SiRD: An Epidemic Model with Social Distancing

A variation on the S.I.R. Model

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- Kermack, W; McKendrick, A (1991). "Contributions to the mathematical theory of epidemics—I". Bulletin of Mathematical Biology. 53 (1–2): 33–55. doi:10.1007/BF02464423. PMID 2059741.
- Kermack, W; McKendrick, A (1991). "Contributions to the mathematical theory of epidemics—II. The problem of endemicity". Bulletin of Mathematical Biology. 53 (1–2): 57–87. doi:10.1007/BF02464424. PMID 2059742.
- Kermack, W; McKendrick, A (1991). "Contributions to the mathematical theory of epidemics—III. Further studies of the problem of endemicity". Bulletin of Mathematical Biology. 53 (1–2): 89–118. doi:10.1007/BF02464425. PMID 2059743.

The S.I.R. Model

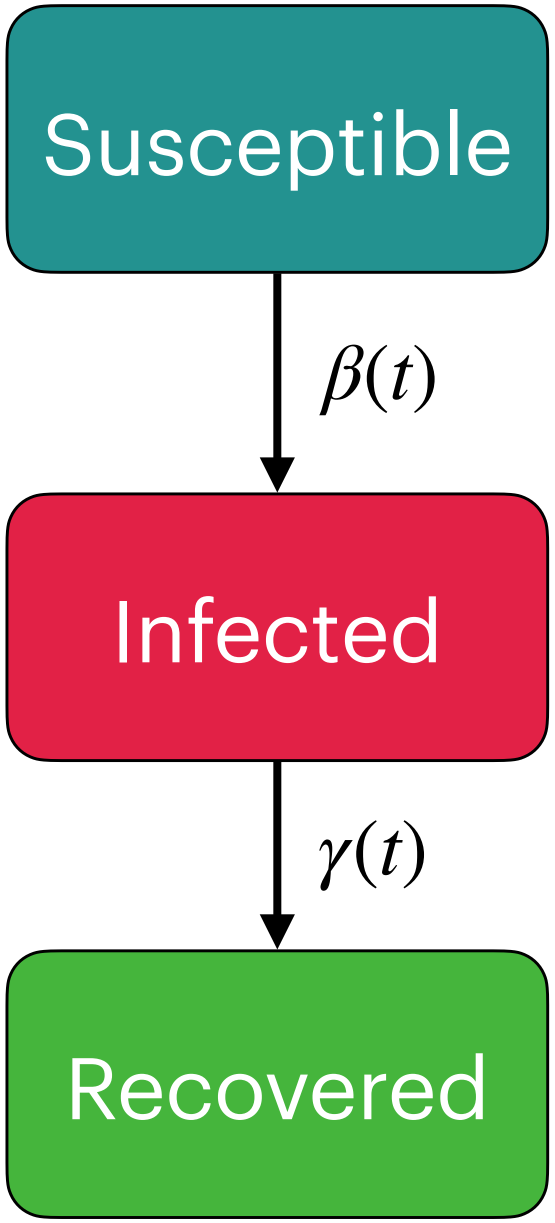
Mathematical modeling at its best

- The **S**usceptible, **I**nfected, **R**ecovered model is an epidemiological model for pandemics
- It was first published in three articles in 1927, 1932, and 1933 by Kermach and McKendrick
- It consists of three categories:
  - A population of **Susceptibles** who are exposed to infected
  - A population of **Infected** who infect susceptibles and later Recover
  - A population of **Recovered**

$$\frac{dS}{dt} = -\beta SI$$

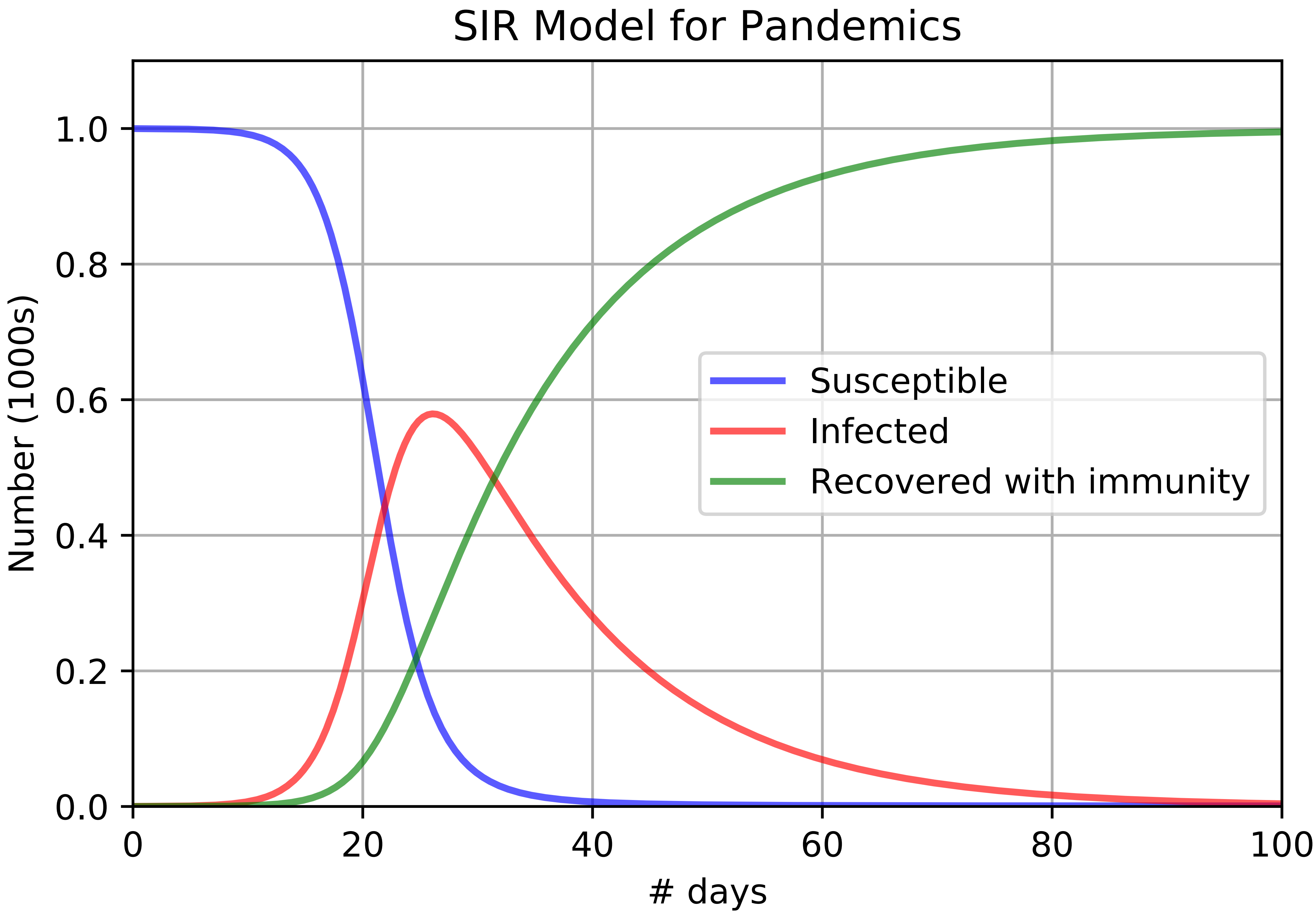
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$



β: average # contacts between **S** and **I**

γ: average fraction of recovered (~ 1/length of sickness)

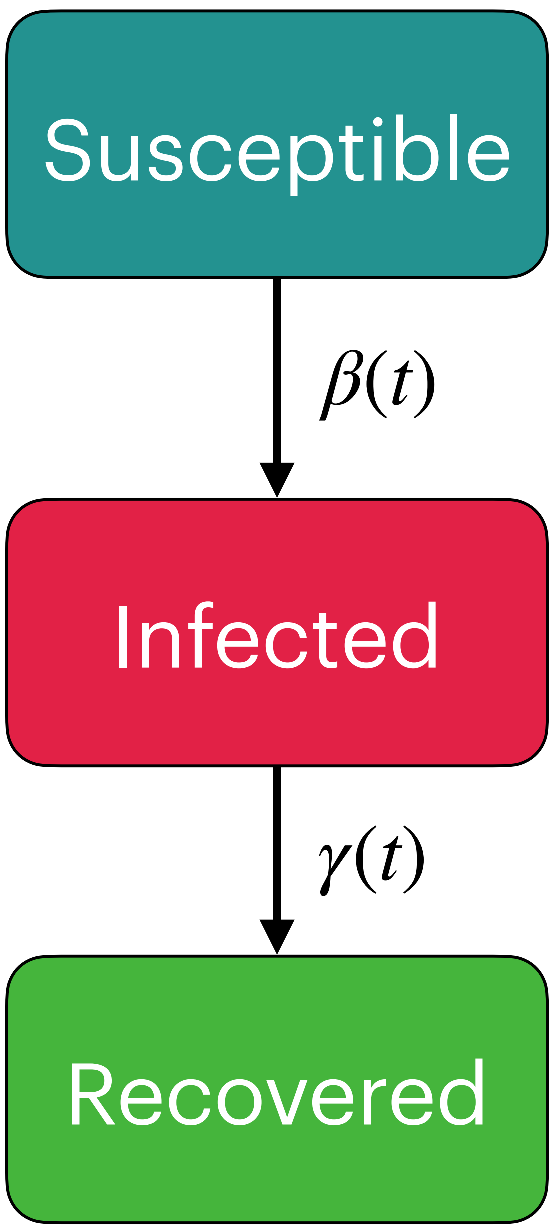


An example solution of the SIR model

The S.I.R. Model

Limitations

- In its original form and other variations, the SIR model does not account for **social distancing**
- I could only identify two papers in the literature that discuss social distancing with SIR
  - Those are done by modeling $\beta(t)$
- I wanted a more elegant alternative that models the behavior of a population and could better address the effects of social distancing



