Chap 12.4

Recall that downsampling/decimation : reduces sampling frequency

decreases number of sample points

- often used to reduce memory requirement

Recall that upsampling/Interpolation : increase sampling frequency

increase number of sample points

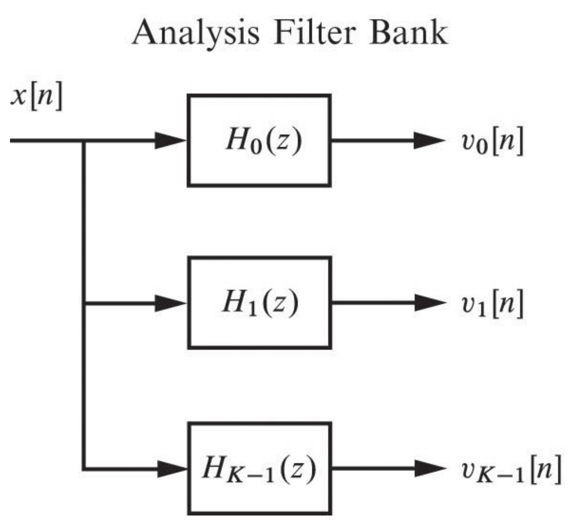
- used to "fill" in missing values

Another important application of decimation and interpolation is filter bank

Filter bank is a collection of filters with a common input or a common output

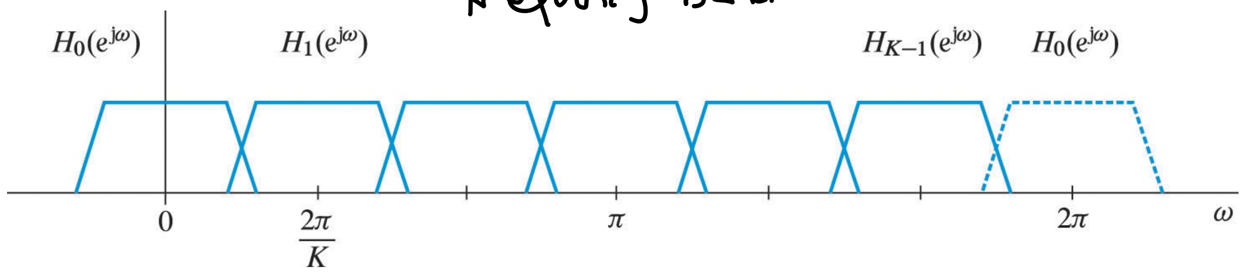
There are two types of filter bank :

analysis filter and synthesis filter band



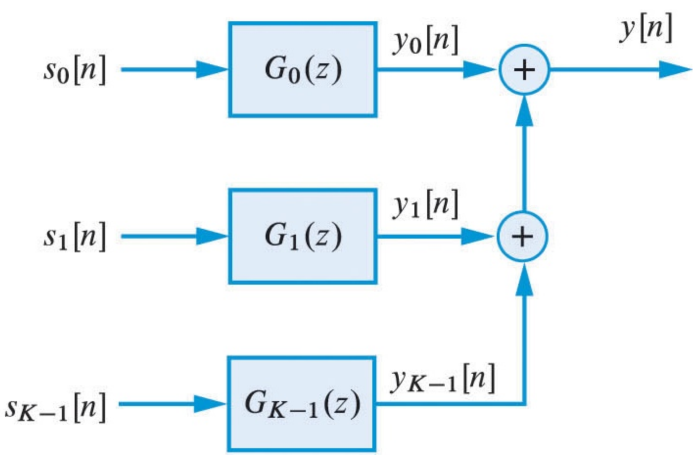
analysis filter splits a signal $x[n]$ into K signals $v_k[n]$ known as sub-band signals using analysis filter $H_k(z)$

Example analysis filter that splits $x[n]$ into different frequency band



- think equalizer in the music player

Synthesis Filter Bank



Synthesis filter bank consists of K synthesis filter $G_k(z)$, which combines K signals $s_k[n]$ into a signal $y[n]$

But there is a big problem with this design. If $x[n]$ has 200 points in the time domain. Then

