Chap 2.3

Discrete-time system = a process that transforms an input sequence $x[n]$ to the output sequence $y[n]$

$$x[n] \xrightarrow{H} y[n] \quad \text{or} \quad y[n] = \mathcal{H}\{x[n]\}$$

In DSP, the system of interests are filters. We need to understand the properties and behaviors of systems in order to design filters.

Discrete-time systems can be studied from 2 perspectives. The reason is that some things are easier in one domain vs. the other

time-domain	Z / frequency domain (i.e. complex exponential)
usually easier to interpret and visualize since we only works with real numbers	easier computation and computing frequency related quantities since we work in the complex domain
	Laplace transform in continuous time system

Discrete-time system describes a mathematical rule that describes how to get out output from input

ex: $y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{2}x[n-2]$

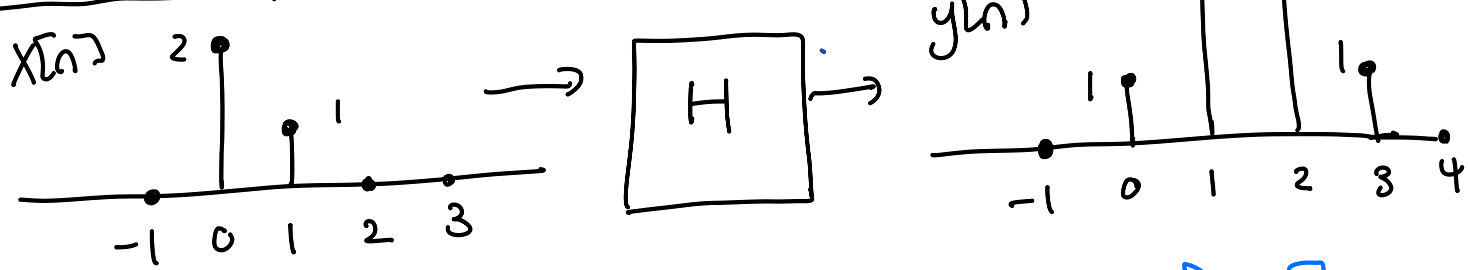
Description of the system in the time-domain

In DSP, we work with a special class of discrete-time systems: Linear, Time-Invariant (LTI) systems

