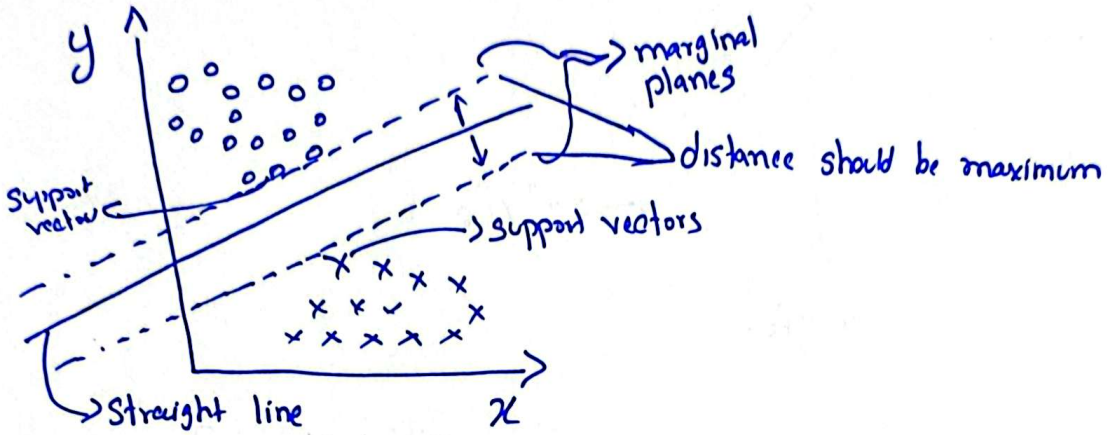
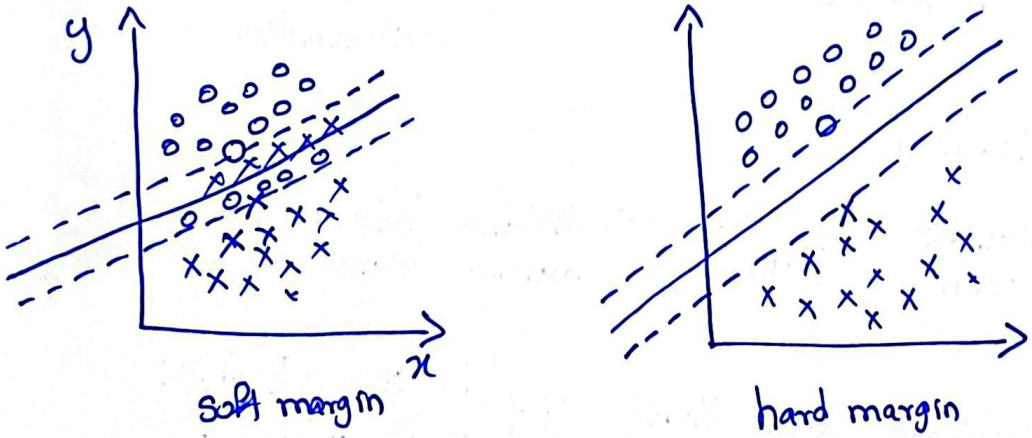
Support Vector Machines

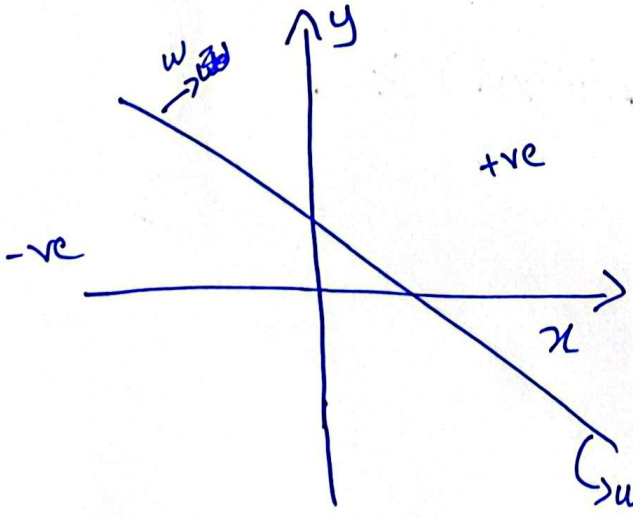
⟶ Support Vector Classifier (SVC)



