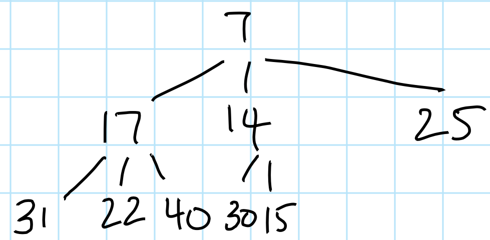
lecture 11 — Sep 28

2.3 d-HEAPS (needed in Assignment 3)

d-heap: generalization of binary heap to a tree in which all nodes have (up to) d children

Example 21. a 3-heap $(d = 3)$



⇝ the larger d, the shallower the heap.

running times

constant

- insert: $\Theta(\log_d N) = \Theta(\log_2 N)$ $\qquad \log_d N = \dfrac{\log_2 N}{\log_2 d}$
- deleteMin: $\Theta(d \log_d (N))$

  $d-1$ comparisons to find min of d children

array implementation still works

BUT: division/multiplication by d (instead of by 2) is no longer just a bit shift, and is therefore computationally more costly.

when to use d-heaps?

- when # insertions >> # deleteMins
- when priority queue needs to be stored on disk (cf. B-trees vs. binary trees)

2.4. LEFTIST HEAPS

problem with heaps: how to efficiently merge two heaps?

Definition 6. Let n be a node in a BT.

The null path length $npl(n)$ of n is the length of the shortest path from n to a node that has at most 1 child.

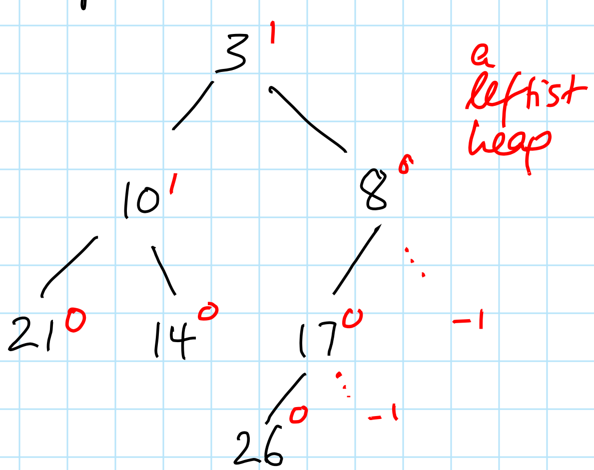
Definition 7. A leftist heap is a BT in which

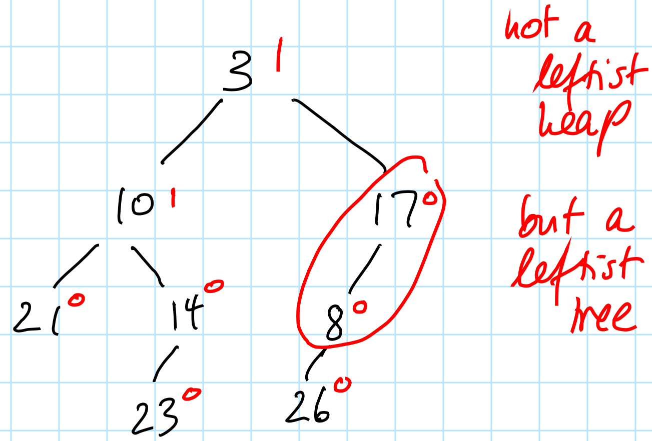
(1) for every node n, the npl of the left child of n is at least as large as -"- right -"-
(where empty children have npl -1)

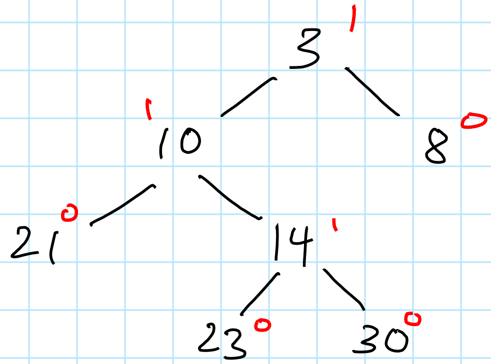
(2) for every node n, except for the root, the entry at n is at least as large as the entry at n's parent.

A BT with (1) is called a leftist tree.

Example 26.



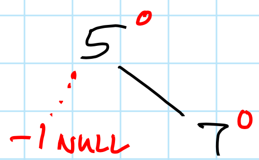




not a leftist tree, since the structural property is violated at the node with entry 10

$\rightsquigarrow$ leftist trees are not balanced

more "weight" on the left



not a leftist tree

NOTE: • The rightmost path in a leftist tree is never longer than any other path in the tree

• Every subtree of a leftist heap (tree) is a leftist heap (tree).

Theorem 7. A leftist tree with q nodes on the rightmost path must have at least $2^q - 1$ nodes.

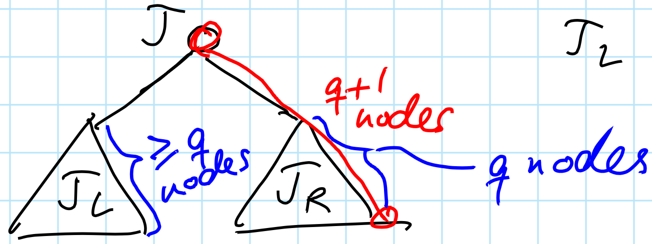
Proof. induction on q.

ind. base $q = 1$. If the rightmost path has 1 node, then there is at least $1 = 2^1 - 1$ node in the tree.

ind. hyp. Suppose theorem holds for a fixed q.

ind. step. $q \rightsquigarrow q+1$.

Let T be a leftist tree with $q+1$ nodes on the rightmost path.



T_L and T_R are leftist trees.

↳ each with at least q nodes on the rightmost path.

By induction hyp. T_L and T_R each have $\geq 2^q - 1$ nodes.

$$\Rightarrow T \text{ has at least } \underset{\text{root}}{1} + \underbrace{2^q - 1}_{T_L} + \underbrace{2^q - 1}_{T_R} = 2^{q+1} - 1$$

□