1) system is time-invariant if and only if

$$y[n] = \mathcal{H}\{x[n]\} \Rightarrow y[n-n_0] = \mathcal{H}\{x[n-n_0]\}$$

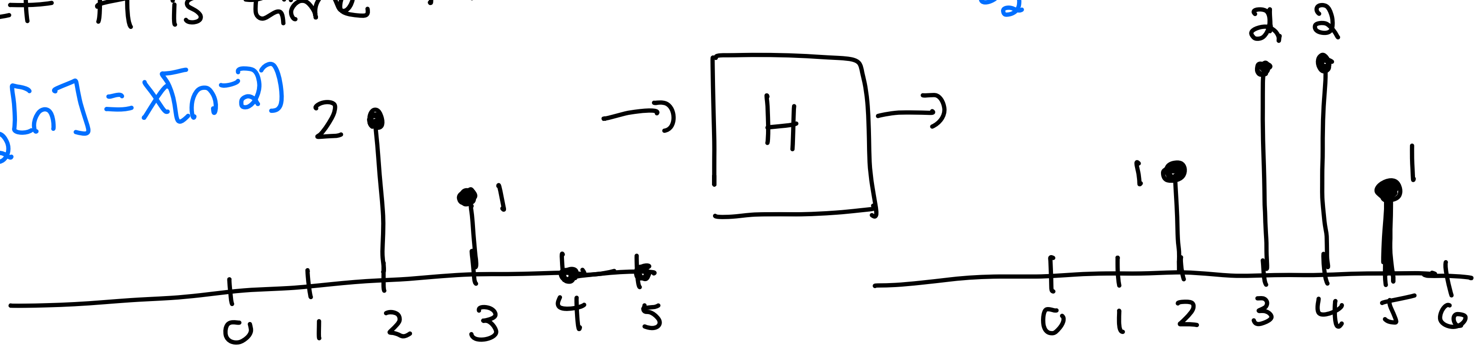
For example



If H is time invariant then

$y_2[n] = y[n-2]$

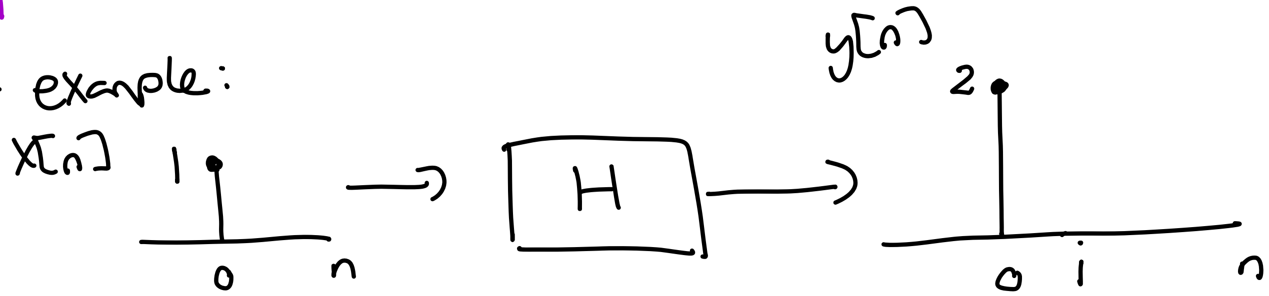
$x_2[n] = x[n-2]$



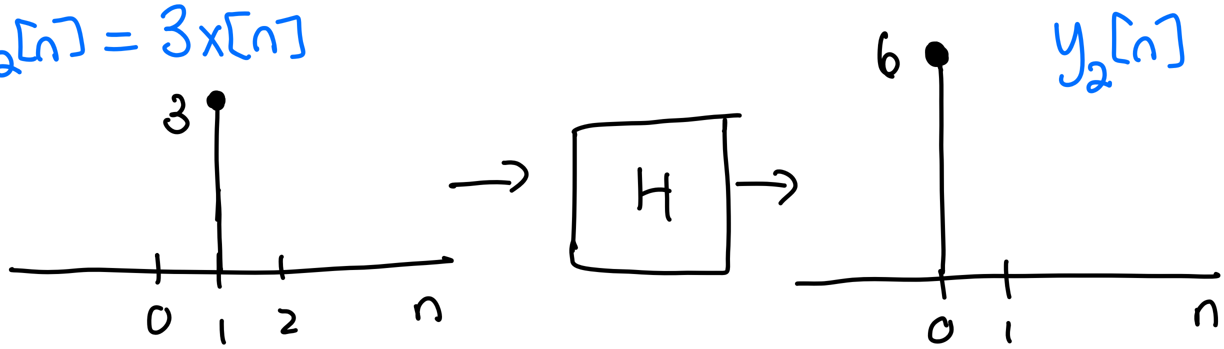
Time-invariance is a good property because we can determine $y_2[n]$ from $y[n]$ instead of having to actually compute the output using $x_2[n]$

2) Linearity is an extremely property because the System would behave the way we "expect"

For example:



