ASSIGNMENT-2 : ICSE 12,2019

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PROBLEM 8-A :

Evaluate :

$$\int \frac{x\left(1+x^2\right)}{1+x^4}dx$$

Solution:
Let

$$x^2 = t \tag{1}$$
$$\implies 2xdx = dt \tag{2}$$
$$\implies xdx = \frac{dt}{2} \tag{3}$$

From (1) and (3),

$$\int \frac{x\left(1+x^2\right)}{1+x^4}dx = \int \frac{(1+t)}{2\left(1+t^2\right)}dt \tag{4}$$
$$\int \frac{(1+t)}{2\left(1+t^2\right)}dt = \frac{1}{2}\int \frac{1}{1+t^2}dt + \frac{1}{2}\int \frac{t}{1+t^2}dt \tag{5}$$
$$= \frac{1}{2}\tan^{-1}(t) + C_1 + \frac{1}{2}\int \frac{t}{1+t^2}dt + \tag{6}$$

$$\boxed{\because \int \frac{1}{1+x^2}dx = \tan^{-1}(x) + C}$$

Let

$$1 + t^2 = z \tag{7}$$
$$\implies 2tdt = dz \tag{8}$$
$$\implies tdt = \frac{dz}{2} \tag{9}$$

From (7) and (9)

$$\int \frac{t}{1+t^2}dt = \frac{1}{2}\int \frac{dz}{z} \tag{10}$$
$$= \frac{1}{2}\ln z + C_2 \tag{11}$$

$$\boxed{\because \int \frac{dx}{x} = \ln x + C}$$

From (1), (6), (7) and (11)

$$\int \frac{x\left(1+x^2\right)}{1+x^4}dx = \frac{1}{2}\tan^{-1}(x^2) + \frac{1}{4}\ln\left(1+x^4\right) + C_1 + C_2 \tag{12}$$