Chap 7.3

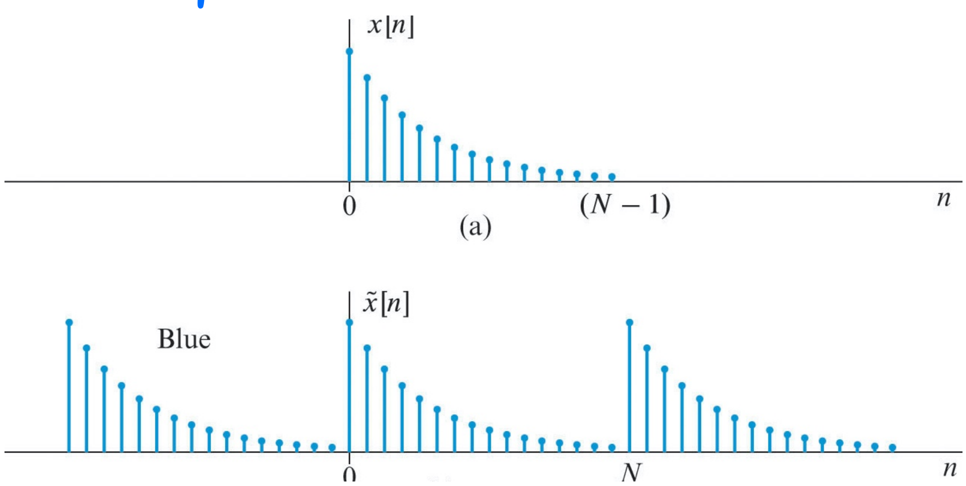
Discret Fouries transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \quad , \quad 0 \leq k$$

Inverse DFT

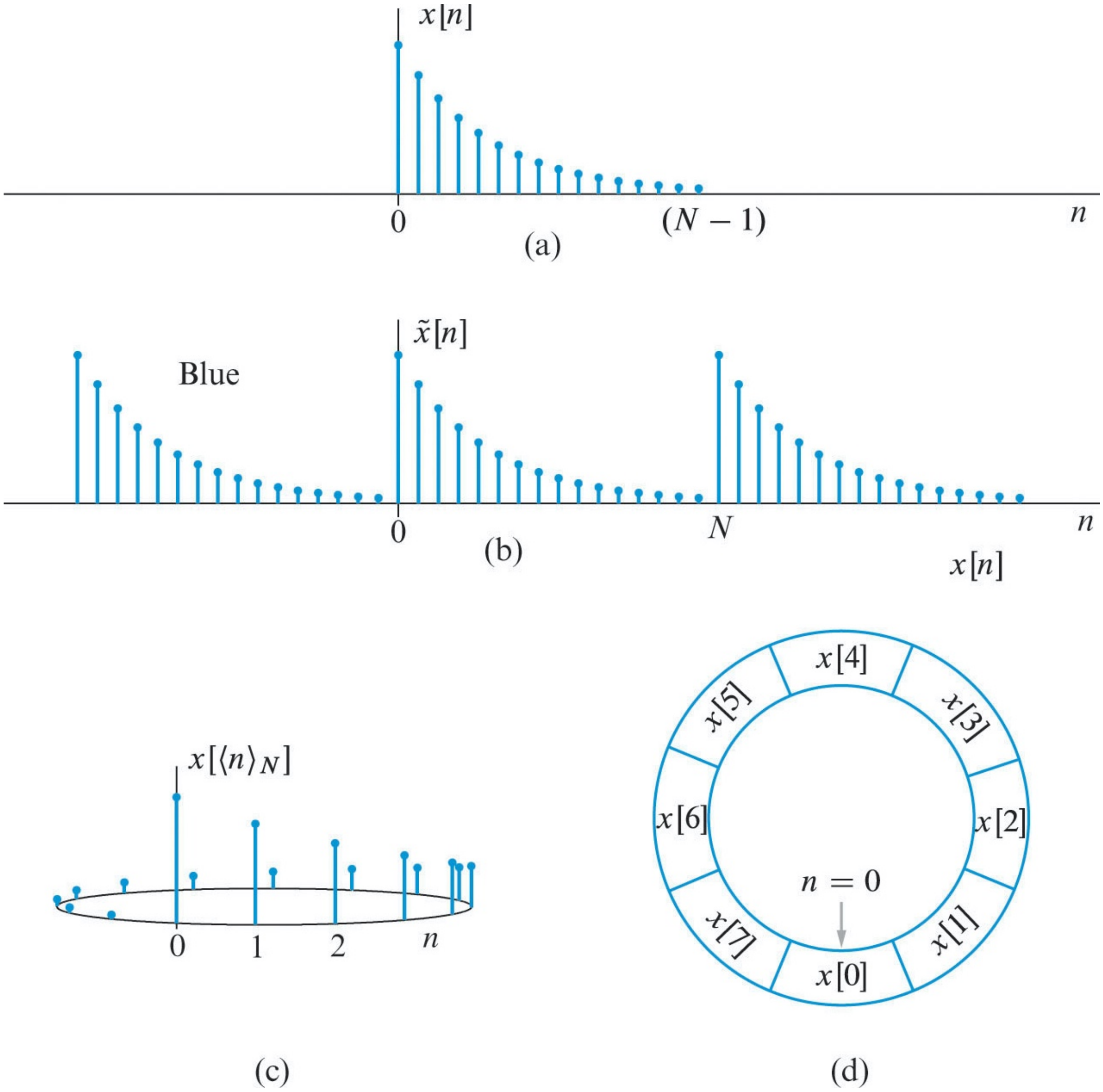
$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} \quad , \quad 0 \leq n$$

