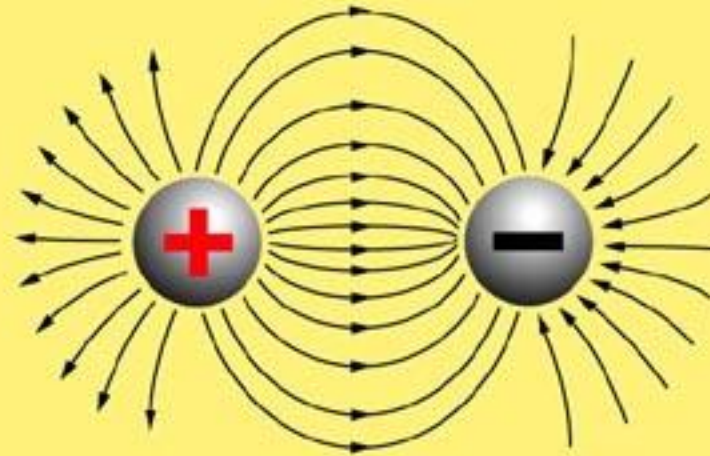
$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$





# Drawing the Electric Field

Field lines point toward negative charges and away from positive charges.



# Type of forces

## Contact forces

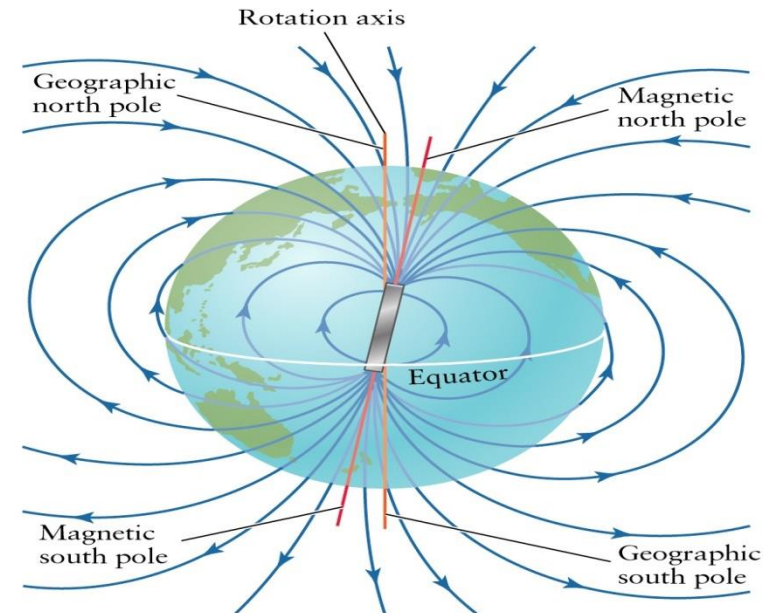
- Pushing a car up a hill or kicking a ball or pushing a desk across a room are some of the everyday **examples** where **contact forces** are at work



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## Field Forces

- **Examples of force fields** include magnetic fields, gravitational fields, and electrical fields.



# Electric Field

- The concept of a field was developed by Michael Faraday (1791–1867) in the context of electric forces.
- An **electric field is said to exist** in the region of space around a charged object, the **source charge**.
- The presence of the electric field can be detected by placing a **test charge in the field and noting the** electric force on it.
- The electric field vector  $E$  at a point in space is defined as the electric force  $F_e$  acting on a positive test charge  $q_0$  placed at that point divided by the test charge:

$$\vec{E} = \frac{\vec{F}_e}{q_0}$$

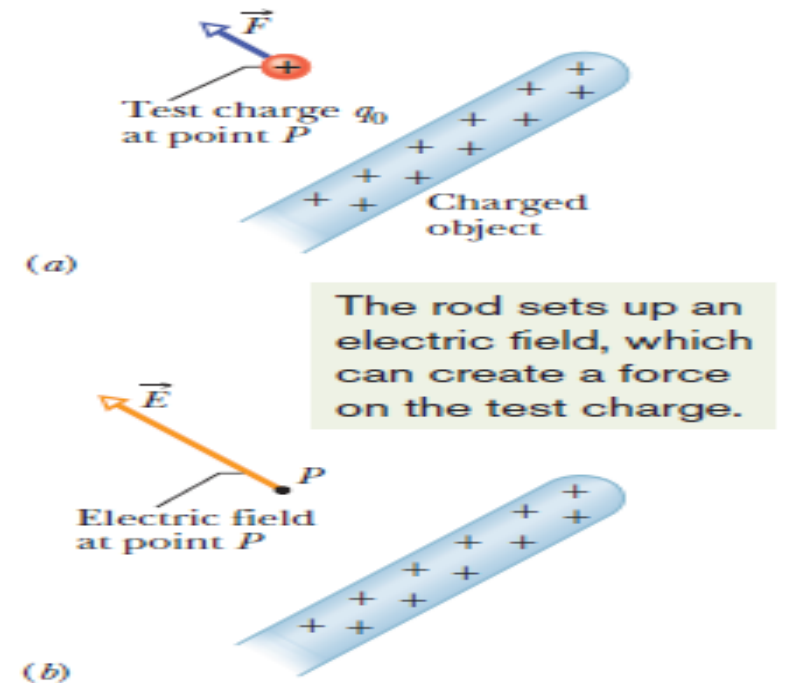
# Electric Field

- It's a vector field
- We define the electric field at point  $P$  *due to the charged object as*

