

## Mock INOI #8

## A. Riding a Bicolour

1 second, 512 megabytes

You're given a grid  $A$  with  $N$  rows and  $M$  columns. Each cell of this grid is coloured either red or blue. You should process  $Q$  queries. In each query, you will be given two cells  $(r_1, c_1)$  and  $(r_2, c_2)$ . Find whether there exists an *alternating path* from  $(r_1, c_1)$  to  $(r_2, c_2)$ .

A path between two cells is a series of cells from the first to the second, such that each pair of adjacent cells on this path share a side. A path is called an *alternating path* if the colours on the path are some contiguous subsequence of  $\{\text{red, blue, red, blue, } \dots\}$ .

**Input**

The first line contains the subtask number  $S$  ( $1 \leq S \leq 6$ ).

The second line contains 3 space-separated integers:  $N M Q$

The next  $N$  lines contain  $M$  space-separated integers. The  $i$ -th of these lines contains integers  $A_{i,1} A_{i,1} \dots A_{i,M}$ .  $A_{i,j} = 1$  indicates that the cell is red.  $A_{i,j} = 0$  indicates that the cell is blue.

The next  $Q$  lines contain 4 integers:  $r_1 c_1 r_2 c_2$ , denoting the starting cell  $(r_1, c_1)$  and the ending cell  $(r_2, c_2)$  for the query.

**Output**

Print  $Q$  lines, each line containing a single integer. The  $i$ -th of these lines should contain 1 if there exists an alternating path for the given starting and ending cell of the  $i$ -th query, and 0 otherwise.

**Scoring**

The input is divided into multiple subtasks. You will be awarded the points for a subtask if you correctly solve every testcase in the subtask.

For all subtasks,  $1 \leq N \cdot M, Q \leq 10^5$ .

- Subtask 1 [10 points]:  $1 \leq N \cdot M \leq 10$  and  $1 \leq Q \leq 5$
- Subtask 2 [10 points]:  $1 \leq N \cdot M \leq 10$  and  $1 \leq Q \leq 100$
- Subtask 3 [5 points]:  $A_{i,j} = 1$  if and only if  $(i + j)$  is even
- Subtask 4 [20 points]:  $1 \leq N, M \leq 50$  and  $1 \leq Q \leq 10^5$
- Subtask 5 [20 points]:  $1 \leq N, M, Q \leq 1000$
- Subtask 6 [35 points]:  $1 \leq N \cdot M \leq 10^5$  and  $1 \leq Q \leq 10^5$

input
1 2 2 2 0 1 0 0 1 1 2 1 1 1 2 2
output
0 1

You're given a grid with 2 rows and columns.

In the first query, you need to find an alternating path from the top-left cell to the bottom left cell. It can be shown that no such path exists, and so you should print 0.

In the second query, you need to find an alternating path from the top-left cell to the bottom-right cell. Such a path is:  $(1, 1), (1, 2), (2, 2)$ . When you write down the colours of the cells on this path, you get the sequence  $\{\text{blue, red, blue}\}$ , which is alternating. You should output 1 for this query.

## B. Frequent Queries

3.4 seconds, 256 megabytes

You're given an array  $V$  with  $N$  integers.

Answer  $Q$  queries of the form  $l_i r_i$ . To answer the  $i$ -th query, print any integer  $X$  such that  $X$  occurs at least  $A$  and at most  $B$  times in the subarray  $V[l_i \dots r_i]$ . If there is no such integer, print  $-1$ .

**Input**

The first line contains the subtask number  $S$  ( $1 \leq S \leq 7$ ).

The second line contains 4 space separated integers:  $N Q A B$

The third line contains  $N$  space separated integers  $V_1 \dots V_N$

The next  $Q$  lines contain two space separated integers. The  $i$ -th of these lines describes  $l_i r_i$ .

**Output**

Print  $Q$  lines, each line containing a single integer. The  $i$ -th of these lines should contain the answer to the  $i$ -th query.

**Scoring**

The input is divided into multiple subtasks. You will be awarded the points for a subtask if you correctly solve every testcase in the subtask.

For all subtasks,  $1 \leq N, Q, V_i \leq 10^6$ .

