

Discrete Structures

Assignment 1

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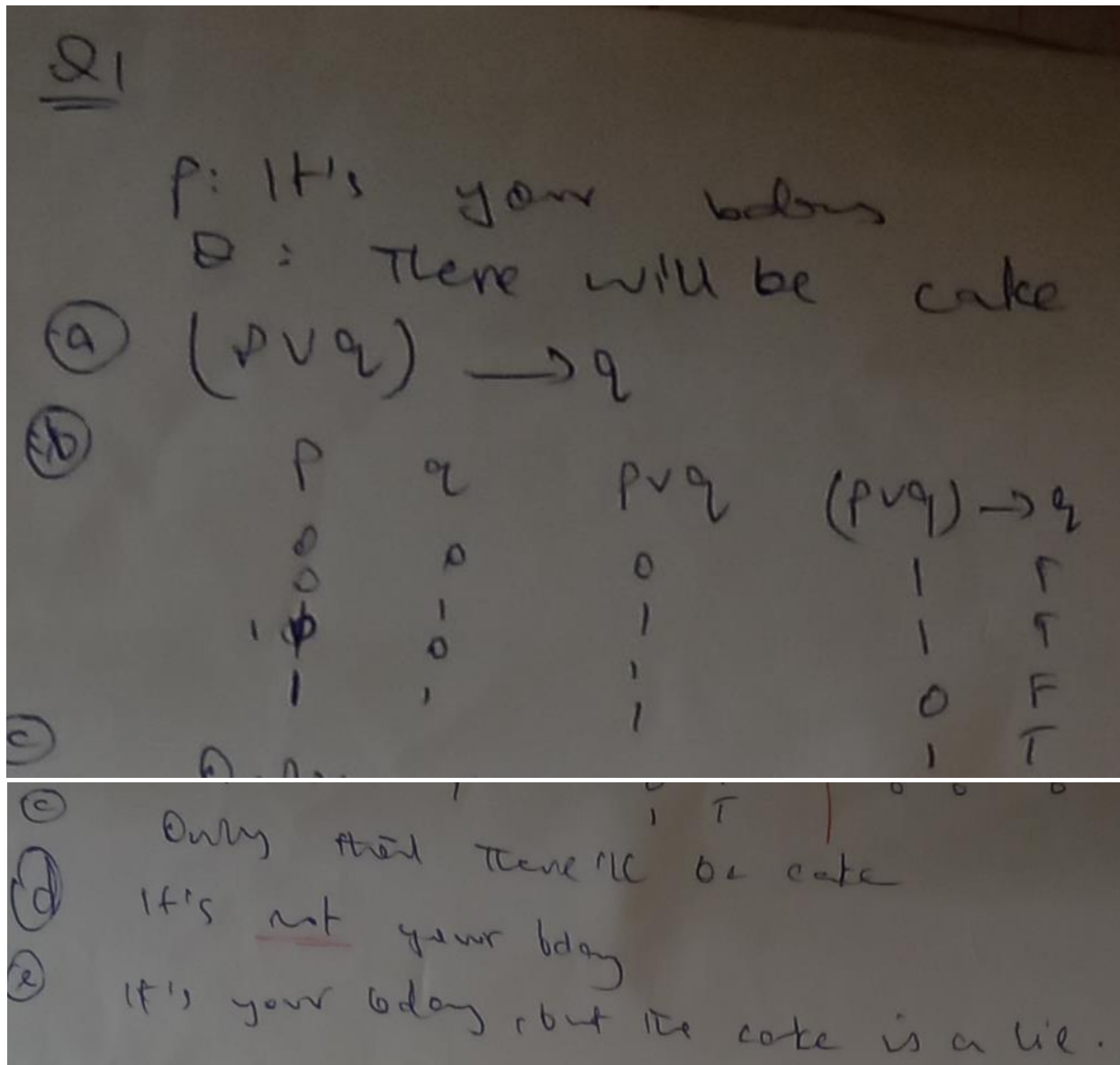
General Guidelines

1. Peer plagiarism will result in zero marks.
2. Late submissions are not allowed in any case.
3. Assignment must be submitted in hardcopy (either handwritten or printed form).
4. Follow the deadline mentioned on Google Classroom.
5. Total marks: 100

Question 1 (1*5 = 5)

Consider the statement about a party, "If it's your birthday or there will be cake, then there will be cake."