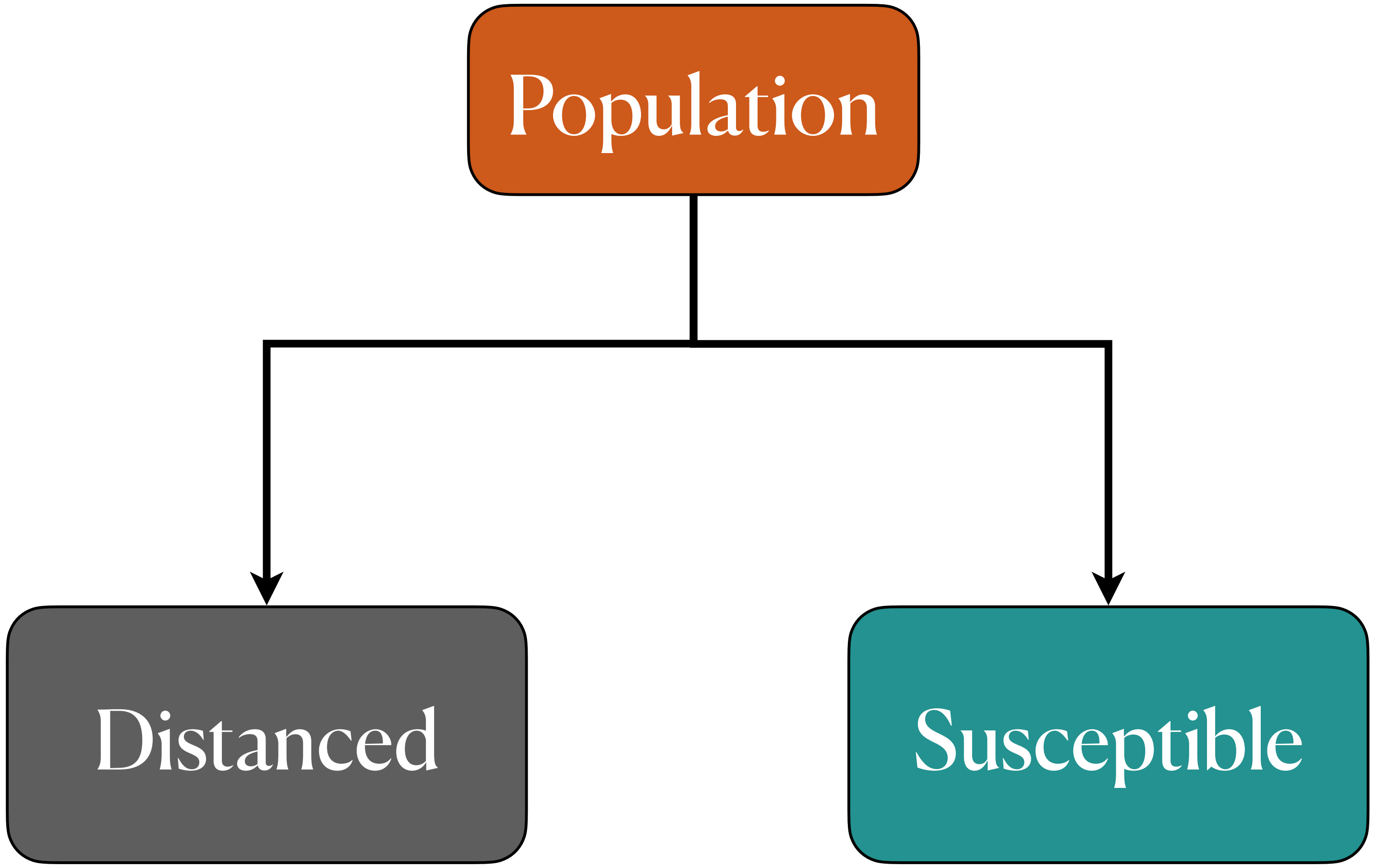
SiRD

A SIR Model with Social and Physical Distancing

S.I.R.D.

A SIR Model with Social and Physical Distancing

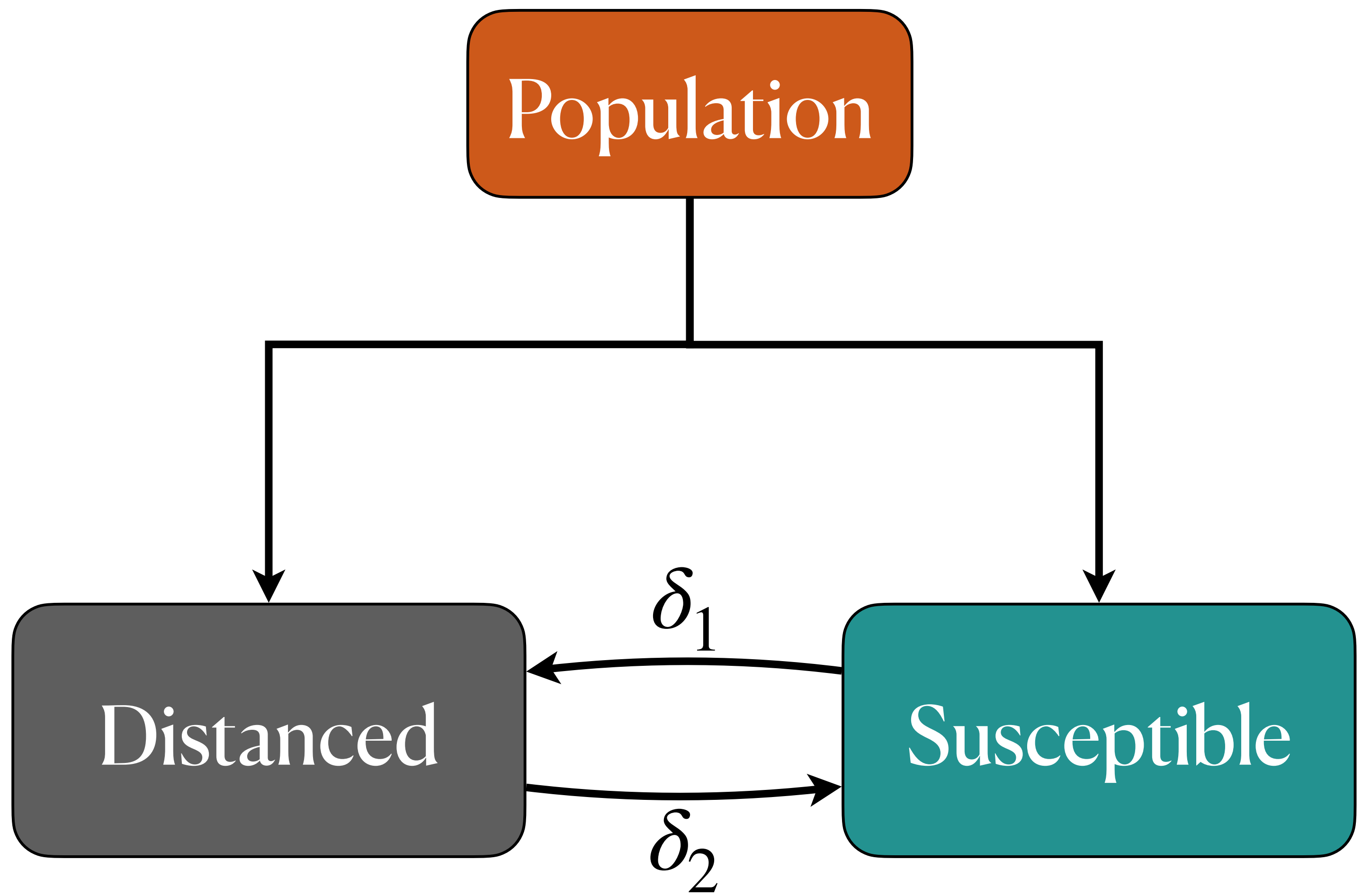
- A population P contains free-roaming individuals (Susceptible) as well as socially and physically Distanced folk.



