Terminology on phase

$$e^{j(\omega n+\phi)} = \cos(\omega n+\phi) + j\sin(\omega n+\phi)$$

cartesian form

phasor form

phase shift

$$e^{j(\omega n+\phi)} = e^{j\omega n}\, e^{j\phi}$$

we look at this to get the frequency component ω

we look at this to get the phase shift

phase of phasor form $= \omega n + \phi$

$$e^{-j\omega n_d}$$

phase of phasor form $= -\omega n_d$