Assignment 7:Papoulis Chapter 9

Saanvi Amrutha-AI21BTECH11026

June 16, 2022

Outline

Question

Show that

1. if $R_x[m_1, m_2] = q[m_1]\delta[m_1 - m_2]$ and $s = \sum_{n=0}^{N} a_n x[n]$ then $E\left\{s^2\right\} = \sum_{n=0}^{N} a_n^2 q[n]$
2. if $R_x\left(t_1, t_2\right) = q\left(t_1\right)\delta\left(t_1 - t_2\right)$ and $s = \int_0^T a\left(t\right)x\left(t\right)dt$ then $E\left\{s^2\right\} = \int_0^T a^2\left(t\right)q\left(t\right)dt$

Solving(a)

1. Given that

$$s = \sum_{n=0}^{N} a_n x[n] \tag{1}$$

$$s^2 = \sum_{n=0}^{N} a_n x[n] \sum_{n=0}^{N} a_n x[n] \tag{2}$$

$$= \sum_{n=0}^{N} a_n x[n] \sum_{m=0}^{N} a_m x[m] \tag{3}$$

$$= \sum_{n=0}^{N} \sum_{m=0}^{N} a_n a_m x[n] x[m] \tag{4}$$

$$E\left\{s^2\right\} = \sum_{n=0}^{N}\sum_{m=0}^{N} a_n a_m E\left\{x[n]x[m]\right\} \tag{5}$$

From,

$$E\left\{x[m_1]x[m_2]\right\} = R_x[m_1, m_2] \tag{6}$$

$$E\left\{s^2\right\} = \sum_{n=0}^{N}\sum_{m=0}^{N} a_n a_m R_x[n, m] \tag{7}$$

$$= \sum_{n=0}^{N}\sum_{m=0}^{N} a_n a_m q[n]\delta[n-m] \tag{8}$$

$$= \sum_{n=0}^{N} a_n^2 q[n] \tag{9}$$

Solving(b)

2. Given that

$$s = \int_0^T a(t)\,x(t)\,dt \tag{10}$$

$$s^2 = \int_0^T a(t)\,x(t)\,dt \int_0^T a(t)\,x(t)\,dt \tag{11}$$

$$= \int_0^T a(t)\,x(t)\,dt \int_0^T a(k)\,x(k)\,dk \tag{12}$$

$$= \int_0^T \int_0^T a(t)\,a(k)\,x(t)\,x(k)\,dkdt \tag{13}$$

$$E\left\{s^2\right\} = \int_0^T \int_0^T a\left(t\right) a\left(k\right) E\left\{x\left(t\right) x\left(k\right)\right\} dkdt \tag{14}$$

From,

$$E\left\{x\left(t_1\right) x\left(t_2\right)\right\} = R_x\left(t_1, t_2\right) \tag{15}$$

$$E\left\{s^2\right\} = \int_0^T \int_0^T a\left(t\right) a\left(k\right) R_x\left(t_1, t_2\right) dkdt \tag{16}$$

$$= \int_0^T \int_0^T a\left(t\right) a\left(k\right) q\left(t\right) \delta\left(t-k\right) dkdt \tag{17}$$

$$= \int_0^T a^2\left(t\right) q\left(t\right) dt \tag{18}$$

Hence proved.