lecture 12 - Sep 30

Theorem 7 A leftist tree with q nodes on the rightmost path has at least $2^q - 1$ nodes.

Corollary 4. The rightmost path in a leftist tree of N nodes has at most

$$\lfloor \log(N+1) \rfloor$$

nodes.

$N = 2^q - 1$

q $\lfloor \log(N+1) \rfloor$

⇝ operations on rightmost path of a leftist heap are "cheap"!

but: need to restore leftist heap property once violated.

Implementation: store npl with a node!

Leftist heap operations.

merge (heap 1, heap 2) (combines two leftist heaps into a leftist heap)

if heap 1 empty, return heap2;

if heap2 empty, return heap1;

if root(heap1) < root(heap2) return merge1 (heap1, heap2)

else return merge1 (heap2, heap1)

merge 1 (heap1, heap2)

[note: the root of heap1 is not larger than the root of heap 2]

if heap1.left empty, append heap2 to the left of heap1.

else

{

append merge (heap1.right, heap2) to the right of heap1; (a)

update npl's

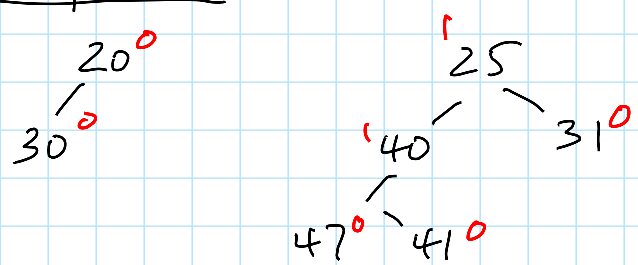
if npl(heap1.left) < npl(heap1.right)

(b) {swap children of heap 1}

npl(heap1) = 1 + npl(heap1.right)

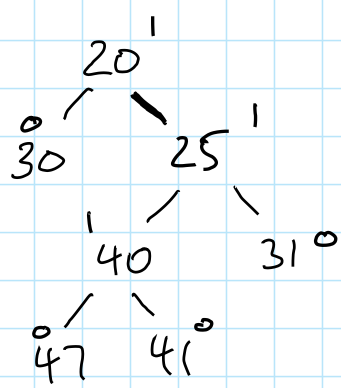
}

Example 23.

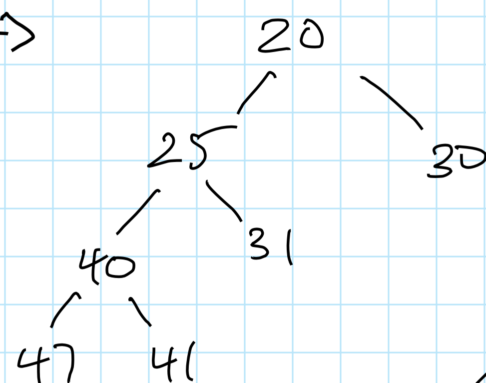


merge?

