Chap 5.3

Note that $Y(e^{jw}) = H(e^{jw})X(e^{jw})$

this means

$$|Y(e^{jw})| = |H(e^{jw})|\,|X(e^{jw})|$$

$$\angle Y(e^{jw}) = \angle H(e^{jw}) + \angle X(e^{jw})$$

The LTI system changes the input signal $x[n]$ into output signal $y[n]$. These changes may be desirable or undesirable (i.e. distortion

A system has distortionless response if $x[n]$ and $y[n]$ have the same "shape". This is possible if the input and output satisfy the condition

$$y[n] = G\,x[n-n_d], \quad G > 0$$

The output $y[n]$ is a scaled (by G) and/or time-shifted (by n_d) version of the input $x[n]$

In this case,

$$|Y(e^{jw})| = G\,|X(e^{jw})|$$

$$\angle Y(e^{jw}) = -wn_d + \angle X(e^{jw})$$

A system introduces magnitude distortion if

$$|H(e^{j\omega})| \neq G \text{ for all } \omega$$

A system introduces phase (or delay) distortion if

$$\angle H(e^{j\omega}) \neq -\omega n_d$$

The quality $\dfrac{\angle H(e^{j\omega})}{\omega}$ shows the time shift (# of samples) experienced by the signal at frequency ω. This is called the phase delay

$$\tau_{pd}(\omega) \equiv -\frac{\angle H(e^{j\omega})}{\omega}$$

(have units of samples)

Note that if $\angle H(e^{j\omega})$ is distortionless, then the phase delay is a constant number

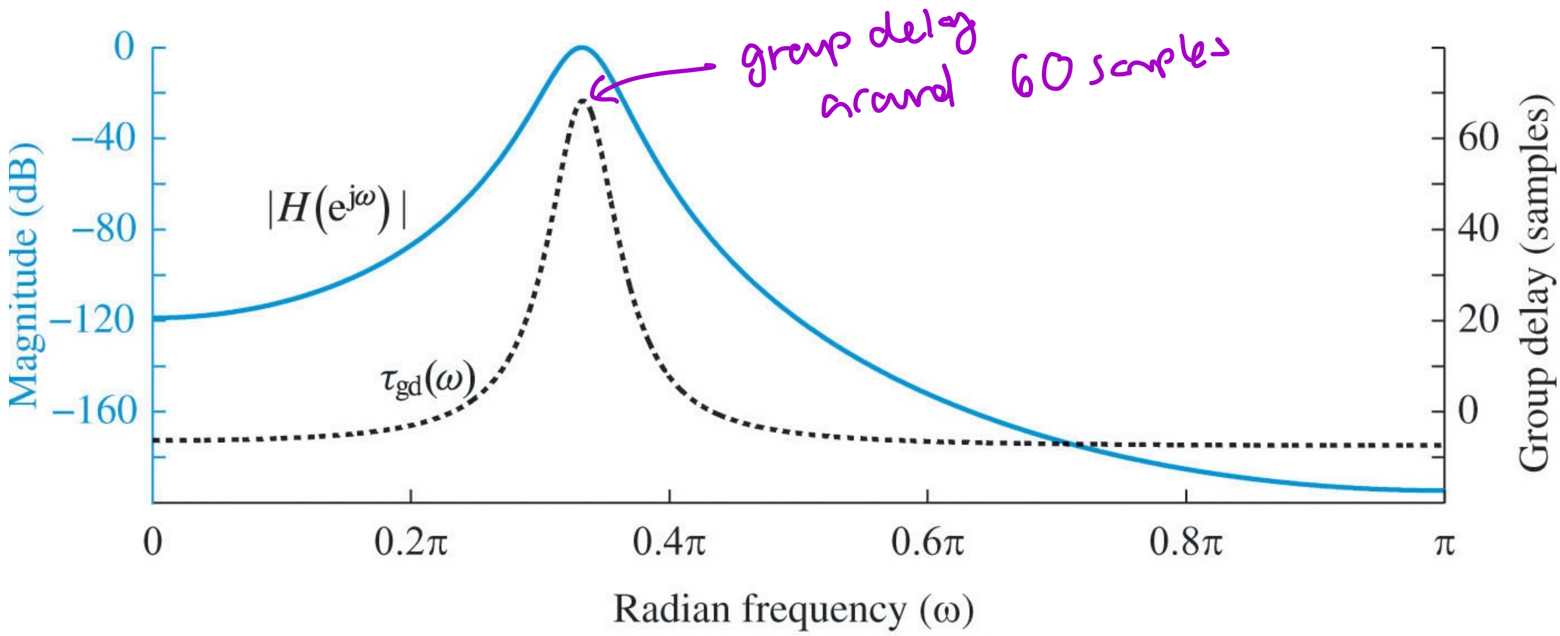
Another way to check linearity of phase response is to use <u>group delay</u>

