

# **Discrete Structures**

## **Lecture # 05**

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“The Consequences of an Act Affect the Probability of its Occurring Again!”  
- B. F. Skinner -

Material Partially Collected by Prof. Dr. A.M. Khan, Slides Prepared by Dr. M. Ahmad

# SET

A well defined **collection** of distinct objects is called a **set**.

The **objects** are called the **elements** or **members** of the **set**.

Sets are **denoted by** capital letters **A,B,C ... X,Y,Z**.

# SET

The elements of a set are **represented** by **lower case** letters **a, b, c, ... , x, y, z.**

If an **object** **x** is a **member** of a set **A** we write  **$x \in A$** , which reads "**x belongs to A**" or "**x is in A**" or "**x is an element of A**"

Otherwise we write  **$x \notin A$** , which reads "**x does not belong to A**" or "**x is not in A**" or "**x is not an element of A**".

# TABULAR FORM

Listing all the elements of a set, separated by commas and enclosed within braces or curly brackets  $\{\}$ .

## EXAMPLES:

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{2, 4, 6, 8, \dots, 50\}$$

$$C = \{1, 3, 5, 7, 9, \dots\}$$

# DISCRIPTIVE FORM

Stating in words the elements of the set.

## EXAMPLES:

A = set of a first five Natural Numbers.

B = set of positive even integers less or equal to fifty.

C = set of positive odd integers.

# SET BUILDER FORM

Writing in symbolic form the **common characteristics** shared by all the elements of the set.

## EXAMPLES

$$A = \{x \in N \mid x \leq 5\} \quad N = \text{Natural Number}$$

$$B = \{y \in E \mid 0 < y \leq 50\} \quad E = \text{Even Number}$$

$$C = \{x \in O \mid x > 0\} \quad O = \text{Odd Number}$$

# SET OF NUMBERS

1. Set of **Natural Numbers**

$$\mathbf{N} = \{1, 2, 3, \dots\}$$

2. Set of **Whole Numbers**

$$\mathbf{W} = \{0, 1, 2, 3, \dots\}$$

3. Set of **Integers**

$$\begin{aligned}\mathbf{Z} &= \{\dots, -3, -2, -1, 0, +1, +2, +3, \dots\} \\ &= \{0, \pm 1, \pm 2, \pm 3, \dots\}\end{aligned}$$

## SET OF NUMBERS

4. Set of **Even Integers**

$$E = \{0, \pm 2, \pm 4, \pm 6, \dots\}$$

5. Set of **Odd Integers**

$$O = \{\pm 1, \pm 3, \pm 5, \dots\}$$

6. Set of **Prime Numbers**

$$P = \{2, 3, 5, 7, 11, 13, 17, 19, \dots\}$$

7. Set of **Rational Numbers**

$$Q = \{x \mid x = p/q ; p, q \in \mathbb{Z}, q \neq 0\}$$

# SUBSET

If A and B are two sets, A is called a **subset** of B, written  $A \subseteq B$ , if, and only if, **every element** of A is **also an element** of B.

Symbolically:

$$A \subseteq B \leftrightarrow \text{if } x \in A \text{ then } x \in B$$

1. So, if A has n elements, the maximum number of subsets of A is  $2^n$
2. On the other hand, the maximum number of non-empty sets is equal to  $2^n - 1$ .



# SUBSET

## REMARKS:

1. When  $A \subseteq B$ , then  $B$  is called a **superset** of  $A$ .
2. When  $A \not\subseteq B$ , then there exist at least one  $x \in A$  such that  $x \notin B$ .
3. Every set is a **subset** of itself.

## EXAMPLE

Let

$$A = \{1, 3, 5\} \quad B = \{1, 2, 3, 4, 5\}$$

$$C = \{1, 2, 3, 4\} \quad D = \{3, 1, 5\}$$

Then

$$A \subseteq B$$

$$A = \{1, 3, 5\}$$

$$A \subseteq D$$

$$D = \{3, 1, 5\}$$

$$A \not\subseteq C$$

$$5 \in A \text{ but } 5 \notin C$$

## PROPER SUBSET

Let  $A$  and  $B$  be sets.  $A$  is a **proper subset** of  $B$ , if, and only if, **every** element of  $A$  is in  $B$  but there is **at least** one element of  $B$  that is **not** in  $A$ .

Symbolically:

$$A \subset B$$

## EQUAL SETS

Two sets  $A$  and  $B$  are **equal** if, and only if, **every element** of  $A$  is in  $B$  and **every element** of  $B$  is in  $A$  and is denoted  $A = B$ .

Symbolically:

$$A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A$$

## EQUAL SETS

### EXAMPLE:

Let  $A = \{1, 2, 3, 6\}$

$B =$  the set of positive divisors of 6

$C = \{3, 1, 6, 2\}$

$D = \{1, 2, 2, 3, 6, 6, 6\}$

Then A, B, C, and D are all equal sets.

### Point to ponder!!

1. Is  $n(A) = n(D)$ ?
2. Are equal sets equivalent and vice versa?

## NULL SET

A **set** which contains **no element** is called a **null set**, or an **empty set** or a **void set**.

Symbolically:

It is denoted by the Greek letter  $\emptyset$ (phi) or  $\{ \}$ .

## NULL SET

### EXAMPLE

$$A = \{x \mid x \text{ is a person taller than 10 feet}\}$$

$$A = \emptyset$$

$$B = \{x \mid x^2 = 4, x \text{ is odd}\}$$

$$B = \emptyset$$

## EXERCISE

(a)	$x$	$\in$	$\{x\}$	TRUE
(b)	$\{x\}$	$\subseteq$	$\{x\}$	TRUE
(c)	$\{x\}$	$\in$	$\{x\}$	FALSE
(d)	$\{x\}$	$\in$	$\{\{x\}\}$	TRUE
(e)	$\emptyset$	$\subseteq$	$\{x\}$	TRUE
(f)	$\emptyset$	$\in$	$\{x\}$	FALSE

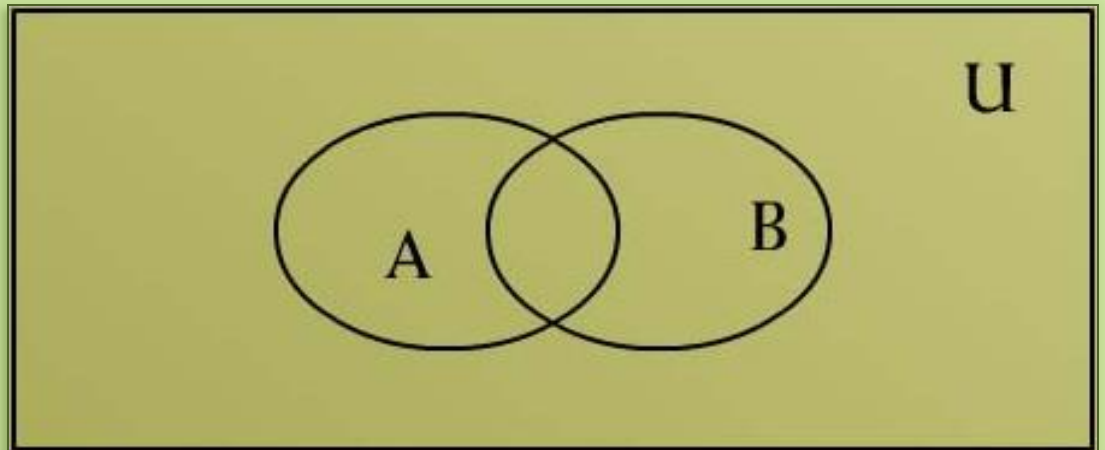
## UNIVERSAL SET

The **set of all elements** under consideration is called the **Universal Set**.

The **Universal Set** is denoted by **U**.

## VENN DIAGRAM

A **Venn diagram** is a graphical representation of **sets by regions** in the plane.





## FINITE AND INFINITE SETS

A set  $S$  is said to be **finite** if it contains **exactly**  $m$  distinct elements where  $m$  denotes some non negative integer.

In such case we write

$$|S| = m \text{ or } n(S) = m$$

A **set** is said to be **infinite** if it is not **finite**.

# FINITE AND INFINITE SETS

## EXAMPLES

1. The set  $S$  of letters of English alphabets is finite and  $|S| = 26$
2. The null set  $\emptyset$  has no elements, is finite and  $|\emptyset| = 0$
3. The set of positive integers  $\{1, 2, 3, \dots\}$  is infinite.

## EXERCISE

1.  $A = \{\text{month in the year}\}$       FINITE
2.  $B = \{\text{even integers}\}$       INFINITE
3.  $C = \{\text{positive integers less than 1}\}$   
FINITE

## MEMBERSHIP TABLE

A **table** displaying the **membership** of elements in sets. To **indicate** that an element is **in a set**, a **1** is used; to **indicate** that an element is **not in a set**, a **0** is used.

$A$	$A^c$
1	0
0	1

## UNION

Let  $A$  and  $B$  be subsets of a universal set  $U$ . The **union** of sets  $A$  and  $B$  is the set of **all elements** in  $U$  that belong to  $A$  **or** to  $B$  or to both, and is denoted  $A \cup B$ .

