

Tabulated supercritical thermodynamics for compressible Navier-Stokes solvers

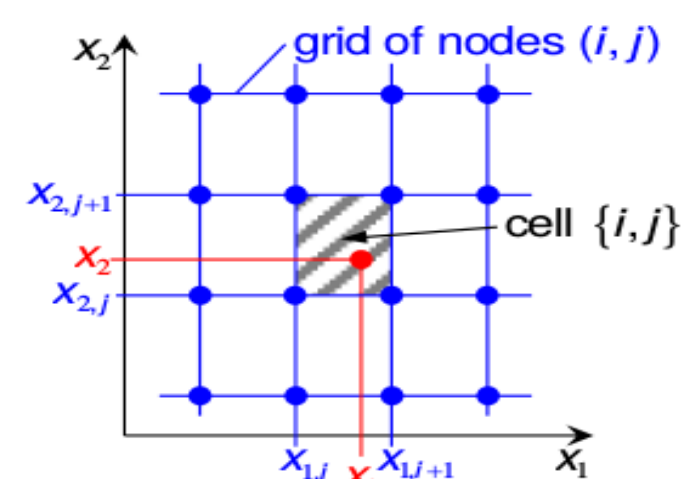


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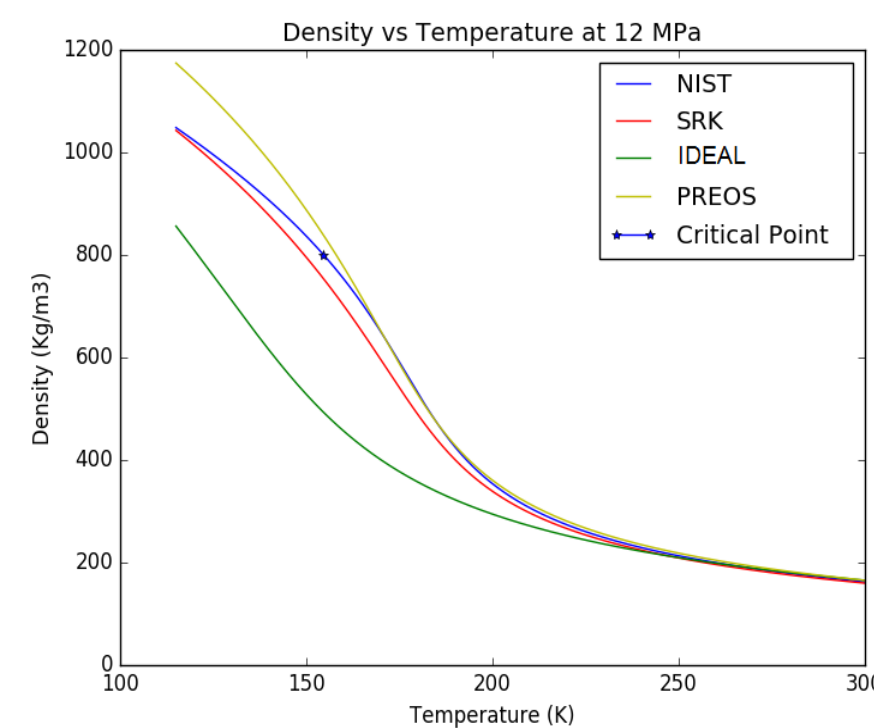
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Introduction

