Chap 6.1

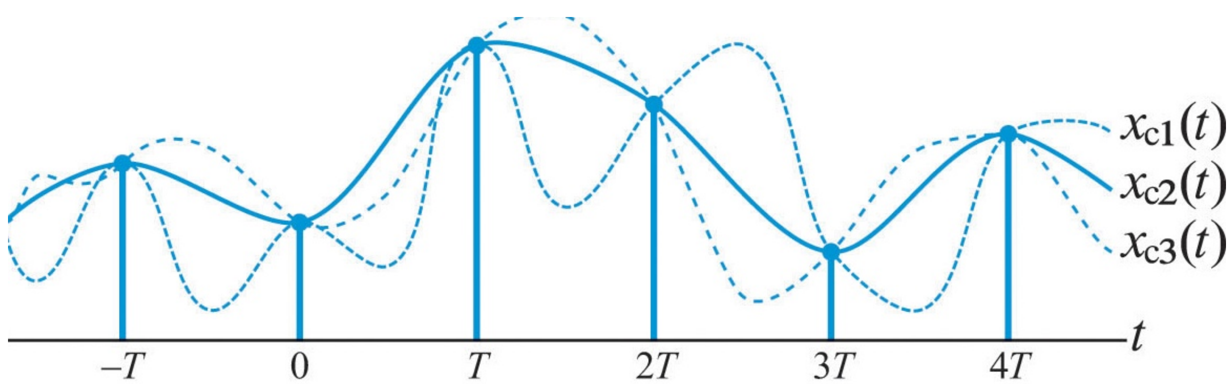
Periodic sampling of continuous-time signal $x_c(t)$ is

$$x[n] \equiv x_c(t)\big|_{t=nT} = x_c(nT), \quad -\infty < n < \infty$$

T = sampling period

$F_s = \frac{1}{T}$ = sampling frequency (with unit of Hertz = $\frac{\text{cycles}}{\text{sec}}$)

A system that perfectly samples the continuous-time signal $x_c(t)$ is the ideal analog to digital converter (ADC)



There are 2 important issues to understand

1) How to pick a good sampling period T

2) What does sampling mean in the frequency domain?

Lets address the frequency question first

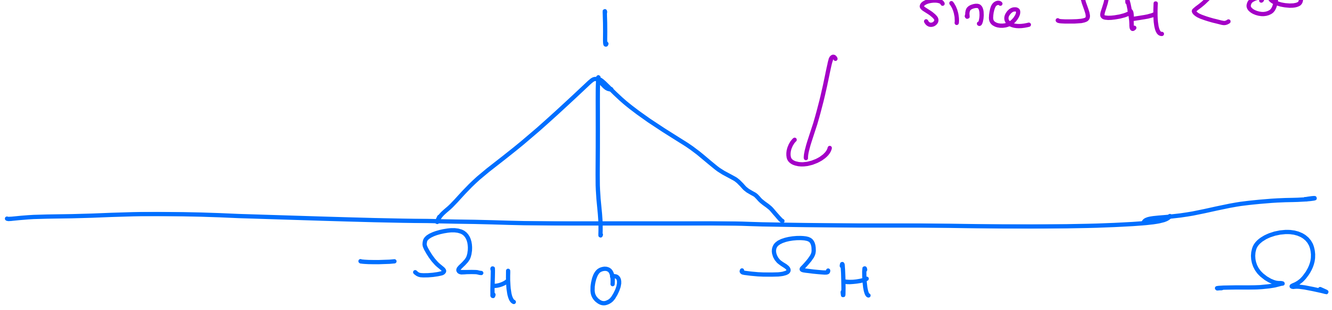
$x_c(t)$ has a continuous - time Fourier transform (CTFT)

$$X_c(j\Omega) = \int_{-\infty}^{\infty} x_c(t)\, e^{-j\Omega t}\, dt$$

We can visualize the magnitude response

$|X_c(j\Omega)|$

bandlimited signal since $\Omega_H < \infty$



What looks difference from the magnitude response of discete time signal

Remember how to convert from analog frequency to normalized frequency

$$\left(\frac{\text{radian}}{\text{sample}}\right) \omega = \Omega T = 2\pi F T = 2\pi \frac{F}{F_s} = 2\pi f$$

