Discrete Probability Distribution

```
#Install package
install.packages("stats")
#To remove package after use
detach("stats", unload = TRUE)
#Load library to load and use package
library(e1071)
library(distr)
```

#Prefix used

- p for "probability", the cumulative distribution function (c. d. f.)
- q for "quantile", the inverse c. d. f.
- d for "density", the density function (p. f. or p. d. f.)
- r for "random", a random variable having the specified distribution

```
#Frequency table
random=sample(1:10, size=1000, replace = TRUE)
t=table(random)
barplot(t)
#How to enter data
rdiscrete( 30, c('0.2','0.5','0.3') )
rdiscrete( 100, c('0.2','0.5','0.3'), c("A","B","C"))

#Example
y= rdiscrete( 100, c(1/4,2/4,1/4), c(0,1,2))
factor(y)
levels(factor(y))
table((factor(y)))
#To find probability associated to any random variable for example
x=1
ddiscrete(1, c(1/4,2/4,1/4), c(0,1,2))

#Example of rolling of die
# generate the vector of probabilities
probability <- rep(1/6, 6)
```

```
# plot the probabilities

barplot(probability, xlab = "outcomes", main = "Probability Distribution")

# generate the vector of cumulative probabilities

cum_probability <- cumsum(probability)


# plot the probabilites

barplot(cum_probability, xlab = "outcomes", main = "Cumulative Probability Distribution")
```

Note: Plots must be customized by using the knowledge of Practical 2.

```
#Mean and variance

X=c(0,1,2,3,4)

P=c(0.1,0.15,0.2,0.55)

XP=X*P

data.frame(X,P,XP)

mean=sum(XP)


#Find unknown for 0.6+6x=1

f <- function(x) (0.6+6*x-1)

uniroot(f, lower=0, upper=1)$root
```

EXERCISE (Programing and problem solving)

1. PDF of random variable X is:

X	1	2	3	4	5	6	7
P(X)	k	2k	3k	k^2	k^2+k	$2k^2$	$4k^2$

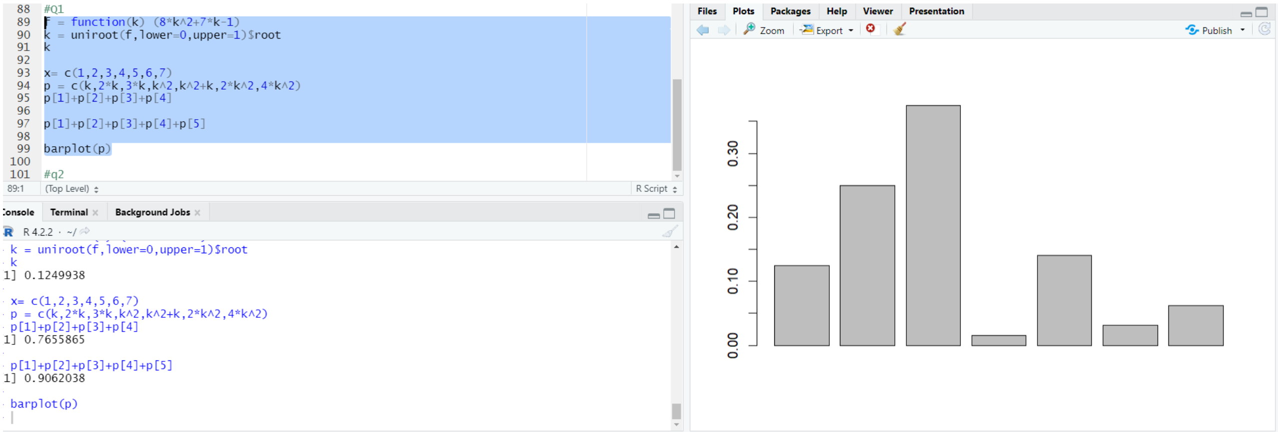
Find k, $P(X$ [illegible] $5)$, $P(1$ [illegible] X [illegible] $5)$