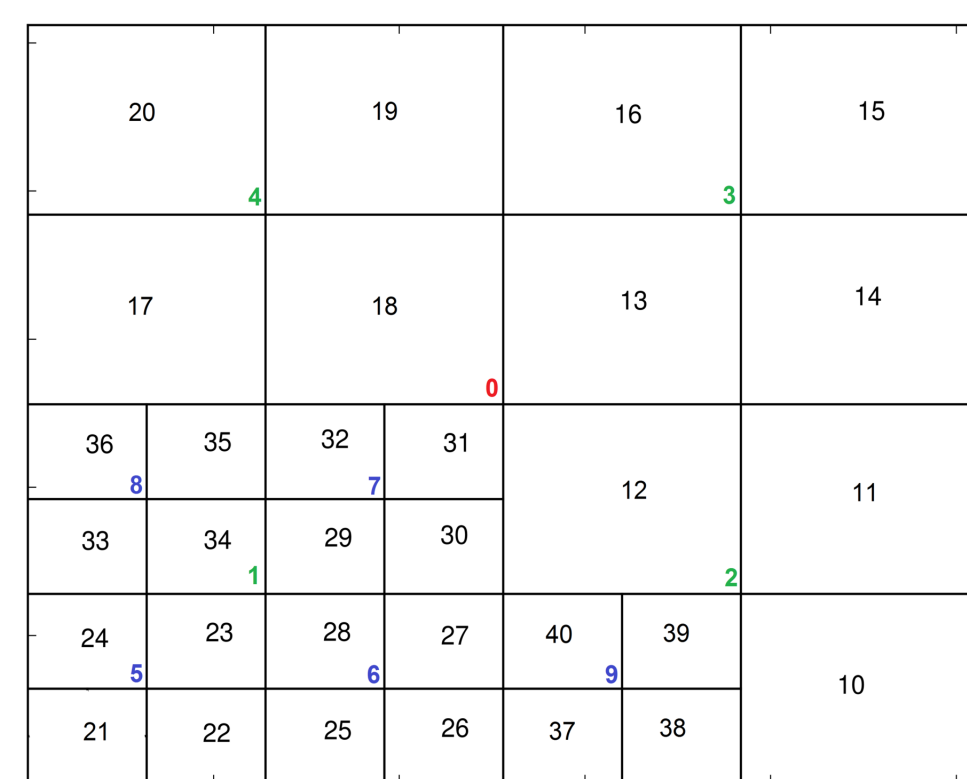
Non-ideal state equations are needed to compute a growing number of engineering-relevant problems. The additional computational overhead from the complex thermodynamics accounts for a significant portion of the total computation, especially the near-critical or transcritical thermodynamic regimes. A compromise between computational speed and the accuracy of the thermodynamic property evaluations results in a propagation of the error from the thermodynamics to the hydrodynamic computations.



Real fluid thermodynamics



Block structured AMR



84	83	80	79	68	67	64	63
81	82	77	78	65	66	61	62
72	71	76	75	56	55	60	59
69	70	73	74	53	54	57	58
36	35	32	31	52	51	48	47
33	34	29	30	49	50	45	46
24	23	28	27	40	39	44	43
21	22	25	26	37	38	41	42