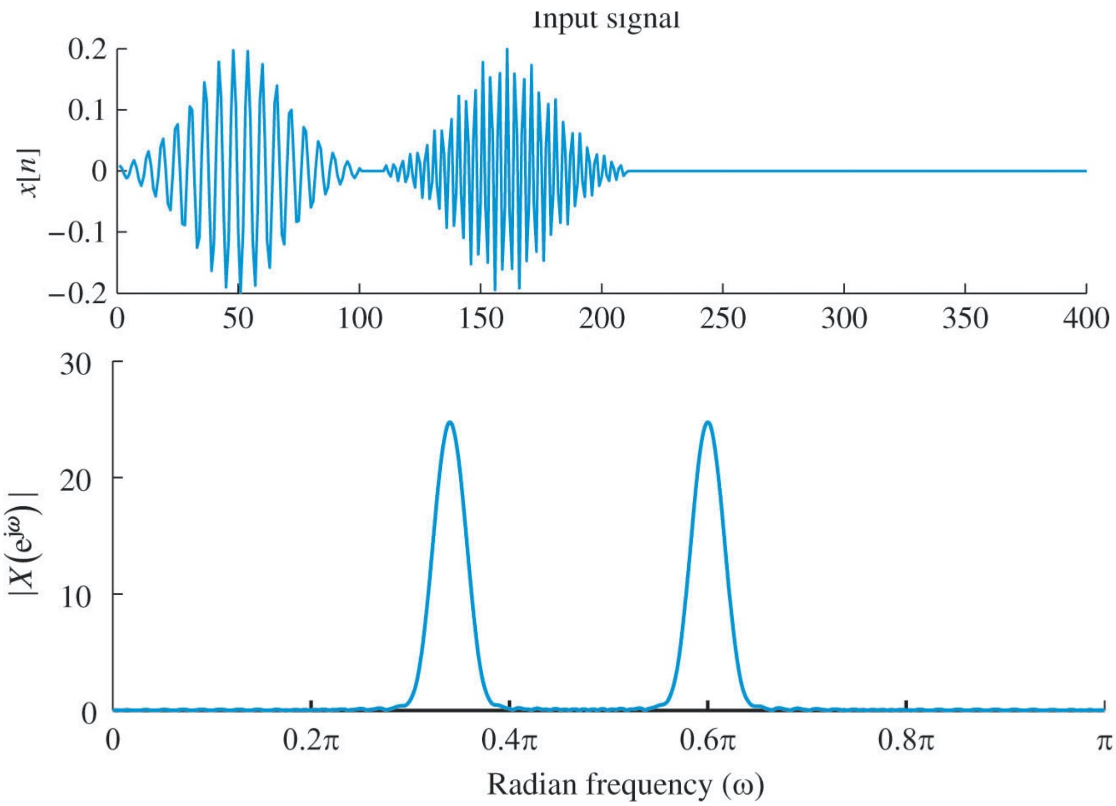
$$\tau_{gd}(\omega) \equiv \frac{d\Psi(\omega)}{d\omega}$$ (have units of samples)

