15 ALGEBRA

Get Set Go!

Working with numbers is known as Arithmetic.

We have learned to perform the following operations on numbers.

- Addition (+)
- Subtraction (–)
- Multiplication (×)
- Division (÷)

We have also learned to solve expressions that involve more than one arithmetic operators.

$3 \times 4 + 1 = 13$

$8 - 5 \times 3 = 9$

Numbers are constant and it is not possible to change the value of numbers.

Let us revise.

1. Add

   a) 34714 + 6320 b) 7034 kg + 2873 kg c) ₹ 235 + Rupee Foradian109

2. Subtract

   a) 9001 – 5923 b) 834 cm – 109 cm c) 874.34 – 623.01

3. Multiply

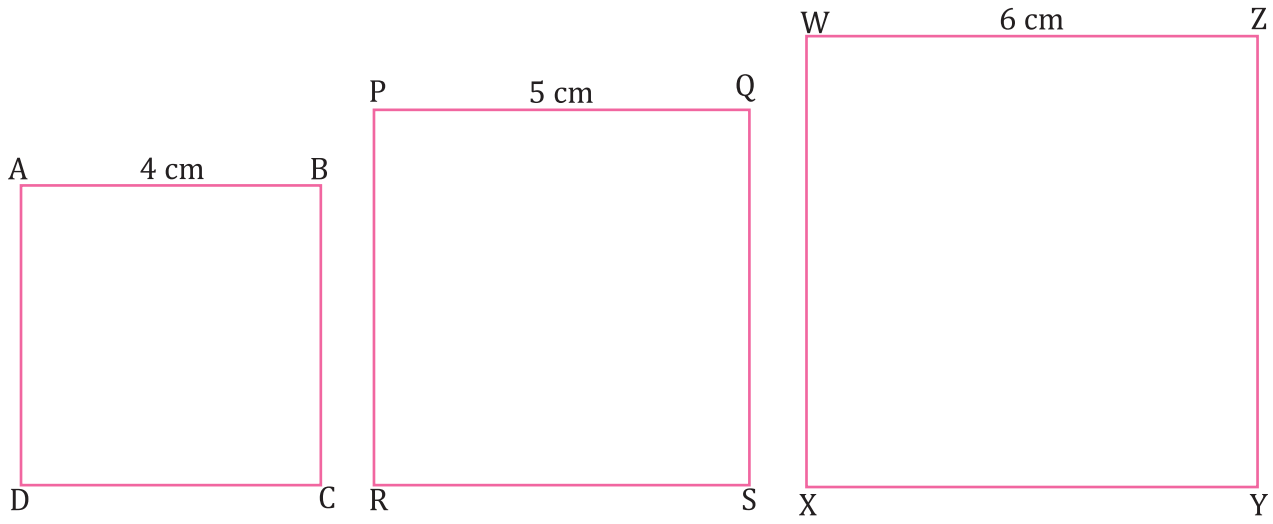
   a) 482 × 22 b) 66.6 × 20 c) 55.76 mm × 20

4. Divide

   a) 6390 ÷ 15 b) 82.768 ÷ 16.3 c) ` 145 ÷ 24

What is Algebra?

Let us consider three squares ABCD, PQRS, WXYZ as shown in the picture below.



Perimeter of square ABCD = 4 cm + 4 cm + 4 cm + 4 cm = 4 × 4 cm = 16 cm

Perimeter of square PQRS = 5 cm + 5 cm + 5 cm + 5 cm = 4 × 5 cm = 20 cm

Perimeter of square WXYZ = 6 cm + 6 cm + 6 cm + 6 cm = 4 × 6 cm = 24 cm

Perimeter of a square = 4 × side of the square

Let us represent 'side of the square' by s, then we can re-write the formula as:

Perimeter of a square = 4 × side

Then for square ABCD, side = 4 cm

square PQRS, side = 5 cm

square WXYZ, side = 6 cm

Here 's' is said to be a variable, which can take any numerical value.