- (a) Translate the above statement into symbols. Clearly state which statement is P and which is Q.
- (b) Make a truth table for the statement.
- (c) Assuming the statement is true, what (if anything) can you conclude if there will be cake?
- (d) Assuming the statement is true, what (if anything) can you conclude if there will not be cake?
- (e) Suppose you found out that the statement was a lie. What can you conclude?



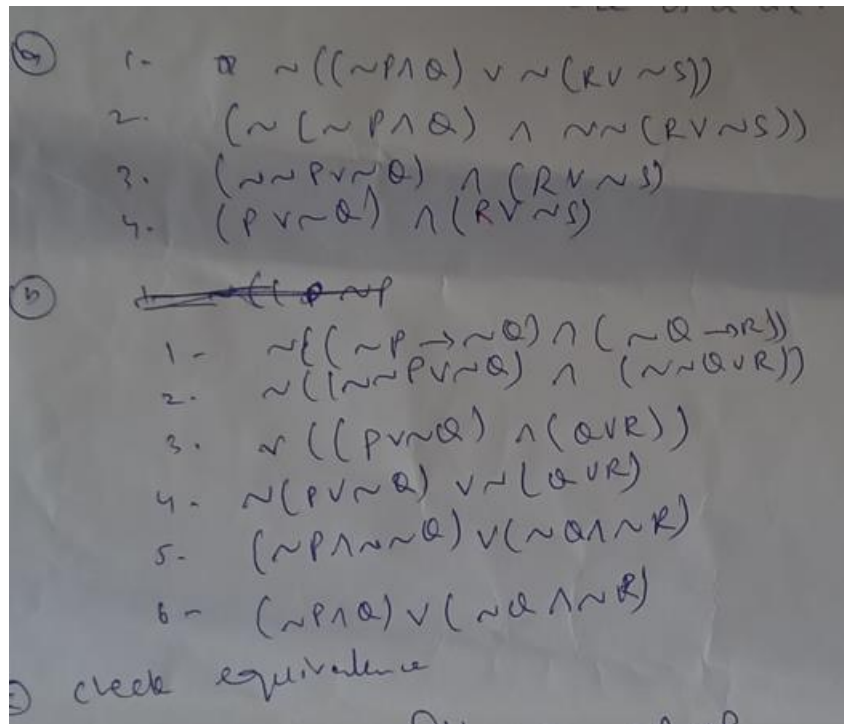
Question 2 (4+4+2=10)

Use De Morgan's Laws, and any other logical equivalence facts you know to simplify the following statements. Show all your steps. Your final statements should have negations only appear directly next to the sentence variables or predicates (P, Q, E(x), etc.), and no double negations. It would be a good idea to use only conjunctions, disjunctions, and negations.

(a) $\neg((\neg P \wedge Q) \vee \neg(R \vee \neg S))$.

(b) $\neg((\neg P \rightarrow \neg Q) \wedge (\neg Q \rightarrow R))$ (careful with the implications).

(c) For both parts above, verify your answers are correct using truth tables. That is, use a truth table to check that the given statement and your proposed simplification are actually logically equivalent



Question 3 (2+2+1)

Geoff Poshington is out at a fancy pizza joint, and decides to order a calzone. When the waiter asks what he would like in it, he replies, "I want either pepperoni or sausage. Also, if I have sausage, then I must also include quail. Oh, and if I have pepperoni or quail then I must also have ricotta cheese."

- (a) Translate Geoff's order into logical symbols.
- (b) The waiter knows that Geoff is either a liar or a truth-teller (so either everything he says is false, or everything is true). Which is it?
- (c) What, if anything, can the waiter conclude about the ingredients in Geoff's desired calzone?

Greoff want Pepperoni - P
 Greoff want Sausage - S
 Greoff include quail - Q
 Greoff have ricotta cheese - R

a) $(P \vee S) \wedge (S \rightarrow Q) \wedge ((P \vee Q) \rightarrow R)$

b) Greoff is true teller ~~then~~ if all the argument is true.

i) $P \vee S$

ii) $S \rightarrow Q$

iii) $(P \vee Q) \rightarrow R$

Let Forc the 1st argument will be true

Let P is true & Q is false then $P \vee S$ is true.

