PS2

3.5-3 Show that a filter with transfer function

$$H(f) = \frac{2(\beta)}{(2\pi f)^2 + \beta^2} e^{-j2\pi f t_0}$$

is unrealizable. Can this filter be made approximately realizable by choosing a sufficiently large t_0? Use your own (reasonable) criterion of approximate realizability to determine t_0.

Hint: Show that the impulse response is noncausal.

$$H(f) = \underbrace{\frac{2(\beta)}{(2\pi f)^2 + \beta^2}}_{F(f)} e^{-j2\pi f t_0}$$

then, $f(t) = e^{-\beta|t|}$

$H(f) = F(f) e^{-j2\pi f t_0}$ ← time shifting

$h(t) = f(t - t_0) = e^{\beta|t - t_0|}$

↑ not causal for $t < 0$

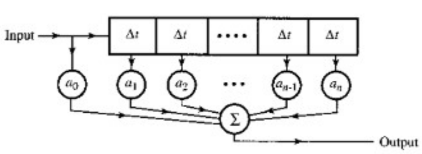
∴ not realizable

3.6-2 The distortion caused by multipath transmission can be partly corrected by a tapped delay-line equalizer. Show that if $\alpha \ll 1$, the distortion in the multipath system in Fig. 3.31a can be approximately corrected if the received signal in Fig. 3.31a is passed through the tapped delay-line equalizer shown in Fig. P3.6-2.

Hint: From Eq. (3.64a), it is clear that the equalizer filter transfer function should be $H_{eq}(f) = 1/(1 + \alpha e^{-j2\pi f \Delta t})$. Use the fact that $1/(1-x) = 1 + x + x^2 + x^3 + \cdots$ if $x \ll 1$ to show what should be the tap parameters a_i to make the resulting transfer function

$$H(f)H_{eq}(f) \approx e^{-j2\pi f t_d}$$

Figure P.3.6-2



equalizer filter (3.65a)

$$H_{eq}(f) = \frac{1}{1 + \alpha e^{-j2\pi f \Delta t}} = \sum_{n=0}^{\infty} (-\alpha)^n e^{-j2\pi f n \Delta t}$$

$$H(f) H_{eq}(f) = e^{-j2\pi f t_d}$$

tapped delay line eq.

$$y(t) = a_0 x(t) + a_1 x(t - \Delta t) + a_2 x(t - 2\Delta t) + \ldots$$

$$= \sum_{m=0}^{n} a_0 x(t - m\Delta t)$$

using time shift prop:

$$Y(t) = \sum_{m=0}^{n} a_m X(f) e^{-j2\pi f m \Delta t}$$

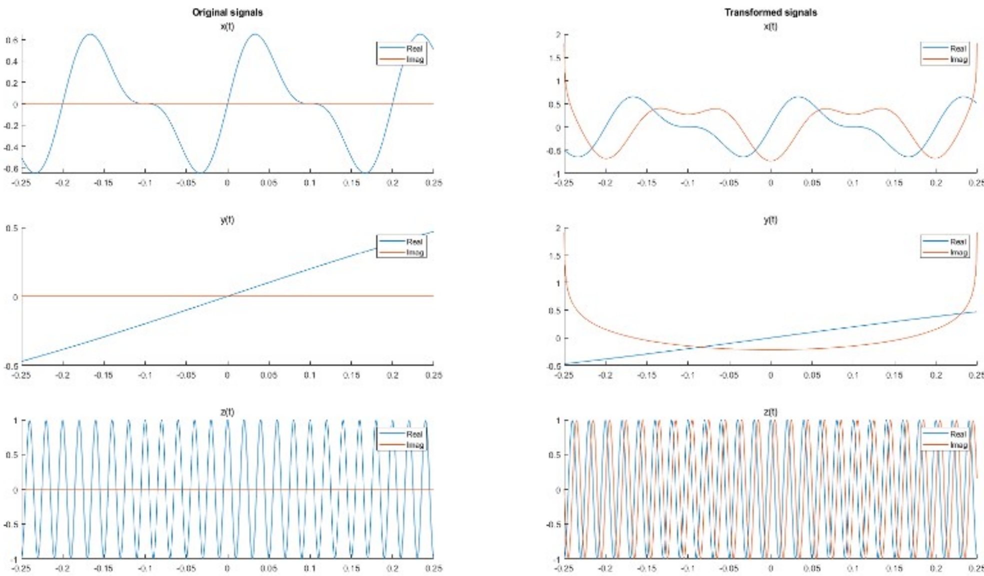
using time shift prop:

$$Y(t) = \sum_{m=0}^{n} \underline{a_m}\, X(f)\, e^{-j2\pi f m \Delta t}$$

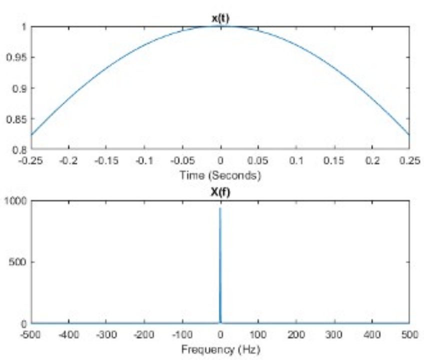
$$H_{eq}(t) = \sum_{n=0}^{\infty} \underline{(-\alpha)^n}\, e^{-j2\pi f n \Delta t}$$

