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EE 416 Midterm Exam I

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Instructions:

- This is a 55 minute exam containing **three** problems.
- Throughout the exam time, you **are required** to turn on your camera in Zoom, and share the entire desktop screen that shows the exam problems and other electronic references (see below). Using additional electronic device **will not** be allowded during the exam.
- This is an open book exam. You **may** have access to your textbooks, lecture notes, and any pre-downloaded reference, but **NOT** web-browsing.
- You **may** use phone and other electronic device to scan your solution. Additional 5 minutes will be given at the end of the exam for scanning your handwritten solution.
- You **may not** communicate with any person other than the official proctor during the exam.
- Sharing your solution via any communication device including cell phone, tablet, and computer **will** result in zero score.

Good luck.

Fact Sheet

- Function definitions

$$\mathrm{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- Continuous-time Fourier transform (CTFT)

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-\mathrm{i}2\pi f t} \,\mathrm{d}t$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{\mathrm{i}2\pi f t} \,\mathrm{d}f$$

- Continuous-space Fourier transform (CSFT)

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-\mathrm{i}2\pi(ux+vy)} \,\mathrm{d}x \,\mathrm{d}y$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{\mathrm{i}2\pi(ux+vy)} \,\mathrm{d}u \,\mathrm{d}v$$

- CTFT pairs

$$\mathrm{rect}(t) \overset{\mathrm{CTFT}}{\Longleftrightarrow} \mathrm{sinc}(f)$$

$$\mathrm{sinc}(t) \overset{\mathrm{CTFT}}{\Longleftrightarrow} \mathrm{rect}(f)$$

- CTFT properties

$$ax(t) + by(t) \overset{\mathrm{CTFT}}{\Longleftrightarrow} aX(f) + bY(f)$$

$$x(at) \overset{\mathrm{CTFT}}{\Longleftrightarrow} \frac{1}{|a|} X(f/a)$$

$$x(t-t_0) \overset{\mathrm{CTFT}}{\Longleftrightarrow} e^{-\mathrm{i}2\pi f t_0} X(f)$$

$$e^{\mathrm{i}2\pi f_0 t} x(t) \overset{\mathrm{CTFT}}{\Longleftrightarrow} X(f-f_0)$$

$$x(t) \circledast y(t) \overset{\mathrm{CTFT}}{\Longleftrightarrow} X(f)Y(f)$$

$$x(t)y(t) \overset{\mathrm{CTFT}}{\Longleftrightarrow} X(f) \circledast Y(f)$$

$$X(t) \overset{\mathrm{CTFT}}{\Longleftrightarrow} x(-f)$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) \,\mathrm{d}t = \int_{-\infty}^{\infty} X(f) Y^*(f) \,\mathrm{d}f$$

- Discrete-time Fourier transform (DTFT)

$$X(e^{\mathrm{i}\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-\mathrm{i}\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{\mathrm{i}\omega}) e^{\mathrm{i}\omega n} \,\mathrm{d}\omega$$

- DTFT pairs

$$a^n u[n] \overset{\mathrm{DTFT}}{\Longleftrightarrow} \frac{1}{1 - ae^{-\mathrm{i}\omega}}$$

- Discrete-space Fourier transform (DSFT)

$$F(e^{\mathrm{i}\mu}, e^{\mathrm{i}\nu}) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} f[m,n] e^{-\mathrm{i}(\mu m + \nu n)}$$

$$f[m,n] = \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} F(e^{\mathrm{i}\mu}, e^{\mathrm{i}\nu}) e^{\mathrm{i}(\mu m + \nu n)} \,\mathrm{d}\mu \,\mathrm{d}\nu$$

- Rep and Comb relations

$$\mathrm{rep}_\Delta\{x(t)\} = \sum_{k=-\infty}^{\infty} x(t - k\Delta)$$

$$\mathrm{comb}_\Delta\{x(t)\} = x(t) \sum_{k=-\infty}^{\infty} \delta(t - k\Delta)$$

$$\mathrm{comb}_\Delta\{x(t)\} \overset{\mathrm{CTFT}}{\Longleftrightarrow} \frac{1}{\Delta}\mathrm{rep}_{\frac{1}{\Delta}}\{X(f)\}$$

$$\mathrm{rep}_\Delta\{x(t)\} \overset{\mathrm{CTFT}}{\Longleftrightarrow} \frac{1}{\Delta}\mathrm{comb}_{\frac{1}{\Delta}}\{X(f)\}$$