Forc the 2nd argument S is false so $S \rightarrow Q$ is true.

Forc the 3rd argument R is true & $P \vee Q$ is true so $(P \vee Q) \rightarrow R$ is true.

So Greoff is true teller.

c) desired colzone with pepperoni and ricotta cheese.
or, desired colzone with sausage, quail & ricotta cheese.

Question 4 (1.25*4=5)

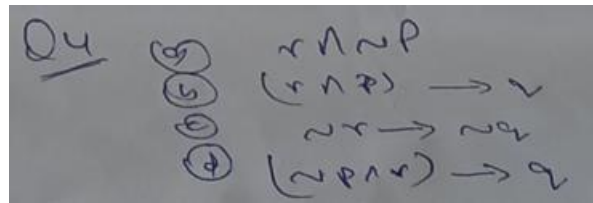
Express these system specifications using the propositions p "The user enters a valid password," q "Access is granted," and r "The user has paid the subscription fee" and logical connectives (including negations).

a) "The user has paid the subscription fee, but does not enter a valid password."

b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."

c) "Access is denied if the user has not paid the subscription fee."

d) "If the user has not entered a valid password but has Paid the subscription fee, then access is granted."



Question 5 (5)

Find premises and conclusion and determine the validity of the following argument:

"Robbery was the motive for the crime only if the victim had money in his pockets. But robbery or vengeance was the motive for the crime. Therefore, vengeance must have been the motive for the crime."

Consider

p : the victim had money in his pockets

q : robbery was the motive for the crime

r : vengence was the motive for the crime

So, the given argument can be written as below in symbol form

$$p \rightarrow \neg q$$

$$q \vee r$$

$$\neg p$$

$$\therefore r$$

The truth table for given argument is given as below.

p	q	r	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$q \vee r$
T	T	T	F	F	F	T
T	T	F	F	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F * T	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	F

In the row which has been stated, all premises are true, but the conclusion is false.

So, the given argument is invalid.

Question 6 (2*6=12)

State the converse, contrapositive, and inverse of each of these conditional statements.

- a) If it snows tonight, then I will stay at home.
- b) I go to the beach whenever it is a sunny summer day.
- c) When I stay up late, it is necessary that I sleep until noon.

1. If it snows tonight, then I will stay at home.

converse = If I will stay at home then it snows tonight.

contrapositive, = If I will not stay at home then it does not snow tonight.

inverse = If it does not snow tonight, then I will not stay at home.

2. I go to the beach whenever it is a sunny summer day.

converse = It is a sunny summer day whenever I go to the beach.

contrapositive, = it is not a sunny summer day whenever I do not go to the beach.

inverse = I do not go to the beach whenever it is not a sunny summer day.

3. we can re write the statement like if i stay up late then i sleep until noon .

converse : if i sleep until noon then i stay up late .

contrapositive : if i dont sleep until noon then i did not stay up late

inverse : if i didnt stay up late then i dont sleep until noon

Question 7 (1*12=12)

Write inverse, converse and contrapositive of the following statements.

1. If P is a square, then P is a rectangle,
2. If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
3. If n is prime, then n is odd or n is 2.
4. If today is New Year's Eve, then tomorrow is January

Write a converse and inverse statement for the following sentence:

If P is a square, then P is a rectangle.

- Converse: If p is a rectangle, then p is a square.
- Inverse: If p is not a square, then p is not a rectangle.

Write a contrapositive for the following statement:

If P is a square, then P is a rectangle.

If p is not a rectangle, then p is not a square.

Write a contrapositive for the following statement:

If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.

If Jim is not her uncle and Sue is not her aunt, then Tom is not Ann's father.

Write a converse and inverse statement for the following sentence:

If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.

- Converse: If Jim is her uncle and Sue is her aunt, then Tom is Ann's father.
- Inverse: If Tom is not Ann's father, then either Jim is not her uncle or Sue is not her aunt.

Write a converse and inverse statement for the following sentence:

If n is prime, then n is odd or n is 2.

- Converse: If n is odd or n is 2, then n is prime.
- Inverse: If n is not prime, then n is neither odd nor 2.

Write a contrapositive for the following statement:

If n is prime, then n is odd or n is 2.

If n is not odd and n is not 2, then n is not prime.

