What is a set?

A **set** X is a collection of items. A set X can be used to define a group of numbers, such as the set $\mathbb{R}$ of all real numbers.
Let's look at some examples of a **set**.
What do the following sets represent?

- $\mathbb{Q}$
- $\mathbb{Z}$
- $\mathbb{N}$

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Subsets

A **subset** V is a set such that all elements are contained in another set. A

subset C of V can be written as $C \subset V$. This can read as V contains C. What are some example subsets of the following sets?

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Set Builder Notation

Set builder notation is a notation that can be used to describe sets neatly and quickly. Set builder notation generally follows the following conventions.

$$V = \{ \text{ element } \mid \text{ condition } \}$$

Some textbooks may use a colon $:$ instead of the bar, $|$. Essentially, this reads the set V contains all elements such that the condition is satisfied. Set builder notation is very useful for describing sets such that all terms satisfy a condition or pattern.

Let's do some examples.

Set Builder Notation Examples

Turn and talk to a partner about the following sets.

What do these sets mean?

- $X = \{x \mid x = 2n - 1\}$, $n \in \mathbb{N}$
- $X = \{x \mid x = 2n\}$, $n \in \mathbb{N}$

  Consider X to be a arbitrary set. Define V to be a subset of X.
- $V =$

A topology τ on a set X is a group of subsets of X that satisfy 3 conditions. A subset that satisfies these conditions is called *open*. Consider the sets U and V.

- The total set X and the empty set $\emptyset$ must be open sets.
- The union of two open sets must be an open set. For example, $U \cup V$ must be an open set.
- The intersection of two open sets must be an open set. For example, $U \cap V$ must be an open set.

A topological space is a pair (X, τ) where X is a set and τ is a topology on X.

Some Exercises

Let (X, τ) be a topological space. Prove each of the following statements *true* or *false*.

- The intersection of any 3 open sets is an open set.
- The intersection of finitely many open sets is open.
- The intersection of open sets is open.