Question 1: A die is rolled. What is the probability of getting:

(a) An even number

(b) A number greater than 4

Ans. **Explanation:** A standard six-sided die has a sample space of possible outcomes

S= {1,2,3,4,5,6}. The total number of possible outcomes is N=6. The probability of any single event E is given by the formula,

P (E)= (Number of favorable outcomes)/ (Total number of outcomes)

a) An even number

A standard die has 6 equally likely outcomes: {1,2,3,4,5,6}. There are 3 even numbers in this set: {2,4,6}.

The probability is calculated as the number of favorable outcomes divided by the total number of outcomes:

P (even)= (Number of favorable outcomes)/ (Total number of outcomes) =3/6=½

b) A number greater than 4

The numbers greater than 4 in the set of outcomes are {5,6}. There are 2 such outcomes.

The probability is calculated similarly:

P (greater than 4) = (Number of outcomes greater than 4)/ (Total outcomes) =2/6=1/3

Question 2: In a class of 50 students:

- **20 like Mathematics (M)**
- **15 like Science (S)**
- **5 like both subjects**

What is the probability that a student chosen at random likes Mathematics or Science?

Ans. **Calculate the number of students who like Mathematics**

Or science

Using the principle of inclusion-exclusion, the number of students who like Mathematics (M) or Science (S) is calculated using the formula

$N(M \cup S) = N(M) + N(S) - N(M \cap S)$

$N(M \cup S) = 20+15-5=30$ students

Calculate the probability

The probability of choosing a student who likes Mathematics or Science is the number of favorable outcomes divided by the total number of students:

$P(M \cup S) = N(M \cup S)/N(\text{Total})$.

$P(M \cup S) = 30/50 = 3/5$

The probability is $P(M \cup S) = 0.6$ or 3/5

Question 3: A bag has 3 red and 2 blue balls. If one ball is drawn randomly and is red, what is the probability that the next ball is also red (without replacement)?

Ans. **<u>Initial Conditions:</u>**

Initially, the bag contains 3 red balls and 2 blue balls, for a total of 5 balls. The probability of drawing a red ball first is $P(R_1) = 3/5$

<u>Conditions After First Draw:</u>

The problem states that the first ball drawn *was* red and that drawing occurs without replacement. This means the composition of the bag for the second draw changes.
After drawing one red ball, the remaining number of balls is:

- Remaining red balls: 3-1=2
- Remaining blue balls: 2
- Total remaining balls: 5-1=4

<u>Determine the Probability</u>

The probability that the next ball (the second ball) is also red, given the first was red, is the ratio of remaining red balls to total remaining balls. This conditional probability is calculated as:

$P(R_2 \mid R_1)$ *=Remaining Red Balls/Total Remaining Balls=2/4*

The probability that the next ball is also red, without replacement, is 2/4 =½ or 0.5

Question 4: The population of a school is divided into 60% boys and 40% girls. If you want equal representation of both genders in the sample, which method should you use: Simple Random Sampling or Stratified Sampling? Why?

Ans. To get equal representation of boys and girls when the population is 60% boys and 40% girls, **Stratified Sampling is the correct method**, not simple random sampling, because it allows us to divide the population into gender groups (strata) and draw samples from each, guaranteeing representation for both genders and ensuring the sample accurately reflects the school's composition, unlike simple random sampling, which might miss one group entirely.

Why Stratified Sampling is Better:

- **Guaranteed Representation:** Stratified sampling ensures both boys (60%) and girls (40%) are included in the sample, making it more representative of the actual school population.
- **Reduces Bias:** It prevents the chance that a simple random sample would over-represent one gender or exclude the other, leading to more precise results.
- **Subgroup Analysis**: It's ideal for when you want to study characteristics within specific subgroups (like comparing boys' opinions to girls' opinions).

Why Simple Random Sampling Isn't Ideal Here:

- **Chance-Based:** In simple random sampling, every student has an equal chance, but there's no guarantee that the sample will mirror the 60/40 split; you could end up with mostly boys or mostly girls, skewing the results.

Question 5: The average height of 1000 students = 160 cm. A sample of 100 students shows an average height = 158 cm.

Find the sampling error.

Ans. Sampling error is the natural difference between a sample's characteristics and the true characteristics of the entire population it's drawn from, occurring because a sample is only a subset and never a perfect replica, leading to potential inaccuracies in research results. It's the gap between a sample statistic (like a sample mean) and the actual, unknown population parameter, and it's inherent in any survey or study that doesn't survey everyone.

The formula is,

Sampling error= $|\bar{x}-\mu|$

Where, Population mean (μ)=160cm

Sample mean ($\bar{x}$)=158cm

Sampling error=|158-160|=2cm

Question 6: The population mean salary is ₹50,000 with σ = ₹5,000. If we take a sample of 100 employees, what is the standard error of the mean (SEM)?

Ans. **Identify the formula and variables**

The standard error of the mean (SEM) is calculated using the formula SEM=$\frac{\sigma}{\sqrt{n}}$

Where σ is the population standard deviation and n is the sample size.

Given variables:

- Population standard deviation(σ) = 5000
- Sample size (n)=100

Calculate the standard error

Substitute the given values into the formula and solve:

SEM=$\frac{5000}{\sqrt{100}}$

=$\frac{5000}{10}$

=10

Question 7: In a group of 100 students:

- **40 like Cricket (C)**
- **30 like Football (F)**
- **10 like both Cricket and Football**

Find the probability that a student likes at least one sport.

Ans. **<u>Given Information:</u>**

- Total number of students = 100
- Number of students who like Cricket (n(C)) = 40
- Number of students who like Football (n(F)) = 30
- Number of students who like both (n (C ∩ F)) = 10

Find the number of students who like at least one sport (C or F):

The formula to find the number of elements in the union of two sets is:

$n (C \cup F) = n (C)+n (F) - n (C \cap F)$

Substitute the given values into the formula:

$n (C \cup F) = 40+30-10=60$

So, 60 students like at least one sport.

Find the probability:

The probability that a randomly selected student likes at least one sport is the number of students who like at least one sport divided by the total number of students.

P (at least one sport) = $n (C \cup F)$/Total students

$=60/100=3/5=0.6$

The probability that a student likes at least one sport is 60/100, which simplifies to 3/5 or 0.6

Question 8: From a deck of 52 cards, two cards are drawn without replacement. What is the probability that both are Aces?

Ans. **Calculate the probability of the first Ace**

The total number of cards in a deck is 52, and there are 4 Aces. The probability of drawing an Ace as the first card is given by the ratio of Aces to total cards:

$P(Ace_1)=4/52$

Calculate the probability of the second Ace

Since the first card is not replaced, there are now 51 total cards left in the deck. If the first card drawn was an Ace, there are 3 Aces remaining. The probability of drawing a second Ace given the first was an Ace is:

$P(Ace_2 | Ace_1)=3/51$

Calculate the combined probability

To find the probability that both events occur, we multiply the individual probabilities:

$P(\text{Both Aces}) = P(\text{Ace}_1) \times P(\text{Ace}_2 \mid \text{Ace}_1) = (4/52) \times (3/51)$

The calculation yields:

$P(\text{Both Aces}) = 12/2652 = 1/221$

Question 9: A factory produces bulbs with 2% defective rate. If 5 bulbs are chosen at random, what is the probability that all are non-defective?

Ans. Determine the probability of a single non-defective bulb

The probability of a bulb being defective is given as 2%, or $p=0.02$

The probability of a bulb being non-defective is $q=1-p$

$q=1-0.02=0.98$

<u>Calculate the combined probability for all five bulbs</u>

Since each bulb selection is an independent event, the probability that all five are non-defective is the product of their individual probabilities.

P (all non-defective)=q^5

P (all non-defective)=$(0.98)^5$

P (all non-defective) = 0.9039≈0.904

The probability that all 5 bulbs chosen at random are non-defective is approximately 0.904

Question 10: Differentiate between discrete and continuous random variables with examples.

Ans. Discrete random variables have countable, specific values (like coin flips), while continuous variables take any value within a range (like height), the key difference being that discrete values are counted, and continuous values are measured, forming distinct points versus a continuum.

Ans.

Aspect	**Discrete Random Variables**	**Continuous Random Variables**
Definition	A variable that can take on a finite or countably infinite set of distinct, isolated values.	A variable that can take any value within a specified range or interval (uncountably infinite possible values).
Value Acquisition	Values are obtained by counting.	Values are obtained by measuring.
Possible Values	Values are distinct whole numbers or specific points (e.g., 0, 1, 2, 3).	Values can include fractions and decimals; any real number within an interval.
Gaps	There are clear gaps between possible values.	Values form a continuum with no gaps.
Probability Function	Described by a Probability Mass Function (PMF), which assigns probability to each exact value.	Described by a Probability Density Function (PDF), which gives the likelihood of values within a range.
Probability Calculation	Probabilities for a range are calculated by summing the probabilities of individual values.	Probabilities are calculated by finding the area under the curve of the PDF over an interval (using integration).
Probability of a Single Value	The probability of a single specific value	The probability of a single specific value is always zero (P(X=x) = 0).

	is typically non-zero (e.g., P(X=x) > 0).	
Graphical Representation	Often represented by bar graphs of isolated points.	Often represented by line graphs or smooth curves.
Examples	Number of children in a family	Height of a person
	Outcome of rolling a die	Weight of a newborn baby
	Number of cars in a parking lot	Time taken to run a race
	Number of heads in coin tosses	Temperature of a room