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ME17B162

classmate

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ME7223: ASSIGNMENT 7

1) $f = 2x_1^2 + 16x_2^2 - 2x_1x_2 - x_1 - 6x_2 - 5$

$$Q = \begin{bmatrix} \partial^2 f/\partial x_1^2 & \partial^2 f/\partial x_1 \partial x_2 \\ \partial^2 f/\partial x_2 \partial x_1 & \partial^2 f/\partial x_2^2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -2 & 32 \end{bmatrix}$$

To check conjugate vectors, $S_1^T Q S_2$ should be $\underline{0}$.

a) $S_1 = [15, -1]^T$, $S_2 = [1, 1]^T$

$$\therefore S_1^T Q S_2 = \begin{bmatrix} 15 & -1 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 32 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 30 \end{bmatrix} = 30 - 30 = \underline{0}$$

S_1, S_2 satisfy the conjugation criteria

b) $S_1 = [-1, 15]^T$, $S_2 = [1, 1]^T$

$$S_1^T Q S_2 = \begin{bmatrix} -1 & 15 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 32 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 15 \end{bmatrix} \begin{bmatrix} 2 \\ 30 \end{bmatrix} = -2 + 150 \neq 0$$

S_1, S_2 not conjugate.

QUESTIONS 2, 3, 4 done on next pages.

JUPYTER NOTEBOOKS ARE ATTACHED IN THE ZIP FILE.

ME7223: ASSIGNMENT 7

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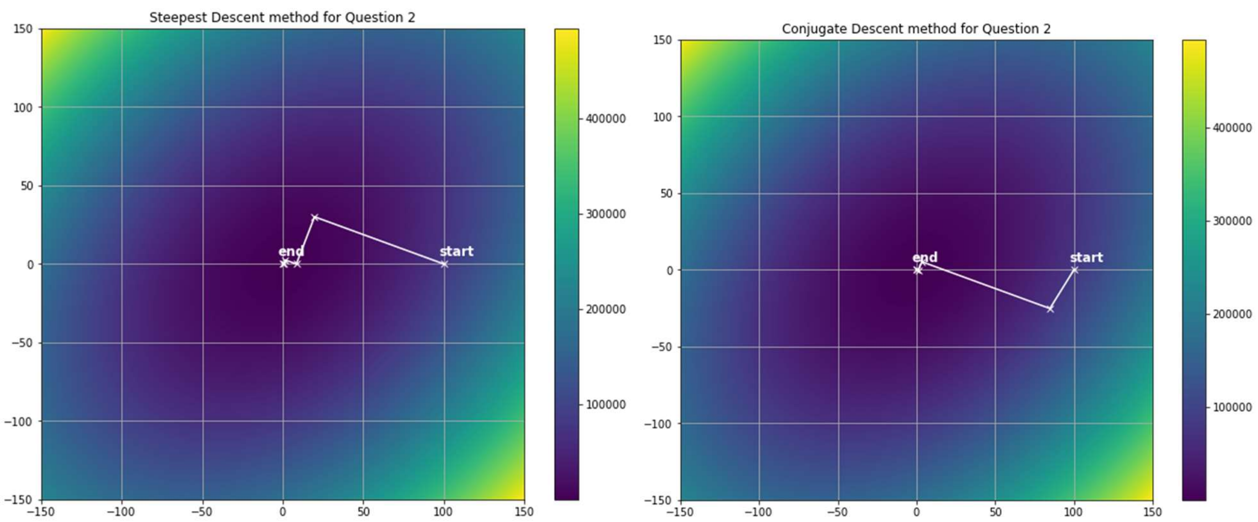
Question 2:

