

PS Experiment 5

1.

A random variable X has the following probability distribution

$X=x$:	0	1	2
$P(X=x)$:	1/3	1/3	1/3,

Find the moment generating function, first four raw moments and the first four central moment.

CODE:

```
# Q 1
x=c(0,1,2)
p=c(1/3,1/3,1/3)
mr0=1
mr1=sum(x*p)
mr2=sum(x*x*p)
mr3=sum(x*x*x*p)
mr4=sum(x*x*x*x*p)
rawm=c(mr0,mr1,mr2,mr3,mr4)
rawm
raw2central(rawm)
```

OUTPUT:

```
# Q 1
x=c(0,1,2)
p=c(1/3,1/3,1/3)
mr0=1
mr1=sum(x*p)
mr2=sum(x*x*p)
mr3=sum(x*x*x*p)
mr4=sum(x*x*x*x*p)
rawm=c(mr0,mr1,mr2,mr3,mr4)
rawm
1] 1.000000 1.000000 1.666667 3.000000 5.666667
raw2central(rawm)
1] 1.0000000 0.0000000 0.6666667 0.0000000 0.6666667
```

Saakshi Jain

PRG No.	
DATE	/ /

EIPS

1) $x = 0 \quad 1 \quad 2$

$P(x) \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$

mom about origin $M_x(t) = E[e^{tx}]$

$$= \sum_{i=1}^n P(x_i) e^{tx_i}$$

$$= \frac{1}{3} [e^0 + e^t + e^{2t}]$$

First 4 moments about origin

$M_{x0} = 1$

$M_{x1} = E[x] = \frac{1}{3} + \frac{2}{3} = 1$

$M_{x2} = E[x^2] = \frac{5}{3}$

$M_{x3} = \frac{9}{3} = 3 \quad M_{x4} = \frac{17}{3}$

First 4 central moments

$M_{c0} = 1 \quad M_{c1} = 0$

$M_{c2} = M_{x2} - (M_{x1})^2 = \frac{5}{3} - 1 = \frac{2}{3}$

$M_{c3} = M_{x3} - 3M_{x1}M_{x2} + 2(M_{x1})^3 = 3 - 3(1)(\frac{5}{3}) + 2 = 0$

$M_{c4} = M_{x4} - 4M_{x1}M_{x3} + 6M_{x1}^2M_{x2} - 3(M_{x1})^4 = \frac{17}{3} - 4(1)(3) + 6(1)(\frac{5}{3}) - 3(1)^4 = \frac{2}{3}$

$M_{c4} = \frac{2}{3}$

2.

The first three moments of the distribution about the value 3 of the random variable are 2, 10, -30 respectively. Find mean variance and skewness.

Write a R program for above problem.

CODE:

```
# Q 2
m0=1
m1=2
m2=10
m3=-30
a=3
v=raw2central(c(m0,m1,m2,m3))
mean=m1+a
cat("Mean = ",mean)
cat("variance = ",v[3])
cat("Skewness = ",v[4])
```

OUTPUT:

```
> # Q 2
> m0=1
> m1=2
> m2=10
> m3=-30
> a=3
> v=raw2central(c(m0,m1,m2,m3))
> mean=m1+a
> cat("Mean = ",mean)
Mean = 5> cat("variance = ",v[3])
variance = 6> cat("Skewness = ",v[4])
Skewness = -74
```

$$\begin{aligned}
 Q2) \quad \mu_2(x=3) &= E[(x-3)^2] \\
 \mu_1 &= E(x-3) = E(x) - 3 = 2 \\
 \therefore E(x) &= 5 \\
 \mu_2 &= E((x-3)^2) = E(x^2) + 9 - (E(x))^2 = 10 \\
 \text{Variance } \mu_{cp} &= E(x^2) - (E(x))^2 = 31 - 25 = 6 \\
 \mu_3 &= E(x-3)^3 = E(x^3 - 27 - 9x^2 + 27x) = -30 \\
 &= E(x^3) = -30 + 27 + 9(31) - 27(5) \\
 E(x^3) &= 141 \\
 \text{Skewness } \mu_{c3} &= \mu_3 - 3\mu_1\mu_2 + 2(\mu_1)^3 \\
 &= E(x^3) - 3E(x)E(x^2) + 2(E(x))^3 \\
 &= 141 - 46.5 + 250 \\
 &= -74
 \end{aligned}$$

3.

