Chap 3.1

When $x[n]$ and $h[n]$ have a finite # of nonzero values, we can do convolution easily (esp with a computer)

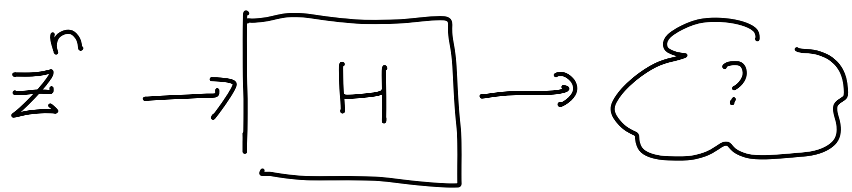
But what if $x[n] = \cos(\omega_0 n)$, $\frac{1}{2}^n$, $e^{j\omega_0 n}$? very hard to compute convolution by hand. We need to go to the frequency domain via the Z-transform

Let Z denote a complex number

$$Z = Re\{z\} + j\,Im\{z\}$$
$$= |Z| e^{j\angle Z}$$

the complex exponential $e^{j\omega_0 n}$ is a specific Z^n, where $|Z| = 1$ and $\angle Z = \omega_0$

So if we want to understand how an LTI System affects an input $e^{j\omega_0 n}$, we can study how



Assuming $x[n] = Z^n$,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k] z^{(n-k)} = \sum_{k=-\infty}^{\infty} h[k] z^{n} z^{-k}$$

$$= \underbrace{\sum_{k=-\infty}^{\infty} \left(h[k] z^{-k}\right)} z^{n} = \sum_{k=-\infty}^{\infty} \left(h[k] z^{-k}\right) x[n]$$

If the summation converges (i.e. adding entries from $-\infty$ to ∞ results in a finite value), we can use this quantity to study the behavior of the system

$H(z)$ = transfer function or system function

$$= \sum_{k=-\infty}^{\infty} \left(h[k] z^{-k}\right)$$

Note that

$$y[n] = \sum_{k=-\infty}^{\infty} \left(h[k] z^{-k}\right) x[n]$$

Note that convolution becomes a convolution when $x[n]$ is a complex number and we have the transfer function $H(z)$

$$= H(z)\, x[n] \text{, for all } n$$

the output of an LTI system when the input $x[n] = z^{n}$, is a complex exponential, is also a complex exponential at the same frequency as $x[n]$

For general $x[n]$ $(x[n] \neq z^n)$, we will see that $Y(z) = H(z)X(z)$

The z-domain is the generalization of the frequency domain

$$z = \text{any complex number}$$

$$\mathrm{Re}\{z\} + j\,\mathrm{Im}\{z\}$$

$$e^{j\omega_0} = \text{frequency domain}$$