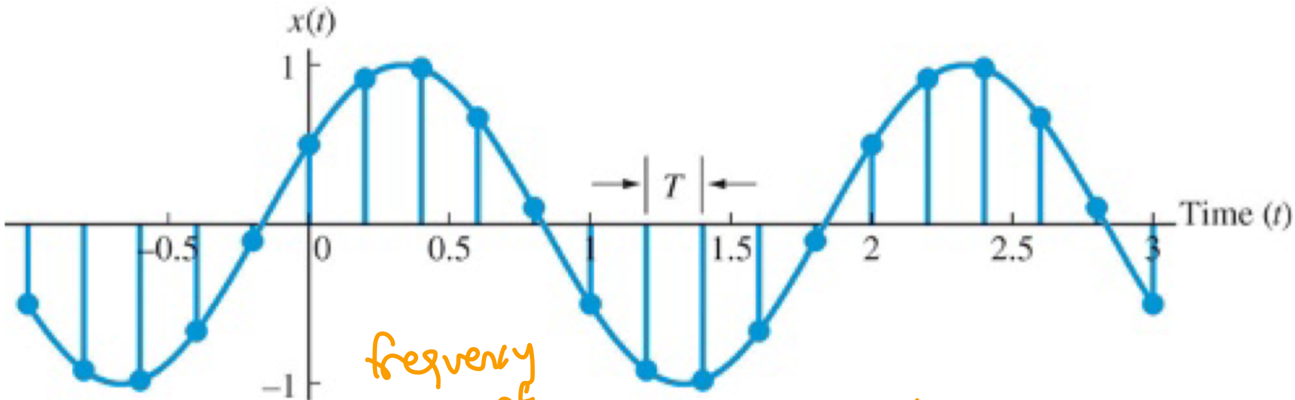
Chap 4.1

Discrete-time Sinusoids

In DSP, we usually consider discrete-time sinusoidal signal as being obtained by sampling the continuous-time sinusoid at equally spaced points $t = nT$

T = sampling period (sec)

$F_s = \frac{1}{T}$ = sampling frequency $\left(\frac{\text{sample}}{\text{sec}}\right)$



frequency of continuous-time signal x(t)

$$x(t) = A\cos(2\pi F t + \theta) = A\cos(\Omega t + \theta), \quad t = nT \Rightarrow$$

$$x[n] \triangleq A\cos(2\pi F nT + \phi) = A\cos(2\pi f n + \phi)$$

$$= A\cos(\Omega nT + \phi) = A\cos(\omega n + \phi)$$

A = amplitude

$f \triangleq FT = \frac{F}{F_s}$ = normalized (cyclic) frequency

$\omega \triangleq \Omega T = 2\pi f$ = normalized angular frequency

ϕ = phase (radians)

There is a close relationship between

<u>continuous-time frequency</u>	<u>discrete-time normalized frequency</u>
F (Hertz $\left(\frac{\text{cycles}}{\text{sec}}\right)$)	f (unitless, kind of)
$\Omega = 2\pi F$ $\left(\frac{\text{radian}}{\text{sec}}\right)$	$\omega = 2\pi f$ (radian)

connected by the sampling period, T or the sampling frequency $F_s = \frac{1}{T_s}$

Since $f = \frac{F}{F_s}$ $\quad \left(\frac{\text{cycles}}{\text{sec}}\right)\left(\frac{\text{sec}}{\text{sample}}\right)$

f can have the unit of $\frac{\text{cycles}}{\text{sample}}$ ... which is hard to interpret IMO

<u>Same as continuous-time system:</u>

complex exponential sequence in discrete-time $\longrightarrow$ $\boxed{\begin{array}{c}\text{LTI}\\ h[n]\end{array}}$ $\longrightarrow$ output is also a complex exponential sequence

$x[n] = e^{j\omega n}$

$y[n] = H(e^{j\omega})e^{j\omega n}$, $\quad -\infty < n < \infty$

Discrete-time sinusoid has two types of periodicity

1) periodicity in frequency

The sequence $x[n] = A\cos(\omega n + \theta)$ is periodic in ω with period 2π (f is periodic with period 1)

$$A\cos(\omega_0 n + \theta) = A\cos(\omega(n + 2\pi) + \theta)$$
$$= A\cos(\omega n + 2\pi\omega + \theta)$$

The fundamental range can be $0 \leq \omega < 2\pi$
or
$-\pi \leq \omega < \pi$

2) periodicity in time

The sequence $x[n] = A\cos(\omega n + \theta) = A\cos(2\pi f n + \theta)$ is periodic in n if $f = \frac{k}{N}$ (i.e. f is a rational number)

$x[n] = x[n+N]$ $\Rightarrow$ periodic in n with period N

$$x[n+N] = A\cos(\omega(n+N) + \theta)$$
$$= A\cos(\omega n + \omega N + \theta)$$

if $\omega N = 2\pi, 4\pi, \ldots$ $(k2\pi)$

$\omega N = k2\pi$ means $\underbrace{\frac{\omega}{2\pi}}_{f} = \frac{k}{N}$,