Goal:

$$\text{minimize} \max_{\omega \in B} \big| \underbrace{H_d(e^{j\omega}) - H(e^{j\omega})}_{\text{approximation error}} \big|$$

ideal filter

practical filter

passband

Recall that a frequency response is

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{j \angle H(e^{j\omega})}$$

$$= \underbrace{A(e^{j\omega})}_{\text{amplitude response}} e^{j \underbrace{\Psi(\omega)}_{\text{unwrapped phase response}}}$$

We can change the goal to

$$\text{minimize} \max_{\omega \in B} \big| A_d(e^{j\omega}) - A(e^{j\omega}) \big|$$

For the 4 different type of linear-phase FIR filters

$$A(e^{j\omega}) = Q(e^{j\omega})P(e^{j\omega})$$

where

$$Q(e^{j\omega}) = \begin{cases} 1 & \text{, if Type 1 filter} \\ \cos(\frac{\omega}{2}) & \text{, if Type 2 filter} \\ \sin(\omega) & \text{, if Type 3 filter} \\ \sin(\frac{\omega}{2}) & \text{, if Type 4 filter} \end{cases}$$

and

$$P(e^{j\omega}) = \sum_{k=0}^{R} p[k]\cos(\omega k),$$

Where $R = \frac{M}{2}$ if filter order M is even

$R = \frac{(M-1)}{2}$ if filter order M is odd

The optimization goal is now

$$\underset{\omega \in B}{\text{minimize max}} \; |A_d(e^{j\omega}) - \underbrace{Q(e^{j\omega})}_{\text{determined by filter type}}\underbrace{P(e^{j\omega})}_{\text{determined by filter length}}|$$

$$P(e^{jw}) = \sum_{k=0}^{R} p[k]\cos(wk) \text{ is}$$

what we can control in our design by choosing filter order M

$P(e^{jw})$ is has special properties that allows us to reduce the optimization goal to a Chebyshev polynomial approximation problem. This will allow us to band the maximum of the error

Matlab function makes designing Parks-McClellan filters very easy