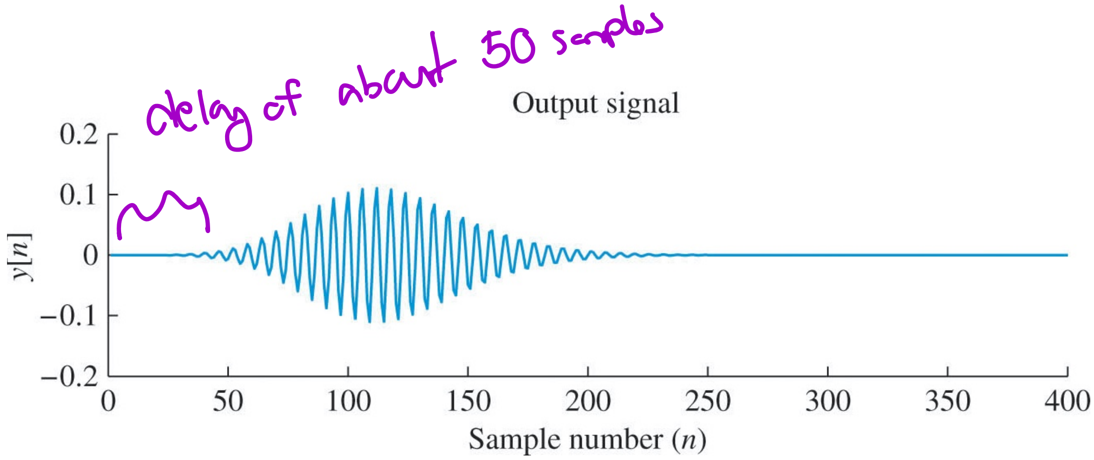
$\Psi(\omega)$ is the unwrapped phase response

Note that if $\angle H(e^{j\omega})$ is distortionless, then the group delay is also a constant number

Group delay is useful in communication application where $y[n] = x[n]\cos(\omega n + \phi)$


group delay around 60 samples
Magnitude (dB)
Group delay (samples)
$|H(e^{j\omega})|$
$\tau_{gd}(\omega)$
0
−40
−80
−120
−160
60
40
20
0
0
0.2π
0.4π
0.6π
0.8π
π
Radian frequency (ω)


Input signal
$x[n]$
0.2
0.1
0
−0.1
−0.2
0
50
100
150
200
250
300
350
400
$|X(e^{j\omega})|$
30
20
10
0
0
0.2π
0.4π
0.6π
0.8π
π
Radian frequency (ω)


delay of about 50 samples
Output signal
$y[n]$
0.2
0.1
0
−0.1
−0.2
0
50
100
150
200
250
300
350
400
Sample number (n)

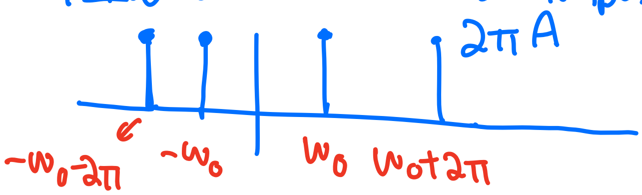
How does LTI system affect a complex exponential input?

$$x[n] = Ae^{j(\omega_0 n + \phi)}$$

we know that the DTFT is

$$X(e^{j\omega}) = Ae^{j\phi} \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - \omega_0 - 2\pi k)$$

$|X(e^{j\omega})|$ is a bunch of impulses



, $\angle X(e^{j\omega}) = \phi$

$$|Y(e^{j\omega})| = |H(e^{j\omega})|\,|X(e^{j\omega})| = |H(e^{j\omega})| \left| A \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - \omega_0 - 2\pi k) \right|$$

$$\angle Y(e^{j\omega}) = \angle H(e^{j\omega}) + \angle X(e^{j\omega})$$

$$= \angle H(e^{j\omega}) + \phi$$ samples (associate phase with time shift)

So we can see that $Y(e^{j\omega})$ is also a bunch of scaled impulse functions at $\pm\omega_0$, $\pm(\omega_0 + 2\pi)$, $\pm(\omega_0 + 4\pi)$, ...