Ω: unit of radian / sec

From the inverse continuous time Fourier transform

$$x_c(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_c(j\Omega)\, e^{j\Omega t}\, d\Omega$$

When we sample, we only do at $t = nT$

$$x_c(nT) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_c(j\Omega) e^{j\Omega nT} d\Omega$$

On the other hand, we know that the inverse DTFT is

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

since $\omega = \Omega T$

constant

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega T}) e^{j(\Omega T)n} d\Omega T$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} T\, X(e^{j\Omega T}) e^{j(\Omega T)n}$$

$x[n] = x_c(nT)$ by definition, so the two expressions on the right hand side has to be the same

With some integration trick, see Chap 6.1 in textbook

The DTFT of x[n] is

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\frac{\omega}{T} - j\frac{2\pi}{T}k\right)$$

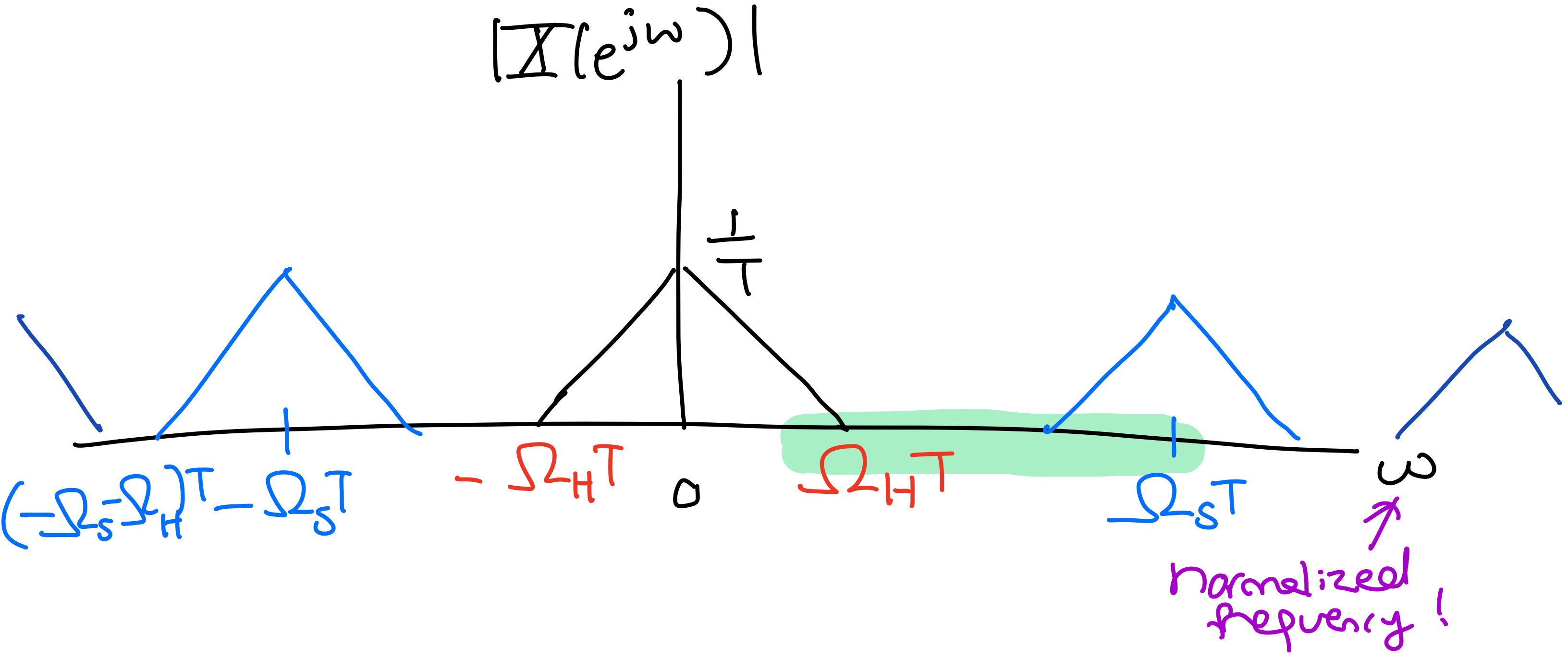
DFTF of x[n]

1) Scales the spectrum of $X_c(t)$ by $\frac{1}{T}$

This scaling only affects the magnitude response, not the phase response. Why?