$$\vec{E} = \frac{\vec{F}}{q_0} \quad (\text{electric field}).$$

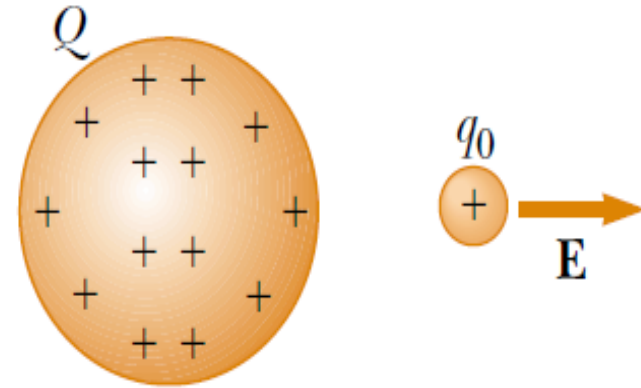
Thus, the magnitude of the electric field  $\vec{E}$  at point  $P$  is  $E = F/q_0$ , and the direction of  $\vec{E}$  is that of the force  $\vec{F}$  that acts on the *positive* test charge. As shown in Fig b

The SI unit for the electric field is the newton per coulomb (N/C).



# Purpose of a Test Charge

- Note that  $E$  is the field produced by some charge or charge distribution separate from the test charge—it is not the field produced by the test charge itself.
- 
- Also, note that the existence of an electric field is a property of its source—the presence of the test charge is **not necessary** for the field to exist.
- The test charge serves as a **detector of the electric field**.



**Figure** A small positive test charge  $q_0$  placed near an object carrying a much larger positive charge  $Q$  experiences an electric field  $\mathbf{E}$  directed as shown.

# Electric Field due to a Point Charge

- Consider a point charge  $Q$  *as a source charge*.
- this charge creates an **electric field** at all points in space surrounding it.
- A **test charge**  $q$  is placed at point  $P$ , a **distance**  $r$  from the **source charge**
- We imagine using the test charge to determine the direction of the electric force and therefore that of the electric field
- However, the electric field does not depend on the existence of the test charge—it is established solely by the source charge.

# Electric Field due to a Point Charge

- According to Coulomb's law, the force exerted by  $q$  on the test charge is:

$$\vec{F}_e = \frac{1}{4\pi\epsilon_o} \frac{Qq}{r^2} \hat{r}$$

- The electric field at  $P$ , the position of the test charge, is defined by,

$$\vec{E} = \frac{\vec{F}_e}{q}$$

- Substituting the value of  $F_e$  in Electric field equation we will get

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{Qq}{r^2} \hat{r} \left( \frac{1}{q} \right)$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{Q}{r^2} \hat{r}}$$

Electric field created by source charge  $Q$



# To calculate E-Field by Group of Charges

- To calculate the electric field at a point P due to a group of point charges, we first calculate the electric field vectors at P individually using Equation,  $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$  and then add them using vector addition method.

Or in other words....

- *at any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.*
- Thus, the electric field at point P due to a group of source charges can be expressed as the vector sum,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_i^n \frac{Q}{r_i^2} \hat{r}_i$$



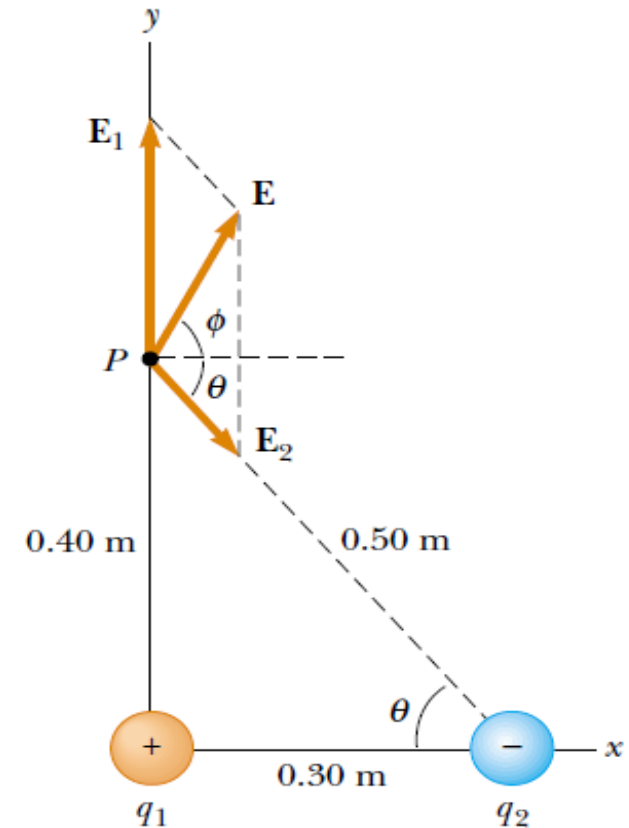
# Field due to Two Charges (due to Point charge)

A charge  $q_1 = 7.0 \mu\text{C}$  is located at the origin, and a second charge  $q_2 = -5.0 \mu\text{C}$  is located on the  $x$  axis, 0.30 m from the origin (Fig.) Find the electric field at the point  $P$ , which has coordinates (0, 0.40) m.

