Assignment 8:Papoulis Chapter 15

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Outline

1. Question
2. Solution

Question

Prove that, in an irreducible Markov chain,all states are of the same type. They are either all transient, all persistent null or all persistent nonnull. All the states are either aperiodic or periodic with the same period.

Solution

The chain is irreducible,and hence every state is accessible from every other state.In that case, for any two states, the series $\sum_n p_{ii}^{(n)}$ and $\sum_n p_{jj}^{(n)}$ converge or diverge together.
Hence,all states are either transient or persistent. If e_i is persistent null, then $p_{ii}^{(n)} \to 0$ as $n \to \infty$ and $p_{jj}^{(n)} \to 0$ as $n \to \infty$ so that e_j and all other states are also persistent null.

Finally if e_i is persistent nonnull and has period T,then $p_{ii}^{(n)} > 0$ whenever n is a multiple of T only.

$$p_{ii}^{(m+r)} \geq p_{ij}^{(m)} p_{ji}^{(r)} = ab > 0 \tag{1}$$

since e_i and e_j are mutually accesible. Here,$(m+r)$ must be a multiple of T.

Finally,

$$p_{jj}^{(n+m+r)} \geq abp_{ii}^{(n)} > 0 \tag{2}$$

where n and $(n+m+r)$ are multiples of T.
Thus, T is also the period of the state e_j.
Hence proved.