	Property	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1.	Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$	At least $R_{x_1} \bigcap R_{x_2}$
2.	Time shifting	$x[n-k]$	$z^{-k}X(z)$	R_x except $z = 0$ or ∞
3.	Scaling	$a^n x[n]$	$X(a^{-1}z)$	$\|a\|R_x$
4.	Differentation	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_x
5.	Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
6.	Real-part	$\mathrm{Re}\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	At least R_x
7.	Imaginary part	$\mathrm{Im}\{x[n]\}$	$\frac{1}{2}[X(z) - X^*(z^*)]$	At least R_x
8.	Folding	$x[-n]$	$X(1/z)$	$1/R_x$
9.	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least $R_{x_1} \bigcap R_{x_2}$
10.	Initial-value theorem	$x[n] = 0$ for $n < 0$	$x[0] = \lim_{z\to\infty} X(z)$	

Table 3.1 Some common z-transform pairs

	Sequence $x[n]$	z-Transform $X(z)$	ROC
1.	$\delta[n]$	1	All z
2.	$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| > 1$
3.	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
4.	$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
5.	$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
6.	$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
7.	$(\cos\omega_0 n)u[n]$	$\dfrac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
8.	$(\sin\omega_0 n)u[n]$	$\dfrac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
9.	$(r^n\cos\omega_0 n)u[n]$	$\dfrac{1-(r\cos\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
10.	$(r^n\sin\omega_0 n)u[n]$	$\dfrac{(r\sin\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$

Common Discrete-Time Fourier Transform Pairs

$f[n]$	$F(\omega)$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - 2\pi k)$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k)$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$a^n u[n]$	$\frac{1}{1-ae^{-j\omega}}, \quad \lvert a\rvert < 1$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 - 2\pi k)$
$p_L[n]$	$\frac{\sin\left(\left(L+\frac{1}{2}\right)\omega\right)}{\sin\left(\frac{\omega}{2}\right)}$
$\frac{L}{\pi}\operatorname{sinc}\left(\frac{Ln}{\pi}\right)$	$\sum_{k=-\infty}^{\infty} p_{2L}(\omega + 2\pi k)$
$\cos(\omega_0 n)$	$\sum_{k=-\infty}^{\infty} \pi\big[\delta(\omega+\omega_0-2\pi k) + \delta(\omega-\omega_0-2\pi k)\big]$
$\sin(\omega_0 n)$	$\sum_{k=-\infty}^{\infty} j\pi\big[\delta(\omega+\omega_0-2\pi k) - \delta(\omega-\omega_0-2\pi k)\big]$

DTFT Theorems and Properties

Property	Time Domain	Frequency Domain
Notation:	$x(n)$	$X(\omega)$
	$x_1(n)$	$X_1(\omega)$
	$x_2(n)$	$X_1(\omega)$
Linearity:	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time shifting:	$x(n-k)$	$e^{-j\omega k}X(\omega)$
Time reversal	$x(-n)$	$X(-\omega)$
Convolution:	$x_1(n) * x_2(n)$	$X_1(\omega)X_2(\omega)$
Multiplication:	$x_1(n)x_2(n)$	$\frac{1}{2\pi}\int_{2\pi} X_1(\lambda)X_2(\omega-\lambda)d\lambda$
Correlation:	$r_{x_1x_2}(l) = x_1(l) * x_2(-l)$	$S_{x_1x_2}(\omega) = X_1(\omega)X_2(-\omega)$
		$= X_1(\omega)X_2^*(\omega)$ [if $x_2(n)$ real]
Frequency Differentiation:	$nx(n)$	$j\frac{dX(\omega)}{d\omega}$
Wiener-Khintchine:	$r_{xx}(l) = x(l) * x(-l)$	$S_{xx}(\omega) = \lvert X(\omega)\rvert^2$

DTFT Symmetry Properties

Time Sequence	DTFT
$x(n)$	$X(\omega)$
$x^*(n)$	$X^*(-\omega)$
$x^*(-n)$	$X^*(\omega)$
$x(-n)$	$X(-\omega)$
$x_R(n)$	$X_e(\omega) = \frac{1}{2}[X(\omega) + X^*(-\omega)]$
$jx_I(n)$	$X_o(\omega) = \frac{1}{2}[X(\omega) - X^*(-\omega)]$
$x(n)$ real	$X(\omega) = X^*(-\omega)$ $X_R(\omega) = X_R(-\omega)$ $X_I(\omega) = -X_I(-\omega)$ $\lvert X(\omega)\rvert = \lvert X(-\omega)\rvert$ $\angle X(\omega) = -\angle X(-\omega)$
$x_e'(n) = \frac{1}{2}[x(n) + x^*(-n)]$	$X_R(\omega)$
$x_o'(n) = \frac{1}{2}[x(n) - x^*(-n)]$	$jX_I(\omega)$

DFT Properties

Property	Time Domain	Frequency Domain
Notation:	$x(n)$	$X(k)$
Periodicity:	$x(n) = x(n+N)$	$X(k) = X(k+N)$
Linearity:	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(k) + a_2X_2(k)$
Time reversal	$x(N-n)$	$X(N-k)$
Circular time shift:	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular frequency shift:	$x(n)e^{j2\pi ln/N}$	$X((k-l))_N$
Complex conjugate:	$x^*(n)$	$X^*(N-k)$
Circular convolution:	$x_1(n) \otimes x_2(n)$	$X_1(k)X_2(k)$
Multiplication:	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k) \otimes X_2(k)$
Parseval's theorem:	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$

Additional Formula

November 10, 2021

1. Discrete-time convolution

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[n]h[n-k], -\infty < n < \infty$$

2. Discrete-Time Fourier Transform

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

3. Inverse Discrete-Time Fourier Transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})e^{j\omega n} d\omega$$

4. Z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

5. Inverse Z-Transform

$$x[n] = \frac{1}{2\pi j} \oint X(z)z^{n-1} dz$$

6. Relative filter specification

$$A_p = 20 \log_{10} \left(\frac{1+\delta_p}{1-\delta_p} \right)$$

$$A_s = 20 \log_{10} \left(\frac{1+\delta_p}{\delta_s} \right)$$