The expression '4 × s' has a constant (number: 4) and a variable (symbol: s).

When an expression involves constants and variables both then it is said to be an algebraic expression.

Remember: Algebra involves the calculation of constant and variables.

Constants and Variable

Numbers have a fixed value, so they are said to be constants.

Hours in a day, days in a year, your date of birth are all constant. You cannot change them.

Can you tell more constants like these?

Unlike constants, symbols can take different values.

As we have seen in the above example symbol 's' can take different values for squares of different sizes.

A symbol in an algebraic expression can have different values, so they are called **variables**.

Note down the minimum and the maximum temperature for the past three days.

What do you observe? Doesn't it vary every day?

Addition and subtraction

The sum of a variable (here x) and a constant is written as: $x + 5$

The sum of two variable, say x and y, is written as: $x + y$

When a constant, say 5, is subtracted from a variable, say x, it is written: $x - 3$

When one variable (y) is subtracted from another variable x, it is written: $x - y$

Multiplication and Division

We know that 3 time 5 is $5 \times 5 \times 5$ i.e 3×5

Similarly, 3 times 'a' is written as $a \times a \times a = 3 \times a$

4 time 's' is $s \times s \times s \times s = 4 \times s$

It is not necessary to write the multiplication sign between a constant and a variable or variable and variable in Algebra.

So, $5 \times p = 5p$

$4 \times x = 4x$

$x \times y = xy$

$10 \times s =$ _____

Remember: m × 4 is $4m$ and not $m4$.

Division expression in Algebra is written as:

$a \div 5 = \frac{a}{5}$

$x \div y = \frac{x}{y}$

$6 \div w = \frac{6}{w}$

Fun with numbers!

1. 7 added to $x = x + 7$
2. 4 subtracted from $a =$ _____
3. 6 added to the product of 3 and $y =$ _____
4. p divided by the product of 9 and $s =$ _____

Coefficients

Consider the algebraic expression: $5x$

The coefficient of 5 is x and the coefficient of x is 5.

Now let us consider the expression: $8xy$

Coefficient of 8 is xy.

Coefficient of x is $8y$.

Coefficient of y is $8x$.

Coefficient of $8x$ is y.

Coefficient of $8y$ is x.

Coefficient of xy is 8.

Remember: Coefficient of any single variable is 1. For example, co-efficient of z is as $z = 1 \times z$.

Expression and terms

When we are in need of something, then we express it through a language we know.

"I want to sleep" is an English sentence, which is expressing one's desire to sleep.

4 subtracted from 5 or $5 - 4$ is an Arithmetic expression as it involves only constants.

But when an expression has both, a constant and a variable, it is an algebraic expression.

Example 1: $4 + s, \frac{2}{r} - 6, 7 + t, 6s - 1$ are all algebraic expressions as they involve both, constants and variables.

5 and x are called the terms of algebraic expression $5 + x$.

$\frac{2}{z}$ and 9 are the terms of algebraic expression $\frac{2}{z} + 9$.

8 and a are the terms of algebraic expression $8 + a$.

$4t$ and 1 are terms of algebraic expression $4t - 1$.

Remember: Terms are separated by + or –, in an algebraic expression.

Value of an algebraic expression

The value of an algebraic expression can be found when the value of all the variables in the expression is given.

Example 2: Find the value of $x + y$, where x = 4 and $y = 5$.

$x + y = 4 + 5 = 9$ **Answer:** 9

Example 3: Find the value of $2(l + b)$, where $l = 3$ and $b = 4$.

$2(l + b) = 2 \times (3 + 4)$

$= 2 \times 7 = 14$ **Answer:** 14

Example 4: Find the value of $4x - 3y$ if $x = 3$ and $y = 2$.

$4x - 3y = 4 \times 3 - 3 \times 2$

$= 12 - 6 = 6$ **Answer:** 6

Example 5: Find the value of $6\frac{x}{y}$ if $x = 5$ and $y = 2$.

