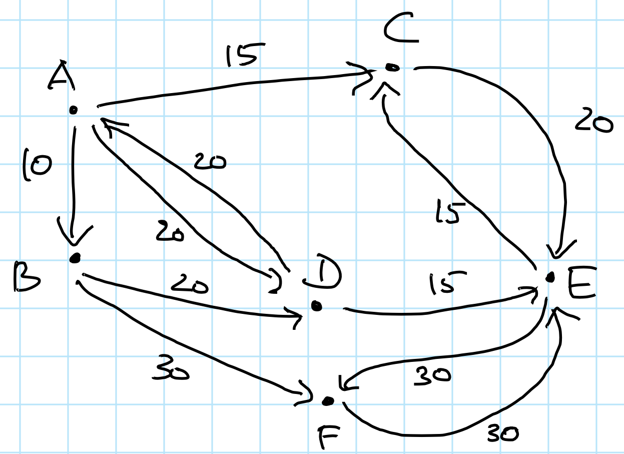
CS 340 - lecture 22 - Oct 28

# 4. GRAPH ALGORITHMS

Many real-world problems can be modeled as problems on graphs

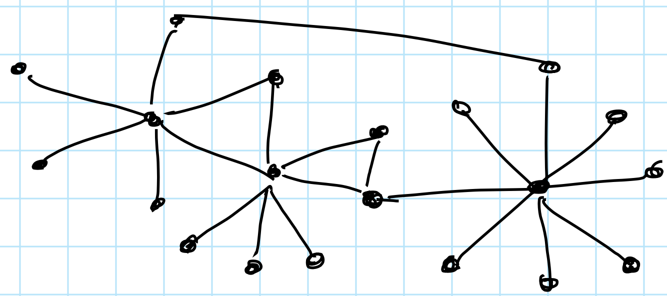
- subway network, road network, airport network: find shortest routes, best connections

$G_1$




- facebook friend network: find all "friends of friends"

$G_2$



## 4.1. TERMINOLOGY

Definition 10. A *graph* is a pair $G = (V, E)$, where:

(1) $V$ is a set. Elements of $V$ are called *vertices* (nodes).

(2) $E \subseteq \{(v,w) \mid v,w \in V\}$. Elements of $E$ are called *edges*. If $(v,w) \in E$, we say $w$ is *adjacent* to $v$.

A graph $G = (V,E)$ is *undirected* if for every edge $(v,w) \in E$ we also have $(w,v) \in E$. Otherwise $G$ is called *directed*.

**Example 39.** $G_1 = (V, E)$, with $V = \{A, B, C, D, E, F\}$ and
$E = \{(A,B), (A,C), (B,D), (C,E), (E,C), \ldots\}$

$G_1$ is directed. $G_2$ is undirected.

**Definition 11.** A <u>weighted graph</u> is a graph $G = (V, E)$ together with a function $c : E \to \mathbb{R}$. For any $(v, w) \in E$, $c(v,w)$ is the <u>cost</u> / <u>weight</u> of $(v, w)$.

**Example 39 ctd.**

$G_1$ is a weighted graph with $c(A,B) = 10$, $c(A,C) = 15$, $c(C,E) = 20$, $c(E,C) = 15$, ...

**Definition 12.** A <u>path</u> in a graph $G = (V, E)$ is a sequence of vertices $[v_1, v_2, \ldots, v_n]$ such that

$(v_i, v_{i+1}) \in E$ for $1 \leq i < n$

(i.e., $(v_1, v_2) \in E$, $(v_2, v_3) \in E$, ..., $(v_{n-1}, v_n) \in E$)

If $p = [v_1, \ldots, v_n]$ is a path in $G$, then:

- The length of $p$ is the number of edges in $p$, here $n-1$
- $p$ is <u>simple</u> if no vertex occurs twice in it with the exception that $v_1$ and $v_n$ could be equal.
- $p$ is a <u>cycle</u> if
  - $v_1 = v_n$ ($G$ directed)

▸ $v_1 = v_n$ and the edges on $p$ are pairwise distinct ($G$ undirected)

- $p$ is a <u>simple cycle</u> if $p$ is a simple path and $p$ is a cycle.

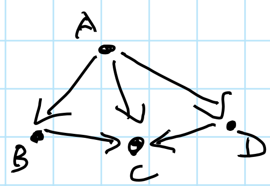
<u>Note</u>: $[v]$ is a path of length 0.

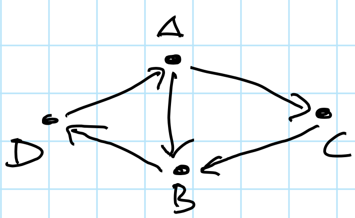
<u>Example 39 ctd.</u> In $G_1$,

- $[B, D, A]$ is a simple path of length 2.
- $[B, D, A, B]$ is a simple cycle of length 3.
- there is no path from $F$ to $A$.
- $[A, B, D, A, C]$ is a path from $A$ to $C$, but it is <u>not</u> simple.

<u>Definition 13</u>. A directed acyclic graph (DAG) is a directed graph without cycles.

<u>Example 40.</u>

- Every tree is a DAG.
- 

A DAG, but not a tree.
- 

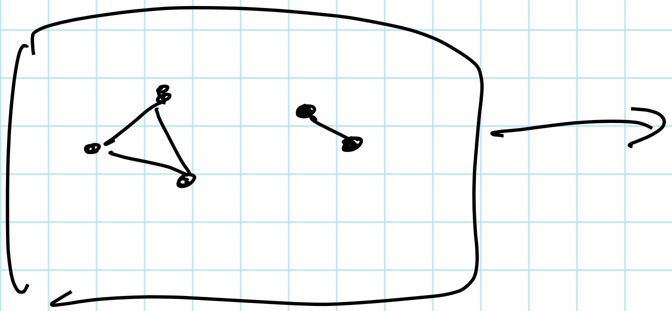
not a DAG: $[A, B, D, A]$ is a cycle.

<u>Definition 14</u>. An undirected graph $G = (V, E)$ is connected if, for all $v, w \in V$, there is a path from $v$ to $w$ in $G$.

A directed graph $G = (V, E)$ is

- strongly connected, if for $v, w \in V$, there is a path from $v$ to $w$ in $G$.
- weakly connected, if the corresponding undirected graph is connected.

Example 41. $G_1$ from Ex. 39 is weakly connected but not strongly connected.

 not connected