Max no of iterations = 4 for all the methods

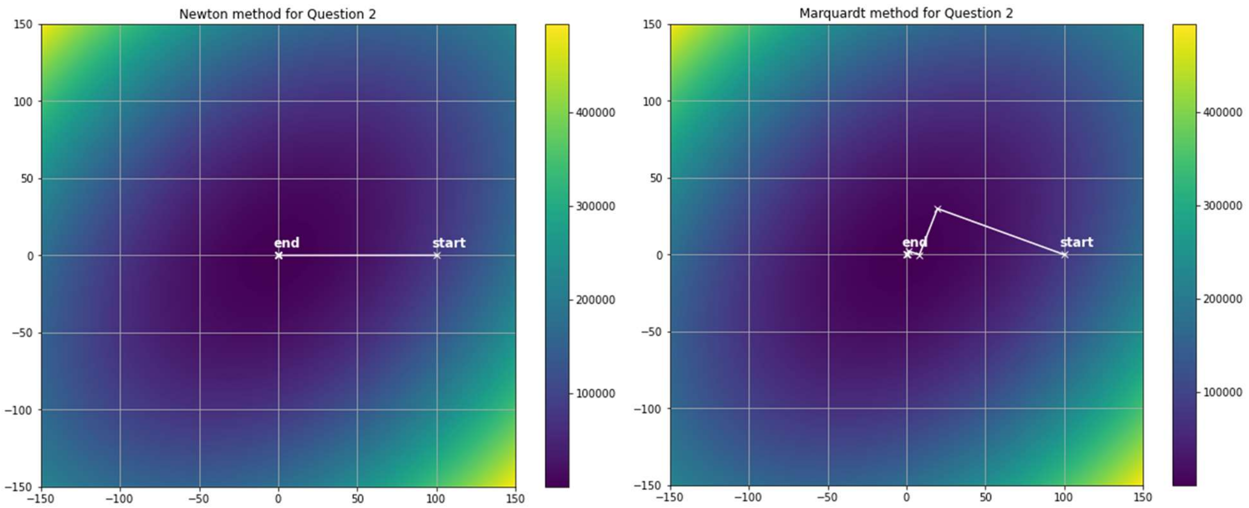
Method used	Initial guess	Final optimal value	No of iterations	f(x*)	Observations
Steepest Descent	[100,0]	[0.7690909 -0.04512547]	4	4.1423141 62813569	f(x) starts with 6798.342 and then moves on to 578.4 to 49.17, 4.14. Uses all 4 iterations
Conjugate Descent	[100,0]	[0.06330873 -0.00536625]	4	-0.034342 263036621 21	f(x) starts with 75262.52 and then moves on to 208.69 to 12.669, -0.034. Used all 4 iterat ions
Newton	[100,0]	[0.04545455 -0.04545455]	1	-0.045	Within the 1st iteration it reaches the best result it could. Rest of the iterations give the same result as the first one
Marquardt (c1 = ¼, c2 = 2)	[100,0]	[0.48867873 -0.08131061]	4	1.6317	f(x) starts with 6738.72 and then moves on t o 544.37 to 39.688, 1.6317. Used all 4 iterati ons

Observations:

- Conjugate descent method gives the most accurate answer of all and also has the fastest convergence rate among all the methods used. It starts with a point giving higher value and then quickly converges to values near the optimal value.
- Newton's method takes the least no of iterations and is also the 2nd most accurate method among the 4. Lacking behind by only -0.011.
- Steepest descent is the least accurate of all the 4 methods used here
- For this question. Marquardt and Steepest descent method tend to follow a similar path for the contour path. It is a combination of Newton and Steepest descent method and have the qualities of both in the algorithm followed.



Steepest Descent, Conjugate Descent Method for initial points: [100,0]



Newton's, Marquardt Method for initial points: [100,0]

Question 3:

Max no of iterations = 4 for all the methods

Method used	Initial guess	Final optimal value	No of iterations	f(x*)	Observations
Steepest Descent	[-1.2,1.0]	[1.44105619 2.0774701]	2	0.194598981010033 23	It closely reaches it's closest optimal point b efore diverging to other points further away after 2 iterations
Conjugate Descent	[-1.2,1.0]	[1.43089261 2.04920713]	4	0.185975905420447 4	f(x) starts with 0.185975 which is very close to the final f(x*) value. The method tends to slowly move towards the optimal point afte r calculating the 1st iteration
Newton	[-1.2,1.0]	[0.99999531 0.94402732]	4	0.313189076116736 35	f(x) starts with 4.7318 and then moves on t o 1411.84 to 0.05596, 0.31318. Used all 4 it erations
Marquardt (c1 = ¼, c2 = 2)	[-1.2,1.0]	[1.46649067 2.14983954]	4	1.64171157	f(x) starts with 0.218358 and then moves on to 0.217670 to 1.347292, 1.64171157. Used all 4 iterations

Observations:

- All the methods give final opt value having higher accuracy than the previous question. The reason for this could be because we started from a point which was closer to the actual optimum
- Steepest descent method gives the most accurate answer of all and with only 2 iterations. This method approaches near to the f(x*) quickly and then reaches the closest opt point before starting to diverge from the 3rd iteration
- Marquardt and Newton's method tend to go follow a zig-zag path in the contour graph. This tells that in the Marquardt method it dynamically keeps on increasing and reducing the 'a' value
- Marquardt method evidently shows both the qualities of Newton's method (in the graph) and Steepest descent method (in the convergence)
- Steepness and Conjugate descent method show similar traits hinting at the similar nature of their algorithms (which is true).

