Started on	Monday, 21 December 2020, 9:12 AM
State	Finished
Completed on	Monday, 21 December 2020, 11:32 AM
Time taken	2 hours 20 mins
Grade	**63.50** out of 100.00

Question **1**

Correct

Mark 1.00 out of 1.00

True or False? Whether or not the complexity class P contains decision problems that are not in the complexity class NP, is an open question.

Select one:

- True
- False ✔

The correct answer is 'False'.

Question **2**

Incorrect

Mark 0.00 out of 1.00

Breadth-first traversal on general graphs corresponds to pre-order traversal on trees.

Select one:

- True ✖
- False

The correct answer is 'False'.

Question **3**

Correct

Mark 1.00 out of 1.00

True or False? Every directed tree has a topological ordering.

Select one:

- True ✔
- False

The correct answer is 'True'.

Question **4**

Incorrect

Mark 0.00 out of 1.00

True or False? If T(N) = 0.3 N^3 * 1,000 N log(N) then T(N) = o(N^5).

Select one:

- True
- False ✖

The correct answer is 'True'.

Question **5**

Correct

Mark 1.00 out of 1.00

True or False? The worst-case running time of Heapsort on input of an array of length N is Theta(N*log(N)).

Select one:

- True ✔
- False

The correct answer is 'True'.

Question **6**

Correct

Mark 1.00 out of 1.00

True or False? Asymptotically, Quicksort has the best possible worst-case running time for a comparison-based sorting algorithm.

Select one:

- True
- False ✔

The correct answer is 'False'.

Question **7**

Correct

Mark 1.00 out of 1.00

True or False? Every undirected graph has a minimum spanning tree.

Select one:

- True
- False ✔

The correct answer is 'False'.

Question **8**

Incorrect

Mark 0.00 out of 1.00

True or False? 2^(4N) = O(2^N).

Select one:

- True ✖
- False

The correct answer is 'False'.

Question **9**

Incorrect

Mark 0.00 out of 1.00

True or False? If a directed graph has a topological ordering, then this graph is a directed tree.

Select one:

- True ✖
- False

The correct answer is 'False'.

Question **10**
Incorrect
Mark 0.00 out of 1.00

True or False? Insertion into skew heaps can be implemented with an amortized running time of O(log(N)).

Select one:
- True
- False ✖

The correct answer is 'True'.

Question **11**
Incorrect
Mark 0.00 out of 1.00

True or False? Quicksort is a typical greedy algorithm.

Select one:
- True ✖
- False

The correct answer is 'False'.

Question **12**
Correct
Mark 1.00 out of 1.00

True or False? The halting problem is reducible to the totality problem.

Select one:
- True ✔
- False

The correct answer is 'True'.

Question **13**
Incorrect
Mark 0.00 out of 1.00

True or False? Every array sorted in decreasing order is a max-heap.

Select one:
- True
- False ✖

The correct answer is 'True'.

Question **14**
Incorrect
Mark 0.00 out of 1.00

True or False? Some NP-complete problems are unsolvable.

Select one:
- True ✖
- False

The correct answer is 'False'.

Question **15**
Incorrect
Mark 0.00 out of 1.00

True or False? Insertion in binomial queues has an average-case running time of O(1).

Select one:
- True
- False ✖

The correct answer is 'True'.

Question **16**
Correct
Mark 1.00 out of 1.00

True or False? The worst-case running time of Shellsort (with Shell's increments) on input of an array of length N is Omega(N*log(N)).

Select one:
- True ✔
- False

The correct answer is 'True'.

Question **17**
Incorrect
Mark 0.00 out of 1.00

True or False? Backtracking is always at least as efficient as a greedy algorithm for the same problem.

Select one:
- True ✖
- False

The correct answer is 'False'.

Question **18**
Correct
Mark 1.00 out of 1.00

True or False? If T(N) = 3 N^2 * 5 N log(N) then T(N) = O(N^3).

Select one:
- True
- False ✔

The correct answer is 'False'.

Question **19**
Correct
Mark 1.00 out of 1.00

True or False? There are cases in which adjacency matrices should be preferred over adjacency lists for graph representation.

Select one:
- True ✔
- False

The correct answer is 'True'.

Question **20**
Correct
Mark 1.00 out of 1.00

True or False? If f(N) = Omega(g(N)), then g(N) = O(0.002 * f(N)).

Select one:
- True ✔
- False

The correct answer is 'True'.

Question **21**

Complete

Mark 0.50 out of 2.00

What are the minimum and maximum numbers of elements in a binary heap of height 4? (No proof required; just provide the two numbers as answer.)

Min = 8 AND Max = 16

Comment:

Question **22**

Correct

Mark 2.00 out of 2.00

Consider the binary heap given by the array [15, 27, 30, 39, 33, 53, 50, 40]. Which array will be obtained after inserting the entry 20 into this heap?

