AI1103 Assignment-1

AI21BTECH11026
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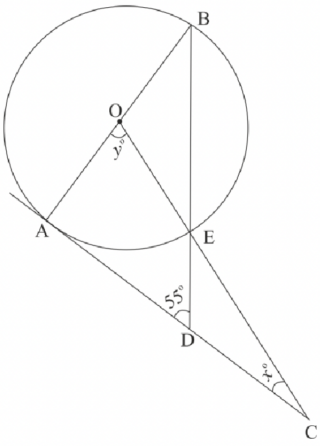
1 QUESTION-7A

1.1. In the given figure AC is a tangent to the circle with centre O. If $\angle ADB = 55°$, find x and y. Give reasons for your answers.



Solution:
Given,
$\angle BDA = 55°$, $\angle OCA = x°$, $\angle AOC = y°$

As AC is a tangent to the given circle,
$\angle OAC = \angle BAD = 90°$

Angle Sum Property for $\triangle OAC$,
$\angle OAC + \angle OCA + \angle AOC = 180°$
$90° + x° + y° = 180°$
$x° + y° = 90°$

Angle Sum Property for $\triangle ABD$,
$\angle ABD + \angle BAD + \angle BDA = 180°$
$\angle ABD + 90° + 55° = 180°$
$\angle ABD = 35°$

$\angle BOE + y° = 180°$
$\angle BOE = 180 - y$

Let the radius of the circle be 'r'.
then,
$OB = OE = r$
Then the length of the chord BE becomes $2rcos35°$.
Applying 'cosine' rule in $\triangle BOE$,

$(BE)^2 = (OB)^2 + (OE)^2 - 2(OB)(OE)cos(\angle BOE)$
$(2rcos35°)^2 = r^2 + r^2 - 2r^2cos(180 - y)$
$4r^2cos^235° = 2r^2 - 2r^2cos(180 - y)$
$2cos^235° = 1 - cos(180 - y)$
$cos(180-y) = -cos70°$ ($\because 1-cos^2\theta = cos2\theta$)
$cos(180 - y) = cos(180 - 70)$
$\therefore y = 70°$
$\therefore x = 20°$ ($\because x = 90 - y$)