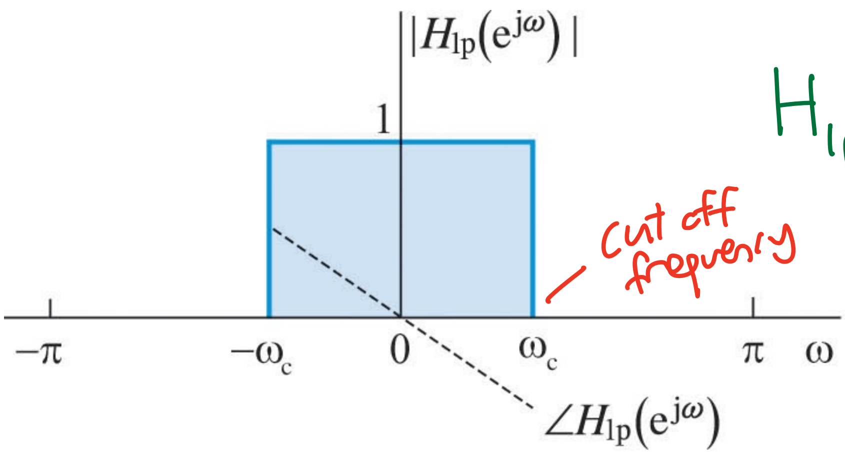
Chap 5.4

Systems that are designed to pass some frequency components without significant distortion while severely or completely eliminating others are known as **frequency-selective filters**

Ideal frequency-selective filter satisfies the requirement for distortionless response over one or more frequency bands

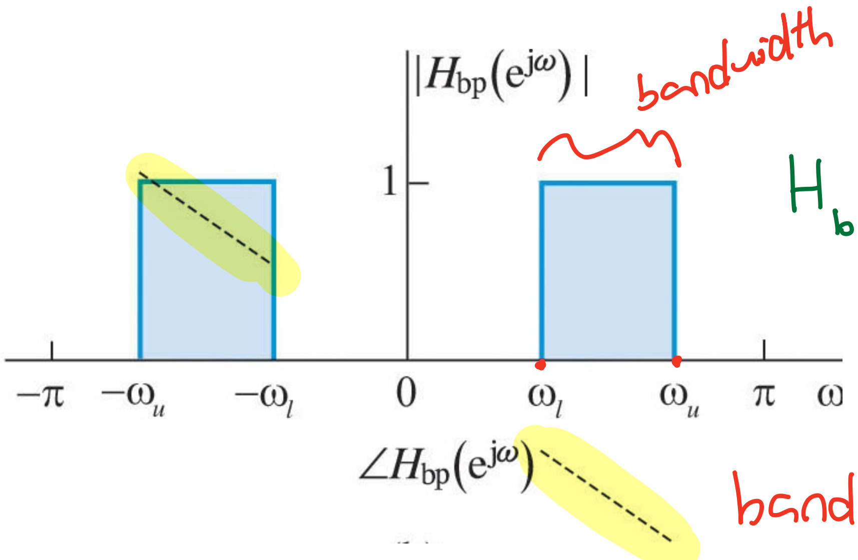
Ideal lowpass filter

$$H_{lp}(e^{j\omega}) = \begin{cases} e^{-j\omega n_0}, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$



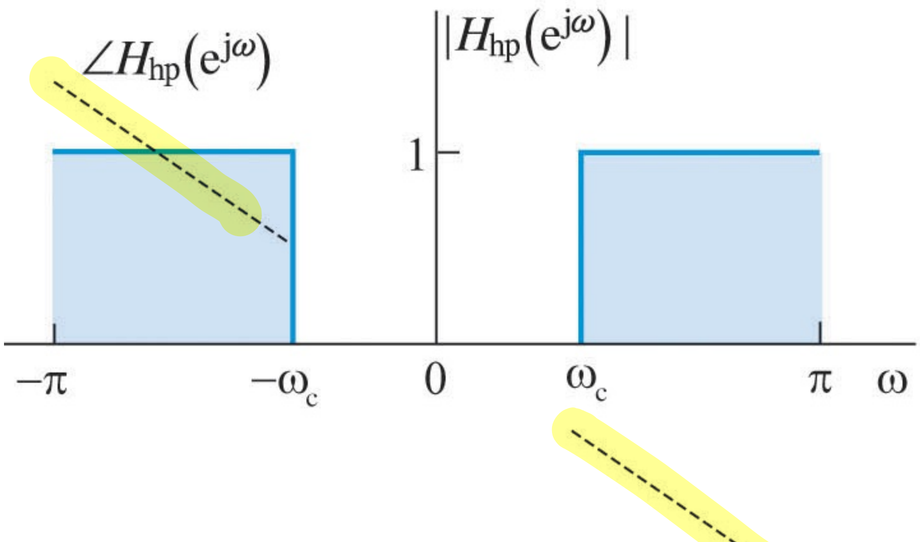
Ideal bandpass filter

$$H_{bp}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & \omega_l \leq |\omega| \leq \omega_u \\ 0, & \text{otherwise} \end{cases}$$



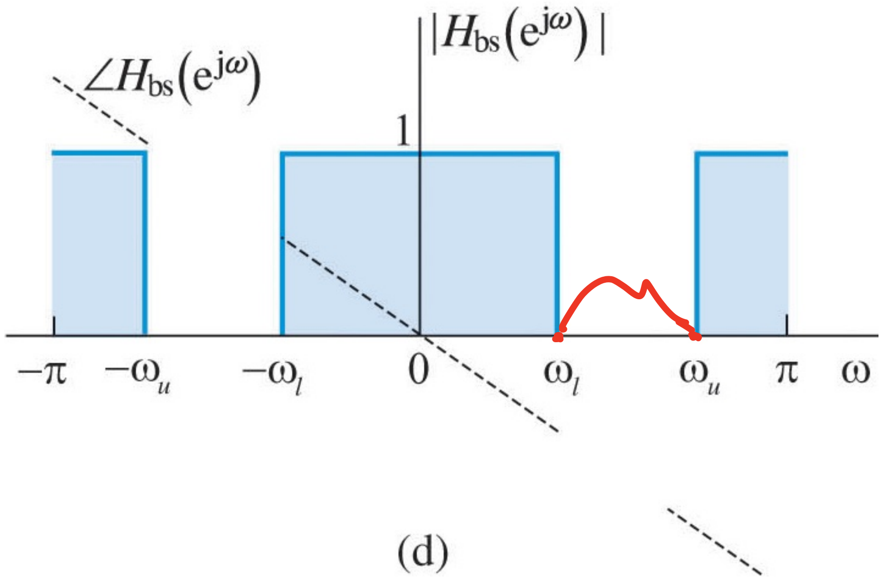
bandwidth $= \omega_u - \omega_l$

Ideal highpass filter



(c)

Ideal bandstop filter



(d)

These filters are all ideal because they can not be realized exactly

The reason is that the signal in the time domain has infinite energy.

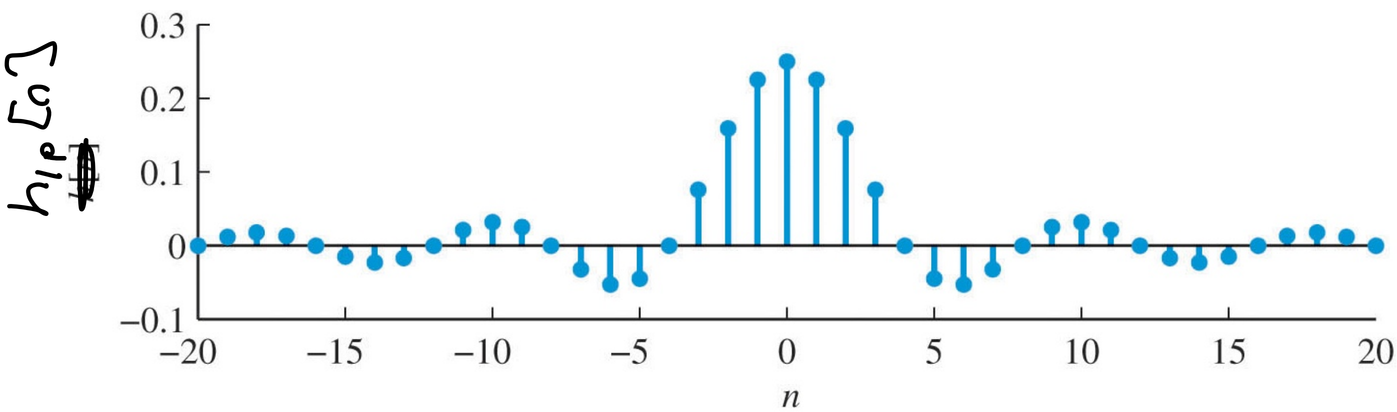
Equivalently, this means the system is unstable

Ideal lowpass filter

$$H_{lp}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

In the time domain

$$h_{lp}[n] = \frac{\sin \omega_c (n-n_d)}{\pi (n-n_d)}, \quad -\infty < n < \infty$$

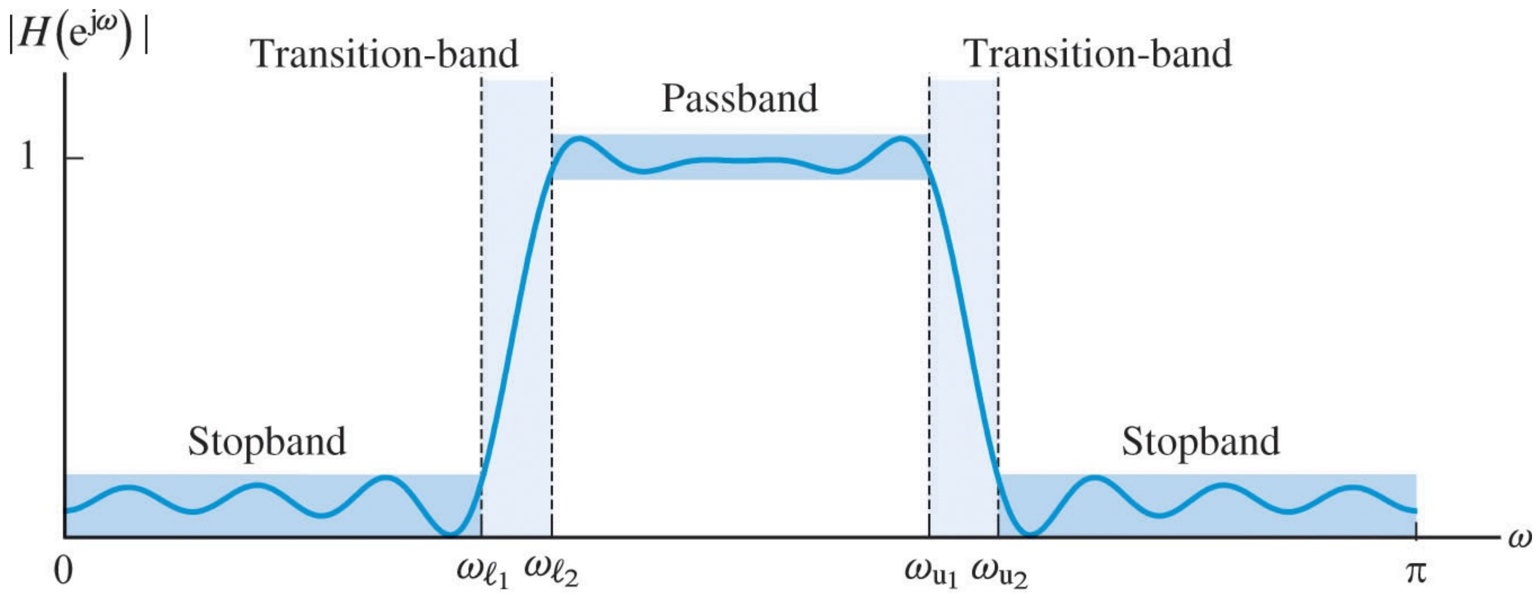


$$\sum_{n=-\infty}^{\infty} |h_{lp}[n]| = \infty$$

Therefore, the idealized lowpass filter is not realizable. None of the idealized filters are realizable in practice

Nonideal filters are approximations of ideal filters

Nonideal filters are not perfect boxes



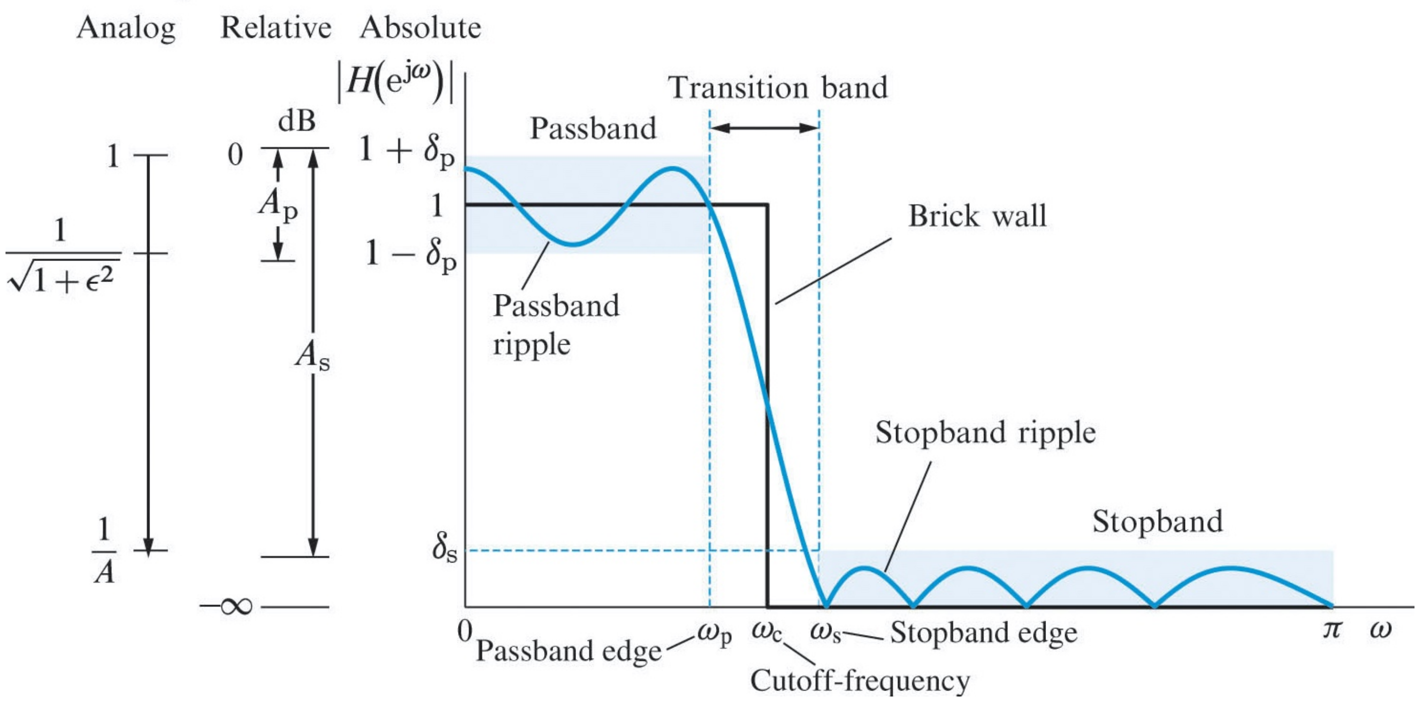
To have nonideal filters that are as close to ideal filters as possible, we want to make

1) stopband and passband as flat as possible

2) transition-band as sharp as possible

The desired characteristic of the nonideal filter is given by the specification.

The magnitude and phase of the filter can not be designed separately



Absolute specification

In the passband:

$$1-\delta_p \le |H(e^{j\omega})| \le 1+\delta_p, \quad 0 \le \omega \le \omega_p$$

where $\delta_p \ll 1$ for a well designed filter

In the stopband:

$$|H(e^{j\omega})| \le \delta_s, \quad \omega_s \le \omega \le \pi$$

where $\delta_s \ll 1$ for a well designed filter

Relative specification – define the allowable ripple

$$\frac{1-\delta_p}{1+\delta_p} \le |H(e^{j\omega})| \le 1, \quad 0 \le \omega \le \omega_p$$

and

$$|H(e^{j\omega})| \le \frac{\delta_s}{1+\delta_p}, \quad \omega_s \le \omega \le \pi$$

There are two types of filters

1) FIR - finite impulse response

filters have $h[n]$ with a finite number of nonzero values

Order = # of nonzero values in $h[n]$.

In general, we want filter with a low a number of order as possible

2) IIR - infinite impulse response

filters have $h[n]$ with an infinite number of nonzero values

you have to make sure IIR filters are stable so

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$