Select one:

- a. [20, 15, 27, 30, 39, 33, 53, 50, 40]
- b. [15, 20, 30, 27, 33, 53, 50, 40, 39] ✔
- c. [15, 20, 27, 30, 39, 33, 53, 50, 40]
- d. [15, 20, 27, 30, 33, 39, 40, 50, 53]
- e. [15, 27, 30, 39, 33, 53, 50, 40, 20]

Your answer is correct.

The correct answer is: [15, 20, 30, 27, 33, 53, 50, 40, 39]

Question **23**

Correct

Mark 2.00 out of 2.00

Which of the following arrays represent min-heaps?

Select all that apply:

- a. [60, 59, 43, 40, 31, 29, 22, 18]
- b. [18, 22, 31, 40, 29, 43, 60, 59] ✔
- c. [18, 22, 29, 40, 31, 60, 59, 43] ✔
- d. [18, 29, 22, 59, 31, 43, 40, 60] ✔
- e. [18, 22, 31, 40, 43, 29, 59, 60]

Your answer is correct.

The correct answers are: [18, 22, 29, 40, 31, 60, 59, 43], [18, 22, 31, 40, 29, 43, 60, 59], [18, 29, 22, 59, 31, 43, 40, 60]

Question **24**

Incorrect

Mark 0.00 out of 2.00

What is the worst-case running time complexity of the best possible algorithm for searching a given entry x in a given heap of N nodes?

Select one:

- a. Theta(N*log(N)) ✖
- b. Theta(N)
- c. Theta(N^2)
- d. Theta(1)
- e. Theta(log(N))

Your answer is incorrect.

The correct answer is: Theta(N)

Question **25**

Correct

Mark 2.00 out of 2.00

Consider the binary heap represented by the array [2, 5, 11, 10, 18, 27, 24, 21, 20]. Suppose the minimum is deleted from this heap. Which array represents the resulting heap?

Select one:

- a. [5, 11, 10, 18, 20, 24, 21, 27]
- b. [5, 10, 11, 20, 18, 27, 24, 21] ✔
- c. [2, 5, 11, 10, 18, 27, 24, 21]
- d. [5, 11, 10, 18, 27, 24, 21, 20]
- e. [20, 5, 11, 10, 18, 27, 24, 21]

Your answer is correct.

The correct answer is: [5, 10, 11, 20, 18, 27, 24, 21]

Question **26**

Complete

Mark 8.00 out of 9.00

Consider sorting the array [11, 27, 5, 31, 15, 35, 41, 17, 8] with the Mergesort procedure as explained in class. For the following three questions, you need to provide only one number each as answer; no explanation is required.

(a) [3 marks] How many merges of two lists does the procedure perform in total when processing this array?

(b) [3 marks] How many comparisons are made for the final merge of two lists when processing this array?

(c) [3 marks] How many comparisons are made in total when processing this array?

(a) 8

(b) 7

(c) 20

Comment:

Question **27**

Complete

Mark 6.00 out of 14.00

(a) [4 marks] What is a spanning tree of a graph?

(b) [4 marks] Suppose T is a minimum spanning tree of a weighted graph G. Prove that there is no spanning tree (not even a non-minimal one) whose largest edge weight is smaller than the largest edge weight of T.

(c) [6 marks] Give an efficient algorithm that, given a weighted graph G = (V,E), represented as an adjacency list, with cost function c, and an integer b, determines whether G has a spanning tree whose largest edge weight is at most b.

(a) Spanning tree is a subset of a graph that has a minimum number of edges with all the possible edges. It cannot be cycling and will not be disconnected. So all the graph which undirected and connected has at least one spanning tree.

(b) An MST Algorithm process the edge weight. Therefore there will only be an edge of weight w as there are no other weight lesser than that edge. So in other words there has to be a spanning tree with the largest value of an edge as a minimum of the other possibilities to consider it as a minimum spanning tree.

(c)

```
//Assuming we have a function 'edgeValid(u, v, vector<bool> MST)'  that return true if the edge u - v is valid or not. than only it will include it in MST
// also assuming we have a cost array that is 2d with all the cost value;

span(int cost[][V]){
 vector<bool> MST(v, false); // in build function in C++
 MST[0] = true; //Initializing with index 0 with true
 while (int)
}
```

Comment:
(b) too vague

Question **28**

Complete

Mark 9.00 out of 9.00

Consider sorting the array [5, 13, 2, 25, 7, 17, 20, 8, 4] in increasing order with Heapsort. Which arrays do you obtain ...

(a) ... after the initial build(Max)Heap operation?

(b) ... after the first deleteMax?

(c) ... after the second deleteMax?

You only need to list one array each per question. No explanation is required.