Write a R program for the above problem. Also write a R program to plot probability distribution.

```
f = function(k) (8*k^2+7*k-1)
k = uniroot(f,lower=0,upper=1)$root
k

x= c(1,2,3,4,5,6,7)
p = c(k,2*k,3*k,k^2,k^2+k,2*k^2,4*k^2)
p[1]+p[2]+p[3]+p[4]
```

p[1]+p[2]+p[3]+p[4]+p[5]

barplot(p)




```
> cd
[1] 0.1249938 0.3749815 0.7499631 0.7655865 0.9062038 0.9374508 0.9999446
> barplot(cd)
> cd[c(4)]
[1] 0.7655865
> barplot(p)
> f = function(k) (8*k^2+7*k-1)
> k = uniroot(f,lower=0,upper=1)$root
> k
[1] 0.1249938
>
> x= c(1,2,3,4,5,6,7)
> p = c(k,2*k,3*k,k^2,k^2+k,2*k^2,4*k^2)
> p[1]+p[2]+p[3]+p[4]
[1] 0.7655865
>
> p[1]+p[2]+p[3]+p[4]+p[5]
[1] 0.9062038
>
> barplot(p)
>
```

Ans 1

X	1	2	3	4	5	6	7
P(X)	k	$2k$	$3k$	k^2	k^2+k	$2k^2$	$4k^2$

PDF of random variable x

Since $\sum p(x) = 1$

$\therefore k+2k+3k+k^2+k+2k^2+4k^2 = 1$

$8k^2 + 7k = 1$

$8k^2 + 7k - 1 = 0$

$8k^2 + 8k - k - 1 = 0$

$8k(k+1) - 1(k+1) = 0$

$(8k-1)(k+1) = 0$

$k = -1, \frac{1}{8}$

Since probability is +ve

negative $k = -1$

$\therefore \boxed{k} = \frac{1}{8} = \boxed{0.125}$ Ans

$\boxed{P(X<5)} = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= k + 2k + 3k + k^2$

$= 6k + k^2$

$= 6(0.125) + (0.125)^2$

$= \boxed{0.765625}$ Ans

$P(1 \le x \le 5) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= k + 2k + 3k + k^2 + k^2 + k$

$= 7k + 2k^2$

$= 7(0.125) + 2(0.125)^2$

$= \boxed{0.90625}$ Ans

2. A random variable X has the following pdf

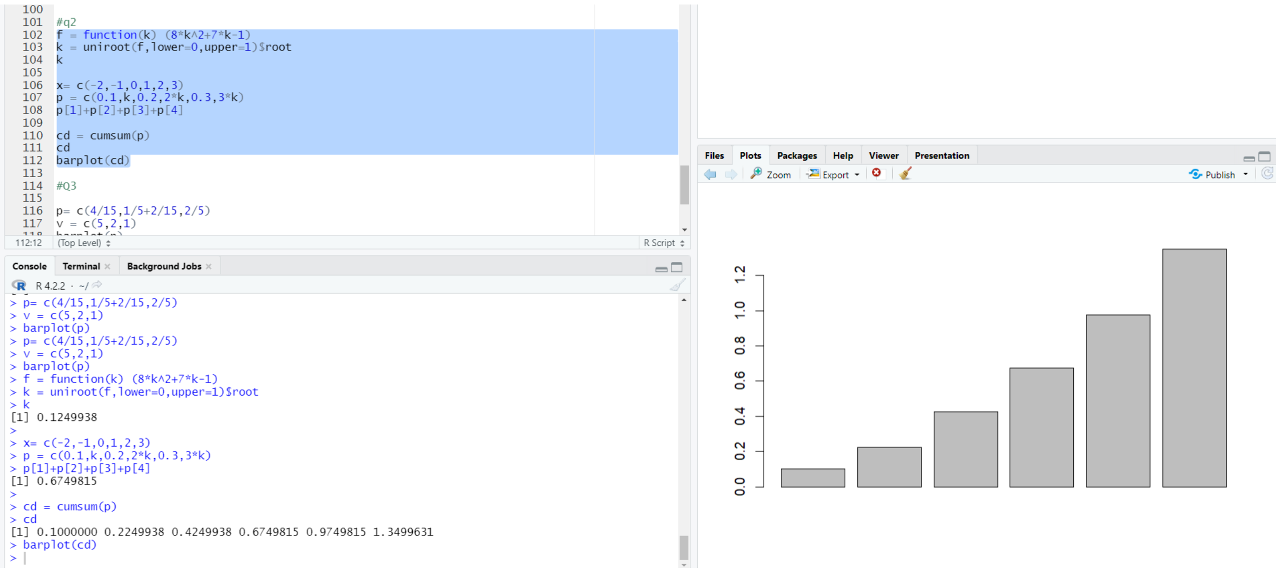
X	-2	-1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	3k

Find k, p(X <2), c.d.f.