Symbolically:

$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$$

# UNION

EXAMPLE:

Let

$$U = \{a, b, c, d, e, f, g\}$$

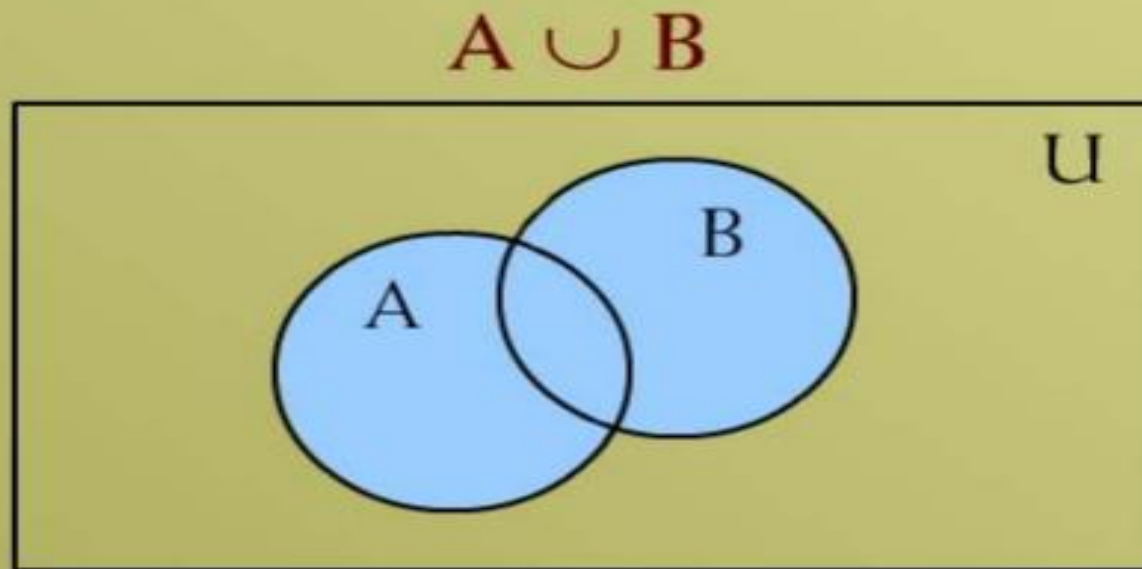
$$A = \{a, c, e, g\}$$

$$B = \{d, e, f, g\}$$

Then

$$\begin{aligned} A \cup B &= \{a, c, e, g\} \cup \{d, e, f, g\} \\ &= \{a, c, d, e, f, g\} \end{aligned}$$

## VENN DIAGRAM FOR



### REMARK

1.  $A \cup B = B \cup A$
2.  $A \subseteq A \cup B$  and  $B \subseteq A \cup B$

## MEMBERSHIP TABLE FOR

$$A \cup B$$

A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0



# INTERSECTION

Let  $A$  and  $B$  subsets of a universal set  $U$ . The intersection of sets  $A$  and  $B$  is the set of all elements in  $U$  that belong to both  $A$  and  $B$  and is denoted  $A \cap B$ .

Symbolically:

$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$$

# INTERSECTION

## EXAMPLE

Let  $U = \{a, b, c, d, e, f, g\}$

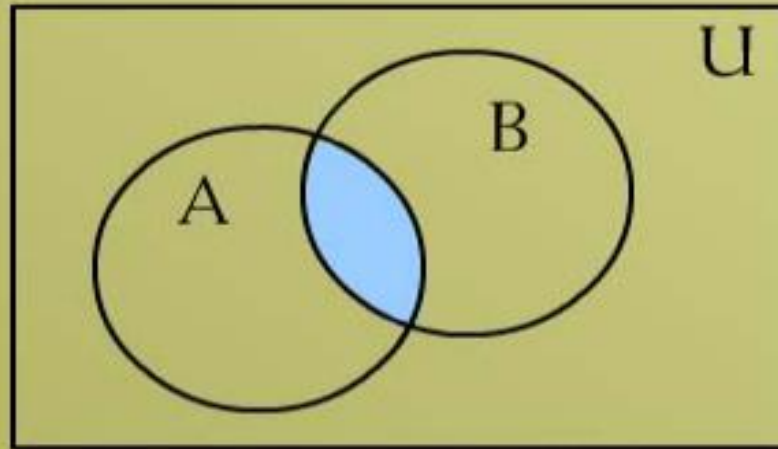
$$A = \{a, c, e, g\}$$

$$B = \{d, e, f, g\}$$

Then

$$\begin{aligned} A \cap B &= \{a, c, e, g\} \cap \{d, e, f, g\} \\ &= \{e, g\} \end{aligned}$$

# VENN DIAGRAM



$A \cap B$  is shaded

## REMARK

1.  $A \cap B = B \cap A$
2.  $A \cap B \subseteq A$  and  $A \cap B \subseteq B$
3. If  $A \cap B = \emptyset$   
then A & B are called **disjoint sets**.

## MEMBERSHIP TABLE FOR

$$A \cap B$$

A	B	$A \cap B$
1	1	1
1	0	0
0	1	0
0	0	0

# SET DIFFERENCE

Let  $A$  and  $B$  be subsets of a universal set  $U$ . The **difference** of “ $A$  and  $B$ ” (or relative complement of  $B$  in  $A$ ) is the set of all element in  $U$  that belong to  $A$  but not to  $B$ , and is denoted by  $A-B$  or  $A/B$ .

**Symbolically:**

$$A - B = \{x \in U \mid x \in A \text{ and } x \notin B\}$$

# SET DIFFERENCE

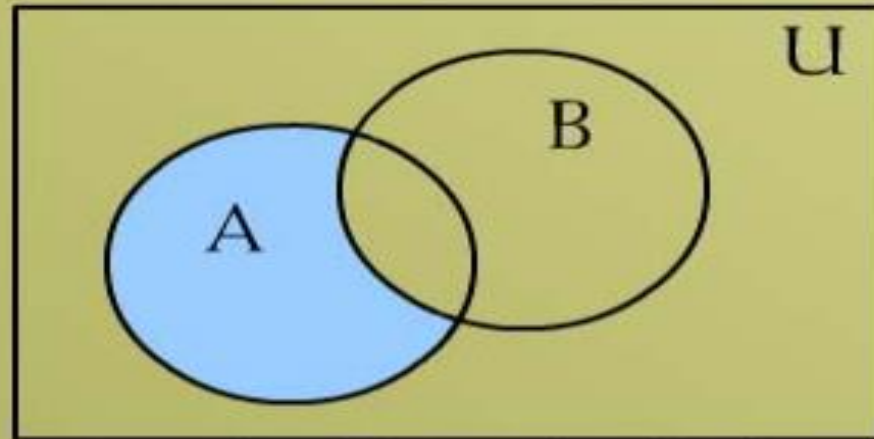
## EXAMPLE:

Let  $U = \{a, b, c, d, e, f, g\}$   
 $A = \{a, c, e, g\}$   
 $B = \{d, e, f, g\}$

Then:

$$\begin{aligned} A - B &= \{a, c, e, g\} - \{d, e, f, g\} \\ &= \{a, c\} \end{aligned}$$

## VENN DIAGRAM



$A - B$  is shaded

### REMARKS:

1.  $A - B \neq B - A$
2.  $A - B \subseteq A$
3.  $A - B$ ,  $A \cap B$  and  $B - A$  are mutually disjoint sets.

## MEMBERSHIP TABLE FOR

$A - B$

A	B	$A - B$
1	1	0
1	0	1
0	1	0
0	0	0



## COMPLEMENT

Let  $A$  be a subset of universal set  $U$ . The complement of  $A$  is the set of all element in  $U$  that do not belong to  $A$ , and is denoted  $A^c$ ,  $\overline{A}$  or  $A'$

Symbolically:

$$A' = \{x \in U \mid x \notin A\}$$

# COMPLEMENT

## EXMAPLE

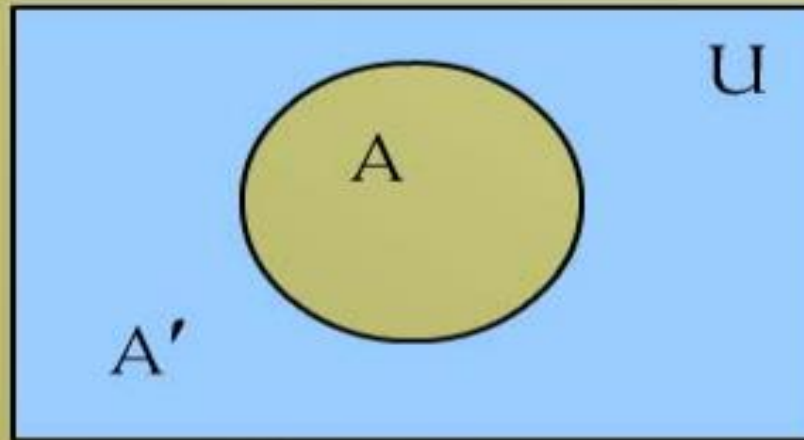
Let  $U = \{a, b, c, d, e, f, g\}$

$$A = \{a, c, e, g\}$$

Then

$$\begin{aligned} A' &= \{a, b, c, d, e, f, g\} - \{a, c, e, g\} \\ &= \{b, d, f\} \end{aligned}$$

# VENN DIAGRAM



REMARKS:

$A'$  is shaded

1.  $A' = U - A$
2.  $A \cap A' = \emptyset$
3.  $A \cup A' = U$

## MEMBERSHIP TABLE FOR

$A'$

A	$A'$
1	0
0	1

## EXERCISE

Let  $U = \{1, 2, 3, \dots, 10\}$   
 $X = \{1, 2, 3, 4, 5\}$   
 $Y = \{y \mid y = 2x, x \in X\}$   
 $Z = \{z \mid z^2 - 9z + 14 = 0\}$

Enumerate:

(i)  $X \cap Y$

(ii)  $Y \cup Z$

(iii)  $X - Z$

(iv)  $Y'$

(v)  $X' - Z'$

(vi)  $(X - Z)'$

**Note that “y” and “z” both belongs to Universal set “U”.**

## SOLUTION

Given

$$U = \{1, 2, 3, \dots, 10\}$$

$$X = \{1, 2, 3, 4, 5\}$$

$$\begin{aligned} Y &= \{y \in U \mid y = 2x, x \in X\} \\ &= \{2, 4, 6, 8, 10\} \end{aligned}$$

$$\begin{aligned} Z &= \{z \in U \mid z^2 - 9z + 14 = 0\} \\ &= \{2, 7\} \end{aligned}$$