$6\frac{x}{y} = 6 \times \frac{5}{2} = 15$ **Answer:** 15

Exercise 15.1

1. Find the coefficient of the variables given in brackets.

a) $5x$ (x) b) $50d$ (d) c) $4bd$ (bd)

d) $3y$ (y) e) $7abc$ (ab) f) $10pq$ (q)

2. Write all the terms of the following algebraic expressions.

a) 4x + 5y b) 6a – bc c) 5x + 8y

d) 5x + 4y + 3z e) ac + bd + cd f) 8pq – 9ab

3. If $x = 4$ and $y = 5$ find the value of the following expressions.

a) x + y b) 3x + 2y c) 5xy

d)4x – 3y e) 7 (x + y) f) $\frac{4}{x} - \frac{5}{y}$

Like and unlike terms

Like terms have the same variable coefficients as in the following examples.

a) 3a, 4a, 10a b) 5x, 7x, 9x c) 10ab, 6ab, 20ab

Unlike terms do not have the same variable coefficients.

Examples of unlike terms are:

a) 3a, 4d, 8c, 2q b) 3g, 8p, x, 2y c) 2ab, 4bc, 5ac

Remember: An algebraic expression may have both like and unlike terms. But we can add or subtract only like terms. Coefficients of only like terms are added or subtracted.

Example 6:

$4a + a = 5a$

$11xy + 3xy = 14xy$

$10s - 3s = 7s$

$18mn - 5mn = 13mn$

Unlike terms can't be subtracted or added.

Expressions such as $5x + 6y - 2z$ cannot be simplified further.

Simplification of an algebraic expression

Steps to simplify algebraic expressions are as follows:

Step 1: Group all the like terms together.

Step 2: Add or subtract the coefficients of like terms.

Example 7: Simplify $5x + 2y + 3x + 4y$

Step 1: Group the like terms together.

$5x + 2y + 3x + 4y = (5x + 3x) + (2y + 4y)$

Step 2: Add the coefficients of like terms.

$(5x + 3x) + (2y + 4y) = 8x + 6y$ **Answer:** $8x + 6y$

Example 8: Simplify $3x + 3y + 3z + 2x - 2y + 2z - x - z$.

$3x + 3y + 3z + 2x - 2y + 2z - x - z = (3x + 2x - x) + (3y - 2y) + (3z + 2z - z)$

$= 4x + y + 4z$

Exercise 15.2

1. Write L for like and U for unlike for the following set of terms.

a) ab, ba b) abc, acb, cba c) $3, 13w$

d) $6pq, 16qp, 60pq$ e) $2, 3x, 6y$ f) $10a, 10b, 10c$

2. Group like terms together.

a) $2a, 4b, 5a$ b) $3x, 2y, 2x, 3y$ c) $4z, 3x, 2y, 2x, 3y, 6z$

d) $5ab, 2bc, ba, 6cb$ e) $7x, 2y, 5z, 12x, 7y, 9z$ f) $ab, 3bc, 4ca, 5ba, 2cb, 2ac$

3. Simplify.

a) $a + a + a + b + b + b + b + c + c + c$ b) $6a + 2b + 3c + 4a - 5b - 2c$

c) $7x + 3y + 2z - x - y - z$ d) $2x + 3y + 2y + 2z + 2x + 4x$

e) $7x + y + 4z - 3x - y - 3z$ f) $10ab + 4bc + 2ca + 5ab + 4bc - 2ca - 5ab + 4ca$

Thinking Hat

Write the algebraic expressions for the following.

1. Three times x added to thrice of y.
2. 4 subtracted from the product of x and y.
3. Nine times x divided by four times y.
4. Seven times the product of xyz added to a quarter of z.
5. 6 divided by the sum of 5 and q.
6. Twice a number p is added to thrice a number y.

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If a = 0, b = 1 and c = 2 find the value of following expressions.

i. $1 + a + b + c$

ii. $b - a + c + 5$

iii. $3a + 2b + 4c + abc$

iv. $b - a/c$

v. $\frac{1}{2}bc$

vi. $abc + cab + bca$