Chap 3.4

Properties of the z-transform : (Table 3.2)

Sequence	z-transform	ROC
$x[n]$	$X(z)$	R_x
$x_1[n]$	$X_1(z)$	R_{x_1}
$x_2[n]$	$X_2(z)$	R_{x_2}

1) Linearity

$$a_1 x_1[n] + a_2 x_2[n] \xleftrightarrow{z} a_1 X_1(z) + a_2 X_2(z)$$

$\boxed{\text{ROC contain atleast } R_{x_1} \cap R_{x_2}}$

2) Time-shifting

$$x[n-k] \xleftrightarrow{z} z^{-k} X(z), \quad \boxed{\text{ROC} = R_x \text{ except } z=0 \text{ or } \infty}$$

3) Scaling

$$a^n x[n] \xleftrightarrow{z} X(a^{-1} z), \quad \text{ROC: } |a| R_x$$

4) differentiation

$$n\, x[n] \xleftrightarrow{z} -z \frac{d X(z)}{dz} \qquad \text{ROC: } R_x$$

5) folding

$$x[-n] \xleftrightarrow{z} X\left(\frac{1}{z}\right) \qquad \text{ROC: } \frac{1}{R_x}$$

6) Convolution

$$x_1[n] * x_2[n] \overset{z}{\longleftrightarrow} X_1(z)X_2(z)$$

ROC: atleast $R_{x_1} \cap R_{x_2}$

Ex)

$$x[n] = \begin{cases} 1 & , 0 \le n \le M \\ 0 & , \text{otherwise} \end{cases}$$

on the one hand, we know this is a finite duration sequence so we know

$$X(z) = \sum_{n=0}^{M} x[n] z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + \dots z^{-M},$$

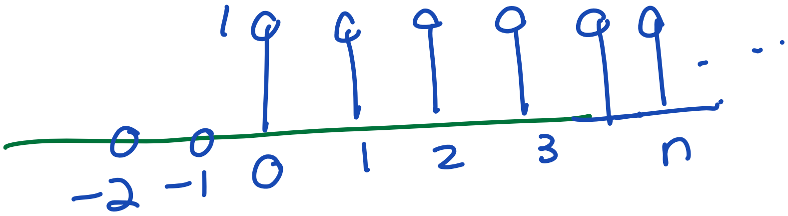
ROC: all of z except $z=0$

Why is $z=0$ a pole??

On the other hand, we know

$x[n] = x_1[n] - x_2[n]$, where

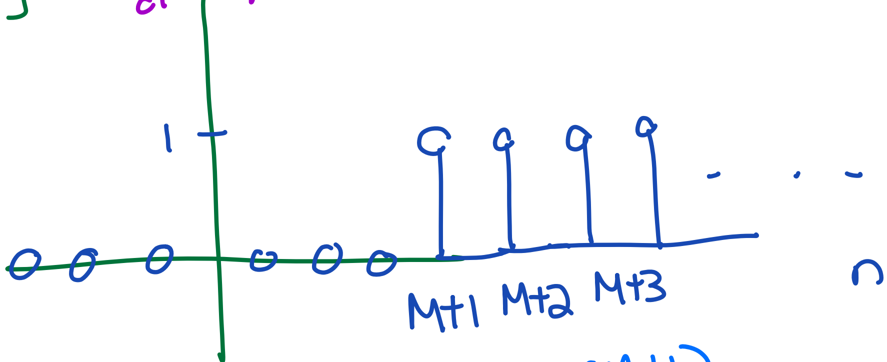
$x_1[n] = u[n]$



$X_1(z) = \frac{1}{1-z^{-1}}$

ROC: $|z|>1$

$x_2[n] = u[n-(M+1)]$ = time shifted version of $x_1[n]$



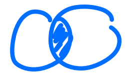
$$X_2(z) = z^{-(M+1)} X_1(z) = \frac{z^{-(M+1)}}{1-z^{-1}}$$

ROC: ROC of $x_1[n]$ except $z=0$ = $|z|>1$

By linearity property of z - transform

$$X(z) = X_1(z) - X_2(z)$$

$$= \frac{1}{1-z^{-1}} - z^{-(M+1)}\left(\frac{1}{1-z^{-1}}\right) = \frac{1-z^{-(M+1)}}{1-z^{-1}}$$

 intersection

ROC of $X(z)$ is atleast ROC of $X_1(z)$ $\cap$ ROC of $X_2(z)$

ROC of $X_1(z)$, we know is $|z| > 1$

ROC of $X_2(z)$ we know is ROC of $x_1[n]$ (since its a time-shifted version of $x_1[n]$) except $z = 0$

ROC of $x[n]$ is atleast

$(|z| > 1) \cap (|z| > 1 \text{ except } z = 0)$

$= |z| > 1$

ex)

$$x[n] = \underbrace{2^n u[n]}_{X_1(z)} + \underbrace{3\left(\tfrac{1}{2}\right)^n u[n]}_{X_2(z)}$$

$$X(z) = \frac{1}{1-2z^{-1}} + \frac{3}{1-\frac{1}{2}z^{-1}}$$

$$= \frac{(1-\frac{1}{2}z^{-1})}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})} + \frac{3(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$\boxed{= \frac{4-\frac{13}{2}z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}}$$

partial fraction expansion

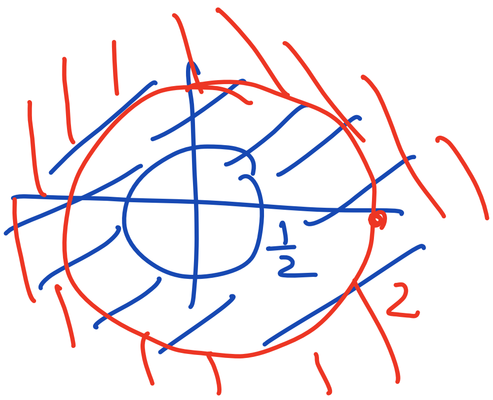
What is the ROC?

ROC of $X_1(z)$: $|z| > 2$

ROC of $X_2(z)$: $|z| > \frac{1}{2}$

(they're both right-sided functions)

ROC of $X(z) = \{|z| > 2\} \cap \{|z| > \frac{1}{2}\}$



$X(z)$ have to be convegene for both region =

ROC : $|z| > 2$

	Property	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1.	Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$	At least $R_{x_1} \bigcap R_{x_2}$
2.	Time shifting	$x[n-k]$	$z^{-k}X(z)$	R_x except $z = 0$ or ∞
3.	Scaling	$a^n x[n]$	$X(a^{-1}z)$	$\|a\|R_x$
4.	Differentation	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_x
5.	Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
6.	Real-part	$\text{Re}\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	At least R_x
7.	Imaginary part	$\text{Im}\{x[n]\}$	$\frac{1}{2}[X(z) - X^*(z^*)]$	At least R_x
8.	Folding	$x[-n]$	$X(1/z)$	$1/R_x$
9.	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least $R_{x_1} \bigcap R_{x_2}$
10.	Initial-value theorem	$x[n] = 0$ for $n < 0$	$x[0] = \lim_{z\to\infty} X(z)$	