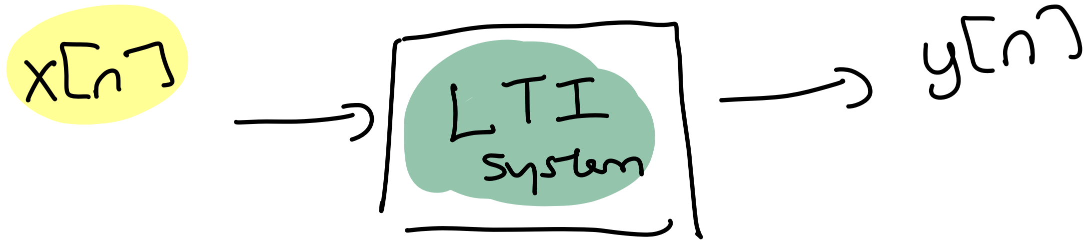
Discrete-time signals passing through a linear, time-invariant (LTI) system



- $x[n]$ is assumed to be a signal sampled from a continuous-time signal $x_c(t)$ with sampling period T, sampling frequency $F_s = \frac{1}{T}$ Hz

- We can analyze $x[n]$ in the frequency domain by taking the discrete-time Fourier transform

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn},$$

which is a special case of the z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

- LTI system can be completely described by

impulse response, $h[n]$ ← time domain

(also called transfer function) System function, $H(z)$

frequency response, $H(e^{jw})$

frequency domain

What makes LTI system LTI?

1) Time-invariance property

if $y[n] = \mathcal{H}\{x[n]\}$ then

$$\mathcal{H}\{x[n-n_0]\} = y[n-n_0]$$

2) Linearity property

$$\mathcal{H}\{a\,x_1[n] + b\,x_2[n]\} = a\,\mathcal{H}\{x_1[n]\} + b\,\mathcal{H}\{x_2[n]\}$$

Additional properties of interests are

1) causality (a system may or may not be causal)

$y[n]$ does not depend on future value like $x[n+1], x[n+2], \ldots$

equivalent, a right-sided input $x[n]=0$, for $n \leq n_0$ result in a right-sided output $y[n]=0$, $n \leq n_0$

2) stability (we always want the system to be stable)

System is stable if $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

LTI systems do not change the fundamental frequencies of inputs

For LTI systems:

$$\text{output, } y[n] = \underbrace{x[n]}_{\text{input}} \underbrace{*}_{\text{convolution}} h[n]$$

In the frequency domain

$$Y(z) = X(z)\,H(z)$$

$$Y(e^{jw}) = X(e^{jw})\,H(e^{jw})$$

so we can find $H(z) = \frac{Y(z)}{X(z)}$ and $H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})}$

We spent some time studying the system function / z-transform of $h[n]$

Z-transform is the formula and the region of convergence (ROC)

ROC: values of z where $H(z) < \infty$

ROC help us determine if a system is stable and causal

1) LTI system is stable if and only if ROC of $H(z)$ includes the unit circle

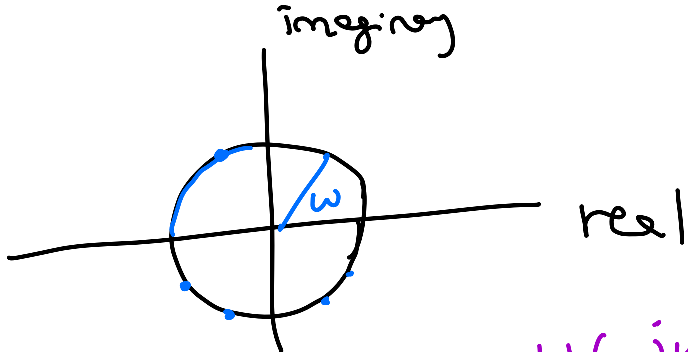
2) LTI with rational $H(z)$ is causal and stable if and only if all the poles are inside the unit circle and ROC is on the exterior of the circle going to infinity

zeros : values of z where $H(z) = 0$

poles : values of z where $H(z) = \infty$

Can ROC contain poles?

Frequency response $(H(e^{jw}))$ is the special case of $H(z)$, where z is restricted to the values along the unit circle



When we look at $H(e^{jw})$, it is often to consider seperate

$$H(e^{jw}) = |H(e^{jw})| \, e^{j \angle H(e^{jw})}$$

magnitude response (amplitude change)

phase response (time shift)

DTFT are functions with 2π periodicity because ω repeats every 2π

LTI systems does not change the fundamental frequencies of inputs

- Magnitude response is an even function
- phase response is an odd function

- what are wrapped and unwrapped phase response?

Filters are LTI systems that we design to have specific magnitude and phase response

filters are specified by transition band, passband ripple, stopband attenuation

It is important to understand how a discrete-time signal is obtained from continuous-time signal

$$x[n] = x_c(nT) \qquad F_s = \frac{1}{T}$$

the normalized frequency of continuous-time signal is

$$f = \frac{F}{F_s}$$

continuous-time frequency

$$\omega = 2\pi f = \frac{2\pi F}{F_s} = \frac{\Omega}{F_s}$$