

# Discrete Structures

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## Assignment 1

Dr. Muhammad Ahmad



**National University of Computer & Emerging Sciences**  
**Department of Computer Science**

## **General Guidelines**

1. Peer plagiarism will result in zero marks.
2. Late submissions are not allowed in any case.
3. Assignment must be submitted in hardcopy (either handwritten or printed form).
4. Follow the deadline mentioned on Google Classroom.
5. Total marks: 100

### Question 1

Consider the statement about a party, "If it's your birthday or there will be cake, then there will be cake."

- (a) Translate the above statement into symbols. Clearly state which statement is P and which is Q.
- (b) Make a truth table for the statement.
- (c) Assuming the statement is true, what (if anything) can you conclude if there will be cake?
- (d) Assuming the statement is true, what (if anything) can you conclude if there will not be cake?
- (e) Suppose you found out that the statement was a lie. What can you conclude?

### Question 2

Use De Morgan's Laws, and any other logical equivalence facts you know to simplify the following statements. Show all your steps. Your final statements should have negations only appear directly next to the sentence variables or predicates (P, Q, E(x), etc.), and no double negations. It would be a good idea to use only conjunctions, disjunctions, and negations.

- (a)  $\neg((\neg P \wedge Q) \vee \neg(R \vee \neg S))$ .
- (b)  $\neg((\neg P \rightarrow \neg Q) \wedge (\neg Q \rightarrow R))$  (careful with the implications).
- (c) For both parts above, verify your answers are correct using truth tables. That is, use a truth table to check that the given statement and your proposed simplification are actually logically equivalent

### Question 3

Geoff Poshington is out at a fancy pizza joint, and decides to order a calzone. When the waiter asks what he would like in it, he replies, "I want either pepperoni or sausage. Also, if I have sausage, then I must also include quail. Oh, and if I have pepperoni or quail then I must also have ricotta cheese."

- (a) Translate Geoff's order into logical symbols.
- (b) The waiter knows that Geoff is either a liar or a truth-teller (so either everything he says is false, or everything is true). Which is it?
- (c) What, if anything, can the waiter conclude about the ingredients in Geoff's desired calzone?

### Question 4

Express these system specifications using the propositions p "The user enters a valid password," q "Access is granted," and r "The user has paid the subscription fee" and logical connectives (including negations).

- a) "The user has paid the subscription fee, but does not enter a valid password."
- b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
- c) "Access is denied if the user has not paid the subscription fee."
- d) "If the user has not entered a valid password but has Paid the subscription fee, then access is granted."

### Question 5

Find premises and conclusion and determine the validity of the following argument:

"Robbery was the motive for the crime only if the victim had money in his pockets. But robbery or vengeance was the motive for the crime. Therefore, vengeance must have been the motive for the crime."

### Question 6

State the converse, contrapositive, and inverse of each of these conditional statements.

- a) If it snows tonight, then I will stay at home.
- b) I go to the beach whenever it is a sunny summer day.
- c) When I stay up late, it is necessary that I sleep until noon.

### Question 7

Write inverse, converse and contrapositive of the following statements.

- 1. If P is a square, then P is a rectangle,
- 2. If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
- 3. If n is prime, then n is odd or n is 2.
- 4. If today is New Year's Eve, then tomorrow is January

### Question 8

What rule of inference is used in each of these arguments?

- a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
- b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
- c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

### Question 9

Mark or circle Row members that make the argument valid or invalid

Premises:

$$A \wedge (B \rightarrow C).$$

$$(A \wedge B) \rightarrow (\neg C).$$

$$B \rightarrow D.$$

**Conclusion:**  $A \rightarrow D$

### Question 10

Draw the **Venn diagram** for the given equation of set A, B, C and D. Make Venn diagram for each step and then show the final Venn diagram.

$$A - (B \cap C \cap D)$$

### Question 11

Determine the reflexivity, symmetry, and transitivity of the following relations:

- (a) Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$ .
- (b) If  $A = \{1, 2, 3, 4\}$  then  $R = \{(1, 1), (2, 2), (1, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$ .
- (c) Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 2), (2, 2), (3, 1), (1, 3)\}$
- (d) Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 1), (2, 2), (1, 2), (2, 1), (2, 3), (3, 2)\}$ .
- (e) Let  $A = \{0, 1, 2, 4, 6\}$  and  $R = \{(0, 1), (1, 0), (2, 4), (4, 6), (6, 4), (4, 2)\}$ .

