~~2 31 30 12 33 1 39 35 40 15~~

2 31 12 30 ~~1 33 35 39 15 40~~

$\boxed{\text{2 31 30}}\boxed{\text{12 33}}$ ~~1 39 35 40 15~~

~~2 12~~

$\underbrace{\text{2 31 30 12 33}}$ | $\underbrace{\text{1 39 35 40 15}}$

left part larger than right part

2 31 30 | 12 33 | 1 39 35 | 40 15

right part bigger than left part

2 | 31 30 | 12 33 | 1 | 39 35 | 40 15 |

2 30 31 | 12 33 | 1 | 35 39 | 15 40 | → not good!

make either always

left ≥ right

always or

right ≥ left

2 12 30 31 33 | 1 35 39 | 15 40 |

2 12 30 31 33 | 1 15 35 39 40

1 2 12 15 30 31 33 35 39 40

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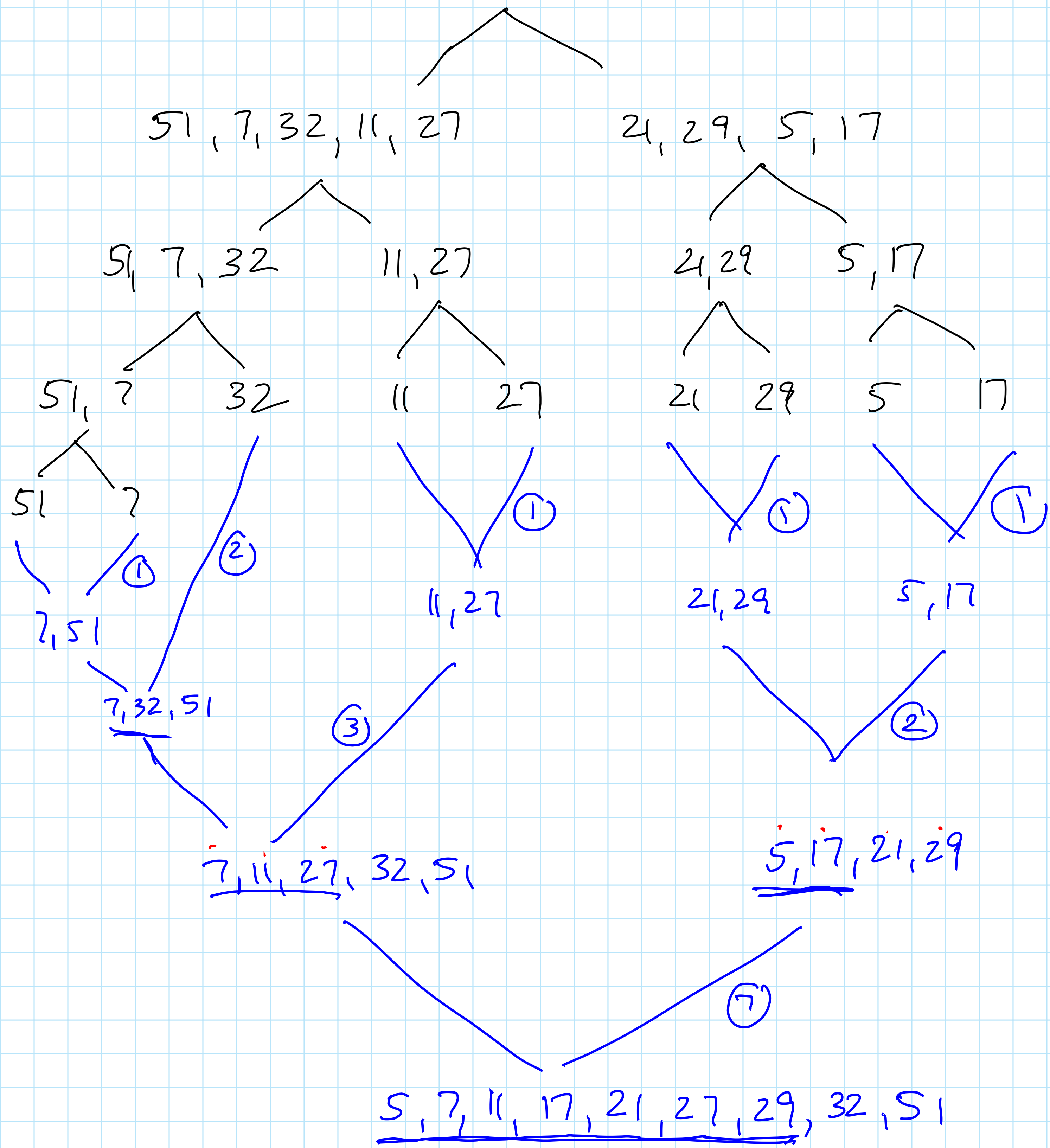
Example 35. $N = 9$

