ASSIGNMENT-4

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Excerscise 13.3

PROBLEM-11 :

A manufacturer has three machine operators A,B and C. The first operator A produces 1% defective items,where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Solution:

Let $X \in \{0,1,2\}, Y \in \{0,1\}$ be random variables.

Random Variable	Description
X=0	Operator A is on the job
X=1	Operator B is on the job
X=2	Operator C is on the job
Y=0	Produced item is non-defective
Y=1	Produced item is defective

TABLE I

Given that,

$$\Pr(X=0) = \frac{50}{100} \tag{1}$$
$$\Pr(X=1) = \frac{30}{100} \tag{2}$$
$$\Pr(X=2) = \frac{20}{100} \tag{3}$$
$$\Pr(Y=1|X=0) = \frac{1}{100} \tag{4}$$
$$\Pr(Y=1|X=1) = \frac{5}{100} \tag{5}$$
$$\Pr(Y=1|X=2) = \frac{7}{100} \tag{6}$$

Then,

$$\Pr(Y=1) = \sum_{i=0}^{2} \Pr(X=i)\Pr(Y=1/X=i) \tag{7}$$
$$= \frac{50}{100}\times\frac{1}{100} + \frac{30}{100}\times\frac{5}{100} + \frac{20}{100}\times\frac{7}{100} \tag{8}$$
$$= \frac{340}{10000} \tag{9}$$

The probability that the defective item was produced by A is given by $\Pr(X=0|Y=1)$.
Now we can write,

$$\Pr(X=0|Y=1) = \frac{\Pr((X=0)\cap(Y=1))}{\Pr(Y=1)} \tag{10}$$
$$= \frac{\Pr(X=0)\Pr(Y=1|X=0)}{\Pr(Y=1)} \tag{11}$$

From(1),(4) and(9)

$$\Pr(X=0|Y=1) = \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{340}{10000}} \tag{12}$$
$$= \frac{50}{340} \tag{13}$$
$$= \frac{5}{34} \tag{14}$$

$\therefore$ The probability that the defective item was produced by A is $\frac{5}{34}$