Chap 12.1

Case 2, sampling frequency increases by an integer factor

$$T_o = \frac{T}{I}$$

$$x[n] = x_c(nT)$$

$$x_I[n] = x_c(nT_o) = x_c\left(n\frac{T}{I}\right)$$

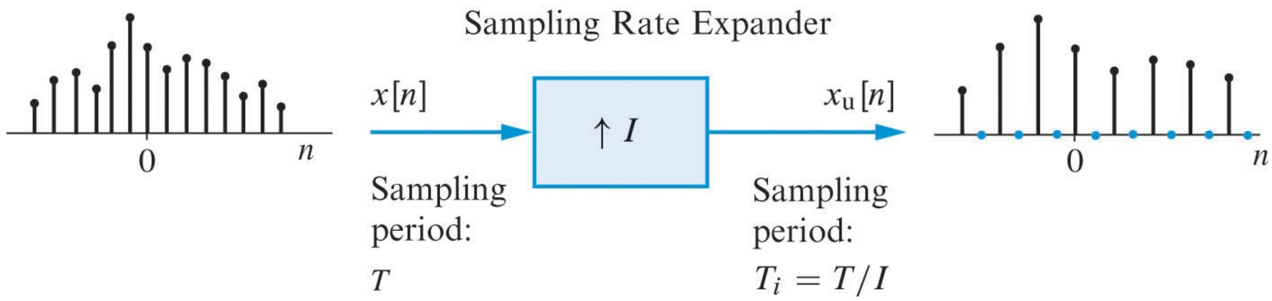
Note that $x_I[n]$ has more entries than $x[n]$

First, let's define an upsampled signal

$$x_u[n] \equiv \begin{cases} x\left[\frac{n}{I}\right], & n \text{ is a multiple of } I \\ 0, & \text{otherwise} \end{cases}$$

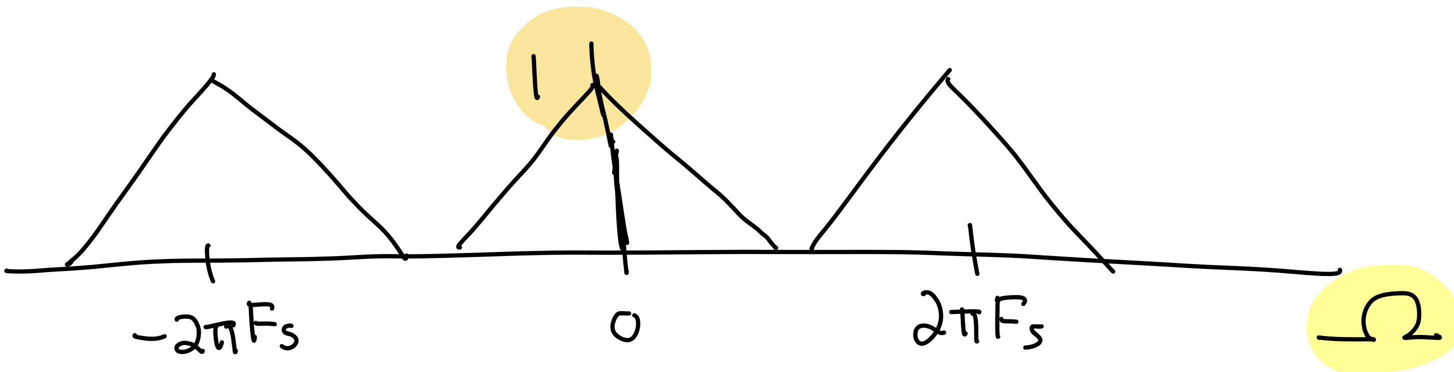
$x_u[n]$ has the same length as $x_I[n]$ except the "missing" values are all 0

$x_u[n]$ is the upsampled signal. Obviously just upsampling is not enough.



