HW Ch. #1

Solution

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Prob. 1. (a) $h(x;x') = \delta(x - Mx')$

10 (b) $g(x) = \frac{1}{|M|} f(\frac{x}{M})$ $\left(g(x) = \int_{-\infty}^{\infty} f(x')\delta(x - Mx')dx' \underbrace{=}_{\text{by scaling property of } \delta(x)} \frac{1}{|M|}\int_{-\infty}^{\infty} f(x')\delta(x' - \frac{1}{M}x)dx'\right.$

$\left. = \frac{1}{|M|} f(\frac{1}{M}x) \text{ by sifting property of } \delta(x)\right)$.

10 (c) $f(x) \to \boxed{S} \xrightarrow{g(x)} \boxed{\text{shift } x_0} \to g_1(x) = g(x - x_0) = \frac{1}{|M|} f(\frac{x - x_0}{M})$

$f(x) \to \boxed{\text{shift } x_0} \xrightarrow{} \boxed{S} \to g_2(x) = \frac{1}{|M|} f(\frac{x}{M} - x_0)$

$f_2(x) = f(x - x_0)$

$g_1(x) \neq g_2(x)$ ∴ Shift ~~In~~variant //

Prob 2. (a) $CTFT\{g(t-a)\} = \int_{-\infty}^{\infty} g(t-a) e^{-j2\pi ft} dt$

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$= \int_{-\infty}^{\infty} g(t') e^{-j2\pi f(t - t_0)} dt'$ by $dt' = t - a$.

$= e^{-j2\pi fa} \int_{-\infty}^{\infty} g(t') e^{-j2\pi ft'} dt'$

$= e^{-j2\pi fa} G(f)$

$(x' = \frac{x}{a}, y' = \frac{y}{b})$

10 (b) when $a > 0$, $b > 0$, $CSFT\{g(\frac{x}{a}, \frac{y}{b})\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x', y') e^{j2\pi(uax' + vby')} a\,dx'\,b\,dy'$

$= ab\, G(au, bv)$

when $a < 0$, $b > 0$, $CSFT\{g(\frac{x}{a}, \frac{y}{b})\} = \int_{-\infty}^{\infty}\int_{\infty}^{-\infty} g(x', y') e^{j2\pi(uax' + vby')} a\,dx'\,b\,dy'$

$= -ab\, G(au, bv)$

when $a > 0$, $b < 0$, $CSFT\{g(\frac{x}{a}, \frac{y}{b})\} = -ab\, G(au, bv)$

when $a < 0$, $b < 0$, $CSFT\{g(\frac{x}{a}, \frac{y}{b})\} = ab\, G(au, bv)$

$CSFT\{g(\frac{x}{a}, \frac{y}{b})\}$ ∴ $= |ab|\, G(au, bv)$

(c) $CSFT\{g(A\begin{bmatrix} x \\ y \end{bmatrix})\} = \int_{\mathbb{R}^2} g(Ar) e^{-j2\pi r^T f} dr$.

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by $r = \begin{bmatrix} x \\ y \end{bmatrix}$ & $f = \begin{bmatrix} u \\ v \end{bmatrix}$

$= \int_{\mathbb{R}^2} g(r') e^{-j2\pi (A^{-1}r')^T f} |\det A^{-1}|\, dr'$

by $r' = Ar$, $r = A^{-1}r'$

$= \int_{\mathbb{R}^2} g(r') e^{-j2\pi (A^{-1}r')^T f} \cdot |\det(A)|^{-1} dr'$

by $|\det A^{-1}| = |\det(A)^{-1}| = |\det(A)|^{-1}$

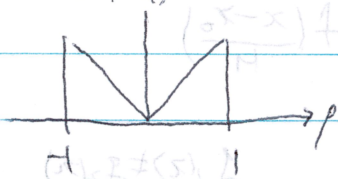
$= |\det A|^{-1} \int_{\mathbb{R}^2} g(r') e^{-j2\pi (r')^T (A^{-1})^T f} dr'$

$= |\det A|^{-1} G((A^{-1})^T f) = |\det A|^{-1} G((A^{-1})^T \begin{bmatrix} u \\ v \end{bmatrix})$. //

Prob 3. (a) $H(\rho) = |\rho|$.

(b) $H(\rho)$ in (a) is not bandlimited (i.e., not square integrable), so not realistic.

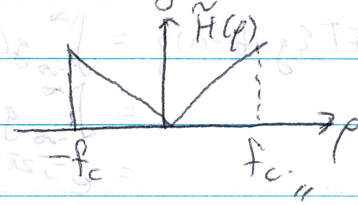
(c) $\tilde{H}(\rho)$



It is a realistic filter.

However, it can soften image, and can't change band limiting frequency...

(Using the following filter is recommended.



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