HW Ch. 3

Solution.

Prob 1. a) (10) $\left\| \left[\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y} \right] \right\| > T$ & $\frac{\partial^2 f(x,y)}{\partial x} + \frac{\partial^2 f(x,y)}{\partial y} = 0$.

b) (10) Remind that $\left. \frac{df(t)}{dt} \right|_{t=n\Delta} \approx \frac{f[n] - f[n-1]}{\Delta}$

$$\frac{\partial f(x,y)}{\partial x} \approx s[m+1,n] - s[m,n] \qquad \begin{bmatrix} \boxed{-1} & 1 \\ 0 & 0 \end{bmatrix}$$

or

$$\frac{\partial f(x,y)}{\partial y} \approx s[m,n+1] - s[m,n] \qquad \begin{bmatrix} \boxed{-1} & 0 \\ 1 & 0 \end{bmatrix}.$$

c) (10) $\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} \approx \nabla^2 f(x,y) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & \boxed{-4} & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\frac{\partial^2 f(x,y)}{\partial x^2} \approx \begin{bmatrix} 0 & 0 & 0 \\ 1 & \boxed{-2} & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \frac{\partial^2 f(x,y)}{\partial y^2} \approx \begin{bmatrix} 0 & 1 & 0 \\ 0 & \boxed{-2} & 0 \\ 0 & 1 & 0 \end{bmatrix}$

d) (10) $(s[m+1,n] - s[m,n])^2 + (s[m,n+1] - s[m,n])^2 > T^2$

$g[m,n]\, g[m+1,n] < 0$ or $g[m,n]\, g[m,n+1] < 0$,

where $g[m,n] \triangleq -4\delta[m,n] + \delta[m-1,n] + \delta[m,n-1] + \delta[m+1,n] + \delta[m,n+1]$.

e) (10) As T increases, probability of detection goes down;
probability of false alarm goes down

As T decreases, probability of detection goes up;
probability of false alarm goes up.

Prob. 2. (10) a) computer proj.

(15) b) computer proj.

(10) c) computer proj.