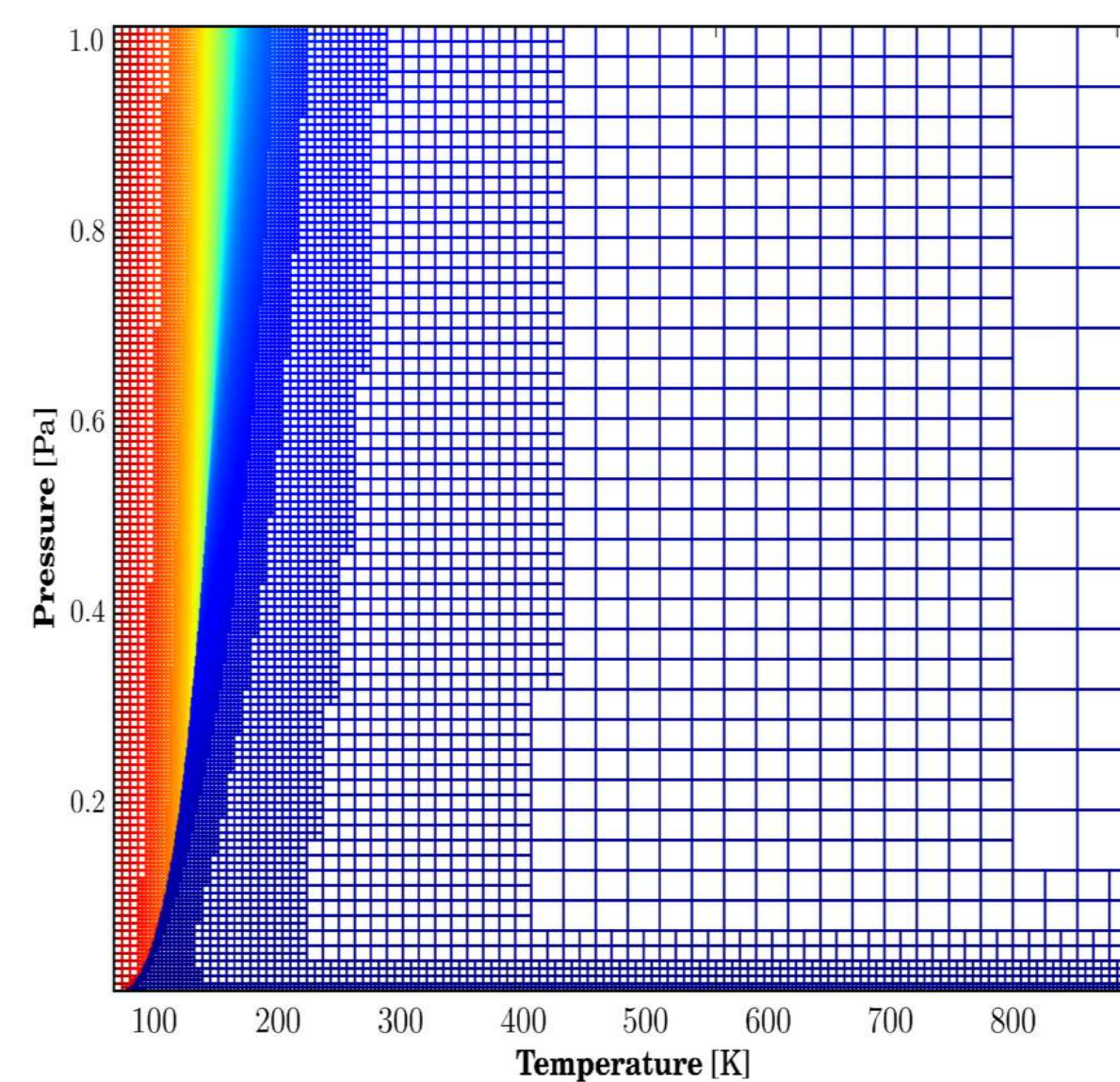
Figure 1: Uniform index for a maximum refinement level of 3

Bezier Patch based AMR

In the block structured AMR table, the underlying criteria for refinement is based on the bivariate approximation. Rather than tabulating solely on the basis of bilinear reconstruction error we chose to implement Bezier patches that are approximated surfaces. The approximated function of two independent variables is given by [1]