51, 7, 32, 11, 27, 21, 29, 5, 17



18 comparisons

analysis

merging needs at most N comparisons

$$T(1) = 1$$

$$T(N) = \underbrace{2 \cdot T\left(\frac{N}{2}\right)}_{\text{2 rec. calls}} + \underbrace{N}_{\text{cost for merge}} \qquad 2 \cdot T(2^{k-1}) + 2^k$$

assume N is a power of 2, i.e., $N = 2^k$ for some k ...

(cf. Rule 5 for analyzing algorithms)

$T(2^0) =$

$T(2^1) =$

$\vdots \qquad \vdots$

find pattern:

$$T(2^k) = (k+1) \cdot 2^k$$

verify with induction

If $N = 2^k$ then $k = \log N$

$\rightsquigarrow \quad T_{worst}(N) = O\big((\log(N)+1) \cdot N\big) = O(N \log N)$

(works also when N not a power of 2.)

NOTE: The implementation suggested is <u>not</u> in-place.

- uses extra memory
- copying costs time

There are Mergesort implementations avoiding copying.

BUT using more comparisons. Trade-off...

Analyzing recurrence relations

Theorem 11 Master Theorem

Let $a \geq 1$ and $b > 1$ be constants, $f: \mathbb{N} \to \mathbb{N}$, $T: \mathbb{N} \to \mathbb{N}$, with

$$\boxed{T(N) = a \cdot T\left(\frac{N}{b}\right) + f(N).}$$

Let $z = \log_b a$.

(1) If $f(N) = O(N^x)$ for some $x < z$, then $T(N) = \Theta(N^z)$

(2) If $f(N) = \Theta(N^z)$, then $T(N) = \Theta(N^z \log N)$

(3) If $f(N) = \Omega(N^x)$ for some $x > z$, and if there are $n_0 \in \mathbb{N}$, $c < 1$ such that $a \cdot f\left(\frac{N}{b}\right) \leq c \cdot f(N)$ for all $N \geq n_0$, then $T(N) = \Theta(f(N))$.

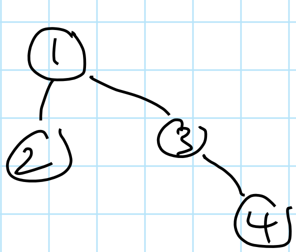
$$\boxed{T(N) = 2 \cdot T\left(\frac{N}{2}\right) + N}$$ (Mergesort)

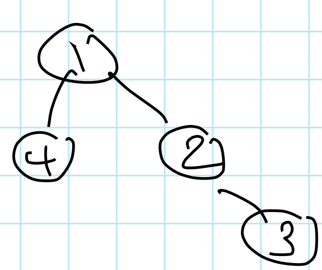
$$\left.\begin{array}{l} a = 2 \\ b = 2 \end{array}\right\} \Rightarrow \log_b a = 1, \quad f(N) = N$$