What is the relationship in the frequency domain between $x[n]$ and $x_u[n]$

DTFT of $x[n]$

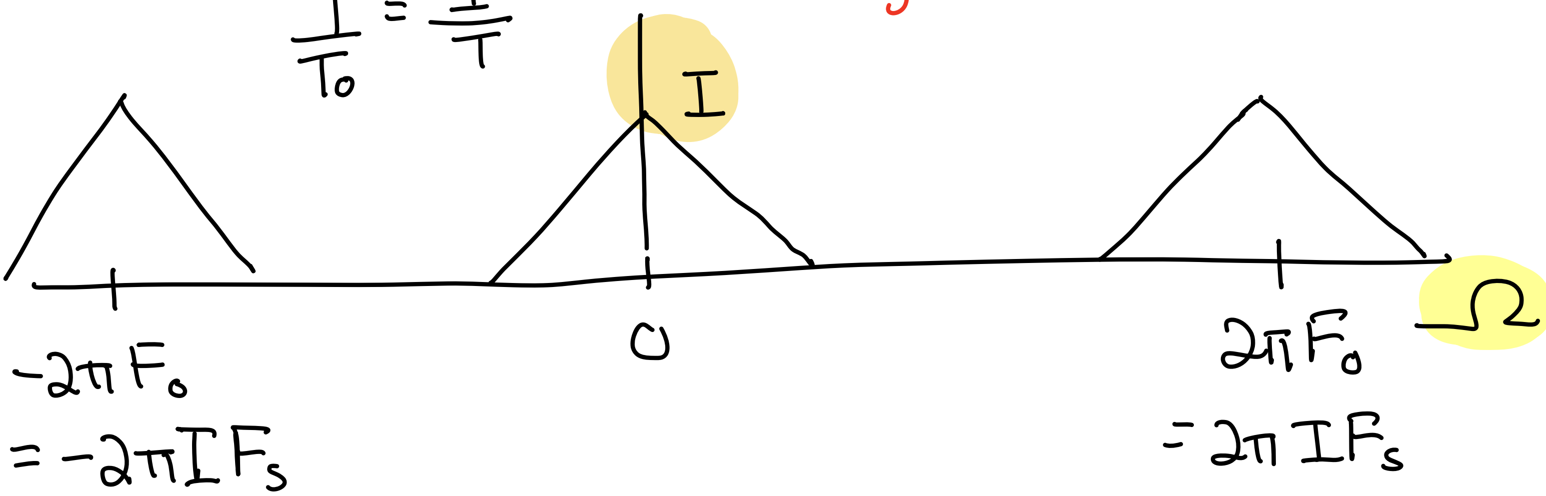


DTFT of $x_u[n]$

$$F_o = \frac{1}{T_o} = \frac{I}{T} = I F_s$$

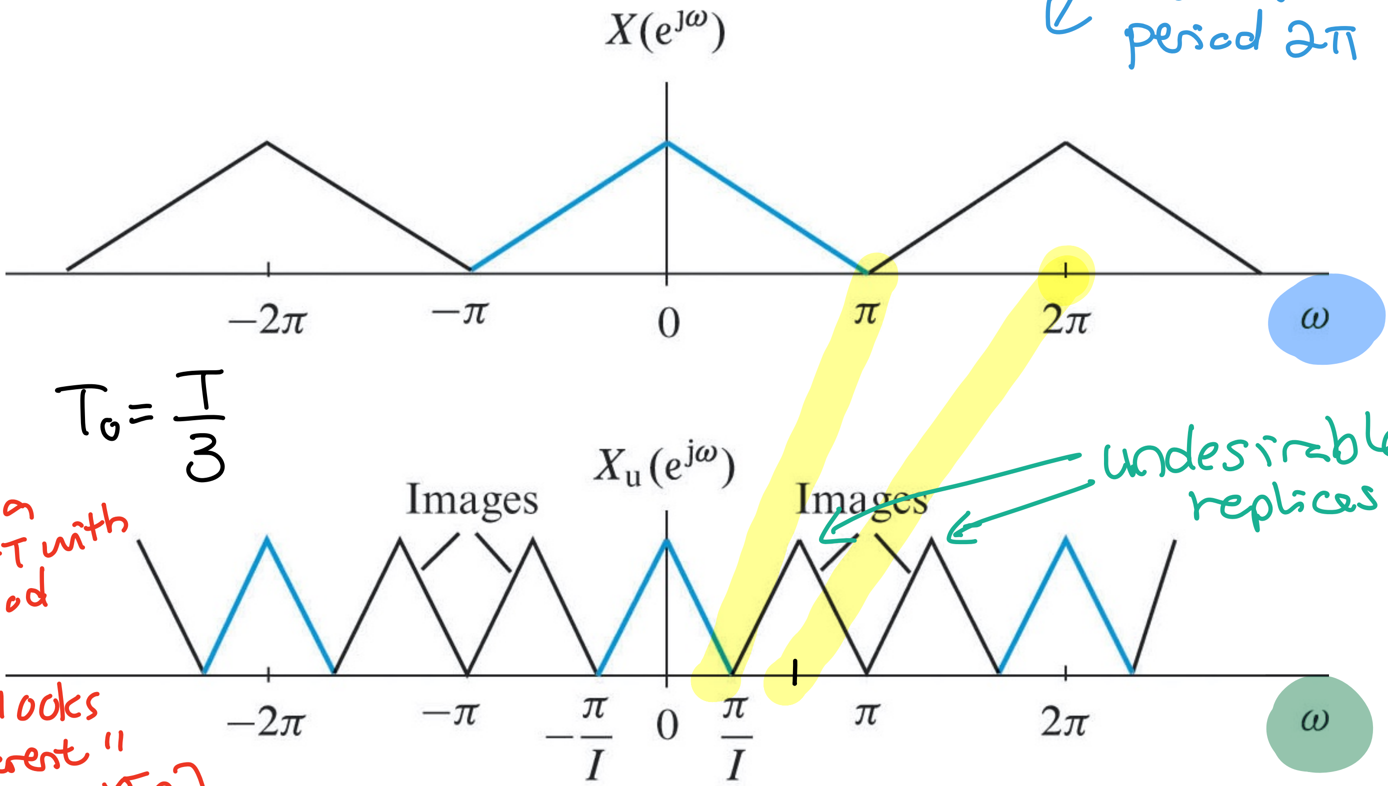
amplitude gain

$$\frac{1}{T_o} = \frac{I}{T}$$



Again, we want to consider how the DTFT are related with a normalized frequency axis

regular DTFT with period 2π



$T_0 = \frac{T}{3}$

undesirable replicas

Still a DTFT with period 2π but looks "different" from $x[n]$

π in $X(e^{jw})$ becomes $\frac{\pi}{I}$ in $X_u(e^{jw})$

2π in $X(e^{jw})$ becomes $\frac{2\pi}{I}$ in $X_u(e^{jw})$

To fill in the missing values in $x_u[n]$, we need to interpolate:

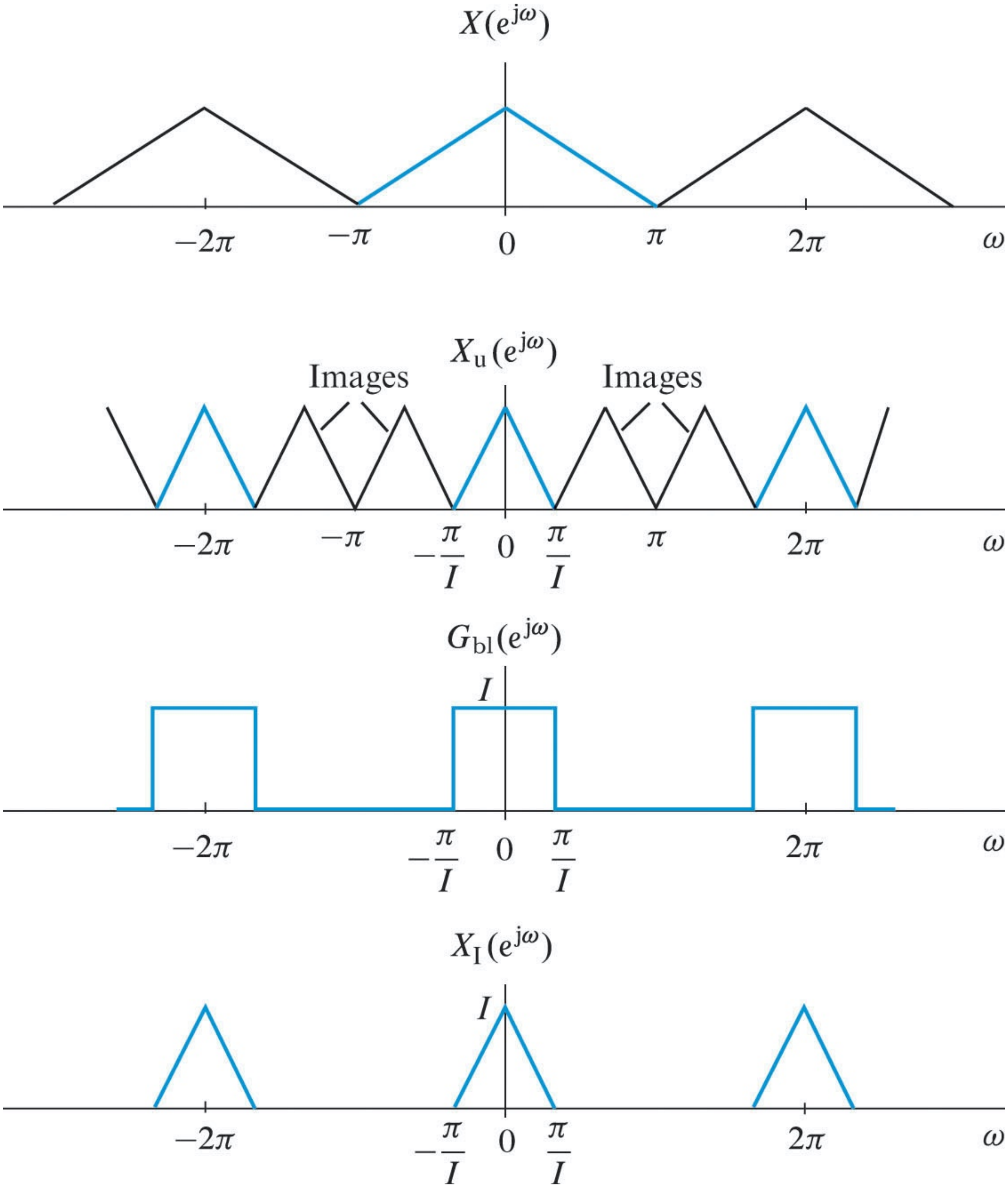
$$x_I[n] = \sum_{k=-\infty}^{\infty} x_u[k]\, g_r[n-k]$$

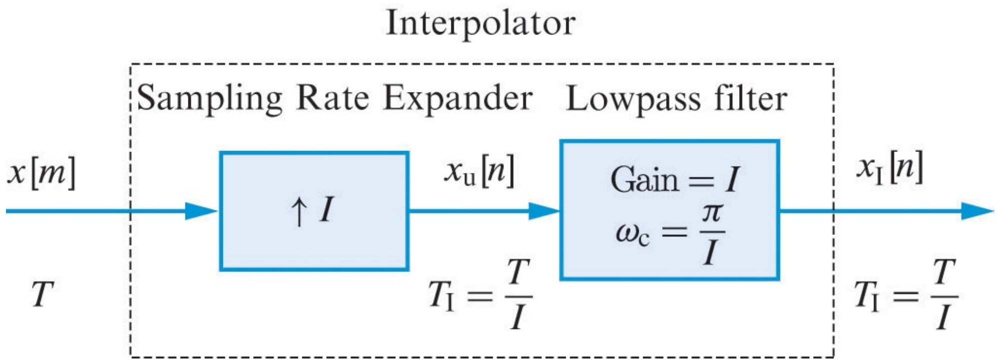
same interpolation function

In the frequency domain, this is

$$X_I(e^{jw}) = X_u(e^{jw})\, G_r(e^{jw})$$

A good interpolation function to choose from is the sinc function, which in the frequency domain is the ideal low pass filter