S.I.R.D.

A SIR Model with Social and Physical Distancing

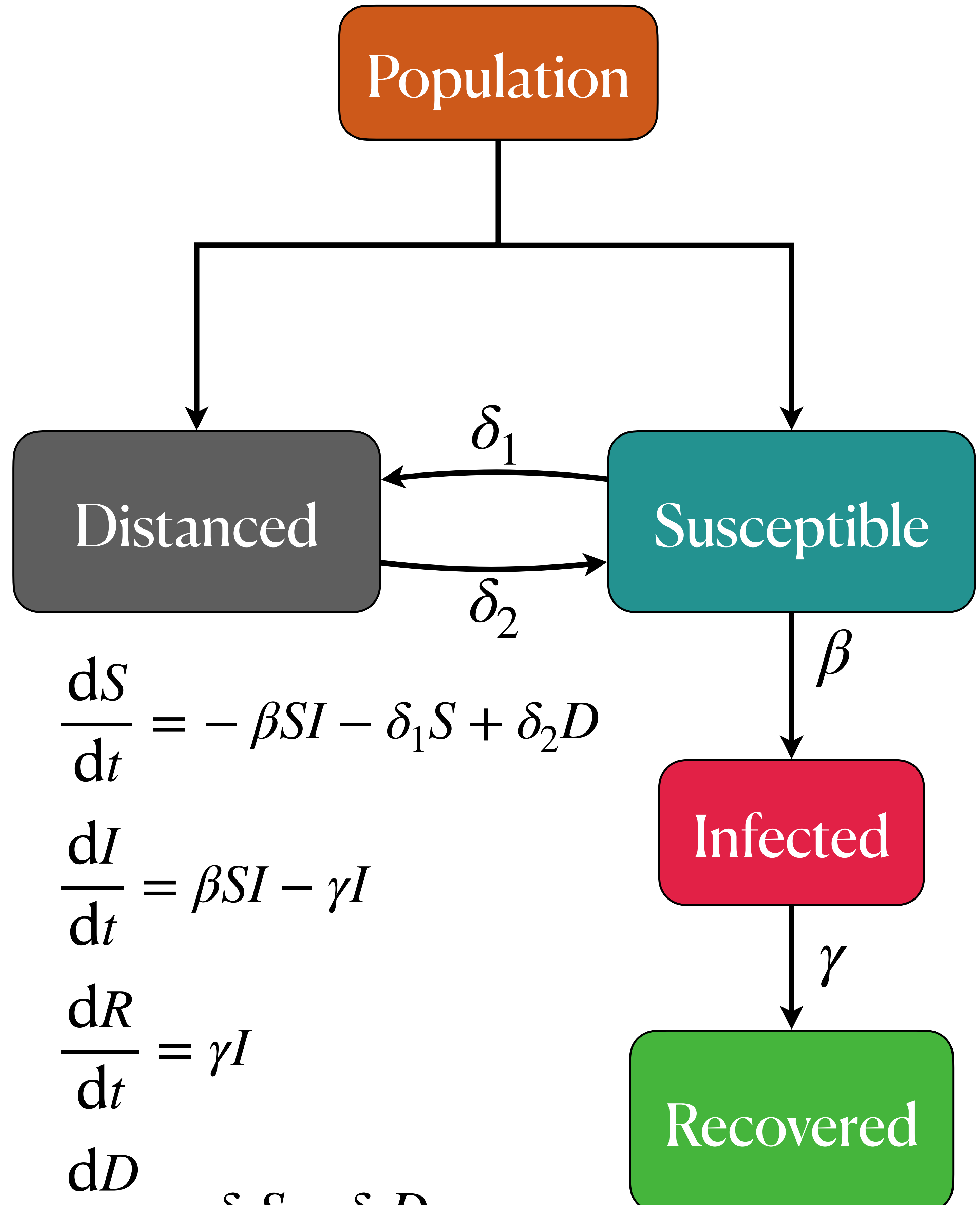
- A population P contains free-roaming individuals (Susceptible) as well as socially and physically Distanced folk.
- These two groups move in and out of those respective categories based on policy, needs, current infection rates etc…



S.I.R.D.

A SIR Model with Social and Physical Distancing

- A population P contains free-roaming individuals (Susceptible) as well as socially and physically Distanced folk.
- These two groups move in and out of those respective categories based on policy, needs, current infection rates etc...
- The Susceptibles have a probability of being infected when in contact with an infected person, as in the SIR model
- The Distanced do not interact at all with the infected and therefore have o likelihood of being infected



$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\beta SI - \delta_1 S + \delta_2 D$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \beta SI - \gamma I$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I$$

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \delta_1 S - \delta_2 D$$

Results

In all subsequent slides we use the following:

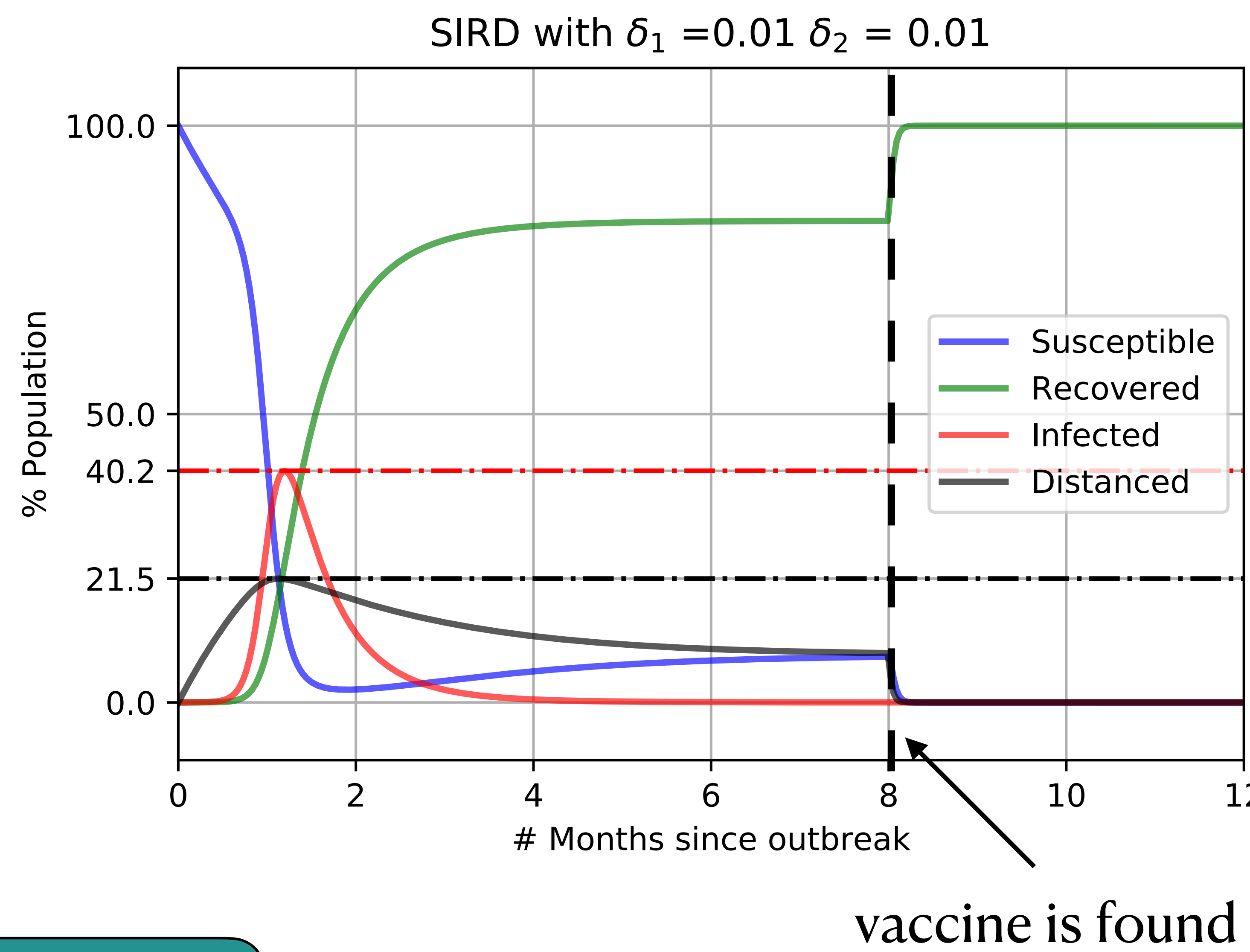
$$\beta = 0.5 \quad \gamma = 1/14$$

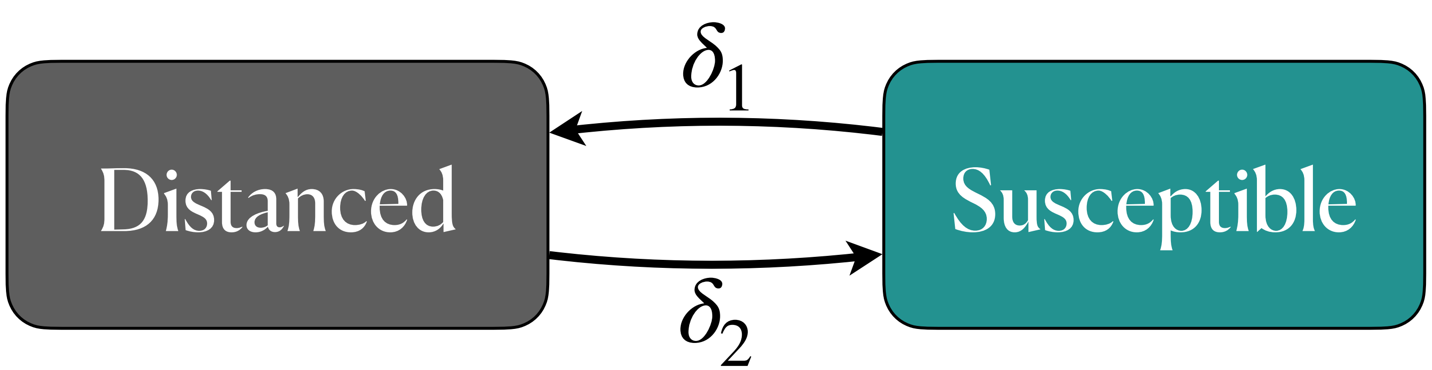
We assume that a vaccine is found after 8 months of the outbreak and mass vaccination takes place

Scenario A

General Social Distancing Until Vaccine is Found

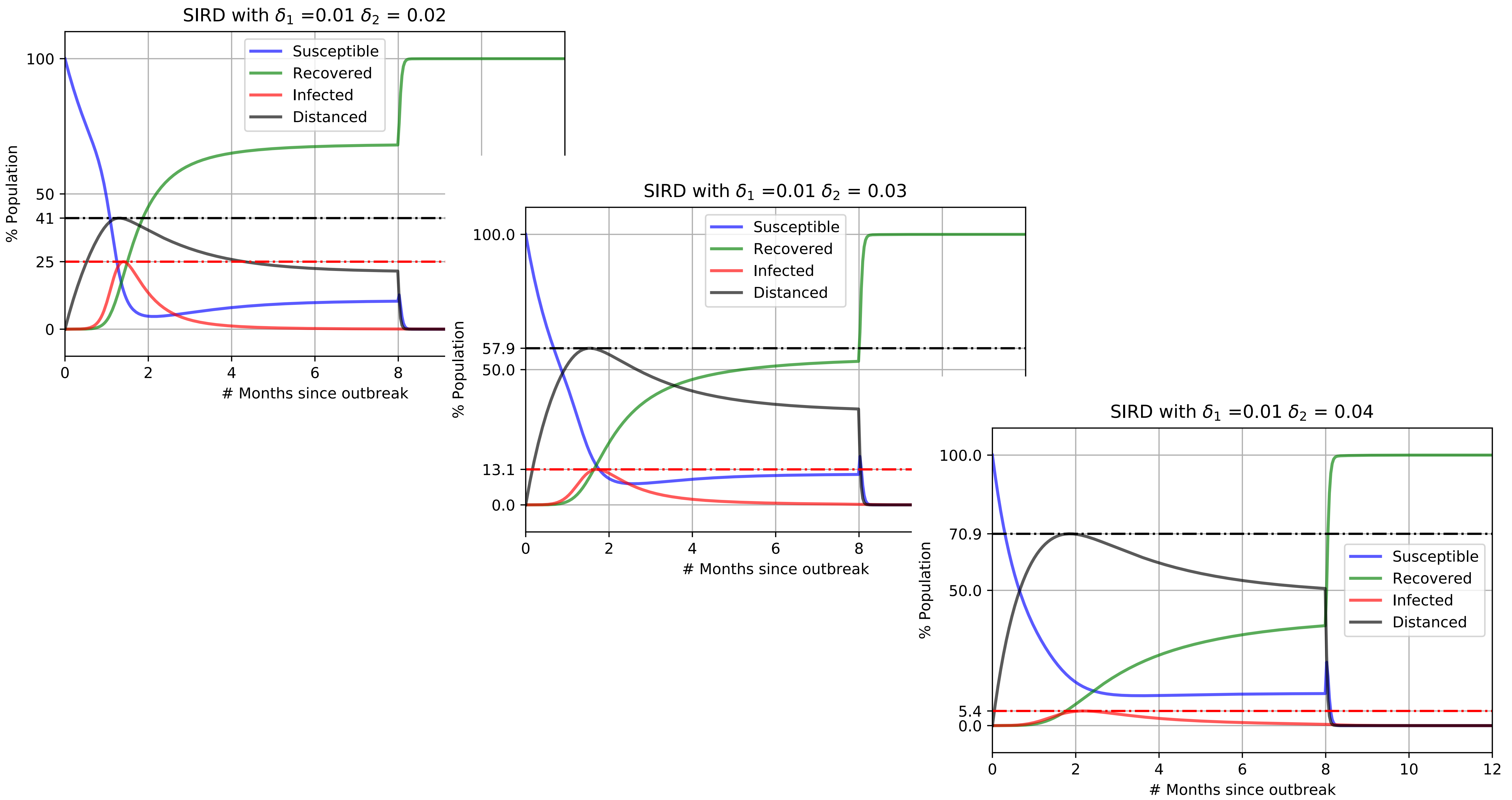
- In this case, there is fixed inflow and outflow of **D** and **S** (δ_1 and δ_2 are constant)





δ_1: movement from **Distanced** back into **Susceptible**

δ_2: movement from **Susceptible** towards **Distanced**



For a fixed δ_1 (movement from D into S) an increase in δ_2 (more social distancing) clearly flattens the curve. But this comes at a great expense - a lengthy period of social distancing until a vaccine is found.

Scenario B

Reactive Distancing

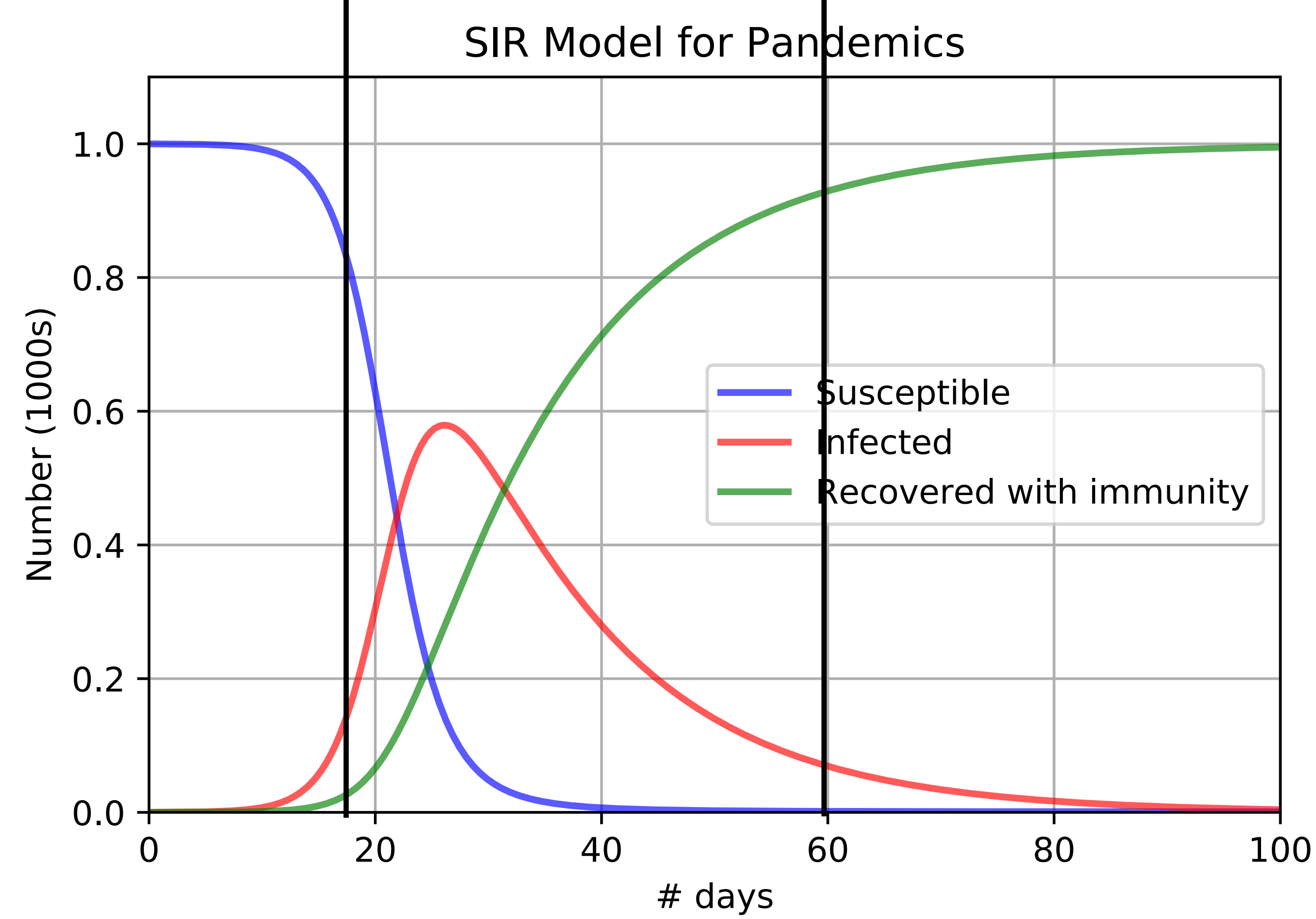
- A better social distancing strategy relies on triggering social distancing when the infected population reaches a certain threshold
- Then, social distancing is relaxed when the infected drop below a certain value