However, the reality is that the inverse
is actually a periodic function with per



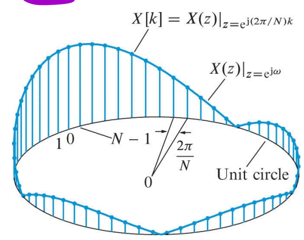
$$\tilde{x}[n] = \sum_{l=-\infty}^{\infty} x[n - lN]$$

Of carse because it is periodic, we only
1 period $x[n]$, $0 \leq n \leq N-1$



x[n]
0
(N − 1)
n
(a)
x̃[n]
Blue
0
N
n
(b)
x[n]
x[⟨n⟩N]
0
1
2
n
(c)
x[4]
x[5]
x[3]
x[6]
x[2]
n = 0
x[7]
x[1]
x[0]
(d)

Why is $\tilde{x}[n]$ a periodic function in time with period N?



This is because the DFT are samples of the DTFT (i.e. N samples for 2π frequency). Since the DTFT is periodic, the samples are also periodic

Time-domain aliasing happens if the length of $x[n]$ if $> N$

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$$x[n] = a^n u[n], \; 0 < a < 1 \xleftrightarrow{\text{DTFT}} X(e^{j\omega}) = \frac{1}{1 - a e^{-j\omega}}$$

$\downarrow$

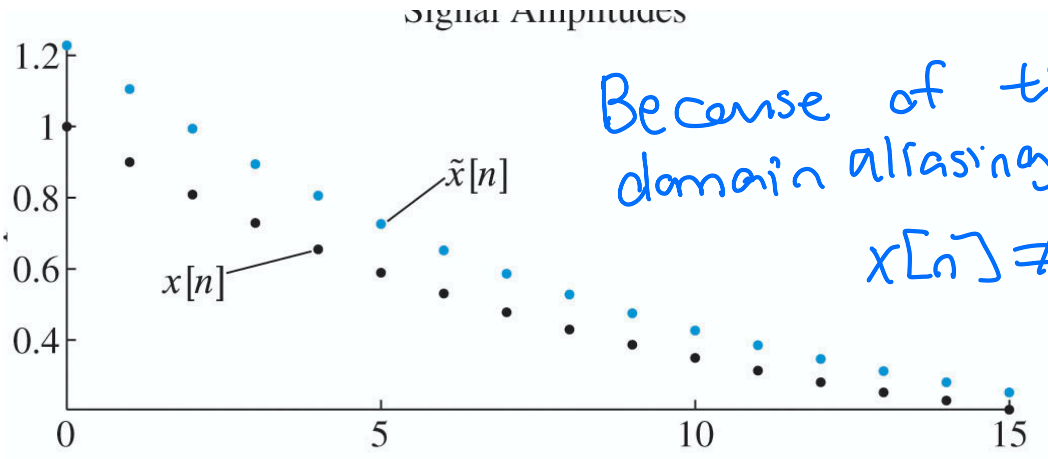
note that $x[n]$ is infinite length, we will know that there will be <u>time-domain aliasing</u>