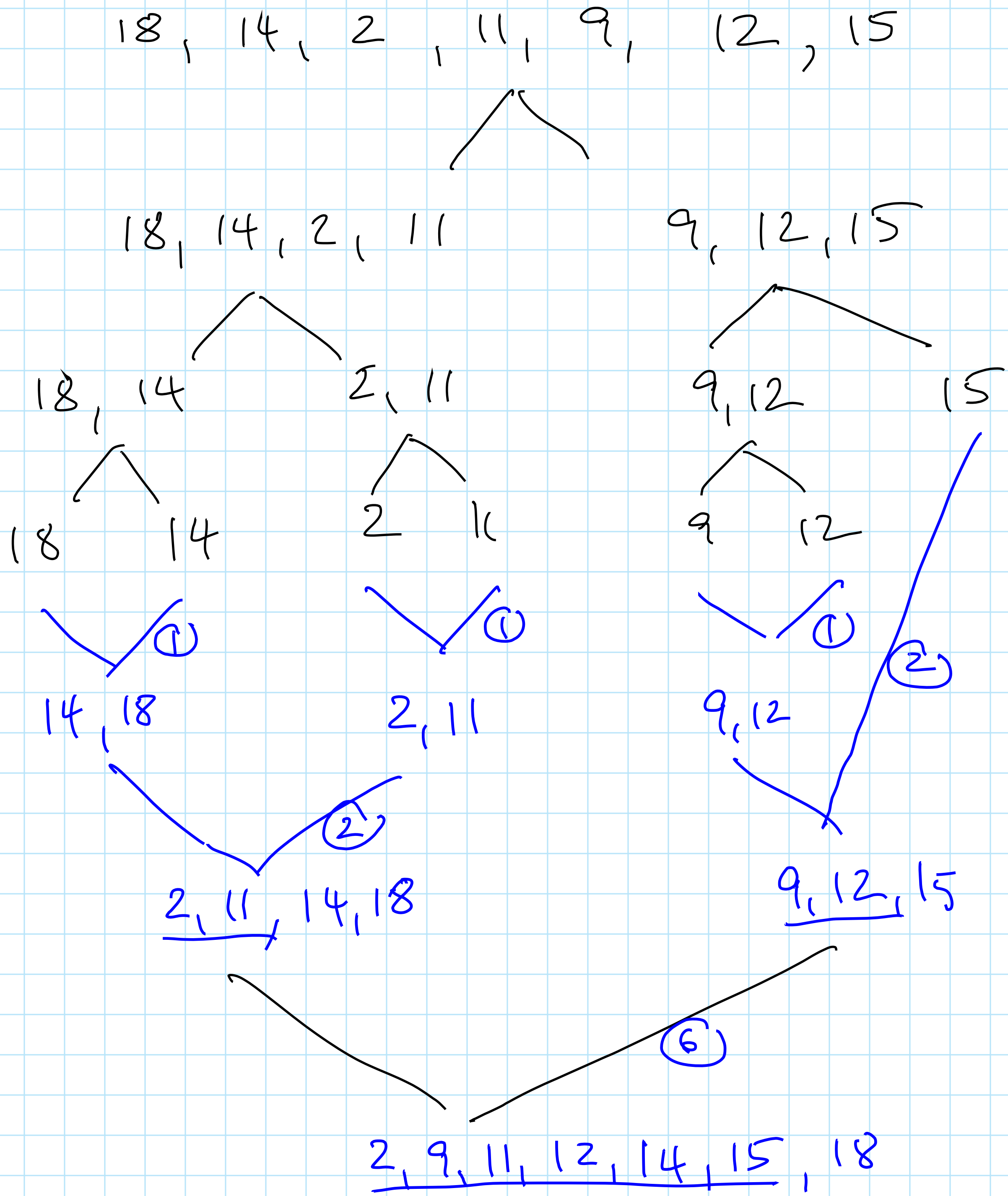
M. T. (2) $f(N) = \Theta(N^1) = \Theta(N^z)$

$$\Longrightarrow T(N) = \Theta(N^z \cdot \log N) = \Theta(N \log N)$$

1, 2, 3, 4






18, 14, 2, 11, 9, 12, 15
18, 14, 2, 11
9, 12, 15
18, 14
2, 11
9, 12
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9
12
1
1
1
2
14, 18
2, 11
9, 12
2
2, 11, 14, 18
9, 12, 15
6
2, 9, 11, 12, 14, 15, 18