Therefore $Y(e^{jw})$ is also a complex exponential with normalized fundamental frequency w_0.

$$|Y(e^{jw})| = A\,|H(e^{jw})|$$

$$\angle Y(e^{jw}) = \angle H(e^{jw}) + \phi$$

therefore $y[n] = A|H(e^{jw})|\, e^{j(w_0 n + \angle H(e^{jw}) + \phi)}$

whereas

$$x[n] = A e^{j(w_0 n + \phi)}$$

The LTI system can change the magnitude and the phase of the complex exponential input, but <u>not</u> the fundamental frequency w_0

(This is used to tell if system is linear or not)

Recall that DTFT breaks down a $x[n]$ in sums of complex sinusoids

$$x[n] = w_0 + w_1 + w_2 + \cdots$$

$H(e^{jw})$ will affect the magnitude and phase of each term (possibly differently, meaning that

$$|H(e^{j\omega_0})| \neq |H(e^{j\omega_1})| \neq |H(e^{j\omega_2})|$$

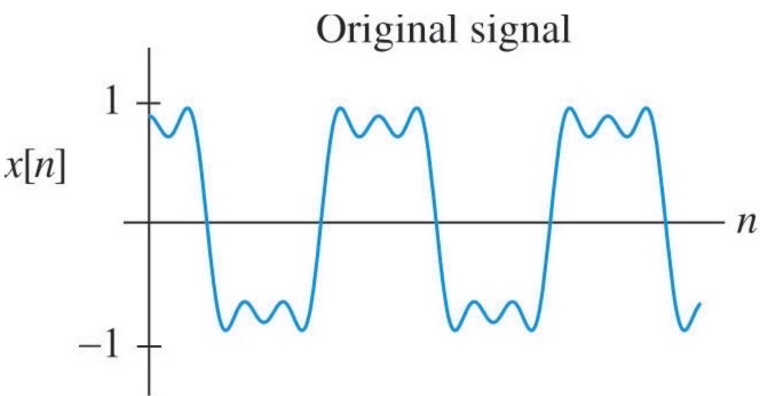
Ex :

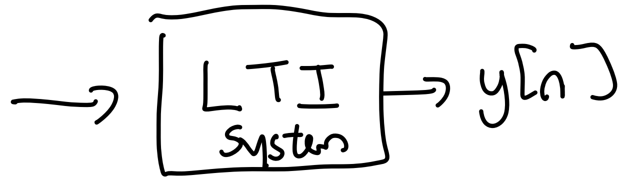
$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n)$$

there are 3 complex exponential terms

$$\omega_0 + \omega_1 + \omega_2$$

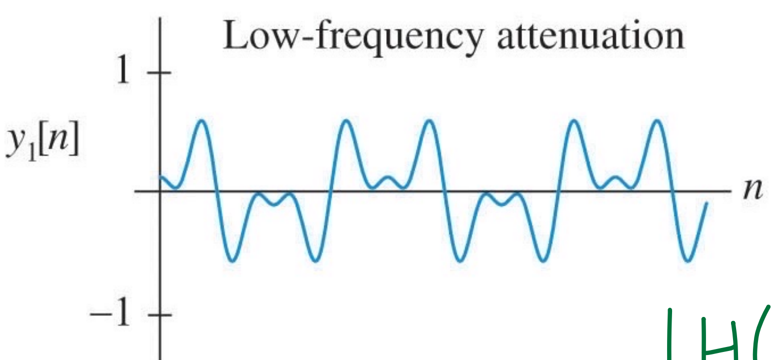
$\omega_1 = 3\omega_0$, $\omega_2 = 5\omega_0$