> Soft Margin and Hard Margin



> Maths Intuition



$$ax + by + c = 0$$

$$\Downarrow$$

$$w_1 x_1 + w_2 x_2 + b = 0$$

$$w^T x + b = 0$$

$$\Downarrow$$

$$b = 0$$

$$w^T x = 0$$

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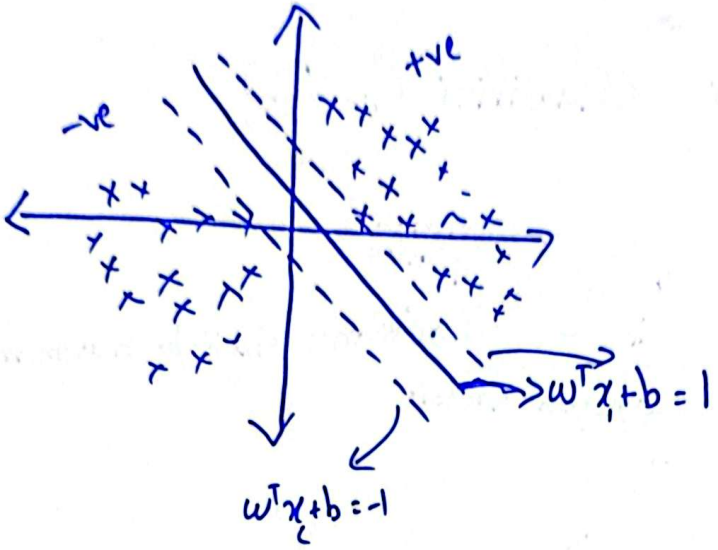

$$\begin{array}{r} w^T x_1 + b = 1 \\ - \; w^T x_2 + b = -1 \\ \hline w^T (x_1 - x_2) = 2 \end{array} \Longrightarrow \frac{w^T (x_1 - x_2)}{\|w\|} = \frac{2}{\|w\|}$$

cost function

Cost Function

Maximize $_{w,b}$ $\frac{2}{\|w\|}$ $\Rightarrow$ distance between marginal planes

Constraint such that $y_i \begin{cases} 1 & \text{if } w^T x + b \geq 1 \\ -1 & \text{if } w^T x + b < 1 \end{cases}$

For all correct points

constraint $\longrightarrow y_i * (w^T x + b) \geq 1$

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> Cost Function

$$\underset{w,b}{\text{Maximize}} \quad \frac{2}{\|w\|} \Rightarrow \text{distance between marginal planes}$$

Constraints:

① $y_i \begin{cases} 1 & \text{if } w^T x + b \geq 1 \\ -1 & \text{if } w^T x + b < 1 \end{cases}$

② for all correct points

$$y_i * (w^T x + b) \geq 1$$

$$\underset{w,b}{\text{Maximize}} \quad \frac{2}{\|w\|} \Rightarrow \frac{\|w\|}{2}$$

Cost Function of SVC

$$\underset{w,b}{\text{Minimize}} \quad \frac{\|w\|}{2} + C_j \sum_{i=1}^{n} \eta_i$$

(η: eta)

$\sum_{i=1}^{n} \eta_i$: hinge loss

$\frac{\|w\|}{2}$ ⇓ Soft Margin

C_j : how many points we want to avoid missclassification

η_i : summation of the distance of the incorrect data points from the margin plane

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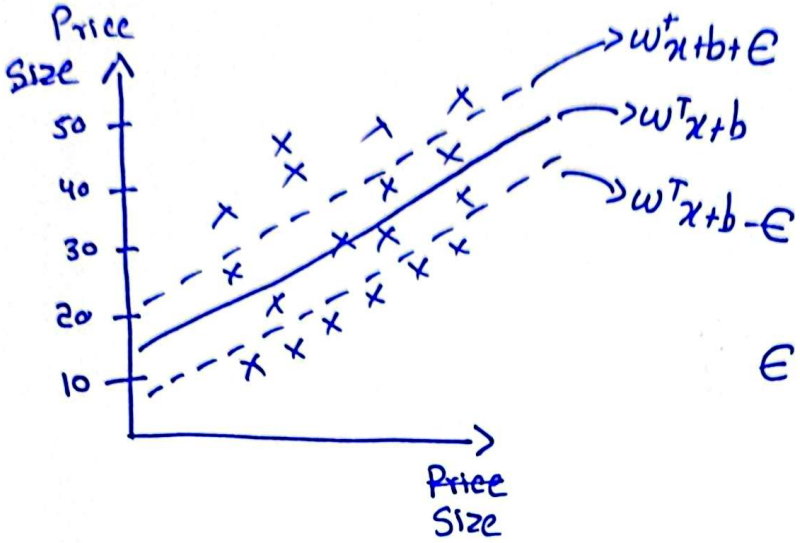
→ Support Vector Regressor