## SOLUTION

$$(i) X \cap Y = \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} \\ = \{2, 4\}$$

$$(ii) Y \cup Z = \{2, 4, 6, 8, 10\} \cup \{2, 7\} \\ = \{2, 4, 6, 7, 8, 10\}$$

$$(iii) X - Z = \{1, 2, 3, 4, 5\} - \{2, 7\} \\ = \{1, 3, 4, 5\}$$

## SOLUTION

$$(iv) Y' = U - Y$$

$$= \{1, 2, 3, \dots, 10\} - \{2, 4, 6, 8, 10\}$$

$$= \{1, 3, 5, 7, 9\}$$

$$(v) X' - Z'$$

$$= \{6, 7, 8, 9, 10\} - \{1, 3, 4, 5, 6, 8, 9, 10\}$$

$$= \{7\}$$

$$(vi) (X - Z)'$$

$$= U - (X - Z)$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 3, 4, 5\}$$

$$= \{2, 6, 7, 8, 9, 10\}$$



## EXERCISE

$$U = \{ x \in \mathbb{Z}, 0 \leq x \leq 10 \}$$

$$P = \{ x \in U \mid x \text{ is a prime number} \}$$

$$Q = \{ x \in U \mid x^2 < 70 \}$$

(i) Draw a Venn diagram for the above

(ii) List the elements in  $P^c \cap Q$

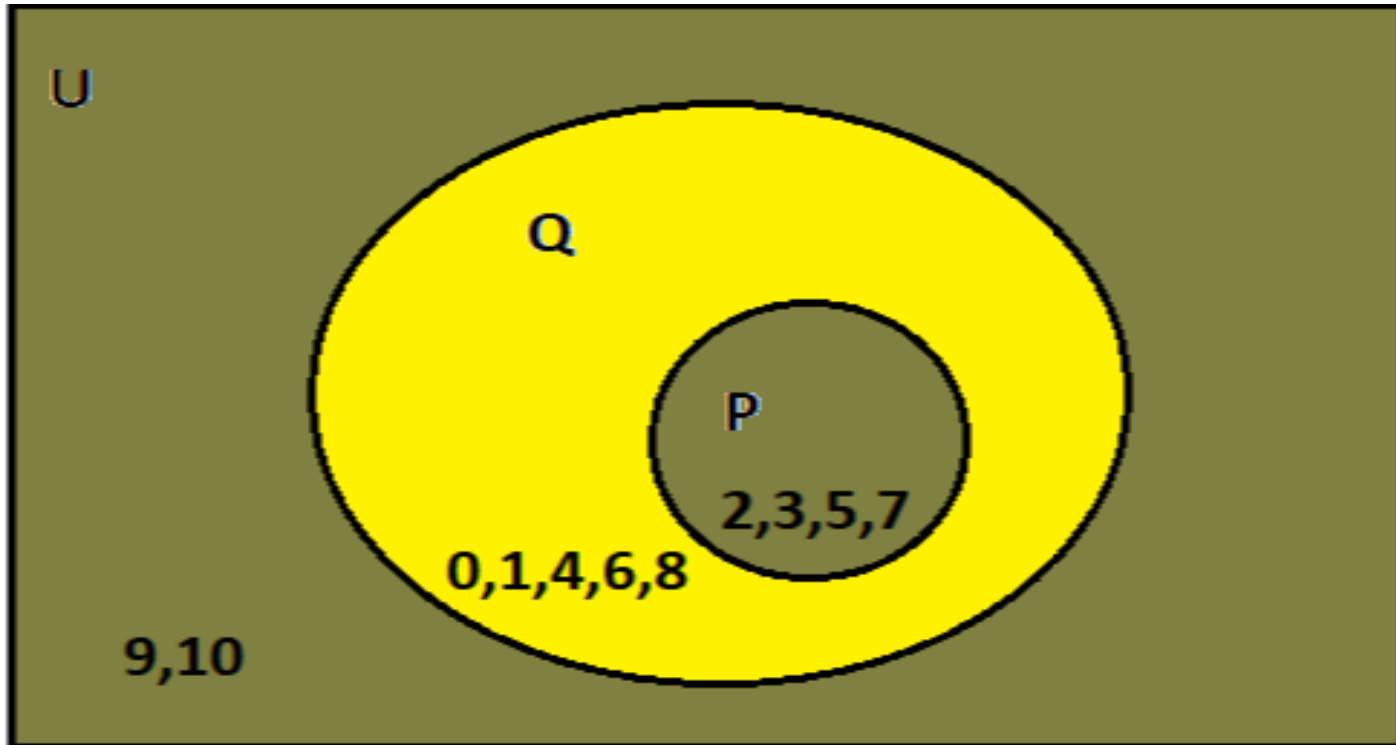
## SOLUTION

$$\begin{aligned} U &= \{ x \in \mathbb{Z}, 0 \leq x \leq 10 \} \\ &= \{0, 1, 2, 3, \dots, 10\} \end{aligned}$$

$$\begin{aligned} P &= \{x \in U \mid x \text{ is a prime number}\} \\ &= \{2, 3, 5, 7\} \end{aligned}$$

$$\begin{aligned} Q &= \{x \in U \mid x^2 < 70\} \\ &= \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \end{aligned}$$

# VENN DIAGRAM



The yellow shaded region is the desired result.

## ELEMENTS OF

$$(ii) P' \cap Q$$

$$P' = U - P$$

$$= \{0, 1, 2, 3, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{0, 1, 4, 6, 8, 9, 10\}$$

and

$$P' \cap Q$$

$$= \{0, 1, 4, 6, 8, 9, 10\} \cap \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

$$= \{0, 1, 4, 6, 8\}$$

## EXERCISE

Let  $U = \{1, 2, 3, 4, 5\}$        $C = \{1, 3\}$

Where  $A$  and  $B$  are **non empty** sets. Find  $A$  in each of the following:

(i)  $A \cup B = U$     $A \cap B = \emptyset$  and  $B = \{1\}$

## EXERCISE

(ii)  $A \subset B$  and  $A \cup B = \{4, 5\}$

(iii)  $A \cap B = \{3\}$   $A \cup B = \{2, 3, 4\}$   
and  $B \cup C = \{1, 2, 3\}$

(iv)  $A$  and  $B$  are disjoint,  $B$  and  $C$  are disjoint,  
and the union of  $A$  and  $B$  is the set  $\{1, 2\}$ .

## SOLUTION

$$(i) \quad A \cup B = U \quad A \cap B = \emptyset \quad \text{and} \quad B = \{1\}$$

**SOLUTION:**

$$\begin{aligned} \text{Since } A \cup B &= U \\ &= \{1, 2, 3, 4, 5\} \end{aligned}$$

$$\text{and } A \cap B = \emptyset$$

$$\begin{aligned} \text{Therefore} \quad A &= B' \\ &= \{1\}' \\ &= \{2, 3, 4, 5\} \end{aligned}$$

## SOLUTION

(ii)  $A \subset B$  and  $A \cup B = \{4, 5\}$  also  $C = \{1, 3\}$

**SOLUTION:**

When  $A \subset B$

$$\begin{aligned}\text{then } A \cup B &= B \\ &= \{4, 5\}\end{aligned}$$

Also  $A$  being a proper subset of  $B$  implies

$$A = \{4\} \qquad \text{or} \qquad A = \{5\}$$



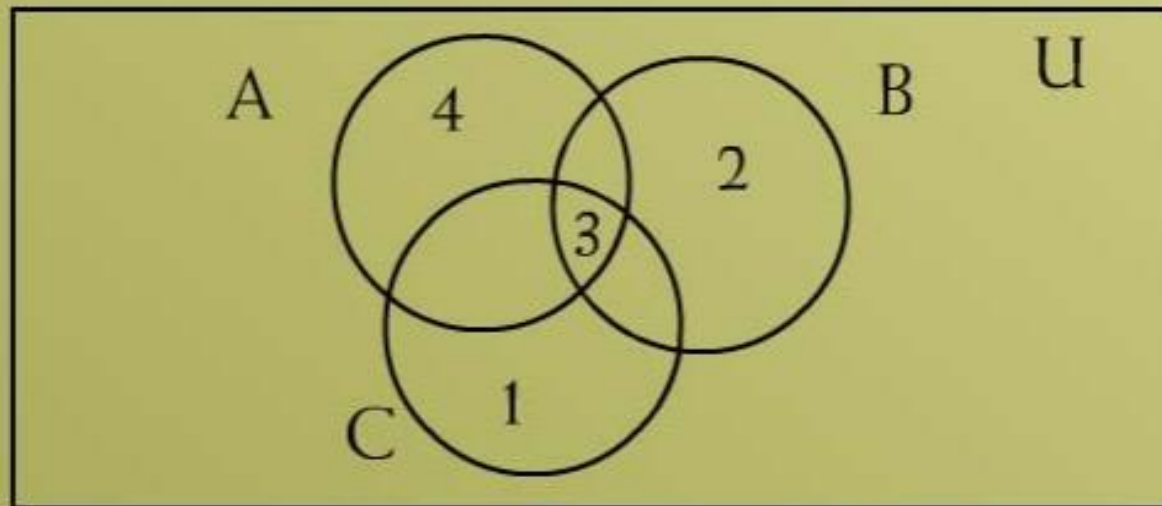
## Solution contd...