### Question 12

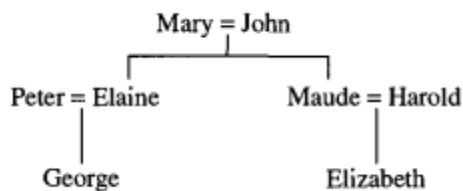
**Draw a Venn diagram to show the information.**

In a survey of 500 students at a university, it was found that 49% students liked watching football, 53% liked watching hockey and 62% liked watching basketball. Also, 27% liked watching football and hockey both, 29% liked watching basketball and hockey both and 28% liked watching football and basketball both. 5% liked watching none of the game.

- (a) Make Venn diagram of this question.
- (b) How many students like watching all the three games?
- (c) Find the ratio of number of students who like watching only football to those who like watching only hockey.
- (d) Find the number of students who like watching only one of the three given games.
- (e) Find the number of students who like watching at least two of the given games.

### Question 13

Although the marriage of a descendent is indicated by an = sign and the name of the partner, no further information is given about these partners. For example, in the family tree of Mary and John shown in the figure, even if Peter and Harold were brothers, this would not be shown. A family tree is a rich source of information about a number of relations.



For the family tree shown in Figure 3.2, identify the elements of the relations

- (a) IsMarriedTo,
- (b) IsParentOf, and
- (c) IsSameGeneration.

### Question 14

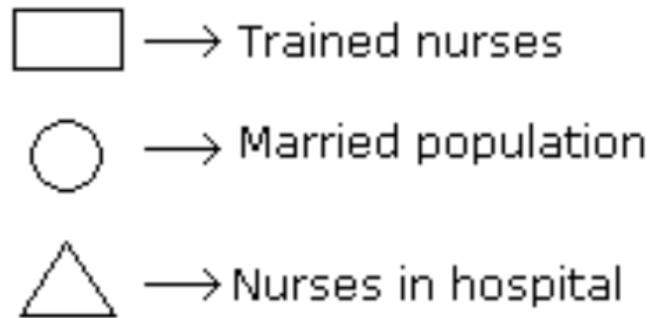
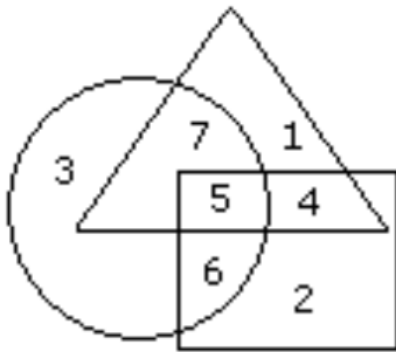
A group of 62 students were surveyed, and it was found that each of the students surveyed liked at least one of the following three fruits: apricots, bananas, and cantaloupes.

- 34 liked apricots.
- 30 liked bananas.
- 33 liked cantaloupes.
- 11 liked apricots and bananas.
- 15 liked bananas and cantaloupes.
- 17 liked apricots and cantaloupes.
- 19 liked exactly two of the following fruits: apricots, bananas, and cantaloupes.

A) How many students liked all of the following three fruits: apricots, bananas, and cantaloupes?

B) How many students liked apricots and cantaloupes, but not bananas?

### Question 15



Give answers with explanation.

- (a) How many are only married trained nurses?
- (b) By which number, married but untrained nurses in the hospital are represented?
- (c) By which numbers trained nurses are represented?
- (d) What is represented by the number 7?
- (e) By which number, the trained unmarried nurses in the hospital are represented?