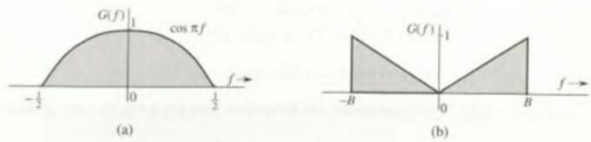
PS1

3.1-6 From definition (3.9b), find the inverse Fourier transforms of the spectra shown in Fig. P3.1-6.

Figure P.3.1-6



a. $G(f) = \cos \pi f$

$$g(t) = \int_{-1/2}^{1/2} \cos \pi f \cdot e^{j2\pi f t}\, df$$

$$= \int_{-1/2}^{1/2} \frac{1}{2}\left(e^{j\pi f} + e^{-j\pi f}\right) e^{j2\pi f t}\, df$$

$$= \frac{1}{2}\int_{-1/2}^{1/2} e^{j\pi f} e^{j\pi f 2t} + e^{-j\pi f} e^{j\pi f 2t}\, df$$

$$= \frac{1}{2}\int_{-1/2}^{1/2} e^{(j\pi + j2\pi t)f} + e^{(-j\pi + j2\pi t)f}\, df$$

$$= \frac{1}{2}\left(\frac{2\cos(\pi t)}{\pi(2t+1)} - \frac{2\cos(\pi t)}{\pi(2t-1)}\right)$$

$$g(t) = \frac{\cos(\pi t)}{\pi(2t+1)} - \frac{\cos(\pi t)}{\pi(2t-1)}$$

b. $$g(t) = \int_{-B}^{0} -\frac{f}{B} e^{j2\pi f t}\, df + \int_{0}^{B} \frac{f}{B} e^{j2\pi f t}\, df$$

$$= 2\int_{0}^{B} \frac{f}{B} e^{j2\pi f t}\, df$$

$$= \frac{2}{B}\int_{0}^{B} f e^{j2\pi t f}\, df = \frac{2}{B}\left(\frac{e^{j2\pi t f}(j2\pi t f + 1)}{(j2\pi t)^2}\right)\Bigg|_{0}^{B}$$

$$= \frac{2}{B}\left(\frac{e^{j2\pi B t}(j2\pi B t + 1)}{(j2\pi t)^2} - \frac{j2\pi t B + 1}{(j2\pi t)^2}\right)$$

$$g(t) = \frac{2(j2\pi B t + 1)}{(j2\pi t)^2}\left(e^{j2\pi B t} - 1\right)$$

$$g(t) = \frac{\cdots}{(j2\pi t)^2}\left(e^{\cdots} - 1\right)$$

3.4-3 For a linear system with transfer function

$$H(f) = \frac{1}{1 + j2\pi f}$$

find the output signal $y(t)$ of this system when the input signal is given by

$$x(t) = 1 + 2\delta(t - t_0) - \cos(\omega_0 t) + \sum_{i=1}^{n} A_i e^{-a_i(t - t_i)} u(t - t_i) \qquad a_i > 0$$

$$x(t) = 1 + 2\delta(t - t_0) - \cos(2\pi f_0 t) + \sum_{i=1}^{n} A_i e^{-a_i(t - t_i)} u(t - t_i)$$

$$X(f) = \delta(f) + 2e^{-j2\pi f t_0} - \frac{1}{2}\left(\delta(f + f_0) + \delta(f - f_0)\right) + \sum_{i=1}^{n} A_i \left(\frac{1}{a_i + j2\pi f}\right) e^{-j2\pi f t_i}$$

$$H(f) = \frac{1}{1 + j2\pi f}$$

$$Y(f) = X(f)H(f) = \left(\delta(f) + 2e^{-j2\pi f t_0} - \frac{1}{2}\left(\delta(f + f_0) + \delta(f - f_0)\right) + \sum_{i=1}^{n} A_i \left(\frac{1}{a_i + j2\pi f}\right) e^{-j2\pi f t_i}\right) \frac{1}{1 + j2\pi f}$$

$$y(t) = \int_{-\infty}^{\infty} \frac{1}{1 + j2\pi f}\left[\delta(t) + 2e^{-j2\pi f t_0} - \frac{1}{2}\left(\delta(f + f_0) + \delta(f - f_0)\right) + \sum_{i=1}^{n} A_i \left(\frac{1}{a_i + j2\pi f}\right) e^{-j2\pi f t_i}\right] df$$

3.7-1 Use Parseval's theorem to solve the following integral:

$$\int_{-\infty}^{\infty} \mathrm{sinc}^4(kt)\, dt$$

let $f(t) = g(t) = \mathrm{sinc}^2(kt)$

$$\mathrm{sinc}^2(kt) \Rightarrow \frac{\pi}{k} \Delta\left(\frac{\pi f}{2k}\right)$$

$$\int_{-\infty}^{\infty} \mathrm{sinc}^4(kt)\, dt = \int_{-\infty}^{\infty} \left(\frac{\pi}{k} \Delta\left(\frac{\pi f}{2k}\right)\right)^2 df$$

$$= \left(\frac{\pi}{k}\right)^2 \int_{-\infty}^{\infty} \left(\Delta\left(\frac{\pi f}{2k}\right)\right)^2 df$$

$$\int_{-\infty}^{\infty} \operatorname{sinc}^4(kt)\,dt = \int_{-\infty}^{\infty} \left(\frac{\pi}{k} \Delta\left(\frac{\pi f}{2k}\right)\right)^2 df$$

$$= \left(\frac{\pi}{k}\right)^2 \int_{-\infty}^{\infty} \left(\Delta\left(\frac{\pi f}{2k}\right)\right)^2 df$$

3.