(iii)  $A \cap B = \{3\}$

$A \cup B = \{2, 3, 4\}$

and

$B \cup C = \{1, 2, 3\}$  Also  $C = \{1, 3\}$

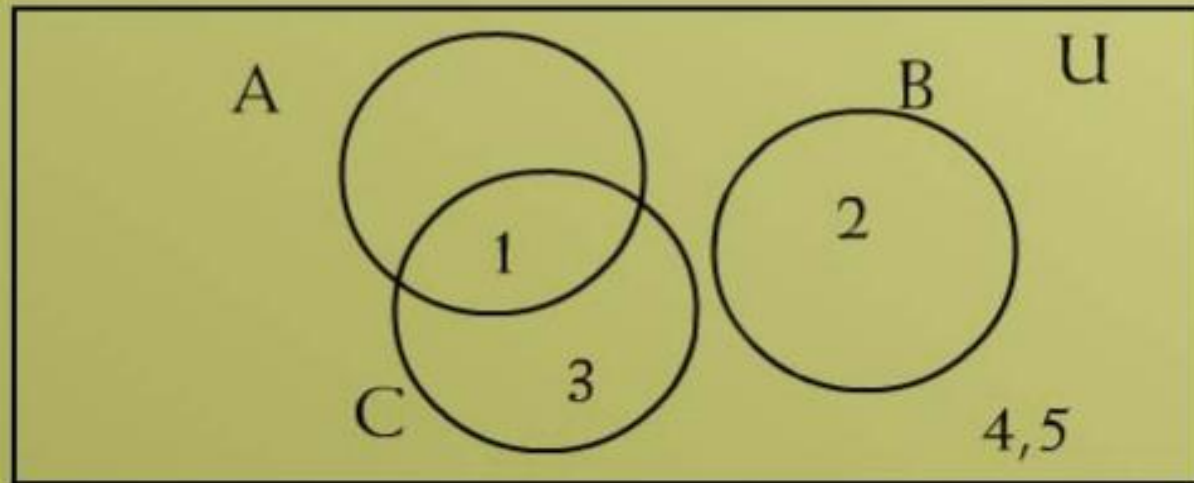


$A = \{3, 4\}$   $B = \{2, 3\}$

## Solution contd...

$$(iv) A \cap B = \emptyset \quad B \cap C = \emptyset$$

$$A \cup B = \{1, 2\} \quad \text{Also} \quad C = \{1, 3\}$$



$$A = \{1\}$$

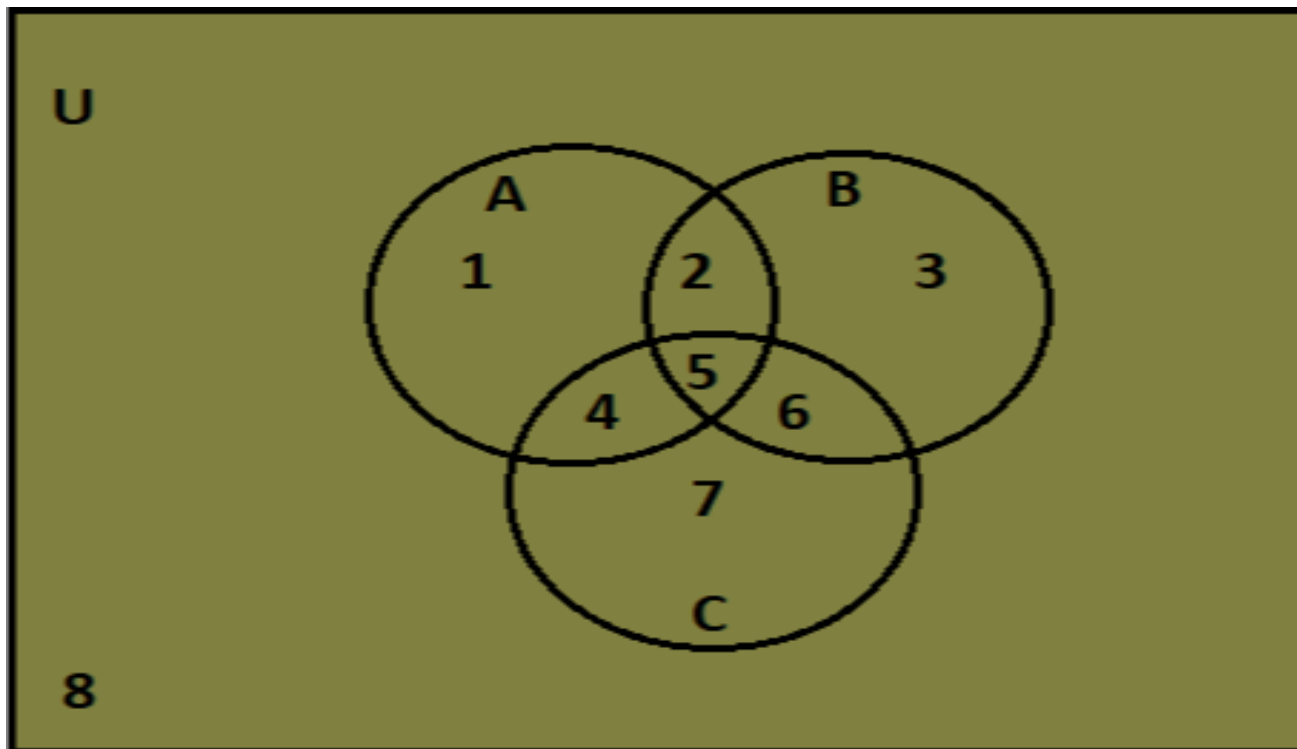
## EXERCISE

(i)  $(A \cap B) \cap C'$

(ii)  $A' \cup (B \cup C)$

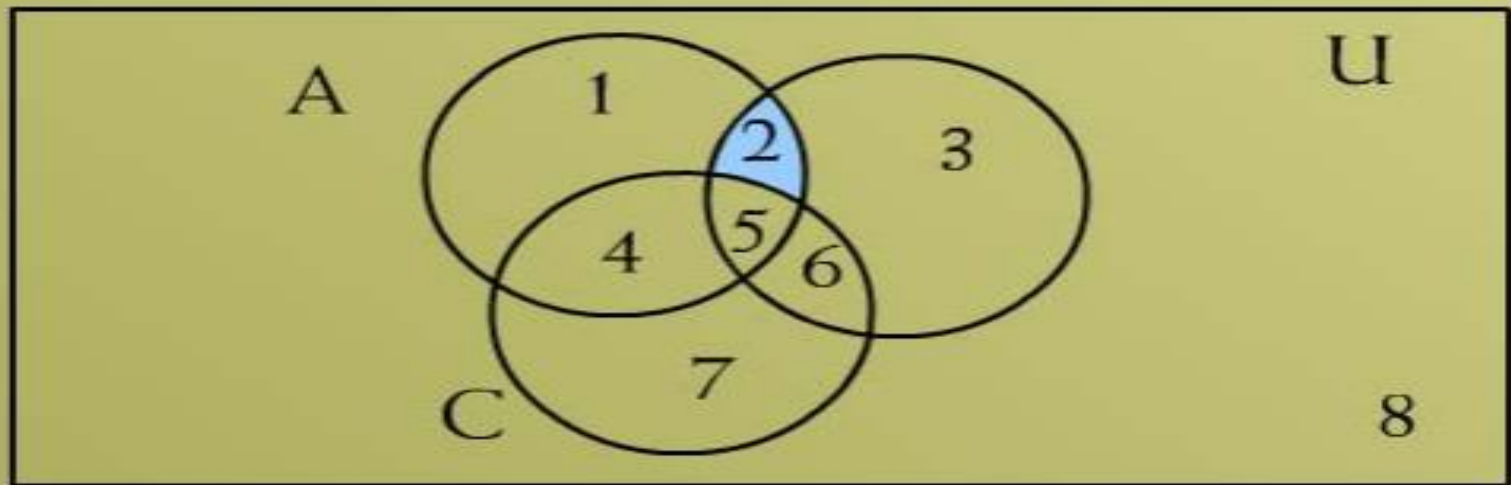
(iii)  $(A - B) \cap C$

(iv)  $(A \cap B') \cup C'$



## VENN DIAGRAM FOR

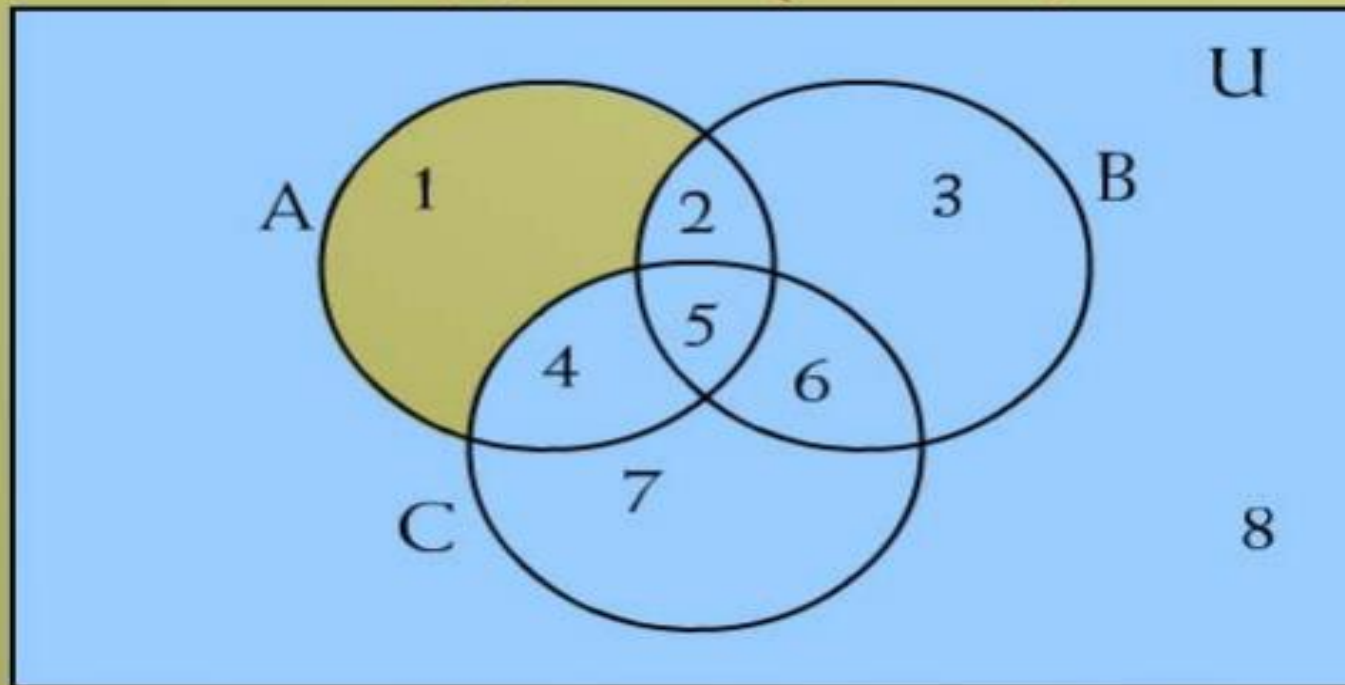
(i)  $(A \cap B) \cap C'$



$$(A \cap B) \cap C' = \{2\}$$

## VENN DIAGRAM FOR

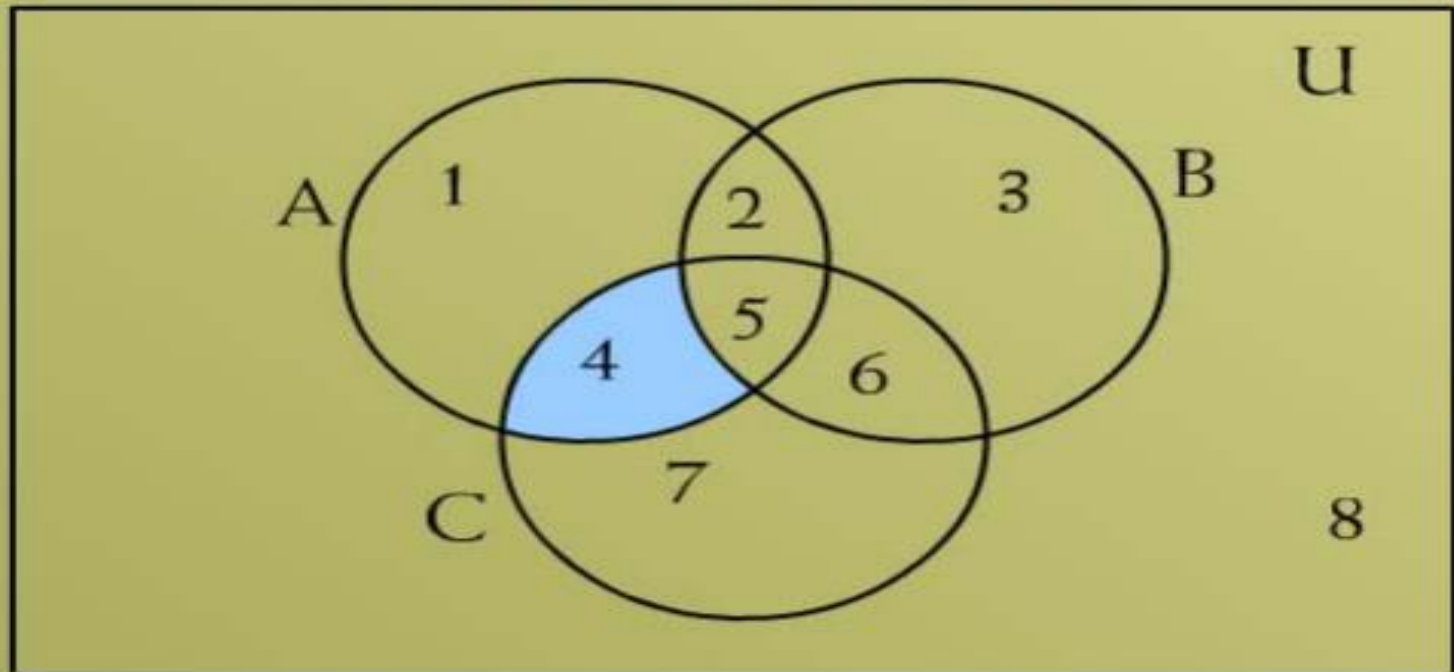
(ii)  $A' \cup (B \cup C)$



$$A' \cup (B \cup C) = \{2, 3, 4, 5, 6, 7, 8\}$$

## VENN DIAGRAM FOR

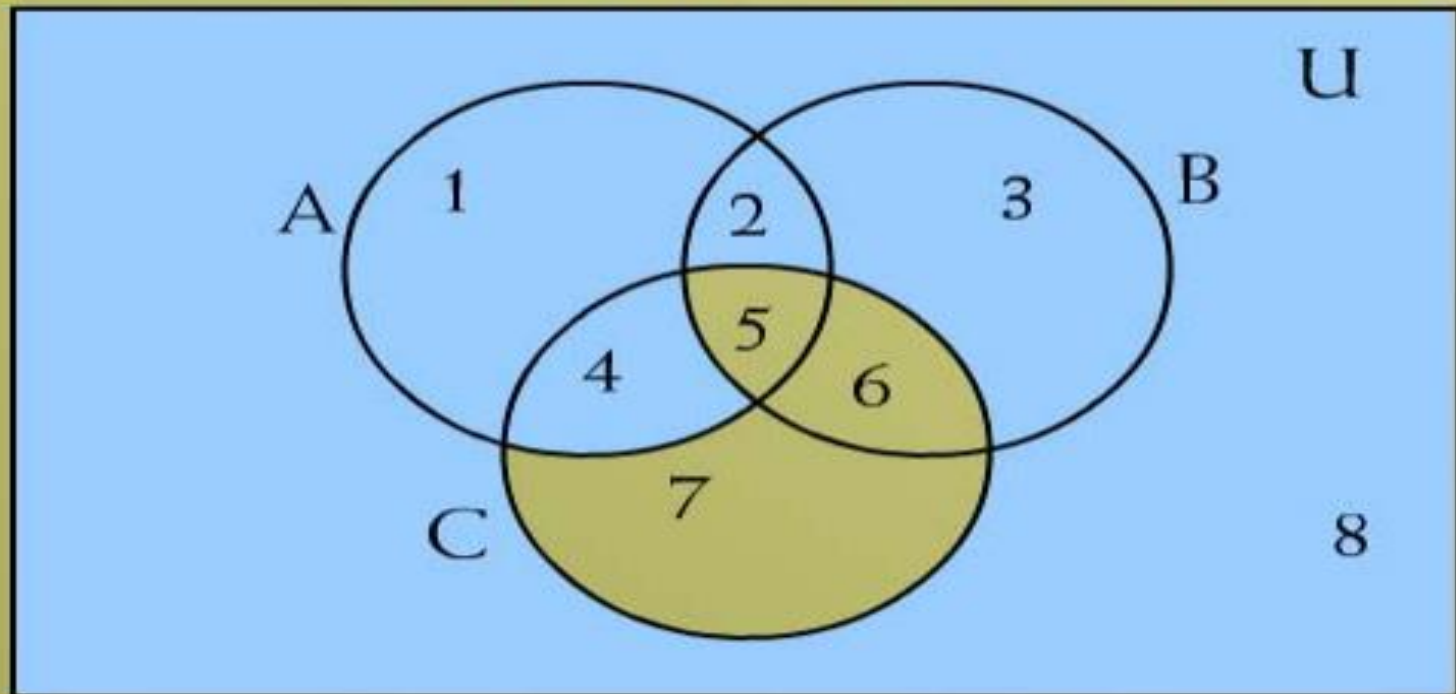
$$(iii) (A - B) \cap C$$



$$(A - B) \cap C = \{4\}$$

## VENN DIAGRAM FOR

$$(iv) (A \cap B') \cup C'$$



$$(A \cap B') \cup C' = \{1, 2, 3, 4, 8\}$$

## PROOFS USING VENN DIAGRAM

$$(i) \quad A - (A - B) = A \cap B$$

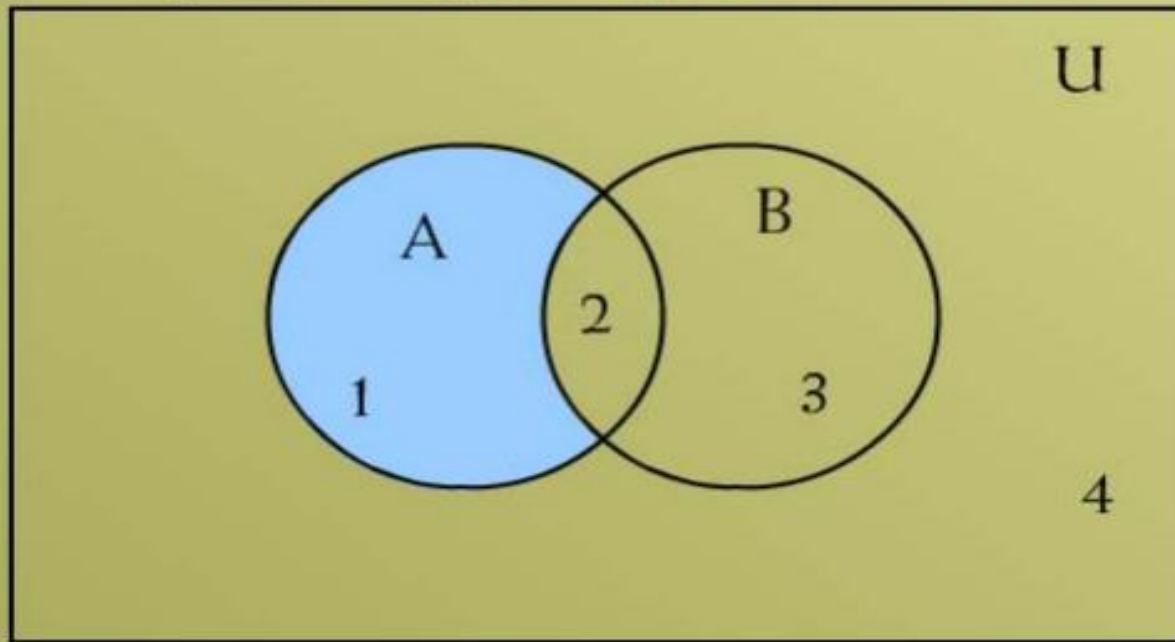
$$(ii) \quad (A \cap B)' = A' \cup B'$$

