ASSIGNMENT 6

PROBLEM 1.

20 13 17 25 18 2 29 14 8 pivot=18

20 13 17 25 8 2 29 14 18

i j

14 13 17 25 8 2 29 20 18

i i j j

14 13 17 2 8 25 29 20 18

i j ij

14 13 17 2 8 | 18 | 29 20 25 pivots = 14, 25

8 13 17 2 14 | 18 | 29 20 25

i i j i j

8 13 2 17 14 | 18 | 20 29 25

ij ij ij ij

8 13 2 | 14 | 17 | 18 | 20 | 25 | 29 pivot=8

2 13 8 | 14 | 17 | 18 | 20 | 25 | 29

ij ij

2 | 8 | 13 | 14 | 17 | 18 | 20 | 25 | 29

[subtract 1 mark for each mistake]

PROBLEM 2. The students have to find one correct topological ordering. There are several possibilities:

s, G, D, A, B, H, E, I, F, C, t
s, G, H, D, A, B, E, I, F, C, t
s, G, D, H, A, B, E, I, F, C, t
s, G, D, A, H, B, E, I, F, C, t
s, G, H, D, A, E, B, I, F, C, t
s, G, D, H, A, E, B, I, F, C, t
s, G, D, A, H, E, B, I, F, C, t
s, G, H, D, A, E, I, B, F, C, t
s, G, D, H, A, E, I, B, F, C, t
s, G, D, A, H, E, I, B, F, C, t
s, G, H, D, A, E, I, F, B, C, t
s, G, D, H, A, E, I, F, B, C, t
s, G, D, A, H, E, I, F, B, C, t

[subtract 1 mark for each mistake]

NOTE: the graph corresponding to the given adjacency list is the one shown in Figure 9.81 (page 437) of the textbook. This is a weighted graph, and the weights are given in the adjacency matrix, even though they are completely irrelevant to the problem. The students have to figure out themselves that the only thing that matters is where the edges are (and in which direction they go), but not what the weights of the edges are.

Some students might try to illustrate the algorithm with which they obtain the topological order, but it's not required for this question.

PROBLEM 3.

3.1 Suppose G has a cycle, $[w_1, \dots, w_n]$, where $w_1 = w_n$.
Assume, by way of contradiction, that G has a topological ordering $v_1, \dots, v_z$. Let $i, j \in \{1, \dots, z\}$ such that $w_1 = v_i$, $w_2 = v_j$.
Since $[w_1, w_2]$ is a path in G, and $v_1, \dots, v_z$ is a top. ordering, we have

$$i < j. \quad (*)$$

Since $[w_2, w_3, \dots, w_{n-1}, w_n]$ $(= [w_2, w_3, \dots, w_{n-1}, w_1])$ is a path in G, and $v_1, \dots, v_z$ is a top. ordering, we have $\quad j < i. \quad (**)$

(*) and (**) contradict each other. Therefore G has no top. ordering.

[give 1 mark if only a small part of the reasoning is missing]

3.2 If G has no top. ordering then there is a <u>subgraph</u> of G in which <u>no</u> vertex has in-degree zero (otherwise the alg. given in class produces a top. ordering for G). Let $v_1, \dots, v_k$ be the vertices in this subgraph.
For each $v \in \{v_1, \dots, v_k\}$ there is a $pred(v) \in \{v_1, \dots, v_k\}$ such that

$$(pred(v), v) \in E.$$

Hence $[\underbrace{pred(pred(\dots(pred(v))\dots))}_{k}, \underbrace{pred(\dots pred(v)\dots)}_{k-1}, \dots, pred(v), v]$

is a path in G using only vertices in $\{v_1, \dots, v_k\}$. Since the path has length k, at least one vertex in $\{v_1, \dots, v_k\}$ occurs twice on this path.
Therefore the path contains a cycle as a subpath.
Hence G has a cycle.

[give 2 marks if only a small part of the reasoning is missing,
is there]
give 1 mark