Information and Communication (EC4.102)

Assignment 2

Spring-24, February 17, 2024

---

1. Complete the assignment all by yourself. You can discuss approaches to the questions but the solution should be yours.
2. Draw neat and well labelled plots wherever asked. Incomplete plots will be penalized.
3. For MATLAB questions, write clean and well-commented code. Label the axes appropriately with proper titles. Un-labelled plots will get 0.

Deadline: 21st Feb 11:59 PM

---

1. In digital communications, sampling is the operation of converting a continuous-time signal into a discrete-time signal. Sampling involves multiplying the continuous-time signal with an impulse train. An impulse train is written as:

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

where, T_s is the sampling period.

(a) Prove that an impulse train in the time domain is also an impulse train in the frequency domain with some period f_s (called the sampling frequency). Find the relation between f_s and T_s. [3+1]

(b) Given a time signal $m(t)$ whose Fourier spectrum is $\tilde{m}(f)$, find a mathematical form for the Fourier transform of the sampled signal $m_s(t) = m(t) \cdot p(t)$. Clearly mention the property used in each step. [3]

(c) Let the Fourier transform of $m(t)$ be given in Fig. 1. Recall and discuss the problem of aliasing by taking the appropriate labelled Fourier spectrum diagrams. (*Hint: vary* f_s). Thereby prove that in order to avoid aliasing, the maximum permissible value of T_s (called the Nyquist interval) is $\frac{1}{2B}$. [3+1]

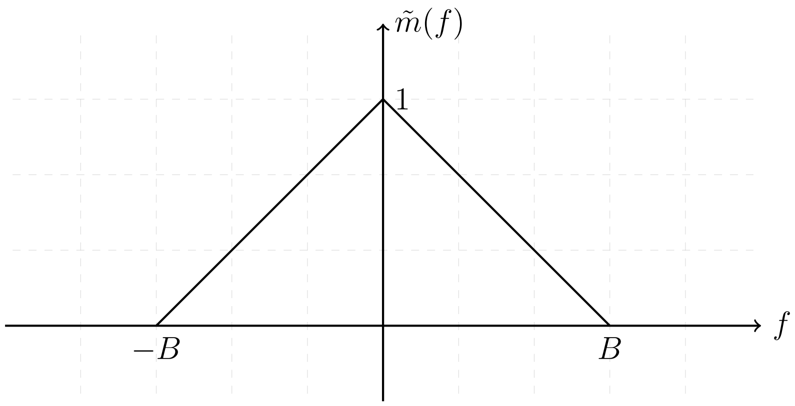


Figure 1

2. Let the signal $x(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$. Then:

(a) Solving the Fourier transform or otherwise, obtain and draw the frequency domain representation for the signal $x(t)$. What is (i) the bandwidth of this signal, (ii) the Nyquist rate of sampling? [3+1+1]

(b) Take the sampling frequency $f_s = 5000Hz$. For this sampling frequency, draw the frequency domain representation for the sampled signal for x-axis restricted to the range $f \in [0, 3f_s]$. Neatly label the diagram. Also, mark the aliasing locations in the diagram, if any. [3+1]

3. Let $x(t)$ be a signal with Nyquist rate f_0. Determine the Nyquist rate for each of the following signals: [2+2+2+2+2]

   (a) $x(t) + x(t-1)$

   (b) $\frac{dx(t)}{dt}$

   (c) $x^2(t)$

   (d) $x(t)cos(2\pi f_0 t)$

   (e) The output $y(t)$ after passing $x(t)$ through a low-pass filter with bandwidth B_1.

4. The message signal $m(t) = 2cos(400t)+4sin(500t+\pi/3)$ modulates the carrier signal $c(t) = Acos(8000\pi t)$, using DSB-SC amplitude modulation. Find the time domain and frequency domain representation of the modulated signal. Plot the spectrum (Fourier transform) of the modulated signal. [3+2]

5. In a DSB system the carrier is $c(t) = Acos(2\pi f_c t)$ and the message signal is given by $m(t) = sinc(t) + sinc^2(t)$. Find the frequency domain representation and the bandwidth of the modulated signal. Here, $sinc(t)$ is given as: [2+1]

$$sinc(t) = \frac{sin(\pi t)}{\pi t}$$

6. A message signal $m(t) = cos(2000\pi t) + 2cos(4000\pi t)$ modulates the carrier $c(t) = 100cos(2\pi f_c t)$ where $f_c = 1$ MHz, to produce the DSB signal $m(t) \cdot c(t)$.

   (a) Determine the expression for the upper sideband (USB) signal. [2]

   (b) Determine and sketch the spectrum of the USB signal. [3]

7. An FM signal is given as:

$$u(t) = 100\cos\left(2\pi f_c t + 100\int_{-\infty}^{t} m(\tau)d\tau\right),$$

   where $m(t)$ is given as in Fig. 2. Sketch the instantaneous frequency as a function of time. Determine the peak frequency deviation, i.e., the difference between the maximum instantaneous frequency and the minimum instantaneous frequency. [3+1]

8. You flip three identical fair coins, with each flip being independent of the others. Define the events:

$$\begin{aligned}
\text{A} &= \{\text{all tosses give the same outcome}\}\\
\text{B} &= \{\text{there is at most one head}\}\\
\text{C} &= \{\text{there is at least one head and one tail}\}
\end{aligned}$$