$$|Y(e^{j\omega})| = |H(e^{j\omega})|\,|X(e^{j\omega})|$$

$$\angle Y(e^{j\omega}) = \angle H(e^{j\omega}) + \angle X(e^{j\omega})$$



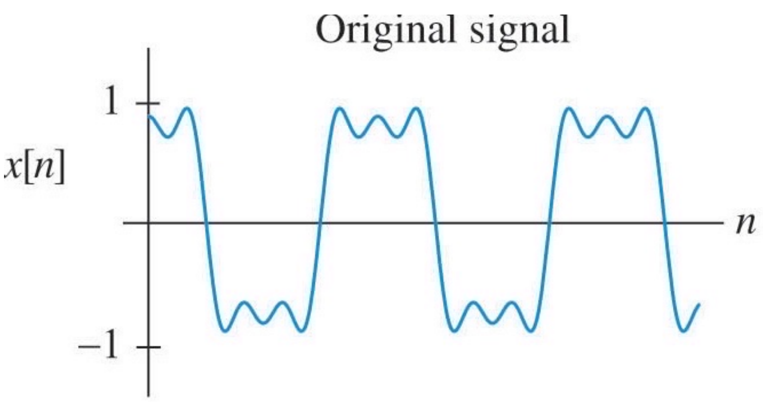
$$\angle Y(e^{j\omega}) = \angle X(e^{j\omega})$$

so $\angle H(e^{j\omega}) = 0$

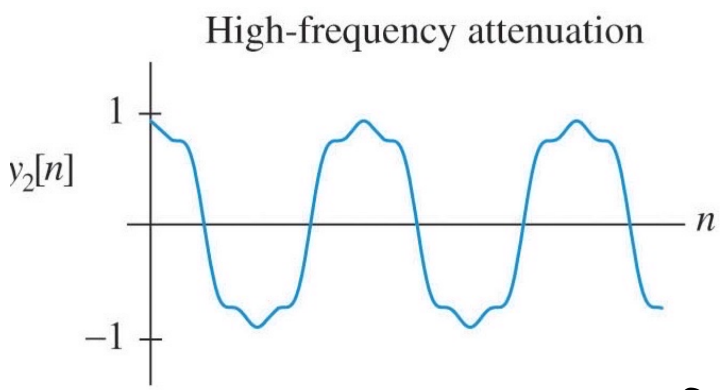
$$\omega_0 < \omega_1 < \omega_2$$

$$|H(e^{j\omega_0})| < |H(e^{j\omega_1})| < |H(e^{j\omega_2})|$$

we are attenuating the low frequency component of $X(e^{j\omega})$



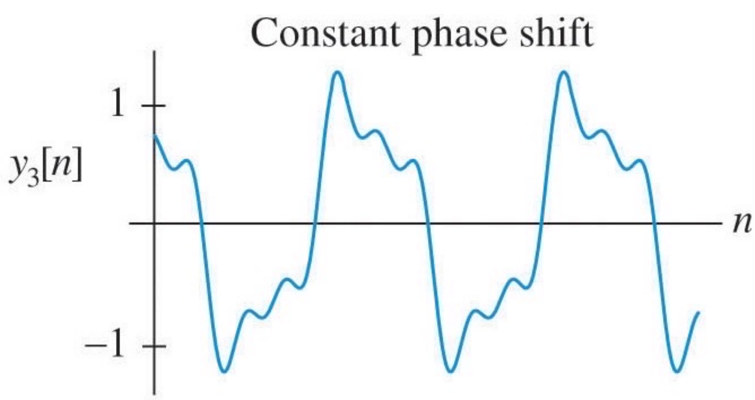
$\rightarrow$ $\boxed{\text{LTI system}}$ $\rightarrow y[n]$