- Sampling and reconstruction

$$s[n] = f(n\Delta)$$

$$S(e^{\mathrm{i}\omega}) = \frac{1}{\Delta}\sum_{k=-\infty}^{\infty} F\left(\frac{\omega - 2\pi k}{2\pi\Delta}\right)$$

$$r(t) = \sum_{k=-\infty}^{\infty} s[k]\delta(t - k\Delta)$$

$$R(f) = S(e^{\mathrm{i}2\pi f\Delta})$$

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Problem 1. (20pt) Assume that you measure the function

$$p(x) = \int_{-\infty}^{\infty} f(x, y) \, dy.$$

Using the definitions of the Fourier transform, derive an expression for $F(u, 0)$ in terms of the function $p(x)$, where $F(u, v)$ is the CSFT of $f(x, y)$.

$$F(u,0) = \int\int f(x,y) e^{-j2\pi xu} \, dx \, dy$$

$$= \int e^{-j2\pi xu} \int f(x,y) \, dy \, dx$$

$$F(u,0) = \int_{-\infty}^{\infty} p(x) e^{-j2\pi xu} \, dx$$

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Problem 2. (40pt) Consider a 2D linear space-invariant filter with input $x[m,n]$, output $y[m,n]$:, and impulse response $h[m,n]$, so that

$$y[m,n] = h[m,n] * x[m,n]$$

Furthermore, let the impulse response be given by

$$h[m,n] = \begin{cases} \frac{1}{81}, & |m| \le 4, |n| \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

a) Calculate the DC gain of this filter, $H(e^{i0}, e^{i0})$, where $H(e^{i\mu}, e^{i\nu})$ is the transfer function of $h[m,n]$. Justify your answer.
b) Suppose that you implement this filter directly with 2D convolution. How many multipliers are required per *each output value*?
c) Find a separable decomposition of $h[m,n]$ so that $h[m,n] = f[m]g[n]$, where $f[m]$ and $g[n]$ are 1D functions.
d) Explain how the functions $f[m]$ and $g[n]$ can be used to compute $y[m,n]$. What is the advantage of this approach? Be specific, i.e., specify how many multipliers are required per *each output value*.

$H_u = 2\cos(4u) + 2\cos(3u) + 2\cos(2u) + 2\cos(u) + 1$
$H_v = 2\cos(4v) + 2\cos(3v) + 2\cos(2v) + 2\cos(v) + 1$

$H(e^{ju}, e^{jv}) = H_u \cdot H_v$

(a) $H(e^{j0}, e^{j0}) = (2+2+2+2+1)(2+2+2+2+1)$
$= (9)(9) = 81$

(b) 81 multipliers

(c) $f[m] = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad g[n] = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$

(d) by splitting $h[m,n]$ into $f[m]g[n]$,
$y[m,n]$ becomes

$$y[m,n] = f[m]\,g[n] * x[m,n]$$

then by CSFT

$$Y(u,v) = F(u)\,G(v)\,X(u,v)$$
$$= F(u)\big(G(v)\,X(u,v)\big)$$

with this method, $x[m,n]$ convolves with $g[n]$ first (9 multipliers), then the intermediate value convolves with $f[m]$ (9 multipliers) to obtain $y[m,n]$

This brings the total multipliers per pixel from 81 to 18.

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Problem 3. (40pt) Below is a pseudo-code description of a subroutine called ConnectedSet for labeling all pixels connected to the pixel s_0. Let $c(s)$ be the set of connected neighbors to the pixel s, and let Y be the image containing the label for each pixel.

$ClassLabel = 1$
Initialize $Y_r = 0$ for all $r \in S$
ConnectedSet$(s_0, Y, ClassLabel)${
  $B \leftarrow \{s_0\}$
  While(B is not empty){
    $s \leftarrow$ any element of B
    [**Missing pseudo-code**]
    Y(s) = ClassLabel
    $B \leftarrow c(s)$
  }
  return(Y)
}

a) Fill in the above missing pseudo-code with the correct operations.

For the following problems, let $c(s) = \{r \in \mathcal{N}s : X_r = X_s\}$, where $\mathcal{N}s$ is the set of neighbors for the pixel s, and the image X is given as follows:

the image X:

n \ m	0	1	2	3	4
0	1	1	2	2	2
1	0	0	1	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

b) Use the following table to specify the values of Y returned by the subroutine when $s_0 = (m, n) = (0, 0)$ and the 4-point neighborhood system is used.

the segmented image Y:

n \ m	0	1	2	3	4
0	1	1	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0

c) Use the following table to specify the values of Y returned by the subroutine when $s_0 = (m, n) = (0, 0)$ and the 8-point neighborhood system is used.

the segmented image Y:

		m 0	1	2	3	4
n	0	1	1	0	0	0
	1	0	0	1	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

d) Consider the case when $c(s) = \{r \in \mathcal{N}s : |X_r - X_s| \leq T\}$. Explain the disadvantage of choosing T to be large, and the disadvantage of choosing T to be small.

If T is large, the set c(s) may contain more connected neighbors than what is desired.

If T is small, the set c(s) may be missing some connected neighbors that the system considers not connected

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