EXP 4

1.For the continuous random variable X, the probability density function given below $f(x) = \{ k(2 - x); 0 \le x < 2\ kx(x-2); 2 \le x < 3\ 0$; otherwise . Find k and mean of the distribution. Write a R program for the above problem.

Ans:

Exp 4

Q1) $f(x) = \begin{cases} k(2-x) & 0 \le x < 2 \\ kx(x-2) & 2 \le x < 3 \\ 0 & \text{otherwise} \end{cases}$

To find k:

$$\int_0^2 k(2-x)\,dx = k\int_0^2 (2-x)\,dx$$

$$= k\left[2x - \frac{x^2}{2}\right]_0^2 = \boxed{2k} \quad (1)$$

$$\int_0^3 kx(x-2)\,dx = k\int_2^3 x(x-2)\,dx$$

$$= k\left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_2^3 = k\left[\frac{27}{3} - 9 - \frac{8}{3} + 4\right]$$

$$= k\left(\frac{4}{3}\right) \quad - (2)$$

Now adding (1) & (2) and equating to 1 as area under pdf equals to 1

$\therefore 2k + \frac{4k}{3} = 1 \qquad \therefore k = 3/10$

$$\text{Mean} = \int_0^2 kx(2-x)\,dx + \int_2^3 kx^2(x-2)\,dx$$

$$= k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 + k\left[\frac{x^4}{4} - \frac{2x^3}{3}\right]_2^3$$

$$= k\left[4 - \frac{8}{3}\right] + k\left[\frac{81}{4} - 18 - 4 + \frac{16}{3}\right]$$

$$= 0.3\left[4 - \frac{8}{3}\right] + 0.3\left[\frac{81}{4} - 22 + \frac{16}{3}\right]$$

$$= 1.475$$

main.r

Run

```r
#question1
f1 = function(x) (2 - x)
f2 = function(x) ( x * (x - 2))
v1 =integrate(f1,lower= 0, upper=2)$value
v2 =integrate(f2,lower= 2,upper= 3)$value
k=1/(v1+v2)
k
mean = integrate(function(x)k* x * f1(x),lower= 0, upper=2)$value + integrate(function(x) k*x * f2(x),lower= 2,upper= 3)$value
print(mean)
```

```
Rscript /tmp/hozrFr0mUU.r
[1] 0.3
[1] 1.475
```

```
#question1

f1 = function(x) (2 - x)

f2 = function(x) ( x * (x - 2))

v1       =integrate(f1,lower=       0,

upper=2)$value                         v2

=integrate(f2,lower=      2,upper=

3)$value k=1/(v1+v2)

k

mean = integrate(function(x)k* x * f1(x),lower= 0, upper=2)$value +
integrate(function(x) k*x * f2(x),lower= 2,upper= 3)$value

print(mean)
```

2.A daily consumption of electric power (in million kWh) is a random variable X with probability density function given below /3 , 0 ( ) 0 0 x kxe x f x x } ⇄ Find (i) k (ii) expectation of X (iii) Probability that on a given day, the electric consumption is more than expected value. Write a R program for the above problem.

Ans:

Q2) $f(x) = \begin{cases} kxe^{-x/3} & x > 0 \\ 0 & x \le 0 \end{cases}$

To find $k = \dfrac{1}{\int_0^\infty x e^{-x/3} \, dx} = \dfrac{1}{t}$

$t = x \int_0^\infty e^{-x/3} \, dx + \int_0^\infty 1 \int_0^\infty e^{-x/3} \, dx$

$= \left[\dfrac{x e^{-x/3}}{-1/3} \right]_0^\infty + \left[\dfrac{e^{-x/3}}{1/9} \right]_0^\infty$

$= 0 + 9$

$k = \dfrac{1}{9} = 0.1111$

Mean $= \int_0^\infty x \, kx e^{-x/3} \, dx$

$= \int_0^\infty x^2 k e^{-x/3} \, dx$

$= k \left[\dfrac{2x e^{-x/3}}{-1/3} \right]_0^\infty + k \left[\int_0^\infty 2x \int_0^\infty e^{-x/3} \, dx \right]$

$= 0 + k \left[\int_0^\infty \dfrac{2x e^{-x/3}}{(-1/3)} \right]$

$$\text{Mean} = k(-6)\left[\left(\frac{x e^{-x/3}}{-1/3}\right)_0^{\infty} + \int_0^{\infty} 1 \int_0^{\infty} \frac{e^{-x/3}}{(-1/3)}\, dx\, dx\right]$$

$$= -6k(-9)$$

$$\text{Mean} = 5.999 \approx 6$$

$$\text{Probability} = \int_6^{\infty} k x e^{-x/3}\, dx$$

$$= k \int_6^{\infty} x e^{-x/3}\, dx$$

$$= k\left[\frac{x e^{-x/3}}{-(1/3)}\right]_6^{\infty} + k\left[\int_6^{\infty} 1 \int_6^{\infty} \frac{e^{-x/3}}{(-1/3)}\, dx\, dx\right]$$

$$= 6k e^{-x}(+3) + (9e^{-2})$$

$$= 0.4060$$

```
#question2
k = 1 / integrate(function(x) x * exp(-x/3), 0, Inf)$value
mean = integrate(function(x) x * k * x * exp(-x/3), 0, Inf)$value
prob = integrate(function(x)  k * x * exp(-x/3), 6, Inf)$value
k
mean
prob
```

Output

```
Rscript /tmp/hozrFr0mUU.r
[1] 0.1111111
[1] 6
[1] 0.4060058
```

#question2

k = 1 / integrate(function(x) x * exp(-x/3), 0, Inf)$value

mean = integrate(function(x) x * k * x * exp(-x/3),

0, Inf)$value prob = integrate(function(x) k * x *

exp(-x/3), 6, Inf)$value

k

mea

n

prob

3.A continuous random variable has probability density function $f(x) = 6(x - x^2)$, $0 \leq x \leq 1$. Find
mean and variance and also find $P(|x - \mu| < \sigma)$. Write a R program for

the above problem. Ans:

Q3) $fx = 6(x - x^2) \qquad 0 \leq x \leq 1$

$$\text{mean} = \int_0^1 6x(x - x^2)\,dx$$

$$= \int_0^1 6x^2 - 6x^3\,dx$$

$$= \left[2x^3\right]_0^1 - \left[\frac{3x^4}{2}\right]_0^1$$

$$= 2 - \frac{3}{2} = \frac{1}{2} = 0.5$$

$$\text{Variance} = \int_0^1 x^2 6(x - x^2)\,dx - (0.5)^2$$

$$= \int_0^1 6x^3 - 6x^4\,dx - 0.25$$

$$= \left[\frac{3x^4}{2}\right]_0^1 - \left[\frac{6x^5}{5}\right]_0^1 - 0.25$$

$$= \frac{3}{2} - \frac{0.25}{5}$$

$$= 0.05$$

$$\sigma = \sqrt{\text{Variance}} = \sqrt{0.05} = 0.626609$$

```r
#question3
f = function(x) 6 * (x - x^2)
mean = integrate(function(x) x * f(x), 0, 1)$value
variance = integrate(function(x) (x - mean)^2 * f(x), 0, 1)$value
sigma = sqrt(variance)
prob = integrate(f, mean - sigma, mean + sigma)$value
mean
sigma
prob
```

Output

```
Rscript /tmp/hozrFrOmUU.r
[1] 0.5
[1] 0.2236068
[1] 0.626099
```

#question3

f = function(x) 6 * (x - x^2)

mean = integrate(function(x) x * f(x), 0, 1)$value

variance = integrate(function(x) (x - mean)^2 *

f(x), 0, 1)$value sigma = sqrt(variance)