$$a_m = (-\alpha)^m$$

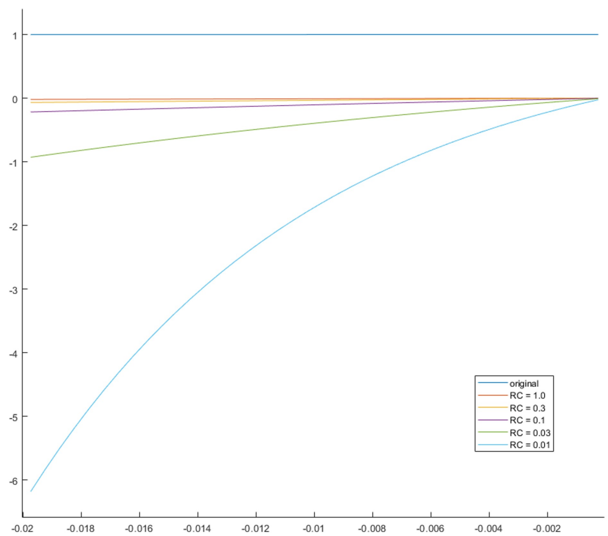
2)



3) a.

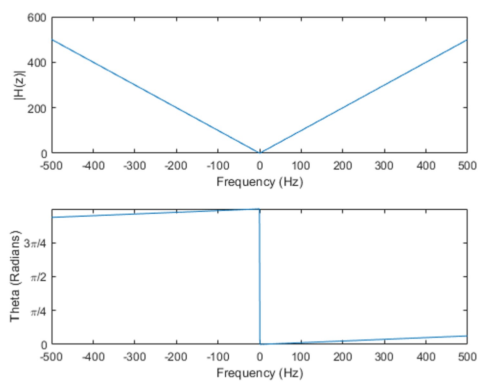


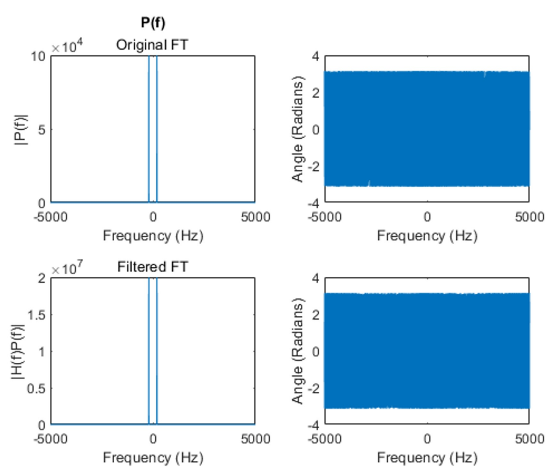
b.

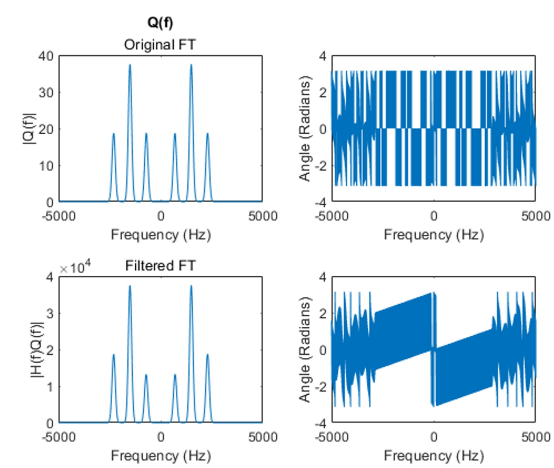


the signal is relatively unchanged at RC = 1.0

c.

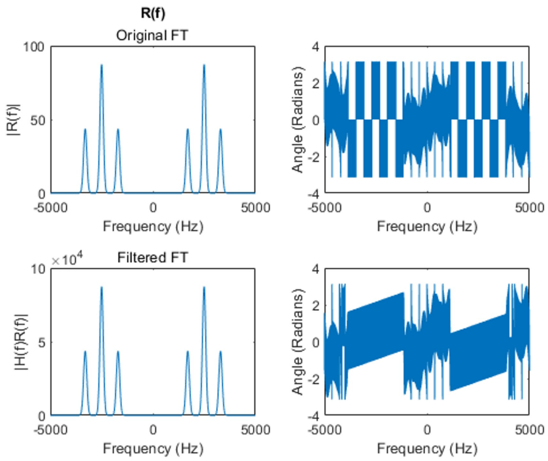


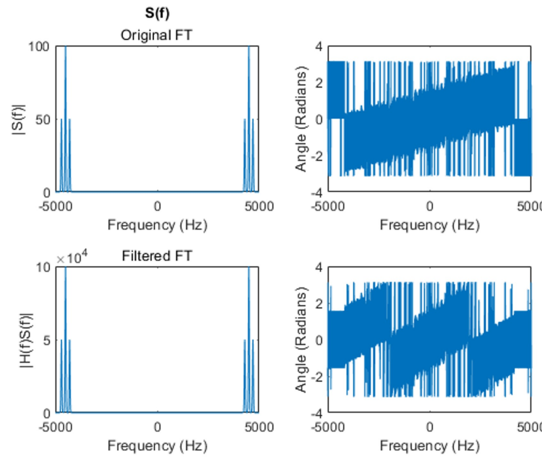




P(f) → magnitude distortion

Q(f) → phase & magnitude distortion





$R(f) \rightarrow$ phase distortion

$S(f) \rightarrow$ phase distortion

4) $M(f) = \Pi(f)\cos(2\pi f)$

$$m(t) = \int_{-0.5}^{0.5} \cos(2\pi f)\, e^{-j2\pi f}\, df$$