Chap 7.4 (Matlab's fft function by default set N to be the same length as input, not 256)

The DFT is its own transform. Therefore, certain properties can be used to find the DFT <u>without</u> having to compute it from scratch

1) Linearity

$$a_1 x_1[n] + a_2 x_2[n] \overset{DFT}{\longleftrightarrow} a_1 X_1[k] + a_2 X_2[k],$$

$$0 \leq k \leq N-1$$

2) Time-shifting (circular shifts)

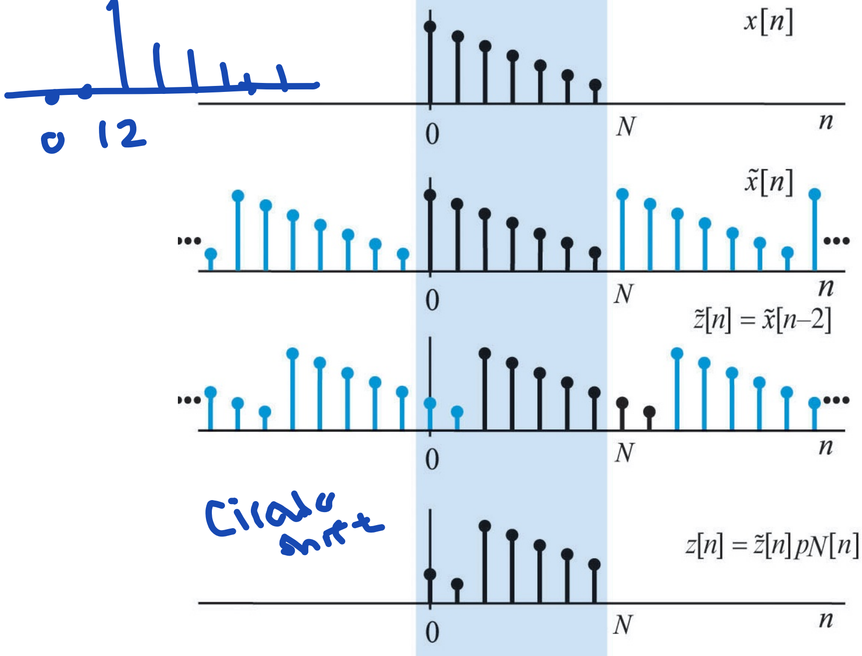
$$x[\langle n-m \rangle_N] \overset{DFT}{\longleftrightarrow} e^{-j\frac{2\pi}{N}km} \; X[k]$$

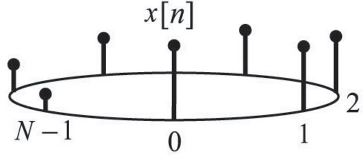
New operation because x[n] for DFT is really a periodic sequence with period N

$$\langle n \rangle_N \equiv n \text{ modulo } N$$ = always a number between 0 and N-1

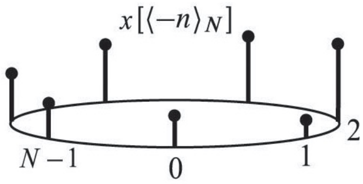
$$n = \ell N + r \quad , 0 \leq r \leq N-1$$