**Solution** First, let us find the magnitude of the electric field at  $P$  due to each charge. The fields  $\mathbf{E}_1$  due to the  $7.0\text{-}\mu\text{C}$  charge and  $\mathbf{E}_2$  due to the  $-5.0\text{-}\mu\text{C}$  charge are shown in Their magnitudes are

$$\begin{aligned} E_1 &= k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(7.0 \times 10^{-6} \text{ C})}{(0.40 \text{ m})^2} \\ &= 3.9 \times 10^5 \text{ N/C} \end{aligned}$$

$$\begin{aligned} E_2 &= k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(5.0 \times 10^{-6} \text{ C})}{(0.50 \text{ m})^2} \\ &= 1.8 \times 10^5 \text{ N/C} \end{aligned}$$



The total electric field  $\mathbf{E}$  at  $P$  equals the vector sum  $\mathbf{E}_1 + \mathbf{E}_2$ , where  $\mathbf{E}_1$  is the field due to the positive charge  $q_1$  and  $\mathbf{E}_2$  is the field due to the negative charge  $q_2$ .

The vector  $\mathbf{E}_2$  has an  $x$  component given by

$$E_2 \cos \theta = \frac{3}{5} E_2$$

and a negative  $y$  component given by

$$-E_2 \sin \theta = -\frac{4}{5} E_2$$

Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}} \text{ N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

The resultant field  $\mathbf{E}$  at  $P$  is the superposition of  $\mathbf{E}_1$  and  $\mathbf{E}_2$ :

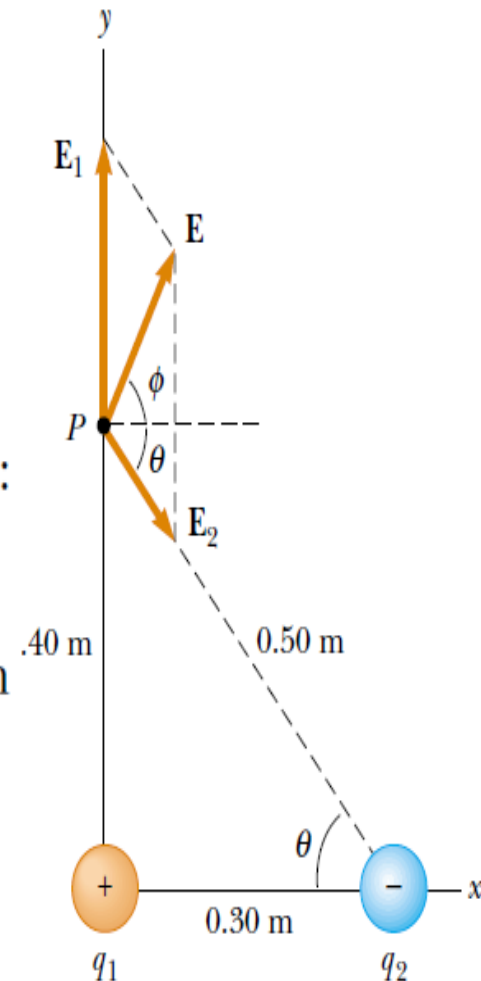
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

From this result, we find that  $\mathbf{E}$  makes an angle  $\phi$  of  $66^\circ$  with the positive  $x$  axis and has a magnitude of  $2.7 \times 10^5 \text{ N/C}$ .

