Information and Communication (EC4.102)

Assignment 1

Spring-24, January 23, 2024

1. Complete the assignment all by yourself. You can discuss approaches to the questions but the solution should be yours.
2. Draw neat and well labelled plots wherever asked. Incomplete plots will be penalized.
3. For MATLAB questions, write clean and well-commented code. Label the axes appropriately with proper titles. Un-labelled plots will get 0.

Deadline: 27th Jan 11:59 PM

1. Compute the Fourier transform for the following signals:

   (a) $u(t)$ (Unit Step Function)

   (b) $[e^{-\alpha t} cos(2\pi f_o t)]u(t)$, $\alpha > 0$

   (c) $e^{2+t}u(-t+1)$

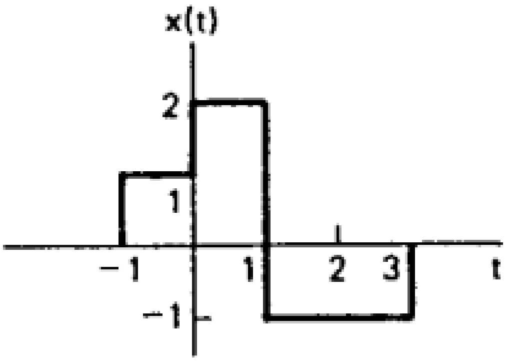
   (d) $e^{-3|t|}sin(2t)$

   (e) $\sum_{k=0}^{\infty} \alpha^k \delta(t-kT)$, $|\alpha| < 1$

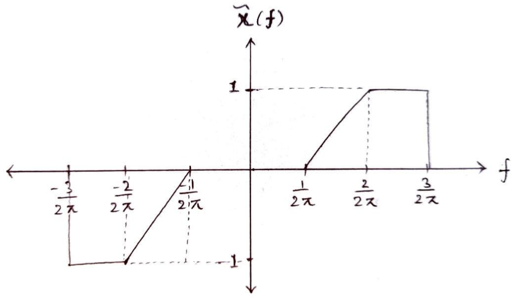
   (f) $[te^{-2t}sin(4t)]u(t)$

   (g) $x(t)$ as in Figure 1

   HINT : *For (a), use the differentiation in time domain property. Calculate $\tilde{u}(f)$ for $f = 0$ (DC component) separately and combine the results.*



(a) Figure 1



(b) Figure 2

2. The following are the Fourier transforms of continuous-time signals. Determine the continuous-time signal corresponding to each transform.

   (a) $\tilde{x}(f) = \frac{2sin[3(2\pi f - 2\pi)]}{(2\pi f - 2\pi)}$

   (b) $\tilde{x}(f) = cos(8\pi f + \frac{\pi}{3})$

   (c) $\tilde{x}(f) = 2[\delta(2\pi f - 1) - \delta(2\pi f + 1)] + 3[\delta(2\pi f - 2\pi) + \delta(2\pi f + 2\pi)]$

   (d) $\tilde{x}(f)$ as in Figure 2

3. A real, continuous-time function $x(t)$ has a Fourier transform $X(f)$ whose magnitude obeys the relation

$$\ln|X(f)| = -|f|$$

   Find $x(t)$ if $x(t)$ is known to be:

(a) an even function of time

(b) an odd function of time

4. (a) Prove that convolution in time-domain is equivalent to multiplication in frequency domain.

$$x_1(t) * x_2(t) = \tilde{x_1}(f) \cdot \tilde{x_2}(f)$$

(b) Calculate the fourier transform of $sinc(t)$

(c) Using the above property, prove that

$$sinc(t) * sinc(t) = sinc(t)$$

5. Let $x(t)$ be a time-domain signal, whose Fourier transform is given according to the figure below :-

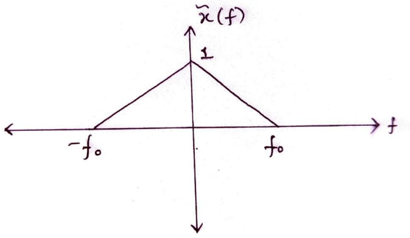


Figure 2: Fourier transform of $x(t)$

Now consider the signal $y(t) = \sum_{n=-\infty}^{\infty} x(t) \cdot e^{2j\pi n f_0 t}$. Find the Fourier transform of $y(t)$ and plot it (on paper). From this, obtain a simplified expression for $y(t)$. HINT : Modulation Property

6. The impulse train $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ can be used for sampling continuous-time signals. Its Fourier Transform is:

$$\tilde{p}(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

(a) Sketch $\tilde{p}(f)$ for the sampling period $T = 2$.

(b) Sketch the spectrum of the sampled signal $y(t) = x(t)p(t)$ where the spectrum of $x(t)$:

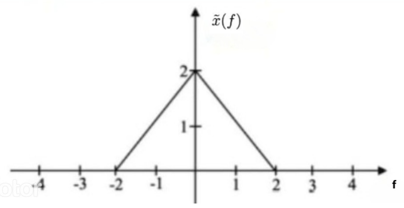


Figure 3: Fourier transform of $x(t)$

Hint: Multiplication in the time domain results in convolution in the frequency domain as follows:

$$x(t)p(t) \leftrightarrow \tilde{x}(f) * \tilde{p}(f)$$

(c) Sketch the spectrum of the sampled signal $y(t)$ for the sampling period $T = \frac{2\pi}{3}$.

(d) Explain whether you can recover the signal $x(t)$ from the sampled signal in parts (b) and (c).

Coding Questions (in MATLAB only)

1. Consider the continuous time signal $x(t) = Asin(2\pi f_o t)$. It is given that A = 5 and $f_o = 20$ Hz with a sampling rate of 1000 Hz. Plot the real and imaginary part of the Fourier transform of $x(t)$ along with it's magnitude and phase all in a single plot.(Use the subplot function in MATLAB to plot these.)

2. Consider the message signal $m(t) = A_m \cdot sin(2\pi f_m t)$ and the carrier signal $c(t) = A_c \cdot cos(2\pi f_c t)$. Find out the modulated signal $x(t)$ when $m(t)$ is modulated with carrier $c(t)$ for the following values;

   - $A_c = 1$ and $A_m = 2$
   - $f_c = 100$ Hz and $f_m = 10$ Hz

   Plot the Magnitude Spectrum for Fourier transform of $m(t), c(t)$ and $x(t)$. What do you observe? Write briefly (as comment in the code itself).

3. Consider the signal
$$x(t) = \frac{sin(\pi t)}{\pi t}$$
   Calculate the fourier transform of the following signals :-

   1. $e^{j2\pi f_c t} \cdot x(t)$ where $f_c = 10$ Hz
   2. $x(t/A)$ where $A = 20$
   3. $x(t)^2$

   Plot the Fourier transform of the above signals alongwith $\tilde{x}(f)$ in 2×2 subplot in MATLAB, verifying and naming the properties demonstrated in each plot.
