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Practice Problems on Proof by Induction and Recurrence Relations (not for grade)

Problem 1. Prove by induction that
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
for all $n \in \mathbb{N}$ with $n \geq 1$.

Problem 2. Consider the following recurrence.
$$
\begin{aligned}
T(1) &= 1 \,, \\
T(N) &= 2T(N-1) + 1 \text{ for } N > 1 \,.
\end{aligned}
$$

Use the guess and verify method to solve this recurrence, i.e., to express $T(N)$ in a formula that does not contain T in the righthand side. What is the corresponding growth rate class?

Problem 3. Consider the following recurrence.
$$
\begin{aligned}
T(1) &= 1 \,, \\
T(N) &= T(\frac{N}{2}) + 1 \text{ for } N > 1 \,.
\end{aligned}
$$

Use the guess and verify method to solve this recurrence, i.e., to express $T(N)$ in a formula that does not contain T in the righthand side. Focus on the case $N = 2^k$ for some $k \in \mathbb{N}$. What is the corresponding growth rate class? Do you know of a standard recursive algorithm that leads to the above recurrence relation?