Chap 10.3

$M =$ filter order

$h[n] =$ impulse response of FIR filter

$$= \underbrace{h_0 \delta[n] + h_1 \delta[n-1] + \dots h_M \delta[n-M]}_{\text{note that there are } M+1 \text{ nonzero coefficients}}$$

$$H(e^{j\omega}) = \underbrace{\sum_{n=0}^{M} h[0]\, e^{-j\omega n}}_{\text{also has } M+1 \text{ nonzero elements}}$$

$L =$ filter length, also called filter taps

$= M+1$

Ideal low pass filter in the frequency domain

$$H_{lp}(e^{jw}) = \begin{cases} e^{-j\alpha w}, & |w| < w_c \\ 0, & w_c < |w| \leq \pi \end{cases}$$

It has a magnitude response

$$|H_{lp}(e^{jw})| = \begin{cases} 1, & |w| < w_c \\ 0, & w_c < |w| \leq \pi \end{cases}$$

and phase response

$$\angle H_{lp}(e^{jw}) = \begin{cases} -\alpha w, & |w| < w_c \\ 0, & w_c < |w| \leq \pi \end{cases}$$

The ideal lowpass filter in the time domain is the sinc function

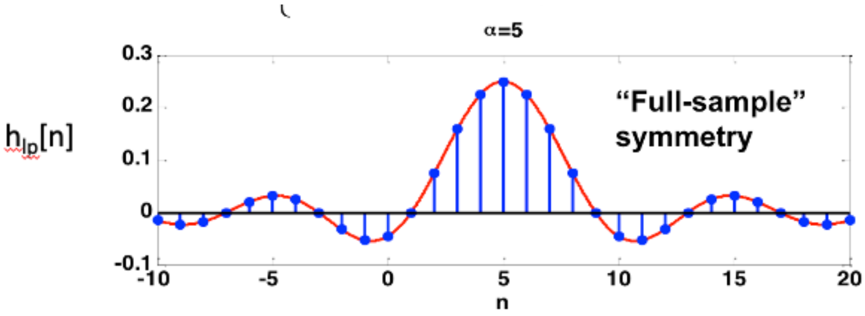
$$h_{lp}[n] = \frac{\sin w_c(n-\alpha)}{\pi(n-\alpha)}, \quad -\infty < n, < \infty$$

α is known as the delay variable

For window FIR filters $\alpha = \frac{M}{2}$ ← filter order

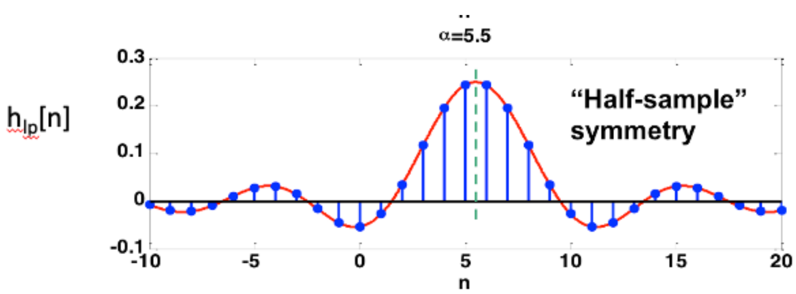
$\alpha = 5$

$$\frac{\sin w_c (n-5)}{\pi (n-5)}$$



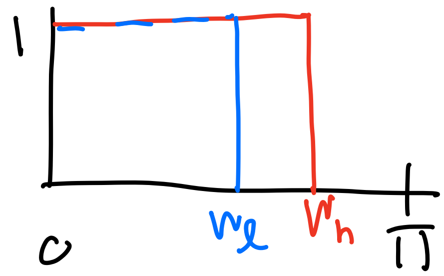
$\alpha = 5.5$

$$\frac{\sin w_c (n-5.5)}{\pi (n-5.5)}$$



Bandpass filter can be formed by subtracting 2 low pass filter

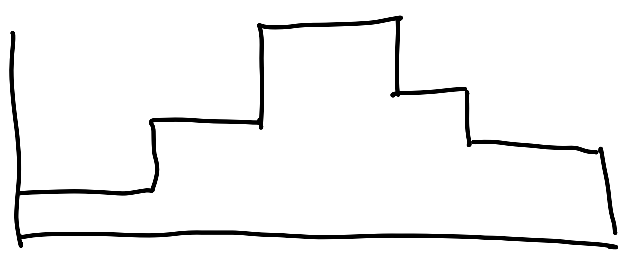
red - blue is



In the time domain, the ideal bandpass filter is

$$h_{bp} = \frac{\sin[w_h(n-\alpha)]}{\pi(n-\alpha)} - \frac{\sin[w_l(n-\alpha)]}{\pi(n-\alpha)}$$

We can use this ideal to for general multiband filters like



by composing multiple ideal low pass filters

For linear-phase FIR filters, the frequency response can be written in two ways

$$H(e^{jw}) = \underbrace{|H(e^{jw})|}_{\text{magnitude response}} \, e^{j\underbrace{\angle H(e^{jw})}_{\text{phase response}}}$$

$$= \underbrace{A(e^{jw})}_{\text{amplitude response}} \, e^{j\underbrace{\Psi(w)}_{\text{unwrapped phase response}}}$$

Amplitude response is like the magnitude response except it can take negative values