$$E_1 = 3.9 \times 10^5 \text{ N/C}$$

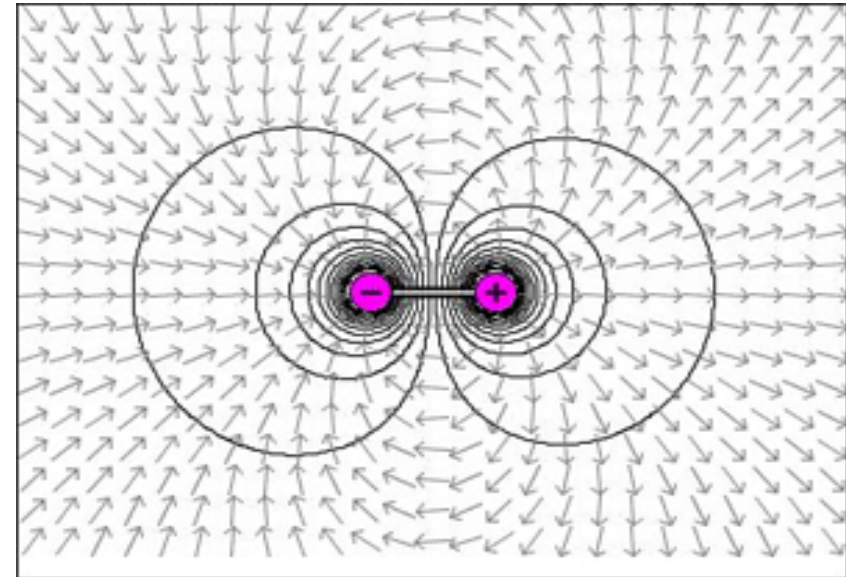
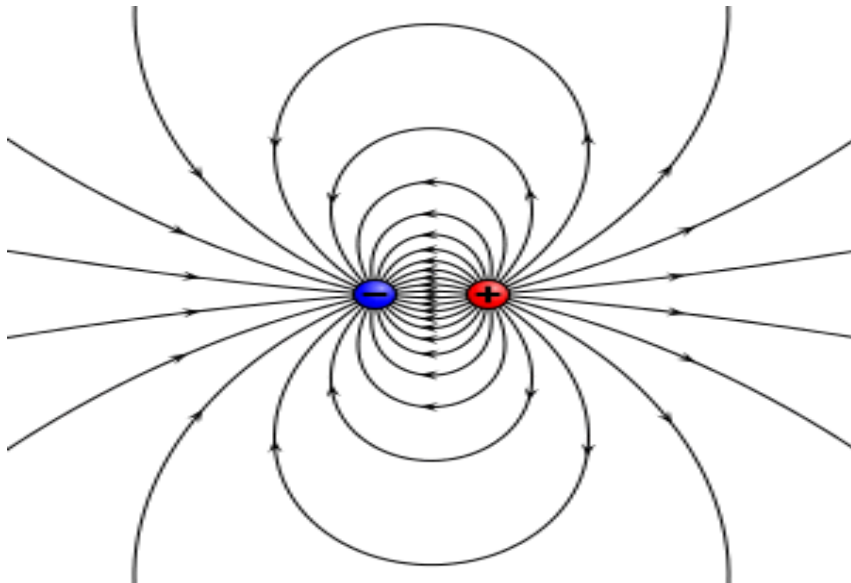
$$E_2 = 1.8 \times 10^5 \text{ N/C}$$

The vector  $\mathbf{E}_1$  has only a  $y$  component.  
coordinates (0, 0.40) m. <sup>9</sup>



# Electric Dipole & Dipole Moment

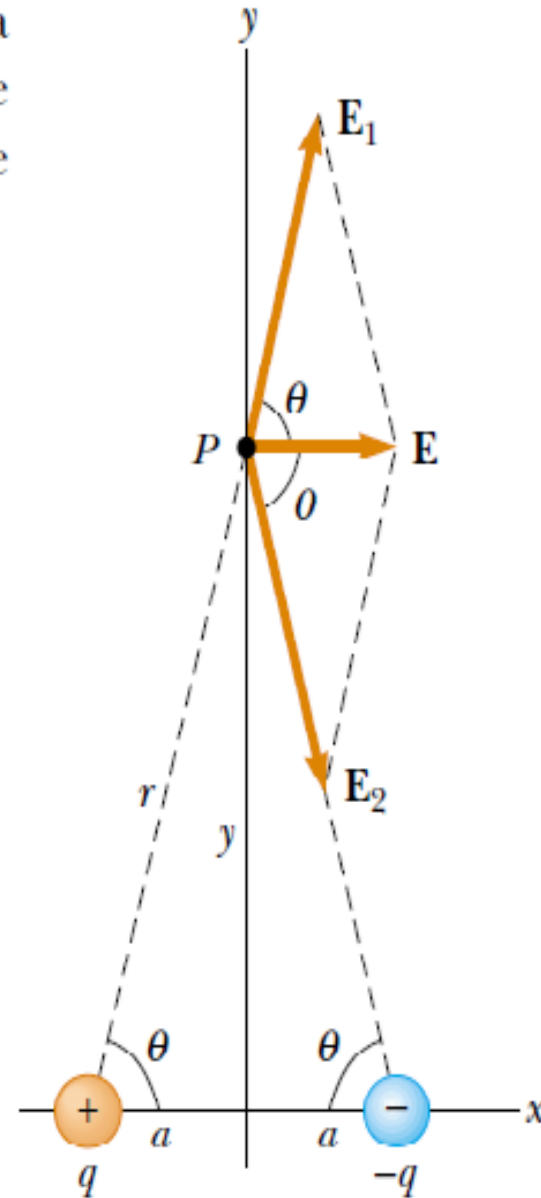
- **Electric Dipole:** Two **equal and opposite** point charges attached at a **fixed distance** is called Electric dipole.
- **Dipole Moment:** when the pair of these charges are placed in electric field they experience a turning effect this turning effect is known as **dipole moment**.  $\vec{P} = 2ql$



# Electric Field due to Dipole

An **electric dipole** is defined as a positive charge  $q$  and a negative charge  $-q$  separated by a distance  $2a$ . For the dipole shown in Figure , find the electric field  $\mathbf{E}$  at  $P$  due to the dipole, where  $P$  is a distance  $y \gg a$  from the origin.

**Figure** The total electric field  $\mathbf{E}$  at  $P$  due to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum  $\mathbf{E}_1 + \mathbf{E}_2$ . The field  $\mathbf{E}_1$  is due to the positive charge  $q$ , and  $\mathbf{E}_2$  is the field due to the negative charge  $-q$ .



