AI1110 Assignment-1
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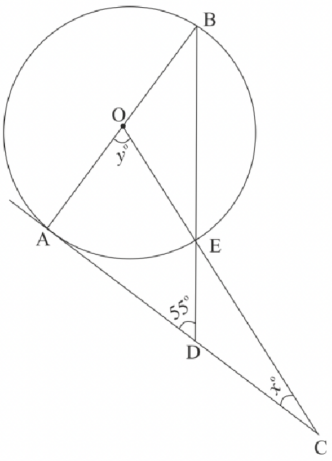
Question 7(a)
In the given figure AC is a tangent to the circle with centre O. If $\angle ADB = 55°$, find x and y. Give reasons for your answers.



Solution:
Given,

$$\angle BDA = 55° \tag{1}$$
$$\angle OCA = x° \tag{2}$$
$$\angle AOE = y° \tag{3}$$

As AC is a tangent to the given circle,

$$\angle OAC = \angle BAD = 90° \tag{4}$$

Angle Sum Property for $\triangle OAC$,

$$\angle OAC + \angle OCA + \angle AOC = 180° \tag{5}$$
$$90° + x° + y° = 180° \tag{6}$$
$$x° + y° = 90° \tag{7}$$

Angle Sum Property for $\triangle ABD$,

$$\angle ABD + \angle BAD + \angle BDA = 180° \tag{8}$$
$$\angle ABD + 90° + 55° = 180° \tag{9}$$
$$\angle ABD = 35° \tag{10}$$

Let the radius of the circle be r.

$$\implies OB = OE = r \tag{11}$$

Then, $\triangle BOE$ is an isosceles triangle.

$$\implies \angle ABD = \angle BEO \tag{12}$$

In a triangle, exterior angle is equal to sum of the two opposite interior angles.

$$\implies \angle AOE = \angle ABD + \angle BEO \tag{13}$$
$$\implies y = 2(\angle ABD) \tag{14}$$
$$\implies y = 70° \tag{15}$$
$$\implies x = 20° \tag{16}$$

METHOD-2
Steps for construction:
i) Draw a circle with centre at origin **O** and radius r.
Assume $r = 5$.

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{17}$$

ii) Consider points

$$\mathbf{A} = \begin{pmatrix} 0 \\ -r \end{pmatrix} \tag{18}$$
$$\implies \mathbf{A} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \tag{19}$$
$$\mathbf{B} = \begin{pmatrix} 0 \\ r \end{pmatrix} \tag{20}$$
$$\implies \mathbf{B} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \tag{21}$$

Join **A**,**B**.
iii) Consider a point **C** where

$$\mathbf{C} = \begin{pmatrix} AC \\ -r \end{pmatrix} \tag{22}$$

From the given figure in the question,

$$\tan\angle AOC = \frac{AC}{OA} \tag{23}$$

$$\tan y = \frac{AC}{r} \tag{24}$$

$$AC = r\tan y \tag{25}$$

$$AC = 5\tan 70^\circ \tag{26}$$

$$\implies \mathbf{C} = \begin{pmatrix} 5\tan 70^\circ \\ -5 \end{pmatrix} \tag{27}$$

Join $\mathbf{A}$,$\mathbf{C}$.
Join $\mathbf{O}$,$\mathbf{C}$.
iv) Consider a point D where,

$$\mathbf{D} = \begin{pmatrix} AD \\ -r \end{pmatrix} \tag{28}$$

From the given figure,

$$\tan\angle ABD = \frac{AB}{AD} \tag{29}$$

$$\tan 35^\circ = \frac{2r}{AD} \tag{30}$$

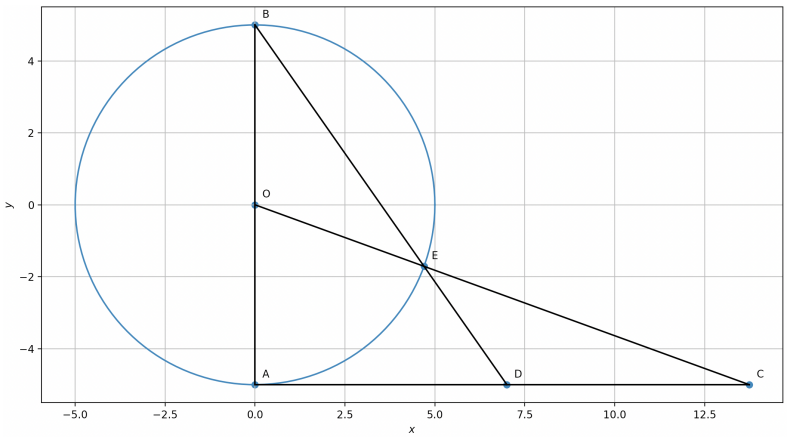
$$AD = 10\tan 35^\circ \tag{31}$$

$$\implies \mathbf{D} = \begin{pmatrix} 10\tan 35^\circ \\ -5 \end{pmatrix} \tag{32}$$

Join $\mathbf{B}$,$\mathbf{D}$.

Using the above steps of construction, generate the figure using python.

Generated Figure:



The input and output parameters required for drawing the figure are available in the below table.

Variable	Value	Input/Output
r	5	Input
$\angle AOC$	70°	Input
$\angle ABD$	35°	Input
O	**0**	Input
A	$\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	Input
B	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$	Input
C	$\begin{pmatrix} 5\tan 70^\circ \\ -5 \end{pmatrix}$	Output
D	$\begin{pmatrix} 10\tan 35^\circ \\ -5 \end{pmatrix}$	Output
E	$\begin{pmatrix} 5\sin 70^\circ \\ 5\cos 70^\circ \end{pmatrix}$	Output