Write a R program for the above problem. Also write a R program to plot cumulative distribution function.

```
f = function(k) (8*k^2+7*k-1)

k = uniroot(f,lower=0,upper=1)$root

k
```

```
x= c(-2,-1,0,1,2,3)

p = c(0.1,k,0.2,2*k,0.3,3*k)

p[1]+p[2]+p[3]+p[4]


cd = cumsum(p)

cd

barplot(cd)
```



PDF of a random Variable X

X	-2	-1	0	1	2	3
P(x)	0·1	k	0·2	2k	0·3	3k

Since $\Sigma p(x) = 1$

$\therefore 0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$

$0.6 + 6k = 1$

$6k = 0.4$

$\boxed{k = \frac{0.2}{3}} = \boxed{0.06667}$

$P(X<2) = P(X=-2) + P(X=-1) + P(x=0) + P(X=1)$

$= 0.1 + k + 0.2 + 2k$

$= 0.3 + 3k$

$= 0.3 + 3\left(\frac{0.2}{3}\right)$

$= 0.3 + 0.2$

$= \boxed{0.5}$ Ans

X	-2	-1	0	1	2	3
P(x)	0·1	$\frac{0.2}{3}$ = 0·6667	0·2	$\frac{0.4}{3}$ = 1·333	0·3	$\frac{0.2\times 3}{3}$ = 0·2
c.d.f	0·1	0·16667	0·366	0·5	0·8	1

3. A RV X has the following probability distribution:

X	-2	-1	0	1	2
P(X=x)	1/5	1/5	2/5	2/15	1/15
V	5	2	1	2	5
P(V)	4/15	5/15	6/15		

Find the probability distribution of $V = X^2 + 1$.

Write a R program for the above problem and also draw the plot.

```
p= c(4/15,1/5+2/15,2/5)
v = c(5,2,1)
barplot(p)
```

```
#Q3

p= c(4/15,1/5+2/15,2/5)
v = c(5,2,1)
barplot(p)

#Q4

x = c(-3,-2,-1,0,1,2)
p=  c(0.05,0.1,0.2,0.3,0.2,0.15)
m = sum(x*p)
m
var = sum(x^2*p) - m^2
var
```

116:1 (Top Level) R Script

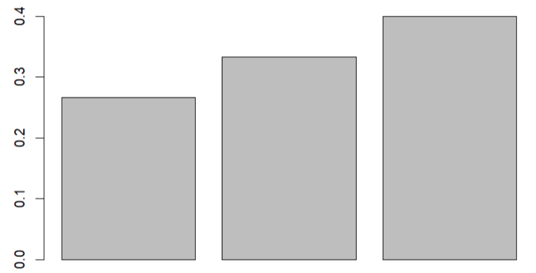
Console Terminal Background Jobs

R 4.2.2 · ~/

```
> x = c(-3,-2,-1,0,1,2)
> p=    c(0.05,0.1,0.2,0.3,0.2,0.15)
> m = x*p
> var = sum(x^2*p) - m^2
> var
[1] 1.8275 1.8100 1.8100 1.8500 1.8100 1.7600
> x = c(-3,-2,-1,0,1,2)
> p=    c(0.05,0.1,0.2,0.3,0.2,0.15)
> m = sum(x*p)
> m
[1] -0.05
> var = sum(x^2*p) - m^2
> var
[1] 1.8475
> p= c(4/15,1/5+2/15,2/5)
> v = c(5,2,1)
> barplot(p)
> p= c(4/15,1/5+2/15,2/5)
> v = c(5,2,1)
> barplot(p)
>
```

