GR10.jpg

Chap 3.7

A rational system functions with distinct poles can be decomposed as

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = \underbrace{\sum_{k=0}^{M-N} C_k z^{-1}}_{\text{only exist if } M>N} + \sum_{k=1}^{N} \frac{A_k}{1-P_k z^{-1}}$$

C_k are always real but A_k may be real or complex

- If P_i and P_j are complex conjugate poles, then A_i and A_j are complex conjugate of one another

- To avoid A_k being complex numbers, we can combine

$$\frac{A}{1-pz^{-1}} + \frac{A^*}{1-p^*z^{-1}} = \frac{b_0+b_1 z^{-1}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

so $b_0 = 2\,\mathrm{Re}\{A\} = 2|A|\cos\theta$

$b_1 = -2\,\mathrm{Re}\{Ap^*\} = -2r|A|\cos(\omega_0-\theta)$

$a_1 = -2\,\mathrm{Re}\{p\} = -2r\cos\omega_0$

$a_2 = |p|^2 = r^2$

So we can say

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = \sum_{k=0}^{M-N} C_k z^{-1} + \sum_{k=1}^{N} \frac{A_k}{1-P_k z^{-1}}$$

$$= \sum_{k=0}^{M-1} C_k z^{-1} + \underbrace{\underbrace{\sum_{k=1}^{K_1} \frac{A_k}{1-P_k z^{-1}}}_{\text{term with all real poles}}}_{\text{first order system}} + \underbrace{\underbrace{\sum_{k=1}^{K_2} \frac{b_{k0}+b_{k1} z^{-1}}{1+a_{k1} z^{-1}+a_{k2} z^{-2}}}_{\text{term with complex conjugate poles}}}_{\text{second-order system}}$$

$$K_1 + K_2 = N$$

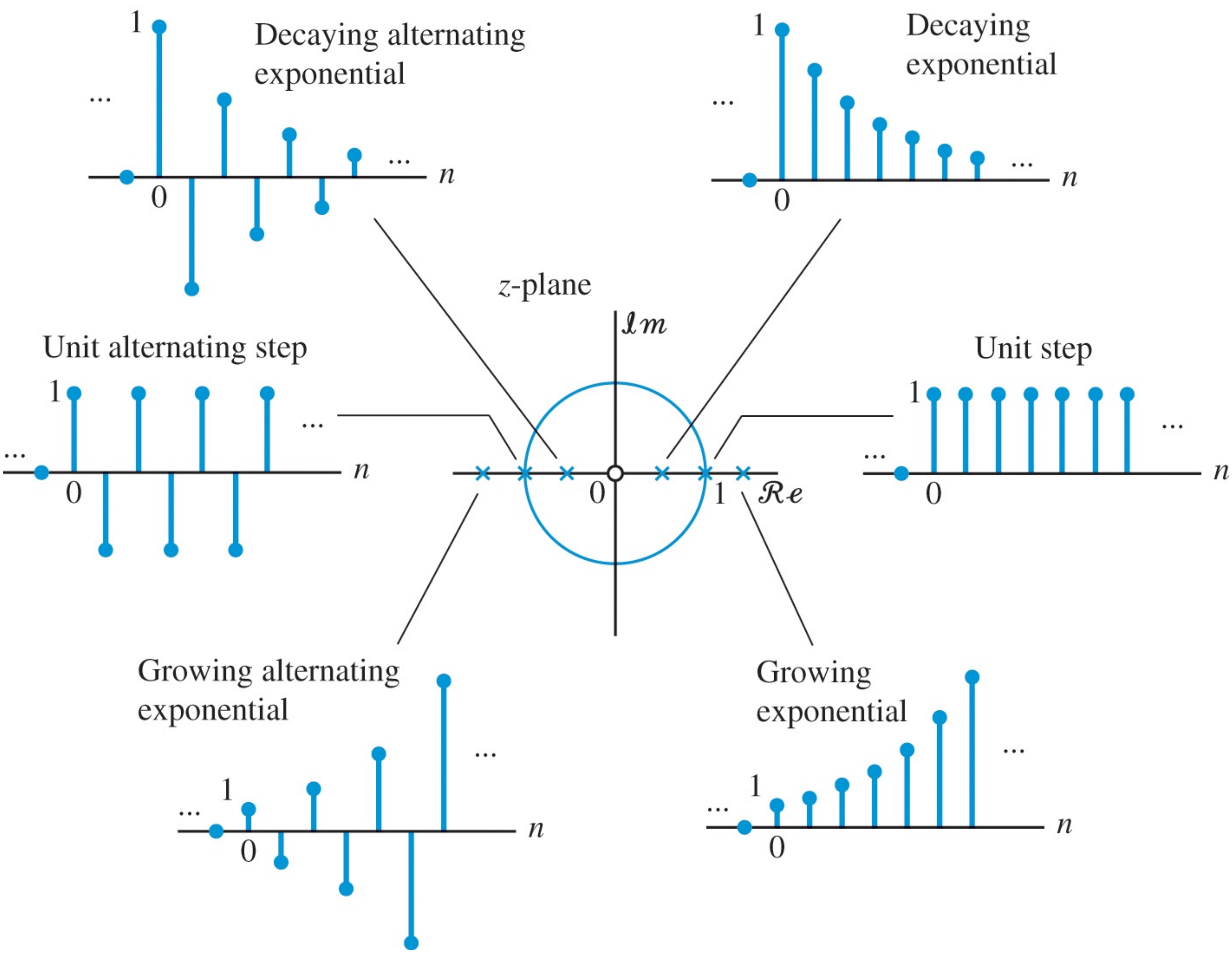
The behavior of a system with a rational system function can be understood in terms of the behavior of a <u>first order system with real poles</u> and a <u>second order system with complex conjugate poles</u>

First order system

$$H(z) = \frac{b}{1-az^{-1}}$$, a and b are real numbers

assuming a causal system

$$h[n] = b\,a^n\,u[n]$$



Second-order system

We are particularly interested in second order systems with complex-conjugate poles

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{A}{1 - p z^{-1}} + \frac{A^*}{1 - p^* z^{-1}}$$

$$h[n] = A p^n u[n] + A^* (p^*)^n u[n]$$

$$= 2|A| r^n \cos(\omega_0 n + \theta) u[n]$$

where $r^2 = |p|^2$