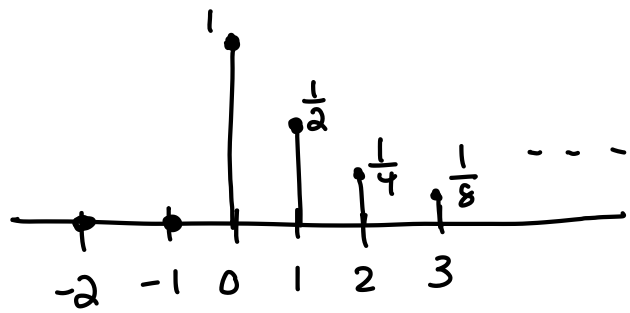
Chap 2.1 - 2.2

- Discrete-time signal $x[n]$ is a sequence of numbers (n can only be integer numbers)

- Discrete-time signal and sequence may be used interchangeably

For example

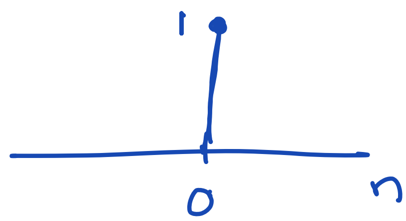
$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



<u>Elementary discrete-time signals</u> (infinite length sequences)

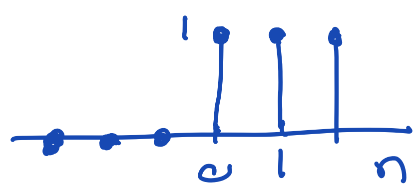
1) unit-impulse

$$\delta[n] \triangleq \begin{cases} 1, & n = 0 \\ 0 & n \neq 0 \end{cases}$$



2) unit-step

$$u[n] \triangleq \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



3) exponential sequence

$$x[n] = a^n, \quad n > 0$$

4) sinusoidal sequence

$$x[n] = A\cos(\omega_0 n + \phi), \quad -\infty < n < \infty$$

amplitude (no unit)

phase (no unit)

frequency (radian per sampling interval)

5) complex sinusoidal sequences

$$x[n] = Ae^{j\omega_0 n} = A\cos(\omega_0 n) + jA\sin(\omega_0 n)$$

by Euler's formula

6) A sequence $x[n]$ is called periodic if

$$x[n] = x[n+N] \quad \text{for all } n$$

The smallest value of N for which this holds is known as the period of $x[n]$

$$x[n] = \{ \underset{\uparrow}{1}, \tfrac{1}{2}, \tfrac{1}{4}, 1, \tfrac{1}{2}, \tfrac{1}{4} \}$$

$$x[0] = x[3]$$

N = period = 3

Energy of a sequence $x[n]$ is

$$\mathcal{E}_x \triangleq \sum_{n=-\infty}^{\infty} |x[n]|^2$$

A signal where $\mathcal{E}_x < \infty$ is a finite energy signal.

Is $\delta[n]$ finite energy?

Is $u[n]$ finite energy?

Mathematical operations can be performed on discrete-time signals to obtain new signals

$$y[n] = x_1[n] + x_2[n]$$

$$y[n] = x_1[n] - x_2[n]$$

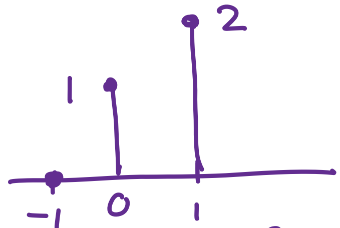
$$y[n] = a\, x_2[n]$$, where a is a real #. Signal scaling

There are two important time based transforms

1) Time-reversal (folding)

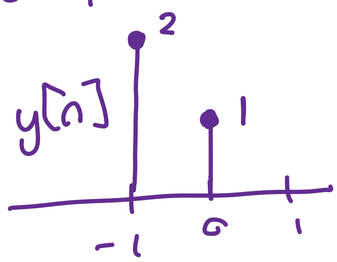
$$y[n] = x[-n]$$

$x[n] =$



$y[-1] = x[1]$
$y[0] = x[0]$
$y[1] = x[-1]$

$y[n]$

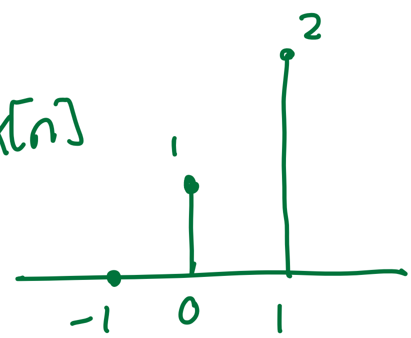


2) Time-shift

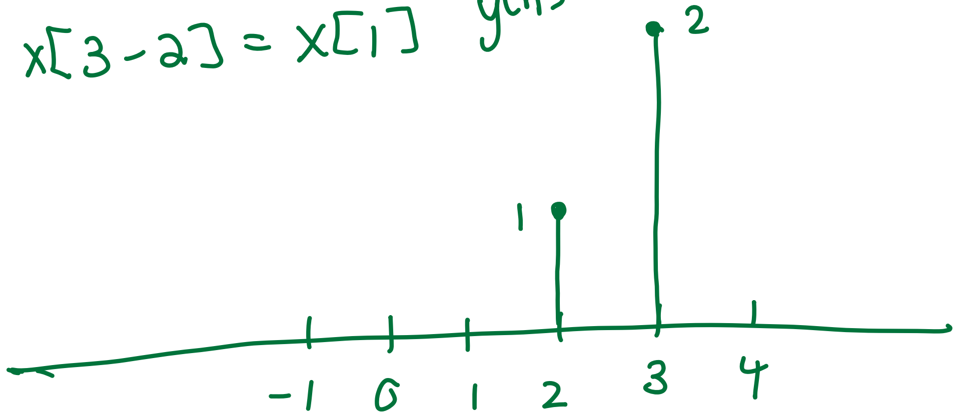
$$y[n] = x[n - n_0]$$

if $n_0 > 0$, the sequence $x[n]$ is shifted to the right. The sequence 'appears later', so it is called a time-delay

Ex $\quad y[n] = x[n-2] \quad$ where $\quad x[n]$



$y[0] = x[0-2] = x[-2]$
$y[1] = x[1-2] = x[-1]$
$y[2] = x[2-2] = x[0]$
$y[3] = x[3-2] = x[1]$ $\quad y[n]$

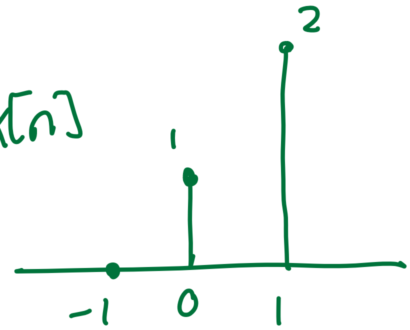


signal appears 'later' in time

if $n_0 < 0$, the sequence is shifted to left. The sequence appears 'earlier', so it is called a time advance

Ex $\quad y[n] = x[n-(-3)] = x[n+3]$

where $x[n]$



$y[-4] = x[-4+3] = x[-1]$

$y[-3] = x[-3+3] = x[0]$

$y[-2] = x[-2+3] = x[1]$

$y[n]$

signal 'appeared' earlier in time

The operation of shifting and folding are not commutative. Order matters!

$$x[n] \xrightarrow{\text{shift}} x[n-3] \xrightarrow{\text{fold}} x[-n-3]$$

is not the same as

$$x[n] \xrightarrow{\text{fold}} x[-n] \xrightarrow{\text{shift}} x[-(n-3)] = x[-n+3]$$

See Matlab example