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- **Equally Likely Events:** Events that have the same chance of occurring

$$P(E) = \frac{The\ number\ of\ outcomes\ in\ E}{The\ Total\ Number\ of\ Outcomes\ in\ S}$$

$$P(E) = \frac{|E|}{|S|}, \text{where E be the equally likely event, and } S \text{ be the S.} S$$

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http://www.futureaccountant.com/probability/study-notes/equally-likely-exhaustive-events.php#.WBEEcPl97IU

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The identification of the sample space depends on the problem at hand. For instance, in the exercise of forecasting tomorrow weather, the sample space consists of all meteorological situations: rain (R), sun (S), cloud (C), typhoon (T) etc.

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In this case, we are sure that if A occurs then B cannot. Clearly, we have $A, \bar{A} = \emptyset$.

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Axiom (3) was known as countable additivity and it is rejected by a school of probability who replace the countable additivity by finite additivity.

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- Compute event probability

Applying Four-Step Method

Four Steps To Compute The Prob..

Perform the following four steps to compute the probability:

- Find the sample space
- Define event of interest
- Compute outcome probabilities
- Compute event probability

Applying Four-Step Method

http://www.maths.qmul.ac.uk/~pjc/notes/prob.pdf

https://services.math.duke.edu/~rtd/PTE/PTE4_1.pdf

http://users.uoa.gr/~dcheliotis/Stirzaker%20D.%20-%20Probability%20and%20random%20variables.%20A%20beginner's%20guide%20-%20CUP%201999.pdf

https://cran.r-project.org/web/packages/IPSUR/vignettes/IPSUR.pdf

https://people.ucsc.edu/~abrsvn/intro_prob_1.pdf

Probability without and with Replacement

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Formula-With Replacement: The formula for finding the combined probability of events obtained with replacement, we simply find the product of the probabilities of the event directly:

$$\boldsymbol{P(A \cap B) = P(A)\,P(B)}$$

Probability without Replacement

Example 1: We have a bag containing 4 yellow, 5 green and 6 orange candies. We draw two candies without replacement. Find the probability of getting both candies green.

Solution: Clearly n(S) = 4 + 5 + 6 = 15

Let A be the event of drawing a green candy first.

Then P (A) = 5/15

Now since we are not replacing back, thus, number of green candies left in bag now is 4 and total number of candies is 14.

Let B be the vent of drawing a green candy again.

Then P (B|A) = 4/14.

Thus the probability of getting both candies green = P (A & B)

= P (A) * P (B|A)

= 5/15 * 4/14

= 1/3 * 2/7 = 2/21

Probability without Replacement

Example 2: A jar contains 10 blue balls and 11 red balls. Two balls are drawn without replacement. What is the probability of getting two red balls.

Solution: Total number of balls = 10 + 11 = 21

Let P(A) = Probability of getting first red ball and P(B) = Probability of getting second red ball

Therefore: P(A) = 11/21

After first withdraw we are left with 20 balls, So P(B) = 10/20

Probability of getting two red balls = P(A)P(B) = 11/21 × 10/20 = 110/420 = 11/42

Probability with Replacement

Example 1: Given is a basket of fruits containing 4 oranges, 5 apples and 1 pears. We pick three fruits with replacement from the basket. Find the probability of getting an orange and two apples.

Solution: Here, n (S) = 4 + 5 + 1 = 10

Let A be the event of drawing n orange first.

Then P (A) = 4/10.

As we have replaced back after every pick, so the events are completely independent.

Let B be the event of drawing an apple.

Then P (B) = 5/10.

Let C be the event of drawing second apple.

Then P (C) = 5/10

Thus the probability of getting an orange and two apples with replacement

= P (A & B & C)

= P (A) * P (B) * P (C)

= 4/10 * 5/10 * 5/10

= 2/5 * 1/2* 1/2 = 1/2

Probability with Replacement

Example 2: A basket contains 3 apples and 7 oranges. A fruit is drawn and put back in the basket. This process is repeated 3 times. What is the probability that selected three fruits are orange?

Solution: A basket contains 3 apples and 7 oranges.

Total number of fruits = 3 + 7 = 10

Since fruit was put back in the basket after each drawn, clearly this is the case of experiment "with replacement".

The probability for each drawn = 7/10 i.e. P(orange) = 7/10. Therefore probability of 3 oranges = 7/10 × 7/10 × 7/10 or P(orange ∩ orange ∩ orange) = 7/10 × 7/10 × 7/10 = 343/1000 = 0.343

http://www.probabilityformula.org/probability-with-replacement.html#

Probability with Replacement

N Choose K or NcK.

$$\binom{\boldsymbol{n}}{\boldsymbol{c}\atop\boldsymbol{k}} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{k})!\,\boldsymbol{k}!}$$