2) copies of the scaled spectrum $\frac{1}{T}X_c(j\Omega)$ at all integer multiples of the sampling frequence

$\Omega_s = 2\pi F_s$ $\frac{\text{radian}}{\text{samples}}$



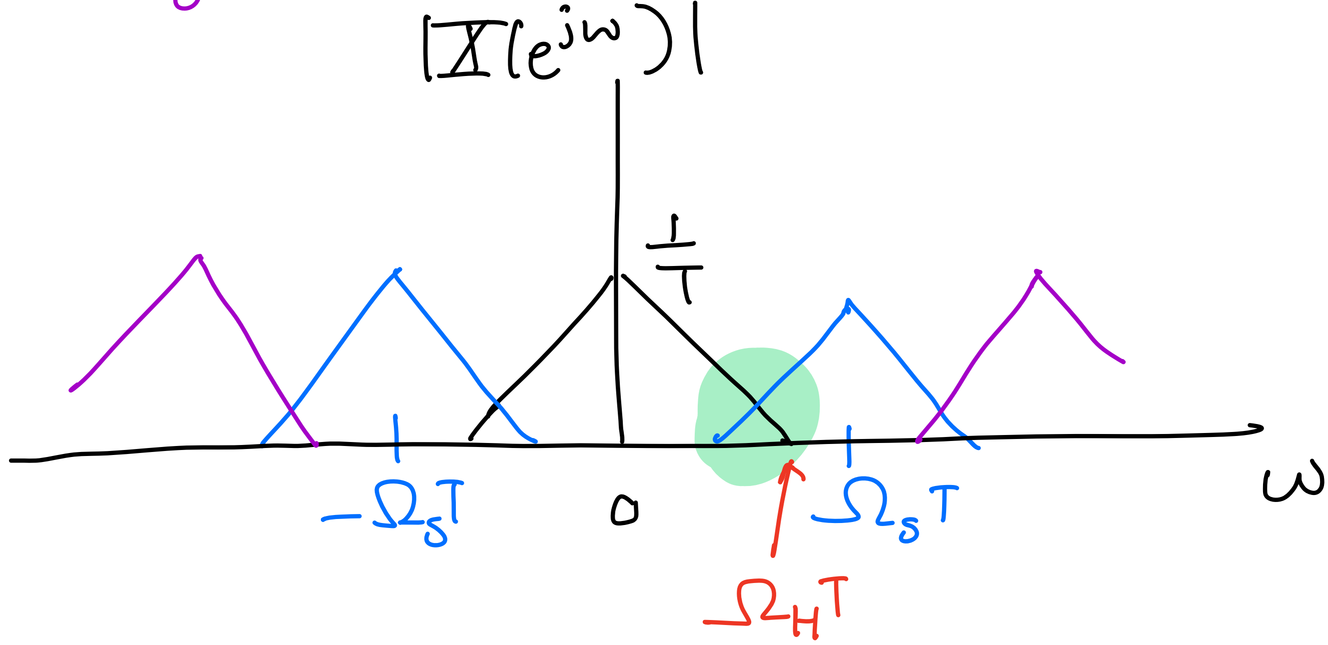
What is $\Omega_s T$?

$\Omega_s = 2\pi F_s$

$T = \frac{1}{F_s}$

$\Omega_s T = 2\pi$ This is also why DTFT has 2π periodicity

We have to pick the sampling frequency carefully. If Ω_s is too small, we get



This problem is call aliasing. Under what condition can we avoid this?

Nyquist sampling theorem:

Let $x_c(t)$ be a continuous-time bandlimited ($\Omega_H T < 2\pi$) with Fourier transform

$X_c(j\Omega)$

$x_c(t)$ can be uniquely determined by its samples $x[n] = x_c(nT)$ if the sampling frequency, Ω_s satisfies the condition

$$\Omega_s \geq 2\Omega_H$$