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CS383, Algorithms Spring 2009 HW9

1. You will design a dynamic programming algorithm for the following computational task. The inputs are two arrays. The desired output is the length of the longest subsequence that appears in both of the input arrays. Elements of the shared subsequence need not occupy immediately consecutive locations in each array. For example, for the two arrays $\{12, 3, -20, 2, -8, 17, 40, 2, 30\}$ and $\{-50, 2, -8, 17, 2, 30, 40\}$, the desired output is 5, since the two input arrays share a subsequence of length 5 (e.g., $\{2, -8, 17, 2, 30\}$) but no subsequence of length greater than 5.
 - (a) Specify each of the ingredients of a dynamic programming solution that we discussed in class. Explain in detail in each case.
 - i. supproblem definition
 - ii. objective function, value function (optimized objective function)
 - iii. recurrence relation for the value function
 - iv. domain of dependence of a generic subproblem
 - v. boundary conditions, and
 - vi. update order
 - (b) Building on your work in 1a, provide detailed pseudocode for a dynamic programming algorithm that solves the computational task described above.
 - (c) Analyze the running time of your algorithm from 1b.
 - (d) Trace the execution of your algorithm from 1b for the particular input instance described above at the very beginning of task 1. Provide the updated values of the value function at all points in the domain of dependence of the target problem.
2. A manufacturing plant needs to hire workers to meet a sudden rise in demand. The total budget for salaries is \$402,000. Due to the cost of health benefits, at most 100 workers can be hired. Two companies are offering temporary staff: *A* offers up to 66 workers who earn a \$5,000 salary and who are expected to produce 60 units each; *B* offers up to 72 workers who will earn \$3,000 and are expected to produce 50 units each. Because of pre-existing agreements with the two temp companies, at least one out of every seven hires must come from each company. The plant's goal is to determine how many workers to hire from each company to maximize the total number of units produced.
 - (a) Cast the factory hiring problem as a linear programming problem in the standard form discussed in class. Specify the variables (and their meanings), the objective function, and

- all constraints. The objective function must be given as a specific linear combination of the variables. Similarly, each constraint must be expressed as an inequality that provides an upper bound on some specific linear combination of the variables.
- (b) Sketch the feasible region for the linear programming problem from 2a. How many vertices does the feasible region have? Include the equation of each boundary and the numerical coordinates of each of the vertices.
 - (c) Explain the steps followed by a greedy simplex search in finding the solution to the linear programming problem in 2a. Provide detailed calculations of all of the relevant values. Give the optimal hiring decisions and the associated number of units produced.
 - (d) Would the solution to the factory hiring problem change if company *A*'s workers produced 100 units each (all other amounts remain equal)? Explain in detail.
3. Recall the cargo plane example that we discussed in class (exercise 7.3 in the textbook by Dasgupta et al). A plane has weight capacity 100 tons and volume capacity 60 cubic meters. Three materials are available for loading, in limited amounts:
- $40m^3$ of material m_1 , with density 2 tons/ m^3 and value \$1,000/ m^3
 - $30m^3$ of material m_2 , with density 1 ton/ m^3 and value \$1,200/ m^3
 - $20m^3$ of material m_3 , with density 3 tons/ m^3 and value \$12,000/ m^3

We wish to maximize the total value of the shipment subject to the above constraints.

- (a) Formulate the cargo plane task as a linear programming problem in the standard matrix form described in class (see below). In detail, specify the contents of each of the matrices c, A, b . Your answers should make it clear what the size of each matrix is, and what specific numerical value appears in each (row, column) combination. Explain briefly.

Standard matrix form of LP problem.

$$\min c'x \quad \text{subject to } Ax \leq b,$$

where x is an n -dimensional column vector of unknowns, c is an n -dimensional vector containing the coefficients of the objective function, A is an $m \times n$ matrix describing m linear expressions (one constraint per row) in the n unknowns, and b is an n -dimensional column vector corresponding to the right-hand sides (upper bounds) of the m inequality constraints whose left-hand sides correspond to the rows of A . Note carefully that the standard form allows minimization only and that all constraints must be inequalities of the form described above.

- (b) Building on your work in 3a, use Matlab to solve the cargo plane problem. Provide both the coordinates of the optimal vertex, as well as the optimal value of the objective function (both of these should be computed in Matlab). Include a printout of the commands used to enter the data and to compute the solution.

4. Consider the maximum flow problem for the network shown in exercise 7.10 of the textbook, with S as source and T as sink. In this task you will cast this problem as a linear programming problem in standard form and use Matlab to solve it.
 - (a) For the maximum flow problem for the network in exercise 7.10 of the textbook, explain carefully how many variables (unknowns) are needed, and what the meaning of each of them is. Label the variables individually as x_1, \dots, x_n and give the meaning of each in terms of relevant quantities in the network itself.
 - (b) For the same network, state what the objective function is. Write the objective function explicitly in terms of appropriate variables from among those that you described in 4a, using the same variable names as before.
 - (c) State the necessary inequality constraints for the network flow problem for the above network. Describe how many constraints are needed, and of what types. List the constraints explicitly, using the notation $x_1 \dots x_n$ for the variables from 4a.
 - (d) Cast the maximum network flow problem for the above network in the standard matrix-vector form described in 3a. Specify the column vectors b and c , and the matrix A in detail (show all of the elements of each, and what position each element occupies). You should retain the same labeling/ordering of variables that you described above in 4a and that you used for the preceding parts of this task.
 - (e) Use the `linprog` function in Matlab to solve the linear programming problem from 4d. Interpret the results in terms of the original network. What is the resulting size (value) of the maximum flow?