

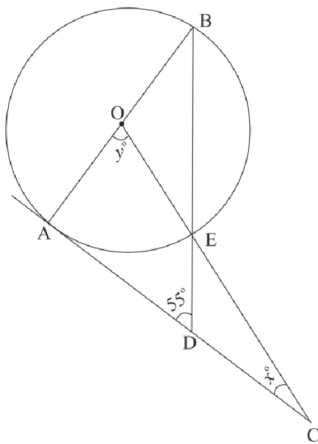
AI1103 Assignment-1

AI21BTECH11026
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CONTENTS

0.1. Question 7a.

In the given figure AC is a tangent to the circle with centre O. If $\angle ADB = 55^\circ$, find x and y. Give reasons for your answers.



Solution: Given,

$$\angle BDA = 55^\circ, \angle OCA = x^\circ, \angle AOC = y^\circ$$

As AC is a tangent to the given circle,
 $\angle OAC = \angle BAD = 90^\circ$

Angle Sum Property for $\triangle OAC$,

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ$$

$$90^\circ + x^\circ + y^\circ = 180^\circ$$

$$x^\circ + y^\circ = 90^\circ$$

Angle Sum Property for $\triangle ABD$,

$$\angle ABD + \angle BAD + \angle BDA = 180^\circ$$

$$\angle ABD + 90^\circ + 55^\circ = 180^\circ$$

$$\angle ABD = 35^\circ$$

$$\angle BOE + y^\circ = 180^\circ$$

$$\angle BOE = 180 - y$$

Let the radius of the circle be 'r'.

then,

$$OB = OE = r$$

Then the length of the chord BE becomes $2r \cos 35^\circ$.

Applying 'cosine' rule in $\triangle BOE$,

$$(BE)^2 = (OB)^2 + (OE)^2 - 2(OB)(OE) \cos(\angle BOE)$$

$$(2r \cos 35^\circ)^2 = r^2 + r^2 - 2r.r \cos(180 - y)$$

$$4r^2 \cos^2 35^\circ = 2r^2 - 2r^2 \cos(180 - y)$$

$$2 \cos^2 35^\circ = 1 - \cos(180 - y)$$

$$\cos(180 - y) = -\cos 70^\circ (\because 1 - \cos^2 \theta = \cos 2\theta)$$

$$\cos(180 - y) = \cos(180 - 70)$$

$$\therefore y = 70^\circ$$

$$\therefore x = 20^\circ (\because x = 90 - y)$$