Chap 4.1

The goal of Fourier analysis of signals is to break up all signals into summations of sinusoidal components

Continuous-time sinusoids

continuous time sinusoidal signal can be represented as a function of time t

$$x(t) = A\cos(2\pi F_0 t + \theta), \quad -\infty < t < \infty$$

A = amplitude

θ = phase in radian

F_0 = frequency. If we assume t is measured in seconds. Then unit of F_0 is Hertz $\left(\frac{\text{cycles}}{\text{sec.}}\right)$
(ex. 300MHz, 2kHz)

T_0 = fundamental period $= \frac{1}{F_0}$, unit is in time (seconds, or minutes)

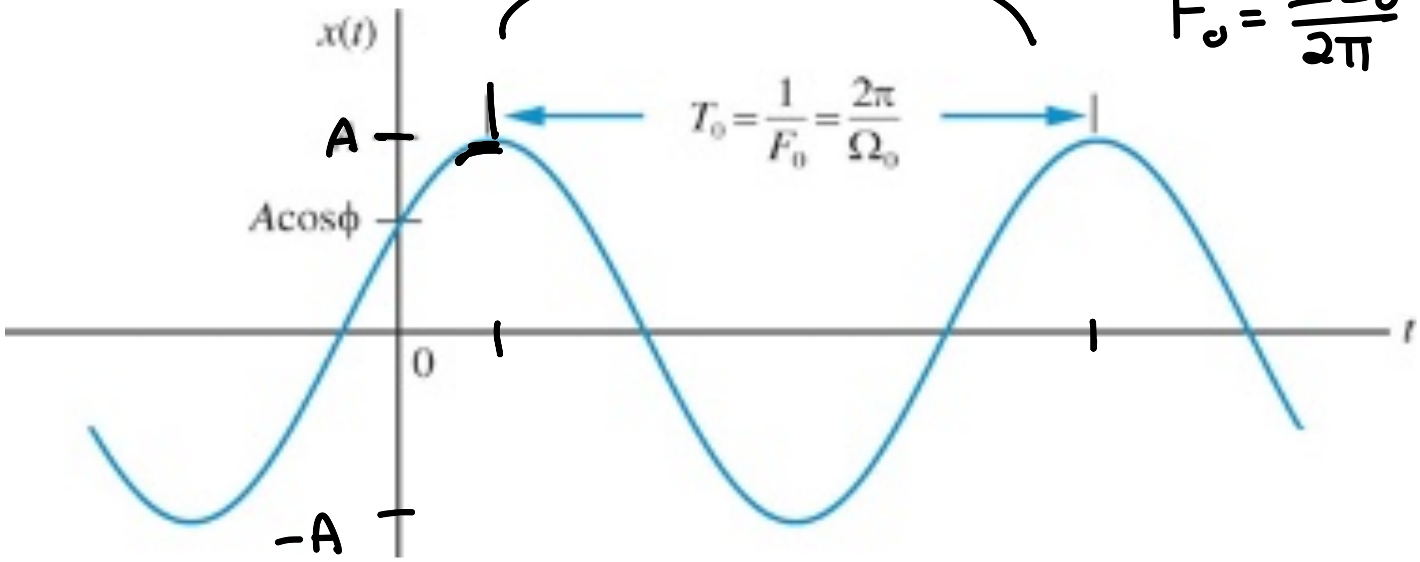
In analysis it is more convenient to use angular frequency instead of frequency

