2. Cauchy-Schwarz Inequality

(a) $\|\vec{v}\| = \sqrt{\gamma^2(\cos^2\theta + \sin^2\theta)}$

$= \sqrt{\gamma^2(1)}$

$= \sqrt{\gamma^2}$

$= \sqrt{x_i^2 + y_i^2}$

(b) $\|\vec{\omega}\| = \sqrt{t^2(\cos^2\phi + \sin^2\phi)}$

$= \sqrt{t^2(1)}$

$= \sqrt{t^2}$

$= \sqrt{x_2^2 + y_2^2}$

(c) $\langle \vec{v}, \vec{\omega} \rangle = \vec{v}^T \cdot \vec{\omega}$

$$[\gamma\cos\theta \quad \gamma\sin\theta]\begin{bmatrix} t\cos\phi \\ t\sin\phi \end{bmatrix}$$

$\langle \vec{v}, \vec{\omega} \rangle = (\alpha\cos\theta)(\beta\cos\phi) + (\alpha\sin\theta)(\beta\sin\phi)$

$= \alpha\beta(\cos\theta\cos\phi + \sin\theta\sin\phi)$

$= \alpha\beta \cos(\theta - \phi)$ (Trig Identity)

$= \|\vec{v}\| \cdot \|\vec{\omega}\| \cos(\theta - \phi) \leq \|\vec{v}\| \cdot \|\vec{\omega}\| \cdot 1$ $\quad (\cos(\theta - \phi) \leq 1)$

$\Rightarrow$ if $\theta - \phi = 90°$, $\cos(\theta - \phi) = 0$, then $\langle \vec{v}, \vec{\omega} \rangle = 0$ "ortlagonal"

$\Rightarrow$ Cauchy Swartz Inequality

$\langle \vec{v}, \vec{\omega} \rangle \leq \|\vec{v}\| \cdot \|\vec{\omega}\|$

$\Rightarrow$ this will hold for any vector in $\mathbb{R}^2$ since $\vec{v}$ and $\vec{\omega}$ are general vectors in $\mathbb{R}^2$.