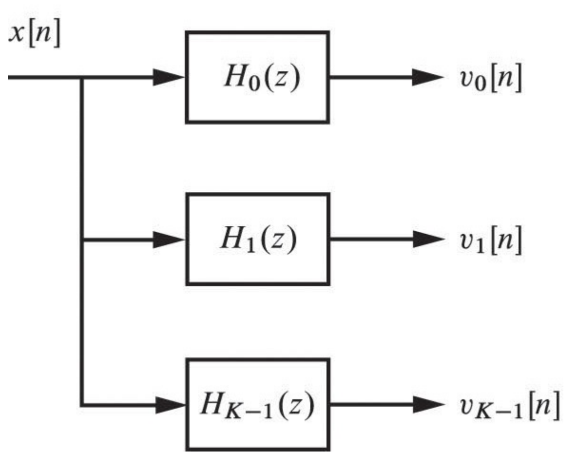
$v_0[n]$ has 200 points in the time domain

$v_1[n]$ has 200 points in the time domain

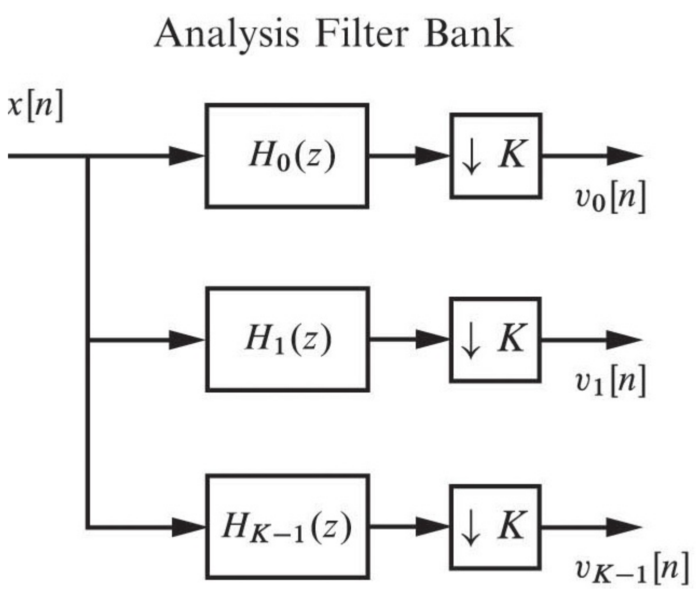
⋮

we have $(200)K$ points to store

Analysis Filter Bank



Practical analysis filter bank



$X[n]$ has 200 points

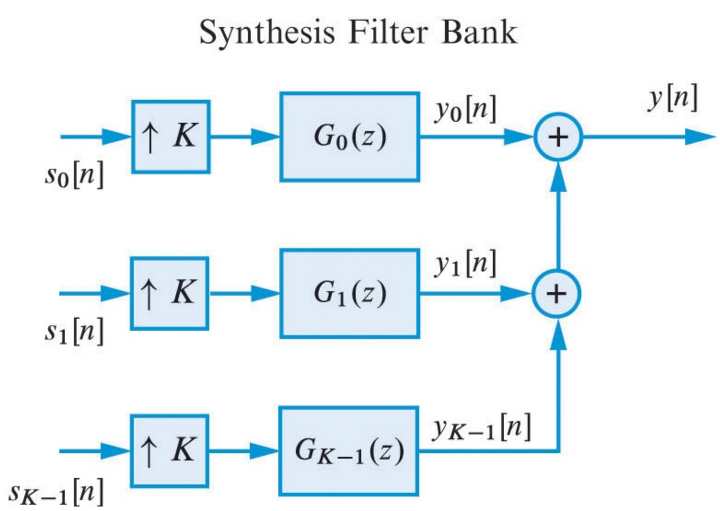
$V_0[n]$ has $\frac{200}{K}$ points

$V_1[n]$ has $\frac{200}{K}$ points

⋮

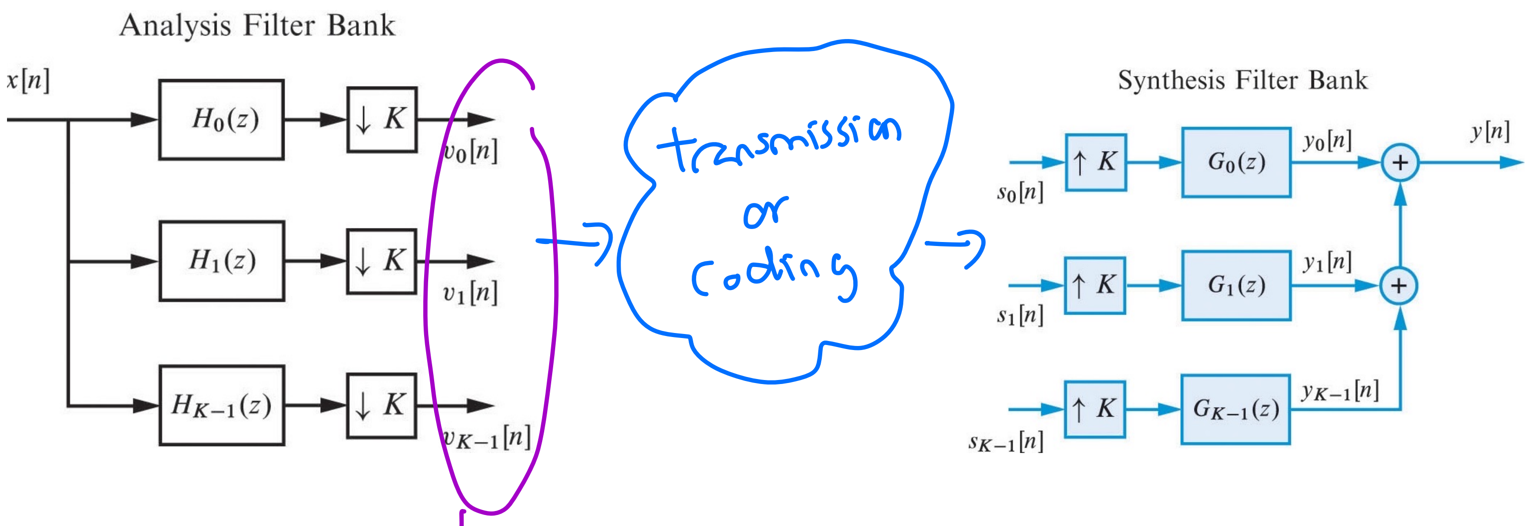
We have $\left(\frac{200}{K}\right)K = 200$ points to store

Analysis filter is followed by a synthesis filter.
So practical synthesis filter have an interpolation step before the synthesis filter



If synthesis filter "undo" the work of analysis filter, why do we use filter bank?

For efficient transmission of information



more compact representation than $x[n]$. Also can amplify or attenuate specific frequency band.

If filter banks are used for transmission, then we want $y[n] \approx x[n]$. We can handle