prob = integrate(f, mean - sigma, mean +

sigma)$value mean

sigm

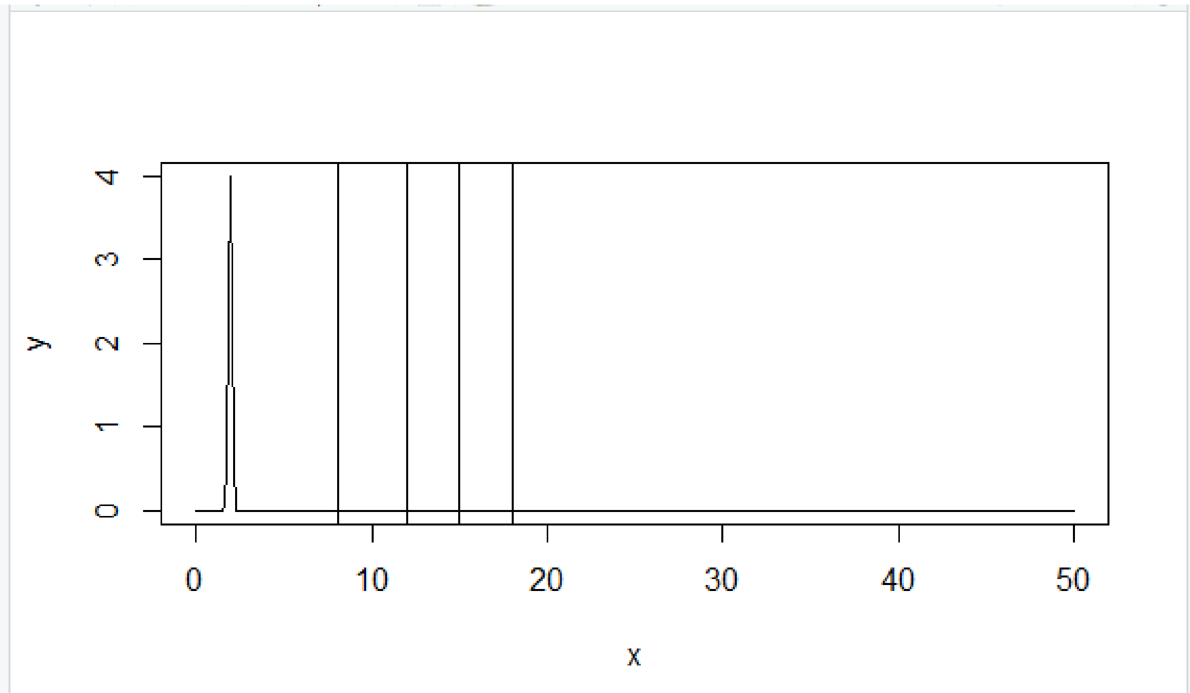
a

prob

4.In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find i. How many students score between 12 and 15? ii. How many score above 18? iii. How many score below 8? Write a R program for the above problem. Also plot the graph for each case.

```
m1 = 14
sd = 2.5
size = 1000
p1 = pnorm(15, mean = m1, sd = sd) - pnorm(12, mean = m1, sd = sd)
p2 = 1 - pnorm(18, mean = m1, sd = sd)
p3 = pnorm(8, mean = m1, sd = sd)
c1 = round(p1 * size)
c2 = round(p2 * size)
c3 = round(p3 * size)
c1
c2
c3
x=seq(0,50,by=0.1)
y= dnorm(x,m1,sd)
plot(x, y, type="l")
abline(v = 12)
abline(v = 15)
abline(v = 18)
abline(v = 8)
```

```
> c2 = round(p2 * size)
> c3 = round(p3 * size)
> c1
[1] 444
> c2
[1] 55
> c3
[1] 8
>
```



m1 = 14

sd = 2.5

size = 1000

p1 = pnorm(15, mean = m1, sd = sd) - pnorm(12,

mean = m1, sd = sd) p2 = 1 - pnorm(18, mean = m1,

sd = sd)

p3 = pnorm(8, mean = m1,

sd = sd) c1 = round(p1 *

size)

c2 = round(p2 *

size) c3 =

round(p3 * size)

c1

c

2

c

3

x=seq(0,50,by

=0.1) y=

dnorm(x,m1,sd

) plot(x, y,

type="l")

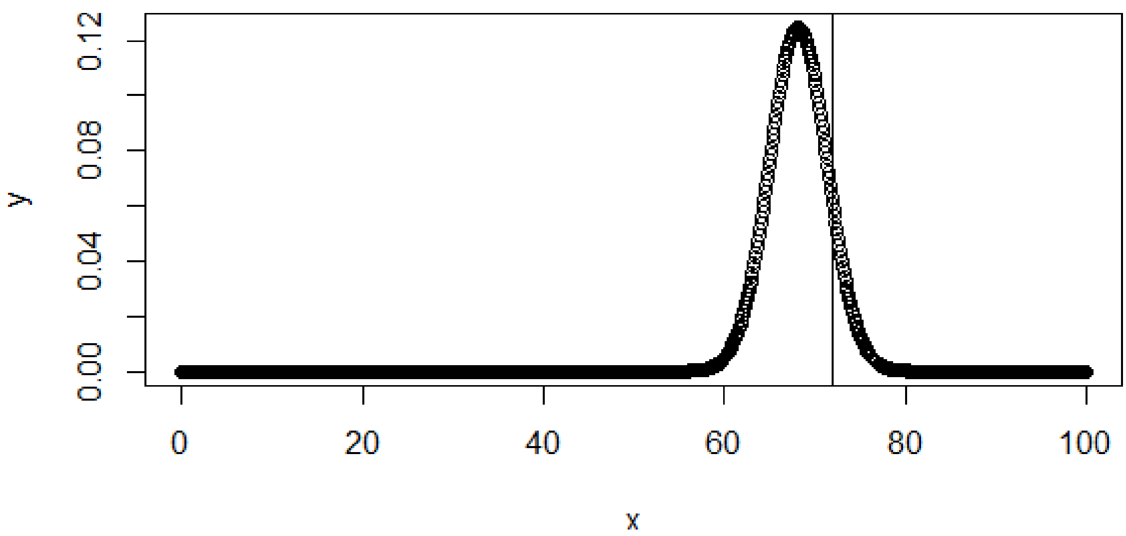
```
abline(v = 12)
abline(v = 15)
abline(v =18)
```

abline(v = 8)