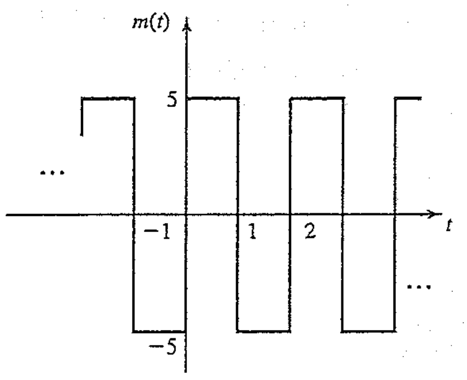


Figure 2

(a) Find whether (i) A is independent of B, (ii) B is independent of C, (iii) A is independent of C. [3]

(b) Investigate whether these results hold if the coins are not fair, but still identical (i.e., all coins have $P(H) = p$). [3]

9. You repeatedly flip a coin with $P(H) = p$. Show that a head eventually turns up with probability one. [2]

10. A memoryless source has the alphabet $A = \{-5, -3, -1, 0, 1, 3, 5\}$ with corresponding probabilities $\{0.05, 0.1, 0.1, 0.15, 0.05, 0.25, 0.3\}$. The entropy $H(A)$ is given by:

$$H(A) = -\sum_{i=1}^{7} p_i \cdot \log_2(p_i)$$

where p_i is the probability of the i^{th} symbol in the alphabet.

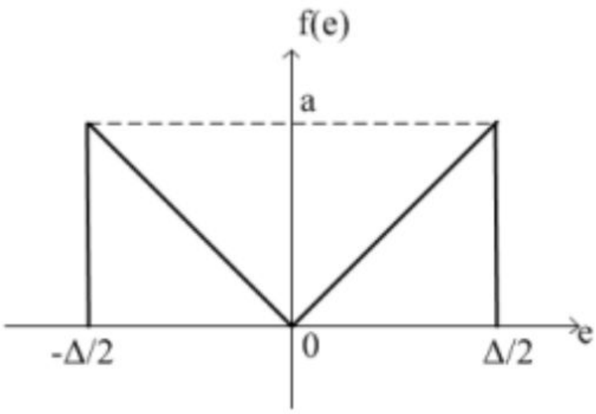
(a) Find the entropy of the source. [2]

(b) Assuming that the source is quantized according to the quantization rule:

$$q(-5) = g(-3) = -4, \quad q(-1) = q(0) = q(1) = 0, \quad q(3) = q(5) = 4$$

Find the entropy of the quantized source. [2]

11. Assuming the input signal lies in the range $[-4, 4]$ and you are doing 4 levels of quantization and the probability density function (pdf) $f_\epsilon(e)$ of the quantization error is given by (where Δ is the step size):



(a) Determine a so that $f_\epsilon(e)$ is a valid pdf. [2]

(b) Determine the mean quantization error ϵ, defined as: [3]

$$E[\epsilon] = \int_{-\infty}^{\infty} \epsilon f_\epsilon(\epsilon)\, d\epsilon.$$

(c) Determine the variance of the random variable E, defined as: [3]

$$\sigma^2 = \int_{-\infty}^{\infty} (\epsilon - E[\epsilon])^2 f_\epsilon(\epsilon)\, d\epsilon.$$

Coding Question (in MATLAB only)

1. Consider a continuous-time message signal $m(t)$ and a carrier signal $c(t)$ defined as:

$$m(t) = \sin(2\pi \cdot 100 \cdot t) + \sin(2\pi \cdot 200 \cdot t)$$
$$c(t) = \cos(2\pi \cdot 1000 \cdot t)$$

(a) Write MATLAB code to generate and plot the message signal $m(t)$ and the magnitude spectrum of its Fourier transform. [5]

(b) Write MATLAB code to generate and plot the carrier signal $c(t)$ and the magnitude spectrum of its Fourier spectrum. [5]

(c) Perform Amplitude Modulation (AM) on the message signal $m(t)$ using the carrier signal $c(t)$ with a modulation index of $\mu = 0.5$ and plot the modulated signal $s(t)$ and the magnitude spectrum of its Fourier transform. Refer to this link to understand what is AM and how is it performed. [5]

(d) Perform Double-Sideband Suppressed Carrier (DSB-SC) modulation on the message signal $m(t)$ using the carrier signal $c(t)$ and plot the DSB-SC modulated signal $s_{\text{dsb-sc}}(t)$ and the magnitude spectrum of its Fourier transform. Refer to this link to understand what is DSB-SC Modulation and how is it performed. [5]

(e) Write down your observations in both the modulation schemes and explain when each modulation technique is preferred. [3]

(f) Try experimenting with different values of μ and observe the results. [**NOT** needed to be submitted (not graded)]

Instructions : Please **strictly** follow the following for the above question :

- Write the whole question in one script only.
- $m(t)$, $\tilde{m}(t)$, $c(t)$, and $\tilde{c}(t)$ should be in a single 4×4 plot.
- $s(t)$, $\tilde{s}(t)$, $s_{dsb-sc}(t)$, and $\tilde{s}_{dsb-sc}(t)$ should be in a single 4×4 plot.
- Save both the plots as **.png** images only in the **results** folder.
- For (e), write down your answer concisely as comments in the end of the script.
- Label and limit the axes for all your plots clearly for better visualization.
- Prefer using `fftshift()` along with `fft()` while calculating FFT. Read more about it here.