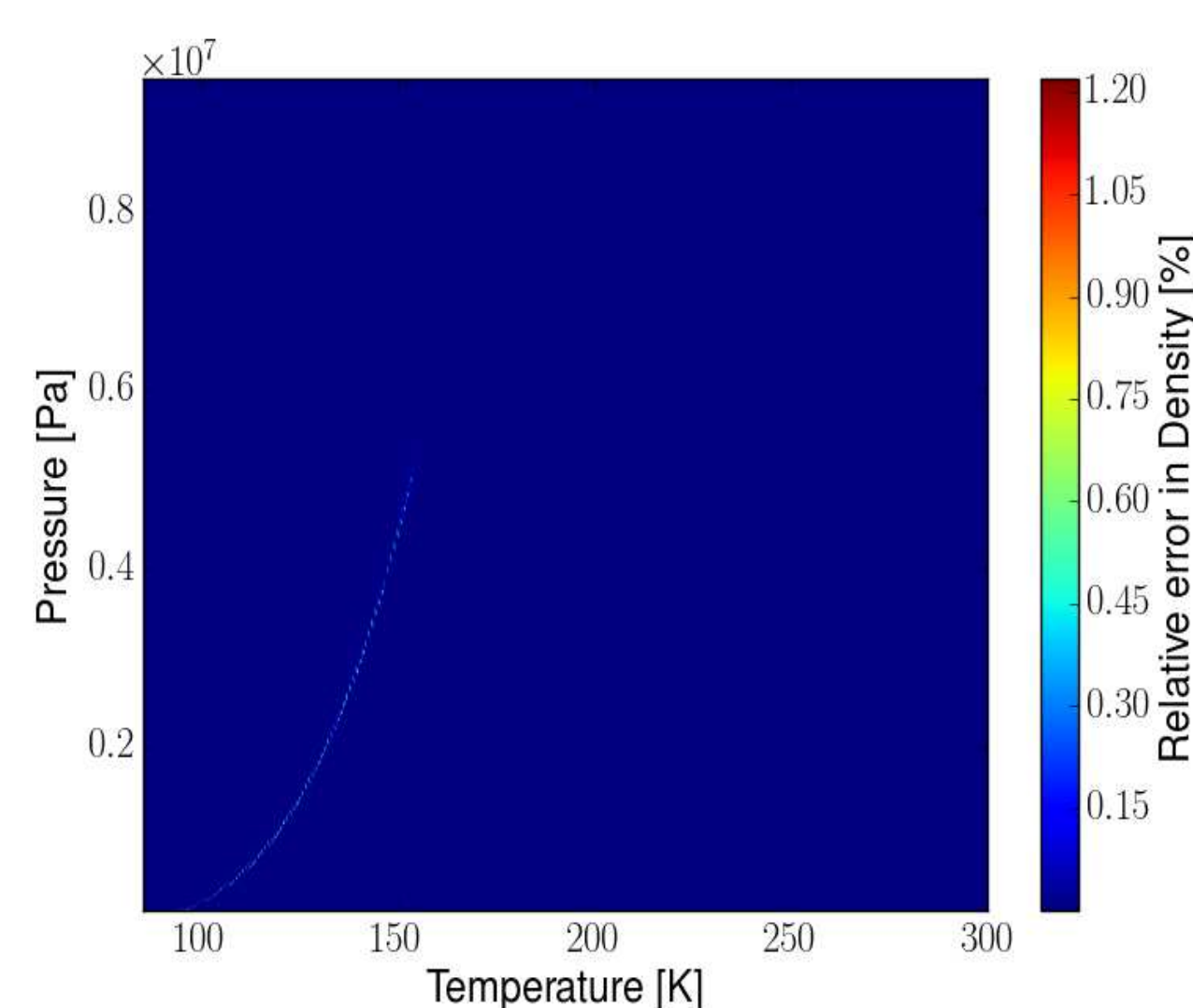
$$F(u, v) = \sum_{i=0}^3 \sum_{j=0}^3 B_i^3(u) B_j^3(v) b_{ij}$$