5.In a male population of 1000, the mean height is 68.16 inches and standard deviation is 3.2 inches. How many men may be more than 6 feet (72 inches)? Write a R program for the above problem. Also plot the graph.

```
m1= 68.16
sd = 3.2
size = 1000
height = 72
p1 = 1 - pnorm(height, mean = m1, sd = sd)
c1 = round(p1 * size)
p1
c1
x=seq(0,100,by=0.1)
y=dnorm(x,m1,sd)
plot(x,y)
abline(v = 72)
```

```
> c1 = round(p1 * size)
> p1
[1] 0.1150697
> c1
[1] 115
>
```

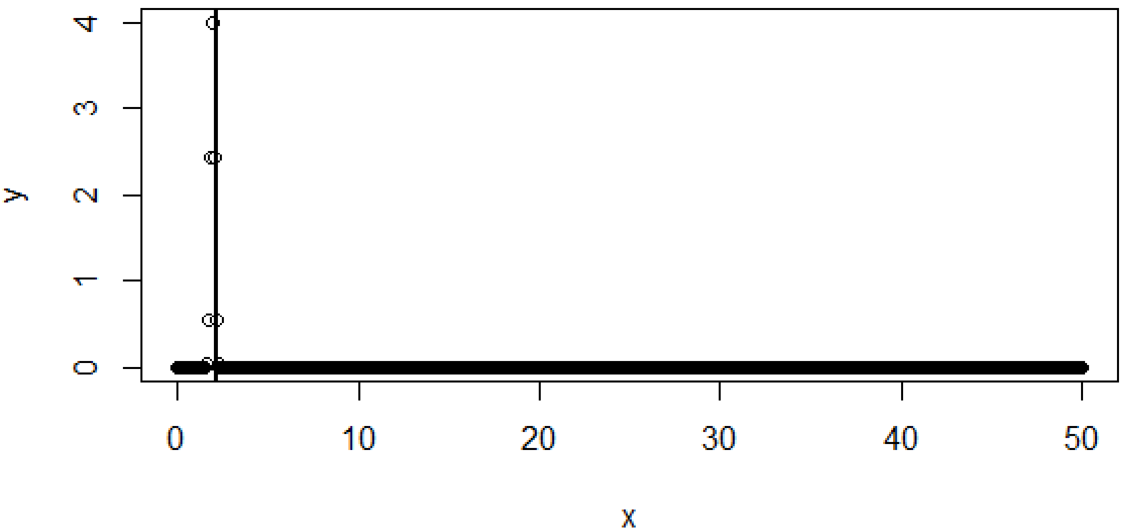


```
m1= 68.16

sd = 3.2

size = 1000

height = 72

p1 = 1 - pnorm(height, mean =

m1, sd = sd) c1 = round(p1 * size)

p

1

c

1

x=seq(0,100,by

=0.1)

y=dnorm(x,m1,

sd) plot(x,y)

abline(v = 72)
```

6.A manufacturer of envelopes known that the weight of the envelopes is normally distributed with mean 1.9 gm and variance 0.01 gm. Find how many envelops weighting (i) 2 gm or more (ii) 2.1 gm or more can be expected in a given packet of 1000 envelopes. Write a R program for the above problem. Also plot the graph.

```
m1= 1.9
var = 0.01
size = 1000
w1 = 2
w2= 2.1
sd = sqrt(var)
p1 = 1 - pnorm(w1, mean = m1, sd = sd)
p2 = 1 - pnorm(w2, mean = m1, sd = sd)
c1 = round(p1 * size)
c2 = round(p2 * size)
c1
c2
x=seq(0,50,by=0.1)
y=dnorm(x,m1,sd)
plot(x,y)
abline( v= 1.99999)
abline( v= 2.09999)
```

```
> c1 = round(p1 * size)
> c2 = round(p2 * size)
> c1
[1] 159
> c2
[1] 23
>
```



```
m1= 1.9
var = 0.01
size = 1000
w1 = 2
w2= 2.1
sd = sqrt(var)
p1 = 1 - pnorm(w1, mean =
m1, sd = sd) p2 = 1 -
pnorm(w2, mean = m1, sd =
sd) c1 = round(p1 * size)
c2 = round(p2 *
size) c1
c2
x=seq(0,50,by
=0.1)
y=dnorm(x,m1
,sd) plot(x,y)
abline( v= 1.99999)
abline( v= 2.09999)
```