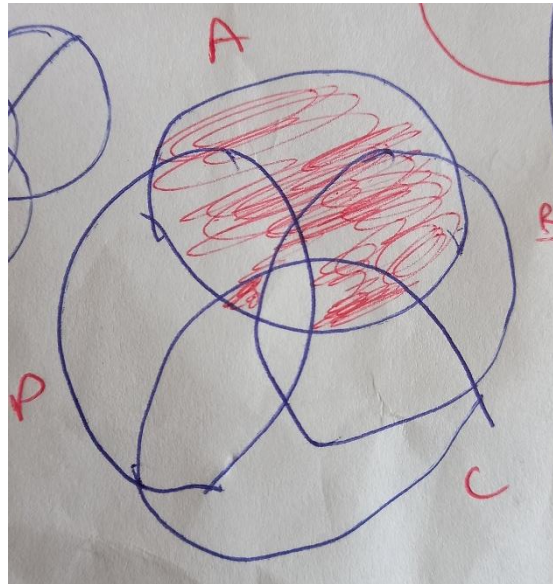
Contrapositive: If tomorrow is not January, then today is not New Year's Eve. Converse: If tomorrow is January, then today is New Year's Eve. Inverse: **If today is not New Year's Eve, then tomorrow is not January.**

Question 8 (2*3=6)

What rule of inference is used in each of these arguments?

- Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
- It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
- Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

$$A - (B \cap C \cap D)$$



Question 11 (2*5=10)

Determine the reflexivity, symmetry, and transitivity of the following relations:

(a) Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$.

Symmetric, transitive

(b) If $A = \{1, 2, 3, 4\}$ then $R = \{(1, 1), (2, 2), (1, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$.

Reflexive

(c) Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 2), (3, 1), (1, 3)\}$

Symmetric

(d) Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (1, 2), (2, 1), (2, 3), (3, 2)\}$.

Symmetric

(e) Let $A = \{0, 1, 2, 4, 6\}$ and $R = \{(0, 1), (1, 0), (2, 4), (4, 6), (6, 4), (4, 2)\}$.

Symmetric

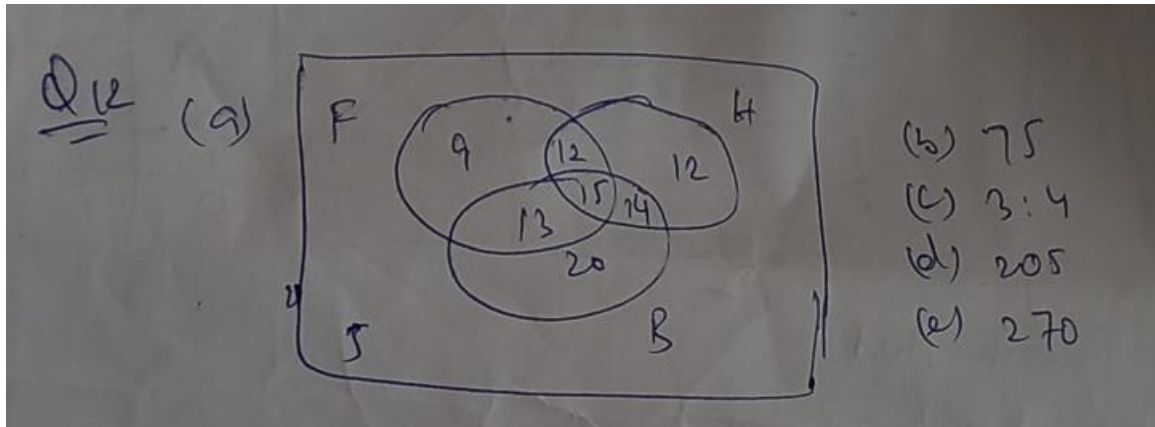
Question 12 (1*5=5)

Draw a Venn diagram to show the information.

In a survey of 500 students at a university, it was found that 49% students liked watching football, 53% liked watching hockey and 62% liked watching basketball. Also, 27% liked watching football and hockey both, 29% liked

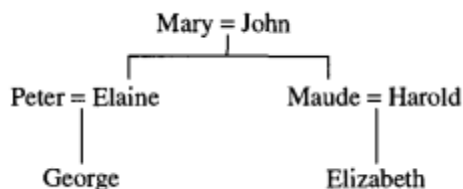
watching basketball and hockey both and 28% liked watching football and basketball both. 5% liked watching none of the game.

