CS 340 - lecture 29 — Nov 20

reduction of problem P to problem P' for which an algorithm A' is known

Example 49. P = unweighted SSSPP

P' = weighted SSSPP

A' = Dijkstra's alg.

problem instance for P:
directed graph $G=(V,E)$
source vertex $v \in V$

$\xrightarrow{\text{red}}$ can be implemented

problem instance for P':
directed graph $G=(V,E)$,
$v \in V$ source,
$c(v',w') = 1$ for all $(v',w') \in E$

Dijkstra $\downarrow A'$

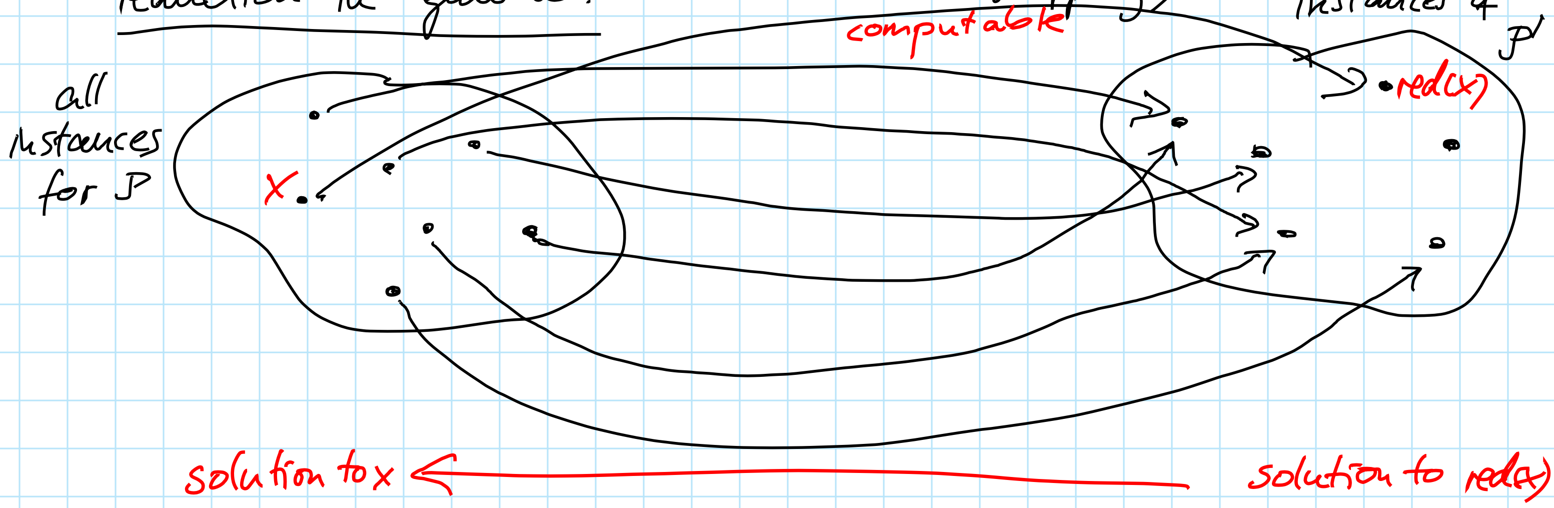
shortest weighted paths from v to any vertex $w \in V$

$\xleftarrow{\text{(ignore weights)}}$ do nothing

shortest unweighted paths from v to any $w \in V$

algorithm that solves P

reduction in general:



Reduction principle

(1) If $\mathcal{P}$ is reducible to $\mathcal{P}'$ and $\mathcal{P}'$ is solvable, then $\mathcal{P}$ is solvable.

(2) If $\mathcal{P}$ is reducible to $\mathcal{P}'$ and $\mathcal{P}$ is not solvable, then $\mathcal{P}'$ is not solvable.

5.2. Unsolvable problems

▷ Hilbert's 10th problem (1900)

diophantine equations

e.g., $3x^2 + 5y^4 = -7z^3$

polynomial equation with integer coefficients

Hilbert asked for an algorithm to solve the following problem:

given: diophantine equ. D

task: determine whether or not D has an integer solution

Yuri Matiyasevich (Russian math.) proved in 1970 that this is unsolvable.

▷ The halting problem HP

given: a computer program (e.g., in C++), called π, an input x to π

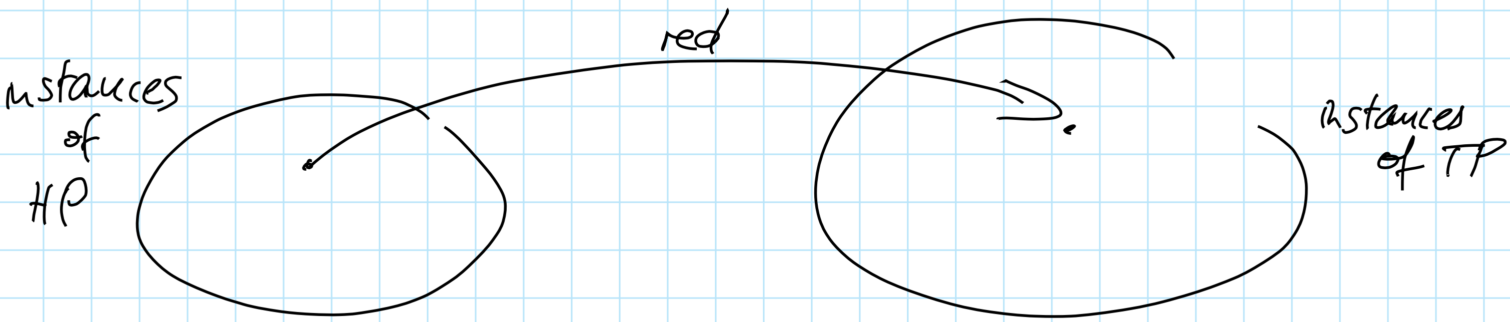
task: determine whether or not π terminates on input x.

Alan Turing proved in 1936 that this problem is unsolvable.

▷ The totality problem TP

given: a computer program π'

task: determine whether or not π' terminates on <u>all</u> possible inputs



$$(\pi, x) \xrightarrow{\text{red}} \pi' = \text{red}(\pi, x)$$

π' on input y does the following:

- ignore y
- simulate π on x
- terminate if and only if π terminates on x

π' terminates on all inputs

<u>if and only if</u>

π terminates on x

⇝ If TP were solvable, then the TP-solution to π' would be an HP-solution to (π, x), and so HP would be solvable.

⇝ TP is <u>not</u> solvable.

<u>Note</u> unsolvability here is not caused by running time or memory limitations.

Hilbert's 10th problem, HP, TP are unsolvable even when assuming unlimited time/memory.