$\angle Y(e^{j\omega}) = \angle X(e^{j\omega})$

$|H(e^{j\omega_0})| > |H(e^{j\omega_1})| > |H(e^{j\omega_2})|$

we are attenuating high frequency component of $X(e^{j\omega})$



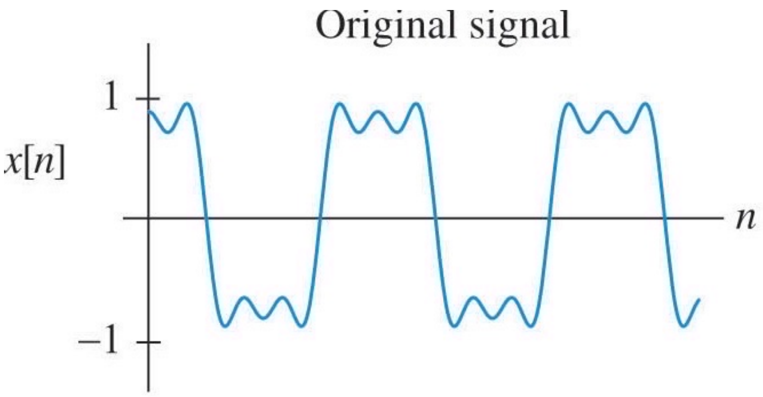
$|Y(e^{j\omega})| = |X(e^{j\omega})|$

$|H(e^{j\omega})| = 5$ for all ω

same constant

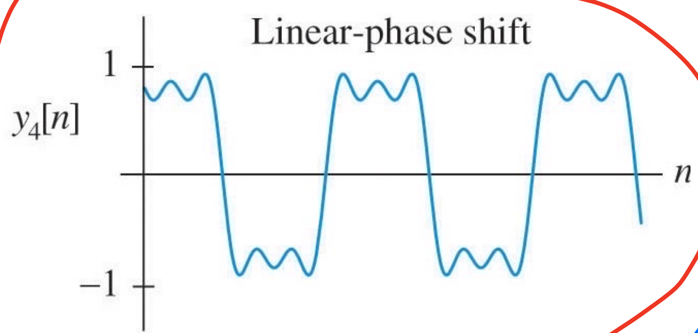
$\angle Y(e^{j\omega}) = 5 + \angle X(e^{j\omega})$

$y[n] = |X(e^{j\omega})|\,|H(e^{j\omega})| \cos(\omega n + 5 + \angle X(e^{j\omega}))$



$\rightarrow$ LTI system $\rightarrow y[n]$

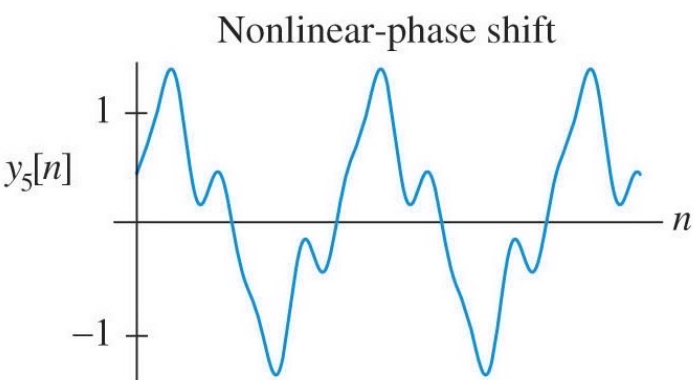
Distortionless



$$|Y(e^{j\omega})| = |X(e^{j\omega})|$$

$$\angle H(e^{j\omega}) = -\omega \underbrace{n_d}_{\text{some integer}}$$

$$\angle Y(e^{j\omega}) = -\omega n_d + \angle X(e^{j\omega})$$



$$|Y(e^{j\omega})| = |X(e^{j\omega})|$$

$\angle H(e^{j\omega})$ is a nonlinear function of ω