Abstract

Topology Packet Answers.

1 The definition of Topology

- Exercise 1.2
  a) $\tau = \{V \subset \mathbb{Z} \mid 0 \in V\}$.
  This is not a topology on $\mathbb{Z}$ because all sets must contain 0, therefore the empty set will not be included and τ is not a valid topology.
  b) $\tau = \{V \subseteq \mathbb{Z} \mid 0 \in V\}$.
  This is a topology on $\mathbb{Z}$.
  c) $\tau = \{V \subset \mathbb{Z} \mid 0 \in V and 1 \in V\}$.
  This is not a topology on $\mathbb{Z}$ becuase all sets must contain 0 and 1, therefore the empty set will not be included and τ is not a valid topology.

  d)$\tau = \{V \subset \mathbb{Z} \mid 0 \in V and 1 \in V\}$.
  This is a topology on $\mathbb{Z}$ because the empty set will be included because the elements 0 and 1 do not have to be in every open set.

  e)$\tau = \{V \subset \mathbb{Z} \mid V \text{ is finite. } \}$.
  This is not a topology on $\mathbb{Z}$ becuase the total set will not be included, becuase $\mathbb{Z}$ is not an finite set. f) $\tau = \{V \subset \mathbb{Z} \mid V \text{ is infinite. } \}$.
  This is not a topology on $\mathbb{Z}$ becuase the empty set will not be included.

  We can further generalize this by saying that all topologies on $\mathbb{Z}$ that require a certain element to exist cannot be topologies becuase they lack the existence of the empty set. We can also say that all topologies τ that require a certain element does not exist within all sets of τ is considered a valid topology.
  We can also say that if all sets within the topology τ must be infinite or finite, these will not be considered valid topologies.

- Exercise 1.3
  Among the following, which ones are topologies on the set $\mathbb{R}$ and which ones are not?
  a)$\tau = \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\phi, \mathbb{R}\}$.

  b)$\tau = \{[a, \infty) \mid a \in \mathbb{R}\} \cup \{\phi, \mathbb{R}\}$.
  Both of these are valid topologies on $\mathbb{R}$ as they include the total set and the empty set. The only differce is that one topology contains a and the other one does not. u

- Exercise 1.4
  Let X be any set.

  - a) What is the topology on X that has the most open sets? This is called the disceret topology on X.

Answer:
The topology with the most amount of open sets on X will include the total set, the empty set, and every subset, and the powet set $\mathbb{X}$ that includes all the subsets of X.

- b) What is the topology on X that has the least open sets? This is called the indiscete topology on X.
  Answer:
  The topology with the least amount of open sets would only contain the total set and the empty set.

- Exercise 1.5 Let X be an arbitrary set. Which ones of the following are topologies?
  - a) The *cofinite* topology: A set $V \subseteq X$ is open iff $[X \backslash V$ is finite or $V = \phi$. This topology can be defined on all sets X because first it includes $/phi$. Now we must prove this condition accounts for the total set. If V is empty, then $X \backslash V$ will be the total set, therefore this topology exists.
  - b) The *coinifnite* topology: A set $V \subseteq X$ is open iff $[X \backslash V$ is infinite or $V = \phi$ or $V = X$.
  - c) The *cocountable* topology: A set $V \subseteq X$ is open iff $[X \backslash V$ is countable or $V = \phi \text{or} V = X$.