A random variable X has the probability distribution

$$P(X=x) = \frac{1}{8} {}^3C_x, \quad X=0,1,2,3, \text{ Find the moment generating function}$$

of X and then find mean and variance.

Write a R program for above problem.

CODE:

```
# Q 3
x=c(0,1,2,3)
p=c(1/8,3/8,3/8,1/8)
mr0=1
mr1=sum(x*p)
mr2=sum(x*x*p)
mr3=sum(x*x*x*p)
v=row2central(c(mr0,mr1,mr2,mr3))
cat("Mean = ",mr1)
cat("variance = ",v[3])
```

OUTPUT:

```
> # Q 3
> x=c(0,1,2,3)
> p=c(1/8,3/8,3/8,1/8)
> mr0=1
> mr1=sum(x*p)
> mr2=sum(x*x*p)
> mr3=sum(x*x*x*p)
> v=row2central(c(mr0,mr1,mr2,mr3))
> cat("Mean = ",mr1)
Mean = 1.5> cat("variance = ",v[3])
variance = 0.75
\
```

(Q3) $P(X=x) = \frac{1}{8} {}^3C_x$

X	0	1	2	3
P(X)	$\frac{1}{8} {}^3C_0 = \frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean = $\sum_{i=1}^n x_i \cdot P_i = \frac{3+6+3}{8} = \frac{12}{8} = 1.5$

Variance = $E(X^2) - [E(X)]^2 = \frac{24}{8} - \frac{9}{4} = \frac{3}{4} = 0.75$

MuF = $M_X(t) = E(e^{tx})$

$= \sum_{i=1}^n P_i(x) e^{tx_i}$

$= \frac{1}{8} e^0 + \frac{3}{8} e^t + \frac{3}{8} e^{2t} + \frac{1}{8} e^{3t}$

$= \frac{1}{8} (e^0 + 3e^t + 3e^{2t} + e^{3t})$

4.

Find the first four moments about mean of the random variable X whose probability mass function is given by

X: -2 3 1
P(X): 1/3 1/2 1/6
Write a R program for above problem.

CODE:

```
# Q 4
x=c(-2,3,1)
p=c(1/3,1/2,1/6)
mr0=1
mr1=sum(x*p)
mr2=sum(x*x*p)
mr3=sum(x*x*x*p)
mr4=sum(x*x*x*x*p)
raw2central(c(mr0,mr1,mr2,mr3,mr4))
```

OUTPUT:

```
> # Q 4
> x=c(-2,3,1)
> p=c(1/3,1/2,1/6)
> mr0=1
> mr1=sum(x*p)
> mr2=sum(x*x*p)
> mr3=sum(x*x*x*p)
> mr4=sum(x*x*x*x*p)
> raw2central(c(mr0,mr1,mr2,mr3,mr4))
[1] 1 0 5 -5 35
```

	X	-2	3	1
P(X)		1/3	1/2	1/6
μ_0		1		
μ_1		0		
μ_2		$E(X^2) - [E(X)]^2 = 36/6 - (1/6)^2 = 5$		
μ_3		$E(X^3) - 3[E(X)E(X^2)] + 2(E(X))^3$		
		$= 11 - 18 + 2$		
		$= -5$		
μ_4		$E(X^4) - 4[E(X)E(X^3)] + 6[E(X)E(X^2)]^2 - 3(E(X))^4$		
		$= 46 - 44 + 36 - 3$		
		$= 35$		

