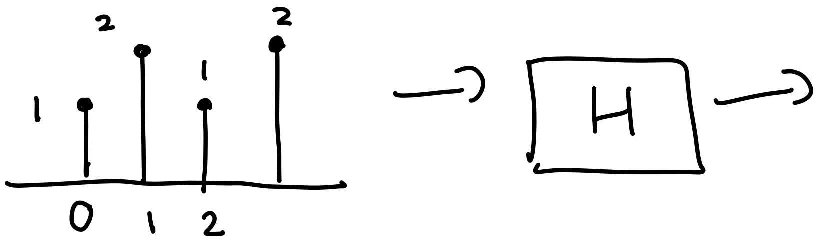
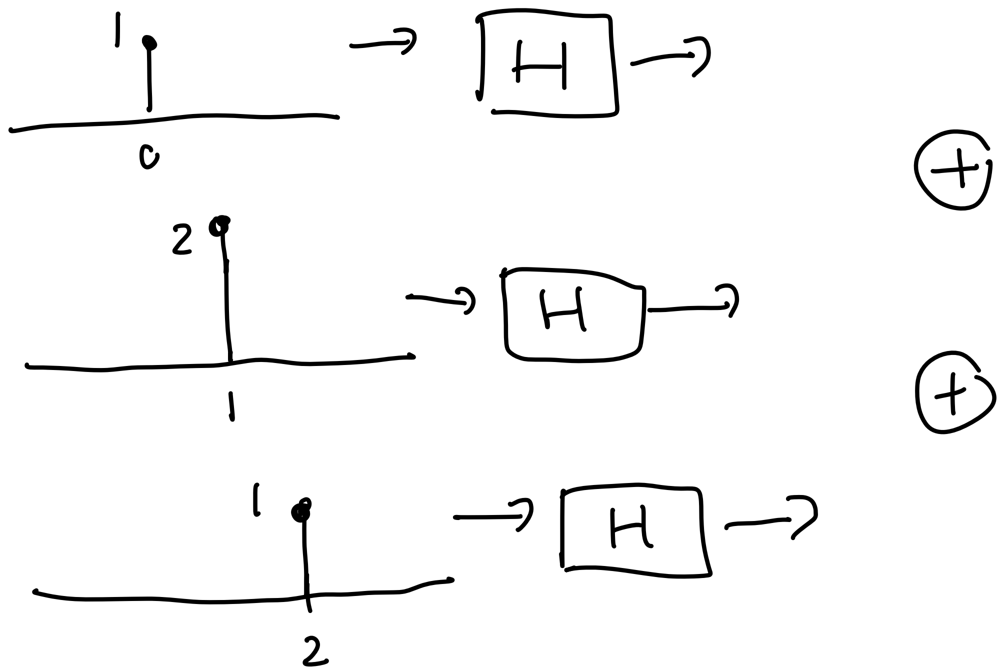
Linear Time-invariant (LTI) systems are both linear and time-invariant

Because of these nice properties. We know that



this output is the same as looking at



For LTI system, the response of the system to any input is characterized by its response $h[n]$ to the unit sample sequence $\delta[n]$

$h[n]$ is therefore called the <u>impulse response</u>

We can consider any input sequence

$$x[n] = \sum_{k} x_k[n] \quad \text{where}$$

$$x_k[n] = \begin{cases} x[k], & n = k \\ 0, & n \neq k \end{cases}$$

$$= x[k]\,\delta[n-k]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k], \quad -\infty < n < \infty$$

= any input sequence is a sum of scaled unit-impulse functions

Therefore, when we want to find the output of a LTI system

$$y[n] = \mathcal{H}\{x[n]\} = \mathcal{H}\left\{\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right\}$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,\underbrace{\mathcal{H}\{\delta[n-k]\}}_{h[n-k]}$$

$h[n-k]$ = impulse response of the system to $\delta[n-k]$

$$= \underbrace{\sum_{k=-\infty}^{\infty} x[k]\, h[n-k], \quad -\infty < n < \infty}$$

this operation is called convolution sum or just convolution

For simplicity, we often write

$$y[n] = x[n] * h[n]$$

The output of an LTI system to <u>any</u> input $x[n]$ can be computed <u>if</u> we know the impulse response.

the step response, $s[n]$, is the output of an LTI when the input is a step function $u[n]$

Properties of LTI system - some properties of LTI system can be derived by looking at the impulse response

1) A LTI system with impulse response $h[n]$ is causal if

$$h[n] = 0 \text{ for } n < 0$$

2) A LTI system with impulse response $h[n]$ is stable if and only if

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

If $h[n]$ is a finite length sequence, then $\mathcal{H}$ is a FIR (finite impulse response) filter

If $h[n]$ is an infinite length sequence, then $\mathcal{H}$ is an IIR (infinite impulse response) filter

How can $h[n]$ be IIR and stable?