$$(iii) \quad A - B = A \cap B'$$



# SOLUTION

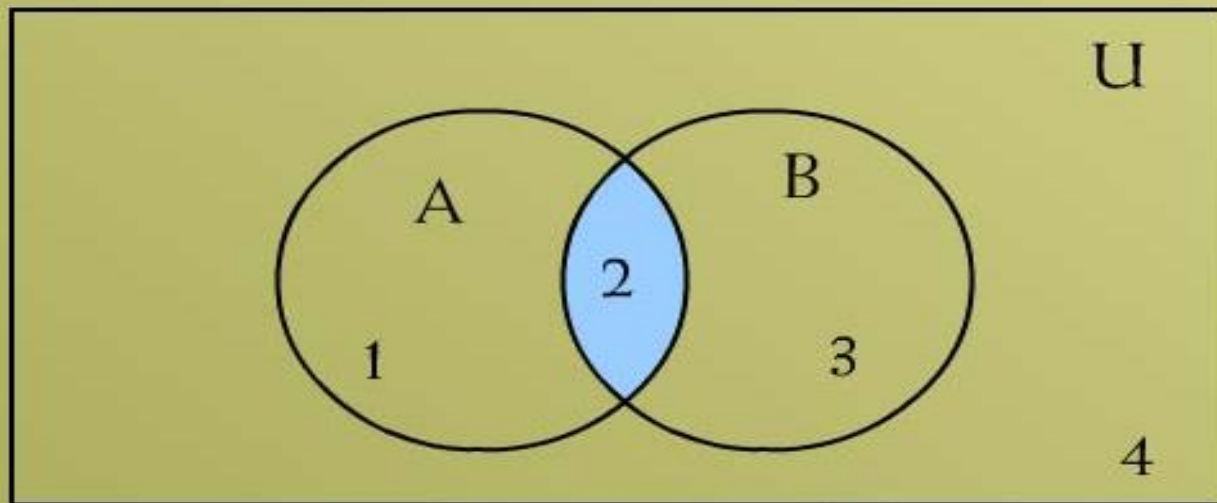
(i)  $A - (A - B) = A \cap B$



$A - B = \{1\}$

## SOLUTION

(i)  $A - (A - B) = A \cap B$

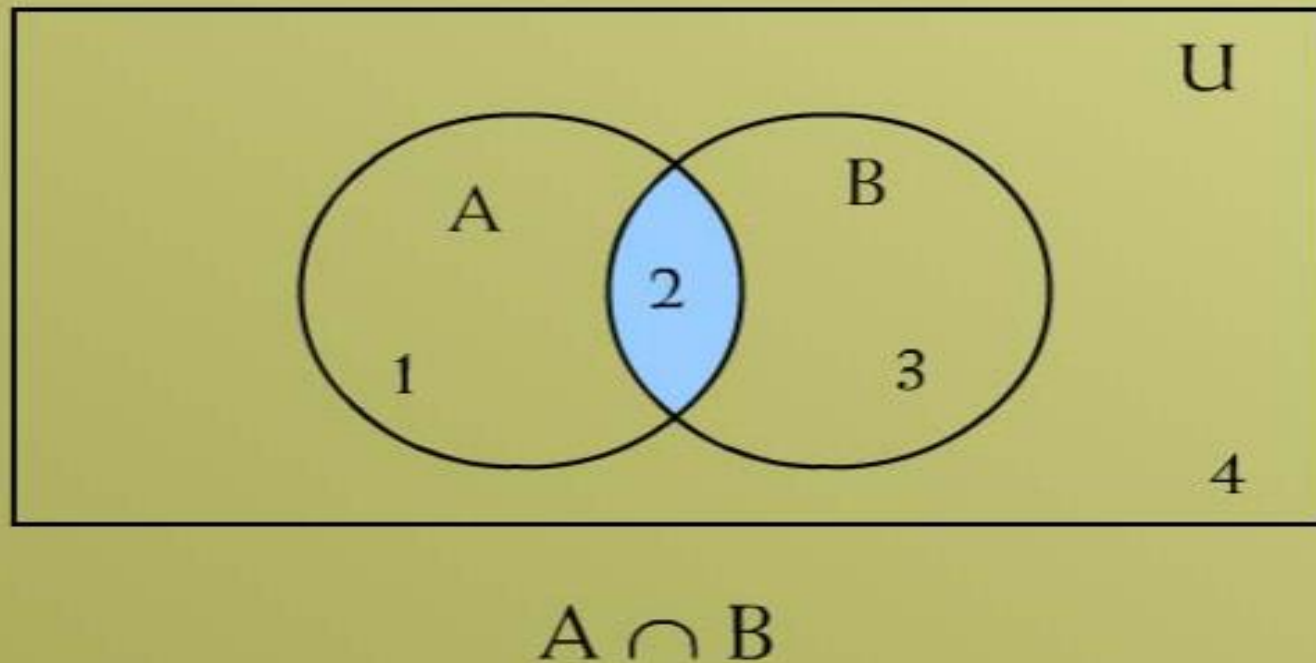


$$A - (A - B) = \{2\}$$

RESULT:  $A - (A - B) = A \cap B$

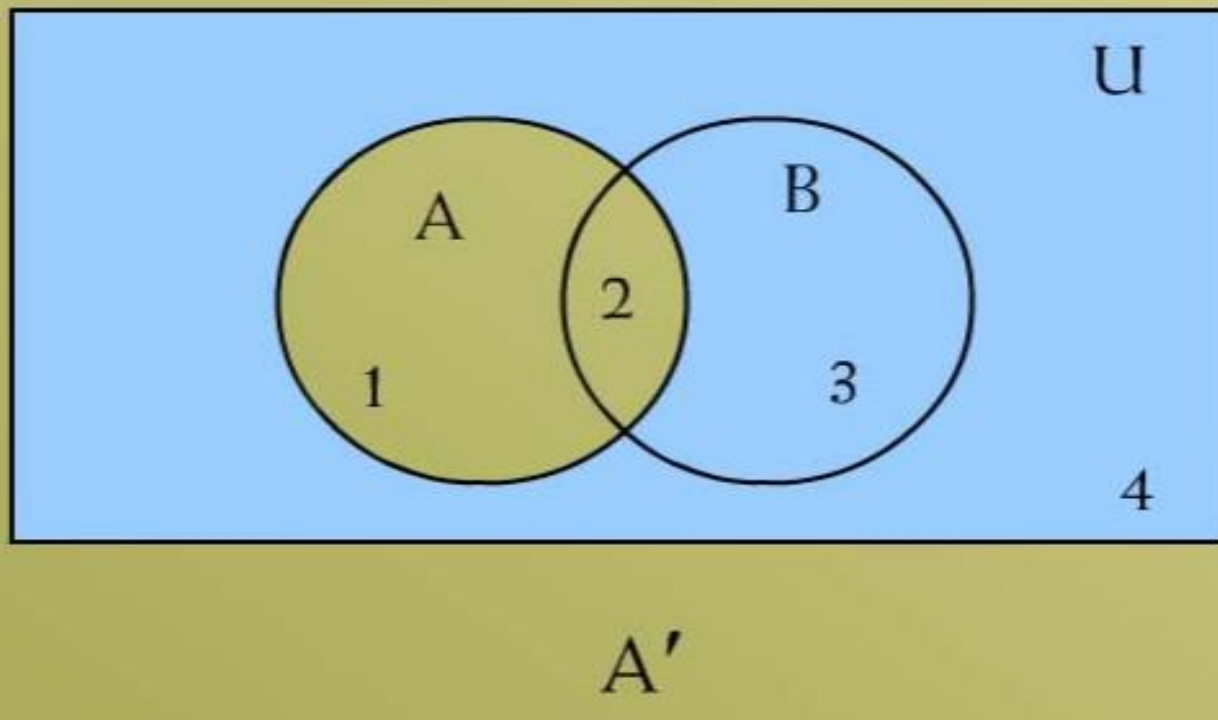
## SOLUTION

(ii)  $(A \cap B)' = A' \cup B'$



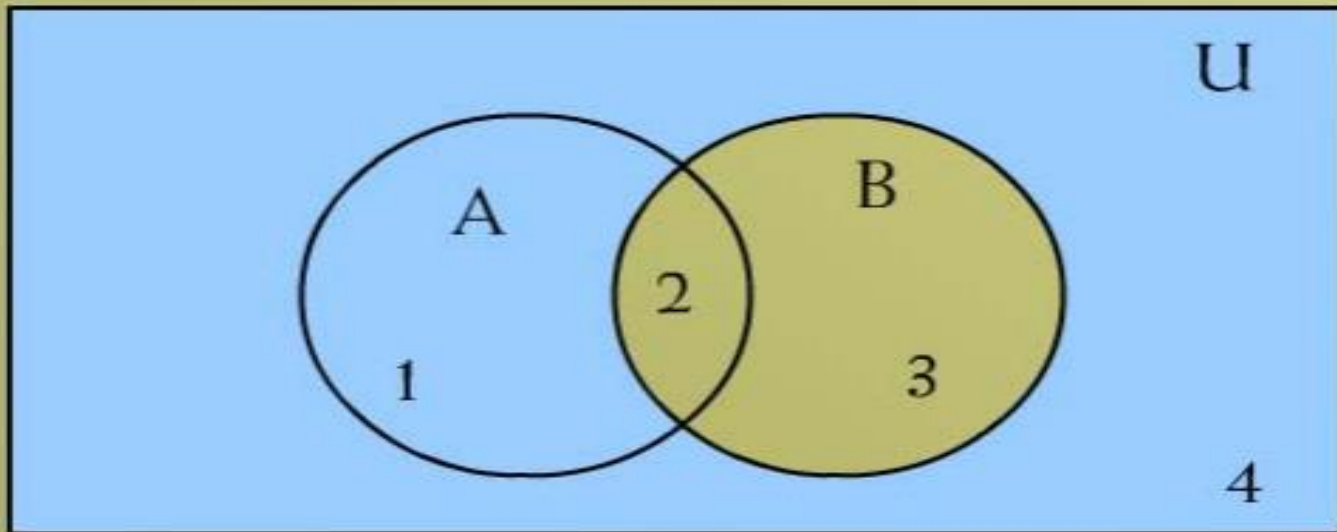
## SOLUTION

$$(ii) \quad (A \cap B)' = A' \cup B'$$



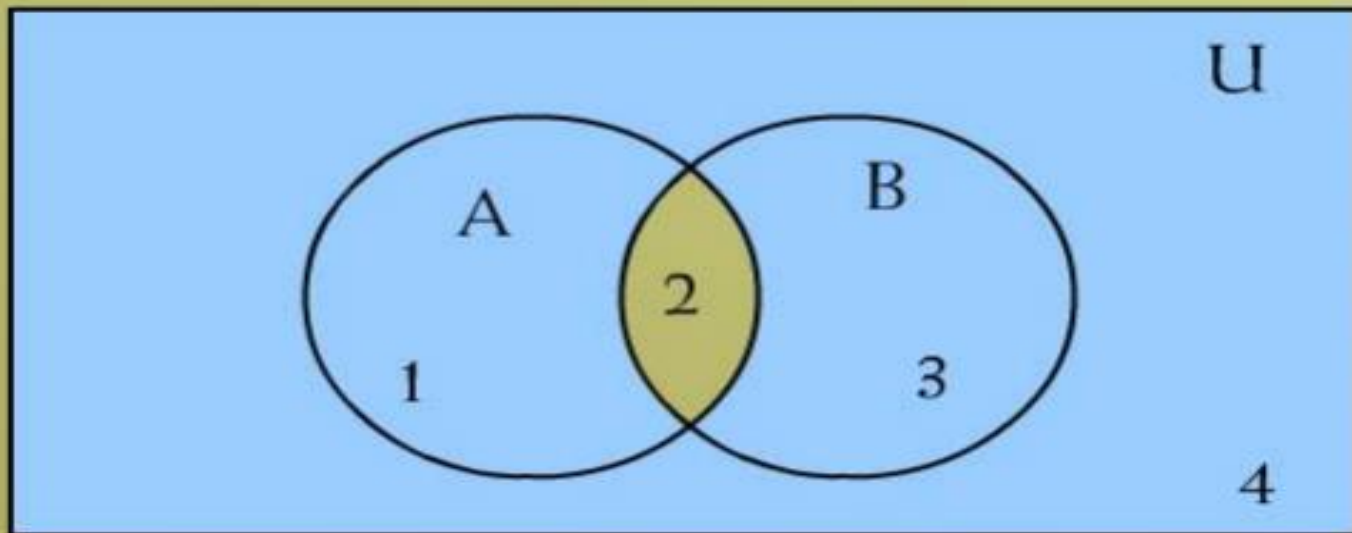
## SOLUTION

(ii)  $(A \cap B)' = A' \cup B'$



## SOLUTION

$$(ii) \quad (A \cap B)' = A' \cup B'$$



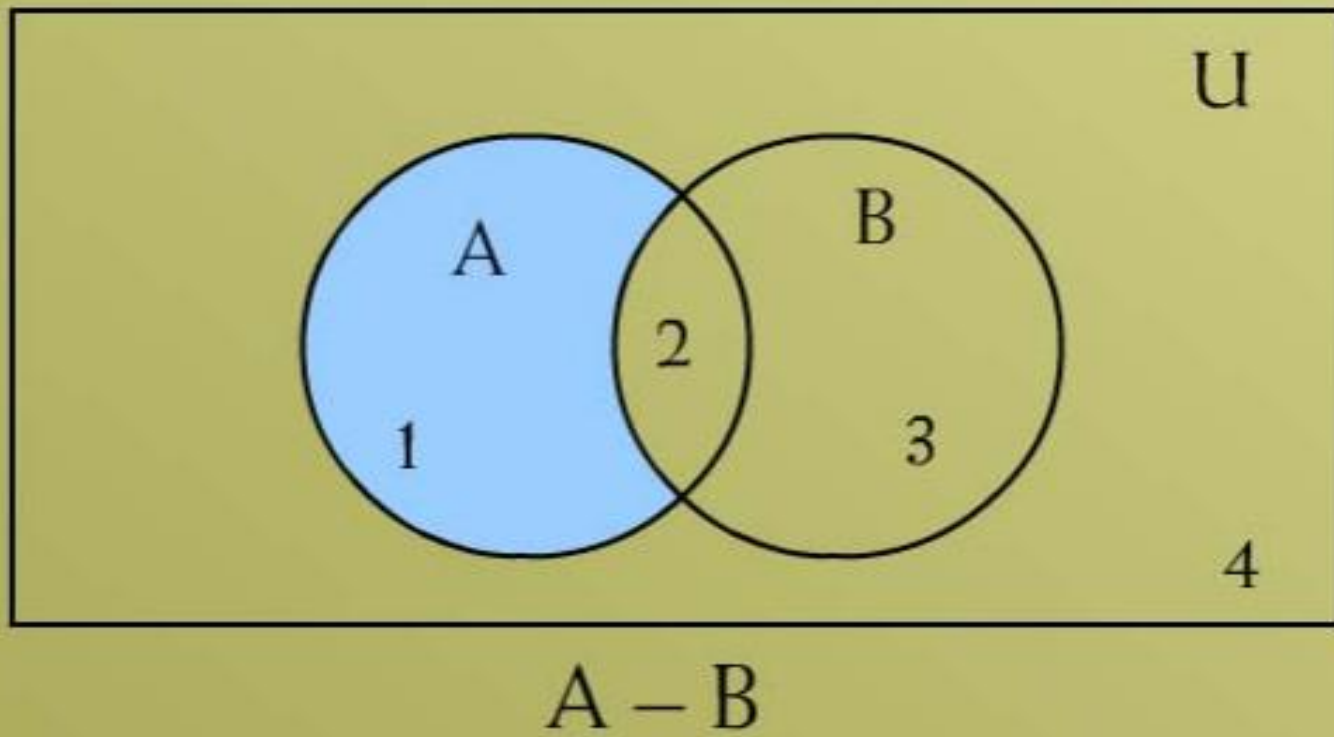
$$A' \cup B'$$

RESULT:

$$(A \cap B)' = A' \cup B'$$

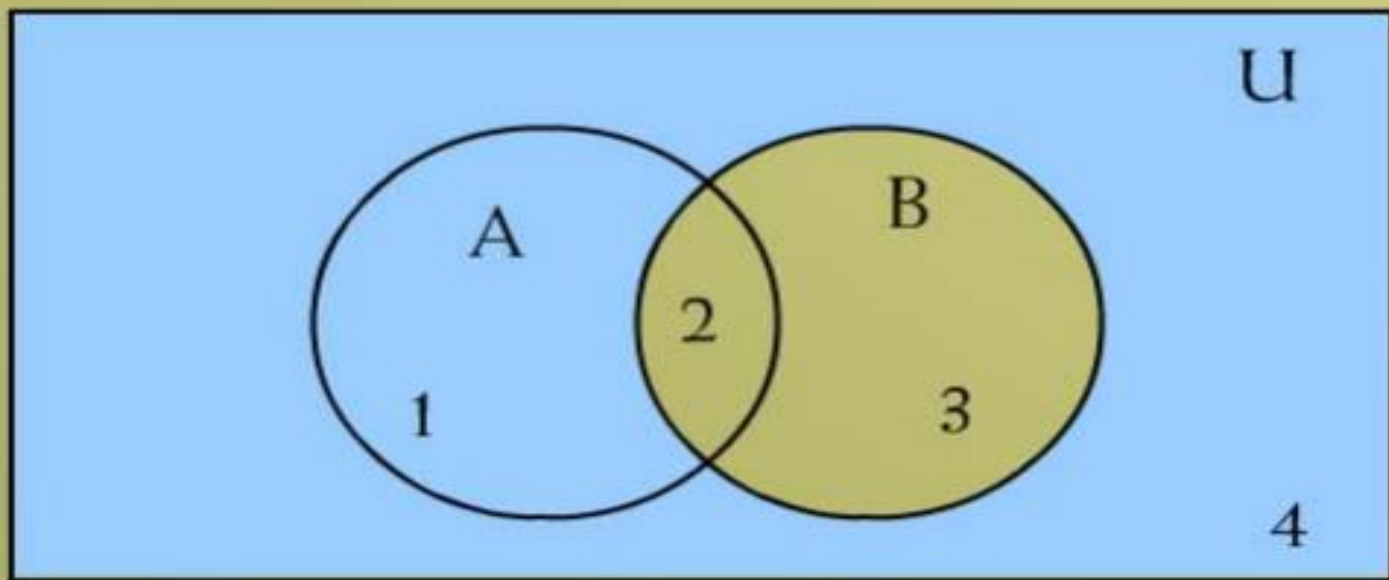
## SOLUTION

(iii)  $A - B = A \cap B'$



## SOLUTION

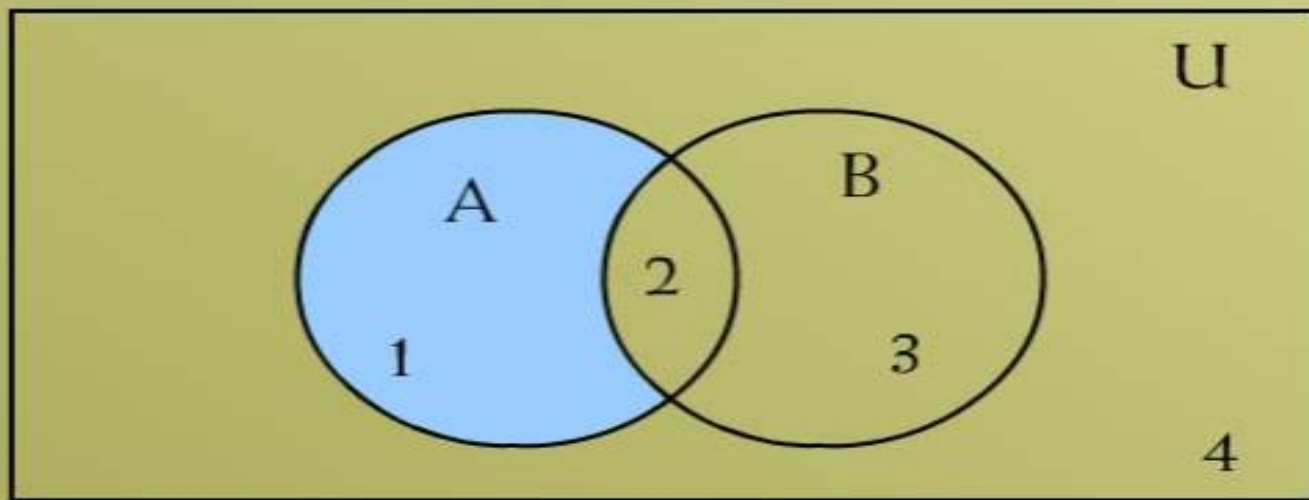
$$(iii) A - B = A \cap B'$$





## SOLUTION

(iii)  $A - B = A \cap B'$



$A \cap B'$

RESULT:  $A - B = A \cap B'$

# SET IDENTITIES

Let  $A, B, C$  be subsets of a universal set  $U$ .

## 1. Idempotent Laws

a.  $A \cup A = A$

b.  $A \cap A = A$

## 2. Commutative Laws

a.  $A \cup B = B \cup A$

