

Assignment 4

Saanvi Amrutha-AI21BTECH11026

May 16, 2022

Outline

- 1 Question
- 2 Defining random variables
- 3 Values of probabilities
- 4 Solving
- 5 Solving

Question

Exercise 13.3.11

A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Solution

Let $X \in \{0, 1, 2\}$, $Y \in \{0, 1\}$ be random variables.

Random Variable	Description
$X=0$	Operator A is on the job
$X=1$	Operator B is on the job
$X=2$	Operator C is on the job
$Y=0$	Produced item is non-defective
$Y=1$	Produced item is defective

Table 1:

Given that,

$$\Pr(X = 0) = \frac{50}{100} \quad (1)$$

$$\Pr(X = 1) = \frac{30}{100} \quad (2)$$

$$\Pr(X = 2) = \frac{20}{100} \quad (3)$$

$$\Pr(Y = 1|X = 0) = \frac{1}{100} \quad (4)$$

$$\Pr(Y = 1|X = 1) = \frac{5}{100} \quad (5)$$

$$\Pr(Y = 1|X = 2) = \frac{7}{100} \quad (6)$$

Then,

$$\Pr(Y = 1) = \sum_{i=0}^2 \Pr(X = i) \Pr(Y = 1|X = i) \quad (7)$$

$$= \frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100} \quad (8)$$

$$= \frac{340}{10000} \quad (9)$$

The probability that the defective item was produced by A is given by $\Pr(X = 0|Y = 1)$.

Now we can write,

$$\Pr(X = 0|Y = 1) = \frac{\Pr(X = 0 \cap Y = 1)}{\Pr(Y = 1)} \quad (10)$$

$$= \frac{\Pr(X = 0) \Pr(Y = 1|X = 0)}{\Pr(Y = 1)} \quad (11)$$

From (1),(4) and (9)

$$\Pr(X = 0|Y = 1) = \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{340}{10000}} \quad (12)$$

$$= \frac{50}{340} \quad (13)$$

$$= \frac{5}{34} \quad (14)$$

\therefore The probability that the defective item was produced by A is $\frac{5}{34}$.