Files Plots Packages Help Viewer Presentation

Zoom Export Publish



Ans3 Pdf of a RV X

X	-2	-1	0	1	2
P(X=x)	1/5	1/5	2/5	2/15	1/5

Given $V = X^2 + 1$

Sol^n $V = 1$ for $x = 0$, $V = 2$ for $x = 1$, $V = 5$ for $x = -2, 2$

$\therefore V = x^2 + 1$

V	1	2	5
P(V)	2/5	$\frac{1}{5} + \frac{2}{15}$	$\frac{1}{5} + \frac{1}{15}$
	$= \frac{2}{5}$	$= \frac{1}{3}$	$= \frac{4}{15}$

P.D. of $V = x^2 + 1$

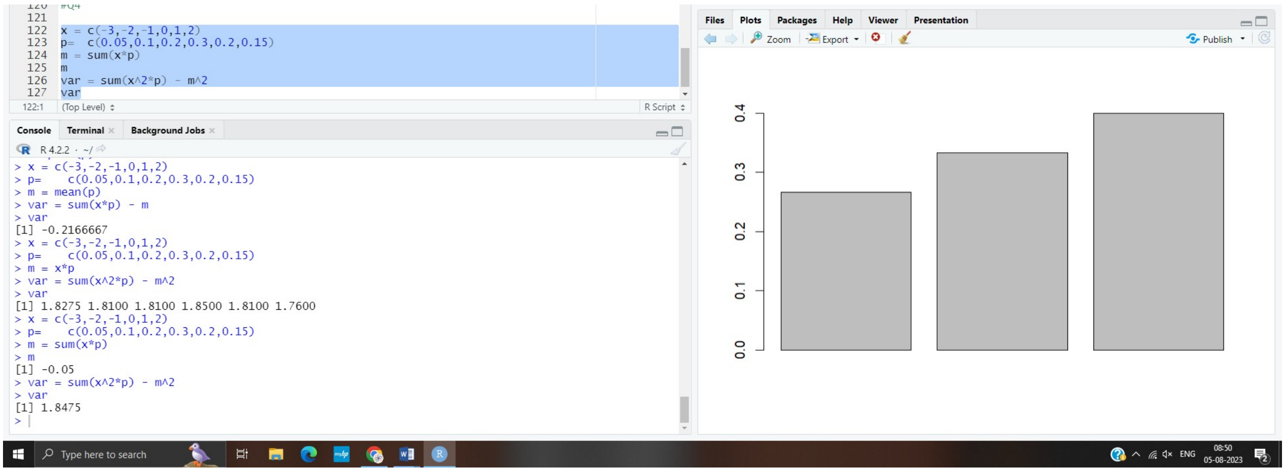
4. Given the following distribution:

x	-3	-2	-1	0	1	2
P(X =x)	0.05	0.1	0.2	0.3	0.2	0.15

Find Mean and Variance.

Write a R program for the above problem.

```
x = c(-3,-2,-1,0,1,2)
p=   c(0.05,0.1,0.2,0.3,0.2,0.15)
m = sum(x*p)
m
var = sum(x^2*p) - m^2
var
```



Ans 4

X	-3	-2	-1	0	1	2
$P(X=x)$	0.05	0.1	0.2	0.3	0.2	0.15

Mean $= E[x] = \sum x_i p_i$

$= -0.15 - 0.20 - 0.20 + 0 + 0.20 + 0.30$

$= -0.05$

$\therefore \boxed{\text{Mean} = -0.05}$ Ans

Variance $= E[x^2] - (E[x])^2$

Now,

$E[x^2] = \sum (x^2 p)$

$= 0.45 + 0.40 + 0.2 + 0 + 0.2 + 0.60$

$= 1.85$

Var $= 1.85 - (-0.05)^2$

$= 1.85 - 0.0025$

$= 1.8475$

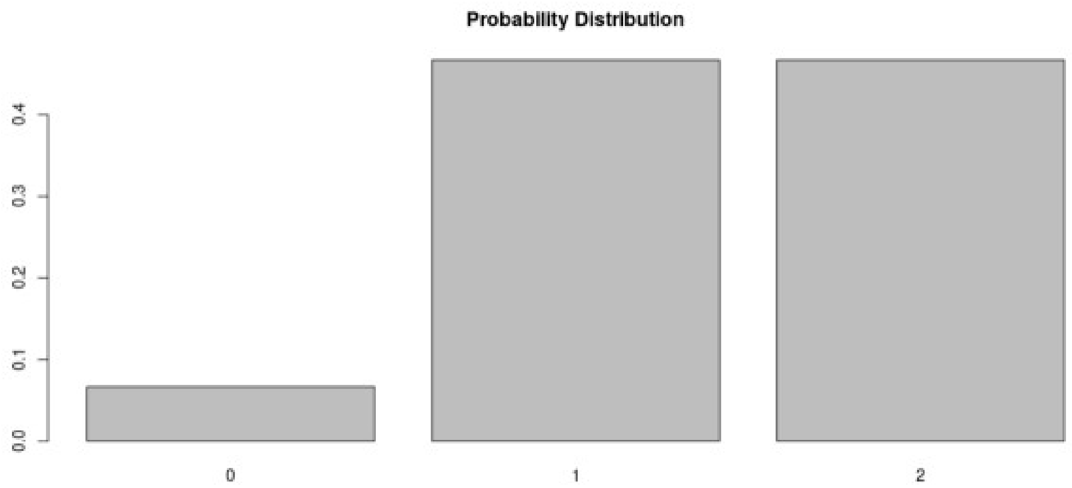
$\therefore \boxed{\text{Var} = 1.8475}$ Ans

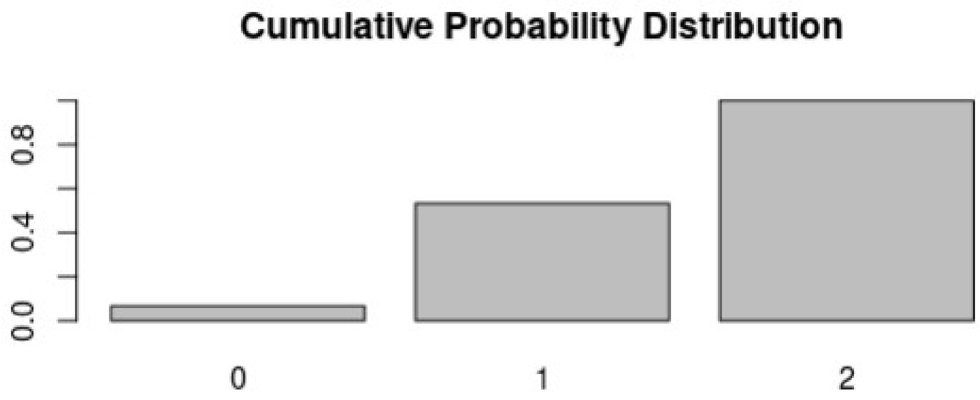
5. An urn contains 7 white and 3 red balls. Two balls are drawn together, at random from this urn. Compute the expected number of white balls drawn

   Write a R program for above problem. Also write a program for to plot probability distribution and cumulative probability distribution.

OUTPUT:

```
> x=c(0,1,2)
> p=c(0.3*2/9,2*0.7*3/9,0.7*2/3)
> sum(x*p)
[1] 1.4
> cd = cumsum(p)
> barplot(names.arg = c(0:2),p,main = "Probability Distribution")
> barplot(names.arg = c(0:2),cd,main = "Cumulative Probability Distribution")
>
```





Ans5 Total balls = 10

Number of white balls = 7

Number of red balls = 3

$\because$ 2 balls are drawn at random

$\therefore$ $n(S) = {}^{10}C_2 = 45$

X	0	1	2
$P(X=x)$	$\frac{^3C_2}{^{10}C_2}$ $= \frac{1}{15}$	$\frac{^7C_1 \times {}^3C_1}{^{10}C_2}$ $= \frac{7}{15}$	$\frac{^7C_2}{^{10}C_2}$ $= \frac{7}{15}$
C.S.d	$\frac{1}{15}$	$\frac{8}{15}$	1

$E[X] \rightarrow$ Expected No of white balls drawn

$$E[X] = \sum x_i p_i = 0 \times \frac{1}{15} + 1 \times \frac{7}{15} + 2 \times \frac{7}{15}$$

$$= 0 + \frac{7}{15} + \frac{14}{15}$$

$$= \frac{21}{15} = 1.4$$

$\therefore$ $E[X] = 1.4$