$$\delta_2(t) = \begin{cases} c_2, & I > I_1 \\ 0 & \text{otherwise} \end{cases}$$

start social distancing

$$\delta_1(t) = \begin{cases} c_1, & I < I_0 \\ 0 & \text{otherwise} \end{cases}$$

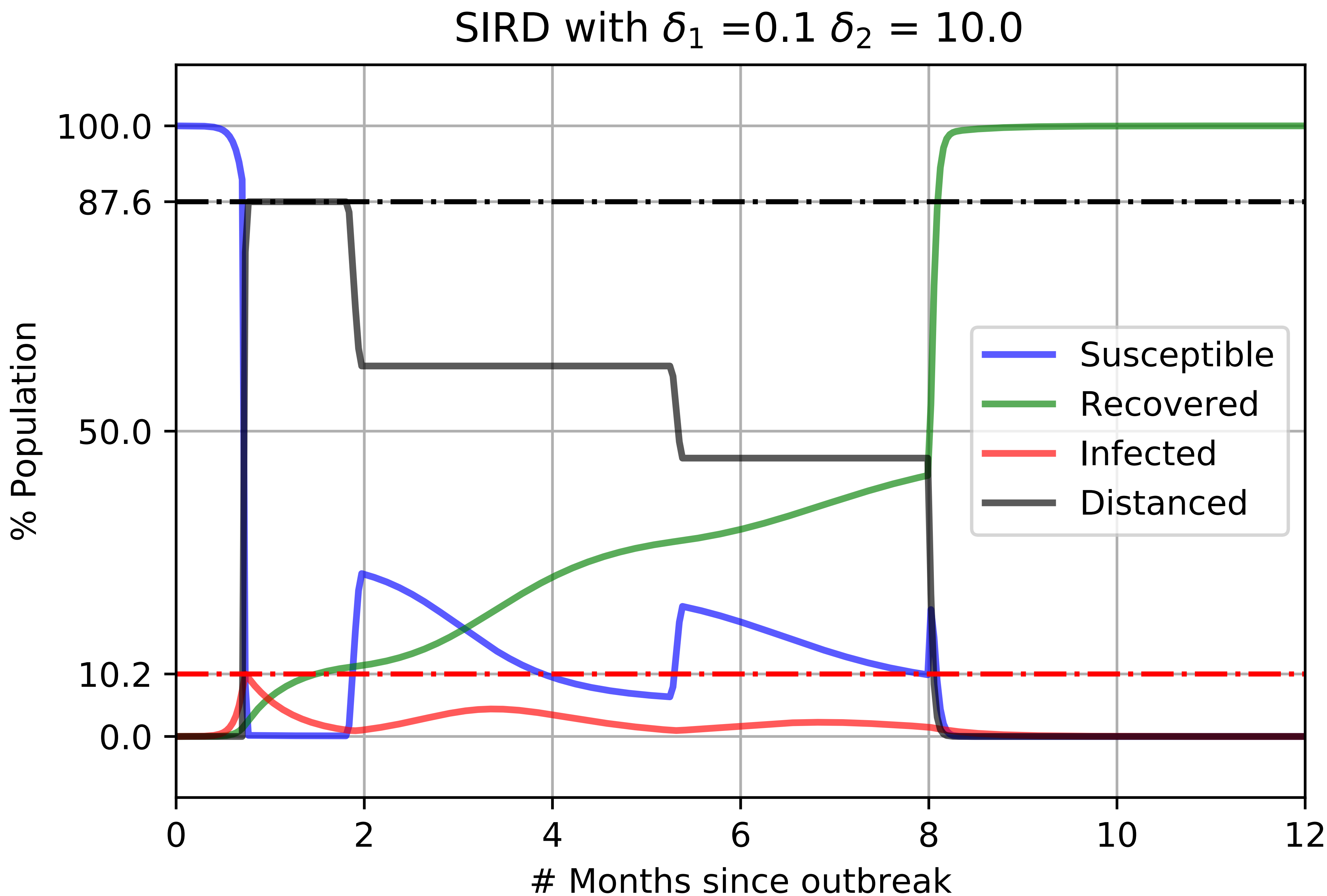
relax social distancing



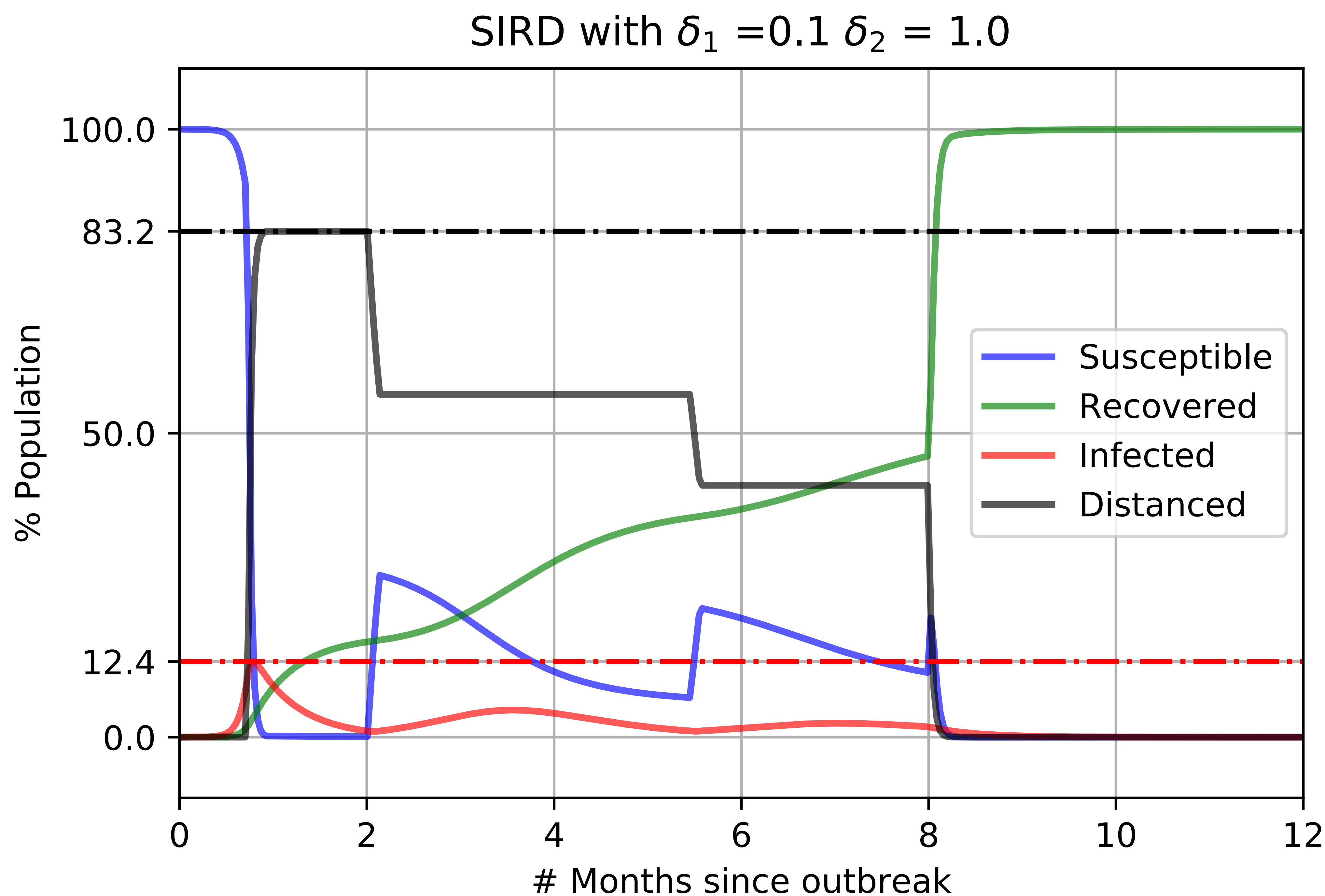
$$\delta_1(t) = \begin{cases} c_1, & I < I_0 \\ 0 & \text{otherwise} \end{cases} \qquad \delta_2(t) = \begin{cases} c_2, & I > I_1 \\ 0 & \text{otherwise} \end{cases}$$