$$\Omega_0 = 2\pi F_0 = \left(\frac{\text{radian}}{\text{sec}}\right)$$

$T_0 = \frac{1}{F_0}$

$\Omega_0 = 2\pi F_0$

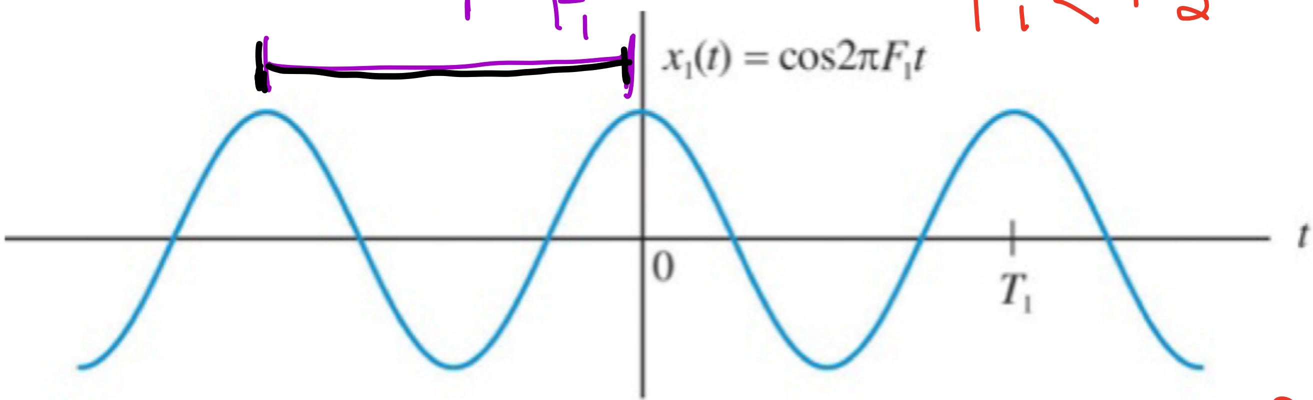
$F_0 = \frac{\Omega_0}{2\pi}$



$T_1 > T_2$

$F_1 < F_2$

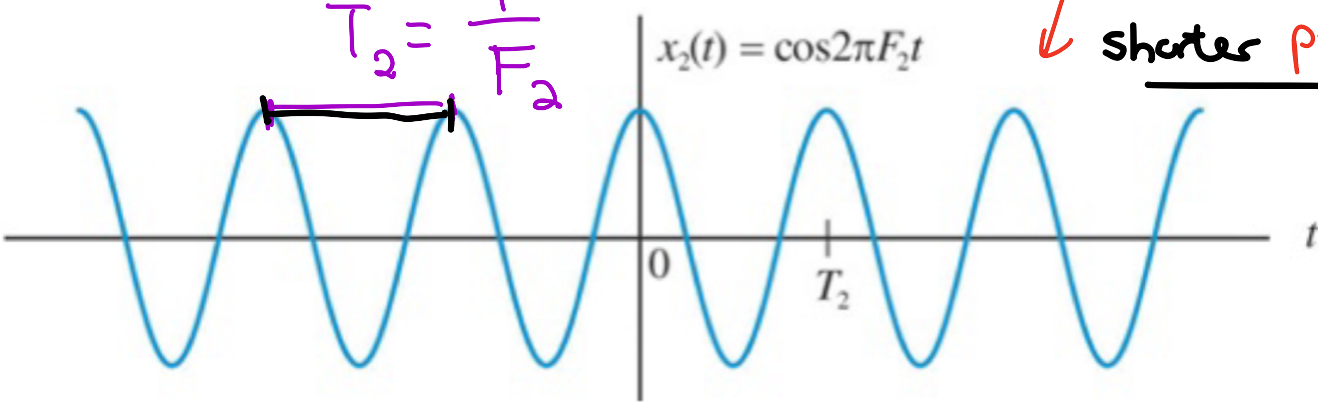
$T_1 = \frac{1}{F_1}$



faster frequency,

shorter period

$T_2 = \frac{1}{F_2}$



Using Euler's idenity, $e^{\pm j w} = \cos w \pm j \sin w$, we can express every sinusoidal signal in term of two complex exponentials with the same frequency:

same frequency

$$A\cos(\Omega_o t + \theta) = \frac{A}{2} e^{j\theta} e^{j\Omega_o t} + \frac{A}{2} e^{-j\theta} e^{-j\Omega_o t}$$

Therefore, we can study the properties of sinusoids by studying the properties of complex exponentials

Complex exponentials are <u>harmonically related</u> if their frequencies are integer multiples of the same fundamental frequency

Set of harmonically related complex exponentials are $e^{jk\Omega_o t}$, $k = 0, \pm 1, \pm 2, \ldots$

$k=1$, $e^{j\Omega_o t}$ is the fundamental harmonic of the set

$e^{jk\Omega_o t}$ is the k^{th} harmonic of the set

- All harmonically related complex exponentials have the same fundamental period T_0

$F_0 = 10\,Hz$

$$x_1(t) = \frac{1}{3}\cos(2\pi F_0 t) - \frac{1}{10}\cos(2\pi 3F_0 t) + \frac{1}{20}\cos(2\pi 5F_0 t)$$

F_0

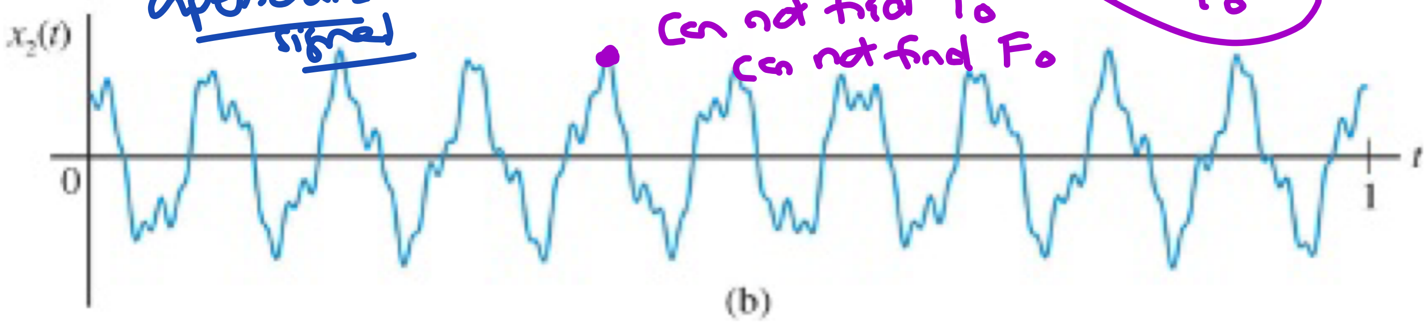
(a)

aperiodic signal

can not find T_0

can not find F_0

$T_0 = \frac{1}{F_0}$



(b)

$$x_2(t) = \frac{1}{3}\cos(2\pi F_0 t) - \frac{1}{10}\cos(2\pi\sqrt{8}F_0 t) + \frac{1}{20}\cos(2\pi\sqrt{51}F_0 t)$$

Complex exponential $\longrightarrow$ LTI system $\longrightarrow$ What is the output $y(t)$?

complex exponential

$x(t) = e^{j\Omega t}$

Complex exponentials are special inputs of LTI system. The output $y(t)$ of LTI system is also a complex exponential at the same frequency as the input

$$y(t) = H(j\Omega)\, e^{j\Omega t}, \quad -\infty < t < \infty$$

There is a special connection between LTI system and (sum of) complex exponential inputs