n modulo N





$\leftarrow$ periodic function with period N



ex $\langle 3 \rangle_5 = 3$

$3 = \underset{0}{l}(5) + r$

ex $\langle 10 \rangle_6 = 4$

$10 = l(6) + r = (1)(6) + 4$

$-5 = -3 + \overset{-2}{r}$

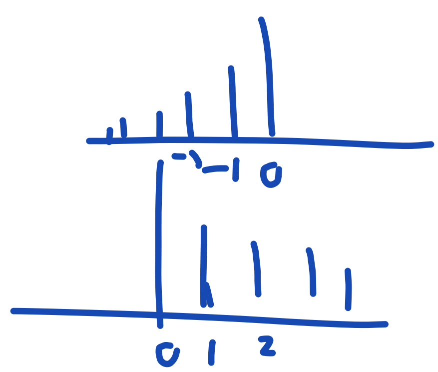
ex $\langle -5 \rangle_3 = 1$

$-5 = l(3) + r = \cancel{(-1)(3) + r}$ but r would have to be negative

$= (-2)(3) + \underline{1}$

ex $\langle -10 \rangle_3 = 2$

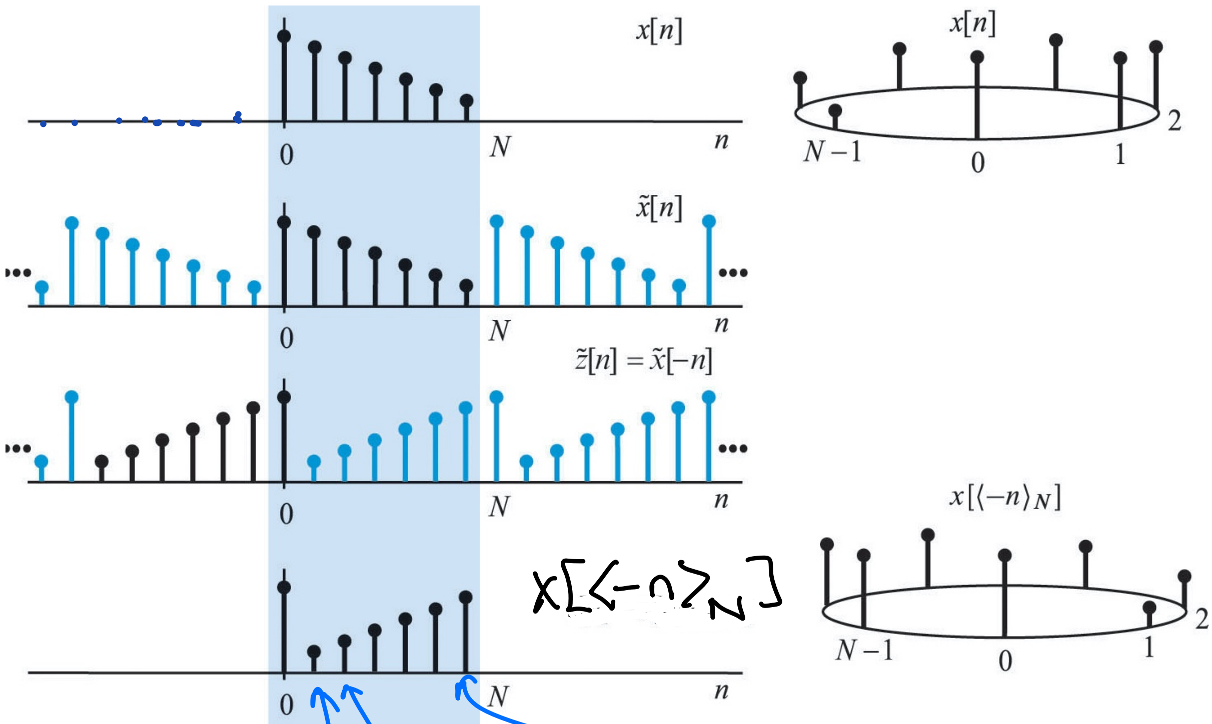
$-10 = l(3) + r = (-4)(3) + 2$



3) Folding (circular folding)

$x[-n]$ no longer makes sense due to periodicity

$$x[\langle -n \rangle_N] \equiv \begin{cases} x[n], & n=0 \\ x[N-n], & 1 \le n \le N-1 \end{cases}$$



$x[0]$ $x[N-1]$ $x[N-2]$ $x[N-N+1] = x[1]$

$$x[\langle -n \rangle_N] \overset{DFT}{\underset{N}{\longleftrightarrow}} X[\langle -k \rangle_N]$$

$$X[\langle -k \rangle_N] \equiv \begin{cases} X[0], & k=0 \\ X[N-k], & 1 \le k \le N-1 \end{cases}$$

4) convolution (circular convolution)

Like time-shift and folding, convolution is also slightly different

$$h[n] \, \textcircled{\small N} \, x[n] \xleftrightarrow[N]{\text{DFT}} H[k]X[k]$$

circular convolution in the time-domain is equivalent to multiplication in the frequency domain

Circular convolution is defined as

$$h[n] \, \textcircled{\small N} \, x[n] = \sum_{m=0}^{N-1} h[m]\, x[\langle n-m \rangle_N], \quad 0 \le n \le N-1$$

circular time shift

Output of circular convolution is also a length N vector

You have to use matlab function cconv to do circular convolution

Ex $x[n] = [2\ 1\ 2]$, $y[n] = [1\ 2\ 3]$

regular convolution (assumes that x[n] and y[n] are infinite length sequences with 0 everywhere else)

$x[n] * y[n] = \text{conv}(x, y)$

$= [2\ \ 5\ \ 10\ \ 7\ \ 6]$

Note that the length of regular convolution is longer than x and y (length = length of x + length of y − 1)

Let N = 3

$x[n] \ (N) \ y[n] = \text{cconv}(x, y, 3)$

$= [9\ \ 11\ \ 10]$

The length of the output of circular convolution is N

Let N = 5, we can zero pad x[n] and y[n] so

$x[n] = [2\ 1\ 2\ 0\ 0]$

$y[n] = [1\ 2\ 3\ 0\ 0]$

$x[n] \ (N) \ y[n] = \text{cconv}(x, y, 5)$

$= [2\ \ 5\ \ 10\ \ 7\ \ 6]$

Same as <u>regular convolution</u> when N is large enough

Let $N=6$, we zeropad $x[n]$ and $y[n]$ again

$$x[n] \; N \; y[n] = \text{cconv}(x, y, 6)$$

$$= [2 \quad 5 \quad 10 \quad 7 \quad 6 \quad 0]$$

5) correlation (circular correlation)

The N-point circular correlation of $x[n]$ and $y[n]$ is

$$r_{xy}[l] \equiv \sum_{n=0}^{N-1} x[n]\, y[\langle n-l \rangle_N], \quad 0 \le l \le N-1$$