ASSIGNMENT-1 : Oppenheim

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PROBLEM 3.3.b :

A casual LTI system has impulse response $h[n]$,for which the z-transform is

$$H(z) = \frac{1+z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)} \tag{1}$$

Is the system stable? Explain.
Solution:
Given,

$$H(z) = \frac{1+z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)} \tag{2}$$

$$= \frac{2}{\left(1-\frac{1}{2}z^{-1}\right)} - \frac{1}{\left(1+\frac{1}{4}z^{-1}\right)} \tag{3}$$

From (3), consider

$$\frac{2}{\left(1-\frac{1}{2}z^{-1}\right)} = 2\sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^{n} \qquad \text{for } |z| > \frac{1}{2} \tag{4}$$

Consider,

$$\frac{1}{\left(1+\frac{1}{4}z^{-1}\right)} = \sum_{n=0}^{\infty}\left(-\frac{1}{4}z^{-1}\right)^{n} \qquad \text{for } |z| > \frac{1}{4} \tag{5}$$

$$\implies H(z) = 2\sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^{n} - \sum_{n=0}^{\infty}\left(-\frac{1}{4}z^{-1}\right)^{n}, |z| > \frac{1}{2} \tag{6}$$

$$= \sum_{n=0}^{\infty}\left(\left(\frac{1}{2}\right)^{n-1} - \left(-\frac{1}{4}\right)^{n}\right)z^{-n} \tag{7}$$

$$= \sum_{n=-\infty}^{\infty}\left(\left(\frac{1}{2}\right)^{n-1} - \left(-\frac{1}{4}\right)^{n}\right)u(n)\, z^{-n} \tag{8}$$

We know that,

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)\, z^{-n} \tag{9}$$

Form (8)and (9),we get

$$h(n) = \left(\left(\frac{1}{2}\right)^{n-1} - \left(-\frac{1}{4}\right)^{n}\right)u(n) \tag{10}$$

For a stable system $\sum_{n=-\infty}^{\infty} h(n) < \infty$

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty}\left(\left(\frac{1}{2}\right)^{n-1} - \left(-\frac{1}{4}\right)^{n}\right)u(n) \tag{11}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n-1} - \sum_{n=0}^{\infty}\left(-\frac{1}{4}\right)^{n} \tag{12}$$

$$= \frac{2}{1-\frac{1}{2}} - \frac{1}{1+\frac{1}{4}} \tag{13}$$

$$= \frac{16}{5} < \infty \tag{14}$$

$\therefore$ The system is stable.