Adaptively refined tabular equation of states for Oxygen



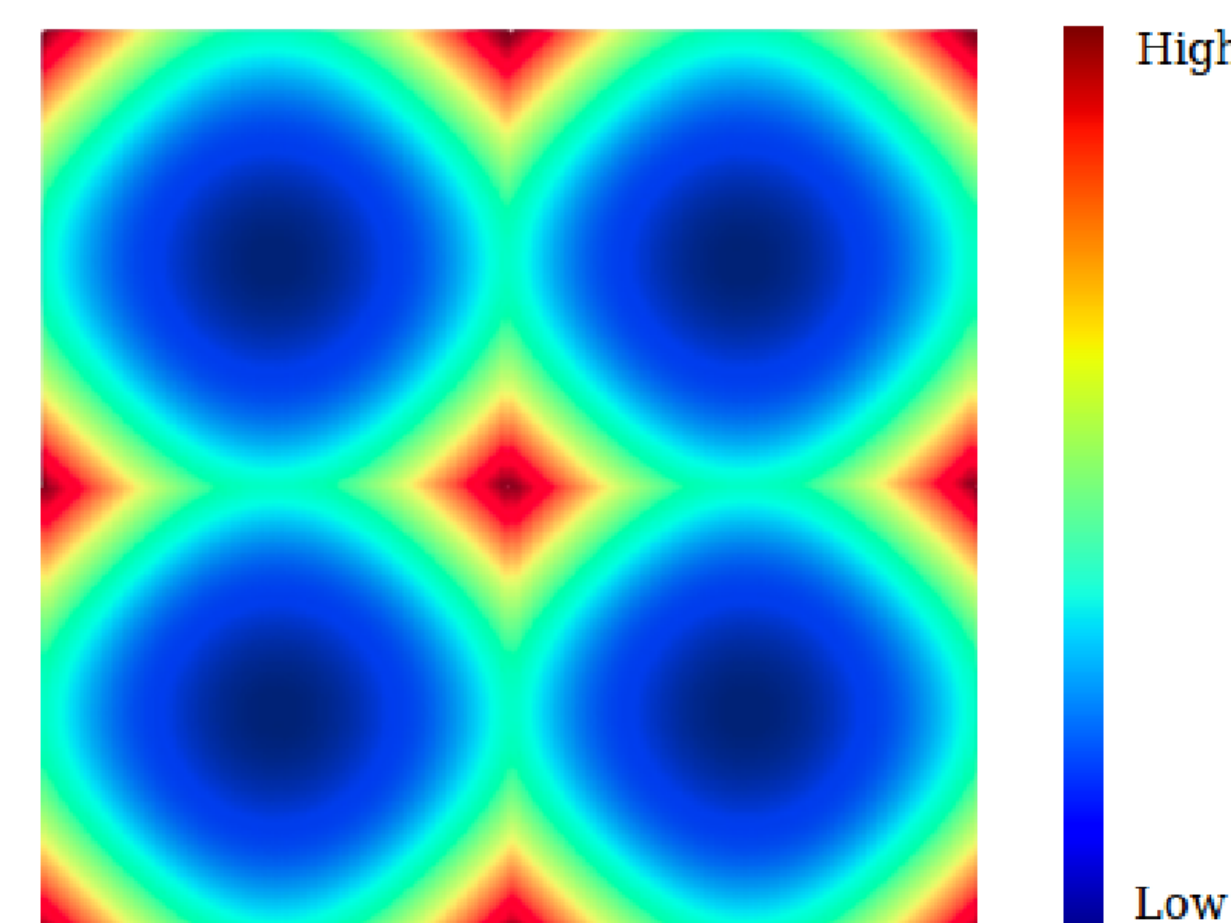
Error Estimates

Density relative error for Oxygen using AMR based tabular EOS

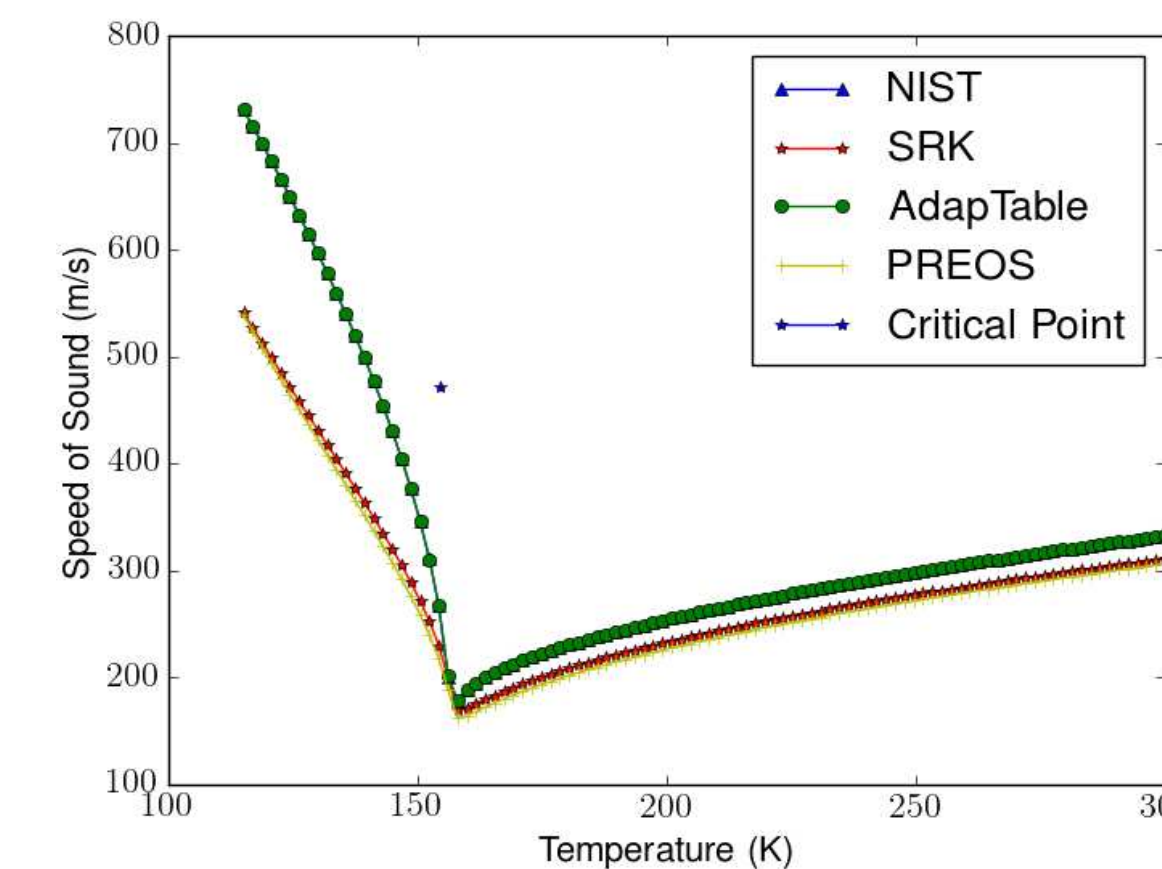


Uncertainty Distribution

Color coded variance as the result of Bilinear interpolation of uncorrelated Gaussian distributed period simulation computational cost comparison with different equation of states