$$\beta = 0.5$$
$$\gamma = 1/14$$

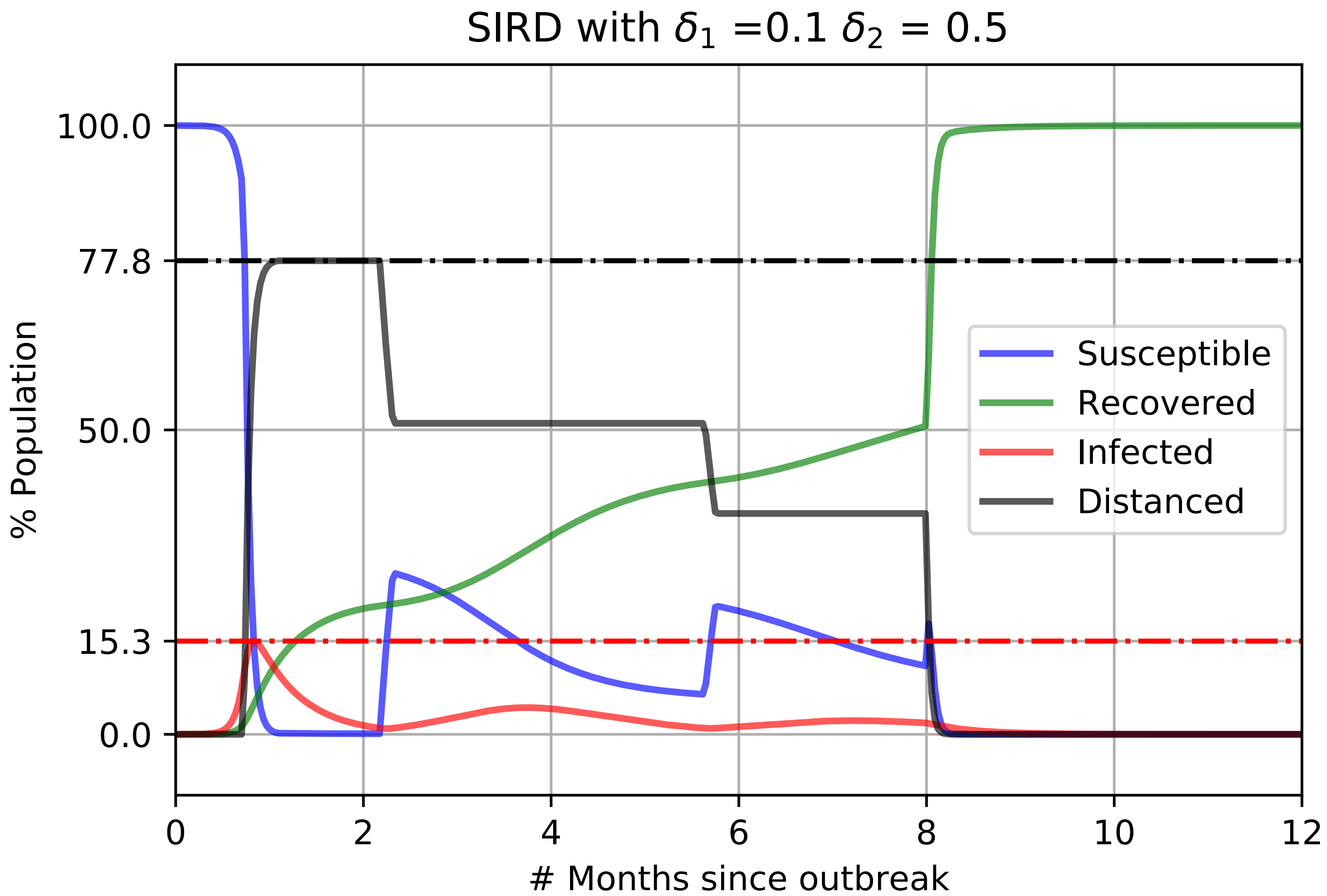
In the next few examples, social distancing is triggered when the # of infected is 10% of the population and relaxed when that number is 1%



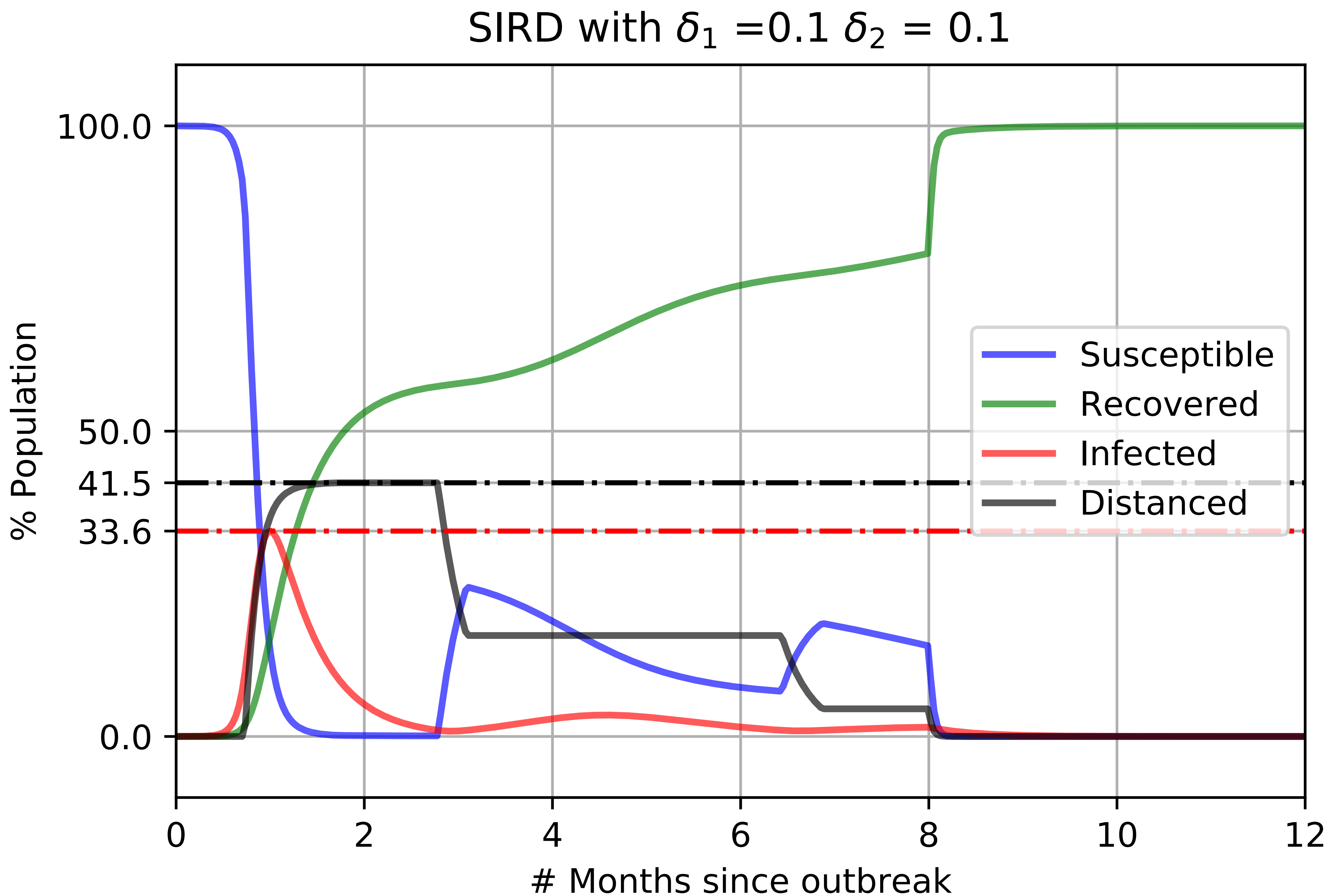
An extreme value for δ_2 (more social distancing) will require about 88% of the population to be isolated for about 1 month.



