

Copying answers and steps are strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. The calculations and answers should be written neatly on paper which is attached as a single pdf. Box your answers where appropriate. Thanks!

Problem 1

Classify the following equations as ordinary or partial differential equations, state the order and degree of each equation and determine whether the equation is linear or nonlinear

(a) $\frac{\partial^3 u}{\partial x^3} + \frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial z}\right)^2 + ux^3 + uy^2 + uz = 0$

(b) $\left(\frac{dy}{dx}\right)^2 = \left(\frac{d^2 y}{dx^2} + y\right)^{\frac{3}{2}}$

Problem 2

Verify that the indicated function is the solutions given differential equation and solve the following initial/boundary value problems.

(a) $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 12y = 0$, $y(0) = -2$, $y'(0) = 6$
 where $y = c_1 e^{4x} + c_2 e^{-3x}$ is the general solution of the given differential equation.

(b) $x^3 \frac{d^3 y}{dx^3} - 3x^2 \frac{d^2 y}{dx^2} + 6x \frac{dy}{dx} - 6y = 0$, $y(2) = 2$, $y'(2) = 2$, $y''(2) = 6$
 where $y = c_1 x + c_2 x^2 + c_3 x^3$ is the general solution of the given differential equation.

(c) $\frac{d^2 y}{dx^2} + y = 0$, $y(0) = 1$, $y'\left(\frac{\pi}{2}\right) = -1$
 whwere $y = c_1 \sin x + c_2 \cos x$ is the general solution of the given differential equation.

Problem 3

Form the differential equation (eliminate the arbitrary constant)

(a) $x^3 + y^3 = 3cxy$

(b) $3y = \frac{4x^3}{x^2 + 1} + \frac{3c}{x^2 + 1}$

Bonus Problem 4

(a) Solve the DE by seperation of variable
 $(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$

(b) Solve the linear DE
 $\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$

(c) Solve the exact DE
 $e^x [y - 3(e^x + 1)^2] dx + (e^x + 1)dy = 0$, $y(0) = 4$