Then $x_2[n] = 3x[n]$



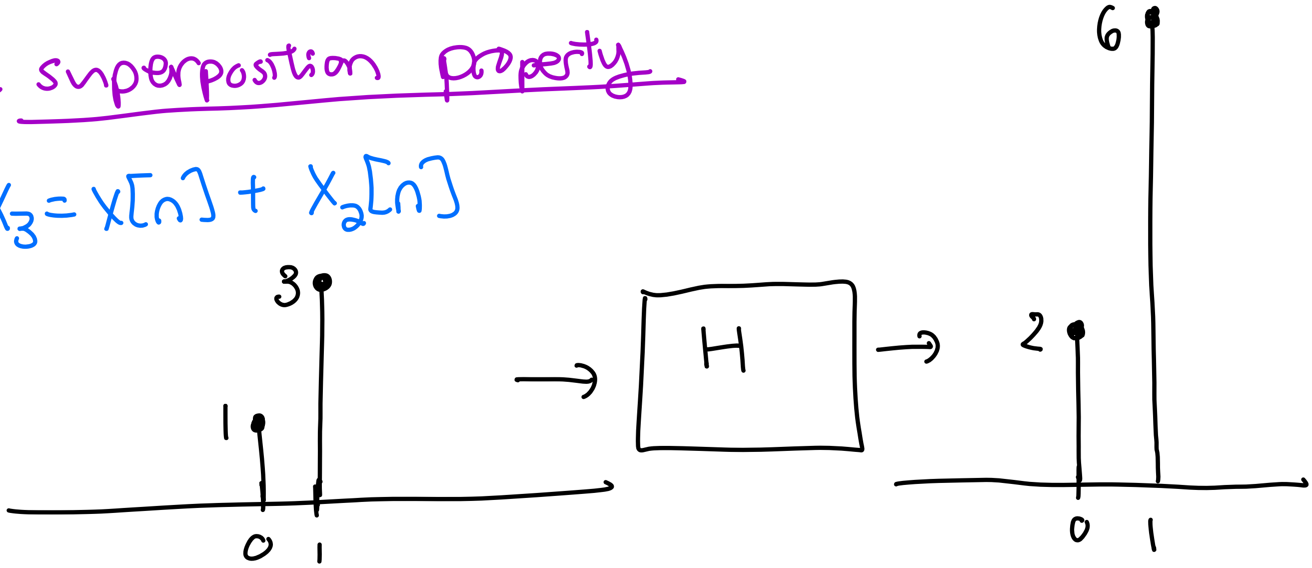
The <u>scaling property</u>:

for every real or complex constant a_1 and a_2 and every input signal $x_1[n]$, $x_2[n]$

$$\mathcal{H}\{a_1 x_1[n]\} = a_1 \mathcal{H}\{x_1 n\}$$

The superposition property

$x_3 = x[n] + x_2[n]$



$$\mathcal{H}\{x_1[n] + x_2[n]\} = \mathcal{H}\{x_1[n]\} + \mathcal{H}\{x_2[n]\}$$

A system is <u>linear</u> if and if it satisfies both scaling and superposition property

$$\mathcal{H}\{a_1 x_1[n] + a_2 x_2[n]\} = a_1 \mathcal{H}\{x_1[n]\} + a_2 \mathcal{H}\{x_2[n]\}$$

Ex.

$y[n] = (x[n])^2$ Is this a linear system

Let $x_2[n] = 3x[n]$. If the system is linear, then

$$\mathcal{H}\{x_2[n]\} = \underbrace{\mathcal{H}\{3x[n]\}}_{\substack{= (3x[n])^2 \\ = 9(x[n])^2}} = \underbrace{3\mathcal{H}\{x[n]\}}_{3y[n] = 3(x[n])^2}$$

Therefore $\mathcal{H}\{3x[n]\} \neq 3\mathcal{H}\{x[n]\}$, so the system is <u>not</u> linear

The filters that we will design are all LTI

In addition, there are two other properties that are important

1) A system is <u>causal</u> if the present value of the output ($y[n]$) does not depend on future value of the input

For example,

$$y[n] = x[n] + 3x[n+2] \text{ is not cause}$$

since to find $y[0]$, we need $x[2]$.

2) a system is <u>stable</u> in the bounded-input bounded-output sense (BIBO) if

$$|x[n]| \leq M_x < \infty \quad \text{results in} \quad |y[n]| \leq M_y < \infty$$

Unstable system generates unbounded output signal from bounded input

When we design filters, we want a stable filter. Depending on the application, we may or may not need a causal filter. When do we not need a causal filter?