**Solution** At  $P$ , the fields  $\mathbf{E}_1$  and  $\mathbf{E}_2$  due to the two charges are equal in magnitude because  $P$  is equidistant from the charges. The total field is  $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$ , where

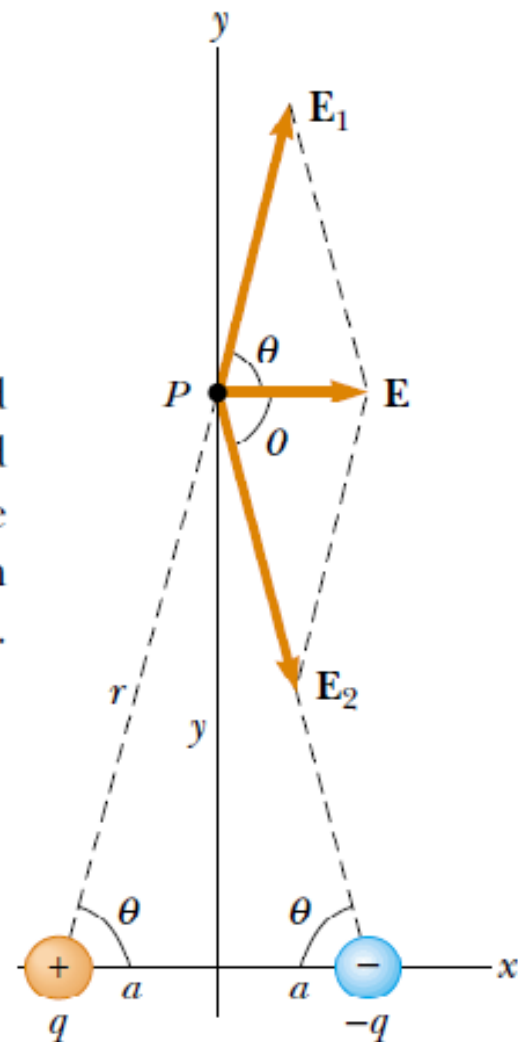
$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

The  $y$  components of  $\mathbf{E}_1$  and  $\mathbf{E}_2$  cancel each other, and the  $x$  components are both in the positive  $x$  direction and have the same magnitude. Therefore,  $\mathbf{E}$  is parallel to the  $x$  axis and has a magnitude equal to  $2E_1 \cos \theta$ . From Figure we see that  $\cos \theta = a/r = a/(y^2 + a^2)^{1/2}$ . Therefore,

$$\begin{aligned} E &= 2E_1 \cos \theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}} \\ &= k_e \frac{2qa}{(y^2 + a^2)^{3/2}} \end{aligned}$$

Because  $y \gg a$ , we can neglect  $a^2$  compared to  $y^2$  and write

$$E \approx k_e \frac{2qa}{y^3}$$



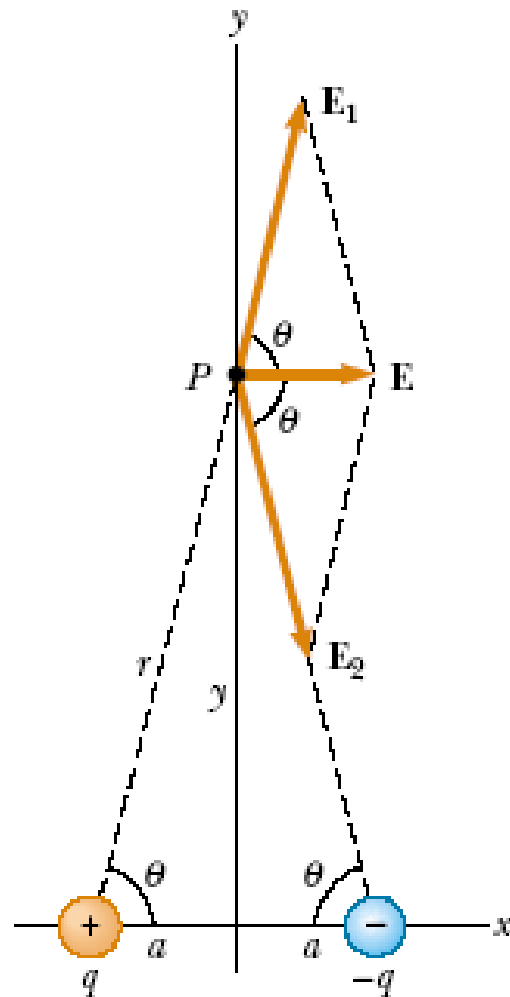
# Electric Field of a Dipole

$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

$$\begin{aligned} E &= 2E_1 \cos \theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}} \\ &= k_e \frac{2qa}{(y^2 + a^2)^{3/2}} \end{aligned}$$

