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ME17B162

ME 7223 : ASSIGNMENT 1

1) a) $f = x_1^2 x_2^3 \quad x_1 > 0 , x_2 > 0$

As $x_1, x_2 > 0$:
$\ln f = 2 \cdot \ln x_1 + 3 \ln x_2$

Let the transformation be :
$y_1 = \ln x_1 , \quad y_2 = \ln x_2$

$\therefore \ln f = 2y_1 + 3y_2$
This separable form.

b) $f = x_1^{x_2} \quad x_1 \geq 0 .$

~~Let $x_1 = e^{a}$~~
$\ln f = x_2 . \ln x_1$
$\ln(\ln f) = \ln x_2 + \ln(\ln x_1)$
This is a separated form.
We need to substitute : $\ln x_1 = y_1$
$\therefore x_1 = e^{y_1}$

$\therefore \ln(\ln f) = \ln(x_2) + \ln(y_1)$
Separated form.
$\ln x_2 = y_2 \Rightarrow x_2 = e^{y_2} , \quad \ln y_1 = z_1 \Rightarrow y_1 = e^{z_1}$

$\therefore \ln(\ln f) = y_2 + z_1 \quad$ where $y_2 = \ln x_2 , \; z_1 = \ln(\ln(x_1))$

e) $h(x) = A\sin(kx) + B\cos(kx)$

$g(x) = g$ = constant

Given constraints are:

$$\left|\frac{d}{dx}(h(x) - g(x))\right| \leq r_1 \Rightarrow |Ak\cos(kx) - Bk\sin(kx)| \leq r_1$$

$$\left|\frac{d^2}{dx^2}(h(x) - g(x))\right| \leq r_2 \Rightarrow |-Ak^2\sin(kx) - Bk^2\cos(kx)| \leq r_2$$

$$\left|\frac{d^3}{dx^3}(h(x) - g(x))\right| \leq r_3 \Rightarrow |-Ak^3\cos(kx) + Bk^3\sin(kx)| \leq r_3$$

Cost for adding ~~the~~ removal:

$$c(x) = \int_0^L |A\sin(kx) - B\cos(kx) - g|\,dx$$

Thus to minimize the cost: we need to minimize A, B, k which are the variables responsible.

$\therefore$ Objective function: Minimize k, A, B and $\int_0^L |A\sin(kx) - B\cos(kx)| \, dx$

constraints: $|Ak\cos(kx) - Bk\sin(kx)| \leq r_1$

$|Ak^2\sin(kx) + Bk^2\cos(kx)| \leq r_2$

$|-Ak^3\cos(kx) + Bk^3\sin(kx)| \leq r_3$

5. The output for the code is given below:

Output

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>

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<stopping criteria details>

diff =

  112.9847
    0.0000
   85.5452
    0.0000
```

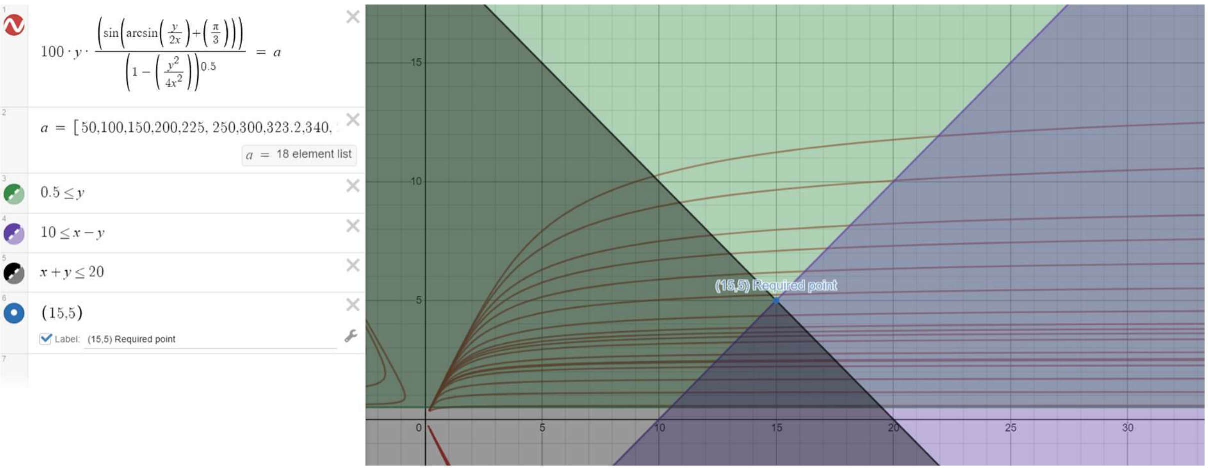
The reasoning for this is attached in the following page

5) 'diff' value is non-zero. This is mainly because there are other local minima's closer to [10, 10, 10, 10] than [0, 0, 0, 0]. Thus, these two are having different optimum value.
Also, another reason could be that max iterations by the fmincon could be reached and that the function chose the nearest global ~~maxima~~ minima or local minima.

6) Optimum value for both methods are equal.
This may not be the case every time and is true for LPs with linear constraints having convex feas. regions.
In case of non-linear it need not hold. But, they may have their opt points within bounds rather than at boundaries.

For concave functions it doesn't hold.
eg. $f = x^4 + y^4$
Constraint : $x \in [-1, 1]$, $y \in [-1, 1]$
Minima at (0,0) and not at a corner point.

6. The contours with the expected region is plotted below:



Here the required point is (15,5) as shown previously in the code