BELLMAN FORD ALGORITHM

-> It is also for shortest path

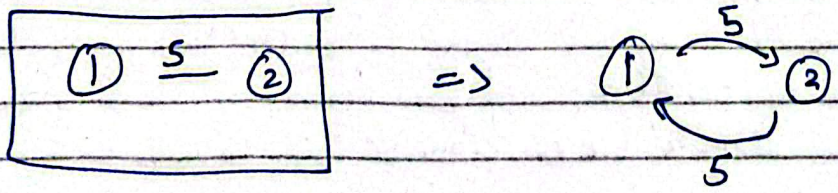
-> Dijkstras' fail if the graph have negative weights.

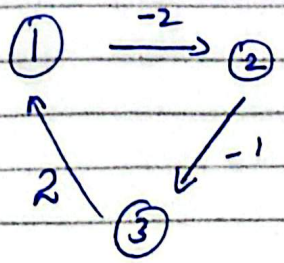
-> Dijkstra gives tle if negative cycles exist.

-> It helps us to detect negative cycles as well.

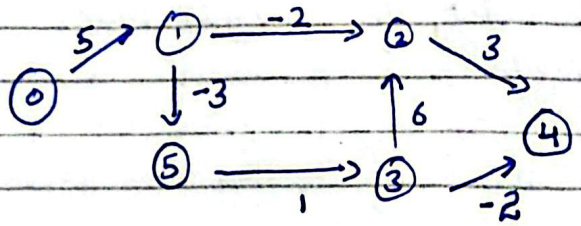
-> It is applicable only in DG directed graphs.

If it is undirected we should convert it to directed.





negative cycle if any weight is negative.



(u, v, wt)
(3, 2, 6)
(5, 3, 1)
(0, 1, 5)
(1, 5, -3)
(1, 2, -2)
(3, 4, -2)
(2, 4, 3)

* Relax all the edges N-1 times.

edges can be in any order

* Relaxation:

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if ( dist[u]+wt < dist[v])
    dist[v] = dist[u]+wt
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Suppose if

dist[s]+1 to reach 3 then

if N = 6

then 5 iterations of relaxation should be done.

dist [0, ∞, ∞, ∞, ∞, ∞]
0 1 2 3 4 5

in every iteration we will go through all the edges.

1 iteration.

if (dist[3] + 6 < dist[2])
∞ < ∞

if (dist[5] + 1 < dist[3])
∞ < ∞

if (dist[0] + 5 < dist[1]) ✓
5 < ∞

So dist becomes
dist = [0, 5, ∞, ∞, ∞, ∞]

if (dist[1] + -3 < dist[5])

So dist becomes
dist [0, 5, ∞, ∞, ∞, 2]
0 1 2 3 4 5

if (dist[1] + (-2) < dist[2])
dist[0, 5, 3, ∞, ∞, 2]
0 1

if (dist[3] - 2 < dist[4])
∞

if [dist[2] + 3 < dist[4])
3+3 < ∞ ✓

dist = [0, 5, 3, ∞, 8, 2]
0 1 2 3 4 5

At the end of first iteration we have

dist [0, 5, 3, ∞, 6, 2]
0 1 2 3 4 5

2nd iteration:

if dist[3] + 6 < dist[2]

if (dist[5] + 1 < dist[3])
2 + 1 < ∞
So dist becomes
dist = [0, 5, 3, 3, 6, 2]

⋮

if (dist[3] - 2 < dist[4])
3 - 2 = 1 < 6
So
dist = [0, 5, 3, 3, 4, 2]

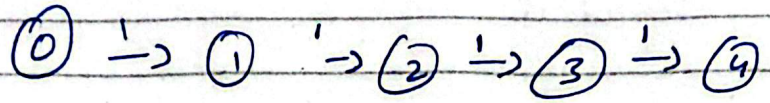
⋮

Similarly we will do 1st, 2nd ... 5th.

Q: why N-1 ?

Q: How to detect negative cycle.

Intuition why N-1 iterations?



(u, v, wt)
(3, 4, 1)
(2, 3, 1)
(1, 2, 1)
(0, 1, 1)

So we have to do 4 iterations.

(1) dist [0, ∞, ∞, ∞, ∞]

dist[3] + 1 < dist[4] ×
∞

dist[2] + 1 < dist[3] ×

dis ist[1] + 1 < dist[2]

dist[0] + 1 < dist[1] ✓

So

dist = [0, 1, ∞, ∞, ∞]

(2) dist = [0, 1, 2, ∞, ∞]

(3) dist = [0, 1, 2, 3, ∞]

(4) dist = [0, 1, 2, 3, 4]

So thats why N-1 iterations.

Q: How to detect Negative cycles?

if after the N-1 iteration still distance reduces then the graph have negative cycle