Differential Equation MT1006 Assignement-3 Spring-22

Copying answers and steps are strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. The calculations and answers should be written neatly on paper which is attached as a single pdf. Box your answers where appropriate. Thanks!

P-1 Show the given set of function is orthogonal and find the Norm of the function in given set

a) $\{\cos x, \cos 3x, \cos 5x, \ldots\}; \quad [0, \pi/2]$

b) $\{\sin x, \sin 3x, \sin 5x, \ldots\}; \quad [0, \pi/2]$

P-2 Sketch the graph of the given function and find the flourier series

$$f(x) = \begin{cases} \pi - x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$

P-3 Check whether function is even or odd and then expand the given function in an appropriate sine or cosine series

a) $$f(x) = \begin{cases} 1, & -2 < x < -1 \\ 0, & -1 < x < 1 \\ 1, & 1 < x < 2 \end{cases}$$

b) $$f(x) = \begin{cases} x + 1, & -1 < x < 0 \\ x - 1, & 0 \le x < 1 \end{cases}$$

P-4 Use separation of variables to find the product solution of the following

a) $$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$$

b) $$a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

P-5 Solve the Heat equation subject to the given condition Assume a rod of length L

$$u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0$$
$$u(x, 0) = x(L - x), \quad 0 < x < L$$

P-6 Solve the Wave equation subject to the given condition

$$u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0$$
$$u(x, 0) = 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = x(L - x), \quad 0 < x < L$$

P-7 Solve the Laplace equation subject to the given boundary condition

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = u(0, y), \quad u(\pi, y) = 1$$
$$u(x, 0) = 0, \quad u(x, \pi) = 0$$

P-8 (Bonus Problem)

Solve Heat , wave and Laplace equations using separation of variables