Discrete Structures

Lecture # 2

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The Foundations: Logic

- **Mathematical Logic** is a tool for working with compound statements
- **Logic** is the study of correct reasoning
- **Use of logic**
  - **In mathematics:** to prove theorems
  - **In computer science:** to prove that programs do what they are supposed to do

Propositional Logic

- Propositional logic: It deals with **propositions**.
- Predicate logic: It deals with **predicates**.

Definition of a Proposition

<u>Definition</u>: A **proposition** (usually denoted by p, q, r, …) is a declarative statement that is either **True** (T) or **False** (F), but not both or somewhere “in between!”.

Propositional Variables

- Variables that represent propositions
- Conventional letters are : p, q, r, s, . . .
- Truth values: T(true), F(false)

Note: Commands and questions are not propositions.

Examples of Propositions

The following are **all** propositions:

- “It is raining” (In a given situation)
- “Amman is the capital of Jordan”
- “1 + 2 = 3” or "2+2 = 3”
- Two plus two is equal to four.
- Toronto is the capital of Canada.
- etc.

Examples of Propositions

But, the following are **NOT** propositions:

- "Who's there?" (Question)
- "La la la la la." (Meaningless)
- "Just do it!" (Command)
- "1 + 2" (Expression with a non-true/false value)
- "$1 + 2 = x$" (Expression with unknown value of x)

Operators / Connectives

An **operator** or **connective** combines one or more **operand** expressions into a larger expression. (e.g., “+” in numeric expression.)

- **Unary** operators take 1 operand (e.g. −3);
- **Binary** operators take 2 operands (e.g. 3 × 4).
- **Propositional** or **Boolean** operators operate on propositions (or their truth values) instead of on numbers.

Compound Statement (Propositions)

- Complicated logical statements build out of simple ones
- Three Symbols
  - ~ (not) --- ~ p (not p)
  - Λ (and) --- pΛq (p and q)
  - V (or) --- pVq (p or q)
- ~ p (Negation), pΛq (Conjunction), pVq (Disjunctions)
- English words to logic
  - "p but q" means "p and q"
  - "neither p nor q" means "~ p and ~ q"

Some Popular Boolean Operators

Formal Name	Nickname	Arity	Symbol
Negation operator	NOT	Unary	$\neg$
Conjunction operator	AND	Binary	$\wedge$
Disjunction operator	OR	Binary	$\vee$
Exclusive-OR operator	XOR	Binary	$\oplus$
Implication operator	IMPLIES	Binary	$\rightarrow$
Biconditional operator	IFF	Binary	$\leftrightarrow$

The Negation Operator

<u>**Definition**:</u> Let p be a proposition then $\neg p$ is the **negation** of p (Not p , it is not the case that p).

e.g. If p = "London is a city"

then $\neg p$ = "London is **not** a city" or " it is not the case that London is a city"

The **truth table** for NOT:

p	$\neg p$
F	T
T	F

Operand column

Result column

T :≡ True; F :≡ False
":≡" means "is defined as".

Examples

1. Let p = “Ahmad’s PC runs Linux”
    - ~ p ?
2. Let H = “It is hot”

   S= “It is Sunny”

   (i). “It is not hot but it is Sunny”

   “_____________________”

   (ii). “It is neither hot nor Sunny”

   “_____________________”

The Conjunction Operator

<u>Definition</u>: Let p and q be propositions, the proposition "p **AND** q" denoted by $(p \wedge q)$ is called the **conjunction** of p and q.

The **conjunction of the statements P** and **Q** is the statement "**P and Q**" and its denoted by **P ∧ Q**. The statement **P ∧ Q** is true only when both P **and** Q are true.

e.g. If p = "I will have salad for lunch" and

q = "I will have steak for dinner", then

$p \wedge q$ = "I will have salad for lunch **and** I will have steak for dinner"

Remember: "$\wedge$" points up like an "A", and it means "AND"

Conjunction Truth Table

- Note that a conjunction $p_1 \wedge p_2 \wedge \ldots \wedge p_n$ of n propositions will have 2^n rows in its truth table.

"And", "But", "In addition to", "Moreover". Ex: The sun is shining but it is raining

Operand columns

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

The Disjunction Operator

<u>Definition</u>: Let p and q be propositions, the proposition "p **OR** q" denoted by $(p \vee q)$ is called the **disjunction** of p and q.

The disjunction of the statements **P** and **Q** is the statement "**P or Q**" and its denoted by **P ∨ Q**. The statement **P ∨ Q** is true only when at least one of P or Q is true.

e.g. p = "My car has a bad engine"

q = "My car has a bad carburetor"

$p \vee q$ = "Either my car has a bad engine **or** my car has a bad carburetor"

Disjunction Truth Table

- Note that $p \vee q$ means that p is true, or q is true, **or both** are true!
- So, this operation is also called **inclusive or**, because it **includes** the possibility that both p and q are true.

p	q	$p \vee q$
F	F	F
F	T	**T**
T	F	**T**
T	T	T

Note the differences from AND

Takeaway

- Rather memorizing, it is easier to remember the rules summarized.