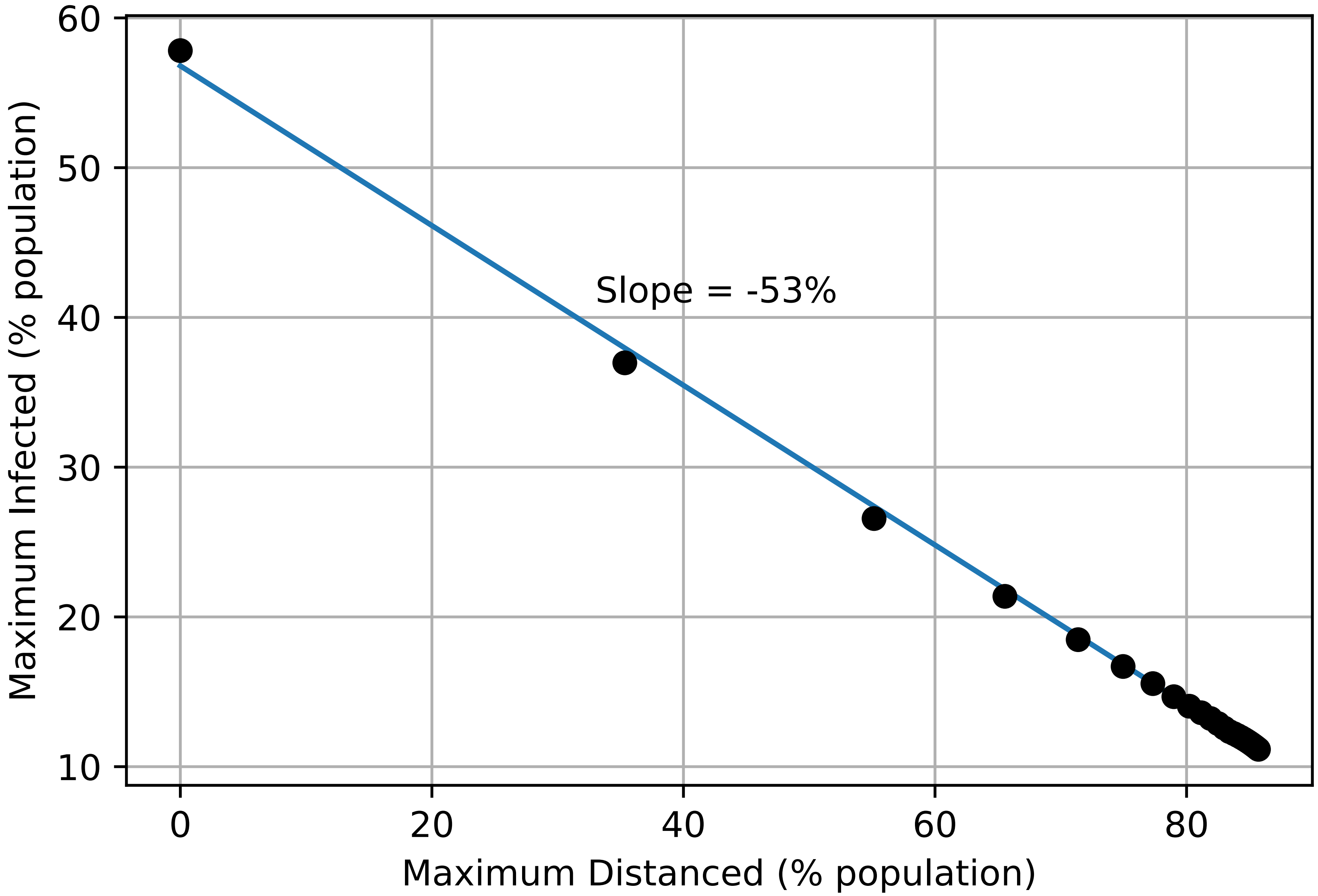
A value of δ_2 one order of magnitude lower than before will require about 83% of the population to be isolated for about 1 month. However, the peak infected reach 12.3% of the population. Note that although social distancing is triggered at 10% infections, the peak infected is NOT at 10% - there is a slight delay.



SIRD with δ_1 =0.1 δ_2 = 0.5
% Population
100.0
77.8
50.0
15.3
0.0
0
2
4
6
8
10
12
Months since outbreak
Susceptible
Recovered
Infected
Distanced



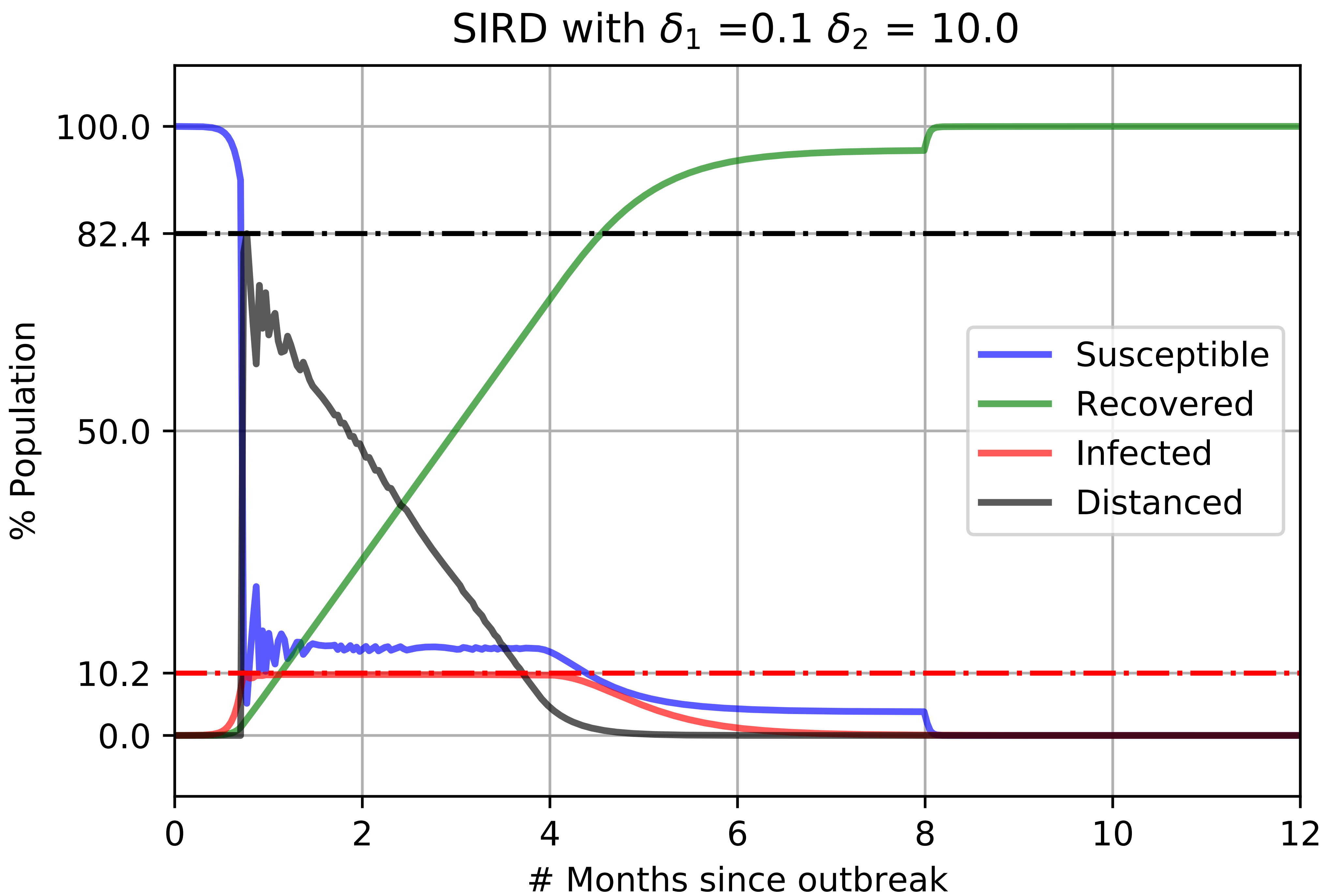
As δ_2 is decreased (less social distancing), the peak # of infected rises before it settles down to lower values.