ϵ : marginal error

Cost Function

$$\min_{w,b} \frac{\|w\|}{2} + C_i \sum_{i=1}^{n} \eta_i$$

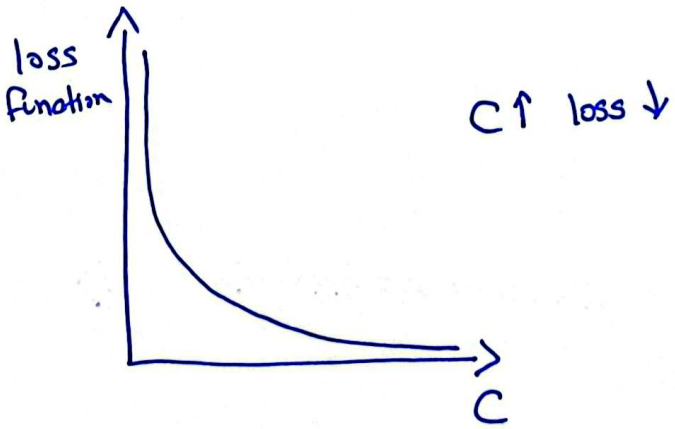
η_i :

Constraints

① $|y_i - w_i x_i| \leq \epsilon + \eta$

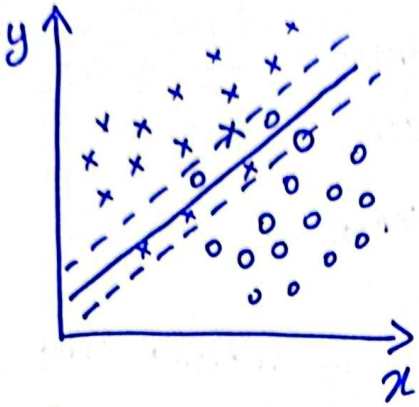
⇓

● loss function

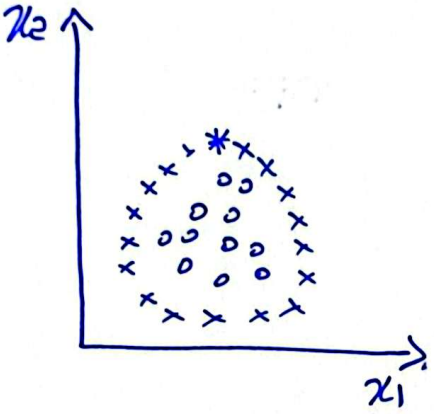


C ↑ loss ↓

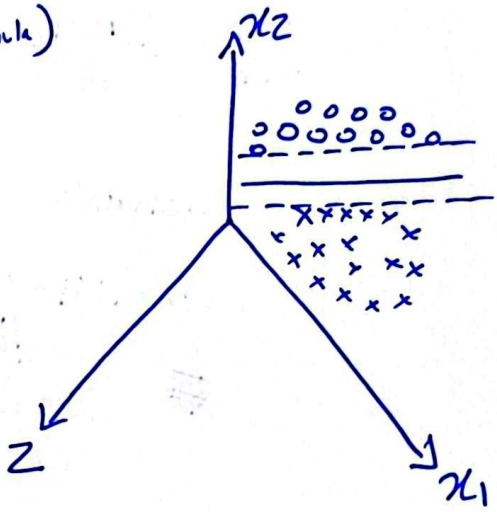
→ SVM Kernels



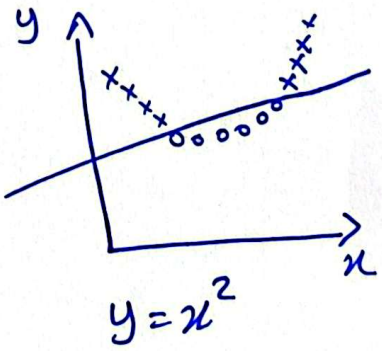
Linear SVC



(Mathematical Formula) Transformation ⟹



⟹



$y = x^2$

Different Types of Kernels

1. RBF Kernel
2. Polynomial Kernel
3. Sigmoid Kernel

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