CS340 – Advanced Data Structures and Algorithm Design – Fall 2020
Assignment 8

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due December 04, 2020, 10.00pm

Problem 1 (6+1 marks).

(a) textbook, Problem 9.15(a) For each algorithm, give a list of the edges in the order in which they are added to the MST.

(b) textbook, Problem 9.15(b)

Problem 2 (2+2 marks). Both (a) and (b) concern the graph in Figure 9.82 (page 438) in your textbook. In both cases, if your traversal algorithm at any point in time has more than one option of which vertex to visit next, choose the alphabetically smaller vertex.

(a) Show the list of the vertices of the graph, as they would be traversed by BFS starting at vertex A.

(b) Show the list of the vertices of the graph, as they would be traversed by DFS starting at vertex A.

Problem 3 (4 marks). Provide an $O(|V|)$ algorithm that solves the following problem.
given: a directed graph $G = (V, E)$, represented as an adjacency *matrix*.
task: determine whether or not there is a vertex $w \in V$ such that, for every other vertex $v \in V$ with $v \neq w$, we have:

$$(v, w) \in E \text{ but } (w, v) \notin E\,.$$

Problem 4 (3 marks). Show that the decision problem $\mathcal{P}$ is reducible to the decision problem $\mathcal{P}'$, using a reduction mapping *red* that runs in time polynomial in the size of its input, i.e., $red(N) = O(f(N))$ where f is some polynomial in N.

Problem $\mathcal{P}$:
given: an undirected graph $G = (V, E)$ and a number $k \in \mathbb{N}$, where $k \leq |V|$
question: is there a set $V' \subseteq V$ of at least k many vertices such that, for all $v, w \in V'$, the (undirected) edge (v, w) belongs to E?

Problem $\mathcal{P}'$:
given: an undirected graph $G = (V, E)$ and a number $k \in \mathbb{N}$
task: is there a set $V' \subseteq V$ of at least k many vertices such that there are no two vertices $v, w \in V'$ for which the (undirected) edge (v, w) belongs to E?

Recall that the size of a graph $G = (V, E)$ is $|V| + |E|$.