One can also measure the effectiveness of social distancing by plotting the maximum infected versus the maximum distanced. The effect is linear with a slope of -0.53.

$$\delta_1(t) = \begin{cases} c_1, & I < I_0 \\ 0 & \text{otherwise} \end{cases} \qquad \delta_2(t) = \begin{cases} c_2, & I > I_1 \\ 0 & \text{otherwise} \end{cases} \qquad \beta = 0.5 \quad \gamma = 1/14$$

An interesting thing happens when $I_0 \to I_1$



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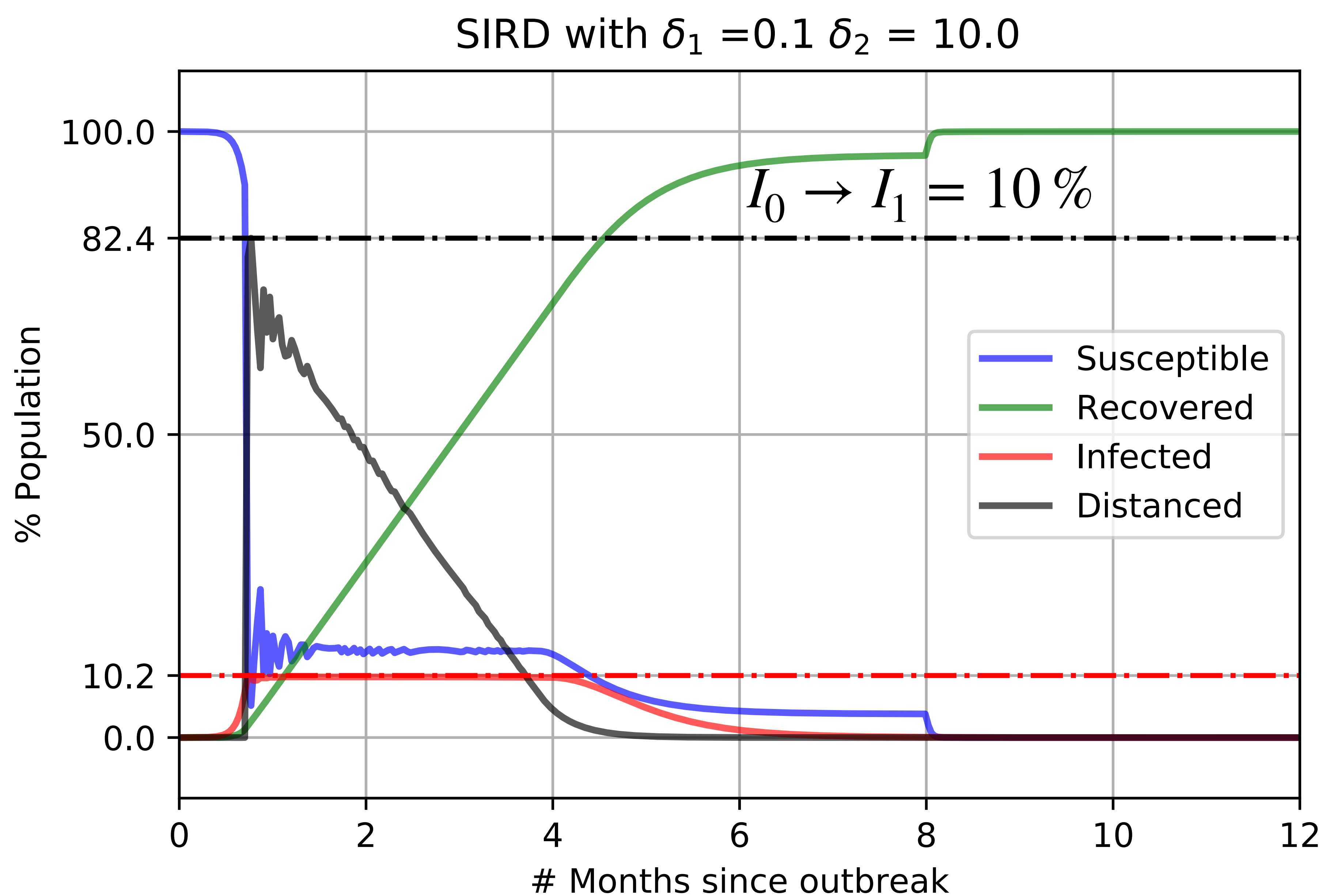
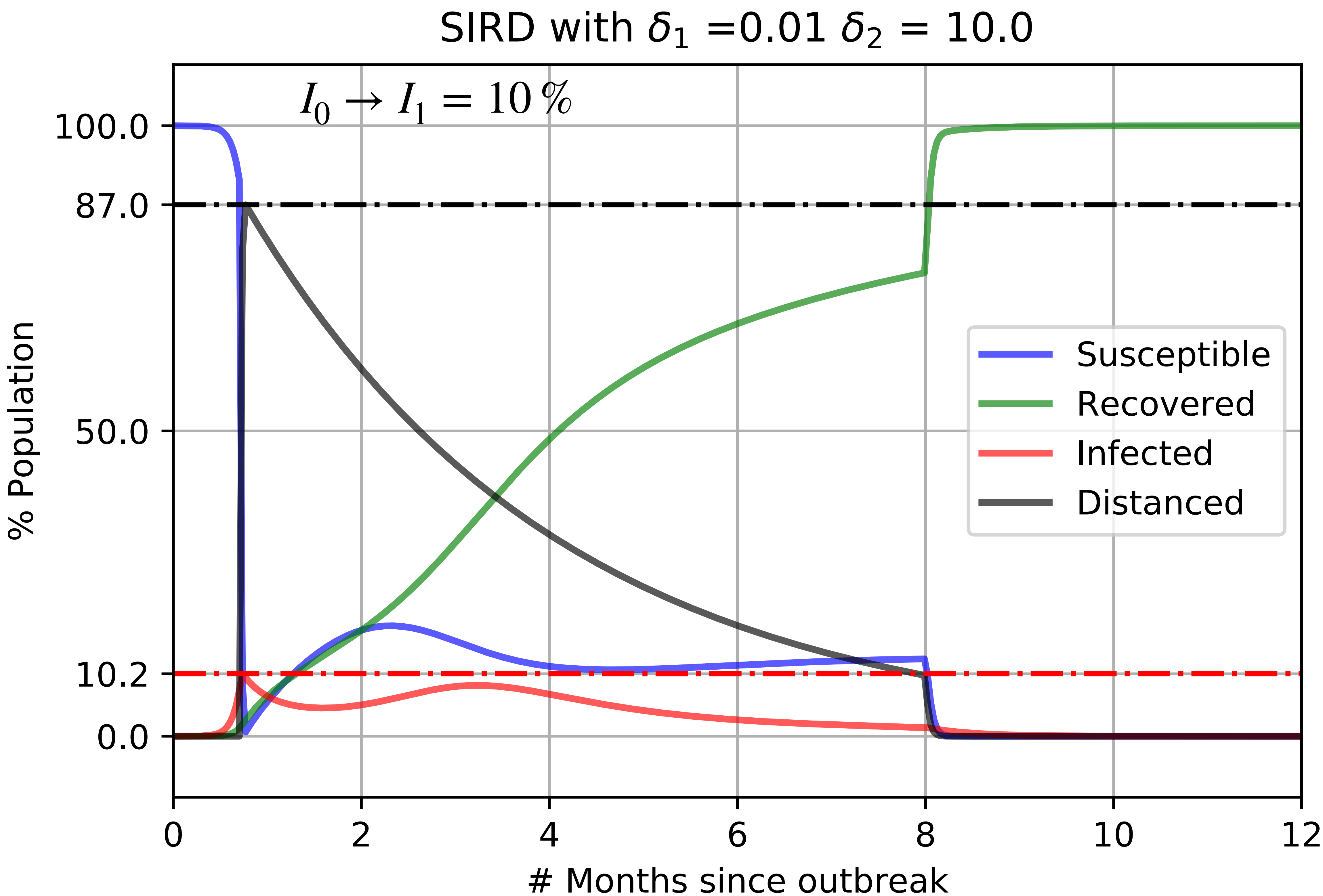


One is able to maintain the maximum # of infected at the desired rate

As $I_0 \to I_1$, reducing the migration from D to S ($\delta_1 \ll 1$) produces a smoother curve



You can test model parameters for yourself here:

http://tonysaad.net/SIRD/SIRD.html

or

https://saadtony.github.io/SIRD/SIRD.html

(click on load widgets - it will take ~30 s for code to load. You may have to refresh.)

Please report any bugs to: tony.saad@chemeng.utah.edu

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Conclusions

- It is possible to use modeling to maintain the total number of infected under the limits of hospitalization capacity
- Controlled social distancing (for example, elderly, those at risk, odd-even driver license # on different days, etc…) will produce more efficient results and a reduced impact on the economy
- It is possible to place a dollar amount on the impact of controlled social distancing
- Pandemic models, like population dynamics, resemble Chemical Kinetics and can be used as motivating examples for our students
- Chemical Engineers can do anything!