(a) [25, 13, 20, 8, 7, 17, 2, 5, 4]

(b) [20, 13, 17, 8, 7, 4, 2, 5]

(c) [17, 13, 5, 8, 7, 4, 2]

Comment:

Question **29**

Not answered

Marked out of 6.00

Determine the Theta-bound for the running time of the following pseudocode fragment, by finding an appropriate recurrence relation and using the Master Theorem (given below). Here "list" is an array sorted in increasing order and "x", "left", "right" are integers with "left" <= "right". For the initial input, assume that

- the length of "list" equals N,

- "left" equals zero,

- "right" equals N-1.

```
search(list,left,right,x)
{
  if list[(left+right)/2] equals x
  then { return (left+right)/2 }
  else
  {
     if list[(left+right)/2] is less than x
     then { return search(list,(left+right)/2 +1,right,x) }
     else { return search(list,left,(left+right)/2 -1,x) }
  }
}
```

Recall the Master Theorem:

Let a>=1 and b>1 be constants. Let f, T be two functions from the natural numbers to the natural numbers, such that T(N) = a * T(N/b) + f(N). Let z = log_b(a).

(1) If f(N) = O(N^x) for some x<z, then T(N) = Theta(N^z).

(2) If f(N) = Theta(N^z), then T(N) = Theta(N^z log(N)).

(3) If f(N) = Omega(N^x) for some x>z, and if there is a natural number n_0 and a real number c<1 such that a * f(N/b) <= c * f(N) for all N>=n_0, then T(N) = Theta(f(N)).

Question **30**

Correct

Mark 12.00 out of 12.00

Consider the directed graph whose adjacency matrix is

	s	v_1	v_2	v_3	v_4	t
s	∞	16	13	∞	∞	∞
v_1	∞	∞	10	12	∞	∞
v_2	∞	4	∞	∞	14	∞
v_3	∞	∞	9	∞	∞	20
v_4	∞	∞	∞	7	∞	4
t	∞	∞	∞	∞	∞	∞

Pick the three correct statements about this graph out of the following.

Select all that apply:

- [ ] The graph has exactly 26 edges.
- [x] The graph has exactly 10 edges. ✔
- [ ] The graph has exactly 6 edges.
- [ ] The maximum flow from s to t in this graph is closer to 18 than to 25.
- [x] The maximum flow from s to t in this graph is closer to 25 than to 18. ✔
- [ ] This graph has exactly one network flow graph witnessing the maximum flow from s to t.
- [ ] This graph has exactly two different network flow graphs witnessing the maximum flow from s to t.
- [x] This graph has more than two different network flow graphs witnessing the maximum flow from s to t. ✔

Your answer is correct.

The correct answers are: The graph has exactly 10 edges., The maximum flow from s to t in this graph is closer to 25 than to 18., This graph has more than two different network flow graphs witnessing the maximum flow from s to t.

Question **31**

Complete

Mark 12.00 out of 14.00

Consider the following recursive algorithm for computing Pascal's triangle (i.e., for computing the binomial coefficient {i choose j}):

```
Pascal(i,j)
{
  if ( i == 0 || j == 0 )  { return 1 }              // since {0 choose j} is always 1 and {i choose 0} is also always 1
  else
    { return Pascal(i,j-1) + Pascal(i-1,j) }   // using a well-known equation on binomial coefficients
}
```

(a) [4 marks] What problem would you expect with this recursive algorithm, and why?

(b) [8 marks] Using dynamic programming, give a more efficient algorithm (in pseudocode or in C++) that has the same input/output behaviour as Pascal(i,j).

(c) [2 marks] Analyze your algorithm from (b) in terms of its worst-case running time.

(a) This recursive algorithm is too slow when the number i and j gets more than 90. It will take time in comparison to the time taken for our whole solar system to be destroyed and sucked in a black hole if we use a generic system. This is because we are solving resolved steps again and again which can be solved using dynamic programming.

(b)

```
//considering we have a min function which return minimum between two nums 'min(i, j)'
void Pascal (int size){
 for(int m = 0 ; m < size; m++){
  for(int n = 0;  n <= m; n++){
    cout << bioCoeff(m, n);
  }
 }
}
int bioCoeff(i, j){
 int arr[i+1][j+1];
 for(int m = 0; m <= i; m++){
  for(int n = 0; n <= min(m, j); m++){
    if(n == 0 || n == m)
      arr[m][n] = 1;
    else { arr[m][n] = arr[m][n-1] + arr[m-1][n]; }
  }
 }
 return arr[i][j];
}
```

(c) The time complexity of this algorithm is O(i ^ j) or O(pow(i, j)).

Comment:
c wrong

Question **32**

Not answered

Marked out of 6.00

Determine the largest value "a" for which T(N) = O(N^4), where T(N) is given by the recurrence T(N) = a * T(N/2) + N^3. Explain your answer.

To obtain your answer, recall the Master Theorem:

Let a>=1 and b>1 be constants. Let f, T be two functions from the natural numbers to the natural numbers, such that T(N) = a * T(N/b) + f(N). Let z = log_b(a).

(1) If f(N) = O(N^x) for some x<z, then T(N) = Theta(N^z).

(2) If f(N) = Theta(N^z), then T(N) = Theta(N^z log(N)).

(3) If f(N) = Omega(N^x) for some x>z, and if there is a natural number n_0 and a real number c<1 such that a * f(N/b) <= c * f(N) for all N>=n_0, then T(N) = Theta(f(N)).