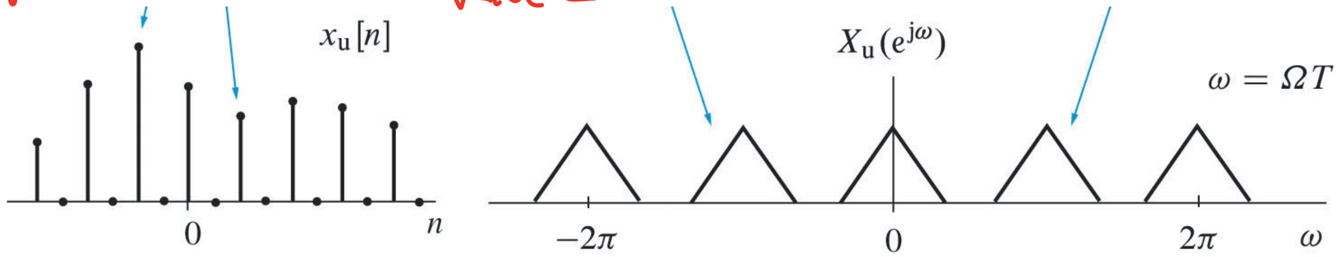


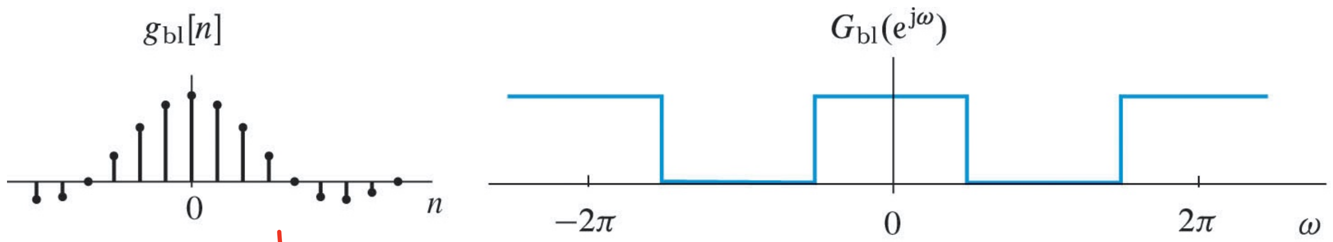


Interpolation is upsampler plus low pass filter

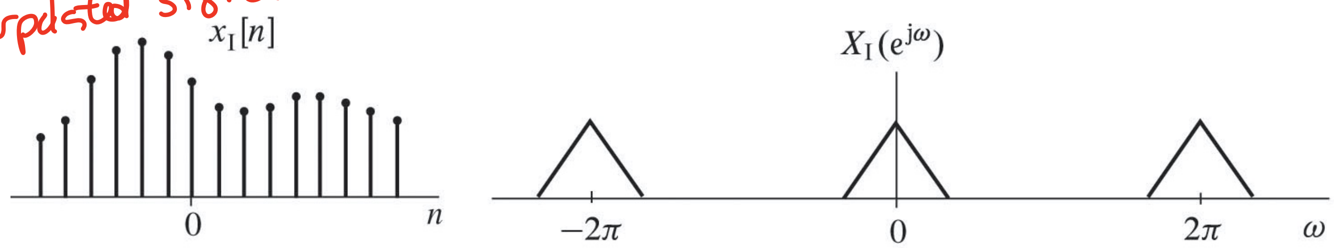
Ex

upsampled signal, missing value are all 0

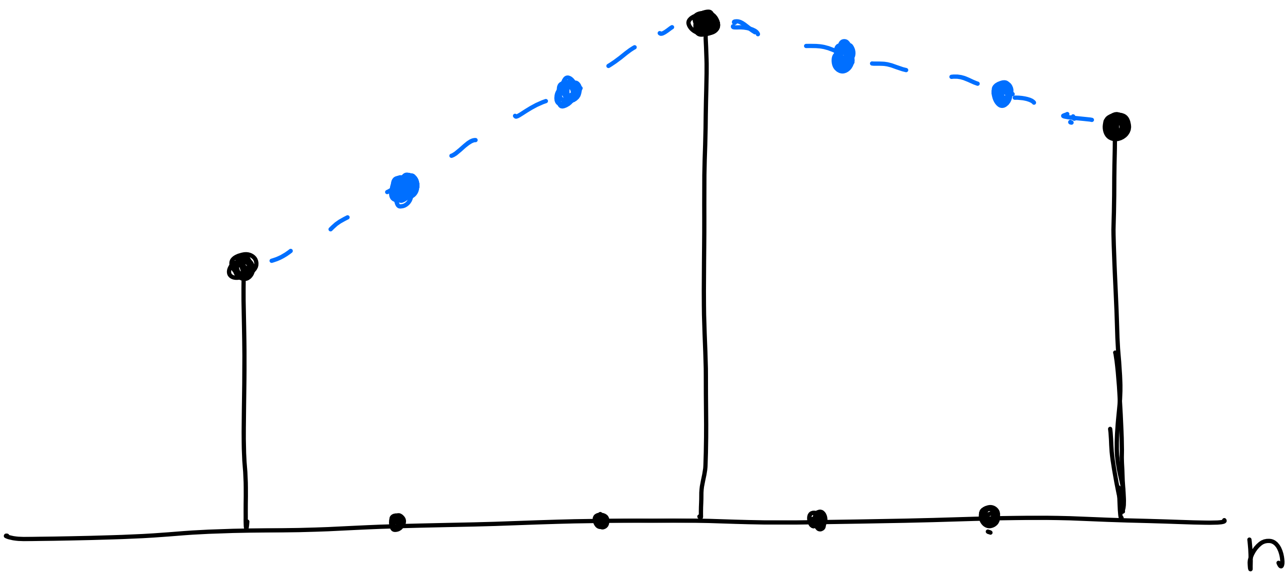




interpolated signal



In the time domain, the most often used interpolator function in practice is linear interpolation

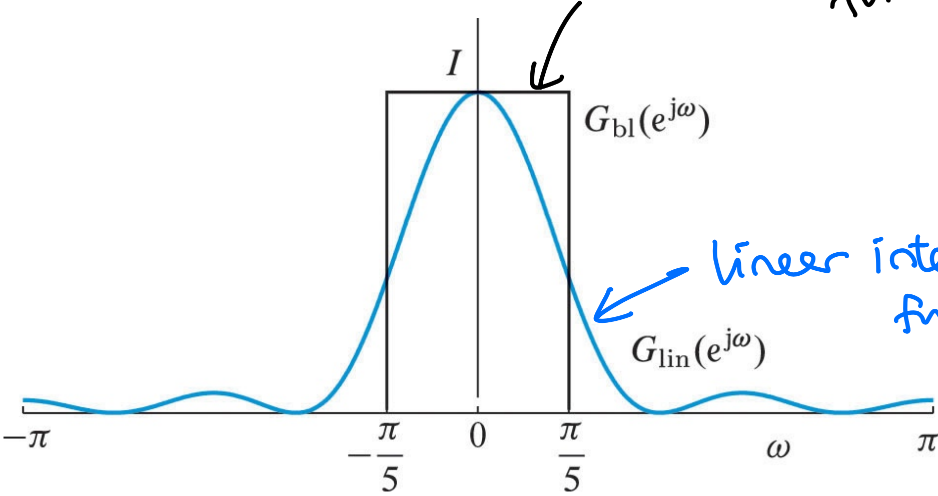


When you do linear interpolation, in the time domain, the interpolation function is

$$g_{lin}[n] = \begin{cases} 1 - \dfrac{|n|}{I}, & -I < n < I \\ 0 & \text{otherwise} \end{cases}$$

In the frequency domain

ideal sinc interpolation function



linear interpolation function