b.  $A \cap B = B \cap A$

## 3. Associative Laws

a.  $A \cup (B \cup C) = (A \cup B) \cup C$

b.  $A \cap (B \cap C) = (A \cap B) \cap C$

# SET IDENTITIES

## 4. Distributive Laws

a.  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b.  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

## 5. Identity Laws

a.  $A \cup \emptyset = A$

b.  $A \cap U = A$

## 6. Complement Laws

a.  $A \cup A' = U$

b.  $A \cap A' = \emptyset$

c.  $U' = \emptyset$

d.  $\emptyset' = U$

## Domination Laws

a.  $A \cup U = U$

b.  $A \cap \emptyset = \emptyset$

## SET IDENTITIES

### 7. Double Complement Law

a.  $(A')' = A$

### 8. DeMorgan's Laws

a.  $(A \cup B)' = A' \cap B'$

b.  $(A \cap B)' = A' \cup B'$

### 9. Alternative Representation for Set Difference

a.  $A - B = A \cap B'$

# SET IDENTITIES

## 10. Subset Laws

a.  $A \cup B \subseteq C$  iff  $A \subseteq C$  and  $B \subseteq C$

b.  $C \subseteq A \cap B$  iff  $C \subseteq A$  and  $C \subseteq B$

## 11. Absorption Laws

a.  $A \cup (A \cap B) = A$

b.  $A \cap (A \cup B) = A$

## EXERCISE

Let  $A$ ,  $B$  and  $C$  be subsets of a universal set  $U$ .  
Show that