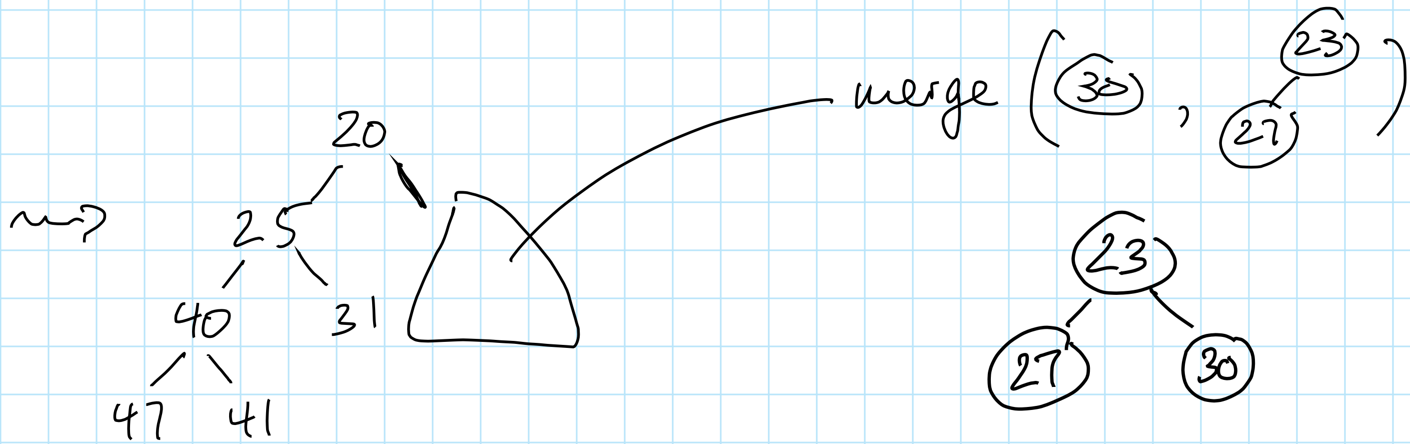
(a) →



(b) →



J



merge
20
25
40
31
47
41
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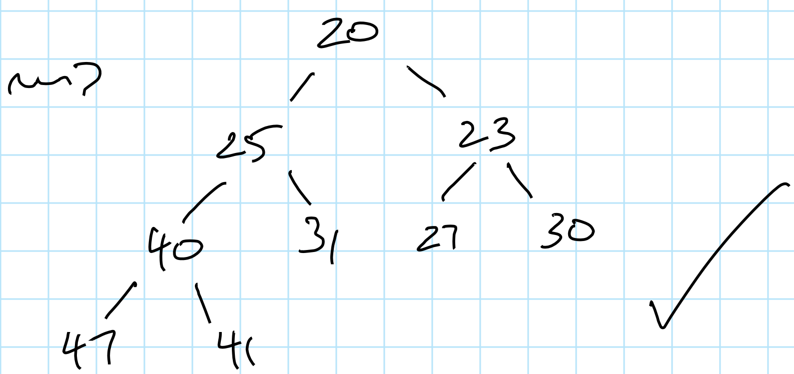
23
27



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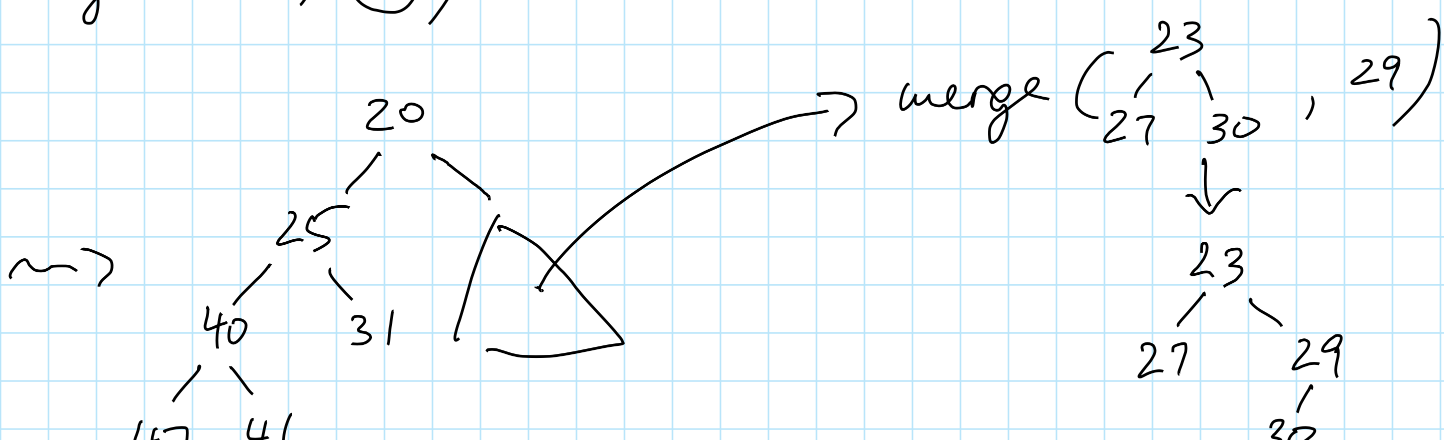
J'

merge (J', 29)



20
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41
merge
23
27
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29

analysis

- at each node visited in rec. calls, constant time needed
- visit only nodes on right path of leftist heap ($O(\log(N))$ many)

$\rightsquigarrow T_{worst}(N) = O(\log(N))$.

insertion : merge of a leftist heap with another leftist heap, of size 1

$\rightsquigarrow T_{worst}(N) = O(\log(N))$

deleteMin : destroy root ($O(1)$) and merge resulting leftist heaps (subtrees of the original root) ($O(\log(N))$)

$\rightsquigarrow T_{worst}(N) = O(\log(N))$

2.5 SKEW HEAPS

version of leftist heap that requires no npl information

$T_{worst}(N) = \Theta(N)$ for all ops, but amortized time $O(\log(N))$ per op.

(cf. AVL tree $\iff$ splay tree
to
leftist heap $\iff$ skew heap

merge : as for leftist heaps, but

(b) **always** swap children of heap1

except for the case that root is the maximal node in rightmost path (empty right child)

<u>NOTE</u> · for a single op., a right path might be long!

· no space for npl information needed

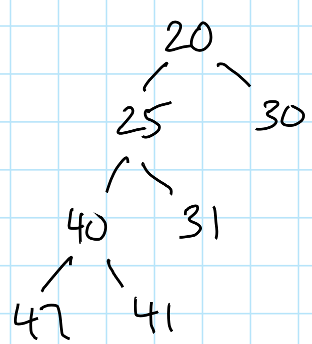
· no tests for npl needed for swaps

Open problem T_{avg} for ops on both leftist and skew heaps.

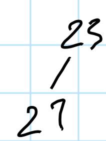
Example for skew heaps
(will be explained next class):

<u>Example 24.</u>

merge



with



⇝