Operator	Symbolic	Summary of Truth Values
Conjunction	$P \wedge Q$	True only when both P and Q are true
Disjunction	$P \vee Q$	False only when both P and Q are false
Negation	($\sim$ or $\neg$) $\sim P$	Opposite truth value of P

Compound Statements

- Let p, q, r be simple statements. We can form other compound statements, such as
  - $(p \vee q) \wedge r$
  - $p \vee (q \wedge r)$
  - $\neg p \vee \neg q$
  - $(p \vee q) \wedge (\neg r \vee s)$
  - and many others…

Truth Table – Example

- Lets try to build table for

1. **$\sim P \wedge Q$**
2. **$\sim P \wedge (Q \vee \sim P)$**
3. **$(P \vee Q) \wedge \sim(P \wedge Q)$**
4. **$\sim(\sim P)$**

P	**Q**	**$\sim P$**	**$\sim P \wedge Q$**
T	T		
T	F		
F	T		
F	F		

Truth Table – Example – Cont..

- Lets try to build table for

1. ~P ∧ Q

P	Q	~P	~P ∧ Q
T		F	
T		F	
F		T	
F		T	

Truth Table – Example – Cont..

- Lets try to build table for

1. ~P ∧ Q

P	Q	~P	~P ∧ Q
	T	F	F
	F	F	F
	T	T	T
	F	T	F

Truth Table – Example – Cont..

- Lets try to build table for

1. ~P ∧ Q

P	Q	~P	~P ∧ Q
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

Truth Table – Example – $(p \lor q) \land r$

p	q	r	$p \lor q$	$(p \lor q) \land r$
F	F	F		
F	F	T		
F	T	F		
F	T	T		
T	F	F		
T	F	T		
T	T	F		
T	T	T		

Truth Table – Example – Cont.. $(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
F	F	F	F	F
F	F	T	F	F
F	T	F	T	F
F	T	T	T	T
T	F	F	T	F
T	F	T	T	T
T	T	F	T	F
T	T	T	T	T

Truth Table – Example

1. $\sim P \wedge (Q \vee \sim R)$

P	Q	R	~R	Q ∨ ~R	~P	~P ∧ (Q ∨~R)
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Truth Table – Example – Cont..

1. $\sim P \wedge (Q \vee \sim R)$

P	Q	R	~R	Q ∨ ~R	~P	~P ∧ (Q ∨~R)
		T	F			
		F	T			
		T	F			
		F	T			
		T	F			
		F	T			
		T	F			
		F	T			

Truth Table – Example – Cont..

1. ~P ∧ (Q ∨~R)

P	Q	R	~R	Q ∨ ~R	~P	~P ∧ (Q ∨~R)
	T		F	T		
	T		T	T		
	F		F	F		
	F		T	T		
	T		F	T		
	T		T	T		
	F		F	F		
	F		T	T		

Truth Table – Example – Cont..

1. ~P ∧ (Q ∨~R)

P	Q	R	~R	Q ∨ ~R	~P	~P ∧ (Q ∨~R)
T					F	
T					F	
T					F	
T					F	
F					T	
F					T	
F					T	
F					T	

Truth Table – Example – Cont..

1. ~P ∧ (Q ∨~R)

P	Q	R	~R	Q ∨ ~R	~P	~P ∧ (Q ∨~R)
				T	F	F
				T	F	F
				F	F	F
				T	F	F
				T	T	T
				T	T	T
				F	T	F
				T	T	T

Truth Table – Example – Cont..

1. $\sim P \wedge (Q \vee \sim R)$

P	Q	R	~R	Q ∨ ~R	~P	~P ∧ (Q ∨~R)
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	T	T	T

Truth Table – Example – Cont..

1. **$(P \vee Q) \wedge \sim(P \wedge Q)$**

P	Q	$P \vee Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge \sim(P \wedge Q)$
T	T				
T	F				
F	T				
F	F				

Truth Table – Example – Cont..

1. **$(P \vee Q) \wedge \sim(P \wedge Q)$**

P	Q	$P \vee Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge \sim(P \wedge Q)$
T	T	T			
T	F	T			
F	T	T			
F	F	F			

Truth Table – Example – Cont..

1. **$(P \lor Q) \land \sim(P \land Q)$**

P	**Q**	**$P \lor Q$**	**$P \land Q$**	**$\sim(P \land Q)$**	**$(P \lor Q) \land \sim(P \land Q)$**
T	T		T		
T	F		F		
F	T		F		
F	F		F		

Truth Table – Example – Cont..

1. $(P \lor Q) \land \sim(P \land Q)$

P	Q	$P \lor Q$	$P \land Q$	$\sim(P \land Q)$	$(P \lor Q) \land \sim(P \land Q)$
T	T		T	F	
T	F		F	T	
F	T		F	T	
F	F		F	T	

Truth Table – Example – Cont..

1. $(P \vee Q) \wedge \sim(P \wedge Q)$

P	**Q**	**$P \vee Q$**	**$P \wedge Q$**	**$\sim(P \wedge Q)$**	**$(P \vee Q) \wedge \sim(P \wedge Q)$**
T	T	T		F	F
T	F	T		T	T
F	T	T		T	T
F	F	F		T	F

Truth Table – Example – Cont..

1. $(P \vee Q) \wedge \sim(P \wedge Q)$

P	Q	$P \vee Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge \sim(P \wedge Q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

A Simple Exercise

Let p = "It rained last night",
q = "The sprinklers came on last night" ,
r = "The grass was wet this morning".

Translate each of the following into English:

$\neg p$ =

$r \wedge \neg p$ =

$\neg r \vee p \vee q$ =

A Simple Exercise

Let p = "It rained last night",
q = "The sprinklers came on last night" ,
r = "The grass was wet this morning".

Translate each of the following into English:

$\neg p$ = **"It didn't rain last night"**

$r \wedge \neg p$ = **"The grass was wet this morning, and it didn't rain last night"**

$\neg r \vee p \vee q$ = **"Either the grass wasn't wet this morning, or it rained last night, or the sprinklers came on last night"**