CS340 – Advanced Data Structures and Algorithm Design – Fall 2020
Assignment 7 – November 16, 2020

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**due November 23, 2020, 10.00pm**

*Problem* 1 (4+4 marks). For this problem, use tables for illustration, as shown on the handout provided in class. Additionally, list the final obtained shortest (weighted or unweighted) path length for every vertex, after termination of the algorithm. NOTE: It is enough if you list all the tables only for one of the two problems (you may choose which one). For the other one, you can opt to show only the final results.

(a) For the graph shown in Figure 1, find the shortest weighted paths from $A$ to the other vertices in the graph (one path for each vertex).

(b) For the graph shown in Figure 1, find the shortest unweighted paths from $B$ to the other vertices in the graph (one path for each vertex).

*Problem* 2 (4 marks). Give an example of a directed weighted graph $G$ that satisfies the following three properties simultaneously. (1) $G$ has no cycle with negative cost; (2) $G$ has an edge with negative cost; (3) Dijkstra's algorithm, applied to $G$, gives an incorrect answer.

*Problem* 3 (6 marks). Provide a modification of Dijkstra's algorithm for the case that the range of the weight function $c$ equals $\{1, 2, 3, \ldots, k\}$ for some $k \in \mathbb{N}$. Your algorithm must have a worst-case running time of $O(k \cdot |V| + |E|)$. Explain the correctness of your modified algorithm and its running time.

*Problem* 4 (4+3 marks).

(a) Provide an algorithm that solves the following problem.

given: an undirected graph $G = (V, E)$ that contains no cycles; a vertex $s \in V$.
task: for each vertex $v \in V$, determine the length of the longest path from $s$ to $v$ in $G$.

Your algorithm must have a worst-case running time of $O(|V| + |E|)$. Explain the correctness of your algorithm and its running time.

(b) Why does the problem in (a) become more difficult if the input graph is no longer known to be acyclic and we ask for the lengths of the longest *simple* paths? What is the running time cost of the most efficient algorithm you can think of?

*Problem* 5 (4 marks). Find the maximum flow from $s$ to $t$ in the graph shown in Figure 2.

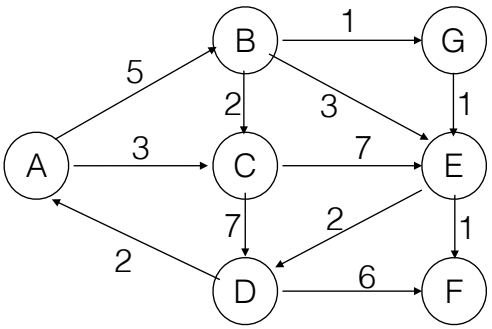


Figure 1: Graph used in Problem 1

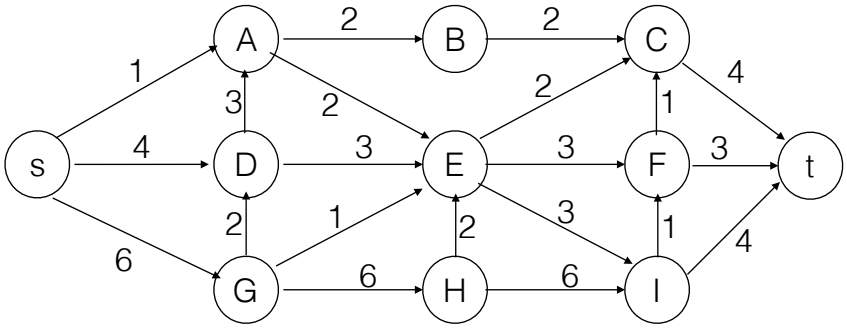


Figure 2: Graph used in Problem 5

(both figures adapted from your textbook)