Assignment 8: Question 2

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Animation GIF is added separately in the submission

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import imageio
```

In [2]:

```
def f(x):
    return (x[0]-5)**2 + (x[1]-5)**2

def g1(x):
    return x[0]+2*x[1]-15

def g2(x):
    return -x[0]+1

def g3(x):
    return -x[1]+1

def g4(x):
    return x[0]-10

def g5(x):
    return x[1]-10
```

In [30]:

```
def cont_score (g1,g2,g3,g4,g5):
    func_vals = [g1,g2,g3,g4,g5]
    abv = np.array([max(i, 0) for i in func_vals])
    return np.sum(abv**2)

def err_diff_f(x):
    diff_x0 = 2*x[0]-10
    diff_x1 = 2*x[1]-10
    return diff_x0**2+diff_x1**2

def check(x):
    if g1(x)<=0 and x[0]>=0 and x[1]>=0:
        return True
    else:
        return False

def centroid(x):
    return np.mean(x,axis=0)

def worst_no(points):
    max_no = 0
    max_val = f(points[0])
    for i in range(len(points)-1):
        if f(points[i+1]) > max_val:
            max_no = i
            max_val = f(points[i])
    return max_no

def complex_pts(points, pt_worst):
    alpha = 1
    while alpha>1e-4:
        pt_new = (1.0 + alpha)*np.array(centroid(points)) - alpha*np.array(pt_worst)
        if (f(pt_new)<f(pt_worst) and check(pt_new) == True):
            points.append(pt_new.tolist())
            return points
        else:
            alpha = alpha/2
    return points.append(pt_worst.tolist())
```

In [44]:

```
ini = [[1.01,1.01],[1.1,1.1],[1.1,1.01],[1.01,1.1]]
mid = centroid(ini)
err = err_diff_f(mid)
points = ini
i = 0
max_iters = 4000
max_err = 1e-4
mid_points = [mid.tolist()]

while err>max_err:
    worst_no = worst_no(points)
    pt_worst = points[worst_no]
    points.remove(pt_worst)
    points = complex_pts(points, pt_worst)
    if points[worst_no(points)] != pt_worst:
        mid = centroid(points)
        mid_points.append(mid.tolist())
        err = err_diff_f(mid)
        i = i+1
    else:
        break
    if i%int(0.0005*max_iters) == 0:
        print('Iteration:',i,' - Centroid Point ', mid,' - Cost:', f(mid))
    if i>max_iters:
        break
```

```
Iteration: 2  - Centroid Point  [1.075 1.135]  - Cost: 30.34385
Iteration: 4  - Centroid Point  [1.13055556 1.19722222]  - Cost: 29.43371913
580247
Iteration: 6  - Centroid Point  [1.17820988 1.26339506]  - Cost: 28.56829621
2467608
Iteration: 8  - Centroid Point  [1.19206447 1.34646776]  - Cost: 27.84867078
5374097
Iteration: 10  - Centroid Point  [1.22133135 1.42183432]  - Cost: 27.0816063
5656894
Iteration: 12  - Centroid Point  [1.28097301 1.4820135 ]  - Cost: 26.2073908
19468337
Iteration: 14  - Centroid Point  [1.35773573 1.51363213]  - Cost: 25.4208498
9609765
Iteration: 16  - Centroid Point  [1.43960231 1.52714329]  - Cost: 24.7371654
47628772
Iteration: 18  - Centroid Point  [1.50321362 1.57323887]  - Cost: 23.9702068
2199426
Iteration: 20  - Centroid Point  [1.56577781 1.63020094]  - Cost: 23.1494277
1582229
Iteration: 22  - Centroid Point  [1.64682996 1.65216382]  - Cost: 22.4517564
15439277
Iteration: 24  - Centroid Point  [1.7248208 1.6710378]  - Cost: 21.808788163
720575
Iteration: 26  - Centroid Point  [1.78500397 1.7255974 ]  - Cost: 21.0579119
0242077
Iteration: 28  - Centroid Point  [1.85220571 1.77531576]  - Cost: 20.3071973
89890625
Iteration: 30  - Centroid Point  [1.93565543 1.79042433]  - Cost: 19.6915836
55935084
Iteration: 32  - Centroid Point  [1.95418034 1.87322275]  - Cost: 19.0537533
73343355
Iteration: 34  - Centroid Point  [2.00708206 1.93881707]  - Cost: 18.3283987
```