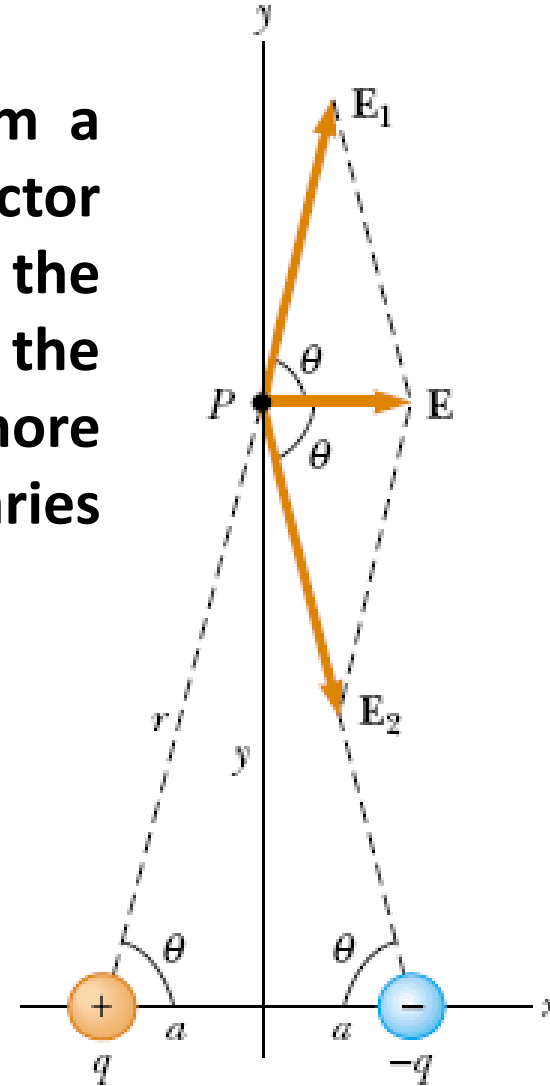
Because  $y \gg a$ , we can neglect  $a^2$  and write

$$E \approx k_e \frac{2qa}{y^3}$$

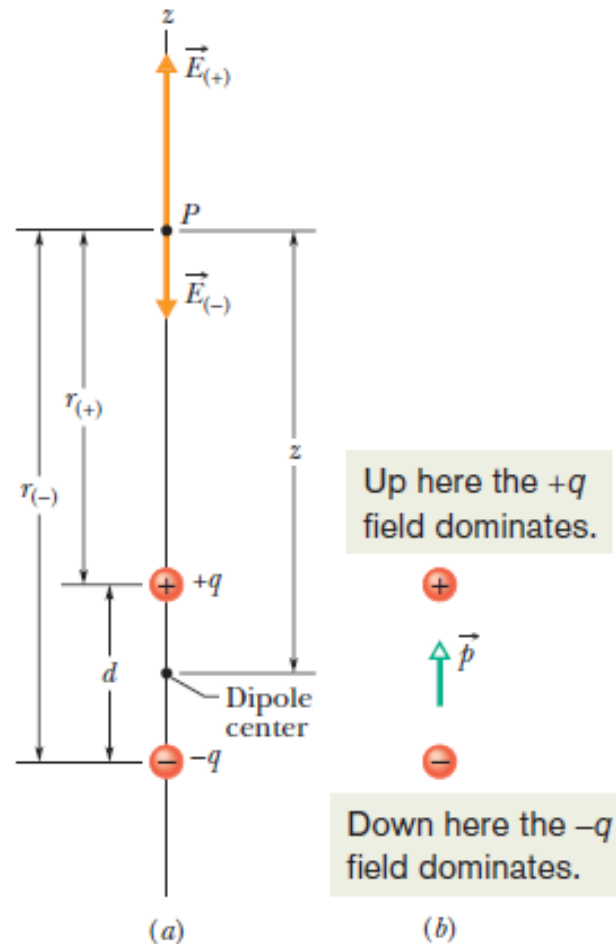


$$\cos \theta = a/r = a/(y^2 + a^2)^{1/2}.$$

Thus, we see that, at distances far from a dipole but along the perpendicular bisector of the line joining the two charges, the magnitude of the electric field created by the dipole varies as  $1/r^3$ , whereas the more slowly varying field of a point charge varies as  $1/r^2$ .



## Example 2 :



**Figure 22-9** (a) An electric dipole. The electric field vectors  $\vec{E}_{(+)}$  and  $\vec{E}_{(-)}$  at point  $P$  on the dipole axis result from the dipole's two charges. Point  $P$  is at distances  $r_{(+)}$  and  $r_{(-)}$  from the individual charges that make up the dipole. (b) The dipole moment  $\vec{p}$  of the dipole points from the negative charge to the positive charge.

$$\begin{aligned}
 E &= E_{(+)} - E_{(-)} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(-)}^2} \\
 &= \frac{q}{4\pi\epsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\epsilon_0(z + \frac{1}{2}d)^2}.
 \end{aligned}$$

After a little algebra, we can rewrite this equation as

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left( \frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2} \right). \quad (22-7)$$

After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\epsilon_0 z^2} \frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\epsilon_0 z^3} \frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}. \quad (22-8)$$

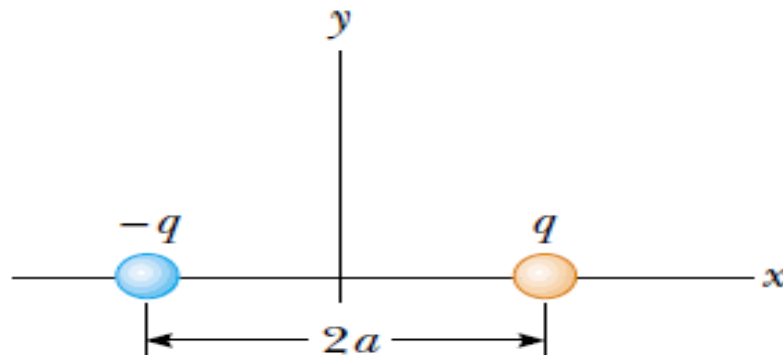
We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that  $z \gg d$ . At such large distances, we have  $d/2z \ll 1$  in Eq. 22-7. Thus, in our approximation, we can neglect the  $d/2z$  term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3}. \quad (22-8)$$



