h[n] has infinite length, therefore it is more convenient to characterize it in the Z-domain

The filter we want to design has a ration Z-transform

$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} , \quad M < N$$

can be infinite length

but M and N are finite values

$$= b_0 \frac{\prod_{k=1}^{M} (z - z_k)}{\prod_{k=1}^{N} (z - p_k)}$$

zeros

poles

N is the IIR filter order

What is N for FIR filter?

By convention, we want to design filters where M<N

In MATLAB, convention is

$$B = [b_0, b_1, \ldots b_M]$$
$$A = [1, a_1, a_2, \ldots a_N]$$

as in freqz(B, A) =

IIR filter uses continuous-time filter in the design process.

Continuous-time filter is characterized in the frequency domain by the Laplace transform

We are interested in filters with rational Laplace transform

$$H(s) = \int_0^\infty h(t) e^{-st} \, dt$$

h(t) is the continuous-time impulse response

h(t) is defined for t = −0.128, t = 0.1318, t = 20.45019, etc. an infinite # of values

(h[n] is defined for n = −2, n = 1, n = 1,000, etc.)

$$H(s) = \int_0^{\infty} h(t) e^{-st}\, dt = \frac{\sum_{k=0}^{M} \beta_k s^k}{1 + \sum_{k=1}^{N} \alpha_k s^k}, \quad M < N$$

N is the continuous-time filter order

$$= b_0 \frac{\prod_{k=1}^{M} (s - \zeta_k)}{\prod_{k=1}^{N} (s - s_k)}$$

zeros

poles

In MATLAB, convention is

$$C = [\beta_0, \beta_1, \dots \beta_M]$$

$$D = [1, \alpha_1, \alpha_2, \dots \alpha_N]$$

- The phase of $H(s)$ is ignored in causal continuous-time filters

- The filter parameter (i.e. the order N) is designed based on $|H(s)|^2$, the square of the magnitude

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

when $\Omega = \Omega_p$ (passband frequency), then $|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}}$

$$= |H(j\Omega_p)|^2$$

In decibel

$$10\log_{10}|H(j\Omega_p)|^2 = A_p = 20\log_{10}|H(j\Omega_p)|$$

This is a 10 instead of 20 because of the square

This means $|H(j\Omega_p)|^2 = 10^{\left(\frac{A_p}{10}\right)}$

when $\Omega = \Omega_s$ (stopband frequency), then $|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}$

$$= |H(j\Omega_s)|^2$$

In decibel

$$10\log_{10}|H(j\Omega_s)|^2 = A_s$$

This means $|H(j\Omega_s)|^2 = 10^{\left(\frac{A_s}{10}\right)}$

We have 2 equations

$$10^{\left(\frac{A_p}{10}\right)} = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}}$$

we know from specifications

the 2 unknowns not given by the specifications

and

$$10^{\left(\frac{A_s}{10}\right)} = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}$$

With algebra

$$N = \left\lceil \frac{\log_{10}\left(\dfrac{10^{\frac{A_p}{10}}-1}{10^{\frac{A_s}{10}}-1}\right)}{2\log_{10}\left(\frac{\Omega_p}{\Omega_s}\right)} \right\rceil$$

round to the latest integer value

$$\frac{\Omega_p}{\left(10^{\frac{A_p}{10}}-1\right)^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_s}{\left(10^{\frac{A_s}{10}}-1\right)^{\frac{1}{2N}}}$$

implemented by buttord function