```
2382863
Iteration: 36  - Centroid Point  [2.06297371 1.99570223]  - Cost: 17.6519285
10072345
Iteration: 38  - Centroid Point  [2.08382413 2.07172685]  - Cost: 17.0788653
06386135
Iteration: 40  - Centroid Point  [2.10947155 2.15220743]  - Cost: 16.4650772
01801027
Iteration: 42  - Centroid Point  [2.16909421 2.21255833]  - Cost: 15.7838586
49304147
Iteration: 44  - Centroid Point  [2.24944263 2.23936823]  - Cost: 15.1866536
19800431
Iteration: 46  - Centroid Point  [2.3242829  2.26020307]  - Cost: 14.6659492
57542621
Iteration: 48  - Centroid Point  [2.37962144 2.31590669]  - Cost: 14.0707407
0870029
Iteration: 50  - Centroid Point  [2.44480185 2.36983676]  - Cost: 13.4467962
84205721
Iteration: 52  - Centroid Point  [2.5272969  2.38929216]  - Cost: 12.9300560
38321652
Iteration: 54  - Centroid Point  [2.54127267 2.47279904]  - Cost: 12.4320847
69386417
Iteration: 56  - Centroid Point  [2.58971344 2.54013567]  - Cost: 11.8604137
97139863
Iteration: 58  - Centroid Point  [2.6441849 2.6016311]  - Cost: 11.302038198
604098
Iteration: 60  - Centroid Point  [2.66285119 2.68059482]  - Cost: 10.8419049
45447805
Iteration: 62  - Centroid Point  [2.74363383 2.70170884]  - Cost: 10.3733305
62696115
Iteration: 64  - Centroid Point  [2.80629111 2.75808276]  - Cost: 9.83855159
6942954
Iteration: 66  - Centroid Point  [2.87105023 2.80466179]  - Cost: 9.35193700
1507014
Iteration: 68  - Centroid Point  [2.95346762 2.81815756]  - Cost: 8.94873122
1982968
Iteration: 70  - Centroid Point  [3.02985912 2.84880005]  - Cost: 8.50911628
2776393
Iteration: 72  - Centroid Point  [3.08993144 2.90871542]  - Cost: 8.02183310
3135465
Iteration: 74  - Centroid Point  [3.12033515 2.98458914]  - Cost: 7.59502086
9598549
Iteration: 76  - Centroid Point  [3.13433011 3.06725384]  - Cost: 7.21623185
0377756
Iteration: 78  - Centroid Point  [3.18164006 3.13245108]  - Cost: 6.79417183
8040764
Iteration: 80  - Centroid Point  [3.23795543 3.194739  ]  - Cost: 6.36376833
6950585
Iteration: 82  - Centroid Point  [3.25895461 3.2751409 ]  - Cost: 6.00637798
5290124
Iteration: 84  - Centroid Point  [3.2787964  3.35442678]  - Cost: 5.67045302
6762626
Iteration: 86  - Centroid Point  [3.33421159 3.4158467 ]  - Cost: 5.28439270
6024013
Iteration: 88  - Centroid Point  [3.38287896 3.48235284]  - Cost: 4.91833336
9152984
Iteration: 90  - Centroid Point  [3.39747246 3.56559467]  - Cost: 4.62561316
6805344
Iteration: 92  - Centroid Point  [3.48003983 3.5842065 ]  - Cost: 4.31475012
8780188
Iteration: 94  - Centroid Point  [3.54580638 3.63766135]  - Cost: 3.97064566
8086627
```

