## Student t Distribution

Student's t-distribution, also known as the t-distribution, is a probability distribution that is used in statistics for making inferences about the population mean when the sample size is small or when the population standard deviation is unknown.

## Formula

$$t = \frac{X - \mu}{\frac{S}{\sqrt{n}}}$$

## Degree of Freedom

$$df = n - 1$$

## Example

Suppose a company claims that the average weight of its packaged cereal is 300 grams. You suspect that the actual weight might be different. To test this, you take a random sample of 10 cereal boxes and measure their weights (in grams):

Sample weights: 298, 302, 299, 301, 297, 303, 300, 298, 299, 301

Step 1: State the hypotheses

$$H_0: \mu = 300$$

$$H_1: \mu \neq 300$$

Step 2: Calculate the sample mean and sample standard deviation

$$\overline{X} = \frac{298 + 302 + 299 + 301 + 297 + 303 + 300 + 298 + 299 + 301}{10} = 299.8$$

$$S = \sqrt{\frac{\sum(X_i - \overline{X})^2}{n-1}} = 1.932$$

$$t = \frac{X - \mu}{\frac{S}{\sqrt{n}}}$$

$$t = \frac{299.8 - 300}{\frac{1.932}{\sqrt{10}}} = -0.327$$

Step 4: Determine the Degrees of Freedom and Critical t-Value

$$df = 10 - 1 = 9$$

For a two-tailed test at the 0.05 significance level, the critical t-value from the t-distribution table with 9 degrees of freedom is approximately ±2.262.

Step 5: Compare the t-Statistic with the Critical t-Value

t-Statistic: -0.327

Critical t-Value: ±2.262

Since -0.327 is within the range of -2.262 to 2.262, we fail to reject the null hypothesis.