$$y[n] = G x[n-n_d]$$

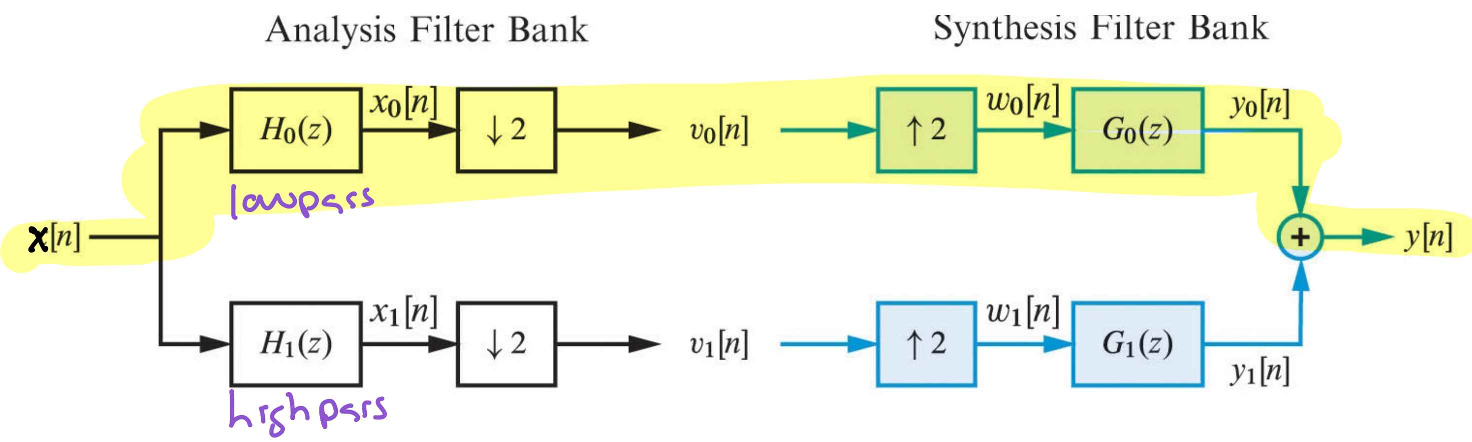
This will restrict the design of $H_k(z)$ and $G_k(z)$

In the frequency domain perfect (or distortionless) reconstruction means

← linear phase response

$$Y(z) = G z^{-n_d} X(z) = G e^{-j\omega n_d} X(z)$$

Two-channel filter bank is the simplest filter bank and will let us see what conditions we need on $H_k(z)$ and $G_k(z)$ to ensure perfect reconstruction.



We don't want to have to deal with convolution in the time domain

Consider the 0th channel. Usually this is the lower frequency channel

analysis filter

$$X_0(z) = X(z)H_0(z)$$

$$V_0(z) = \frac{1}{2}X_0(z^{\frac{1}{2}}) + \frac{1}{2}X_0(-z^{\frac{1}{2}})$$

this is from decimation

Synthesis filter

$$W_0(z) = V_0(z^2)$$

this is from upsampling

$$Y_0(z) = V_0(z^2)G_0(z)$$

$$= \frac{1}{2}\left(H_0(z)X(z) + H_0(-z)X(-z)\right)G_0(z)$$

We can consider the other band the same way and get

$$Y_1(z) = \frac{1}{2}\left(H_1(z)X(z) + H_1(-z)X(-z)\right)G_1(z)$$

The output of the synthesis filter

$$Y(z) = Y_0(z) + Y_1(z)$$

we shuffle terms

$$= \frac{1}{2}\left(T(z)X(z) + A(z)X(-z)\right)$$

$$= \frac{1}{2}\Big[\left(H_0(z)G_0(z) + H_1(z)G_1(z)\right)X(z)$$
$$+ \left(H_0(-z)G_0(z) + H_1(-z)G_1(z)\right)X(-z)\Big]$$

Recall we want distortion $Y(z) = G z^{-n_d} X(z)$

This means we want

$$H_0(z)G_0(z) + H_1(z)G_1(z) = G z^{-n_d}$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$$

We can write this as a system of linear equations

$$\begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix} \begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \begin{bmatrix} G z^{-n_d} \\ 0 \end{bmatrix}$$

(recall this looks like $Ax=b$)

This is only soluable if the determinant of the matrix, $\Delta_m(z) \neq 0$

$$\Delta_m(z) = H_0(z)H_1(-z) - H_0(-z)H_1(z) \neq 0$$

If we can solve the system of equation, we have

$$G_0(z) = \frac{2z^{-n_d}}{\Delta_m(z)} H_1(-z)$$

$$G_1(z) = -\frac{2z^{-n_d}}{\Delta_m(z)} H_0(-z)$$

To guarantee perfect construction, we need

$$z^{n_d} H_0(z) G_0(z) + z^{n_d} H_1(z) G_1(z) = G = 2$$

Let $R(z) \equiv z^{n_d} H_0(z) G_0(z)$, we will call this the product filter

Designing analysis and synthesis filters reduces to first designing $R(z)$ and then finding $H_0(z), H_1(z), \ldots G_0(z), G_1(z)$