$$1. \quad A \subseteq A \cup B$$

$$2. \quad A - B \subseteq A$$

$$3. \quad \text{If } A \subseteq B \text{ and } B \subseteq C \text{ then } A \subseteq C$$

# SOLUTION

$$1. A \subseteq A \cup B$$

SOLUTION:

Let  $x$  be an arbitrary element of  $A$ , that is  $x \in A$ .

$$\Rightarrow x \in A \text{ or } x \in B$$

$$\Rightarrow x \in A \cup B$$

But  $x$  is an arbitrary element of  $A$ .

$$\therefore A \subseteq A \cup B$$

## Solution contd...

$$2. \quad A - B \subseteq A$$

SOLUTION:

Let  $x \in A - B$

$\Rightarrow x \in A$  and  $x \notin B$  (by definition of  $A - B$ )

$\Rightarrow x \in A$  (in particular)

But  $x$  is an arbitrary element of  $A - B$

$$\therefore A - B \subseteq A$$



## Solution contd...

3. If  $A \subseteq B$  and  $B \subseteq C$  then  $A \subseteq C$

SOLUTION:

Suppose that  $A \subseteq B$  and  $B \subseteq C$

Consider  $x \in A$

$\Rightarrow x \in B$  (because  $A \subseteq B$ )

$\Rightarrow x \in C$  (because  $B \subseteq C$ )

But  $x$  is an arbitrary element of  $A$

$\therefore A \subseteq C$

## EQUALITY OF SETS

Two sets are equal if first is subset of second and second is subset of first.