- Make Ven diagram of this question.
- How many students like watching all the three games?
- Find the ratio of number of students who like watching only football to those who like watching only hockey.
- Find the number of students who like watching only one of the three given games.
- Find the number of students who like watching at least two of the given games.



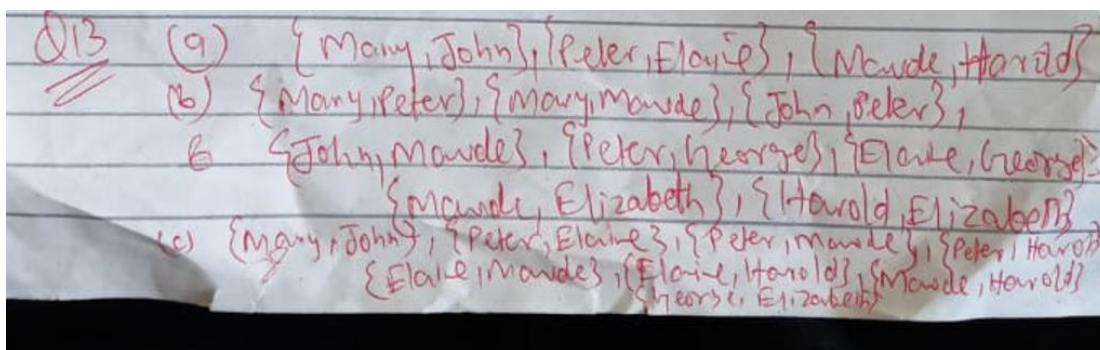
Question 13 (2*3=6)

Although the marriage of a descendent is indicated by an = sign and the name of the partner, no further information is given about these partners. For example, in the family tree of Mary and John shown in the figure, even if Peter and Harold were brothers, this would not be shown. A family tree is a rich source of information about a number of relations.



For the family tree shown in Figure 3.2, identify the elements of the relations

- IsMarriedTo,
- IsParentOf, and
- IsSameGeneration.



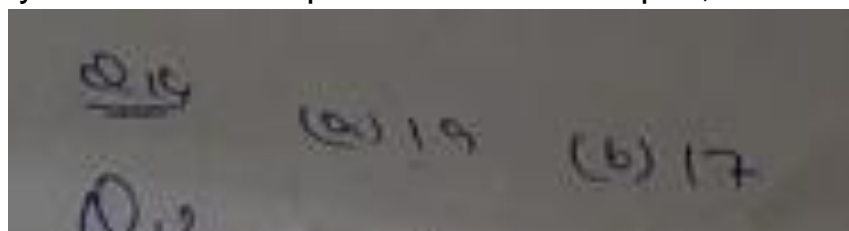
Question 14 (2+2=4)

A group of 62 students were surveyed, and it was found that each of the students surveyed liked at least one of the following three fruits: apricots, bananas, and cantaloupes.

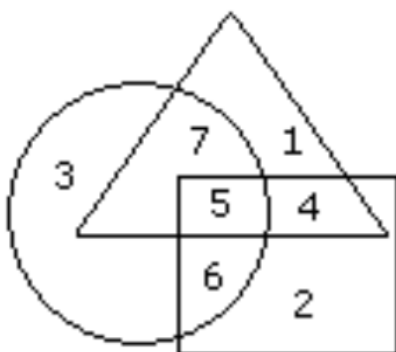
- 34 liked apricots.
- 30 liked bananas.
- 33 liked cantaloupes.
- 11 liked apricots and bananas.
- 15 liked bananas and cantaloupes.
- 17 liked apricots and cantaloupes.
- 19 liked exactly two of the following fruits: apricots, bananas, and cantaloupes.

A) How many students liked all of the following three fruits: apricots, bananas, and cantaloupes?

B) How many students liked apricots and cantaloupes, but not bananas?



Question 15 (1*5=5)



- Trained nurses
 → Married population
 → Nurses in hospital

Give answers with explanation.

- (a) How many are only married trained nurses?
- (b) By which number, married but untrained nurses in the hospital are represented?
- (c) By which numbers trained nurses are represented?
- (d) What is represented by the number 7?
- (e) By which number, the trained unmarried nurses in the hospital are represented?