```
Iteration: 96  - Centroid Point  [3.60098666 3.69217857]  - Cost: 3.66763520
53969277
Iteration: 98  - Centroid Point  [3.67532028 3.71161669]  - Cost: 3.41470792
16964356
Iteration: 100  - Centroid Point  [3.75598386 3.73778779]  - Cost: 3.1407558
05456286
Iteration: 102  - Centroid Point  [3.81608746 3.79754178]  - Cost: 2.8475546
909272844
Iteration: 104  - Centroid Point  [3.8422747  3.87759998]  - Cost: 2.6001096
737728724
Iteration: 106  - Centroid Point  [3.86440364 3.95406332]  - Cost: 2.3835626
30636919
Iteration: 108  - Centroid Point  [3.92135077 4.0110213 ]  - Cost: 2.1415630
23882541
Iteration: 110  - Centroid Point  [3.97461883 4.07590867]  - Cost: 1.9053513
252153496
Iteration: 112  - Centroid Point  [3.99388627 4.15853931]  - Cost: 1.7203209
2371764
Iteration: 114  - Centroid Point  [4.07750412 4.17293756]  - Cost: 1.5350309
237517137
Iteration: 116  - Centroid Point  [4.14541207 4.22232866]  - Cost: 1.3350932
399548783
Iteration: 118  - Centroid Point  [4.20557635 4.27581591]  - Cost: 1.1555515
41785677
Iteration: 120  - Centroid Point  [4.28317134 4.29340004]  - Cost: 1.0131268
232061497
Iteration: 122  - Centroid Point  [4.36463652 4.3148665 ]  - Cost: 0.8730946
630176277
Iteration: 124  - Centroid Point  [4.42781094 4.37137333]  - Cost: 0.7225718
134864023
Iteration: 126  - Centroid Point  [4.45865583 4.45036346]  - Cost: 0.5951538
343227707
Iteration: 128  - Centroid Point  [4.48245188 4.5234693 ]  - Cost: 0.4949375
721740499
Iteration: 130  - Centroid Point  [4.54014385 4.57539945]  - Cost: 0.3917533
002635205
Iteration: 132  - Centroid Point  [4.59735225 4.63791961]  - Cost: 0.2932274
1812759767
Iteration: 134  - Centroid Point  [4.62077218 4.71926211]  - Cost: 0.2226275
0116062485
Iteration: 136  - Centroid Point  [4.70473393 4.72896558]  - Cost: 0.1606417
081989809
Iteration: 138  - Centroid Point  [4.77399728 4.77367735]  - Cost: 0.1022991
699674756
Iteration: 140  - Centroid Point  [4.83853481 4.82538225]  - Cost: 0.0565623
6669501064
Iteration: 142  - Centroid Point  [4.91882092 4.84052481]  - Cost: 0.0320223
7748730252
Iteration: 144  - Centroid Point  [4.94358275 4.91988161]  - Cost: 0.0096018
61734657553
Iteration: 146  - Centroid Point  [4.99706006 4.96482101]  - Cost: 0.0012462
043593770467
Iteration: 148  - Centroid Point  [5.02245754 4.98247788]  - Cost: 0.0008113
656106519422
Iteration: 150  - Centroid Point  [4.98344862 4.99490338]  - Cost: 0.0002999
2365089809937
Iteration: 152  - Centroid Point  [5.0081254  4.98680561]  - Cost: 0.0002401
1404168766996
Iteration: 154  - Centroid Point  [4.99834023 4.99531126]  - Cost: 2.4739134
842279976e-05
```

In [45]:

```
print('Points at each step will be:')
mid_points
```

```
Points at each step will be:
```

Out[45]:

```
[[1.0550000000000002, 1.0550000000000002],
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```

```
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```
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 [4.999816268919273, 4.990032831545186],
 [4.998340231433445, 4.995311257828825]]
```

In [47]:

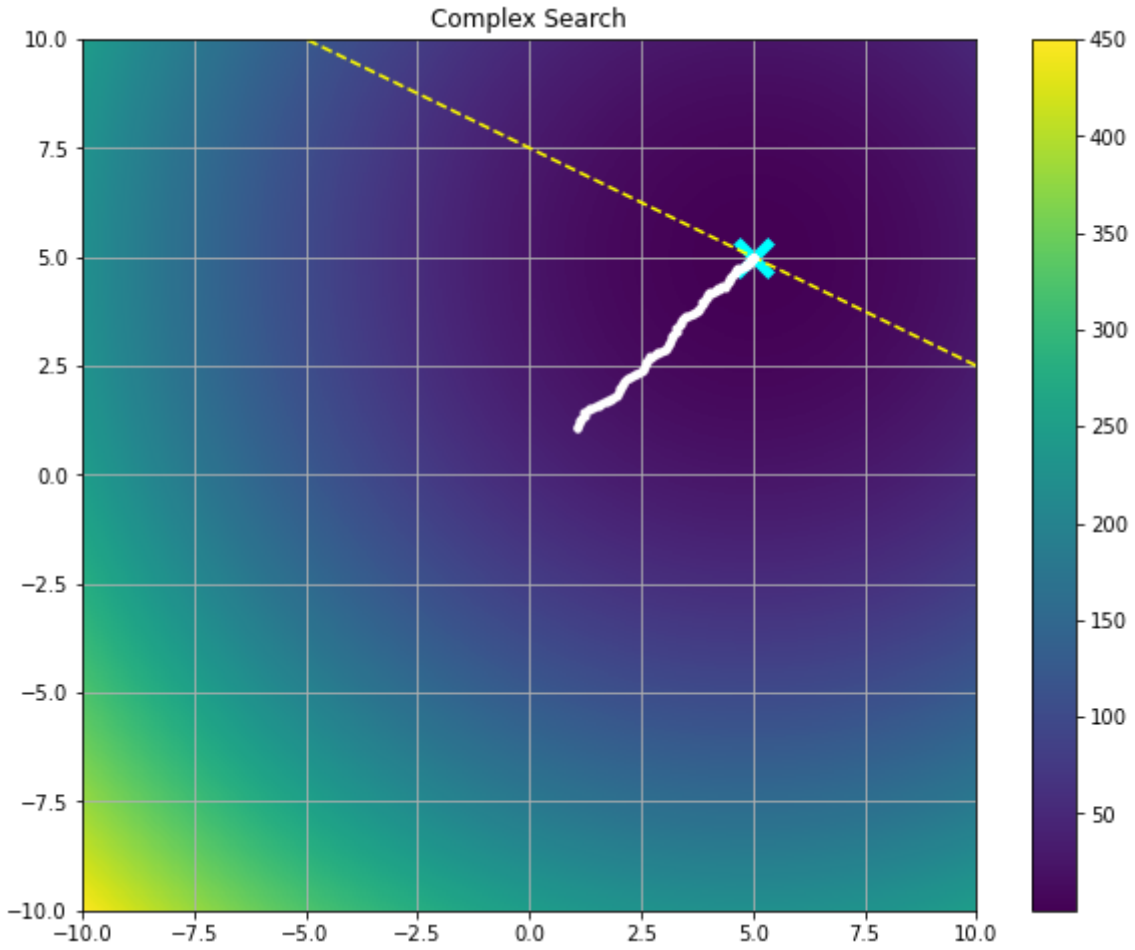
```
print('Final centroid point is:', mid_points[-1], 'and optimal value is:', f(mid_points
```

```
Final centroid point is: [4.998340231433445, 4.995311257828825] and optimal
value is: 2.4739134842279976e-05
```

Final Plot

In [53]:

```
mid_points = np.array(mid_points)
fig = plt.figure(figsize=(10, 8))
x1 = np.linspace(-10, 10, 400)
x2 = np.linspace(-10, 10, 400)
X1, X2 = np.meshgrid(x1, x2)
Y = np.array([f(a) for a in np.c_[X1.ravel(), X2.ravel()]]).reshape(X1.shape)
plt.pcolormesh(X1, X2, Y)
plt.colorbar()
x2 = np.linspace(-10, 10, 400)
y2 = (15-x2)/2
plt.plot(x2, y2, color='yellow', linestyle='--')
plt.xlim(-10, 10)
plt.ylim(-10, 10)
plt.plot(mid_points[:, 0], mid_points[:, 1], color='white', marker='.')
plt.scatter(mid_points[-1, 0], mid_points[-1, 1], c='cyan', marker='x', s=300, linewidt
plt.title('Complex Search')
plt.grid()
plt.show()
```



For Animation

In [61]:

```
for i in range(len(mid_points)):
    x1 = np.linspace(-10, 10, 400)
    x2 = np.linspace(-10, 10, 400)
    X1, X2 = np.meshgrid(x1, x2)
    Y = np.array([f(a) for a in np.c_[X1.ravel(), X2.ravel()]]).reshape(X1.shape)
    plt.pcolormesh(X1, X2, Y)
    plt.colorbar()
    x2 = np.linspace(-10, 10, 400)
    y2 = (15-x2)/2
    plt.plot(x2, y2, color='yellow', linestyle='--')
    plt.xlim(-10, 10)
    plt.ylim(-10, 10)
    plt.plot(mid_points[:i, 0], mid_points[:i, 1], color='white', marker='o')
    if i==len(mid_points)-1:
        plt.scatter(mid_points[-1, 0], mid_points[-1, 1], c='cyan', marker='x', s=300,
    plt.grid()
    plt.title('Complex Search')
    plt.savefig(str(i)+'q2.png')
    plt.clf()
```

```
<Figure size 432x288 with 0 Axes>
```

In [63]:

```
images = []
for i in range(len(mid_points)):
    images.append(imageio.imread(str(i)+'q2.png'))
imageio.mimsave('movie_q2.gif', images)
```