	Property	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1.	Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$	At least $R_{x_1} \bigcap R_{x_2}$
2.	Time shifting	$x[n-k]$	$z^{-k}X(z)$	R_x except $z = 0$ or ∞
3.	Scaling	$a^n x[n]$	$X(a^{-1}z)$	$\lvert a\rvert R_x$
4.	Differentiation	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_x
5.	Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
6.	Real-part	$\mathrm{Re}\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	At least R_x
7.	Imaginary part	$\mathrm{Im}\{x[n]\}$	$\frac{1}{2}[X(z) - X^*(z^*)]$	At least R_x
8.	Folding	$x[-n]$	$X(1/z)$	$1/R_x$
9.	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least $R_{x_1} \bigcap R_{x_2}$
10.	Initial-value theorem	$x[n] = 0$ for $n < 0$	$x[0] = \lim_{z\to\infty} X(z)$	

Table 3.1 Some common z-transform pairs

	Sequence $x[n]$	z-Transform $X(z)$	ROC
1.	$\delta[n]$	1	All z
2.	$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3.	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4.	$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
5.	$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
6.	$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7.	$(\cos\omega_0 n)u[n]$	$\dfrac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
8.	$(\sin\omega_0 n)u[n]$	$\dfrac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
9.	$(r^n\cos\omega_0 n)u[n]$	$\dfrac{1-(r\cos\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
10.	$(r^n\sin\omega_0 n)u[n]$	$\dfrac{(r\sin\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

Modulation

$$x(t) = m(t)\cos(2\pi f_0 t)$$

$$X(f) = \frac{1}{2}\left(M(f-f_0) + M(f+f_0)\right)$$

Magnitude $\quad |Y(f)| = |X(f)||H(f)|$

Phase $\quad \angle Y(f) = \angle X(f) + \angle H(f)$

Eular's Formula

$$\sin x = \frac{1}{j2}\left(e^{jx} - e^{-jx}\right) \qquad e^{jx} - e^{-jx} = j2\sin(x)$$

$$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right) \qquad e^{jx} + e^{-jx} = 2\cos(x)$$

$$e^{jx} = \cos x + j\sin x$$

$$e^{-jx} = \cos x - j\sin x$$

Sampling $\quad x(t) \longrightarrow x[n]$

- Ideal Sampling : let $x(t)$ be a baseband signal with BW $= B_m$.

then : sampling freq. $T = \frac{1}{f_0}$ ← channel freq where $X(f)$ lies

ideal LPF $\quad h(t) = 2BT\,\mathrm{sinc}(2Bt)$

where $B_m \le B \le f_0/2$

TABLE 3.1
Short Table of Fourier Transforms

	$g(t)$	$G(f)$	
	$e^{-at}u(t)$	$\dfrac{1}{a+j2\pi f}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j2\pi f}$	$a>0$
3	$e^{-a\lvert t\rvert}$	$\dfrac{2a}{a^2+(2\pi f)^2}$	
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j2\pi f)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j2\pi f)^{n+1}}$	
6	$\delta(t)$		
7	1	$\delta(f)$	
8	$e^{j2\pi f_0 t}$	$\delta(f-f_0)$	
9	$\cos 2\pi f_0 t$	$0.5\,[\delta(f+f_0)+\delta(f-f_0)]$	
10	$\sin 2\pi f_0 t$	$j0.5\,[\delta(f+f_0)-\delta(f-f_0)]$	
11	$u(t)$	$\frac{1}{2}\delta(f)+\dfrac{1}{j2\pi f}$	
12	$\mathrm{sgn}\,t$	$\dfrac{2}{j2\pi f}$	
	$\cos 2\pi f_0 t\, u(t)$	$\frac{1}{4}[\delta(f-f_0)+\delta(f+f_0)]+\dfrac{j2\pi f}{(2\pi f_0)^2-(2\pi f)^2}$	
14	$\sin 2\pi f_0 t\, u(t)$	$\frac{1}{4j}[\delta(f-f_0)-\delta(f+f_0)]+\dfrac{2\pi f_0}{(2\pi f_0)^2-(2\pi f)^2}$	
15	$e^{-at}\sin 2\pi f_0 t\, u(t)$	$\dfrac{2\pi f_0}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	
16	$e^{-at}\cos 2\pi f_0 t\, u(t)$	$\dfrac{a+j2\pi f}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	
	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\,(\pi f\tau)$	
18	$2B\,\mathrm{sinc}\,(2\pi Bt)$	$\Pi\left(\dfrac{f}{2B}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\,\mathrm{sinc}^2\left(\dfrac{\pi f\tau}{2}\right)$	
20	$B\,\mathrm{sinc}^2\,(\pi Bt)$	$\Delta\left(\dfrac{f}{2B}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT$	$f_0\sum_{n=-\infty}^{\infty}\delta(f-nf_0)$	
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-2(\sigma\pi f)^2}$	