# Home Work

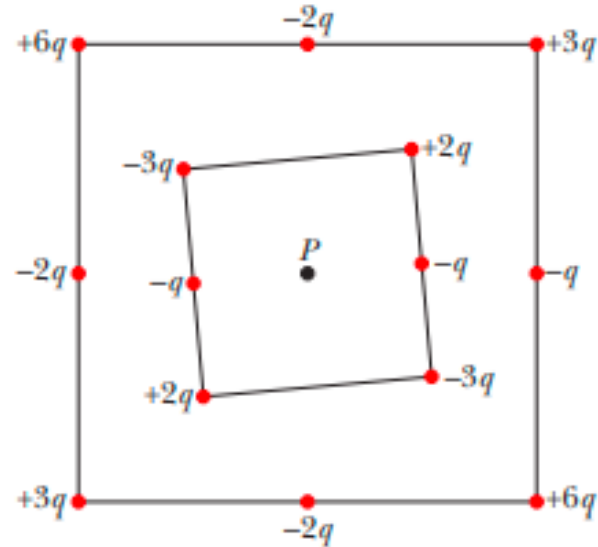
- Q1** Two  $2.00\text{-}\mu\text{C}$  point charges are located on the  $x$  axis. One is at  $x = 1.00\text{ m}$ , and the other is at  $x = -1.00\text{ m}$ . (a) Determine the electric field on the  $y$  axis at  $y = 0.500\text{ m}$ . (b) Calculate the electric force on a  $-3.00\text{-}\mu\text{C}$  charge placed on the  $y$  axis at  $y = 0.500\text{ m}$ .
- Q2** Consider the electric dipole shown in Figure . Show that the electric field at a *distant* point on the  $+x$  axis is  $E_x \approx 4k_e qa/x^3$ .



**Figure**

# Questions: (from textbook)(chapter 22)

**2** Figure 22-23 shows two square arrays of charged particles. The squares, which are centered on point  $P$ , are misaligned. The particles are separated by either  $d$  or  $d/2$  along the perimeters of the squares. What are the magnitude and direction of the net electric field at  $P$ ?



**3** In Fig. 22-24, two particles of charge  $-q$  are arranged symmetrically about the  $y$  axis; each produces an electric field at point  $P$  on that axis. (a) Are the magnitudes of the fields at  $P$  equal? (b) Is each electric field directed toward or away from the charge producing it? (c) Is the magnitude of the net electric field at  $P$  equal to the sum of the magnitudes  $E$  of the two field vectors (is it equal to  $2E$ )? (d) Do the  $x$  components of those two field vectors add or cancel? (e) Do their  $y$  components add or cancel? (f) Is the direction of the net field at  $P$  that of the canceling components or the adding components? (g) What is the direction of the net field?

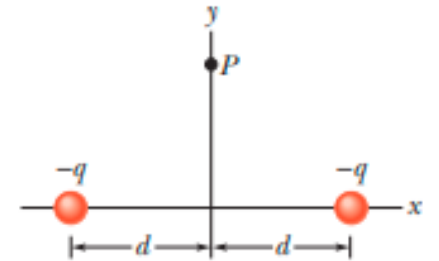
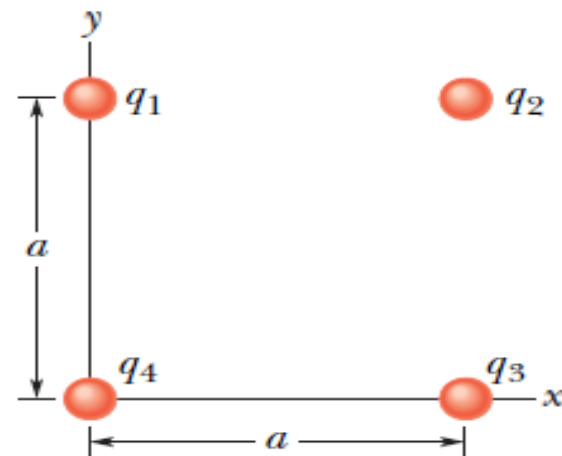



Figure 22-24 Question 3.

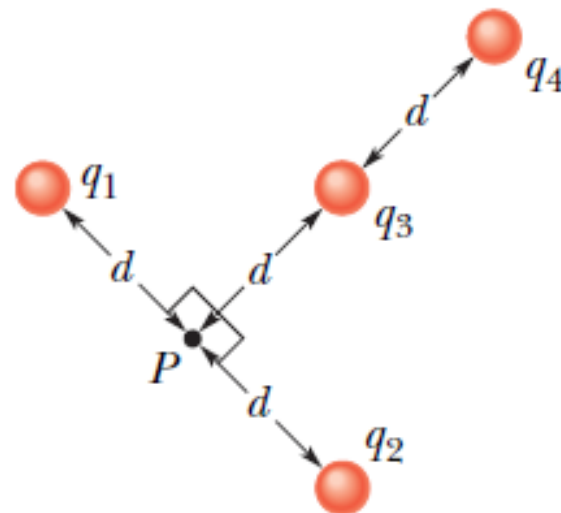
# End of Chapter Problems:

**••7** **SSM** **ILW** **WWW** In Fig. 22-30, the four particles form a square of edge length  $a = 5.00$  cm and have charges  $q_1 = +10.0$  nC,  $q_2 = -20.0$  nC,  $q_3 = +20.0$  nC, and  $q_4 = -10.0$  nC. In unit-vector notation, what net electric field do the particles produce at the square's center?