DFT gives us N samples over one period of the DTFT

$\tilde{X}(e^{j\frac{2\pi}{N}k})$, Let $N = 16$

We can take the inverse DFT to get

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}\left(e^{j\frac{2\pi}{N}k}\right) e^{j\frac{2\pi}{N}kn}$$



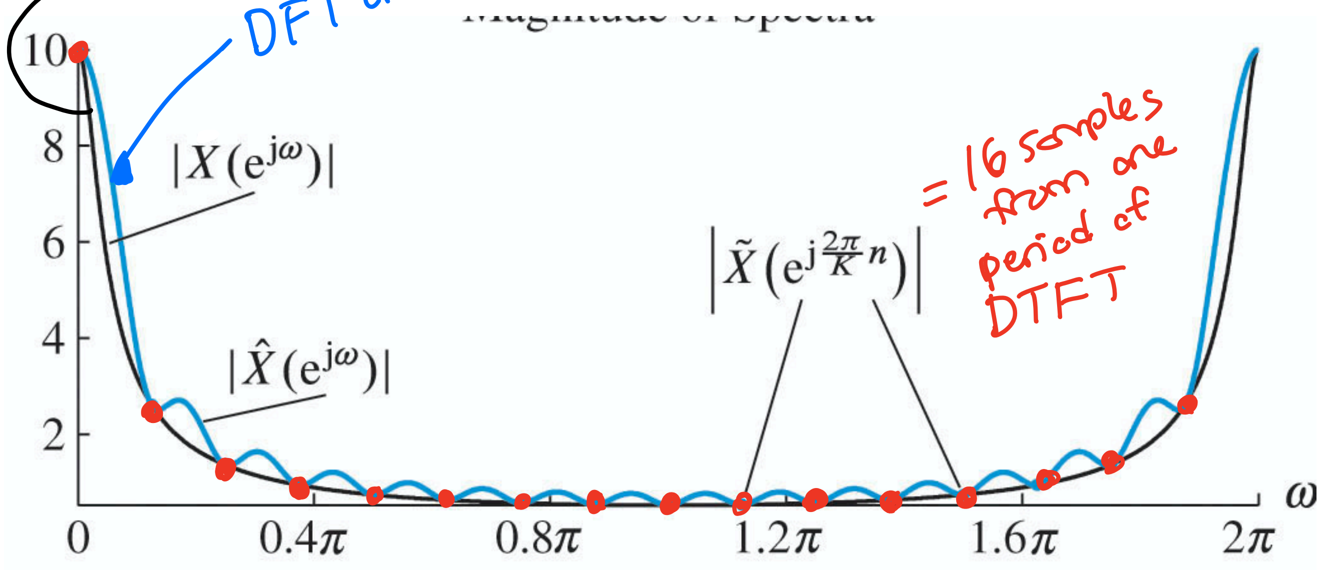
Because of time-domain aliasing

$x[n] \neq \tilde{x}[n]$

We can compute the DFT of $\tilde{x}[n]$

DTFT of $x[n]$

DFT of $\tilde{x}[n]$ will with more points



= 16 samples from one period of DTFT

Aliasing in time is when N is not large enough. We should zero pad $x[n]$ to make sure N is large.

If x[n] is infinite length, DFT will never exactly recover the true DTFT

Practical issues:

1) In many practical applications like speech processing and communication, the input sequence have indefinite length

2) Length of input sequence may be too large for storage and computation

3) Computation of output sequence can not be started until all input signal samples have been collected. This may cause unacceptable delays for many applications