forward FT

- continuous

$$F(f) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}\,dt$$

-discrete

$$F(e^{j\omega}) = \sum_{n=-\infty}^{\infty} f[n]e^{-j\omega n}$$

inverse FT

- continuous

$$f(t) = \int_{-\infty}^{\infty} F(f)e^{j2\pi ft}\,df$$

-discrete

$$f[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{j\omega})e^{j\omega n}\,d\omega$$

$X(f)$ lies

ideal LPF $\quad h(t) = 2BT\,\mathrm{sinc}(2Bt)$

where $B_m \le B \le f_0/2$

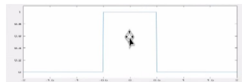
sinusoid $\sin(x)$ / $\cos(x)$

CSFT of a sinusoid → impulse @ that frequency

pulse signal $\Pi(t)$

$$x(t) = \Pi\left(\frac{t}{2T}\right) = u(t-T) - u(t+T)$$

$$X(f) = 2T\,\mathrm{sinc}(2fT)$$



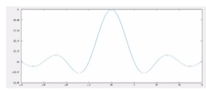
$T = 0.5 \rightarrow \Pi(t) = u(t-0.5) - u(t+0.5)$

CSFT $\{\Pi(t)\} = \mathrm{sinc}(f)$

sinc signal $\mathrm{sinc}(t)$

$$x(t) = \mathrm{sinc}\left(\frac{t}{T}\right) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\pi t/T}$$

$X(f) =$ a pulse signal



$\mathrm{sinc}(0) = 1$

impulse signal $\delta(t)$

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otw} \end{cases} \qquad FT\{\delta(t)\} = 1$$

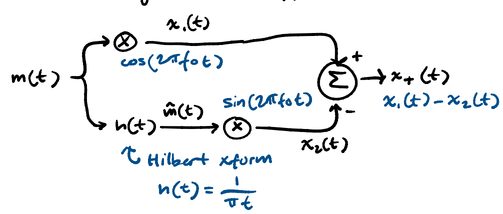
impulse train

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$$X(f) = \sum_{n=-\infty}^{\infty} \frac{1}{T}\,\delta(f - nf_0), \quad f_0 = \frac{1}{T}$$

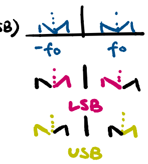
- FT of an impulse train is an impulse train

SSB-SC → Single sideband suppressed carrier



DSB-SC → Double Sideband Suppressed Carrier

- transmits in lower and upper sidebands (LSB/USB)
  - potentially wasted sidebands



$m(t)$: message signal

$x(t) = Am(t)\cos(2\pi f_0 t)$ $\quad$ modulates $m(t)$ to lie at f_0

QAM

$x(t)$ carries info of two message signals $m_1(t)$, $m_2(t)$

$$x(t) = m_1(t)\cos(2\pi f_0 t) + m_2(t)\sin(2\pi f_0 t)$$

in phase term $\qquad$ out-of-phase term