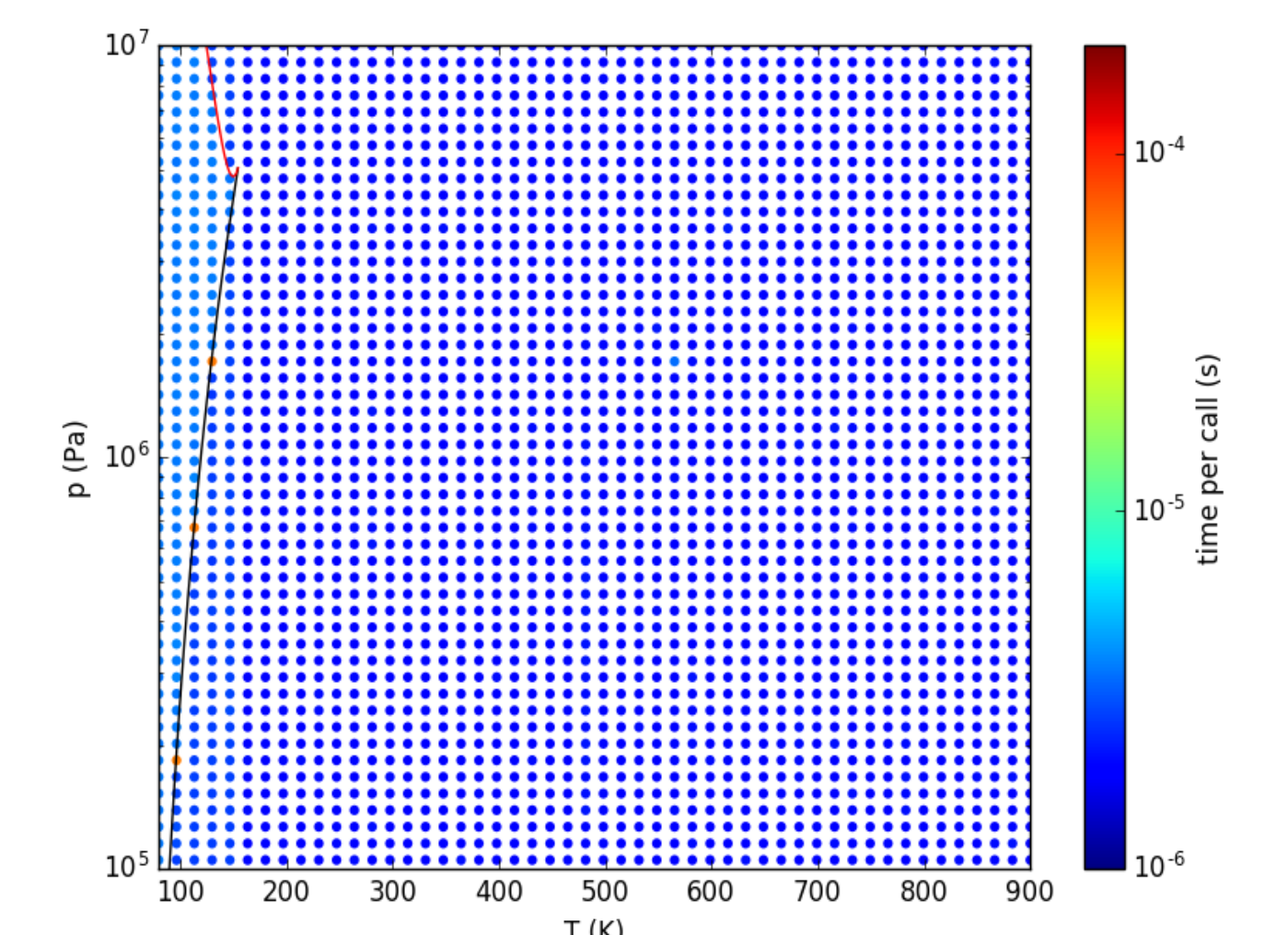


Verification



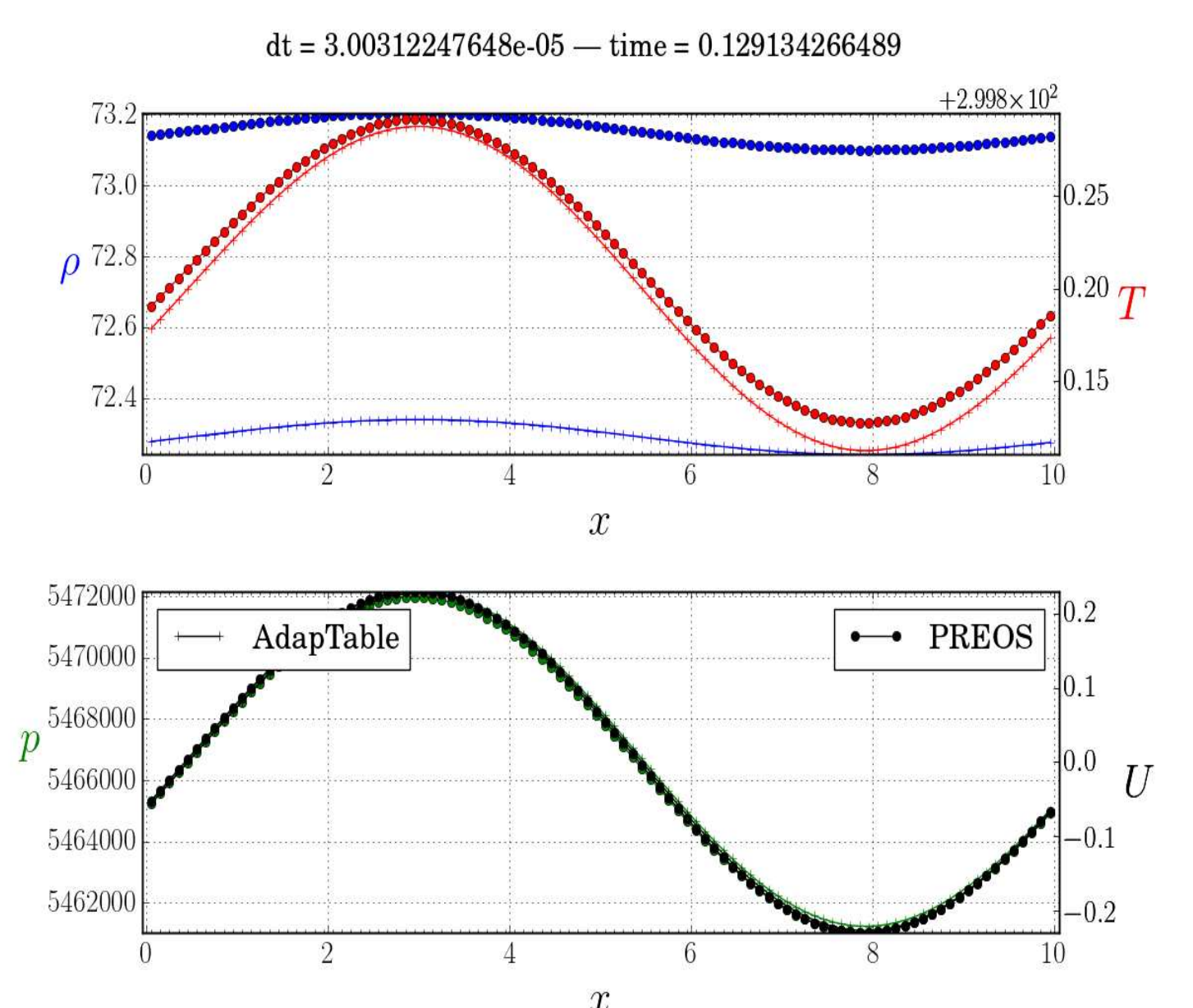
Lookup search estimates

The increase in speed of evaluating properties compared to cubic EOS is one of the primary motivations. Here we show an example of the speedup of computations. When pressure is evaluated from randomly generated ρ, e combination in the supercritical regime. For each thermodynamic property, 100000 loops/calls are executed and the slowest run time is tabulated.



Test case: Acoustic wave

One-dimensional acoustic wave propagation in supercritical fluid case, Using periodic boundary conditions at both sides a harmonic wave is initialized. The computational domain is $x \in [0, 10]$ m and 100 grid points are used, giving a uniform grid spacing $\Delta x = 0.1$ m.



References

- [1] E. Collins and E. Luke. Fast evaluation of complex equations of state. *Electronic Journal of Differential Equations*, 2013.

Computational cost comparison

Travelling Acoustic wave, one flow-through time interpolation computational cost comparison with different equation of states