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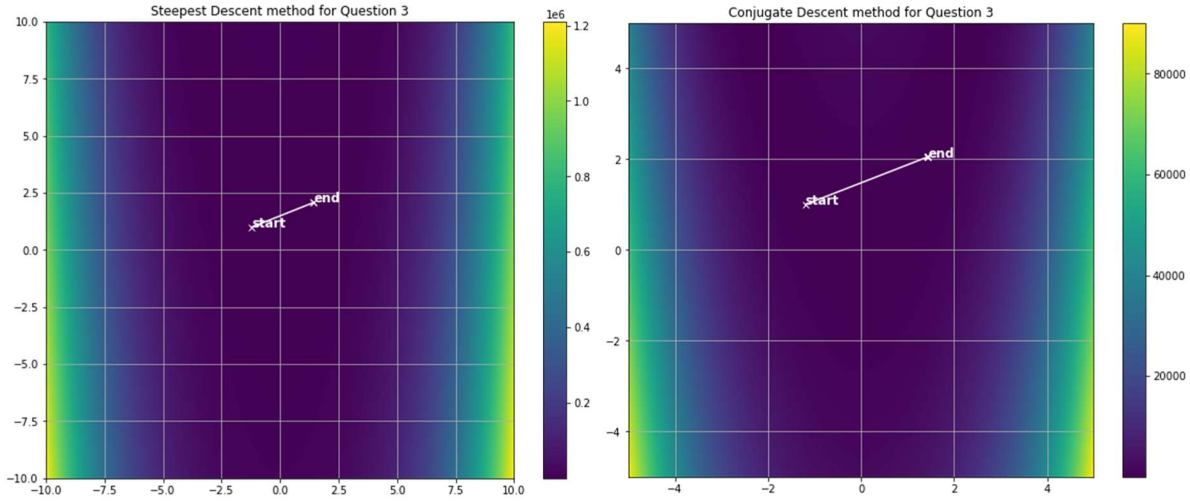
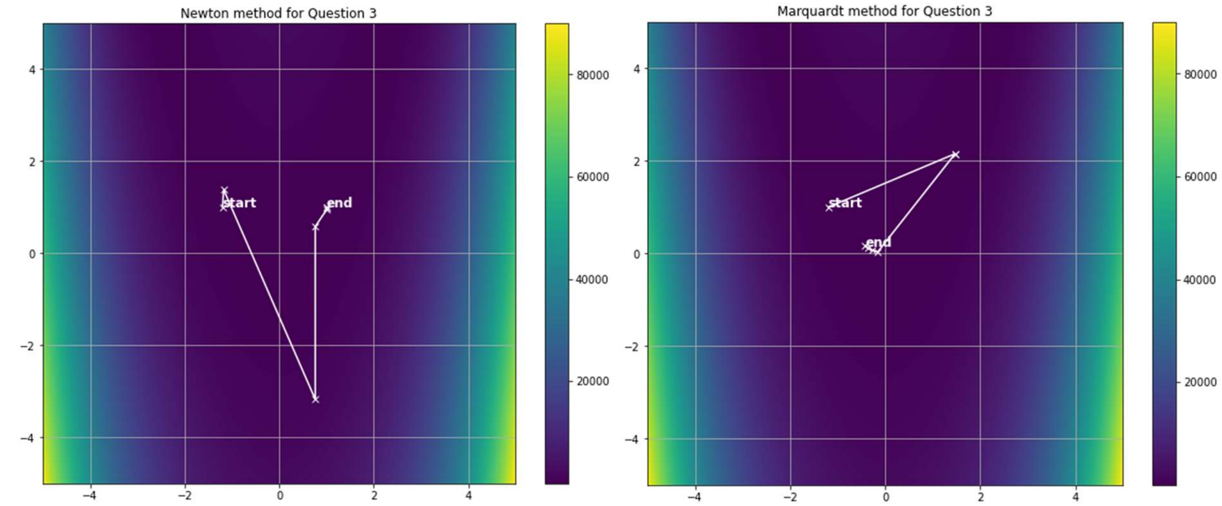


Steepest Descent, Conjugate Descent Method for initial points: [-1.2,1.0]



Newton's, Marquardt Method for initial points: [-1.2,1.0]

Question 4:

- Marquardt method incorporates good parts of both Newton's method, which is the inclusion of Hessian, and Steepness method, which is the inclusion of the 's' value which is the gradient, in the calculation of x values
- This ensures that Marquardt method would give consistent results for higher range of functions which may not be the case for conjugate and Newton's method, as seen in the two questions solved before
- Marquardt method is superior also because it takes into consideration an external parameter 'a' which lets us control the speed of convergence and dynamically control its value using c1, c2.
- This helps us avoid divergence which is likely in the conjugate descent method