Now

$$A - B = A \cap B'$$

$$\Leftrightarrow A - B \subseteq A \cap B' \text{ and } A \cap B' \subseteq A - B$$

## EXERCISE

Let  $A$  and  $B$  be subsets of a universal set  $U$ .  
Prove that

$$A - B = A \cap B'$$

SOLUTION

Let

$$x \in A - B$$

$$\Rightarrow x \in A \text{ and } x \notin B$$

(definition of set difference)

## Solution contd...

$$\Rightarrow x \in A \text{ and } x \in B'$$

(definition of complement)

$$\Rightarrow x \in A \cap B'$$

(definition of intersection)

But  $x$  is an arbitrary element of  $A-B$

$$\therefore A-B \subseteq A \cap B' \dots\dots\dots (1)$$

## Solution contd...

Conversely,

Let  $y \in A \cap B'$

$\Rightarrow y \in A$  and  $y \in B'$

(definition of intersection)

$\Rightarrow y \in A$  and  $y \notin B$

(definition of complement)

## Solution contd...

$$\Rightarrow y \in A - B$$

(definition of set difference)

But  $y$  is an arbitrary element of  $A \cap B'$

$$\therefore A \cap B' \subseteq A - B \dots \dots \dots (2)$$

From (1) and (2) it follows that

$$A - B = A \cap B' \quad \text{(as required)}$$

## DEMORGAN'S LAW

$$(A \cup B)' = A' \cap B'$$

PROOF

First we show that  $(A \cup B)' \subseteq A' \cap B'$

Let  $x \in (A \cup B)'$

$$\Rightarrow x \notin A \cup B$$

(definition of complement)

$$\Rightarrow x \notin A \text{ and } x \notin B$$

(DeMorgan's Law of Logic)

## Proof contd...

$\Rightarrow x \in A' \text{ and } x \in B'$   
(definition of complement)

$\Rightarrow x \in A' \cap B'$   
(definition of intersection)

But  $x$  is an arbitrary element of  $(A \cup B)'$   
 $\therefore (A \cup B)' \subseteq A' \cap B' \dots\dots\dots (1)$



## Proof contd...

Conversely,

Let  $y \in A' \cap B'$

$\Rightarrow y \in A'$  and  $y \in B'$

(definition of intersection)

$\Rightarrow y \notin A$  and  $y \notin B$

(definition of complement)

$\Rightarrow y \notin A \cup B$

(DeMorgan's Law of Logic)

## Proof contd...

$$\Rightarrow y \in (A \cup B)'$$

(definition of complement)

But  $y$  is an arbitrary element of  $A' \cap B'$

$$\therefore A' \cap B' \subseteq (A \cup B)' \dots\dots\dots (2)$$

From (1) and (2) we have

$$(A \cup B)' = A' \cap B' \quad \text{(proved)}$$

## ASSOCIATIVE LAW

$$A \cap (B \cap C) = (A \cap B) \cap C$$

PROOF

First we show that  $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

Consider  $x \in A \cap (B \cap C)$

$$\Rightarrow x \in A \text{ and } x \in B \cap C$$

(definition of intersection)

$$\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in C$$

(definition of intersection)

## Proof contd...

$$\Rightarrow x \in A \cap B \text{ and } x \in C$$

(definition of intersection)

$$\Rightarrow x \in (A \cap B) \cap C$$

(definition of intersection)

But  $x$  is an arbitrary element of  $A \cap (B \cap C)$

$$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C \dots\dots\dots (1)$$

## Proof contd...

Conversely,

Let  $y \in A \cap (B \cap C)$

$\Rightarrow y \in A \cap B$  and  $y \in C$   
(definition of intersection)

$\Rightarrow y \in A$  and  $y \in B$  and  $y \in C$   
(definition of intersection)

$\Rightarrow y \in A$  and  $y \in B \cap C$   
(definition of intersection)

## Proof contd...

$$\Rightarrow y \in A \cap (B \cap C)$$

(definition of intersection)

But  $y$  is an arbitrary element of  $(A \cap B) \cap C$

$$\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C) \dots\dots(2)$$

From (1) and (2) we conclude that

$$A \cap (B \cap C) = (A \cap B) \cap C \quad (\text{proved})$$

## EXERCISE

For all subset A and B of a universal set U, prove that

$$(A - B) \cup (A \cap B) = A$$

PROOF:

$$\text{L.H.S} = (A - B) \cup (A \cap B)$$

$$= (A \cap B') \cup (A \cap B)$$

(Alternative representation  
for set difference)



## Proof contd...

$$= A \cap (B' \cup B)$$

Distributive Law

$$= A \cap U$$

Complement Law

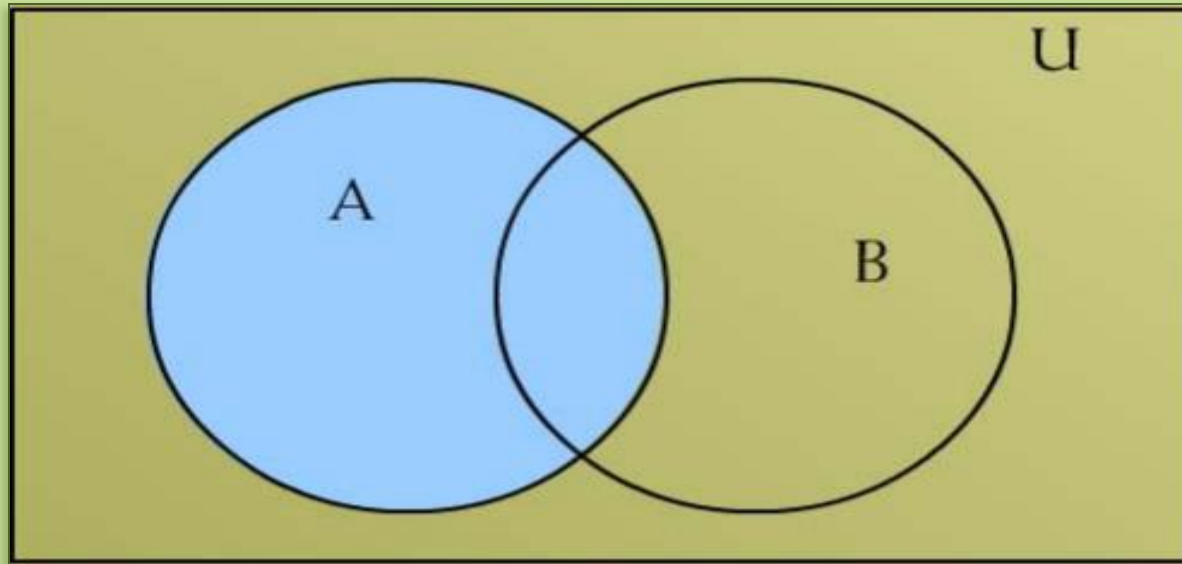
$$= A$$

Identity Law

$$= \text{R.H.S}$$



# VENN DIAGRAM



$$(A - B) \cup (A \cap B) = A$$

## EXERCISE

$$A - (A - B) = A \cap B$$

SOLUTION

$$\text{L.H.S} = A - (A - B)$$

$$= A - (A \cap B')$$

Alternative representation for  
set difference

$$= A \cap (A \cap B')'$$

Alternative representation for  
set difference

## Solution contd...

$$\text{L.H.S} = A \cap (A' \cup (B')')$$

DeMorgan's Law

$$= A \cap (A' \cup B)$$

Double Complement Law

$$= (A \cap A') \cup (A \cap B)$$

Distributive Law

$$= \emptyset \cup (A \cap B)$$

Complement Law

$$= A \cap B$$

Identity Law

$$= \text{R.H.S}$$

## EXERCISE

$$(A - B) - C = (A - C) - B$$

SOLUTION

$$\text{L.H.S} = (A - B) - C$$

$$= (A \cap B') - C$$

Alternative representation of  
set difference

$$= (A \cap B') \cap C'$$

Alternative representation of  
set difference

$$= A \cap (B' \cap C') \quad \text{Associative Law}$$

## Solution contd...

$$\text{LHS} = A \cap (C' \cap B') \quad \text{Commutative Law}$$

$$= (A \cap C') \cap B' \quad \text{Associative Law}$$

$$= (A - C) \cap B'$$

Alternative representation  
of set difference

$$= (A - C) - B$$

Alternative representation  
of set difference

$$= \text{RHS}$$

(proved)

## PROVING SET IDENTITIES BY MEMBERSHIP TABLE

1.  $A - (A - B) = A \cap B$

2.  $(A \cap B)' = A' \cup B'$

3.  $A - B = A \cap B'$

## SOLUTION

$$A - (A - B) = A \cap B$$

A	B	$A - B$	$A - (A - B)$	$A \cap B$
1	1	0	1	1
1	0	1	0	0
0	1	0	0	0
0	0	0	0	0

## SOLUTION

$$A - (A - B) = A \cap B$$

A	B	$A - B$	$A - (A - B)$	$A \cap B$
1	1	0		
1	0	1		
0	1	0		
0	0	0		



## SOLUTION

$$A - (A - B) = A \cap B$$

A	B	$A - B$	$A - (A - B)$	$A \cap B$
1	1	0	1	
1	0	1	0	
0	1	0	0	
0	0	0	0	

## SOLUTION

$$A - (A - B) = A \cap B$$

A	B	$A - B$	$A - (A - B)$	$A \cap B$
1	1			1
1	0			0
0	1			0
0	0			0

## SOLUTION

$$(A \cap B)' = A' \cup B'$$

A	B	$A \cap B$	$(A \cap B)'$	$A'$	$B'$	$A' \cup B'$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

## SOLUTION

$$A - B = A \cap B'$$

A	B	$A - B$	$B'$	$A \cap B'$
1	1	0	0	0
1	0	1	1	1
0	1	0	0	0
0	0	0	1	0