**Fig. 22-30** Problem 7.

••8  In Fig. 22-31, the four particles are fixed in place and have charges  $q_1 = q_2 = +5e$ ,  $q_3 = +3e$ , and  $q_4 = -12e$ . Distance  $d = 5.0 \mu\text{m}$ . What is the magnitude of the net electric field at point  $P$  due to the particles?



Answer: 0  
Field Lines:

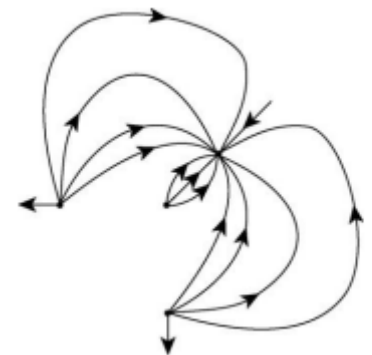


Fig. 22-31 Problem 8.

••11 SSM Two charged particles are fixed to an  $x$  axis: Particle 1 of charge  $q_1 = 2.1 \times 10^{-8} \text{ C}$  is at position  $x = 20 \text{ cm}$  and particle 2 of charge  $q_2 = -4.00q_1$  is at position  $x = 70 \text{ cm}$ . At what coordinate on the axis (other than at infinity) is the net electric field produced by the two particles equal to zero?

••14 In Fig. 22-41, particle 1 of charge  $q_1 = -5.00q$  and particle 2 of charge  $q_2 = +2.00q$  are fixed to an  $x$  axis. (a) As a multiple of distance  $L$ , at what coordinate on the axis is the net electric field of the particles zero? (b) Sketch the net electric field lines between and around the particles.

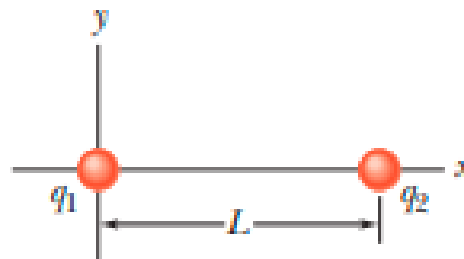
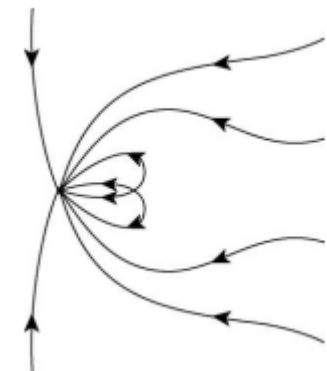


Figure 22-41 Problem 14.

Answer:  
 $x = 2.72 L$   
 Field Lines:





Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).



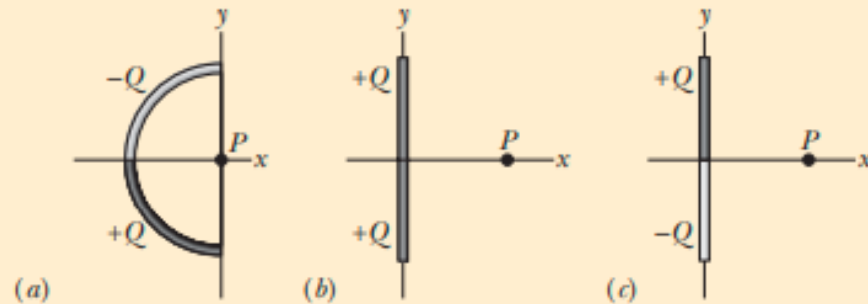
### Checkpoint 1

The figure here shows a proton  $p$  and an electron  $e$  on an  $x$  axis. What is the direction of the electric field due to the electron at (a) point  $S$  and (b) point  $R$ ? What is the direction of the net electric field at (c) point  $R$  and (d) point  $S$ ?



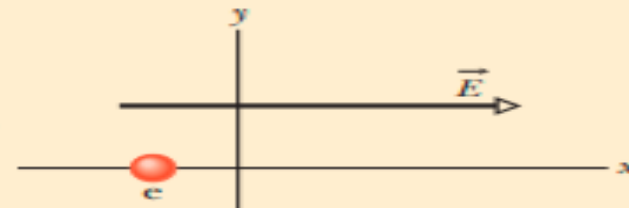
### Checkpoint 2

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude  $Q$  along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point  $P$ ?



### Checkpoint 3

(a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown? (b) In which direction will the electron accelerate if it is moving parallel to the  $y$  axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?



# Adapted from:

Book:

- Fundamentals of Physics by Halliday Resnick & Jearl Walker 10<sup>th</sup> edition