

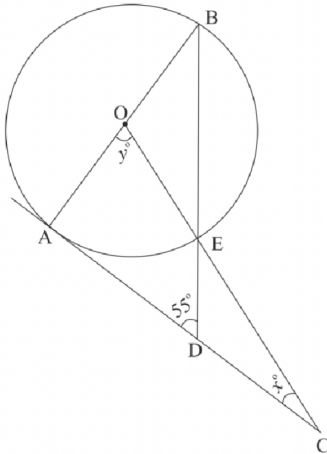
AI1110 Assignment-1

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Question 7(a)

In the given figure AC is a tangent to the circle with centre O . If $\angle ADB = 55^\circ$, find x and y . Give reasons for your answers.



Solution:

Given,

$$\angle BDA = 55^\circ$$

$$\angle OCA = x^\circ$$

$$\angle AOE = y^\circ$$

As AC is a tangent to the given circle,

$$\angle OAC = \angle BAD = 90^\circ$$

Angle Sum Property for $\triangle OAC$,

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ \quad (5)$$

$$90^\circ + x^\circ + y^\circ = 180^\circ \quad (6)$$

$$x^\circ + y^\circ = 90^\circ \quad (7)$$

Angle Sum Property for $\triangle ABD$,

$$\angle ABD + \angle BAD + \angle BDA = 180^\circ \quad (8)$$

$$\angle ABD + 90^\circ + 55^\circ = 180^\circ \quad (9)$$

$$\angle ABD = 35^\circ \quad (10)$$

Let the radius of the circle be r .

$$\Rightarrow OB = OE = r \quad (11)$$

Then, $\triangle BOE$ is an isosceles triangle.

$$\Rightarrow \angle ABD = \angle BEO \quad (12)$$

In a triangle, exterior angle is equal to sum of the two opposite interior angles.

$$\Rightarrow \angle AOE = \angle ABD + \angle BEO \quad (13)$$

$$\Rightarrow y = 2(\angle ABD) \quad (14)$$

$$\Rightarrow y = 70^\circ \quad (15)$$

$$\Rightarrow x = 20^\circ \quad (16)$$

METHOD-2

Steps for construction:

i) Draw a circle with centre at origin O and radius r .

(1) Assume $r = 5$.

$$(2) \quad O = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (17)$$

(3) ii) Consider points

$$(4) \quad A = \begin{pmatrix} 0 \\ r \end{pmatrix} \quad (18)$$

$$\Rightarrow A = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \quad (19)$$

$$B = \begin{pmatrix} 0 \\ -r \end{pmatrix} \quad (20)$$

$$\Rightarrow B = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \quad (21)$$

Join A, B .

iii) Consider a point C where

$$C = \begin{pmatrix} -r \\ AC \end{pmatrix} \quad (22)$$

From the given figure in the question,

$$\tan \angle OCA = \frac{OA}{AC} \quad (23)$$

$$\tan x = \frac{r}{AC} \quad (24)$$

$$AC = r \tan x \quad (25)$$

$$AC = 5 \tan 20^\circ \quad (26)$$

$$\Rightarrow \mathbf{C} = \begin{pmatrix} -5 \\ 5 \tan 20^\circ \end{pmatrix} \quad (27)$$

Join A,C.

Join O,C.

iv) Consider a point D where,

$$\mathbf{D} = \begin{pmatrix} -r \\ AD \end{pmatrix} \quad (28)$$

From the given figure,

$$\tan \angle BDA = \frac{AB}{AD} \quad (29)$$

$$\tan 55^\circ = \frac{2r}{AD} \quad (30)$$

$$AD = 10 \tan 55^\circ \quad (31)$$

$$\Rightarrow \mathbf{D} = \begin{pmatrix} -5 \\ 10 \tan 55^\circ \end{pmatrix} \quad (32)$$

Join B,D.

Using the above steps of construction, generate the figure using python.

Generated Figure:

