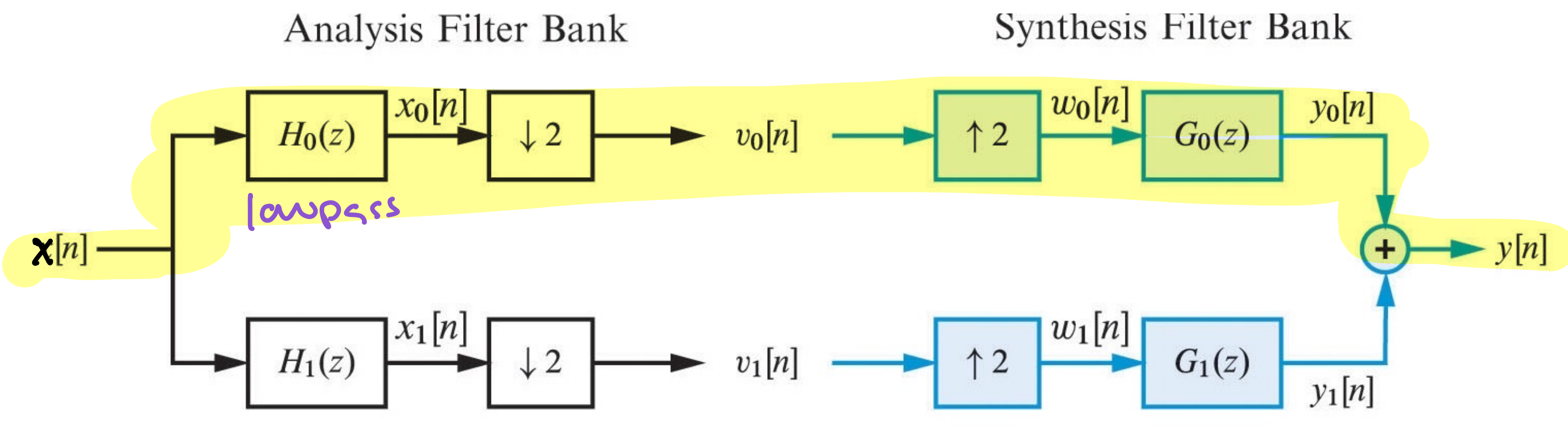
Chap 12.4.3



We want to design a perfect reconstruction two band filter bank

We need to design

$H_0(z)$, $H_1(z)$, $G_0(z)$, $G_1(z)$

Frequency response $H_0(e^{jw})$, $H_1(e^{jw})$, $G_0(e^{jw})$, $G_1(e^{jw})$

Impulse response $h_0[n]$, $h_1[n]$, $g_0[n]$, $g_1[n]$

We can choose to design either FIR or IIR filters. Because we don't want to have to worry about stability, we will choose a design method where all the filters are FIR

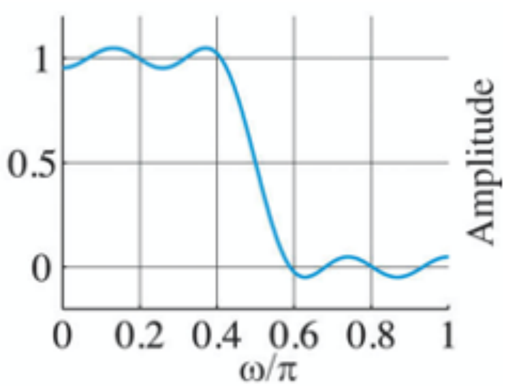
We will use design conjugate quadrature filters (CQFs)

$H(e^{jw})$ and $G(e^{jw})$ must be odd-ordered filters !!

Recall $R_0(z) = z^{n_0} \underbrace{H_0(z)}_{\text{M order filter, M is odd}} \underbrace{G_0(z)}_{\text{M order filter, M is odd}}$

$R_0(z)$ has order $2M$

$R_0(e^{jw})$ is a half band filter, meaning that it is a lowpass filter whose cutoff frequency is 0.5π



Algorithm

1. Design a lowpass zero-phase half-band FIR filter $R_0(z)$ of order $2M$, where the number M *must* be an *odd* integer

2. If the minimum value $\delta_{\min}$ of the real and even function $R_0(e^{j\omega})$ is negative, form a nonnegative function as
$$R_+(e^{j\omega}) = R_0(e^{j\omega}) + |\delta_{\min}| \geq 0$$
This is equivalent to adding the value $|\delta_{\min}|$ to the sample $r_0[0]$, that is,
$$r_+[n] = r_0[n] + |\delta_{\min}|\, \delta[n]$$

3. Scale $R_+(z)$ so that the frequency response is equal to $1/2$ at $\omega = \pi/2$,
$$R(z) = \frac{1/2}{1/2 + |\delta_{\min}|} R_+(z)$$

4. Determine the minimum-phase filter $H(z)$ by solving the spectral factorization problem $R(z) = H(z)H(z^{-1})$

This minimum-phase filter $H(z)$ can then be used to find $H_0(z)$, $H_1(z)$, $G_0(z)$, $G_1(z)$ with the following mapping

$$h_0[n] = h[n] \xleftrightarrow{\text{DTFT}} H_0(e^{j\omega}) = H(e^{j\omega}),$$
$$h_1[n] = (-1)^n h[M-n] \xleftrightarrow{\text{DTFT}} H_0(e^{j\omega}) = -H(e^{-j\omega})e^{-j\omega M},$$
$$g_0[n] = 2h[M-n] \xleftrightarrow{\text{DTFT}} G_0(e^{j\omega}) = 2H(e^{-j\omega})e^{-j\omega M},$$
$$g_1[n] = -2(-1)^n h[n] \xleftrightarrow{\text{DTFT}} G_1(e^{j\omega}) = -2H(e^{-j\omega}).$$

How to do spectral factorization:

We want to find $H(z)$ such that

$$R(z) = H(z)H(z^{-1})$$

If $H(z)$ has a zero at z_k, $H(z^{-1})$ has a zero at $\frac{1}{z_k^*}$ ← complex conjugate

We only work with FIR filters so we don't have to worry about poles

1) Find the poles and zeros of $R(z)$

2) Keep all the zeros inside the unit circle and only one zero from each pair of zeros on the unit circle

Then you can use the function zp2tf (with gain $k=1$) to get the system function $H(z)$