- Subtask 1 [9 points]:  $1 \leq N, Q, V_i \leq 100$
- Subtask 2 [5 points]:  $1 \leq N, Q \leq 2000$
- Subtask 3 [14 points]:  $1 \leq N \leq 1000$  and  $1 \leq Q \leq 5 * 10^5$
- Subtask 4 [12 points]:  $1 \leq N, Q \leq 10^5$  and  $1 \leq V_i \leq 20$
- Subtask 5 [12 points]:  $1 \leq N, Q \leq 4 * 10^4$
- Subtask 6 [24 points]:  $1 \leq N, Q \leq 2 * 10^5$
- Subtask 7 [24 points]: No additional constraints

input
1 6 2 1 2 1 1 2 2 2 3 1 6 3 5
output
1 -1

For the first query, the entire array is considered. 1 occurs 2 times, 2 occurs 3 times, and 3 occurs 1 time. Since the number of occurrences of value  $2 > B$  (that is,  $3 > 2$ ), 2 is not a valid answer for the first query. You can print either of the possible answers 1 or 3.

For the second query, the subarray in consideration is  $[2, 2, 2]$ . There are no valid integers satisfying the conditions in this subarray, so you should print  $-1$ .

## C. Capital Messages

1.2 seconds, 128 megabytes

You're the President of UFDS. Yes, you, not Aruj! Unless you're Aruj.

Your country has  $N$  cities, labelled from 1 through  $N$ . However, only  $K$  ( $K < N$ ) of those are called **special**. These cities are numbered from 1 to  $K$ . But the capital of your country is the city labelled  $K + 1$ . As a result, it is **not special**. There are also  $M$  **unidirectional** roads connecting some pairs of cities. It is guaranteed that starting from each city, you can reach every other city.

You want to divide the special cities into  $G$  ( $1 \leq G \leq K$ ) non-empty groups. Each day, each special city sends a message to every other special city in its group.

The messages are routed in an interesting fashion. If a message is sent by some special city  $X$  to another special city  $Y$ , then the message first goes from  $X$  to the capital (which is the city with label  $K + 1$ ), and then from the capital to city  $Y$ . Messages are allowed to pass through non-special cities as well. Messages can also pass the same cities several times, including the endpoints. The only restriction is that the message must start at  $X$ , pass through the capital, and end at  $Y$ . The cost to send a message is the distance travelled by the message.

Over all possible groupings of the  $K$  special cities into  $G$  disjoint groups, find the minimum cost incurred by the country to send the messages each day.

### Input

The first line contains the subtask number  $S$  ( $1 \leq S \leq 6$ ).

The second line contains 4 space-separated integers:  $N M K G$

The next  $M$  lines contain 3 integers:  $a b w$  ( $1 \leq a, b \leq N$ ,  $1 \leq w \leq 10000$ ). These denote a unidirectional road of length  $w$  from city  $a$  to city  $b$ .

### Output

Print a single line with a single integer, equal to the minimum cost to deliver messages each day over all possible groupings of special cities.

### Scoring

The input is divided into multiple subtasks. You will be awarded the points for a subtask if you correctly solve every testcase in the subtask.

For all subtasks:

$$2 \leq N \leq 5000$$

$$1 \leq G \leq K < N$$

$$1 \leq M \leq 50000$$

Every city is reachable from every other city.

- Subtask 1 [15 points]:  $1 \leq N, G \leq 6$
- Subtask 2 [5 points]:  $G = 1$

- Subtask 3 [5 points]: For all  $1 \leq i \leq K$ , there exists an edge from city  $i$  to  $N$  and an edge from city  $N$  to  $i$ .
- Subtask 4 [10 points]:  $M = N$ . From all cities  $1 \leq i \leq N - 1$ , there exists an edge to  $i + 1$ . An edge from  $N$  to 1 also exists.
- Subtask 5 [30 points]:  $1 \leq N \leq 100$
- Subtask 6 [35 points]: No additional constraints.

input
1 3 4 2 1 1 3 8119 3 2 2708 3 1 8188 2 3 7550
output
26565

There are 3 cities labelled from 1 through 3.  $K = 2$  of these are special, and are numbered from 1 and 2. City 3 is the capital.

You need to put the special cities into  $G = 1$  group. Hence, both special cities go into the same group.

Each day, city 1 sends a message to city 2 passing through the capital, which follows the path  $1 \rightarrow 3 \rightarrow 2$ . This takes a cost of  $8119 + 2708 = 10827$ .

Also, city 2 sends a message to city 1 passing through the capital, which takes the path  $2 \rightarrow 3 \rightarrow 1$ . This takes a cost of  $7550 + 8188 = 15738$ .

The total cost is  $10827 + 15738 = 26565$ .